SUSY naturalness without prejudice

D. Ghilencea*

"Physics Challenges in the face of LHC-14".

17 Sep 2014 - IFT, Madrid.

---

arXiv:1311.6144, NPB 876(2013), NPB 870(2013).

* **sponsored by UEFISCDI PN-II-ID-PCE-2011-3-0607, contract 118/2011.**

- Purpose of the talk:

1). - do NOT assume a criterion of naturalness or a fine-tuning measure (many, controversial, etc)

2). - derive mathematical support and eventually identify these from more general principles. How?

3). - Recall the essence of naturalness that motivated SUSY:

"Fixing" the EW scale ($v \sim m_Z$), at quantum level, to the measured value.

We do this because, unlike SM, in TeV-scale SUSY the EW scale is a prediction.

(I). Naturalness - review:

L. Susskind PRD 20 (1979), 2619; K. Wilson.

• why EW scale $v \sim M_Z \ll M_P$ stable under quantum corrections?, $\left[\frac{G_f h^2}{G_N c^2} = 1.7 \times 10^{33}\right]$

$\delta v^2 \sim \delta m_h^2 \sim f(\alpha_j)\,\Lambda^2$, Λ: scale of new physics; if $\Lambda \sim M_P$: "tune" couplings: $1:10^{33}\,(!)$

Hierarchy Problem $\Leftrightarrow$ "Fine tuning". Unnatural. Ways out: symmetries:

"Naturalness dogma": 't Hooft (1979)

- Then consider:

1. scale (conformal) symmetry....

see for example Bardeen 1995;

2. SUSY: $\delta m_h^2 \sim m_S^2 \ln \Lambda/m_S$, $m_S \sim$TeV... no SUSY seen, $m_S \gg \mathrm{TeV} \rightarrow$ back to SM fine-tuning

.....worse fine tunings: cosmological const. $\left[\frac{\rho_v}{\rho} \approx \frac{(2.3\times 10^{-12}\mathrm{GeV})^4}{(10^{19}\,\mathrm{GeV})^4}\right]$

.....but softly broken TeV-scale SUSY solves hierarchy problem

3. EFT approach, enforce consistency at every loop order.

Note: extra dimensions do not fix this problems...$m_h^2(q^2) \sim y^2/R^2 + \mathcal{O}(q^4 R^2)$.

- General potential, SUSY models:

$$V = m_1^2\,|h_1|^2 + m_2^2\,|h_2|^2 - (B_0\,\mu_0\,h_1\cdot h_2 + h.c.) + \lambda_1\,|h_1|^4 + \lambda_2\,|h_2|^4 + \lambda_3\,|h_1|^2\,|h_2|^2$$
$$+\ \lambda_4\,|h_1\cdot h_2|^2 + \big[\,\lambda_5/2\,(h_1\cdot h_2)^2 + \lambda_6\,|h_1|^2\,(h_1\cdot h_2) + \lambda_7\,|h_2|^2\,(h_1\cdot h_2) + h.c.\big]$$

$$m^2 \equiv m_1^2\,\cos^2\beta + m_2^2\,\sin^2\beta - B_0\,\mu_0\,\sin 2\beta, \quad \mathrm{UV}: m_{1,2}^2 = m_0^2 + \mu_0^2$$
$$\lambda \equiv \lambda_1\,\cos^4\beta + \lambda_2\,\sin^4\beta + \lambda_{345}/4\,\sin^2 2\beta + \sin 2\beta\,\big(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta\big)$$

- The Problem: scales vs. couplings "tension" (or "fine-tuning", whatever its measure...):

$$v^2 = -m^2/\lambda,\ \ \text{with}\ \ v = \mathcal{O}(100\ \text{GeV}), \quad \lambda < 1, \qquad \textbf{but} \quad m_{1,2}, B_0\ \text{and}\ m \sim \mathcal{O}(1\,\text{TeV}).$$

- $m_h > m_Z \leftrightarrow$ large loop effects $\leftrightarrow$ large $m_{1,2}, B_0$...; also a problem of couplings (λ small)

$\Rightarrow$ Solution: - increase λ by 1.- quantum corrections.

2.- (susy) corrections from "new physics" beyond MSSM.

• A closer look: Lagrangian $\mathcal{L}$ of UV parameters $\gamma_i : m_0, \mu_0, A_0, B_0, m_{1/2}, \cdots$.

$$\frac{(g_1^2+g_2^2)\,v^2}{8} = -\mu^2 + \frac{m_1^2 - m_2^2 \tan^2\beta}{\tan^2\beta - 1} + \ldots,$$
$$2\,m_3^2 = (m_1^2 + m_2^2 + 2\mu^2)\,\sin 2\beta + \cdots,$$

so $v = v(\gamma, \beta)$, $\beta = \beta_0(\gamma) \Rightarrow \quad m_Z = m_Z(\gamma, \beta_0(\gamma))$. Taylor expansion

$$m_Z = m_Z^0 + \left(\frac{\partial m_Z}{\partial \gamma_i}\right)_{\gamma_i=\gamma_i^0} (\gamma_i - \gamma_i^0) + \cdots, \qquad m_Z^0 = 91.187\,\mathrm{GeV}$$

$$\Rightarrow \frac{\delta m_Z}{m_Z^0} = \Delta_q\, n^i\, \frac{\delta\gamma_i}{\gamma_i^0} + \mathcal{O}((\delta\gamma_i)^2), \qquad \vec{n} \text{ normal to } m_Z(\gamma_i^0, \beta_0(\gamma_i^0)) = m_Z^0.$$

with notation $\Delta_q \equiv \left\{ \sum_i \left(\frac{\partial \ln m_Z}{\partial \ln \gamma_i}\right)^2_{\gamma_i=\gamma_i^0} \right\}^{1/2}$

$$\frac{\delta m_Z}{m_Z^0} = 4.6 \times 10^{-5}\ (2\sigma), \text{if } \Delta_q \approx 1000 \rightarrow \frac{\delta\gamma_i}{\gamma_i^0} \approx 4.6\times 10^{-8} \rightarrow \gamma_i = 1\,\mathrm{TeV} \rightarrow \delta\gamma_i = 46\,\mathrm{keV}!$$

(II). Fixing EW scale (m_Z) and emergent fine-tuning.

