Higgs production in gluon fusion close to threshold

Claude Duhr

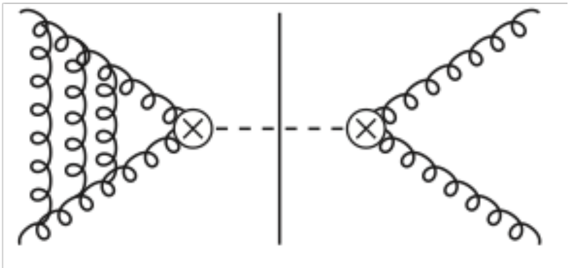
Physics Challenges in the face of LHC-14
Madrid, 18 September 2014

The gluon fusion cross section

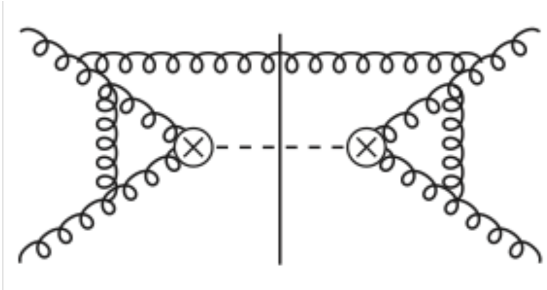
- If we want to reduce the theory uncertainty on the Higgs cross section at the LHC-14, we need to compute the gluon-fusion cross section to N3LO in QCD.
  - ➡ State-of-the-art already reviewed in Anastasiou's talk this morning.
- This talk:
  - ➡ Provide some details on recent computation of N3LO cross section at threshold.
  - ➡ Give outlook/perspectives results away from threshold.
- N3LO is uncharted territory, with new challenges!

The gluon fusion cross section

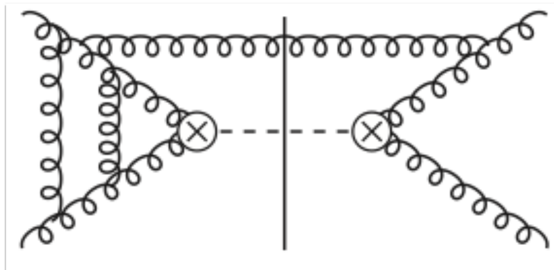
- At N3LO, there are five contributions:



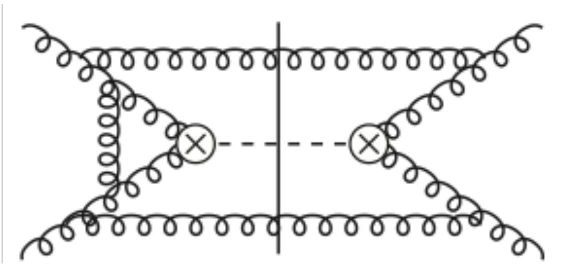
Triple virtual



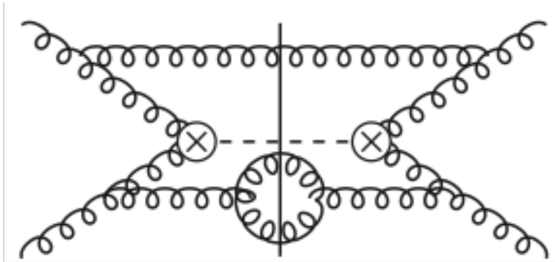
Real-virtual squared



Double virtual real



Double real virtual



Triple real

Higgs production at threshold

- First milestone recently achieved! The soft-virtual term describing Higgs production at threshold at N3LO:

$$\hat{\sigma}(z) = \boxed{\sigma_{-1}} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2 \qquad z = \frac{m^2}{\hat{s}}$$

- The soft-virtual term receives contributions from a 'pole' at $z \sim 1$:

$$(1-z)^{-1+n\epsilon} = \frac{\delta(1-z)}{n\,\epsilon} + \left[\frac{1}{1-z}\right]_+ + n\epsilon\left[\frac{\log(1-z)}{1-z}\right]_+ + \mathcal{O}(\epsilon^2)$$

- The N3LO soft-virtual term includes:
  - ➡ The full three-loop corrections to gluon fusion.
  - ➡ The real corrections from the emission of soft gluons.
  - ➡ Only the gluon channel contributes.

The soft-virtual approximation

- There is a consistent way to compute the soft-virtual contribution to the cross section:
  - ➡ Expand the integrals in soft momenta.
  - ➡ All final-state momenta are soft.
  - ➡ Loop momenta are either soft of hard.
  - ➡ The expanded objects can be interpreted as loop diagrams themselves!

