# Multiple Solutions in the CMSSM

Ben Allanach[a] (University of Cambridge)

[a] BCA, George, Gripaios, `1304.5462`

# Introduction

- Different solutions for same $m_0$, $M_{1/2}$, sign($\mu$), $\tan\beta$ and $A_0$ are *physically distinct*
- MSSM parameters are different between solutions but they satisfy the *same boundary conditions*
- Eventual aim: to explore if current LHC bounds are weakened by the multiple solutions.

# Plan of Talk

- Illustrate the general point with a toy model
- Show usual methodology for obtaining solutions
- Illustrate potential stability problem with such methodology
- Tweak it to show new solutions
- Illustrate with some results

# BKT Theory

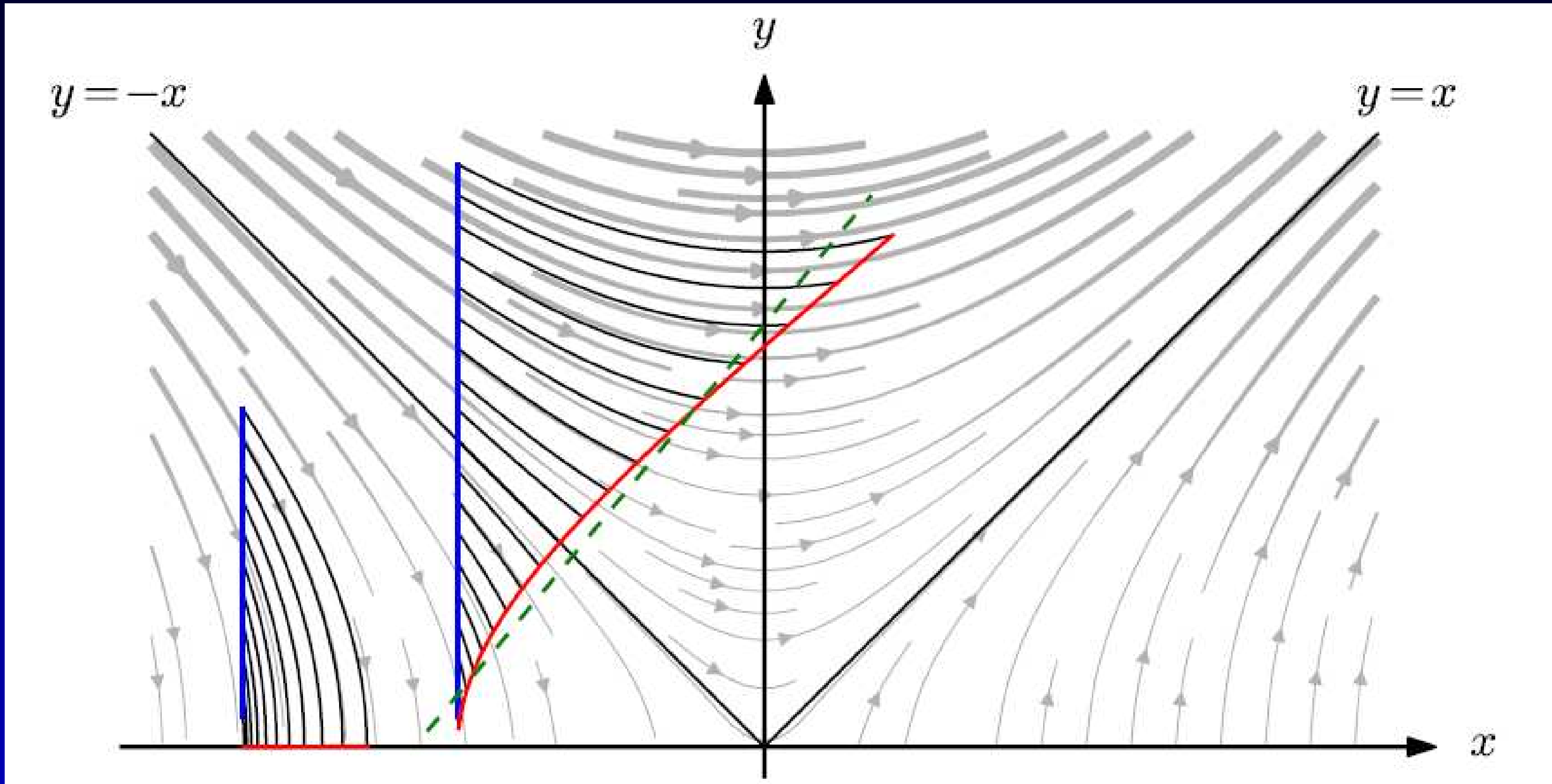
- There are theorems about uniqueness of solutions with systems of ODEs (if $\beta$ fns are continuous, finite and smooth)
- If its a two boundary problem, all bets are off

Berezinsky, Kosterlitz, Thouless theory is a simple QFT that illustrates the phenomenon in compact $U(1)$ gauge theory 2 space dimension. There are 2-couplings $x$, $y$ with RGEs:

$$\frac{dx}{d\ln\mu} = y^2, \qquad \frac{dy}{d\ln\mu} = xy.$$

This toy theory shows that, even with linear boundary conditions you can get multiple solns

# BKT Multiple Solutions




Example linear BCs: $x(\mu = 100GeV) = -2$,
$x(\mu = 200GeV) - y(\mu = 200GeV) = -2$.

