Searching for physics beyond the Standard Model using atomic nuclei

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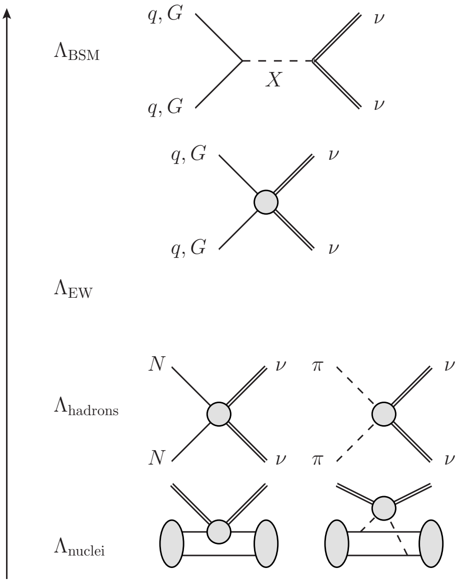
Atomic nuclei as laboratory for BSM searches

	Observable	Physics scope
Precision tests	(Superallowed) β decays	V_{ud} and CKM unitarity, $V-A$
	Parity-violating electron scattering (PVES)	Weak charge, $V-A$
	Coherent elastic neutrino–nucleus scattering (CEνNS)	Weak charge, $V-A$
Null tests	$\mu \to e$ conversion	Lepton flavor violation
	Neutrinoless double-β decay ($0\nu\beta\beta$)	Lepton number violation, ν mass
	Electric dipole moments (EDMs)	*CP* violation
	Direct detection of dark matter	Weakly interacting massive particles

- Atomic nuclei as laboratory for BSM searches
  - Advantage: stable targets, **large statistics** (Avogadro's number)
  - Disadvantage: need to know **nucleon and nuclear matrix elements for interpretation**
- Modern perspective: **effective-field-theory approach**

Scales in the example of CEνNS



1. **BSM scale** Λ_{BSM}: $\mathcal{L}_{\text{BSM}}$
2. **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda^i_{\text{BSM}}} \mathcal{O}_{i,k}$
3. Integrate out **EW physics**

   (start here if only SM)
4. **Hadronic scale**: nucleons and pions

   $\hookrightarrow$ effective interaction Hamiltonian H_I
5. **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$

   $\hookrightarrow$ nuclear wave function

Effective field theory

- Need to deal with large **hierarchy of scales** between BSM and nuclear physics

  $\hookrightarrow$ **effective field theories**
- Matching and RG corrections
  - Standard Model EFT (SMEFT): BSM to EW
  - Low-energy EFT (LEFT): EW to hadronic
  - Chiral perturbation theory (ChPT): single-hadron matrix elements
  - Chiral EFT: few-hadron amplitudes as input for ab-initio nuclear structure calculations
- Non-perturbative matching at low energies:
  - Nucleon matrix elements: ChPT, lattice QCD
  - Nuclear matrix elements: chiral EFT, nuclear many-body techniques
- Important both for limits and, even more so, a possible detection

Outline

1. Superallowed β decays and CKM unitarity

   Cirigliano, Dekens, de Vries, Gandolfi, MH, Mereghetti PRL 133 (2024) 211801, PRC 110 (2024) 055502

2. $\mu \to e$ conversion in nuclei

   Noël, MH JHEP 08 (2024) 052, Heinz, MH, Miyagi, Noël, Schwenk arXiv:2412.04545

CKM unitarity: a precision test of the SM

Unitarity of the CKM matrix

	$V_{ij}V^*_{kj} = \delta_{ik}$	$V_{ji}V^*_{jk} = \delta_{ik}$
$i = k$	$\lvert V_{id}\rvert^2 + \lvert V_{is}\rvert^2 + \lvert V_{ib}\rvert^2 = 1$	$\lvert V_{ui}\rvert^2 + \lvert V_{ci}\rvert^2 + \lvert V_{ti}\rvert^2 = 1$
$i \neq k$	$V_{id}V^*_{kd} + V_{is}V^*_{ks} + V_{ib}V^*_{kb} = 0$	$V_{ui}V^*_{uk} + V_{ci}V^*_{ck} + V_{ti}V^*_{tk} = 0$

- Interesting relations if different terms scale in the same way with Wolfenstein parameter λ

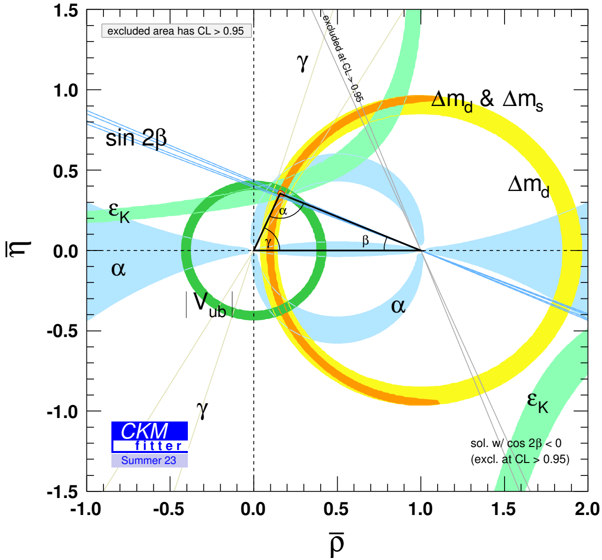
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- **Unitarity triangle** for $i = d$, $k = b$:

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

$$A\lambda^3(\rho + i\eta) - \lambda A\lambda^2 + A\lambda^3(1-\rho-i\eta) = 0$$

The unitarity triangle



$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2} + \ldots\right)$$

$$\bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2} + \ldots\right)$$

$$\bar{\eta} = \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| \sin\gamma$$

$$\bar{\eta} = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| \sin\beta$$

Unitarity circles

Benchmarks numbers for CKM tests from PDG "12. CKM Quark-Mixing Matrix"

first row:	$\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 + \lvert V_{ub}\rvert^2 = 0.9985(7)$
second row:	$\lvert V_{cd}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{cb}\rvert^2 = 1.001(12)$
first column:	$\lvert V_{ud}\rvert^2 + \lvert V_{cd}\rvert^2 + \lvert V_{td}\rvert^2 = 0.9972(20)$
second column:	$\lvert V_{us}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{ts}\rvert^2 = 1.004(12)$