- N.B.: The plus-distribution terms were computed already some years ago by Moch and Vogt from splitting functions.

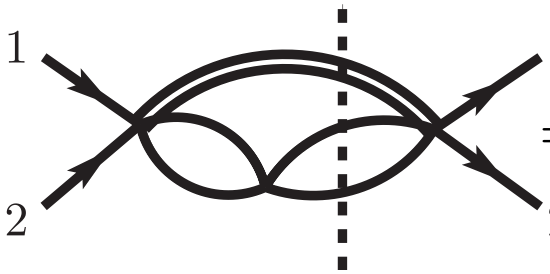
  ➡ Did not include three-loop corrections.

The soft-virtual approximation

- All the integrals can be computed analytically!
  - ➡ 22 three-loop. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - ➡ 3 double-virtual-real. [CD, Gehrmann; Li, Zhu]
  - ➡ 7 real-virtual-squared. [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore]
  - ➡ 10 double-real-virtual. [Anastasiou, CD, Dulat, Furlan, Herzog, Mistlberger]
  - ➡ 8 triple real. [Anastasiou, CD, Dulat, Mistlberger]
- In addition, one needs:
  - ➡ three-loop splitting functions. [Moch, Vogt, Vermaseren]
  - ➡ three-loop beta function. [Tarasov, Vladimirov, Zharkov; Larin, Vermaseren]
  - ➡ three-loop Wilson coefficient. [Chetyrkin, Kniehl, Steinhauser; Schroder, Steinhauser; Chetyrkin, Kuhn, Sturm]

The integrals

- Every integral is individually divergent, and gives rise to poles in dimensional regularisation.
- Many integrals are trivial to compute:

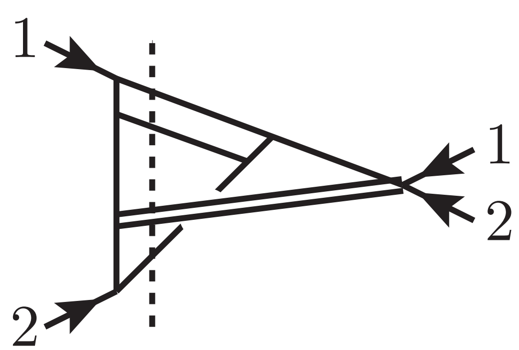


$$= \frac{\Gamma(4-4\epsilon)\Gamma(2-3\epsilon)}{(1-2\epsilon)^2\epsilon\Gamma(4-6\epsilon)\Gamma(1-\epsilon)}$$

$$= \frac{1}{\epsilon} + \frac{14}{3} + (24-6\zeta_2)\,\epsilon + \left(-28\zeta_2 - 42\zeta_3 + \frac{400}{3}\right)\epsilon^2 + (-144\zeta_2 - 196\zeta_3$$
$$-195\zeta_4 + \frac{2320}{3}\bigg)\,\epsilon^3 + (252\zeta_3\zeta_2 - 800\zeta_2 - 1008\zeta_3 - 910\zeta_4 - 1302\zeta_5 + 4576)\,\epsilon^4$$
$$+\bigg(882\zeta_3^2 + 1176\zeta_2\zeta_3 - 5600\zeta_3 - 4640\zeta_2 - 4680\zeta_4 - 6076\zeta_5 - \frac{9219\zeta_6}{2}$$
$$+\frac{81920}{3}\bigg)\,\epsilon^5 + \mathcal{O}(\epsilon^6)\,,$$

The integrals

- Other integrals are … 'less trivial'…



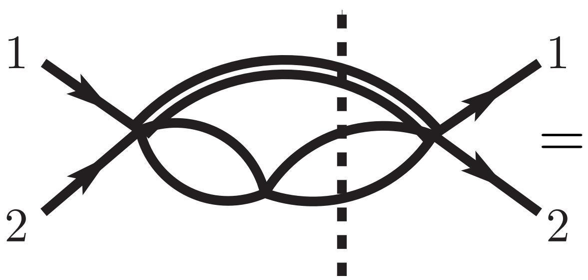
$$= \frac{\Gamma(12-6\epsilon)\Gamma(3-3\epsilon)\Gamma(1-\epsilon)}{\Gamma(5-6\epsilon)\Gamma(2-\epsilon)^4}\Big[\mathcal{I}_{9,1}(\epsilon)+\mathcal{I}_{9,2}(\epsilon)\Big]$$

