Higgs production in gluon fusion close to threshold

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Physics Challenges in the face of LHC-14
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The gluon fusion cross section

- If we want to reduce the theory uncertainty on the Higgs cross section at the LHC-14, we need to compute the gluon-fusion cross section to N3LO in QCD.
  - State-of-the-art already reviewed in Anastasiou’s talk this morning.
- This talk:
  - Provide some details on recent computation of N3LO cross section at threshold.
  - Give outlook/perspectives results away from threshold.
- N3LO is uncharted territory, with new challenges!
At N3LO, there are five contributions:

- Triple virtual
- Real-virtual squared
- Double virtual real
- Double real virtual
- Triple real
Higgs production at threshold

- First milestone recently achieved! The soft-virtual term describing Higgs production at threshold at N3LO:
  \[ \hat{\sigma}(z) = \sigma_0 + (1 - z) \sigma_1 + \mathcal{O}(1 - z)^2 \quad z = \frac{m^2}{\hat{s}} \]

- The soft-virtual term receives contributions from a ‘pole’ at \( z \sim 1 \):
  \[ (1 - z)^{-1 + n\epsilon} = \frac{\delta(1 - z)}{n \epsilon} + \left[ \frac{1}{1 - z} \right]_+ + n\epsilon \left[ \frac{\log(1 - z)}{1 - z} \right]_+ + \mathcal{O}(\epsilon^2) \]

- The N3LO soft-virtual term includes:
  - The full three-loop corrections to gluon fusion.
  - The real corrections from the emission of soft gluons.
  - Only the gluon channel contributes.
The soft-virtual approximation

- There is a consistent way to compute the soft-virtual contribution to the cross section:
  - Expand the integrals in soft momenta.
  - All final-state momenta are soft.
  - Loop momenta are either soft or hard.
  - The expanded objects can be interpreted as loop diagrams themselves!

- N.B.: The plus-distribution terms were computed already some years ago by Moch and Vogt from splitting functions.
  - Did not include three-loop corrections.
The soft-virtual approximation

- All the integrals can be computed analytically!
  - 22 three-loop. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - 3 double-virtual-real. [CD, Gehrmann; Li, Zhu]
  - 7 real-virtual-squared. [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore]
  - 10 double-real-virtual. [Anastasiou, CD, Dulat, Furlan, Herzog, Mistlberger]
  - 8 triple real. [Anastasiou, CD, Dulat, Mistlberger]

- In addition, one needs:
  - three-loop splitting functions. [Moch, Vogt, Vermaseren]
  - three-loop beta function. [Tarasov, Vladimirov, Zharkov; Larin, Vermaseren]
  - three-loop Wilson coefficient. [Chetyrkin, Kniehl, Steinhauser; Schroder, Steinhauser; Chetyrkin, Kuhn, Sturm]
The integrals

- Every integral is individually divergent, and gives rise to poles in dimensional regularisation.
- Many integrals are trivial to compute:
The integrals

- Other integrals are … ‘less trivial’…

\[
\mathcal{I}_{9,1}(\epsilon) = -\int_{0}^{\infty} dt_1 \, dt_2 \int_{0}^{1} dx_1 \, dx_2 \, dx_3 \, t_1^{2-4\epsilon} (1 + t_1)^{\epsilon-1} t_2^{1-2\epsilon} \\
\times x_1^{-\epsilon} (1 - x_1)^{2-4\epsilon} x_2^{1-3\epsilon} (1 - x_2)^{-\epsilon} x_3^{-\epsilon} (1 + t_2 x_3)^{1-3\epsilon} (1 + t_2 x_2 x_3)^{\epsilon} \\
\times \left( t_1 t_2 x_1 x_2 x_3 + t_2^2 x_2 x_3 + t_1 t_2 x_1 x_2 + t_1 t_2 x_3 + t_2 x_2 x_3 + t_2 + t_1 + 1 \right)^{3\epsilon-3},
\]

\[
\mathcal{I}_{9,2}(\epsilon) = \int_{0}^{\infty} dt_1 \, dt_2 \int_{0}^{1} dx_1 \, dx_2 \, dx_3 \, t_1^{2-4\epsilon} (1 + t_1)^{\epsilon-1} t_2^{1-2\epsilon} \\
\times x_1^{1-\epsilon} (1 - x_1)^{2-4\epsilon} x_2^{1-3\epsilon} (1 - x_2)^{-\epsilon} x_3^{-\epsilon} (1 + t_2 x_3)^{1-3\epsilon} (1 + t_2 x_2 x_3)^{\epsilon} \\
\times \left( t_1 t_2 x_1 x_2 x_3 + t_2^2 x_1 x_2 x_3 + t_2 x_1 + t_1 t_2 x_1 x_2 + t_1 t_2 x_3 + t_2 x_1 x_2 x_3 + t_1 + x_1 \right)^{3\epsilon-3},
\]
The integrals

- Every integral is individually divergent, and gives rise to poles in dimensional regularisation.
- Many integrals are trivial to compute:

\[
\begin{align*}
&= \frac{\Gamma(4 - 4\epsilon)\Gamma(2 - 3\epsilon)}{(1 - 2\epsilon)^2\epsilon\Gamma(4 - 6\epsilon)\Gamma(1 - \epsilon)} \\
&= \frac{1}{\epsilon} + \frac{14}{3} + (24 - 6\zeta_2) \epsilon + \left(-28\zeta_2 - 42\zeta_3 + \frac{400}{3}\right) \epsilon^2 + (-144\zeta_2 - 196\zeta_3 \\
&\quad - 195\zeta_4 + \frac{2320}{3}\right) \epsilon^3 + (252\zeta_3\zeta_2 - 800\zeta_2 - 1008\zeta_3 - 910\zeta_4 - 1302\zeta_5 + 4576) \epsilon^4 \\
&\quad + \left(882\zeta_3^2 + 1176\zeta_2\zeta_3 - 5600\zeta_3 - 4640\zeta_2 - 4680\zeta_4 - 6076\zeta_5 - \frac{9219\zeta_6}{2}\right) \epsilon^5 + \mathcal{O}(\epsilon^6),
\end{align*}
\]
There are criteria from number theory that allow to decide when integrals can be evaluated in the ‘naive way’ by doing one integration at the time. [Brown]

