Unitarised fixed order + parton shower schemes

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(DESY)

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Task: Combine multiple fixed-order calculations with each other.
Keep fixed order accuracy for inclusive n-jet cross sections.
Keep fixed-order + resummation goodies for exclusive n-jet cross sections.
Make n-jet observables stable for any $p_{\perp,min}$.

Once this is done, we hope to describe collider data of jet observables with satisfactory precision (and good agreement).

Aim: Combine multiple fixed-order calculations. Approximate giant k-factors from soft W/Z-boson emissions.

$$
\begin{aligned}
\langle\mathcal{O}\rangle \quad &= \mathrm{B}_0\mathcal{O}(S_{+0j}) \;-\; \int d\rho\; \mathrm{B}_1\Theta_>^{(1)}\mathcal{O}(S_{+0j}) \;+\; \int \mathrm{B}_2\Theta_>^{(2)}\mathcal{O}(S_{+0j}) \\
&+ \int \mathrm{B}_1\Theta_>^{(1)}\mathcal{O}(S_{+1j}) \;-\; \sum_{\text{loopings}}\int d\rho\; \mathrm{B}_2\Theta_>^{(2)}\mathcal{O}(S_{+1j}) \\
&+ \int \mathrm{B}_2\Theta_>^{(2)}\mathcal{O}(S_{+2j})
\end{aligned}
$$

where $\Theta_>^{(n)} = \Theta\left(t\left(S_{+n}\right) - t_{\mathrm{MS}}\right)$. Also available at NLO.

LoopSim enforces cancellation of IR logarithms by "unitarity" condition. This unitarity condition is similar to what's done in parton shower algorithms, where

$$P_{no-emission} = 1 - P_{emission}$$

CKKW(-L)

Aim: Combine multiple tree-level calculations with each other and (PS) resummation. Fill in soft and collinear regions with parton shower.

$$
\begin{aligned}
\langle\mathcal{O}\rangle &= \mathrm{B}_0\mathcal{O}(S_{+0j}) \\
&\qquad - \int d\rho\ \mathrm{B}_0 P_0(\rho)\Theta_>^{(1)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho)\mathcal{O}(S_{+0j}) \\
&+ \int \mathrm{B}_1\Theta_>^{(1)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho)\mathcal{O}(S_{+1j}) \\
&\qquad - \int d\rho\ \mathrm{B}_1 P_1(\rho)\Theta_>^{(2)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho_1)\Pi_{S_{+1}}(\rho_1,\rho)\mathcal{O}(S_{+1j}) \\
&+ \int \mathrm{B}_2\Theta_>^{(2)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho_1)\Pi_{S_{+1}}(\rho_1,\rho)\mathcal{O}(S_{+2j})
\end{aligned}
$$

Not unitary
$\Longrightarrow$ Can contain numerically large (sub-leading) logs.
$\Longrightarrow$ Needs fixing!

Bug vs. Feature in CKKW(-L)

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).
These are the improvements that we need to describe multiple hard jets!

If we simply add samples, the “improvements” will degrade the inclusive cross section: σ_{inc} will contain $\ln(t_{\mathrm{MS}})$ terms.

THE INCLUSIVE CROSS SECTION DOES NOT CONTAIN LOGS RELATED TO CUTS ON HIGHER MULTIPLICITIES.

Traditional approach: Don’t use a too small merging scale.
→ Uncancelled terms numerically not important.

Unitary approach[1]:
Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on t_{MS}.