$$\mathcal{I}_{9,1}(\epsilon) = -\int_0^\infty dt_1\, dt_2 \int_0^1 dx_1\, dx_2\, dx_3\, t_1^{2-4\epsilon}\,(1+t_1)^{\epsilon-1}\, t_2^{1-2\epsilon}$$
$$\times\, x_1^{-\epsilon}\,(1-x_1)^{2-4\epsilon}\, x_2^{1-3\epsilon}\,(1-x_2)^{-\epsilon}\, x_3^{-\epsilon}\,(1+t_2x_3)^{1-3\epsilon}\,(1+t_2x_2x_3)^{\epsilon}$$
$$\times\, \big(t_1t_2^2x_1x_2x_3 + t_2^2x_2x_3 + t_1t_2x_1x_2 + t_1t_2x_3 + t_2x_2x_3 + t_2 + t_1 + 1\big)^{3\epsilon-3}\,,$$

$$\mathcal{I}_{9,2}(\epsilon) = \int_0^\infty dt_1\, dt_2 \int_0^1 dx_1\, dx_2\, dx_3\, t_1^{2-4\epsilon}\,(1+t_1)^{\epsilon-1}\, t_2^{1-2\epsilon}$$
$$\times\, x_1^{1-\epsilon}\,(1-x_1)^{2-4\epsilon}\, x_2^{1-3\epsilon}\,(1-x_2)^{-\epsilon}\, x_3^{-\epsilon}\,(1+t_2x_3)^{1-3\epsilon}\,(1+t_2x_2x_3)^{\epsilon}$$
$$\times\, \big(t_1t_2^2x_1x_2x_3 + t_2^2x_1x_2x_3 + t_2x_1 + t_1t_2x_1x_2 + t_1t_2x_3 + t_2x_1x_2x_3 + t_1 + x_1\big)^{3\epsilon-3}\,,$$

The integrals

- Every integral is individually divergent, and gives rise to poles in dimensional regularisation.
- Many integrals are trivial to compute:



$$= \frac{\Gamma(4-4\epsilon)\Gamma(2-3\epsilon)}{(1-2\epsilon)^2\epsilon\Gamma(4-6\epsilon)\Gamma(1-\epsilon)}$$

$$= \frac{1}{\epsilon} + \frac{14}{3} + (24 - 6\zeta_2)\,\epsilon + \left(-28\zeta_2 - 42\zeta_3 + \frac{400}{3}\right)\epsilon^2 + (-144\zeta_2 - 196\zeta_3$$

$$-195\zeta_4 + \frac{2320}{3}\Bigg)\,\epsilon^3 + (252\zeta_3\zeta_2 - 800\zeta_2 - 1008\zeta_3 - 910\zeta_4 - 1302\zeta_5 + 4576)\,\epsilon^4$$

$$+ \left(882\zeta_3^2 + 1176\zeta_2\zeta_3 - 5600\zeta_3 - 4640\zeta_2 - 4680\zeta_4 - 6076\zeta_5 - \frac{9219\zeta_6}{2}\right.$$

$$\left. + \frac{81920}{3}\right)\epsilon^5 + \mathcal{O}(\epsilon^6)\,,$$

The integrals

- There are **criteria from number theory** that allow to decide when integrals can be evaluated in the 'naive way' by doing one integration at the time. [Brown]

- Basic idea: find a sufficient condition such that we can integrate over each variable using the basic definition of multiple polylogarithms:

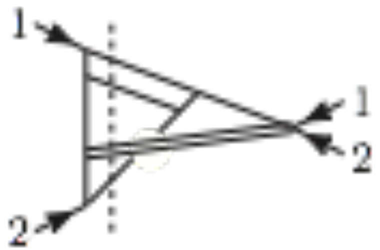
$$G(a_1,\ldots,a_n;z) = \int_0^z \frac{\mathrm{d}t}{t-a_1}\,G(a_2,\ldots,a_n;t)$$

- Moreover, number theory also tells you how to do this in an algorithmic way!

  ➡ Can simply integrate out one variable at a time.

