Carroll physics and flat space holography

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IFT Christmas Workshop, Madrid, December 2024

based on work with Afshar, Aggarwal, Bagchi, Basu, Chakrabortty, Detournay, Fareghbal, Gary, Merbis, Nandi, Parekh, Radhakrishnan, Riegler, Rosseel, Schöller, Simón, Sinha

Outline

Flat space holography

Carroll limit and tantum gravity

Entries in $\mathrm{FS}_3/\mathrm{CCFT}_2$ dictionary

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Motivation: how general is holography?

- holographic principle motivated by black hole entropy

$$S_{\text{BH}} = \frac{A}{4}$$

't Hooft 93; Susskind '95

Motivation: how general is holography?

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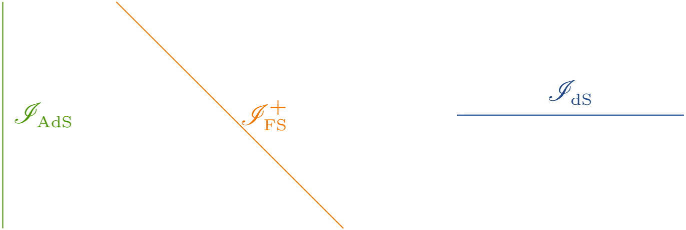

First two diagrams suggest: AdS $\to$ FS is Carroll limit

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- main implementation of holography so far: AdS/CFT

Klebanov and Maldacena, Physics Today 62 (2009) 28

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- which lessons of AdS/CFT are generic for holography?

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- (how) does holography work in asymptotically flat spacetimes?

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For technical simplicity, address these questions in three bulk dimensions

Approaches to flat space holography

1. Flat space holography as limit of AdS/CFT

> **Idea**
>
> Large AdS radius-limit of AdS/CFT

Nice aspect: can exploit AdS/CFT and "only" have to apply $\ell \to \infty$

Witten '98; Polchinski '99; Susskind '99; Giddings '99

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Example 1: naive large ℓ-limit of LaAdS expansion

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \left(e^{2\rho/\ell}\gamma_{\mu\nu}^{(0)} + \gamma_{\mu\nu}^{(2)} + \dots\right)\mathrm{d}x^\mu\,\mathrm{d}x^\nu \quad \overset{\ell\to\infty}{\Rightarrow} \quad \mathrm{d}\rho^2 + \gamma_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

vs. asymptotically flat expansion in Bondi gauge

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Example 2: naive large ℓ-limit of CFT_2 conformal algebra

$$[L^\pm_n, L^\pm_m] = (n-m)L^\pm_{n+m} + \frac{\ell}{8G}n^3\delta_{n,-m} \quad \overset{\ell\to\infty}{\Rightarrow} \quad [\hat{L}^\pm_n, \hat{L}^\pm_m] = \frac{1}{8G}n^3\delta_{n,-m}$$

vs. Galilei limit vs. Carroll limit

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Focus on asymptotic symmetries of $\mathscr{I}$

Nice aspect: can exploit BMS/Brown–Henneaux-type of analyses and "only" have to decipher dual Carroll CFT (CCFT)

Bagchi et al. since 2010; Barnich et al. since 2010

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Problem 1: have one Carroll CFT at $\mathscr{I}^+$ and one at $\mathscr{I}^-$ — what is their relation?

Problem 2: how are holographic observables related to S-matrix observables?

Approaches to flat space holography

1. Flat space holography as limit of AdS/CFT
2. Carroll holography
3. Celestial amplitudes

Idea

IR triangle: soft theorems — BMS symmetries — memory effects

Nice aspect: encodes gravitational S-matrix elements as conformal correlators on celestial sphere

Strominger et al.; Pasterski et al.

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Useful observation: Carroll holography & Celestial amplitudes related
Donnay, Fiorucci, Herfray, Ruzziconi '22; Bagchi, Banerjee, Basu, Dutta '22

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  - correlations functions and $CCFT_2$ Ward id's (2015)
  - thermal properties and Cardyology (2012-2014)
  - entanglement entropy and RT-like prescription (2014-2020)
- brief outlook

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Comment 1: symmetries and Carroll limit

BMS_3/CCFT_2 correspondence:

$$[L_n, L_m] = (n-m)L_{n+m} \qquad [L_n, M_m] = (n-m)M_{n+m} + \underbrace{\frac{1}{4G}}_{=\frac{c_M}{12}} n^3\delta_{n,-m}$$