D.G., Graham Ross, 2012, 2013.

- Max of likelihood to fit EW observables O_j (γ_i UV parameters), factorization:

$$L(\text{data}|\gamma_i; v, \beta) = \prod_{j\geq 1} L(O_j|\gamma_i; v, \beta), \qquad \gamma_i = m_0, m_{1/2}, A_0, B_0, \mu_0, ...(\text{susy}) \;\; (y_t, y_b, ...)$$

- Unlike SM, SUSY predicts $v = v(\gamma_i)$ & EWSB $\Rightarrow$ Regard m_Z as an observable, $m_Z^0 = 91.187$ GeV

$$f_1 \equiv v - (-m^2/\lambda)^{1/2} = 0, \qquad f_2 \equiv \tan\beta - \tan\beta_0(\gamma_i) = 0$$

$$L_{\text{total}}(\text{data}, m_Z|\gamma_i) = \int dv\; d(\tan\beta)\; \delta\left(f_1(\gamma_i; v, \beta)\right)\; \delta\left(f_2(\gamma_i; v, \beta)\right)\; L(\text{data}|\gamma_i; v, \beta)\; \delta(1 - m_Z/m_Z^0)$$

$$= v_0 \left[L\big(\text{data}|\gamma_i;\, v_0,\, \beta\big)\; \delta\big[f_1\big(\gamma_i;\, v_0,\, \beta\big)\big]\right]_{\beta=\beta_0(\gamma_i)}$$

$$= \; L\big(\text{data}|\gamma_i;\, v_0,\, \beta_0(\gamma_i)\big)\; \delta\Big[1 - \frac{m_Z(\gamma_i, \beta_0(\gamma_i))}{m_Z^0}\Big],$$

$$v_0 = 246\ \text{GeV}$$
$$m_Z = gv/2$$
$$m_Z^0 = gv_0/2$$
$$g^2 = g_1^2 + g_2^2$$

D.G., Graham Ross.

All γ_i vary simultaneously:

$$\delta\Big[1-\frac{m_Z(\gamma_i,\beta_0(\gamma_i))}{m_Z^0}\Big]=\frac{1}{\Delta_q}\,\delta\Big[n^j\,(1-\frac{\gamma_j}{\gamma_j^0})\Big],\qquad \Delta_q\equiv\Big[\sum_i\Big(\frac{\partial\ln m_Z(\gamma_i,\beta_0(\gamma_i))}{\partial\ln\gamma_i}\Big)\Big]^{1/2}_{\gamma_i=\gamma_i^0},$$

$$\Rightarrow\quad L_{\rm total}({\rm data},m_Z|\gamma_i^0)=\frac{1}{\Delta_q}\,L({\rm data}|\gamma_i;v_0,\beta_0(\gamma_i))\Big|_{\gamma_i=\gamma_i^0}$$

$\Rightarrow$ Emergent Δ_q as part of total likelihood (not put in by hand!).

$\Rightarrow$ Δ_q: sole consequence of "fixing" EW scale $\Rightarrow$ it makes sense to call it "fine-tuning" from now on.

$\Rightarrow$ should maximize the ratio L/Δ_q.

This shows how to compare models A, B:

A. Good fit but large Δ_q and B: less-good fit but smaller fine tuning.

A. Casas et al 2008

B. Allanach et al 2007, 2009

D.G., H.M. Lee, M. Park 2012

$$\delta(f(\vec z))=\frac{1}{|\nabla_z f|_o}\,\delta\Big[\vec n.(\vec z-\vec z^0)\Big],\quad n_i=\frac{\partial_{z_i}f}{|\nabla f|_o}$$

With $\chi^2 = -2\ln L$: D.G., G. Ross, 2012, 2013.

$$\chi^2_{\text{total}}(\gamma_i) = \Big[\chi^2(\gamma_i) + \underbrace{2\ln\Delta_q(\gamma_i)}_{\chi^2_z} \Big]_{f_1=0, f_2=0}$$

- Good fit: χ^2_{total} /ndf $\approx 1.$ $\Rightarrow$ Naturalness bound: $\Delta_q < \exp(\text{ndf}/2) \sim 100.$

$\Rightarrow$ Good fit and fixing EW scale (m_Z) demands small Δ_q.