# Universality

Reduces number of SUSY breaking parameters from 100 to 3:

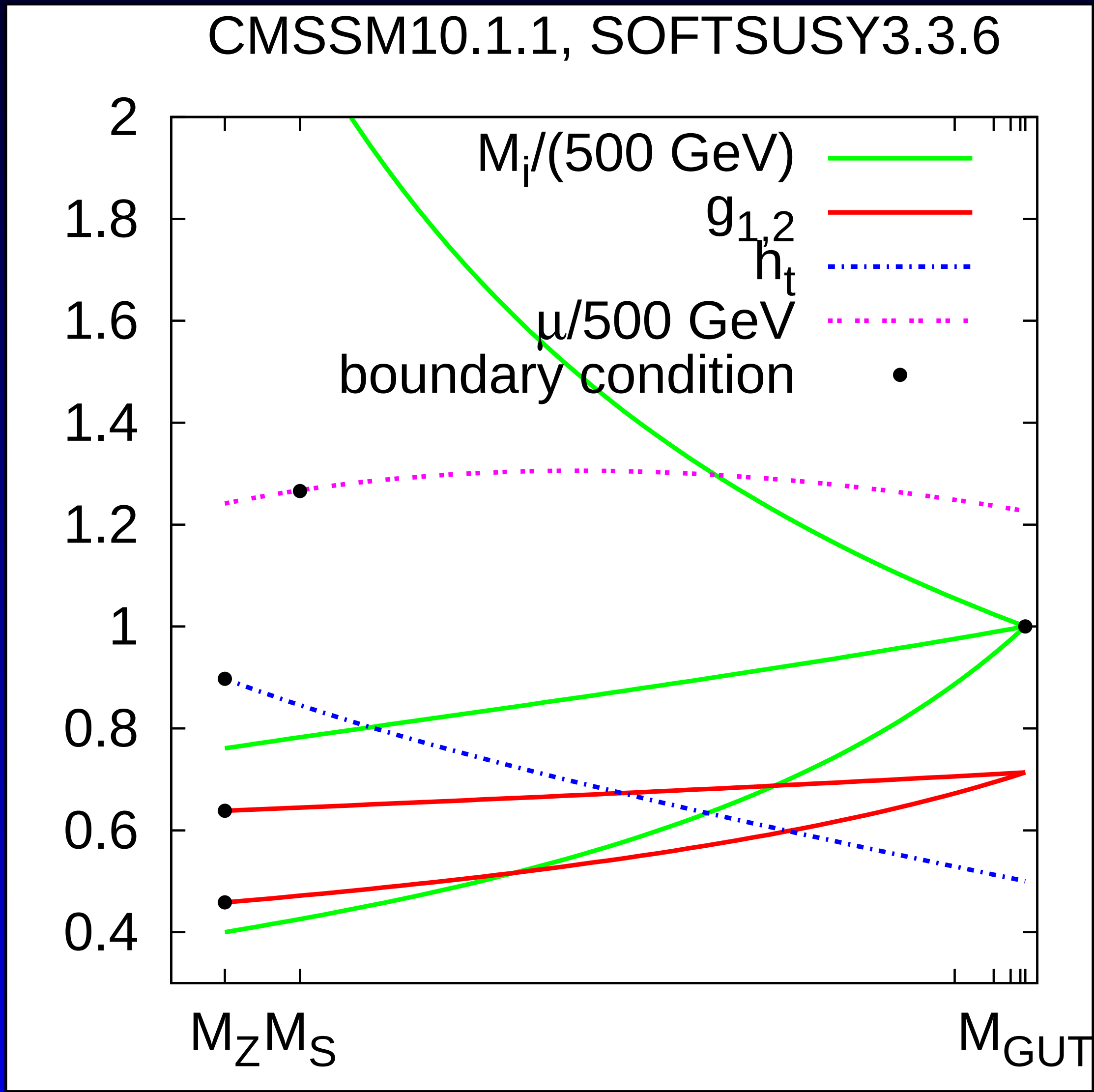
- $\tan\beta \equiv v_2/v_1$
- $m_0$, the common scalar mass (flavour).
- $M_{1/2}$, the common gaugino mass (GUT/string).
- $A_0$, the common trilinear coupling (flavour).

These conditions should be imposed at $M_X \sim O(10^{16-18})$ GeV and receive radiative corrections

$$\propto \ 1/(16\pi^2)\ln(M_X/M_Z).$$

Also, Higgs potential parameter sgn($\mu$)=$\pm$1.

# Multiple Boundary Problem



# Boundary Conditions

$$\tan\beta(M_Z) = \tan\beta(\text{input})$$

$$h_t(M_Z) = \frac{m_t(M_Z)\sqrt{2}}{v(M_Z)\sin\beta}, \qquad h_{b,\tau}(M_Z) = \frac{m_{b,\tau}(M_Z)\sqrt{2}}{v(M_Z)\cos\beta},$$

$$v(M_Z) = 2\sqrt{\frac{M_Z^{\exp 2} + \Pi_{ZZ}^T(M_Z)}{\frac{3}{5}g_1^2(M_Z) + g_2^2(M_Z)}}$$

$$g_1(M_Z) = g_1(\exp), \qquad g_2(M_Z) = g_2(\exp), \qquad g_3(M_Z) = g_3(\exp).$$

$$M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_1}(M_{\text{SUSY}})m_{\tilde{t}_2}(M_{\text{SUSY}})}$$