L_n: superrotations (diff S^1)
M_n: supertranslations

Ashtekar, Bicak, Schmidt '96; Barnich, Compère '06; Bagchi '10; Bagchi, Detournay, DG '12; Duval, Gibbons, Horvathy '14

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CCFT$_2$ algebra from Carroll limit of CFT$_2$ algebra ($\mathrm{Vir}_c \oplus \mathrm{Vir}_{\bar{c}}$):

$$L_n := \mathcal{L}_n - \bar{\mathcal{L}}_{-n} \qquad M_n := \frac{1}{\ell}\left(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}\right)$$

take AdS radius to infinity, $\ell \to \infty$; get CCFT$_2$ algebra with central charges

$$c_L = c - \bar{c} = 0 \qquad c_M = \frac{c + \bar{c}}{\ell} \neq 0 \text{ if } c = \mathcal{O}(\ell) \to \infty$$

Comment 2: tantum gravity limit

implications of $c \to 0$ limit for black hole observables ($E = Mc^2$)

$$T = \frac{\hbar\, c^5}{8\pi\, G_N\, E} \qquad S = \frac{4\pi G_N\, E^2}{\hbar c^5} \qquad r_h = \frac{2G_N E}{c^4}$$

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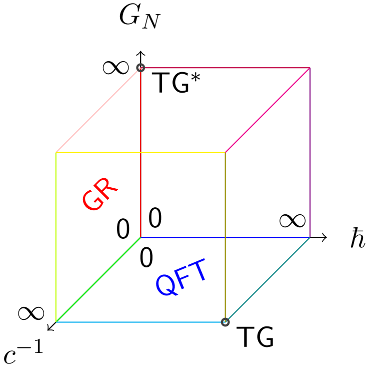
"tantum gravity" limit (see TG corner in compactified Bronstein cube)



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$$\langle T(z_1)T(z_2)\dots T(z_{42})\rangle_{\text{CFT}} \sim \frac{\delta^{42}}{\delta g^{42}}\Gamma_{\text{EH-AdS}}\Big|_{\text{EOM}}$$

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Start slowly with 0-point function

0-point function (on-shell action)

Not check of flat space holography but interesting in its own right

- Calculate the full on-shell action Γ

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- Variational principle?

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - -\frac{1}{8\pi G_N}\int \mathrm{d}^2x\sqrt{\gamma}\,K - I_{\text{counter-term}}$$

with $I_{\text{counter-term}}$ chosen such that

$$\delta\Gamma\big|_{\text{EOM}} = 0$$

for all δg that preserve flat space bc's

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Result (Detournay, DG, Schöller, Simon '14):

$$\Gamma = -\frac{1}{16\pi G_N}\int \mathrm{d}^3x\sqrt{g}\,R - \underbrace{\frac{1}{16\pi G_N}}_{\frac{1}{2}\text{GHY!}}\int \mathrm{d}^2x\sqrt{\gamma}\,K$$

follows also as limit from AdS using Mora, Olea, Troncoso, Zanelli '04
independently confirmed by Barnich, Gonzalez, Maloney, Oblak '15

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- Calculate the full on-shell action Γ
- Variational principle?
- Phase transitions?

  Standard procedure (Gibbons, Hawking '77; Hawking, Page '83)

  Evaluate Euclidean partition function in semi-classical limit

$$Z(T,\,\Omega) = \int \mathcal{D}g\, e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T,\,\Omega)]} \times Z_{\text{fluct.}}$$

  path integral bc's specified by temperature T and angular velocity Ω

  Two Euclidean saddle points in same ensemble if

  - same temperature $T = 1/\beta$ and angular velocity Ω
  - obey flat space boundary conditions
  - solutions without conical singularities

  Periodicities fixed:

$$(\tau_E,\,\varphi) \sim (\tau_E + \beta,\,\varphi + \beta\Omega) \sim (\tau_E,\,\varphi + 2\pi)$$

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- Phase transitions?
  3D Euclidean Einstein gravity: for each T, Ω two saddle points:
  - Hot flat space
    $$\mathrm{d}s^2 = \mathrm{d}\tau_E^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\varphi^2$$
  - Flat space cosmology
    $$\mathrm{d}s^2 = r_+^2\left(1 - \frac{r_0^2}{r^2}\right)\mathrm{d}\tau_E^2 + \frac{r^2\,\mathrm{d}r^2}{r_+^2\,(r^2 - r_0^2)} + r^2\left(\mathrm{d}\varphi - \frac{r_+ r_0}{r^2}\,\mathrm{d}\tau_E\right)^2$$
    shifted-boost orbifold, see Cornalba, Costa '02