- Interesting relations arise for the diagonal tests, except for third row/column $\mathcal{O}(\lambda^4)$
- **Indications for deficit in first row** (and first column)
- Testing consistency of V_{ud} and V_{us} at precision of a few times 10^{-4}, while $\lvert V_{ub}\rvert^2 \simeq 1.5 \times 10^{-5}$
- Here:
  - Status of the first-row test
  - Recent developments for V_{ud} from superallowed β decays

Determination of V_{ud} from superallowed β decays

- Master formula Hardy, Towner 2018

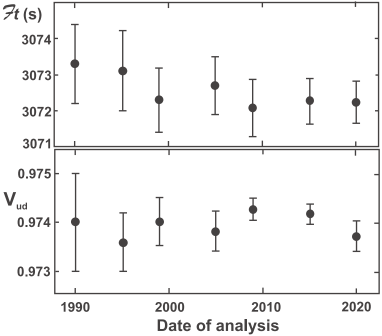
$$|V_{ud}|^2 = \frac{2984.432(3)\,\text{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

with (universal) radiative corrections Δ_R^V

- Value of V_{ud} crucially depends on Δ_R^V:

Ref.	Δ_R^V
Marciano, Sirlin 2006	0.02361(38)
Seng, Gorchtein, Patel, Ramsey-Musolf 2018	0.02467(22)
Czarnecki, Marciano, Sirlin 2019	0.02426(32)
Seng, Feng, Gorchtein, Jin 2020	0.02477(24)
Hayen 2020	0.02474(31)
Shiells, Blunden, Melnitchouk 2021	0.02472(18)
Cirigliano, Crivellin, MH, Moulson 2022	0.02467(27)

$\hookrightarrow$ main uncertainty from Regge region, first lattice calculation largely consistent Ma et al. 2023



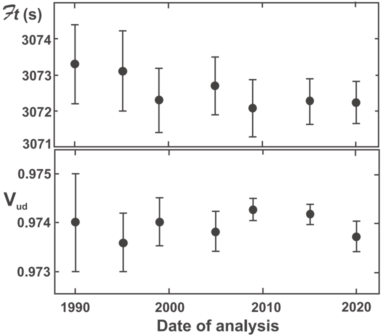
Hardy, Towner 2020

Determination of V_{ud} from superallowed β decays

- Further corrections
  - Isospin breaking Miller, Schwenk 2008, 2009, Condren, Miller 2022, Seng, Gorchtein 2022, Crawford, Miller 2022
  - Nuclear corrections Seng, Gorchtein, Ramsey-Musolf 2018, Gorchtein 2018
- Estimate from Gorchtein 2018 becomes dominant source of uncertainty

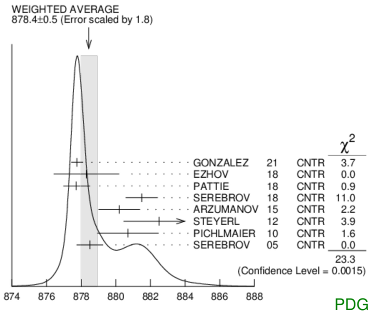
$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_V^R}(27)_{\text{NS}}[32]_{\text{total}}$$

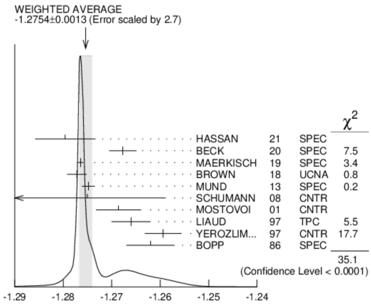
- Improvements from ab-initio nuclear structure necessary to make progress



Hardy, Towner 2020

Determination of V_{ud} from neutron decay





PDG 2022

- Master formula Czarnecki, Marciano, Sirlin 2018

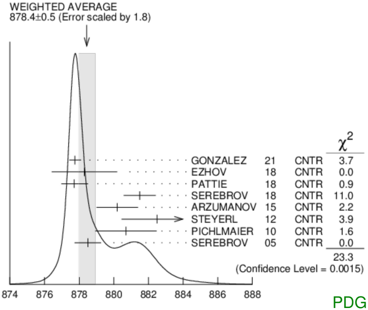
$$|V_{ud}|^2\tau_n(1+3g_A^2)(1+\Delta_{\text{RC}}) = 5099.3(3)\,\text{s}$$

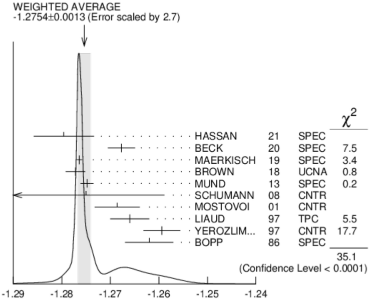
  with radiative corrections Δ_{RC}

  $\hookrightarrow$ need lifetime τ_n and asymmetry $\lambda = g_A/g_V$

- PDG average especially for g_A includes large scale factors

Determination of V_{ud} from neutron decay





PDG 2022

- Results for V_{ud}

$$V_{ud}^{\text{n, PDG}} = 0.97441(3)_f(13)_{\Delta_R}(82)_\lambda(28)_{\tau_n}[88]_{\text{total}}$$

$$V_{ud}^{\text{n, best}} = 0.97413(3)_f(13)_{\Delta_R}(35)_\lambda(20)_{\tau_n}[43]_{\text{total}}$$

$\hookrightarrow$ average of $V_{ud}^{0^+\to 0^+}$ with $V_{ud}^{\text{n, best}}$ gives $V_{ud}^{\beta} = 0.97384(26)$

- Need improved measurements especially for g_A to make progress

Determination of V_{ud} from pion β decay

- Master formula Cirigliano, Knecht, Neufeld, Pichl 2003, Czarnecki, Marciano, Sirlin 2020, Feng et al. 2020

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu_e(\gamma)) = \frac{G_F^2 |V_{ud}|^2 M_{\pi^\pm}^5 |f_+^\pi(0)|^2}{64\pi^3}(1+\Delta_{\text{RC}}^{\pi\ell}) I_{\pi\ell}$$

  $\hookrightarrow$ need branching fraction and pion life time from experiment

- (Theory) inputs
  - Phase space $I_{\pi\ell} = 7.3766(43)\times 10^{-8}$, uncertainty from $\Delta_\pi = M_{\pi^+} - M_{\pi^0}$
  - Form factor $f_+^\pi(0) = 1 - 7\times 10^{-6}$

    $\hookrightarrow$ protected by $SU(2)$ Ademollo–Gatto theorem (Behrends–Sirlin)
  - Radiative corrections $\Delta_{\text{RC}}^{\pi\ell} = 0.0334(10)$ ChPT, Cirigliano et al., $\Delta_{\text{RC}}^{\pi\ell} = 0.0332(3)$ lattice QCD, Feng et al.