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

Unitarised ME+PS

Aim: Combine multiple tree-level calculations with each other and (PS) resummation. Fill in soft and collinear regions with parton shower.

$$
\begin{aligned}
\langle\mathcal{O}\rangle \quad &= \mathrm{B}_0\mathcal{O}(S_{+0j}) \\
&\quad - \int d\rho\ \mathrm{B}_1\Theta_>^{(1)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho)\mathcal{O}(S_{+0j}) - \int d\rho\ \mathrm{B}_2\Theta_>^{(2)}\Theta_<^{(1)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho)\mathcal{O}(S_{+0j}) \\
&+ \int \mathrm{B}_1\Theta_>^{(1)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho)\mathcal{O}(S_{+1j}) \\
&\quad - \int d\rho\ \mathrm{B}_2\Theta_>^{(2)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho_1)\Pi_{S_{+1}}(\rho_1,\rho)\mathcal{O}(S_{+1j}) \\
&+ \int \mathrm{B}_2\Theta_>^{(2)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho_1)\Pi_{S_{+1}}(\rho_1,\rho)\mathcal{O}(S_{+2j}) + \int \mathrm{B}_2\Theta_>^{(2)}\Theta_<^{(1)} w_f w_{\alpha_s}\Pi_{S_{+0}}(\rho_0,\rho_1)\mathcal{O}(S_{+}
\end{aligned}
$$

Inclusive cross sections preserved by construction.
Cancellation between different "jet bins".
$\Longrightarrow$ Statistics needs fixing.

UMEPS, MC@NLO-style (Plätzer)

Aim: Combine multiple tree-level calculations with each other and (PS) resummation. Fill in soft and collinear regions with parton shower.

$$
\begin{aligned}
\langle\mathcal{O}\rangle \quad &= \mathrm{B}_0 \Pi_{S_{+0}}(\rho_0, \rho_{\mathrm{MS}}) \mathcal{O}(S_{+0j}) \\
&\quad - \int d\rho \;\; [\mathrm{B}_1 - \mathrm{B}_0 P_0(\rho)]\, \Theta^{(1)}_{>} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \mathcal{O}(S_{+0j}) \\
&+ \int \mathrm{B}_1 \Theta^{(1)}_{>} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \Pi_{S_{+1}}(\rho, \rho_{\mathrm{MS}}) \mathcal{O}(S_{+1j}) \\
&\quad - \int d\rho \;\; [\mathrm{B}_2 - \mathrm{B}_1 P_1(\rho)]\, \Theta^{(2)}_{>} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+1j}) \\
&+ \int \mathrm{B}_2 \Theta^{(2)}_{>} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+2j}) \; + \int \mathrm{B}_2 \Theta^{(2)}_{>} \Theta^{(1)}_{<} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+}
\end{aligned}
$$

Inclusive cross sections preserved by construction.
Cancellation between different "jet bins" fixed.
$\Longrightarrow$ Statistics okay.

UMEPS results

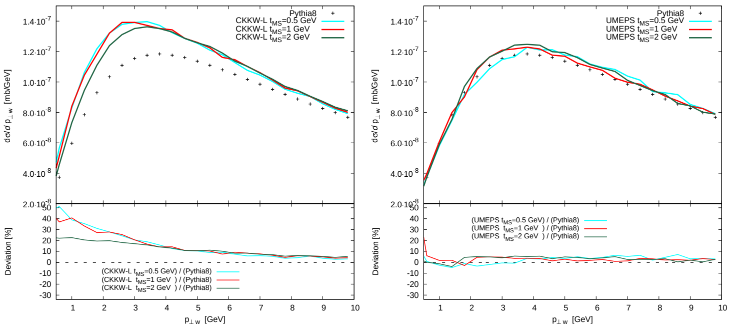


Figure: $p_\perp$ of the W-boson in the Sudakov region (for 2-jet merging, $E_{CM} = 7$ TeV). Lower inset shows the comparison to default PYTHIA 8.

$\Rightarrow$ CKKW-L overshoots for (very) low merging scales due to uncancelled terms.

$\Rightarrow$ UMEPS describes the Sudakov peak nicely.

Aim: Combine multiple exclusive NLO calculations. Extend UMEPS to NLO, get NLO + UMEPS higher orders, and nothing else.

UMEPS @ NLO = UNLOPS

Aim: Combine multiple exclusive NLO calculations. Extend UMEPS to NLO, get NLO + UMEPS higher orders, and nothing else.