Basic idea: find a sufficient condition such that we can integrate over each variable using the basic definition of multiple polylogarithms:

\[ G(a_1, \ldots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \ldots, a_n; t) \]

Moreover, number theory also tells you how to do this in an algorithmic way!

- Can simply integrate out one variable at a time.
At the end of this procedure, one finds

\[
\begin{align*}
&= \frac{160}{\epsilon^5} - \frac{1712}{\epsilon^4} + \frac{1}{\epsilon^3} \left( -120 \zeta_2 + 2784 \right) + \frac{1}{\epsilon^2} \left( -120 \zeta_3 + 1284 \zeta_2 + 31968 \right) \\
&\quad + \frac{1}{\epsilon} \left( 2520 \zeta_4 + 1284 \zeta_3 - 2088 \zeta_2 - 216864 \right) + 15720 \zeta_5 + 1920 \zeta_2 \zeta_3 \\
&\quad - 26964 \zeta_4 - 2088 \zeta_3 - 23976 \zeta_2 + 795744 + \epsilon \left( 82520 \zeta_6 + 9600 \zeta_3^2 \right) \\
&\quad - 168204 \zeta_5 - 20544 \zeta_2 \zeta_3 + 43848 \zeta_4 - 23976 \zeta_3 + 162648 \zeta_2 - 2449440 \right) \\
&\quad + \mathcal{O}(\epsilon^2).
\end{align*}
\]

**Upshot:** we can compute all integrals we need!

\[\Rightarrow\] Computation would have been impossible with the insight from modern number theory!

Putting all the bits and pieces together, we see that all the poles cancel, and we get the final result.
Higgs soft-virtual @ N3LO

\[
\hat{n}^{(3)}(z) = \delta(1-z) \left\{ C_A^3 \left( -\frac{2003}{48} \zeta_6 + \frac{413}{6} \zeta_2^2 - \frac{7579}{144} \zeta_5 + \frac{979}{24} \zeta_2 \zeta_3 - \frac{15257}{864} \zeta_4 - \frac{819}{16} \zeta_3 + \frac{16151}{1296} \zeta_2 + \frac{215131}{5184} \right) + N_F \left[ C_A^2 \left( \frac{869}{72} \zeta_5 - \frac{125}{12} \zeta_3 \zeta_2 + \frac{2629}{432} \zeta_4 + \frac{1231}{216} \zeta_3 - \frac{70}{81} \zeta_2 - \frac{98059}{5184} \right) + C_A C_F \left( \frac{5}{2} \zeta_5 + 3 \zeta_3 \zeta_2 + \frac{11}{12} \zeta_4 + \frac{13}{2} \zeta_3 - \frac{71}{36} \zeta_2 - \frac{63991}{5184} \right) + C_F^2 \left( -5 \zeta_5 + \frac{37}{12} \zeta_3 + \frac{19}{18} \right) \right] + N_F^2 \left[ C_A \left( -\frac{19}{36} \zeta_4 + \frac{43}{108} \zeta_3 - \frac{133}{324} \zeta_2 + \frac{2515}{1728} \right) + C_F \left( -\frac{1}{36} \zeta_4 - \frac{7}{6} \zeta_3 - \frac{23}{72} \zeta_2 + \frac{4481}{2592} \right) \right] \right\}
\]

an [Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]
Going beyond threshold

• Can we do better..?

• Computing the full result requires the computation of probably 1000’s of integrals…
  ➡ NNLO: 28 integrals.
  ➡ Huge jump in complexity!

• Two possible approaches:
  ➡ Compute all contributions exactly.
  ➡ Compute more terms in the threshold expansion.

• In the last couple of minutes: review/outlook of where we stand.
Single-emission contributions

[Anastasiou, CD, Dulat, Herzog, Mistlberger]  [Dulat, Mistlberger; CD, Gehrmann]

• These contributions can easily be calculated exactly!
  ➡ Two-loop matrix elements are known. [Gehrmann, Glover, Jaquier, Koukoutsakis]

• We have computed these contributions in several ways:
  ➡ Perform phase-space integration over special functions appearing in loops, after subtracting all collinear and soft singularities.
  ➡ IBP reduction followed by differential equations for master integrals.
Towards next-to-soft

- The next-to-soft corrections to the triple-real contribution are already known. 
  - Sufficient to expand one order higher in soft momenta.
  - 2 new integrals appear.

- For double-emission at one-loop a new complication arises:
  - Receives contributions from regions where the virtual gluon is collinear to an incoming parton.
  - Needs some rethinking of the technology.

[Anastasiou, CD, Dulat, Mistlberger]
Conclusion

- Computing the gluon-fusion cross section at N3LO is challenging!
  - 1000’s of very complicated integrals.
  - Threshold contribution achieved, more to follow!
- Excellent laboratory to explore new ideas and techniques for multi-loop computations!
  - Expansion by regions, new methods from number theory to do loop integrals, etc.
- We are slowly getting there!
  - Stay tuned!