- Resulting V_{ud} extracted from PIBETA 2004

$$V_{ud}^{\pi,\text{ChPT}} = 0.97376(281)_{\text{BR}}(9)_{\tau_\pi}(47)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[287]_{\text{total}}$$

$$V_{ud}^{\pi,\text{lattice}} = 0.97386(281)_{\text{BR}}(9)_{\tau_\pi}(14)_{\Delta_{\text{RC}}^{\pi\ell}}(28)_{I_{\pi\ell}}[283]_{\text{total}}$$

$\hookrightarrow$ factor 10 possible before other errors creep in (same as for $R_{e/\mu}$)

Summary: V_{ud}

	$+$	$-$
Superallowed β decays	many isotopes to average	nuclear uncertainties
Neutron decay	no nuclear uncertainties	need high precision for τ_n and g_A
Pion β decay	theoretically pristine	experimentally challenging

- Prospects for improvement:
  - Superallowed β decays: need **improved theory**
  - Neutron decay: need **improved measurement of g_A**

    $\hookrightarrow$ PERC, Nab
  - Pion β decay: need an order of magnitude in branching fraction

    $\hookrightarrow$ **PIONEER**
- All approaches rather complementary, several avenues to improve V_{ud}!

Tensions in the V_{ud}–V_{us} plane

- Global-fit point away from unitarity circle

  $(\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 - 1)$

$$V_{ud} = 0.97378(26) \quad V_{us} = 0.22422(36)$$

$$\Delta_{\text{CKM}} = -1.48(53) \times 10^{-3} \quad [2.8\sigma]$$

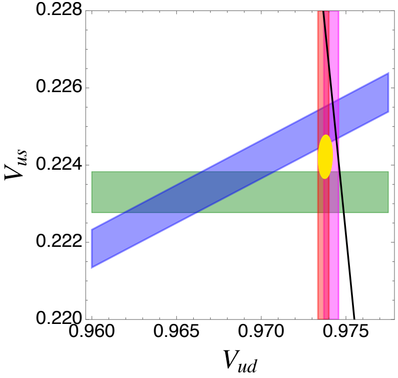
- Three possible measures of the CKM tension

$$\Delta^{(1)}_{\text{CKM}} = |V^{\beta}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1$$
$$= -1.76(56) \times 10^{-3} \quad [3.1\sigma]$$

$$\Delta^{(2)}_{\text{CKM}} = |V^{\beta}_{ud}|^2 + |V^{K_{\ell 2}/\pi_{\ell 2},\, \beta}_{us}|^2 - 1$$
$$= -0.98(58) \times 10^{-3} \quad [1.7\sigma]$$

$$\Delta^{(3)}_{\text{CKM}} = |V^{K_{\ell 2}/\pi_{\ell 2},\, K_{\ell 3}}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1$$
$$= -1.64(63) \times 10^{-2} \quad [2.6\sigma]$$

$\hookrightarrow$ already tension in kaon sector alone 2.6σ



Cirigliano, Crivellin, MH, Moulson 2022

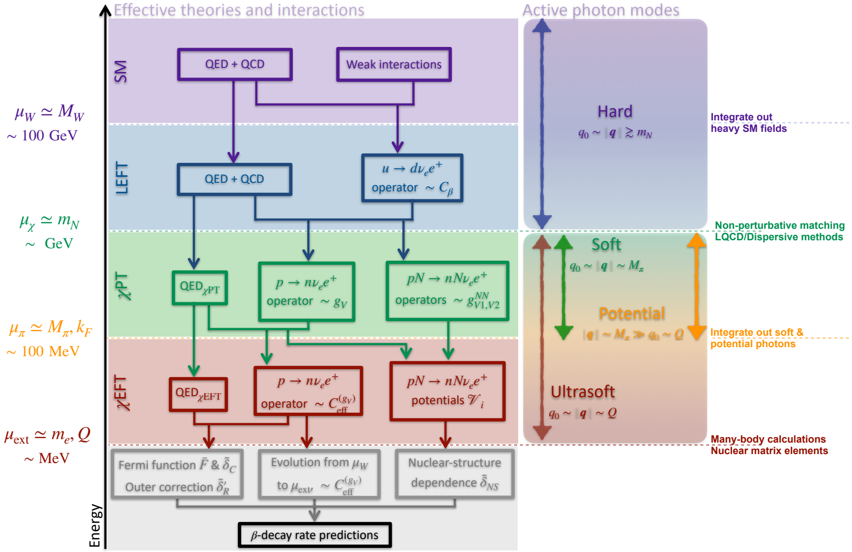
Improving theory for superallowed β decays

- Need to improve **nuclear structure calculations** for radiative corrections
- Two aspects:
  1. Calculate nuclear responses with modern ab-initio techniques
  2. Derive which nuclear responses even need to be calculated

  $\hookrightarrow$ 2. is far from trivial since traditional master formula based on nuclear models
- A large hierarchy of scales: $q_{\text{ext}} \simeq m_e, E_0 \ll M_\pi \ll \Lambda_\chi \ll M_W$

  $\hookrightarrow$ calls for treatment in **effective field theory**

EFT landscape



Hard photons

- **Hard photons**: $\Lambda_\chi^2 \lesssim Q^2 \lesssim M_W^2$
- Integrate out W boson $\Rightarrow$ Low-Energy EFT (LEFT)

$$\mathcal{L}_{\text{LEFT}} = -2\sqrt{2} G_F V_{ud}\, C_\beta^r(\mu)\, \bar{e}_L \gamma_\mu \nu_L\, \bar{u}_L \gamma^\mu d_L + \text{ h.c.}$$