Basic idea: Do NLO multi-jet merging for UMEPS:

- ◇ Subtract approximate UMEPS $\mathcal{O}(\alpha_{\mathrm{s}})$-terms, add back full NLO.
- ◇ To preserve the inclusive (NLO) cross section, add approximate NNLO.

⇒ UNLOPS[1].

For UNLOPS merging, we need exclusive NLO inputs:

$$\widetilde{\mathrm{B}}_n = \mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1|n} \Theta \left(\rho_{\mathrm{MS}} - t \left(S_{+n+1}, \rho \right) \right) - \mathrm{D}_{n+1|n} \right)$$

We can get these e.g. by massaging POWHEG-BOX or MC@NLO output.

[1] JHEP1303(2013)166 (Leif Lönnblad, SP), Similar scheme in JHEP1308(2013)114 (Simon Plätzer)

The UNLOPS method

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \qquad - \int_s \widehat{\mathrm{B}}_{1\to 0} \qquad - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\quad \widehat{\mathrm{B}}_1 \quad - \int_s \widehat{\mathrm{B}}_{2\to 1} \quad \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_{\mathrm{s}}^{n})$- and $\mathcal{O}(\alpha_{\mathrm{s}}^{n+1})$-terms:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(- \left[\int_s \widehat{\mathsf{B}}_{1\to 0} \right]_{-1,2} - \int_s \widehat{\mathsf{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\left[\widehat{\mathsf{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathsf{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathsf{B}}_2 \Bigg\}$$

The UNLOPS method

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \left[\int_s \widehat{\mathrm{B}}_{1\rightarrow 0} \right]_{-1,2} - \int_s \widehat{\mathrm{B}}_{2\rightarrow 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\rightarrow 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Unitarise:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\quad \widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Big\{ \mathcal{O}(S_{+0j}) \Big(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \Big[\int_s \widehat{\mathrm{B}}_{1\to 0}\Big]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Big)$$
$$+ \int \mathcal{O}(S_{+1j}) \Big(\widetilde{\mathrm{B}}_1 + \Big[\widehat{\mathrm{B}}_1\Big]_{-1,2} - \Big[\int_s \widehat{\mathrm{B}}_{2\to 1}\Big]_{-2} \Big) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Big\}$$

Iterate for the case of M different NLO calculations, and N tree-level calculations:

$$\langle\mathcal{O}\rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \Big\{ \widetilde{\mathrm{B}}_m + \Big[\widehat{\mathrm{B}}_m\Big]_{-m,m+1} + \int_s \mathrm{B}_{m+1\to m}$$
$$- \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \Big[\int_s \widehat{\mathrm{B}}_{i\to m}\Big]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}^{\uparrow}_{i+1\to m} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to m} \Big\}$$
$$+ \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Big\{ \widetilde{\mathrm{B}}_M + \Big[\widehat{\mathrm{B}}_M\Big]_{-M,M+1} - \Big[\int_s \widehat{\mathrm{B}}_{M+1\to M}\Big]_{-M} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i+1\to M} \Big\}$$
$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Big\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \Big\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{B}_0 - \int_s \widetilde{B}_{1\to 0} + \int_s B_{1\to 0} - \left[\int_s \widehat{B}_{1\to 0}\right]_{-1,2} - \int_s B^{\uparrow}_{2\to 0} - \int_s \widehat{B}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{B}_1 + \left[\widehat{B}_1\right]_{-1,2} - \left[\int_s \widehat{B}_{2\to 1}\right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j})\widehat{B}_2 \Bigg\}$$