$$\mu^2(M_{\text{SUSY}}) = \frac{m_{\bar{H}_1}^2(M_{\text{SUSY}}) - m_{\bar{H}_2}^2(M_{\text{SUSY}})\tan^2\beta(M_{\text{SUSY}})}{\tan^2\beta(M_{\text{SUSY}}) - 1} - \frac{1}{2}M_Z^2(M_{\text{SUSY}})$$

$$m_3^2(M_{\text{SUSY}}) = \frac{\sin 2\beta(M_{\text{SUSY}})}{2}\left(m_{\bar{H}_1}^2(M_{\text{SUSY}}) + m_{\bar{H}_2}^2(M_{\text{SUSY}}) + 2\mu^2(M_{\text{SUSY}})\right)$$

$$g_1(M_{\text{GUT}}) = g_2(M_{\text{GUT}})$$

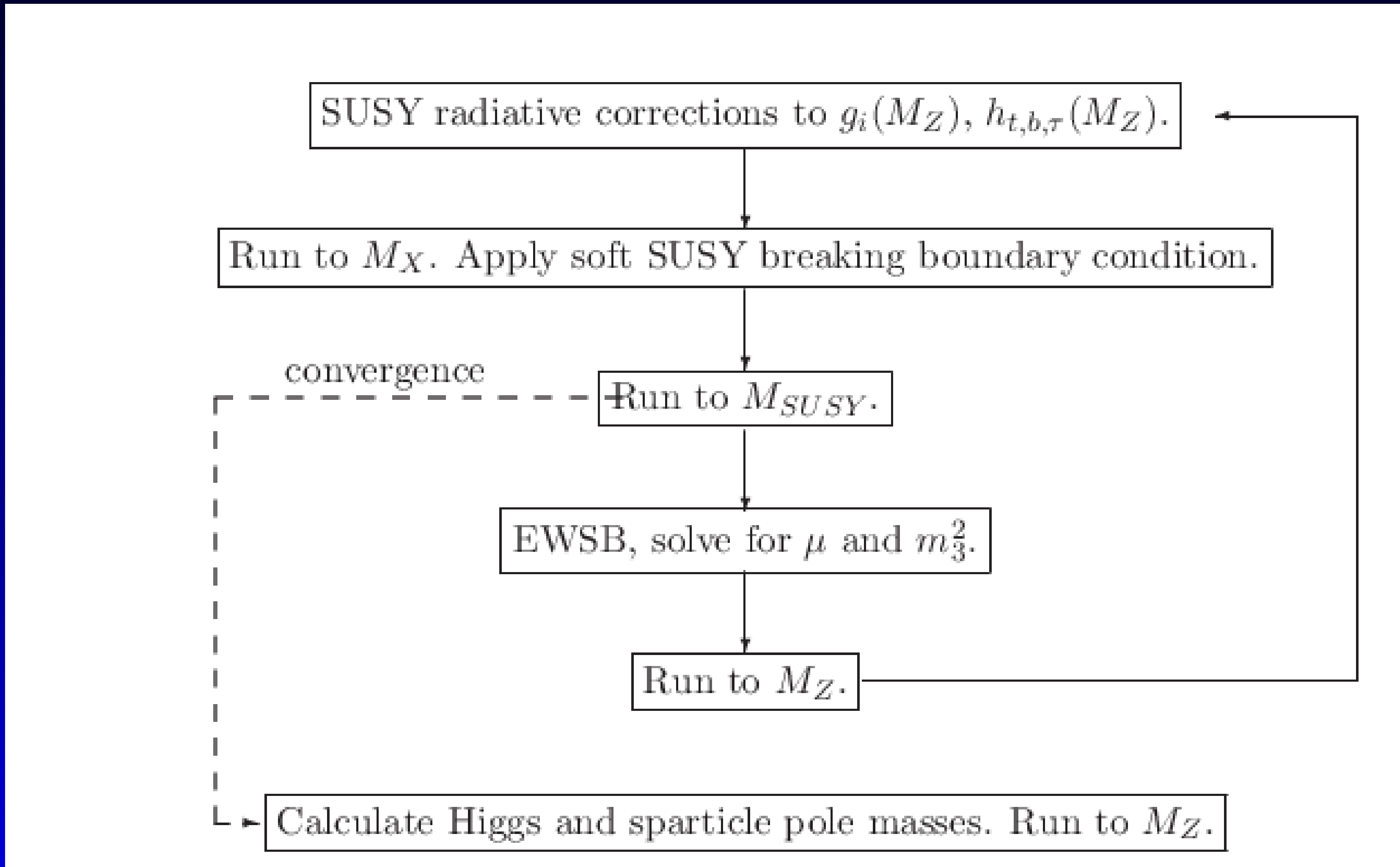
$$M_1(M_{\text{GUT}}) = M_2(M_{\text{GUT}}) = M_3(M_{\text{GUT}}) = M_{1/2}$$

$$m_{\tilde{u}}^2(M_{\text{GUT}}) = m_{\tilde{d}}^2(M_{\text{GUT}}) = m_{\tilde{e}}^2(M_{\text{GUT}}) = m_{\tilde{L}}^2(M_{\text{GUT}}) = m_{\tilde{Q}}^2(M_{\text{GUT}}) = m_0^2 I_3$$

$$m_{H_1}^2(M_{\text{GUT}}) = m_{H_2}^2(M_{\text{GUT}}) = m_0^2$$

$$A_{\tilde{u}}(M_{\text{GUT}}) = A_0 I_3, \qquad A_{\tilde{d}}(M_{\text{GUT}}) = A_0 I_3, \qquad A_{\tilde{e}}(M_{\text{GUT}}) = A_0 I_3.$$