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- Variational principle?
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- Plug two Euclidean saddles in on-shell action and compare free energies

$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

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$$F_{\text{HFS}} = -\frac{1}{8G_N} \qquad F_{\text{FSC}} = -\frac{r_+}{8G_N}$$

- Result of this comparison
  - $r_+ > 1$: FSC dominant saddle
  - $r_+ < 1$: HFS dominant saddle

  Critical temperature:

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}$$

HFS "melts" into FSC at $T > T_c$

Bagchi, Detournay, DG, Simon '13

1-point functions (conserved charges)

First check of entries in holographic dictionary: identification of sources and vevs

In AdS_3:

$$\delta\Gamma\big|_{\text{EOM}} \sim \int_{\partial\mathcal{M}} \text{vev} \times \delta\,\text{source} \sim \int_{\partial\mathcal{M}} T^{\mu\nu}_{\text{BY}} \times \delta g^{\text{NN}}_{\mu\nu}$$

Note that $T^{\mu\nu}_{\text{BY}}$ follows from canonical analysis as well (conserved charges)

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In flat space:

- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

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- non-normalizable solutions to linearized EOM?
- analogue of Brown–York stress tensor?
- comparison with canonical results

everything works (Detournay, DG, Schöller, Simon, '14)

mass and angular momentum:

$$M = \frac{g_{tt}}{8G} \qquad N = \frac{g_{t\varphi}}{4G}$$

full tower of canonical charges: see Barnich, Compere '06

2-point functions (anomalous terms)

First check sensitive to central charges in symmetry algebra

CCFT on cylinder ($\varphi \sim \varphi + 2\pi$):

$$\langle M(u_1,\varphi_1)\,M(u_2,\varphi_2)\rangle = 0$$

$$\langle M(u_1,\varphi_1)\,N(u_2,\varphi_2)\rangle = \frac{c_M}{2s_{12}^4}$$

$$\langle N(u_1,\varphi_1)\,N(u_2,\varphi_2)\rangle = \frac{c_L - 2c_M\tau_{12}}{2s_{12}^4}$$

with $s_{ij} = 2\sin[(\varphi_i - \varphi_j)/2]$, $\tau_{ij} = (u_i - u_j)\,\cot[(\varphi_i - \varphi_j)/2]$

Fourier modes of CCFT stress tensor on cylinder:

$$M := \sum_n M_n e^{-in\varphi} - \frac{c_M}{24}$$

$$N := \sum_n \big(L_n - inuM_n\big)e^{-in\varphi} - \frac{c_L}{24}$$

Conservation equations: $\partial_u M = 0$, $\partial_u N = \partial_\varphi M$

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Summarize first how this works in the AdS case

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Illustrate shortcut in $\text{AdS}_3/\text{CFT}_2$ (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z \, \mu(z, \bar{z}) T(z)$$

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$$\mu(z,\bar{z}) = \epsilon\, \delta^{(2)}(z - z_2, \bar{z} - \bar{z}_2)$$

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- 1-point function in μ-vacuum $\to$ 2-point function in 0-vacuum

$$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon\, \langle T^1\, T^2 \rangle_0 + \mathcal{O}(\epsilon^2)$$

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$$\langle T^1 \rangle_\mu = \langle T^1 \rangle_0 + \epsilon\, \langle T^1\, T^2 \rangle_0 + \mathcal{O}(\epsilon^2)$$

- On gravity side exploit sl(2) CS formulation with chemical potentials

$$A = b^{-1}(\mathrm{d}{+}a)b \qquad\qquad b = e^{\rho L_0}$$

$$a_z = L_+ - \frac{\mathcal{L}}{k} L_- \qquad\qquad a_{\bar{z}} = \mu L_+ + \dots$$

Drinfeld, Sokolov '84, Polyakov '87, H. Verlinde '90
Bañados, Caro '04

2-point functions (anomalous terms)

Illustrate shortcut in AdS_3/CFT_2 (restrict to one holomorphic sector)

- On CFT side deform free action S_0 by source term μ for stress tensor

$$S_\mu = S_0 + \int \mathrm{d}^2 z\, \mu(z,\bar z) T(z)$$

- Localize source

$$\mu(z,\bar z) = \epsilon\, \delta^{(2)}(z - z_2, \bar z - \bar z_2)$$

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$$\bar{\partial}\mathcal{L}^{(1)}(z) = -\frac{k}{2}\,\partial^3 \delta^{(2)}(z - z_2)$$