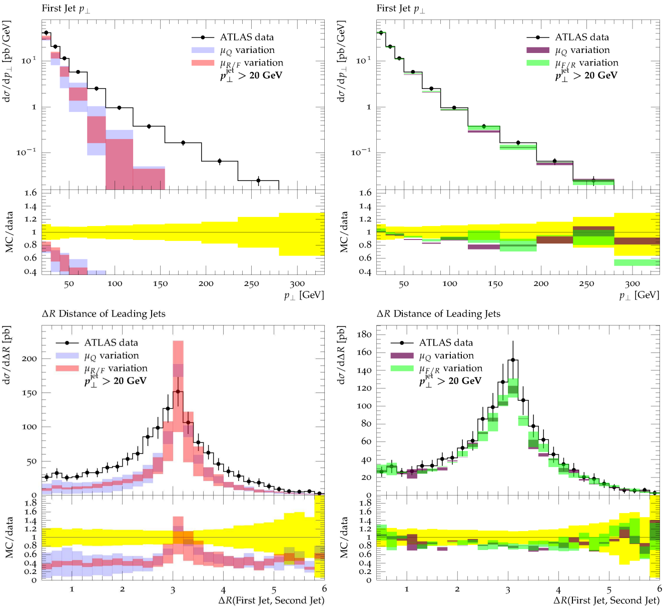
Iterate for the case of M different NLO calculations, and N tree-level calculations:

$$\langle\mathcal{O}\rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int\cdots\int \mathcal{O}(S_{+mj}) \Bigg\{ \widetilde{B}_m + \left[\widehat{B}_m\right]_{-m,m+1} + \int_s B_{m+1\to m} - \sum_{i=m+1}^{M} \int_s \widetilde{B}_{i\to m} - \sum_{i=m+1}^{M} \left[\int_s \widehat{B}_{i\to m}\right]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s B^{\uparrow}_{i+1\to m} - \sum_{i=M+1}^{N} \int_s \widehat{B}_{i\to m} \Bigg\}$$
$$+ \int d\phi_0 \int\cdots\int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{B}_M + \left[\widehat{B}_M\right]_{-M,M+1} - \left[\int_s \widehat{B}_{M+1\to M}\right]_{-M} - \sum_{i=M+1}^{N} \int_s \widehat{B}_{i+1\to M} \Bigg\}$$
$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int\cdots\int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{B}_n - \sum_{i=n+1}^{N} \int_s \widehat{B}_{i\to n} \Bigg\}$$

Inputs (B_n, $\overline{B}_n$) taken from external tools.

Merging done internally in PYTHIA 8.

UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \Big[\int_s \widehat{\mathrm{B}}_{1\to 0} \Big]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \Big[\widehat{\mathrm{B}}_1 \Big]_{-1,2} - \Big[\int_s \widehat{\mathrm{B}}_{2\to 1} \Big]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

. . . however:

$\Big[\widehat{\mathrm{B}}_1\Big]_{-1,2} = \mathrm{B}_1 \Big[\Pi_{S_{+0}}(\rho_0,\rho_1) w_f w_{\alpha_{\mathrm{s}}} - 1 - \Pi_{S_{+0}}(\rho_0,\rho_1)|_{\mathcal{O}(\alpha_s)} - w_f|_{\mathcal{O}(\alpha_s)} - w_{\alpha_{\mathrm{s}}}|_{\mathcal{O}(\alpha_s)} \Big]$ contains

$\Pi_{S_{+0}}(\rho_0,\rho_1)|_{\mathcal{O}(\alpha_s)} = -\int d\rho \alpha_s(\mu_r) P_0(\rho)$, which cancels between "0-" and "1-jet bin".

$\Longrightarrow$ The "1-jet bin" alone is not well-behaved as $\rho_{\mathrm{MS}} \to 0$.
Cancellations make statistics problematic.

UN^{2}LOPS matching

Aim: Combine just two NLO calculations, but go to NNLO matching directly. Amend UNLOPS to truly allow for $\rho_{\text{MS}} \to 0$.

$\Rightarrow$ Inclusive cross section correct to NNLO. Statistics fine.

UN^{2}LOPS matching

Aim: Combine just two NLO calculations, but go to NNLO matching directly. Amend UNLOPS to truly allow for $\rho_{\text{MS}} \to 0$.