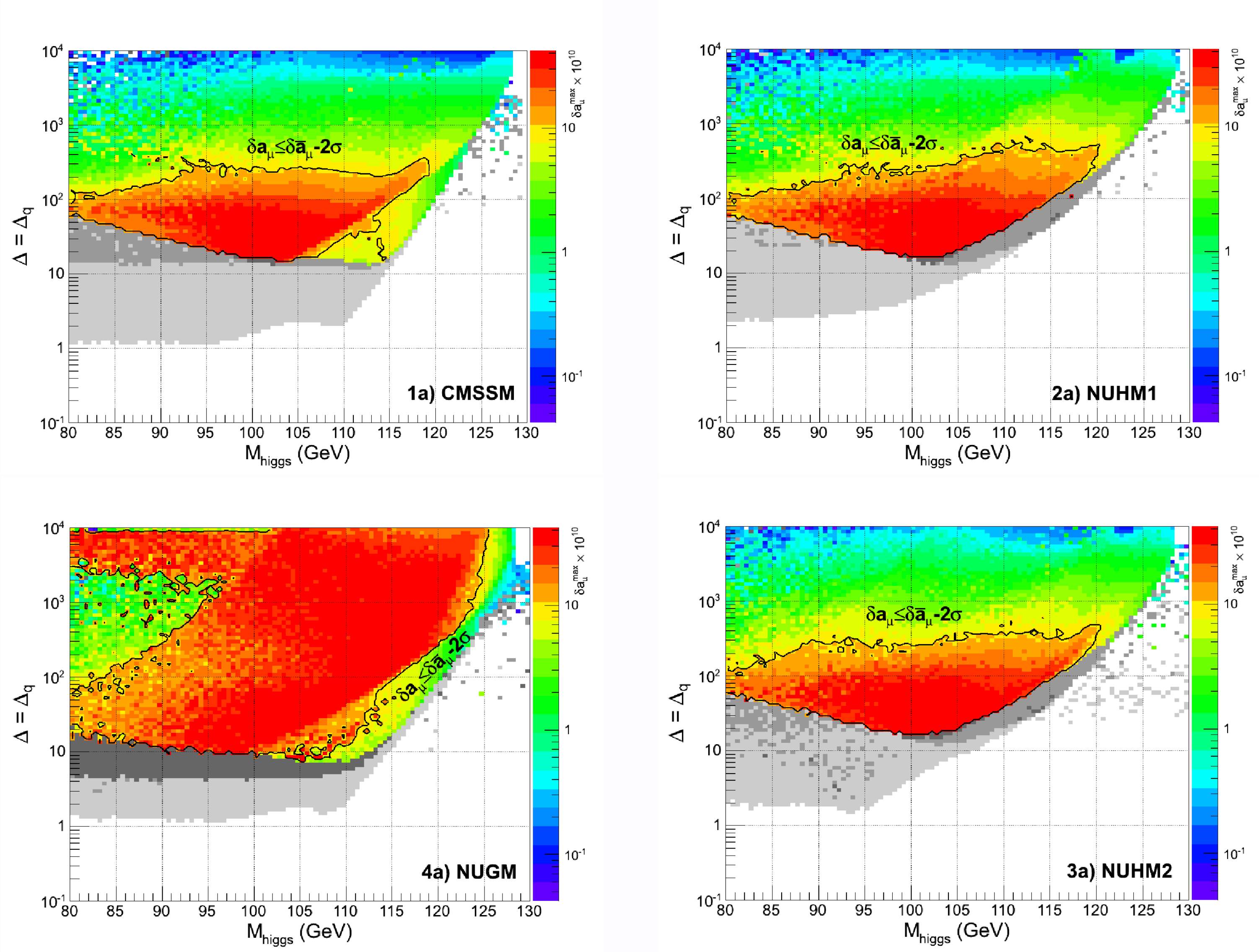
$\Rightarrow$ Implications for SUSY models: $\chi^2_{min} \geq \chi^2_z$:

$$\begin{aligned}
\Delta_q &\approx 10 &\Rightarrow\quad& \chi^2_z/\text{ndf} \approx 0.5 \qquad (\text{ndf}=9)\\
\Delta_q &\approx 100 &\Rightarrow\quad& \chi^2_z/\text{ndf} \approx 1\\
\Delta_q &\approx 1000 &\Rightarrow\quad& \chi^2_z/\text{ndf} \approx 1.5\\
\Delta_q &\approx 2000 &\Rightarrow\quad& \chi^2_z/\text{ndf} \approx 1.7
\end{aligned}$$

- for $m_h \sim 125-126$ GeV: $\Delta_q \sim 500-2000$ (see next). Numerical studies $\chi^2_{min}/\text{ndf} > 1(\sim 2)$..

• Δ in SUSY: Δ_q vs m_h [2-loop, all $\{\gamma, \tan\beta\}$ values]

D.G., H. M. Lee, M. Park



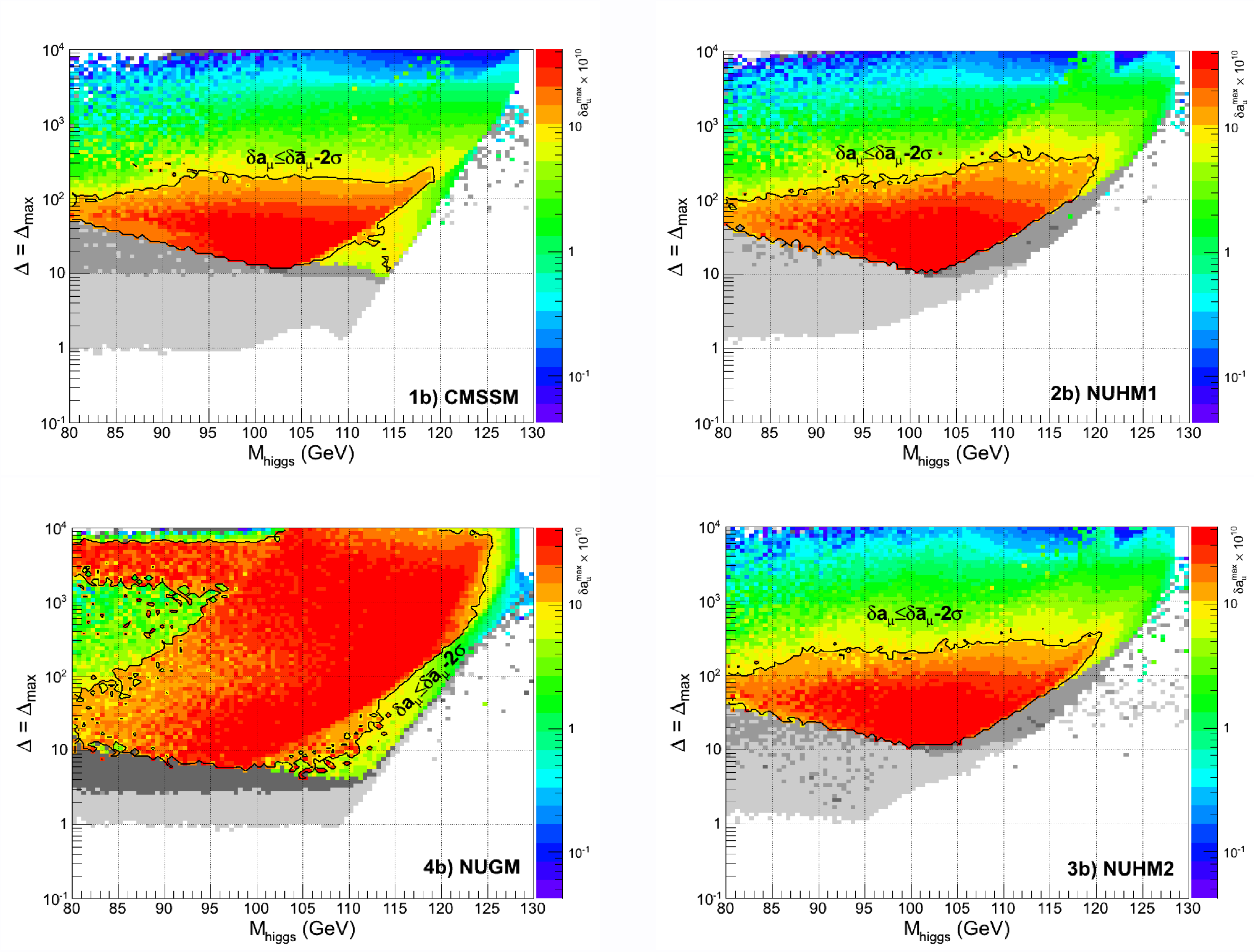
$\Rightarrow$ m_h strongest impact: $\Delta_q \sim \exp(m_h)$. $\Delta_q \sim 1000$ at 125 GeV.