# Solution by Fixed Point Iteration (Fast)




BCA SOFTSUSY3.3.6, hep-ph/0104145

# Stability of Fixed Point Iteration

In 1-dimension, FPI solves $x = f(x)$. We start with a guess $x_1$ of the parameter, and generate (hopefully better) successive approximations $x_n$:

$$x_{n+1} = f(x_n) \Leftrightarrow x - (x - x_{n+1}) = f(x - (x - x_n)).$$

Taylor expanding around $x - x_n = 0$, to first order

$$\cancel{x} - (x - x_{n+1}) = \cancel{f(x)} - (x - x_n)\frac{df}{dx}$$

Stable convergence of FPI requires that, for $x$ sufficiently close to $x_n$, $|x - x_{n+1}| < |x - x_n|$,

$$\Rightarrow \left|\frac{df(x)}{dx}\right| < 1.$$

# An embarassing question

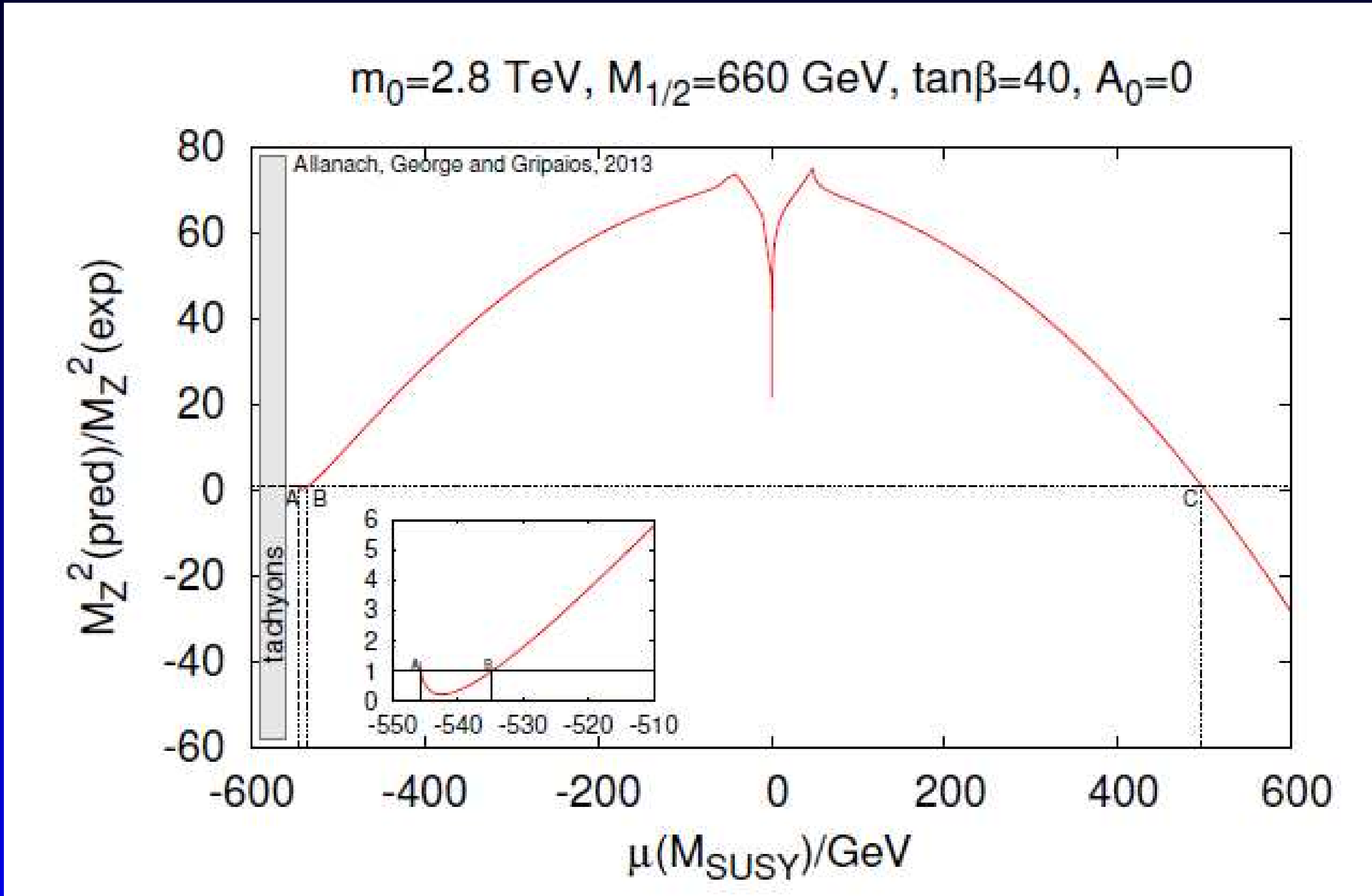
What if there are solutions which are unstable to fixed point iteration?

We invert one of the boundary conditions, scan in $\mu(M_{SUSY})$ and predict $M_Z$ instead and use FPI in the other dimensions.

$$M_Z^2(\text{pred}) = 2\left(\frac{m^2_{\bar{H}_1} - m^2_{\bar{H}_2}\tan^2\beta}{\tan^2\beta - 1} - \mu^2\right)$$

When $M_Z^2(\text{pred})/M_Z^2(\text{exp}) = 1$, the whole system is *solved*.