Basic idea: Start from scratch again and change "higher orders" to something more MC@NLO-style. Thus:

- ◇ Use 0-jet matched (MC@NLO$_0$) and 1-jet matched calculation (MC@NLO$_1$).
- ◇ Remove hard ($q_T > \rho_{\text{MS}}$) reals in MC@NLO$_0$.
- ◇ Reweight B$_1$ of MC@NLO$_1$ with "zero-jet Sudakov" factor $\Pi_{S_{+0}}/\alpha_s$ running.
- ◇ Reweight NLO part $\widetilde{\text{B}}_1^{\text{R}}$ of MC@NLO$_1$ with "zero-jet Sudakov" factor.
- ◇ Subtract erroneous $\mathcal{O}(\alpha_{\text{s}}^{+1})$ terms multiplying B$_1$.
- ◇ Reweight subtractions with $\Pi_{S_{+0}}$ to be able to group them with $\widetilde{\text{B}}_1^{\text{R}}$.
- ◇ Put $\rho_{\text{MS}} \to \rho_c < 1\text{GeV}$. ($\to$ MC@NLO$_0$ becomes exclusive NLO)
- ◇ Unitarise by subtracting the processed MC@NLO$_1'$ from the "zero-q_T bin".
- ◇ Remove all terms up to α_s^2 from the "zero-q_T bin" and add the q_T-vetoed NNLO cross section.

$\Rightarrow$ Inclusive cross section correct to NNLO. Statistics fine.

$\mathrm{UN^2LOPS}$ matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} =\ & \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,O)
\end{aligned}
$$

$$\mathcal{O}^{(\mathrm{UN^2LOPS})} = \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$

Note that this is just an extention of the old Sudakov veto algorithm:
Run trial shower on the reconstructed zero-jet state,
If trial shower produces an emission, keep zero-jet kinematics and stop;
else start PS off one-jet state.

UN^{2}LOPS matching

$$\mathcal{O}^{(\mathrm{UN^2LOPS})} = \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$

Note: $\left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}$ etc. comes from using q_T-vetoed cross sections.

UN^{2}LOPS matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = \; & \int d\Phi_0 \, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2, O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2, O)
\end{aligned}
$$

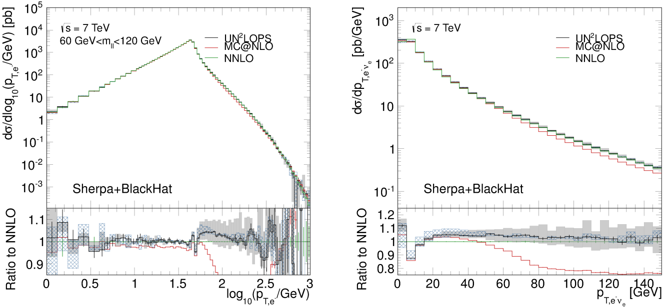
$\bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}} + \tilde{\mathrm{B}}_1^{\mathrm{R}} + \mathrm{H}_1^{\mathrm{R}} + \mathrm{H}_1^{\mathrm{E}} = \mathrm{B}_{\mathrm{NNLO}}$
Other terms drop out in inclusive observables.

UN^{2}LOPS matching

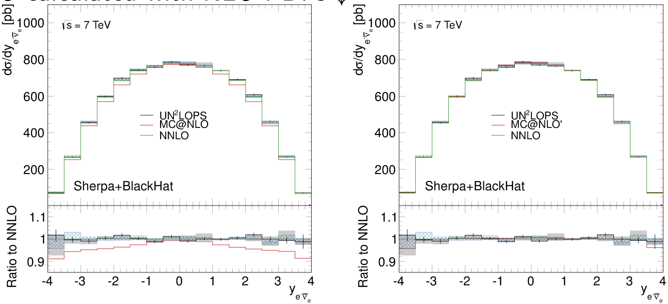
$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0\, \bar{\bar{\bar{\mathrm{B}}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2, O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2, O)
\end{aligned}
$$

Orange terms do not contain any universal α_s corrections present in the PS.
H_1 do not contribute in the soft/collinear limit.
$\Longrightarrow$ PS accuracy is preserved.