• grey 0: excluded by SUSY; grey 1: $b \to s\gamma$, $B_s \to \mu^+\mu^-$, $\delta\rho$; grey 2: excluded by $\delta a_\mu > 0$.

δa_μ: 2σ contour (red) [smaller δa_μ outside]

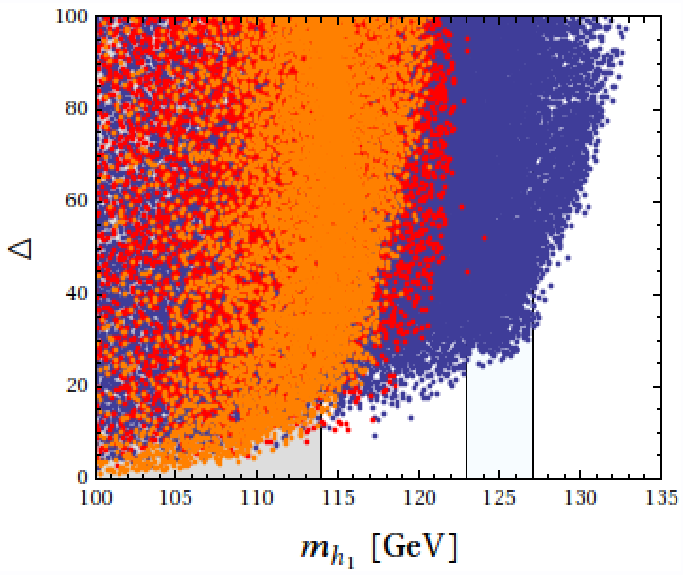
• Δ in SUSY: Δ_{max} vs m_h [2-loop, all $\{\gamma, \tan\beta\}$ values]

D.G., H. M. Lee, M. Park



$\Rightarrow$ min Δ_{max} v. similar to Δ_q where $\Delta_{max} \equiv \max_\gamma |\Delta_\gamma|$, $\quad \Delta_\gamma \equiv \left(\frac{\partial \ln v^2}{\partial \ln \gamma^2}\right)_o$, $\gamma : m_0, m_{1/2}, \mu_0, A_0 ...$

- $\triangle$ in SUSY: Δ_{max} vs m_h in MSSM, NMSSM, GNMSSM. (1-loop).



MSSM

NMSSM: $W = W_Y + \lambda S H_1 H_2 + \kappa S^3, \qquad \Delta \sim 200.$

GNMSSM: $W = W_{NMSSM} + m_{3/2}^2 S + m_{3/2} S^2 + m_{3/2} H_1 H_2.$

from underlying Z_4^R symmetry.

$W = W_Y + (\mu + \lambda S)\, H_1 H_2 + M_*\, S^2 + \kappa S^3$

$\Delta < 30$ for $m_h \leq 126\, GeV.$

G.G. Ross, Schmidt-Hoberg

U. Ellwanger et al; D.G., G.Ross, S. Cassel

- Summary:

Model	Δ_q	$\chi^2_Z(123)$;	Δ_q	$\chi^2_Z(125)$;	Δ_q	$\chi^2_Z(126)$;	Δ_q	$\chi^2_Z(127)$;	ndf	χ^2_Z/ndf (126)
CMSSM	380	11.88	1100	14.01	1800	14.99	3100	16.08	9	1.66
NUHM1	500	12.43	1000	13.82	1500	14.63	2100	15.29	8	1.83
NUHM2	470	12.31	1000	13.82	1300	14.34	2000	15.20	7	2.05
NUGM	230	10.88	700	13.10	1000	13.82	1300	14.34	7	1.97
NUGMd	200	10.59	530	12.55	850	13.49	1300	14.34	9	1.5
NMSSM	>100	9.21	>200	10.59	>200	10.59	>200	10.59	8	1.32
GNMSSM	22	6.18	25	6.43	27	6.59	31	6.87	7	0.95

- error 2-loop m_h: 2-3 GeV; $\Delta_q \sim \exp(m_h/GeV) \Rightarrow \exp(3) \sim 20 \Rightarrow \Delta_q = 20$ or 400 equally "good"

(III). So far we discussed $\chi^2_{min} \sim \chi^2_z = 2\ln\Delta_q$. Let us now address the deviation $\delta\chi^2$ from χ^2_{min}.