- **Hadronization**
  - One nucleon ("one-body"):
    $\mathcal{L}_W^{1b} = -\sqrt{2} G_F V_{ud}\, \bar{e}_L \gamma_\mu \nu_L \bar{N}\big[g_V(\mu) v^\mu - 2 g_A(\mu) S^\mu\big]\tau^+ N + \cdots$
  - Two nucleons ("two-body"):
    $\mathcal{L}_W^{2b} = -\sqrt{2} e^2 G_F V_{ud} \bar{e}_L \gamma_\mu \nu_L v^\mu \Big[g_{V1}^{NN}(\mu) N^\dagger \tau^+ N\, N^\dagger N + g_{V2}^{NN}(\mu) N^\dagger \tau^+ N\, N^\dagger \tau^3 N\Big] + \cdots$
  - Pion-mass difference: $\mathcal{L}_\pi = 2e^2 F_\pi^2 Z_\pi\, \pi^+ \pi^- + \cdots \qquad M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2 F_\pi^2 Z_\pi$
- Renormalization-group evolution important, e.g.

$$g_V(\mu) = \tilde{U}(\mu, \mu_\chi) \times \left[1 + \Box_{\text{had}}^V(\mu_0) - \frac{\alpha(\mu_\chi)}{2\pi}\, \kappa\left(\frac{\mu}{\mu_0}, \frac{\mu_0}{\mu_\chi}\right)\right] \times \left(1 + \frac{\alpha(\mu_\chi)}{\pi} B(a)\right)^{-1} U(\mu_\chi, \mu_W)\, C_\beta^r(\mu_W)$$

Power counting

- **Scales**
  1. Low-energy: $q_{\text{ext}} \simeq m_e \simeq E_0 = \mathcal{O}(1\,\text{MeV})$
  2. Nuclear: $\gamma \simeq R^{-1} \simeq M_\pi \simeq k_F = \mathcal{O}(100\,\text{MeV})$
  3. Chiral/hadronic: $\Lambda_\chi \simeq 4\pi F_\pi \simeq m_N \simeq 1\,\text{GeV}$
  4. Electroweak: $M_W \simeq 100$ GeV

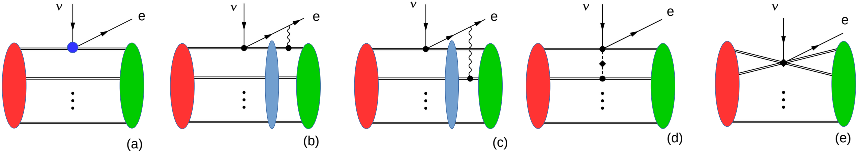
- **Expansion parameters**

$$\epsilon_{\text{recoil}} = \mathcal{O}\left(\frac{q_{\text{ext}}}{\Lambda_\chi}\right) \simeq 0.005 \qquad \epsilon_{\not\pi} = \mathcal{O}\left(\frac{q_{\text{ext}}}{M_\pi}\right) \simeq 0.05 \qquad \epsilon_\chi = \mathcal{O}\left(\frac{M_\pi}{\Lambda_\chi}\right) \simeq 0.1$$

$\hookrightarrow$ need to keep $\mathcal{O}(\alpha\epsilon_{\not\pi})$ and $\mathcal{O}(\alpha\epsilon_\chi)$ for 10^{-4} precision (also some $\mathcal{O}(\alpha^2)$ terms)

- Important lessons:
  - Dominant contributions from "potential" photons: $q^0 \simeq \mathbf{q}^2/m_N \simeq q_{\text{ext}}$, $|\mathbf{q}| \simeq M_\pi$
  - Ultrasoft modes ($q^0 \simeq |\mathbf{q}| \simeq q_{\text{ext}} \simeq M_\pi^2/m_N$) contribute to classic Fermi/Sirlin functions
  - Soft modes ($q^0 \simeq |\mathbf{q}| \simeq M_\pi$) suppressed by $\mathcal{O}(\alpha\epsilon_\chi^2)$
  - g_{V1}^{NN}, g_{V2}^{NN} **enhanced by RG arguments** $\Rightarrow$ need to be kept

Diagrams



- (b), (c), real emission: ultrasoft modes corresponding to Fermi/Sirlin function
- (c): potential modes $\mathcal{O}(\alpha\epsilon_{\not\pi})$ and $\mathcal{O}(\alpha\epsilon_\chi)$
- (d): pion mass splitting $\mathcal{O}(\alpha\epsilon_{\not\pi})$ and $\mathcal{O}(\alpha\epsilon_\chi)$
- (e): contact terms $\mathcal{O}(\alpha\epsilon_\chi)$
- (c,d,e) give rise to "potentials," to be evaluated between nuclear wave functions
- Many more subtleties regarding ultrasoft photons, RG corrections, enhanced $\mathcal{O}(\alpha^2 Z)$, $\mathcal{O}(\alpha^2 \log r)$ terms, . . .

Master formula

Master formula for lifetime

$$\frac{1}{t} = \frac{G_F^2|V_{ud}|^2 m_e^5}{\pi^3 \log 2}\Big[C_{\text{eff}}^{(g_V)}(\mu)\Big]^2 \times [1+\bar{\delta}'_R(\mu)]\,(1+\bar{\delta}_{\text{NS}})\,(1-\bar{\delta}_C)\,\bar{f}(\mu)$$

- Written in a form that resembles the traditional decomposition, but:
  - $\bar{\delta}_{\text{NS}}$ includes contact terms $g_{V1,V2}^{NN}$, so far implicit in high-energy part of matrix elements
  - Only potentials to be evaluated, no sum over intermediate states
  - Large logarithms consistently resummed
  - Two-body currents included, no “quenching” corrections
  - Factorization and scale independence manifest (at the considered order)
- EFT maximally exploits **separation of scales**

QMC evaluation for $^{14}\text{O} \to {}^{14}\text{N}$

- Explicit Quantum Monte Carlo (QMC) calculations for $^{6}\text{Be} \to {}^{6}\text{Li}$, $^{6}\text{Li} \to {}^{6}\text{He}$, and $^{14}\text{O} \to {}^{14}\text{N}$