The integrals

- At the end of this procedure, one finds



$$= \frac{160}{\epsilon^5} - \frac{1712}{\epsilon^4} + \frac{1}{\epsilon^3}\Big(- 120\,\zeta_2 + 2784\Big) + \frac{1}{\epsilon^2}\Big(- 120\,\zeta_3 + 1284\,\zeta_2 + 31968\Big)$$
$$+ \frac{1}{\epsilon}\Big(2520\,\zeta_4 + 1284\,\zeta_3 - 2088\,\zeta_2 - 216864\Big) + 15720\,\zeta_5 + 1920\,\zeta_2\,\zeta_3$$
$$- 26964\,\zeta_4 - 2088\,\zeta_3 - 23976\,\zeta_2 + 795744 + \epsilon\Big(82520\,\zeta_6 + 9600\,\zeta_3^2$$
$$- 168204\,\zeta_5 - 20544\,\zeta_2\,\zeta_3 + 43848\,\zeta_4 - 23976\,\zeta_3 + 162648\,\zeta_2 - 2449440\Big)$$
$$+ \mathcal{O}(\epsilon^2)\,.$$

- Upshot: we can compute all integrals we need!
  - ➡ Computation would have been impossible with the insight from modern number theory!

- Putting all the bits and pieces together, we see that all the poles cancel, and we get the final result.

Higgs soft-virtual @ N3LO

$$\hat{\eta}^{(3)}(z) = \delta(1-z)\Bigg\{C_A^3\left(-\frac{2003}{48}\zeta_6 + \frac{413}{6}\zeta_3^2 - \frac{7579}{144}\zeta_5 + \frac{979}{24}\zeta_2\zeta_3 - \frac{15257}{864}\zeta_4 - \frac{819}{16}\zeta_3 + \frac{16151}{1296}\zeta_2 + \frac{215131}{5184}\right)$$

$$+ N_F\Bigg[C_A^2\left(\frac{869}{72}\zeta_5 - \frac{125}{12}\zeta_3\zeta_2 + \frac{2629}{432}\zeta_4 + \frac{1231}{216}\zeta_3 - \frac{70}{81}\zeta_2 - \frac{98059}{5184}\right)$$

$$+ C_A C_F\left(\frac{5}{2}\zeta_5 + 3\zeta_3\zeta_2 + \frac{11}{72}\zeta_4 + \frac{13}{2}\zeta_3 - \frac{71}{36}\zeta_2 - \frac{63991}{5184}\right) + C_F^2\left(-5\zeta_5 + \frac{37}{12}\zeta_3 + \frac{19}{18}\right)\Bigg]$$

$$+ N_F^2\Bigg[C_A\left(-\frac{19}{36}\zeta_4 + \frac{43}{108}\zeta_3 - \frac{133}{324}\zeta_2 + \frac{2515}{1728}\right) + C_F\left(-\frac{1}{36}\zeta_4 - \frac{7}{6}\zeta_3 - \frac{23}{72}\zeta_2 + \frac{4481}{2592}\right)\Bigg]\Bigg\}$$

$$+ \left[\frac{1}{1-z}\right]_+ \Bigg\{C_A^3\left(186\zeta_5 - \frac{725}{6}\zeta_3\zeta_2 + \frac{253}{24}\zeta_4 + \frac{8941}{108}\zeta_3 + \frac{8563}{324}\zeta_2 - \frac{297029}{23328}\right) + N_F^2 C_A\left(\frac{5}{27}\zeta_3 + \frac{10}{27}\zeta_2 - \frac{58}{729}\right)$$

$$+ N_F\Bigg[C_A^2\left(-\frac{17}{12}\zeta_4 - \frac{475}{36}\zeta_3 - \frac{2173}{324}\zeta_2 + \frac{31313}{11664}\right) + C_A C_F\left(-\frac{1}{2}\zeta_4 - \frac{19}{18}\zeta_3 - \frac{1}{2}\zeta_2 + \frac{1711}{864}\right)\Bigg]\Bigg\}$$

$$+ \left[\frac{\log(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\left(-77\zeta_4 - \frac{352}{3}\zeta_3 - \frac{152}{3}\zeta_2 + \frac{30569}{648}\right) + N_F^2 C_A\left(-\frac{4}{9}\zeta_2 + \frac{25}{81}\right)$$

$$+ N_F\Bigg[C_A^2\left(\frac{46}{3}\zeta_3 + \frac{94}{9}\zeta_2 - \frac{4211}{324}\right) + C_A C_F\left(6\zeta_3 - \frac{63}{8}\right)\Bigg]\Bigg\}$$

$$+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\left(181\zeta_3 + \frac{187}{3}\zeta_2 - \frac{1051}{27}\right) + N_F\Bigg[C_A^2\left(-\frac{34}{3}\zeta_2 + \frac{457}{54}\right) + \frac{1}{2}C_A C_F\Bigg] - \frac{10}{27}N_F^2 C_A\Bigg\}$$

$$+ \left[\frac{\log^3(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\left(-56\zeta_2 + \frac{925}{27}\right) - \frac{164}{27}N_F C_A^2 + \frac{4}{27}N_F^2 C_A\Bigg\}$$