- Many observables depend on $v = v(\gamma)$, so $\mathcal{O}_i = \mathcal{O}_i(\gamma, v(\gamma))$, total $\chi^2_{\textsf{total}} = -2\ln L_{\textsf{tot}}$:
- no distinction between tuning to fit $\mathcal{O}_n = m_Z$ or other $\mathcal{O}_i$. D.G. arXiv:1311.6144

$$L_{tot}(\mathcal{O}|\gamma) \sim \frac{1}{(\det M)^{1/2}} \exp\Big\{ -1/2\,(\mathcal{O}_i - \mathcal{O}_i^0)\,(M^{-1})_{ij}\,(\mathcal{O}_j - \mathcal{O}_j^0)\Big\}$$

$$\sim \frac{1}{\Delta}\,\frac{1}{(\det \tilde M)^{1/2}} \exp\Big\{ -1/2\,(\gamma_\alpha - \gamma_\alpha^0)\,\tilde M^{-1}_{\alpha\beta}\,(\gamma_\beta - \gamma_\beta^0)\Big\}.$$

$$\mathcal{O}_i(\gamma) = \mathcal{O}_i(\gamma^0) + (\gamma_\alpha - \gamma_\alpha^0)\left(\frac{d\mathcal{O}_i}{d\gamma_\alpha}\right)_{\gamma=\gamma^0} + \cdots\,, \qquad \tilde M^{-1} \equiv \mathcal{J}^T M^{-1} \mathcal{J}, \qquad \mathcal{J}_{i\alpha} \equiv \frac{1}{\mathcal{O}_i^0}\left[\frac{d\mathcal{O}_i}{d\ln\gamma_\alpha}\right]_{\gamma=\gamma^0}$$

where $\Delta \equiv \big[\det M \det(\tilde M^{-1})\big]^{\frac{1}{2}}$, "differential entropy". $\tilde M$: covariance matrix, if $M_{ij} = \sigma_i^2\,\delta_{ij}$

$$\tilde M^{-1}_{\alpha\beta} = \left\{\left(\frac{d(\mathcal{O}_i/\sigma_i)}{d\ln\gamma_\alpha}\right)\left(\frac{d(\mathcal{O}_i/\sigma_i)}{d\ln\gamma_\beta}\right)\right\}_{\gamma=\gamma^0}, \qquad \alpha,\beta = 1,2,....s.$$

• With $v = v(\gamma)$, covariance matrix contains info about Δ_q from all $\mathcal{O}_i$!

D.G. arXiv:1311.6144

$$\tilde{M}^{-1}_{\alpha\beta} = \tilde{M}^{-1}_{\alpha\beta}\Big|_{v=const} + \sum_{i=1}^{s}\Big\{\Big(\frac{\partial \mathcal{O}_i/\sigma_i}{\partial \ln v}\Big)^2\Big(\frac{\partial \ln v}{\partial \ln \gamma_\alpha}\Big)\Big(\frac{\partial \ln v}{\partial \ln \gamma_\beta}\Big)\Big\}_{\gamma=\gamma^0}$$
$$+ \sum_{i=1}^{s}\Big\{\Big(\frac{\partial \mathcal{O}_i/\sigma_i}{\partial \ln v}\Big)\Big(\frac{\partial \ln v}{\partial \ln \gamma_\alpha}\Big)\Big(\frac{\partial \mathcal{O}_i/\sigma_i}{\partial \ln \gamma_\beta}\Big) + (\alpha \leftrightarrow \beta)\Big\}_{\gamma=\gamma^0}$$

$\Rightarrow$ includes individual "tunings" $\Delta_\gamma = \partial \ln v / \partial \ln \gamma$ from all observables.

- Also

$$\text{Tr}\, \tilde{M}^{-1} = \sum_{i=1}^{n}\sum_{\alpha=1}^{s}\Big(\frac{d\mathcal{O}_i/\sigma_i}{d \ln \gamma_\alpha}\Big)^2_{\gamma=\gamma^0} = \sum_{i=1}^{n}\Big(\frac{\partial \mathcal{O}_i/\sigma_i}{\partial \ln v}\Big)^2_{\gamma=\gamma^0} \times \underbrace{\sum_{\alpha=1}^{s}\Big(\frac{\partial \ln v}{\partial \ln \gamma_\alpha}\Big)^2_{\gamma=\gamma^0}}_{=\Delta_q^2} + \cdots,$$

$\Rightarrow$ $\tilde{M}$ more fundamental, includes Δ_q in a first approximation!

• Q: do precision data fits (in frequentist approach) account for this effect (v=constant)?

include it in a less manifest way via $\mu = \mu(\gamma, m_Z = m_Z^0)$?....

Bayesian approach: Casas et al.

- The s-standard deviation confidence interval:

$$-2\ln L(\gamma') \leq -2\ln L_{max}(\gamma^0) + s^2, \quad \Rightarrow \quad \sum_{i=1}^{n} \left\{ \left(\frac{d\mathcal{O}_i/\sigma_i}{d\ln\gamma_\alpha} \right)_{\gamma=\gamma^0} (\gamma'_\alpha/\gamma^0_\alpha - 1) \right\}^2 \leq s^2$$

$$\Delta_q \leq \frac{s\,\sigma_z}{m_Z(\gamma^0)} \left| \frac{n^\alpha\,(\gamma'_\alpha - \gamma^0_\alpha}{\gamma^0_\alpha} \right|^{-1} \leq \frac{s\,\sigma_z}{m_Z(\gamma^0)} \left| \frac{n^\alpha\,\sigma_{th,\alpha}}{\gamma^0_\alpha} \right|^{-1}$$

$\Rightarrow \Delta_q$ bound. $|\gamma'_\alpha - \gamma^0_\alpha| > \sigma_{th,\alpha}$.