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Apply shortcut to FS/CCFT (Bagchi, DG, Merbis '15)

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$$M = -k/2 + M^{(1)} \qquad N = N^{(1)}$$

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- Solve with Green function on cylinder

$$M^{(1)} = \frac{6k\epsilon_L}{s_{12}^4} \qquad N^{(1)} = \frac{6k(\epsilon_M - 2\epsilon_L\,\tau_{12})}{s_{12}^4}$$

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- Correct 2-point functions for Einstein gravity with $c_L = 0$, $c_M = 12k$

3-point functions (check of symmetries)

First non-trivial check of consistency with symmetries of dual CCFT

Check of 2-point functions works nicely with shortcut; 3-point too?

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- Iteratively solve EOM

$$\partial_u M = -k\partial_\varphi^3\mu_L + \mu_L\partial_\varphi M + 2M\partial_\varphi\mu_L$$

$$\partial_u N = -k\partial_\varphi^3\mu_M + (1+\mu_M)\partial_\varphi M + 2M\partial_\varphi\mu_M + \mu_L\partial_\varphi N + 2N\partial_\varphi\mu_L$$

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- Result on gravity side matches precisely CCFT results

$$\langle M^1\,N^2\,N^3\rangle=\frac{c_M}{s_{12}^2 s_{13}^2 s_{23}^2}\qquad\langle N^1\,N^2\,N^3\rangle=\frac{c_L-c_M\,\tau_{123}}{s_{12}^2 s_{13}^2 s_{23}^2}$$

provided we choose again the Einstein values $c_L=0$ and $c_M=12k$

4-point functions (enter cross-ratios)

First correlators with non-universal function of cross-ratios

- Repeat this algorithm, localizing the sources at three points

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- Derive 4-point functions for CCFTs (Bagchi, DG, Merbis '15)

$$\langle M^1\, N^2\, N^3\, N^4\rangle = \frac{2c_M\, g_4(\gamma)}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

$$\langle N^1\, N^2\, N^3\, N^4\rangle = \frac{2c_L\, g_4(\gamma) + c_M \Delta_4}{s_{14}^2 s_{23}^2 s_{12} s_{13} s_{24} s_{34}}$$

with the cross-ratio function

$$g_4(\gamma) = \frac{\gamma^2 - \gamma + 1}{\gamma} \qquad \gamma = \frac{s_{12}\, s_{34}}{s_{13}\, s_{24}}$$

and

$$\Delta_4 = 4g_4'(\gamma)\eta_{1234} - (\tau_{1234} + \tau_{14} + \tau_{23})g_4(\gamma)$$

$$\eta_{1234} = \sum(-1)^{1+i-j}(u_i - u_j)\sin(\varphi_k - \varphi_l)/(s_{13}^2 s_{24}^2)$$

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Last new explicit correlators I am showing to you today (I promise)

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$$\langle M^1\, N^2\, N^3\, N^4\, N^5\rangle = \frac{4c_M\, g_5(\gamma,\, \zeta)}{\prod_{1\le i<j\le 5} s_{ij}}$$

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with the previous definitions and ($\zeta = \frac{s_{25}\, s_{34}}{s_{35}\, s_{24}}$)

$$g_5(\gamma,\, \zeta) = \frac{\gamma+\zeta}{2(\gamma-\zeta)} - \frac{(\gamma^2-\gamma\zeta+\zeta^2)}{\gamma(\gamma-1)\zeta(\zeta-1)(\gamma-\zeta)} \times \big([\gamma(\gamma-1)+1][\zeta(\zeta-1)+1]-\gamma\zeta\big)$$

$$\Delta_5 = 4\partial_\gamma g_5(\gamma,\zeta)\eta_{1234} + 4\partial_\zeta g_5(\gamma,\zeta)\eta_{2345} - 2g_5(\gamma,\zeta)\tau_{12345}$$

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n-point functions (holographic Ward identities and recursion relations)

Shortcut to 42 (Bagchi, DG, Merbis '15)

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- After small derivation we find ($c_{ij} := \cot[(\varphi_i - \varphi_j)/2]$)

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- We can also derive same recursion relations on gravity side!

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Consistency check that 3D flat space holography can work!

