Non-perturbative QCD effects in qT spectra of DY and Z-boson production

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based on:

arXive: 1407.3311, with U. D' Alesio (Cagliari), M.G. Echevarría (NIKHEF), S. Melis (Torino)
arXive: 1402.0869, with M.G. Echevarría (NIKHEF), A. Idilbi (Penn U.),

and also PLB726(2013) 795,
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IFT WORKSHOP 2014

Topics and outline

We have finally factorized cross-sections with TMDs (Collins '11, EIS '11-13). How do we know TMDs?

* Transverse momentum distributions involve non-perturbative QCD effects which go beyond the usual PDF formalism. New factorization theorem are required.

* Spin dependent observables and transverse momentum dependent observables need factorization theorems with TMD's

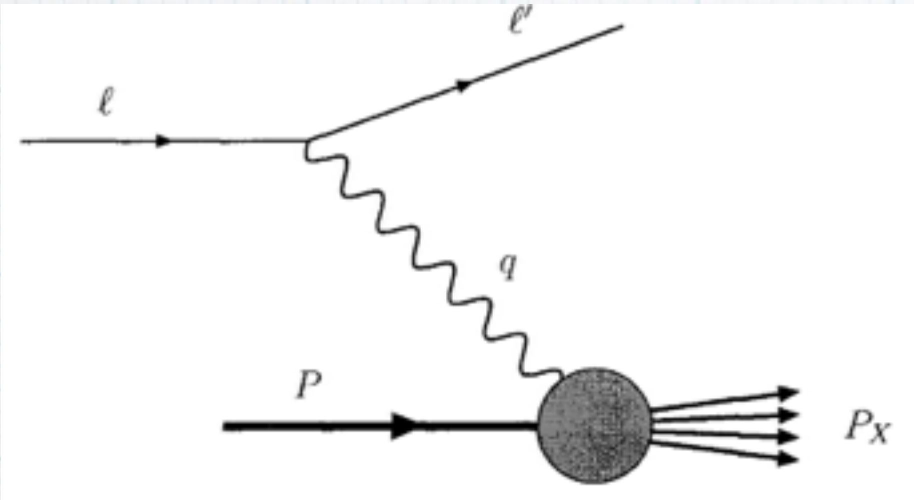
* TMD's are the fundamental non-perturbative objects to be used in factorization theorems in (un-)polarized Drell-Yan, SIDIS, e+e- to 2 jets (multi-jets?). What about LHC?

* Properties of TMD's:
  1) The evolution of all TMD's is universal (alike PDF and FF it is process independent)
  2) The evolution of all TMD's is spin independent and it is the same for TMDPDF and **TMDFF**

* We know the evolution completely at NNLL...

* Extraction of unpolarized TMDPDF from Drell-Yan and Z-boson production using completely resummed TMD's at NNLL

Outline of Factorization theorem



SIDIS as a study case:
both PDF and FF

$$q^2 \gg q_T^2$$

$$l(k) + N(P) \to l'(k') + h(P_h) + X(P_X)$$

Fact. scale

$$W^{\mu\nu} = H(Q^2/\mu^2)\frac{2}{N_c}\sum_q e_q \int d^2k_{n\perp} d^2k_{\bar{n}\perp} \delta^{(2)}(\mathbf{q}_\perp + \mathbf{k}_{\mathbf{n}\perp} - \mathbf{k}_{\bar{\mathbf{n}}\perp})$$

$$\times \mathrm{Tr}\left[F(x, \mathbf{k}_{\mathbf{n}\perp}, S; Q^2/\alpha, \mu^2)\,\gamma^\mu D(z, \hat{P}_{h\perp}, S_h; Q^2\alpha, \mu^2)\gamma^\nu\right]$$

Hard coeff.