- $\tilde{M}$ defines global correlation coefficient:

$$\rho_\alpha = \sqrt{1 - \tilde{M}_{\alpha\alpha}\,(\tilde{M}^{-1})_{\alpha\alpha}}, \qquad 0 \leq \rho_\alpha \leq 1.$$

- $\rho_\alpha = 0$ then γ_α: independent of the rest. $\rho_\alpha = 1$: combination of the rest.

- useful to identify fundamental, independent UV parameters (soft masses, couplings). Not yet studied

Also note: $\rho_{\alpha\beta} \sim \frac{\tilde{M}_{\alpha\beta}}{\sigma_\alpha \sigma_\beta}$ Dreiner et al arXiv:1204.4199

(IV). For model building: ways to achieve smaller Δ_q:

$$v^2 = -\frac{m^2}{\lambda}, \qquad v = \mathcal{O}(100\,\text{GeV}), \quad m \sim \mathcal{O}(\text{TeV}). \quad \lambda < 1.$$

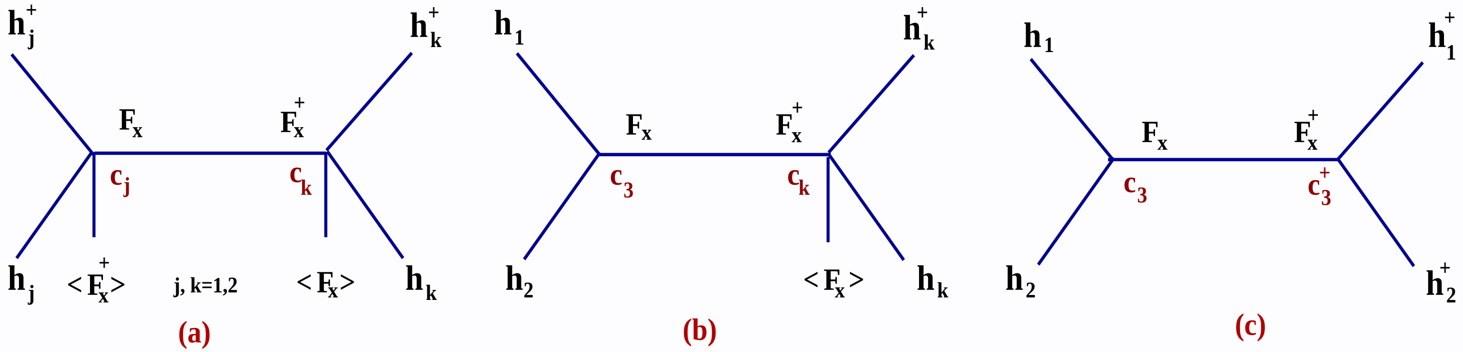
1.- increase λ (effective higgs coupling) by quantum corrections. (m: soft masses combination)

2.- increase λ (and m_h) by classical (susy) corrections from "new physics" beyond MSSM higgs, parametrised by d=5, 6 operators: $\Delta_q(m_h) \approx \exp(-\delta m_h/\text{GeV})\, \Delta_q(m_h)\big|_{\text{CMSSM}}$

Carena et al 2009, D.G., Dudas, Antoniadis 2009, 2010.

3.- increase λ by reducing SUSY breaking scale to $\sqrt{F}_X = 1$ TeV (from "usual" 10^{10} GeV).

Corrections to λ from: Casas et al 2003, D.G., Dudas, Antoniadis 2010, 2014.



No new states added! downside: v. light gravitino, the LSP (mili-eV). Dark matter?

(V) Conclusions:

- χ^2 and Δ_q often studied separately. They are related $\Rightarrow$ Δ_q probabilistic interpretation.

- Δ_q emerges from total likelihood to fit data including $m_Z(\gamma)$ which thus accounts for naturalness:

$\chi^2_{\rm total} = \chi^2 + 2\ln\Delta_q$. Mathematical support for Δ_q as a fine tuning measure

- Bound on Δ_q: good fit: $\chi^2_{\rm z}$ / ndf $\approx 1 \Rightarrow \Delta_q < \exp(\text{ndf}/2) \sim 100$; Δ_q less fundamental than $\tilde M$

- Matrix $\tilde M$ with $v = v(\gamma)$ automatically includes EW fine-tuning effects.

So if you can do a good fit with $v = v(\gamma)$, forget about "fine-tuning"....

- Values of Δ_q:

$\Rightarrow$ Δ_q, $\Delta_{max} \sim \exp(m_h/\text{GeV}) \sim 500 - 1000$, for $m_h \approx 126$ GeV. GNMSSM has $\Delta_q \sim \mathcal{O}(20)$.

Significant error factor: $\sim e^{\sigma_h}$ (=7.5 to 20) of Δ_q due to theoretical $\sigma_h = 2 - 3$ GeV.

• Naturalness in the Bayesian approach. $p(a|b)\,p(b) = p(a\cap b) = p(b|a)\,p(a)$

Bayes theorem: [initial belief + data $\rightarrow$ updated belief].