Cardyology

Bagchi, Detournay, Fareghbal, Simon '12; Barnich '12

- essence of Cardy-formula: S-duality

  high-temperature partition function (dominated by black holes) equivalent to low-temperature partition function (dominated by ground state)

  key assumptions: gap in spectrum, modular invariance of partition function

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$$\sigma \to \frac{a\sigma + b}{c\sigma + d} \qquad\qquad \rho \to \frac{\rho}{(c\sigma + d)^2}$$

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- S- and T-trafos:

$$S:\ \sigma \to -\frac{1}{\sigma} \qquad \rho \to \frac{\rho}{\sigma^2} \qquad\qquad T:\ \sigma \to \sigma + 1 \qquad \rho \to \rho$$

obey usual braiding relations $S^2 = \mathbb{1} = (ST)^3$

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$$Z_{\text{CCFT}}(\sigma,\rho)=Z_{\text{CCFT}}(-1/\sigma,\rho/\sigma^2)$$

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- CCFT_2 Cardy-like entropy formula

$$S_{\text{CCFT}} = (1 - \rho\partial_\rho - \sigma\partial\sigma)Z_{\text{CCFT}}(\sigma, \rho) = 2\pi L_0\sqrt{\frac{c_M}{24M_0}} = \frac{2\pi r_0}{4G} = S_{\text{BH}}$$

Entanglement entropy (EE)

Bagchi, Basu, DG, Riegler '14

- CCFT_2: follow Holzhey, Wilzcek, Larsen '94; Cardy, Calabrese '04

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- EE for CCFT_2 on plane with intervals Δu and Δx

$$S_{\text{EE}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\Delta x}{\epsilon_x}}_{S_L} + \underbrace{\frac{c_M}{6}\Big(\frac{\Delta u}{\Delta x} - \frac{\epsilon_u}{\epsilon_x}\Big)}_{S_M}$$

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$c_M = 0$: recover chiral CFT_2 result for EE on plane, $S_{\text{EE}} = S_L$

Δx: size of entangling region

ϵ_x: UV cutoff

Entanglement entropy (EE)

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$$S_{\text{EE}} = \underbrace{\frac{c_L}{6}\,\ln\frac{\Delta x}{\epsilon_x}}_{S_L} + \underbrace{\frac{c_M}{6}\Big(\frac{\Delta u}{\Delta x} - \frac{\epsilon_u}{\epsilon_x}\Big)}_{S_M}$$

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  DG, Parekh, Riegler '19
  conceptually the same as uniformization maps in $\text{AdS}_3/\text{CFT}_2$ using solutions of Hill's equation $\Rightarrow$ constructed flat Hill's equation
  example: EE for global Minkowski/CCFT on cylinder ($\varphi \sim \varphi + 2\pi$)

$$S_{\text{EE}} = \frac{c_L}{6}\,\ln\frac{2\sin\frac{\Delta\varphi}{2}}{\epsilon_\varphi} + \frac{c_M}{6}\Big(\frac{\Delta u}{2}\,\cot\frac{\Delta\varphi}{2} - \frac{\epsilon_u}{\epsilon_\varphi}\Big)$$

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$$x \to \xi(\varphi) = \varphi + \underbrace{\sigma(\varphi)}_{\text{superrotation}} \qquad\qquad u \to \zeta(u,\varphi) = \underbrace{\eta(\varphi)}_{\text{supertranslation}} + u\,\xi'(\varphi)$$

transform CCFT$_2$ EE as

$$\delta S_L = \sigma S_L' - \frac{c_L}{12}\,\sigma' \qquad\qquad \delta S_M = \sigma S_M' + \zeta \dot{S}_M - \frac{c_M}{12}\,\zeta'$$

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analogous to holographic EE for gravity/higher spin theories in AdS_3
Ammon, Castro, Iqbal '13; de Boer, Jottar '13

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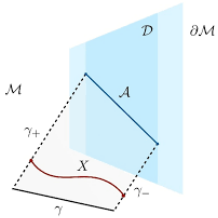
- Jiang, Song, Wen '17; Hijano, Rabideau '17; Apolo, Jiang, Song, Zhong '20: yes, using swing surfaces



$\partial\mathcal{M}$: asymptotic boundary
$\mathcal{A}$: entangling region in CCFT_2
$\gamma_\pm$: null geodesics ("ropes")
X, γ: spacelike surfaces ("bench")
$\gamma_+ \cup \gamma \cup \gamma_-$: extremal surface ("swing")
S_{EE}: area of swing surface; reproduces EE above

Outlook

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- bound on chaos Bagchi, Chakrabortty, DG, Radhakrishnan, Riegler, Sinha '21
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Lesson from 3d for 4d

- Consider geometries with horizons (black holes, cosmologies) in 4d
- Establish states in CCFT_3 dual to these geometries and do physics