  $\hookrightarrow$ power counting expectations largely confirmed

- $^{14}\text{O} \to {}^{14}\text{N}$ is a physical transition, first numerical analysis

$$V_{ud} = 0.97364(10)_{\text{exp}}(12)_{g_V}(22)_{\mu}(12)_{\delta_C}(43)_{g_V^{NN}}(20)_{\delta_{\text{NS}}^E}[56]_{\text{total}}$$

  $\hookrightarrow$ dominant error from contact terms, next residual scale dependence

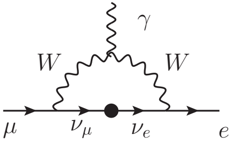
- Points of comparison:
  - Neutron decay in EFT Cirigliano et al. 2023: $V_{ud}^{\text{neutron}} = 0.97402(42)$
  - Superallowed β decays (global) Hardy, Towner 2020: $V_{ud}^{\text{HT}} = 0.97373(31)$
  - $^{14}\text{O} \to {}^{14}\text{N}$ Hardy, Towner 2020: $V_{ud}^{\text{HT},\,^{14}\text{O}} = 0.97405(13)_{\text{exp}}(9)_{\Delta_R^V}(12)_{\delta_C}(31)_{\delta_{\text{NS}}}[37]_{\text{total}}$

- Extraction from single isotope could become competitive if contact terms were known

Future improvements

- Strategies for determination of contact terms
  - Cottingham approach
  - Lattice QCD
  - Global fit to superallowed β decays, with V_{ud} and $g^{NN}_{V1,V2}$ as free parameters
- Ab-initio calculations for more isotopes
- Subleading contributions
  - Two- and three-body $\mathcal{O}(\alpha\epsilon_\chi^2)$ diagrams
  - Subleading terms in the Fermi function $\mathcal{O}(\alpha^2 Z)$
  - Shape corrections in phase-space evaluation

Why lepton flavor violation?



- **Lepton flavor symmetry**
  - Lepton flavor conserved in SM with massless neutrinos
  - **Neutrino oscillations** sign of lepton flavor violation (LFV) in neutral sector
  - Propagates to charged sector via **mass insertions** in loops, but, e.g.,

$$\mathrm{Br}[\mu \to e\gamma] \simeq \left(\frac{\Delta m_\nu^2}{M_W^2}\right)^2 \simeq 10^{-50}$$

$\hookrightarrow$ unobservably small in SM!

  - Lepton flavor **"accidental" symmetry** of SM

$\hookrightarrow$ LFV expected to occur for a wide range of BSM scenarios

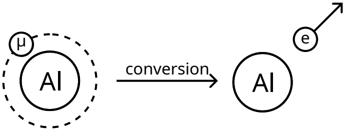
  - In practice: LFV highly sensitive null test

Why $\mu \to e$ conversion in nuclei?

LFV process	current limit on Br	(planned) experiments
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	MEG II
$\mu \to 3e$	$< 1.0 \times 10^{-12}$ SINDRUM	Mu3e
$\tau \to \ell\gamma, 3\ell, \ell P, \ldots$	$\lesssim 10^{-8}$ Belle, LHCb, ...	Belle 2, ...
$K \to \mu e, \mu e\pi, \mu e\pi\pi$	$\lesssim 10^{-11}$ KTeV, NA62, BNL	KOTO, LHCb
$\pi^0 \to \bar{\mu}e$	$< 3.6 \times 10^{-10}$ KTeV	JEF, REDTOP (?)
$\eta \to \bar{\mu}e$	$< 6 \times 10^{-6}$ SPEC	JEF, REDTOP (?)
$\eta' \to \bar{\mu}e$	$< 4.7 \times 10^{-4}$ CLEO II	JEF, REDTOP (?)
Au $\mu^- \to$ Au e^-	$< 7 \times 10^{-13}$ SINDRUM II	
Ti $\mu^- \to$ Ti e^-	$< 6.1 \times 10^{-13}$ SINDRUM II	
Al $\mu^- \to$ Al e^-	$\lesssim 10^{-17}$ (projected)	Mu2e, COMET

$\hookrightarrow$ major experimental improvements expected in coming years!

What is $\mu \to e$ conversion?



- What is $\mu \to e$ conversion? A theorist's perspective:
  - Muon bound in $1s$ level of nucleus
  - Muon converts to electron within Coulomb field of nucleus

$$\bar{e}\sigma^{\alpha\beta}\mu F_{\alpha\beta}, \qquad \bar{e}\mu\,\bar{q}q, \qquad \dots$$

$\hookrightarrow$ both long- and short-range BSM mechanisms possible

  - Electron ejected with energy of muon–electron mass difference (minus binding energy)

$\hookrightarrow$ **very clear experimental signature**

  - Background: muon decay in orbit $\mu \to e\nu_\mu\bar{\nu}_e$
  - Normalization: muon capture $\mu(A, Z) \to \nu_\mu(A, Z-1)$

Master formulae

Very schematic master formula for $\mu \to e$ conversion

$$\Gamma[\mu(A,Z) \to e(A,Z)] \simeq \text{BSM Wilson coefficients} \otimes \text{hadronic matrix elements} \otimes \text{nuclear responses} \otimes \text{bound-state solution}$$

- Latter two traditionally combined into **overlap integrals** Kitano et al. 2002

$$S^{(N)} = \frac{\#N}{2\sqrt{2}} \int_0^\infty dr\, \rho_N(r)\big[g^{(e)}_{-1}(r) g^{\mu}_{-1}(r) - f^{e}_{-1}(r) f^{\mu}_{-1}(r)\big] \qquad V^{(N)} = \frac{\#N}{2\sqrt{2}} \int_0^\infty dr\, \rho_N(r)\big[g^{(e)}_{-1}(r) g^{\mu}_{-1}(r) + f^{e}_{-1}(r) f^{\mu}_{-1}(r)\big]$$

$$D = \frac{-4m_\mu}{\sqrt{2}} \int_0^\infty dr\, E(r)\big[g^{e}_{-1}(r)\, f^{\mu}_{-1}(r) + f^{e}_{-1}(r)\, g^{\mu}_{-1}(r)\big]$$

$\hookrightarrow$ covers coherently enhanced **spin-independent** responses, $\Gamma_{\text{SI}} \propto \#N^2$