Thomas Bayes (1761), Laplace (1812)

$$p(\gamma|\text{data}) = \frac{L(\text{data}|\gamma)\; p(\gamma)}{p(\text{data})}, \qquad p(\text{data}) = \int L(\text{data}|\gamma)\, p(\gamma) d\gamma, \qquad \gamma : \{m_0, m_{1/2}, \mu_0, A_0, B_0; y_t, y_b, ...\}.$$

- p(data): "evidence". Models $1, 2$: $p_1(\text{data})/p_2(\text{data})$. $p(\gamma)$=priors.
- EW constraints: $f_1(\gamma; v, \beta) = f_2(\gamma; v, \beta) = 0$, $\Rightarrow$ $v(\gamma, ...), \tan\beta_0(\gamma ...)$.

$$p(\text{data}) = \int d\gamma\; p(\gamma)\, dv\; d(\tan\beta)\; \delta(m_Z - m_Z^0)\; \delta\left(f_1(\gamma; v, \beta)\right)\; \delta\left(f_2(\gamma; v, \beta)\right) L(\text{data}|\gamma; \beta, v),$$

$$= \int_{f_{1,2}=0} dS_\gamma\; \frac{L(\text{data}|\gamma)}{\Delta_q(\gamma)}\; p(\gamma), \qquad \Delta_q \equiv \Big[\sum_\gamma \Delta_\gamma^2\Big]^{1/2}, \;\; d\gamma \equiv \prod_i d\gamma_i, \;\; p(\gamma) = p(\gamma_1, ...\gamma_i).$$

D.G., H. M. Lee, M. Park, arXiv:1203:0569

$\Rightarrow$ Δ_q due to fixing EW scale, in addition to/independent of priors $p(\gamma)$! large L/Δ_q needed.

$$\int_{R^n} h(z_1, ..., z_n)\, \delta(g(z_1, ..., z_n))\, dz_1 ... dz_n = \int_{S_{n-1}} dS_{n-1}\, h(z_1, ... z_n)\, \frac{1}{|\nabla_{z_i} g|},$$

• Reducing Δ_q by physics beyond MSSM higgs sector: MSSM + d=6 operators

$$\mathcal{O}_j = \int d^4\theta \; \mathcal{Z}_j \, (H_j^\dagger e^{V_j} H_j)^2, \quad (j=1,2). \qquad \mathcal{O}_3 = \int d^4\theta \; \mathcal{Z}_3 \, (H_1^\dagger e^{V_1} H_1)(H_2^\dagger e^{V_2} H_2)$$

$$\mathcal{O}_4 = \int d^4\theta \; \mathcal{Z}_4 \, (H_2 H_1)(H_2 H_1)^\dagger, \qquad \mathcal{O}_k = \int d^4\theta \; \mathcal{Z}_k \, (H_k^\dagger e^{V_k} H_k) H_2 H_1 + h.c., \text{ (k=5,6)}$$

$$\mathcal{O}_7 = \int d^2\theta \; \mathcal{Z}_7 \, \mathrm{Tr}\, W_i^\alpha W_{i,\alpha} (H_2 H_1) + h.c.,$$

where $\mathcal{Z}_j(S, S^\dagger) = \alpha_{j0} + \alpha_{j1} S + \alpha_{j1}^* S^\dagger + \alpha_{j2} m_0^2 S S^\dagger, \qquad \alpha_{jk} \sim 1/M_*^2, \;\; S = m_0 \theta\theta$

$\mathcal{O}_{1,2,3}$: generated by massive T, U(1); $\mathcal{O}_4$: singlet, T. $\mathcal{O}_{5,6}$: 2 D, singlet.

$$\Rightarrow \delta m_h^2 = -2 v^2 \left[(\alpha_{30}+\alpha_{40})\mu_0^2 - \alpha_{20} m_Z^2\right] - \frac{(2\zeta_0 \mu_0)^2 \, v^4}{m_A^2 - m_Z^2} + \frac{v^2 \cot\beta}{m_A^2 - m_Z^2}\Big[4 m_A^2 \mu_0^2 \, (2\alpha_{50}+\alpha_{60}) - (2\alpha_{60} - 3\alpha_{70}) m_A^2 m_Z^2 - (2\alpha_{60}+\alpha_{70}) m_Z^4\Big] + \mathcal{O}\big(1/(M_*^2 \tan^2\beta)\big)$$

$\Rightarrow \alpha_{j0}$ (choice?) $\Rightarrow$ increase m_h, reduce fine-tuning by:

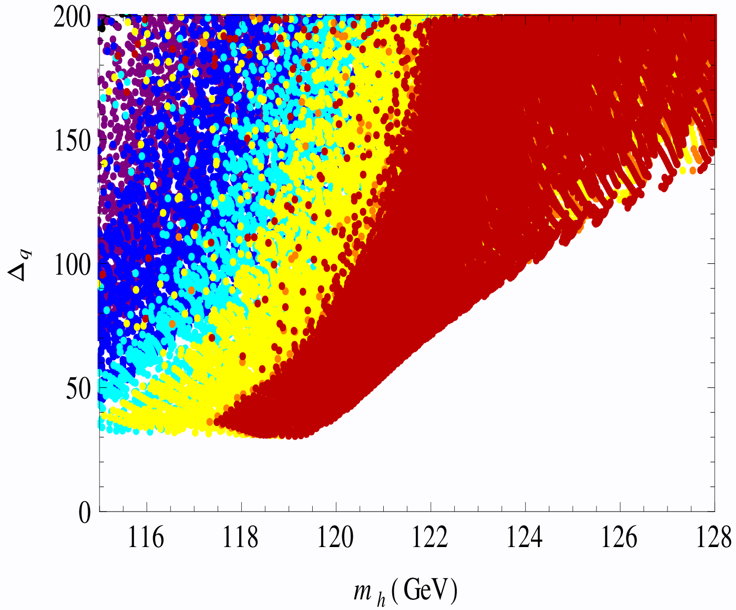
$$\Delta_q(m_h) \approx \exp(-\delta m_h/\text{GeV}) \, \Delta_q(m_h)\Big|_{\text{CMSSM}}$$