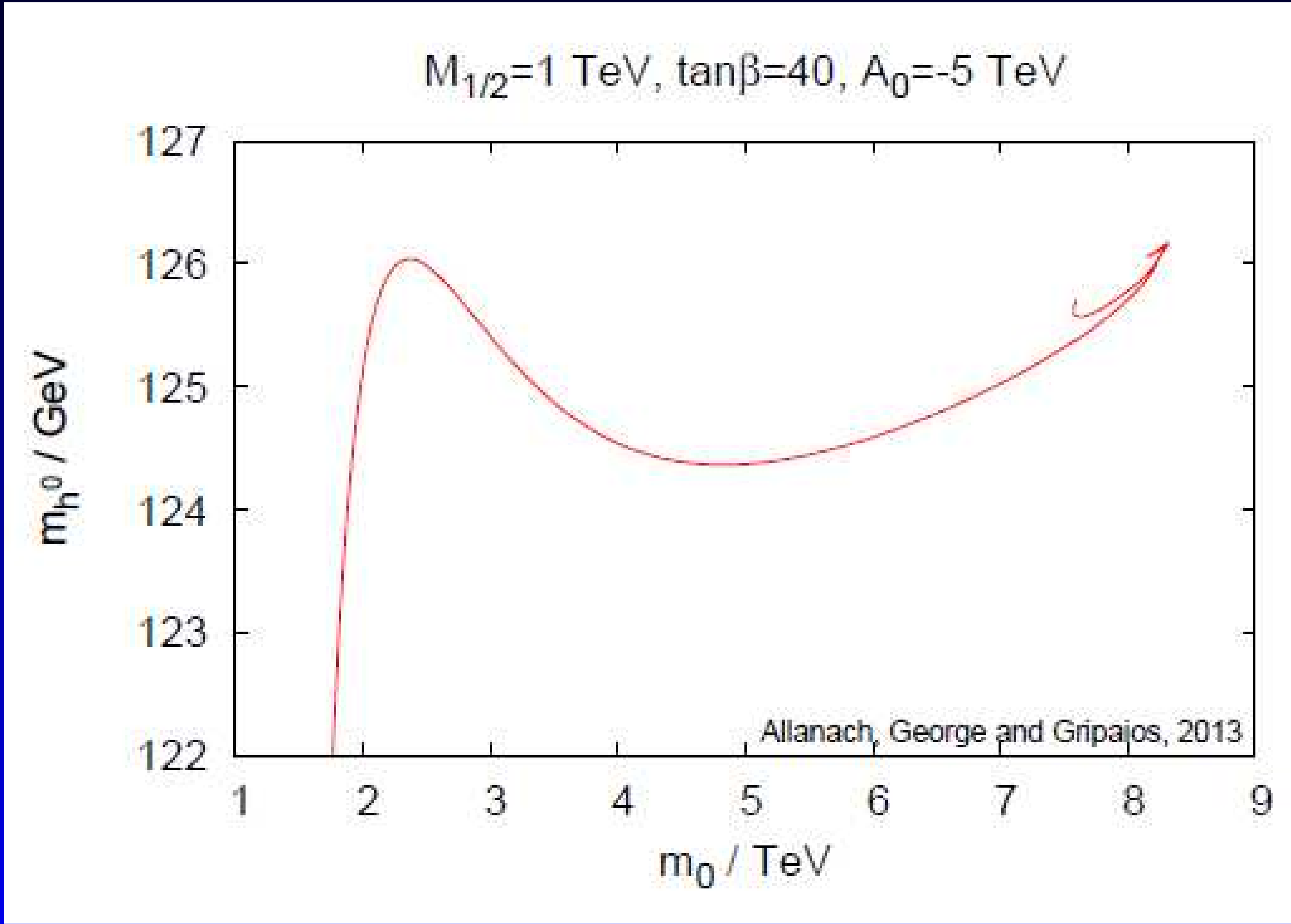
# Example point



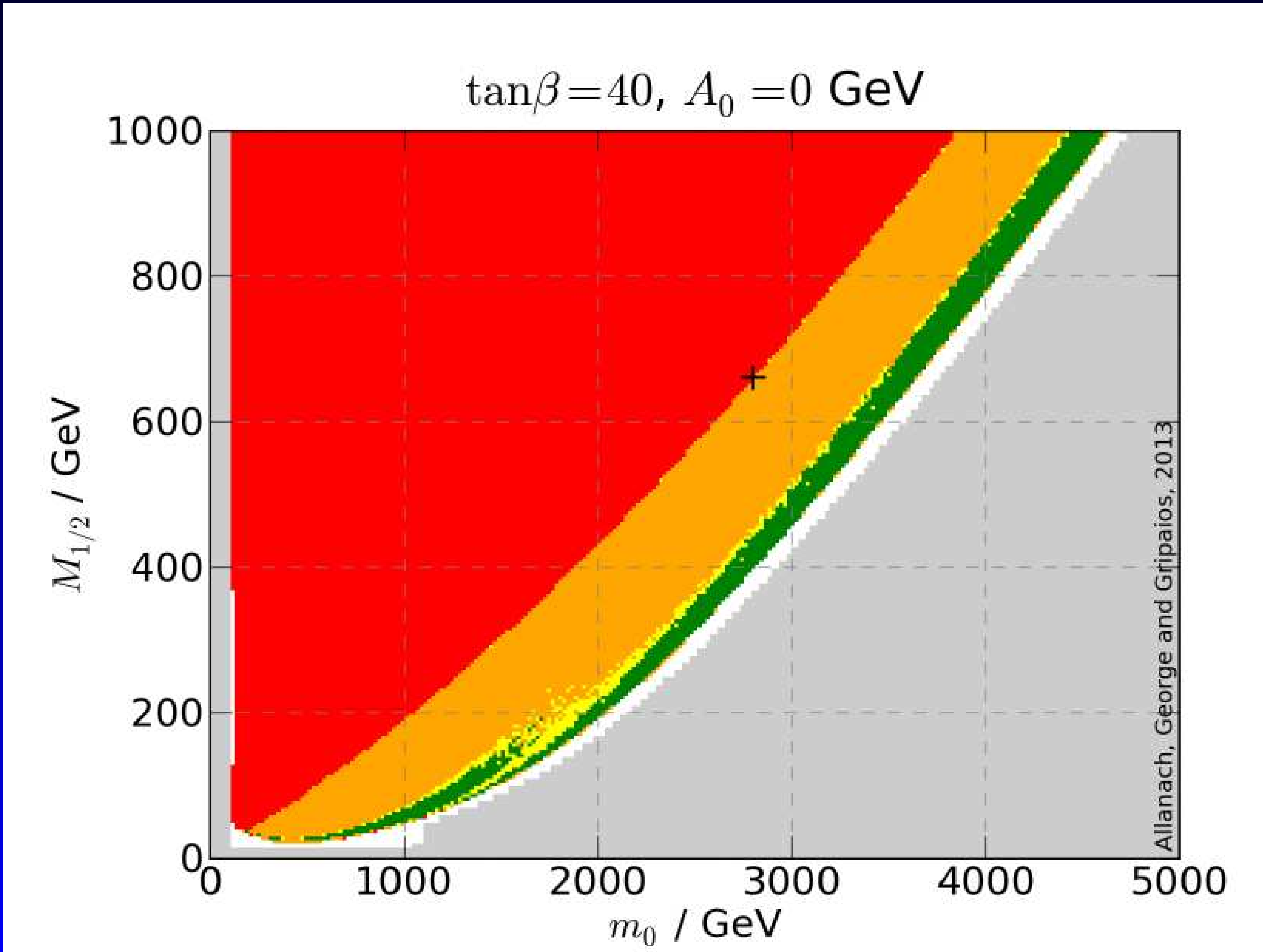
# Properties

| quantity | solution A | solution B | solution C |
|---|---|---|---|
| $\mu(M_{\rm SUSY})/\rm GeV$ | -545 | -535 | 497 |
| $M_{\chi_1^0}/\rm GeV$ | 282 | 282 | 281 |
| $M_{\chi_2^0}/\rm GeV$ | 502 | 497 | 471 |
| $M_{\chi_3^0}/\rm GeV$ | 558 | 548 | 510 |
| $M_{\chi_4^0}/\rm GeV$ | 610 | 605 | 593 |
| $M_{\chi_1^\pm}/\rm GeV$ | 503 | 497 | 470 |
| $M_{\chi_2^\pm}/\rm GeV$ | 609 | 604 | 592 |
| $m_{\tilde g}/\rm GeV$ | 1612 | 1612 | 1612 |
| $m_3^2(M_{\rm SUSY})/10^5\ \rm GeV^2$ | 0.800 | 0.809 | 1.07 |
| $m_{H_2}^2(M_{\rm SUSY})/10^5\ \rm GeV^2$ | -1.94 | -1.83 | -1.42 |
| $h_t(M_{\rm SUSY})$ | 0.840 | 0.839 | 0.836 |
| $A_t(M_{\rm SUSY})/\rm GeV$ | -1056 | -1057 | -1064 |
| $M_X/10^{16}\ \rm GeV$ | 1.94 | 1.93 | 1.89 |
| $g_1(M_Z)$ | 0.460 | 0.470 | 0.456 |
| $g_2(M_Z)$ | 0.634 | 0.640 | 0.633 |