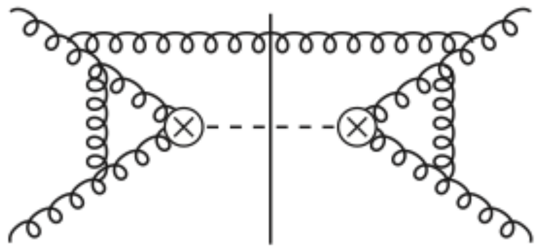
$$+ \left[\frac{\log^4(1-z)}{1-z}\right]_+ \left(\frac{20}{9}N_F C_A^2 - \frac{110}{9}C_A^3\right) + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 8C_A^3 .$$

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

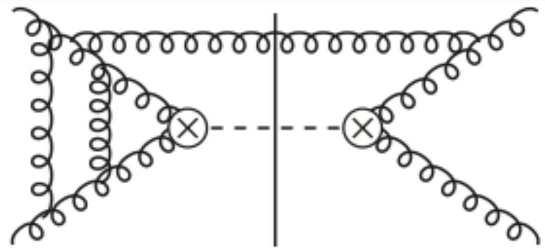
Going beyond threshold

- Can we do better..?
- Computing the full result requires the computation of probably 1000's of integrals…
  - ➡ NNLO: 28 integrals.
  - ➡ Huge jump in complexity!
- Two possible approaches:
  - ➡ Compute all contributions exactly.
  - ➡ Compute more terms in the threshold expansion.
- In the last couple of minutes: review/outlook of where we stand.

Single-emission contributions



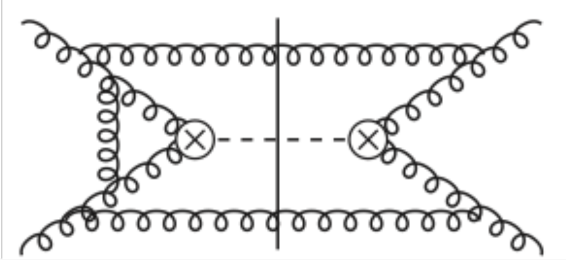
[Anastasiou, CD, Dulat, Herzog, Mistlberger]

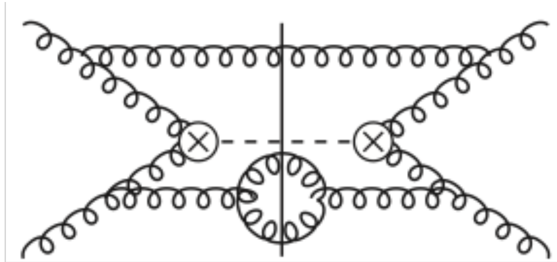


[Dulat, Mistlberger; CD, Gehrmann]

- These contributions can easily be calculated exactly!
  - ➡ Two-loop matrix elements are known. [Gehrmann, Glover, Jaquier, Koukoutsakis]
- We have computed these contributions in several ways:
  - ➡ Perform phase-space integration over special functions appearing in loops, after subtracting all collinear and soft singularities.
  - ➡ IBP reduction followed by differential equations for master integrals.

Towards next-to-soft





- The next-to-soft corrections to the triple-real contribution are already known. [Anastasiou, CD, Dulat, Mistlberger]
  - ➡ Sufficient to expand one order higher in soft momenta.
  - ➡ 2 new integrals appear.
- For double-emission at one-loop a new complication arises:
  - ➡ Receives contributions from regions where the virtual gluon is collinear to an incoming parton.
  - ➡ Needs some rethinking of the technology.

Conclusion

- Computing the gluon-fusion cross section at N3LO is challenging!
  - ➡ 1000's of very complicated integrals.
  - ➡ Threshold contribution achieved, more to follow!
- Excellent laboratory to explore new ideas and techniques for multi-loop computations!
  - ➡ Expansion by regions, new methods from number theory to do loop integrals, etc.
- We are slowly getting there!
  - ➡ Stay tuned!