- Similarly, dominant **spin-dependent** contribution from nuclear responses finite for $q = 0$, e.g., for axial-vector operators $\Gamma_{\text{SD}} \propto g_A^2$ Davidson et al. 2018
- Focus on spin-independent responses for now

Uncertainty quantification for nuclear responses

- So far: nuclear responses calculated in (phenomenological) **nuclear shell model**

  $\hookrightarrow$ uncertainty estimates difficult, especially for neutron responses
- **Ab-initio approaches**
  - Often uncertainties dominated by chiral Hamiltonian, not by many-body solution
  - Often correlations between different responses much more stable

    Hagen et al. 2015, Payne et al. 2019
- Need for **charge distributions with quantified uncertainties**
  - Solution of the Dirac equation
  - Input for correlation analysis in ab-initio approaches
- Charge distributions extracted from electron scattering **without uncertainties (!)**

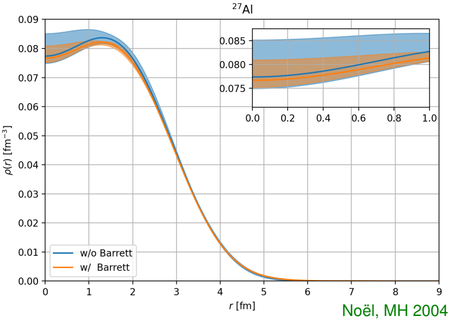
  $\hookrightarrow$ **Fourier–Bessel expansions** Dreher et al. 1974, de Vries et al. 1987

$$\rho(r) = \begin{cases} \sum_{n=1}^{N} a_n j_0(q_n r) & r \leq R \\ 0 & r > R \end{cases} \quad \text{with} \quad q_n = \frac{\pi n}{R}$$

Extracting charge distributions from electron scattering

- Practical challenges of re-analysis:
  - Most data taken in the 70s+80s
  - Many data sets not available at all ("private communication"), or only published in PhD theses
  - Documentation of uncertainties rudimentary
- Propagation of uncertainties computationally intensive
  $\hookrightarrow$ truncation errors in R, N
- Carried this program out for ^{27}Al, ^{40,48}Ca, and ^{48,50}Ti
- Results available as `python` notebook

2406.06677



Noël, MH 2004

Solving the Coulomb problem

- For a realistic description, need to resum **Coulomb phase shifts**

  $\hookrightarrow$ phase shift model

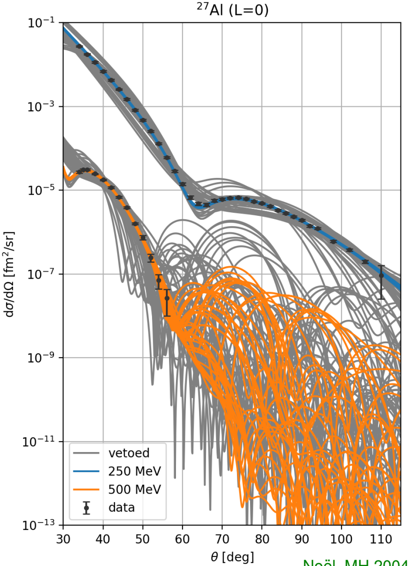
- Fits carried out over large grid of (N, R), using veto on oscillations and asymptotics to prevent overparameterization

- Constraints from **Barrett moments** measured in $2p \to 1s$ transitions of muonic atoms

$$\left\langle r^k e^{-\alpha r} \right\rangle = \frac{4\pi}{Z} \int_0^\infty dr\, r^{k+2} \rho(r) e^{-\alpha r}$$

- Similar approach also applies to Coulomb corrections elsewhere

  $\hookrightarrow$ parity-violating electron scattering



Noël, MH 2004

Results for dipole overlap integrals

Dipole overlap integrals

$D(^{27}\text{Al}) = 0.0359(2)$ $D(^{40}\text{Ca}) = 0.07531(5)$ $D(^{48}\text{Ca}) = 0.07479(10)$

$D(^{48}\text{Ti}) = 0.0864(1)$ $D(^{50}\text{Ti}) = 0.0870(3)$

- Dipole overlap integrals

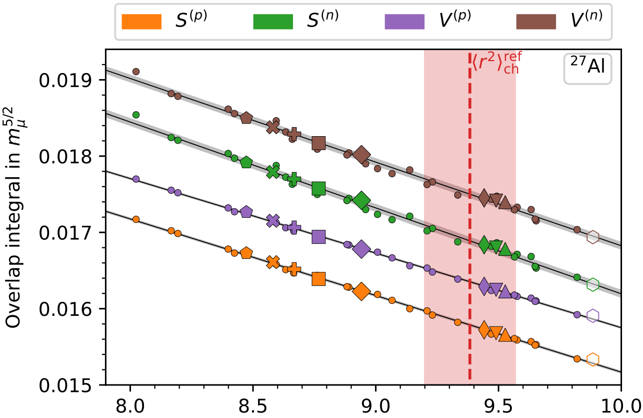
$$D = \frac{-4m_\mu}{\sqrt{2}} \int_0^\infty dr\, E(r) \left[g^e_{-1}(r)\, f^\mu_{-1}(r) + f^e_{-1}(r)\, g^\mu_{-1}(r) \right]$$

$\hookrightarrow$ only depends on charge distributions, electric field $E(r) = \frac{\sqrt{4\pi\alpha}}{r^2} \int_0^r dr'\, r'^2 \rho(r')$

- For the first time, **fully quantified uncertainties**
- For $S^{(N)}$, $V^{(N)}$ other nuclear responses contribute

$\hookrightarrow$ interplay with ab-initio methods

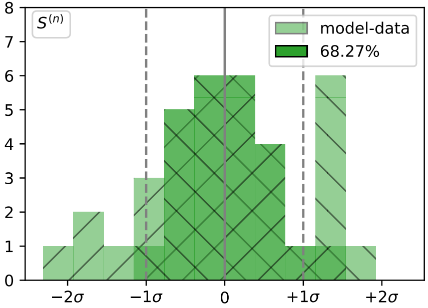
Ab-initio calculations for $\mu \to e$ conversion