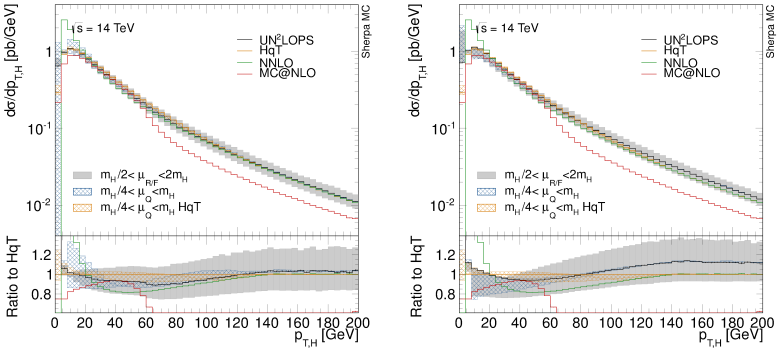
UN^{2}LOPS (Drell-Yan)



NLO calculated with NLO PDFs ↓ ↓ NLO calculated with NNLO PDFs



UN^{2}LOPS (Higgs)



$p_\perp$ of the Higgs-boson for two different matching schemes in UN^{2}LOPS: "individual" matching and "factorised" matching.

Overview

- Within the MC community, unitary fixed-order + parton shower schemes are standard.
- 80's ME corrections[1] are unitary.
- POWHEG and MC@NLO matching[1] unitary.
- MiNLO matching[1] preserves NNLO cross section. As does UN^{2}LOPS matching.
- Unitary LO/NLO merging schemes are now used in HERWIG++, PYTHIA8 and SHERPA
- VINCIA iterated (NLO) ME corrections[1] are unitary.

We claim to have a good understanding of the fixed-order accuracy of our methods (up to NNLO). We should work on the shower next.

Since this is a discussion session, now to some questions. (Apologies if most of these are stupid.)

[1] Apologies for not mentioning these schemes before.

The PS rests on the unitarity condition

$$P_{no-resolved-emission} = 1 - P_{resolved-emission}$$

$\Longrightarrow$ The fixed-order cross section is always preserved. Resummation is not applied outside of phase space boundaries.

Resummed+matched calculations of q_T (thrust) of Catani et al. apply the same contraint by changing the logs to

$$\ln\left(\frac{M^2 b^2}{b_0^2}\right) \longrightarrow \ln\left(\frac{M^2 b^2}{b_0^2} + 1\right)$$

$\Longrightarrow$ Only power corrections, okay to any log accuracy. Resummation vanishes at phase space boundaries, fixed-order cross section preserved.
$\Longrightarrow$ Turns off resummation at high q_T.

Questions: Resummation and (PS) unitarity II

Disclaimer: This slide is pure guesswork. So how sensible is a unitary approach to resummation?

Threshold resummation changes the fixed-order cross section because $L = \ln\left(1 - \frac{M^2}{\hat{s}}\right) = \ln(1-z)$ is integrated from $z_{min} = x$ to 1 and the result does not vanish at the lower limit. Would a replacement like

$$L \to \widetilde{L} = \ln\left[(1-z)\left(1 + \frac{z_{min}}{1 - z_{min}}\right)\right]$$

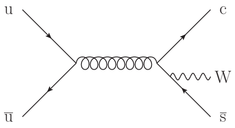
be permissable?

...and how about exponentiation of finite (π^2) terms? Could be okay if these are contained in an exponentiated 1j@NLO-ME-correction?

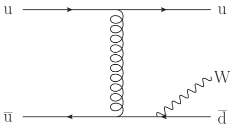
Most subtleties/problems in fixed-order + parton shower schemes come from a limited PS accuracy. Should we keep to the unitarity paradigm when trying to improve the PS resummation?

Questions: New processes

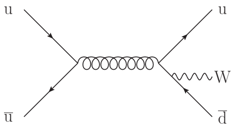
Now we can claim better fixed-order accuracy, but. . .