# Example Higgs Mass Predictions



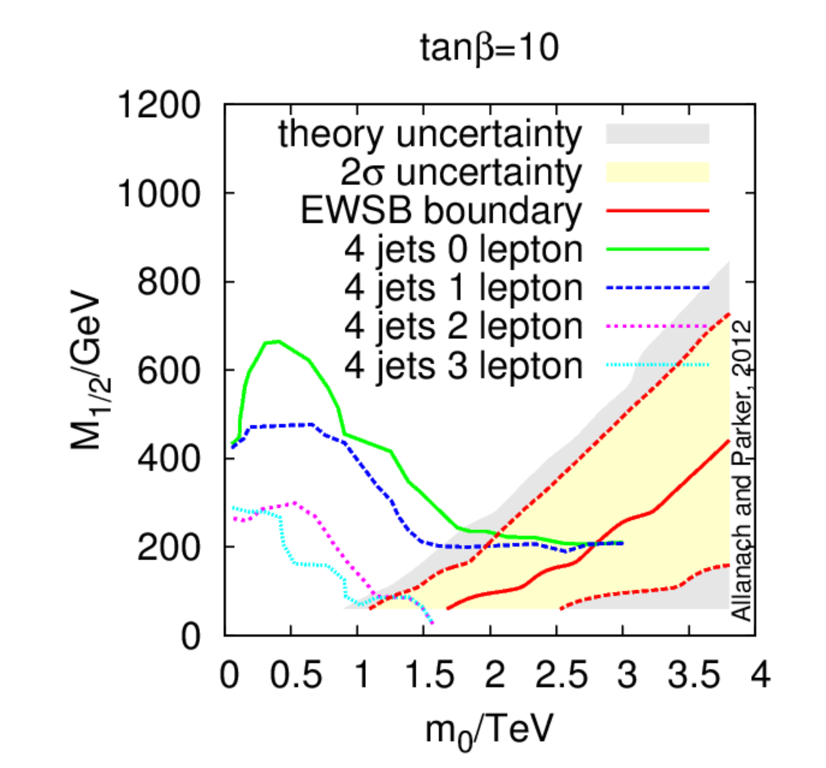
# Number of Solutions

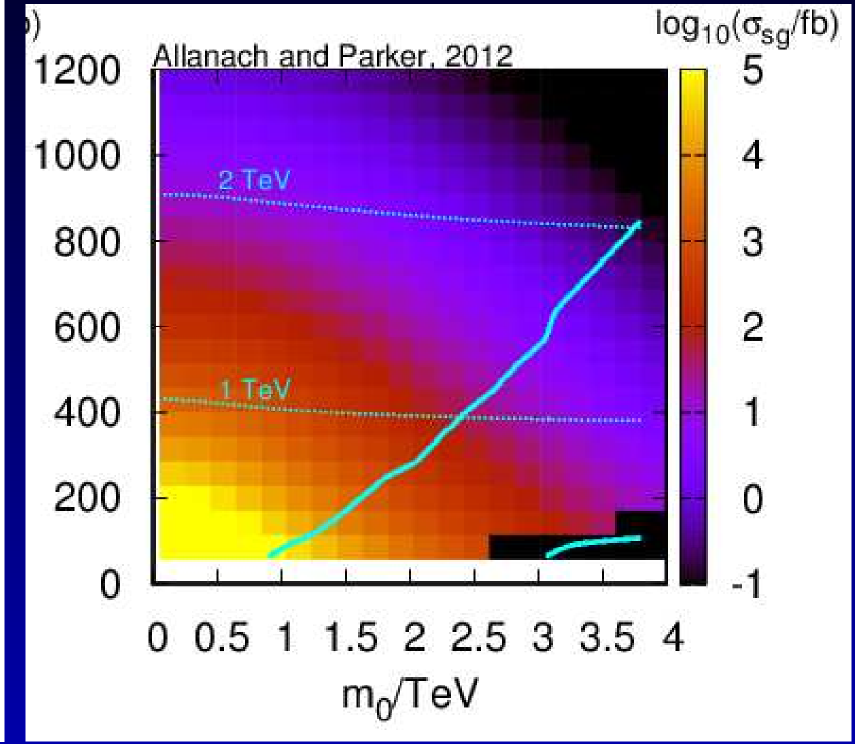





Red=1, orange=2, yellow=3 and green=4

# EWSB in the CMSSM at 14 TeV, 1 fb$^{-1}$, 5 $\sigma$




The position of the EWSB boundary is extremely uncertain due to the error in $m_t = 172.5 \pm 1$ GeV. *Experiments should use the higher value (plus theory error) when making grids of MSSM spectra*[a]

[a] BCA, Parker `1211.3231`

# Full Dimensionality

We[a] are currently investigating the phenomenology of the additional solutions. Also, we are performing a purely *top-down* analysis in order to find potential additional solutions which are unstable to fixed-point iteration.

The input parameters would be:

$V_1 = \tan\beta(M_{GUT})$, $V_2 = \ln M_{GUT}$, $V_3 = g_1(M_{GUT})$, $V_4 = g_3(M_{GUT})$, $V_5 = \mu(M_{GUT})$, $V_6 = m_3^2(M_{GUT})$, $V_7 = Y_t(M_{GUT})$, $V_8 = Y_t(M_{GUT})$, $V_9 = Y_t(M_{GUT})$, $V_{10} = v(M_{GUT})$ and $V_{11} = M_{SUSY}$.

For a given CMSSM point, these 11 parameters should be varied to solve the 11 boundary conditions in as many different ways as possible.