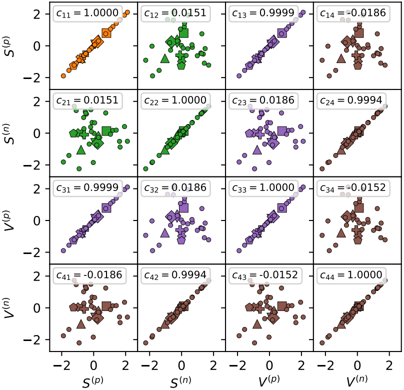


Heinz, MH, Miyagi, Noël, Schwenk arXiv:2412.04545

- Calculations performed using VS-IMSRG for ^{27}Al for various chiral Hamiltonians
  $\hookrightarrow$ tight correlation with charge radius
- Shell-model result actually falls onto correlation

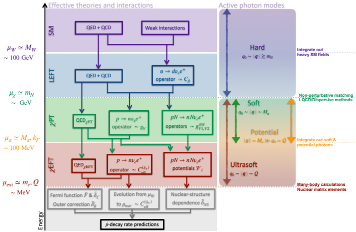
Residuals and correlations

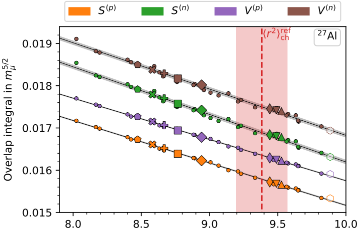




Conclusions

- Atomic nuclei as laboratory for BSM searches
- **Superallowed β decays**
    - 3σ hint for tension with unitarity
    - Derived EFT-based master formula
    - Power counting, RG corrections, factorization, ...
    - Clarified role of residue contribution
    - First numerical analysis for $^{14}O \rightarrow {}^{14}N$
    - Analysis strategy for EFT determination of V_{ud}
- **$\mu \rightarrow e$ conversion in nuclei**
    - Control over nuclear responses critical to disentangle LFV mechanism
    - Tight correlation between proton, neutron, and charge densities
    - Complete set of overlap integrals, including covariances





Determination of V_{us}/V_{ud} from kaon decays: $K_{\ell 2}/\pi_{\ell 2}$

- **$K_{\ell 2}$ decays**: $K \to \ell\nu_\ell$

$$\frac{V_{us}}{V_{ud}}\frac{F_K}{F_\pi} = \left(\frac{\Gamma(K^+ \to \mu^+\nu_\mu(\gamma))M_\pi}{\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma))M_K}\right)^{1/2} \frac{1-\frac{m_\mu^2}{M_\pi^2}}{1-\frac{m_\mu^2}{M_K^2}} \Bigg(1 - \underbrace{\frac{\Delta_{\text{RC}}^K - \Delta_{\text{RC}}^\pi}{2}}_{\Delta_{\text{RC}}^{K\pi}/2}\Bigg)$$

- Consider the ratio over $\pi_{\mu 2}$ because
  - Only need ratio of decay constant
  - Certain structure-dependent radiative corrections cancel
- Need theory input for:
  - **Decay constants** in isospin limit: $F_K/F_\pi = 1.1978(22)$ HPQCD 2013, Fermilab/MILC 2017, CalLat 2020, ETMC 2021
  - **Isospin-breaking corrections**: $\Delta_{\text{RC}}^{K\pi} = -0.0112(21)$ ChPT, Cirigliano, Neufeld 2011, $\Delta_{\text{RC}}^{K\pi} = -0.0126(14)$ lattice, Di Carlo et al. 2019
- Result:

$$\frac{V_{us}}{V_{ud}}\bigg|_{K_{\ell 2}/\pi_{\ell 2}} = 0.23108(23)_{\text{exp}}(42)_{F_K/F_\pi}(16)_{\text{IB}}[51]_{\text{total}}$$

Determination of V_{us} from kaon decays: $K_{\ell 3}$

- **$K_{\ell 3}$ decays**: $K \to \pi \ell \nu_\ell$

$$\Gamma(K \to \pi \ell \nu_\ell(\gamma)) = \frac{C_K^2 G_F^2 |V_{us}|^2 M_K^5 |f_+^{K\pi}(0)|^2}{192\pi^3}\Big(1 + \underbrace{\Delta_{\text{RC}}^{K\ell}}_{\Delta_{\text{EM}}^{K\ell}+\Delta_{SU(2)}}\Big) I_{K\ell}$$

$\hookrightarrow \ell = \mu, e$ and two charge channels

- Need theory input for:
  - **Form factor**: $f_+^{K\pi}(0) = 0.9698(17)$ ETMC 2016, Fermilab/MILC 2019
  - **Radiative corrections**: $\Delta_{SU(2)} = 0.0252(11)$ Cirigliano et al. 2002, $\Delta_{\text{EM}}^{K^0 e} = 0.0116(3)$, $\Delta_{\text{EM}}^{K^+ e} = 0.0021(5)$, $\Delta_{\text{EM}}^{K^0 \mu} = 0.0154(4)$, $\Delta_{\text{EM}}^{K^+ \mu} = 0.0005(5)$ Seng et al. 2022

- Result:

$$V_{us}^{K_{\ell 3}} = 0.22330(35)_{\text{exp}}(39)_{f_+}(8)_{\text{IB}}[53]_{\text{total}}$$

What can we do to clarify the situation?

- Corroborating V_{ud}
  - Nuclear-structure corrections for superallowed β decays
  - Improved neutron-decay measurements (g_A, τ_n)
  - Pion β decay: **PIONEER**
- Corroborating V_{us}
  - Improved lattice calculations of F_K/F_π
  - **A new measurement of $K_{\mu 3}/K_{\mu 2}$**
  - τ and hyperon decays sensitive to V_{us}, but feasible at the relevant level of accuracy?

A new measurement of $K_{\mu 3}/K_{\mu 2}$, why?