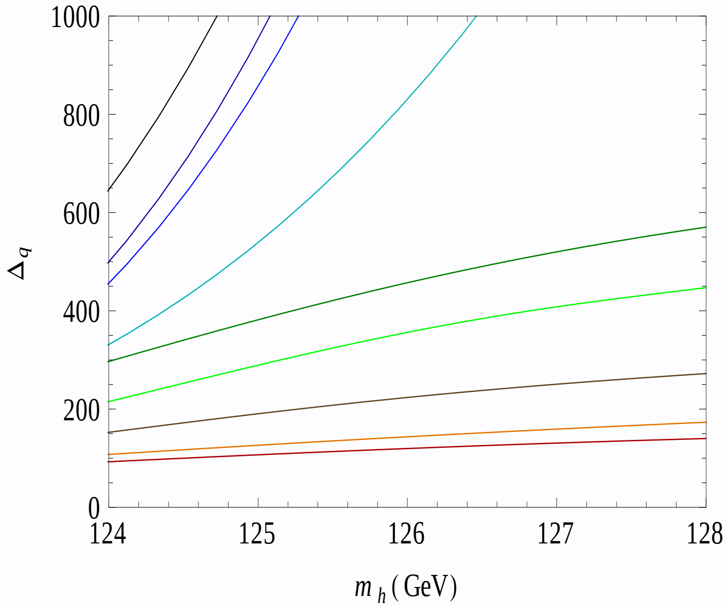
D.G. et al, NPB 848(2011), NPB 831(2010),

M. Carena et al, PRD 85(2012), PRD 81, 82(2010),

F. Boudjema et al, PRD 85 (2012)

- "Non-linear" MSSM: MSSM with low-scale of SUSY breaking: $\sqrt{F} \sim \mathcal{O}(1)$ TeV.



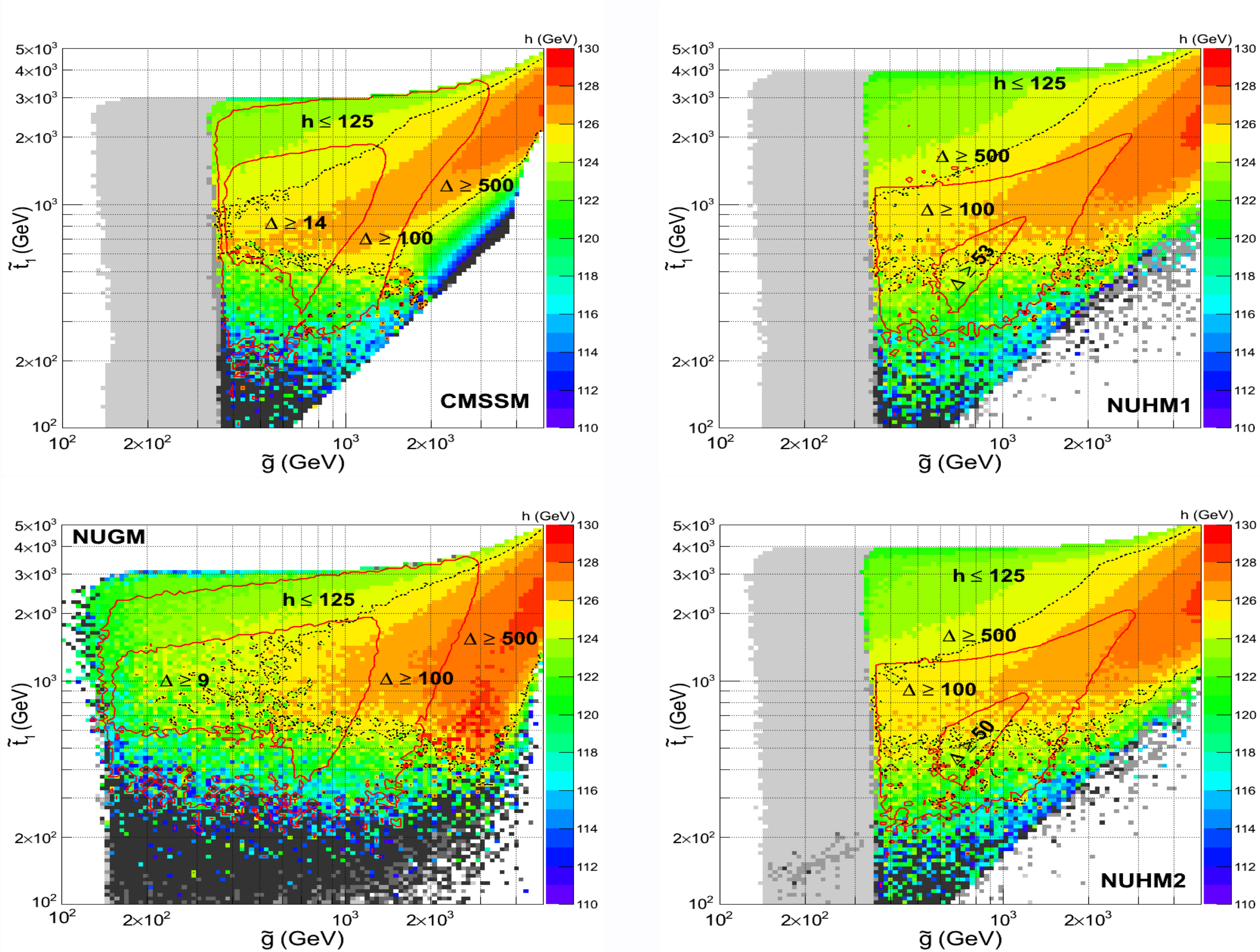


D.G., Dudas, Antoniadis 2011, 2014.

- left plot: Δ_q vs. m_h for $\sqrt{F}$ =3.2 TeV. $\Delta_q \sim 100$.
- right plot: Δ_q vs m_h for $\sqrt{F} = 3.2$ TeV (lower curve) to 9 TeV (top curve).
- MSSM recovered at large $\sqrt{F}$.

• Stop vs Gluino with largest m_h and min Δ_q. [$\{\gamma, \tan\beta\}$ all values]

D.G., H. M. Lee, M. Park



⇒ constraints on m_h strongly reduce the viable regions.