New Born configuration



Standard shower history

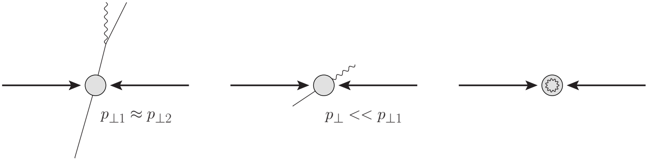


???

- . . . what do we do with new Born states? What's a new Born state?
- How do we attach the QCD resummation (Sudakovs, α_s scales. . .)?
- If these are "weak corrections" to dijet states, should we merge multiple weak emissions?
  $\Longrightarrow$ Resum weak $\ln\left(\frac{\hat{s}}{M_B}\right)$ logs?

Questions: Unordered states

...and the trouble with weak bosons continues:



If a QCD-like history is enforced on this state, it will often be unordered.

Unordered integrations are beyond the accuracy of PS resummation. We cannot currently treat the resummation of unordered shower splittings, and don't have guidelines for choosing α_S scales.

Questions: Unordered states

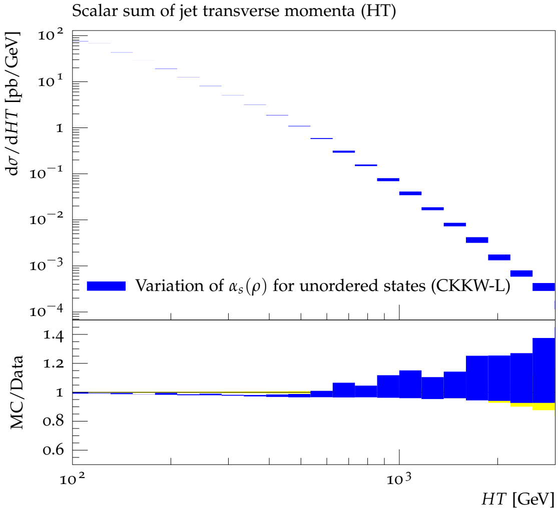
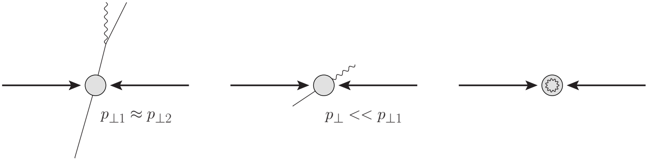


Figure: H_T in CKKW-L merging for Z+jets events @ 100 TeV

Questions: Unordered states

. . . and the trouble with weak bosons continues:

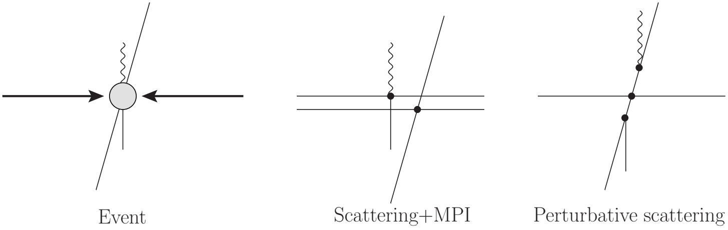


If a QCD-like history is enforced on this state, it will often be unordered.

Unordered integrations are beyond the accuracy of PS resummation. We cannot currently treat the resummation of unordered shower splittings, and don't have guidelines for choosing α_S scales.

$\Longrightarrow$ Need unordered shower emissions (NNLL?) to improve this.

Questions: Competition with MPI

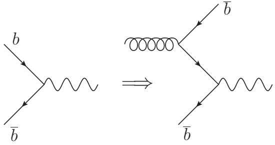


Assume we understand weak showers and sub-leading QCD logs. We still only model the competition between MPI and perturbative QCD!

At LHC, jets from MPI are relatively soft. $\Rightarrow$ Small effects.
At 100 TeV, MPI jets can be relatively hard. $\Rightarrow$ Competition must be understood!