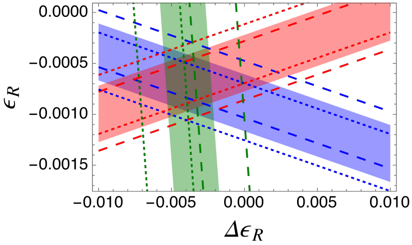
	current fit	$K_{\mu 3}/K_{\mu 2}$ BR at 0.5%			$K_{\mu 3}/K_{\mu 2}$ BR at 0.2%		
		central	$+2\sigma$	-2σ	central	$+2\sigma$	-2σ
$\frac{V_{us}}{V_{ud}}\Big\vert_{K_{\ell 2}/\pi_{\ell 2}}$	0.23108(51)	0.23108(50)	0.23085(51)	0.23133(51)	0.23108(49)	0.23071(51)	0.23147(52)
$V_{us}^{K_{\ell 3}}$	0.22330(53)	0.22337(51)	0.22360(52)	0.22309(54)	0.22342(49)	0.22386(52)	0.22287(52)
$10^2 \Delta_{\text{CKM}}^{(3)}$	$-1.64(63)$	$-1.57(60)$	$-1.18(62)$	$-2.02(63)$	$-1.53(59)$	$-0.83(62)$	$-2.33(62)$
	-2.6σ	-2.6σ	-1.9σ	-3.2σ	-2.6σ	-1.4σ	-3.8σ

- Is the $K_{\ell 3}$ vs. $K_{\ell 2}$ tension real or an experimental problem?
  - $K_{\ell 2}$ data base completely dominated by KLOE 2006
  - Global fit to kaon data not great, p-value $\simeq 1\%$
- This can be clarified with **a new precision measurement of $K_{\mu 3}/K_{\mu 2}$**:
  - In case the tension were of experimental origin, there should be a positive shift compared to current fit

    $\hookrightarrow$ $\Delta_{\text{CKM}}^{(3)}$ would move from -2.6σ to -1.4σ for a $+2\sigma$ shift with a 0.2% measurement
  - In case the tension were of BSM origin, the current value would be confirmed (or move further in the other direction)
- In progress at NA62: one week of minimum-bias data taken

  $\hookrightarrow$ last chance after HIKE cancellation

Interpretation in terms of right-handed currents



- Can parameterize the tensions via **right-handed currents** ϵ_R and $\epsilon_R^{(s)} = \epsilon_R + \Delta\epsilon_R$

  $\hookrightarrow$ modify (axial-)vector processes via $1 \pm \epsilon_R$
- Projections

current:	$\epsilon_R = -0.69(27) \times 10^{-3}$	$[2.5\sigma]$	$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$	$[2.4\sigma]$
0.2% 2σ above:	$\epsilon_R = -0.67(27) \times 10^{-3}$	$[2.5\sigma]$	$\Delta\epsilon_R = -1.8(1.6) \times 10^{-3}$	$[1.1\sigma]$
0.2% 2σ below:	$\epsilon_R = -0.70(27) \times 10^{-3}$	$[2.6\sigma]$	$\Delta\epsilon_R = -5.7(1.6) \times 10^{-3}$	$[3.5\sigma]$

$\hookrightarrow$ should decide whether $\Delta\epsilon_R$ is needed

Residue contribution

- Master formula in **dispersive approach** Seng, Gorchtein 2023

$$\square_{\gamma W} = -\frac{e^2}{M_{\rm F}^{(0)}}\int\frac{d^4q}{(2\pi)^4}\frac{M_W^2}{Q^2+M_W^2}\frac{T_3(\nu,Q^2)}{(p_e-q)^2Q^2}\frac{Q^2+M\nu\frac{p_e\cdot q}{p\cdot p_e}}{M\nu}$$

with $\nu = p\cdot q/M = q^0$, $Q^2=-q^2$

- $T_3(\nu,Q^2)$ scalar function in decomposition of "Compton tensor"

$$T^{\mu\nu}(q;p',p) = \frac{1}{2}\int d^4x\, e^{iq\cdot x}\langle f(p')|T\{J^\mu_{\rm em}(x)J^\nu(0)\}|i(p)\rangle = \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2M\nu}T_3(\nu,Q^2)$$

- **Wick rotation** $\nu\to i\nu$ to perform loop integral
- Not possible in presence of low-lying nuclear states

  $\hookrightarrow$ example: 3^+ and 1^+ levels of ^{10}B in $^{10}\text{C}\to{}^{10}\text{B}$

- Need to subtract **residue**, but singular for $E_e\to 0$ Seng, Gorchtein 2023

  $\hookrightarrow$ enhanced sensitivity to IR scales at odds with EFT power counting?

Toy example

- Consider toy example

$$\frac{iT_3^{\text{toy}}(\nu, Q^2)}{M\nu} = \frac{M}{m_N}\frac{g_A g_M}{s - \bar{M}^2 + i\epsilon}$$

with $s = M^2 + \nu^2 - \mathbf{q}^2 + 2M\nu$ and $M^2 - \bar{M}^2 = 2M\Delta > 0$

- Direct evaluation of loop integral

$$\Box_{\gamma W}^{\text{toy},\Delta} = \frac{3g_A g_M}{4M_{\text{F}}^{(0)}}\frac{\alpha}{\pi}\frac{\Delta}{m_N}\log\frac{2\Delta}{M} + \mathcal{O}(\Delta^2) = \mathcal{O}(\alpha\epsilon_{\text{recoil}})$$

- Residue contribution

$$\Box_{\gamma W}^{\text{toy, res}} = \frac{g_A g_M}{M_{\text{F}}^{(0)}}\sqrt{\frac{M}{m_N}}\frac{\alpha}{\pi}\sqrt{\frac{2\Delta}{m_N}} + \mathcal{O}(\Delta^{3/2}) = \mathcal{O}(\alpha\sqrt{\epsilon_{\text{recoil}}})$$

- Residue violates power counting, but can show explicitly

$$\Box_{\gamma W}^{\text{toy}} = \Box_{\gamma W}^{\text{toy, Wick}} - \Box_{\gamma W}^{\text{toy, res}}$$

- **Power-counting-violating terms cancel between residue and Wick-rotated contribution**

Full master formula: subleading nuclear responses

- To improve SD and even higher responses:
  - **Multipole decomposition** of one-body terms known with respect to fixed momentum transfer q Serot 1978
  - Here: need to combine with **bound-state physics**

    $\hookrightarrow$ perform Fourier transform of leptonic current numerically
  - Uncertainty quantification using ab-initio techniques in combination with data input
- **Two-body corrections**
  - Known to be important for some channels, e.g., **scalar operators** Cirigliano et al. 2022
  - So far evaluated within approximations (“normal ordering” in Fermi gas)
  - Techniques for multipole decomposition of two-body currents becoming available