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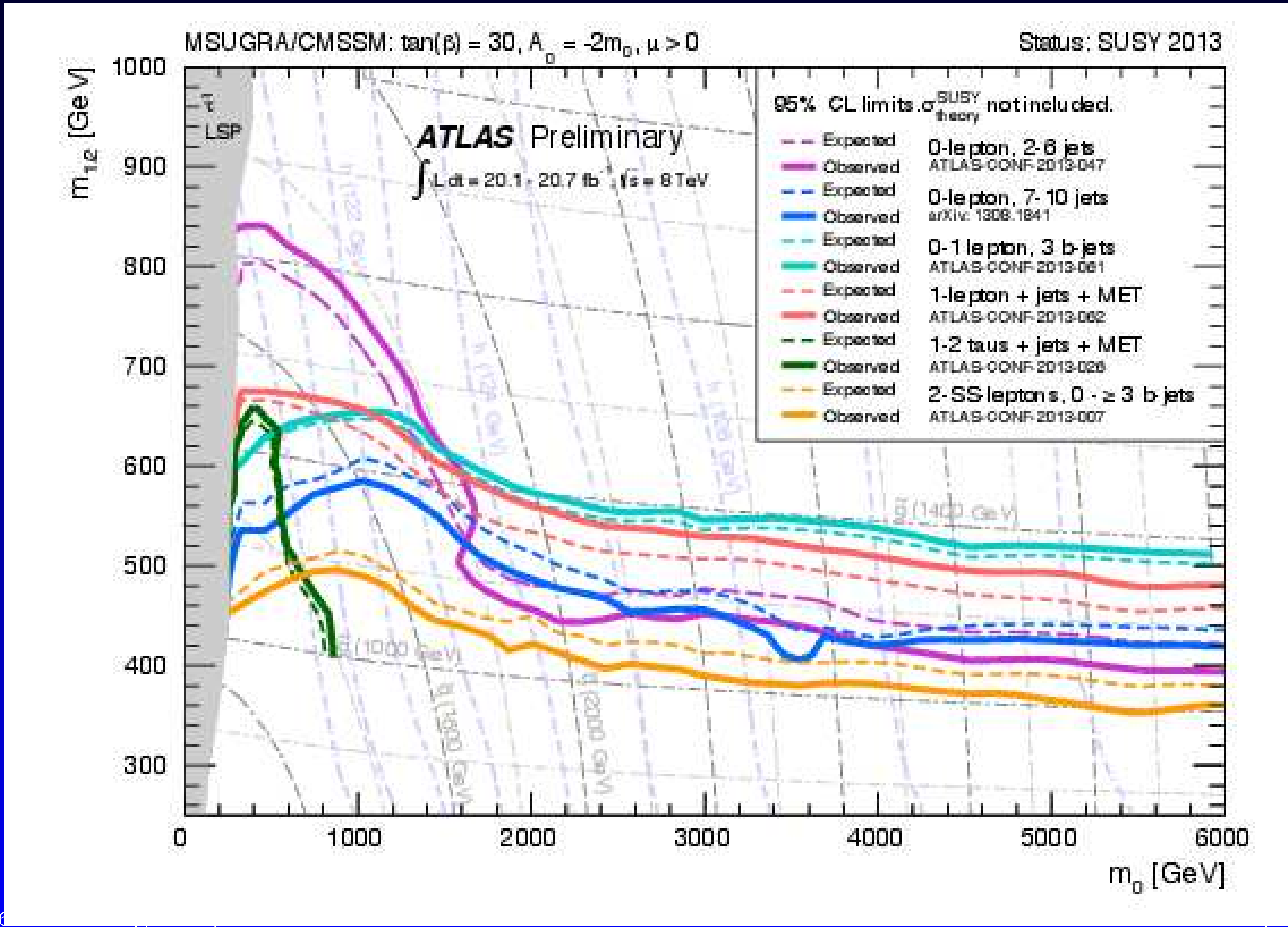
[a] BCA, D George, B Nachmann, work in progress

# CMSSM LHC Limits



# Summary

- CMSSM is just one example to illustrate a more general point
- Multiple solutions to RGEs and boundary conditions can exist for theories (even with linear boundary conditions), where there is more than one parameter, and more than one scale at which boundary conditions are set
- Need to scan over $m_t$ when doing CMSSM searches in order to take uncertainties in position of EWSB boundary into account
- Now investigating any additional solutions in a fully multi-dimensional sense
- You can never be sure that you have found *all* extra solutions in numerical scans of this type

# Supplementary Slides

# Multi-Dimensional Stability

$$\mathbf{x} = \mathbf{f}(\mathbf{x}). \tag{3.5}$$

Proceeding similarly as we did in the one-dimensional case, we solve this system with successive approxiations $\mathbf{x_n}$, where

$$\mathbf{x}_{n+1} = \mathbf{f}(\mathbf{x}_n) \Leftrightarrow \mathbf{x} - (x - \mathbf{x}_{n+1}) = \mathbf{f}(\mathbf{x} - (\mathbf{x} - \mathbf{x}_n)). \tag{3.6}$$

Taylor expanding the right hand side around $\mathbf{x} - \mathbf{x}_n = \mathbf{0}$, we have, to first order in $(\mathbf{x} - \mathbf{x}_n)$,

$$\cancel{x} - (\mathbf{x} - \mathbf{x}_{n+1}) = \cancel{\mathbf{f}(x)} - [(\mathbf{x} - \mathbf{x}_n) \cdot \nabla]\, \mathbf{f}(\mathbf{x}) \Leftrightarrow (\mathbf{x} - \mathbf{x}_{n+1}) = (\mathbf{x} - \mathbf{x}_n)\mathbf{D}, \tag{3.7}$$

where $\mathbf{D}$ is the *Jacobian matrix* of $\mathbf{f}$. Defining the *matrix norm* $||\mathbf{D}||$ of $\mathbf{D}$,

$$||\mathbf{D}|| \equiv \max_{\mathbf{x}:\ ||\mathbf{x}||=\mathbf{1}} \{\ ||\mathbf{D}\mathbf{x}||\ \} = \max_{\mathbf{x}:\ \mathbf{x} \neq \mathbf{0}} \left\{ \frac{||\mathbf{D}\mathbf{x}||}{||\mathbf{x}||} \right\}$$
$$\Rightarrow ||(\mathbf{x} - \mathbf{x}_n)\mathbf{D}|| \leq ||(\mathbf{x} - \mathbf{x}_n)||\ ||\mathbf{D}||. \tag{3.8}$$

Then Eq. 3.7 yields

$$||\mathbf{D}|| < 1 \Rightarrow ||\mathbf{x} - \mathbf{x}_{n+1}|| < ||\mathbf{x} - \mathbf{x}_n|| \tag{3.9}$$

is a stability condition. Conversely, from Eq. 3.7, we have

$$\min_{\mathbf{x}:||\mathbf{x}=\mathbf{1}||} \{||\mathbf{D}\mathbf{x}||\} > 1 \Rightarrow ||\mathbf{x} - \mathbf{x_{n+1}}|| > ||\mathbf{x} - \mathbf{x_n}||. \tag{3.10}$$