- Can we simply only look at jets with large $p_\perp$, i.e ignore competition?
- Do we need to ME-correct MPI jets?
- Do we need weak bosons from MPI?

Showers include a treatment of flavour thresholds. These have usually been considered part of the shower.



Questions: Flavour thresholds

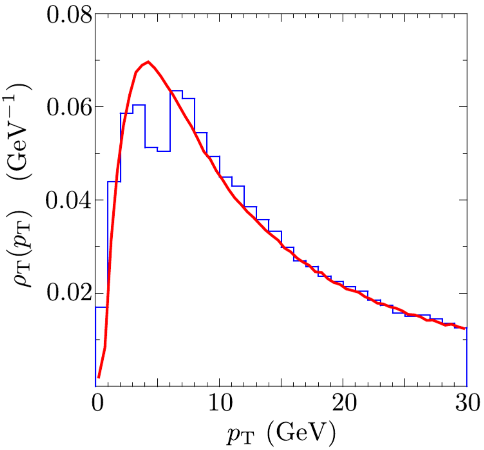
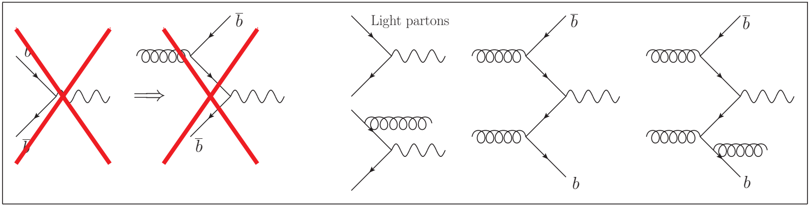


Figure ($p_\perp$ of b-quark in $\mathrm{b}\bar{\mathrm{b}} \to Z$, boosted Z) taken from arXiv:1401.6364[hep-ph] by Nagy, Soper

Questions: Flavour thresholds

Showers include a treatment of flavour thresholds. These have usually been considered part of the shower.

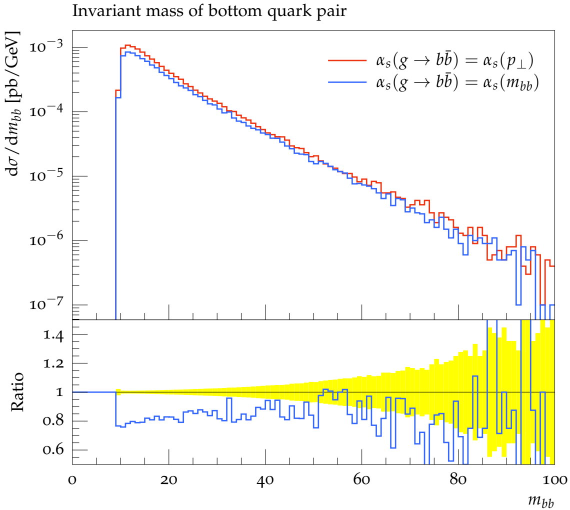


Should we also use matrix elements to describe the flavour thresholds? This only works if the logs are not large[1].

$\Longrightarrow$ To include the flavour thresholds correctly, work in a 4F scheme, i.e. combine multi-light-jet MEs with multi-light/heavy-jet MEs. Question: How many b's do we need from the ME?

[1] see e.g. JHEP1207(2012)022 by Maltoni, Ridolfi, Ubiali

Questions: Flavour thresholds



Note: This is LEP. Question: What scale to choose for $\alpha_s(\mathrm{g} \to \mathrm{b}\bar{\mathrm{b}})$?

Summary

- Unitary fixed-order + parton shower schemes are the norm.
- Basically all schemes except older merging schemes preserve (selected) inclusive cross sections.
- Most of these schemes are in fact not limited to PS resummation and could be used elsewhere.
- PS improvements will directly improve FO+PS schemes.
- But is a unitary paradigm a good idea?

  …I believe there are many subtleties

  …but we're now in the position to address these since we've gotten some fixed-order issues out of the way!