TMDPDF

TMDFF

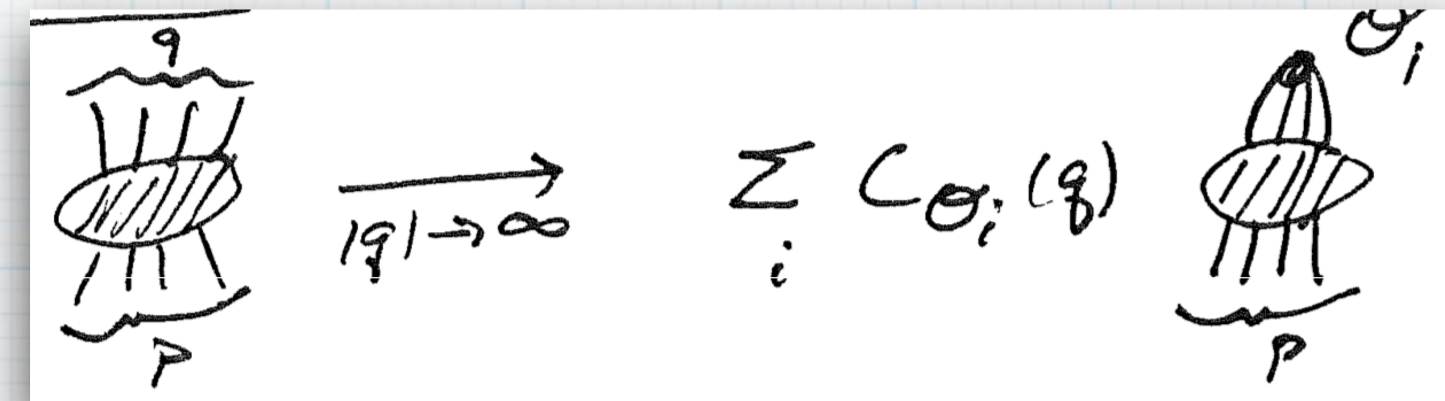
Soft splitting number

$$\mathbf{k}_{\bar{\mathbf{n}}\perp} = -\hat{\mathbf{P}}_{\mathbf{h}\perp}/z$$

$$\zeta_F = Q^2/\alpha$$

$$\zeta_D = \alpha\, Q^2$$

Origin of factorization and EFT



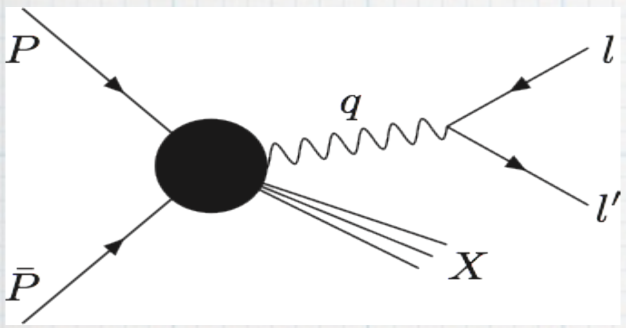
Drawing from J. Preskill lectures

In Green function, when the momenta "q" of some fields are much larger then the others we can "factor out" the hard momenta onto Wilson coefficient and effective matrix elements and use RGE on coefficients to resum large logs (Wilson, Zimmerman, Callan, Symanzyk '70-'72)

Wilson coefficients and hard factors depend only on the high scale "q" and the factorization scale "μ" : no other (IR-sensitive) scale.

Same philosophy in the construction of EFT: identification of modes, effective field theory Lagrangians, factorized cross sections, Wilson coefficients, ...

Energy scales: DY example



Fundamental condition for factorization

$$q^2 = Q^2 \gg q_T^2$$

Smooth transition

$q_T^2 \sim \Lambda_{QCD}^2$ ⇒ $$\tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

$q_T^2 \gg \Lambda_{QCD}^2$ ⇒ $$\tilde{M} = H(Q^2/\mu^2)\, \tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\, \tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\, f_n(x_n; \mu^2)\, f_{\bar{n}}(x_{\bar{n}}; \mu^2)$$

$$\tilde{C}(b^2\mu^2, Q^2/\mu^2)$$

Matching (Wilson) coefficients are extracted regulating consistently two theories above and below the (matching) scale

Processes with different energy scales are more easily treated with EFT: same results with Pert. QCD

$$\mathrm{QCD} \xrightarrow{q_T^2 \gg \Lambda_{QCD}^2} \mathrm{SCET}_{q_T} \xrightarrow{q_T^2 \sim \Lambda_{QCD}^2} \mathrm{SCET}_{\Lambda_{\mathrm{QCD}}}$$

Evolution kernel for TMD's

$$\tilde{F}^{[\Gamma]}_{f/N}(x, \mathbf{b}_\perp, S; \zeta_{F,f}, \mu_f^2) = \tilde{F}^{[\Gamma]}_{f/N}(x, \mathbf{b}_\perp, S; \zeta_{F,i}, \mu_i^2)\, \tilde{R}\left(b_T; \zeta_{F,i}, \mu_i^2, \zeta_{F,f}, \mu_f^2\right) ,$$

$$\tilde{D}^{[\Gamma]}_{h/f}(z, \mathbf{b}_\perp, S_h; \zeta_{D,f}, \mu_f^2) = \tilde{D}^{[\Gamma]}_{h/f}(z, \mathbf{b}_\perp, S_h; \zeta_{D,i}, \mu_i^2)\, \tilde{R}\left(b_T; \zeta_{D,i}, \mu_i^2, \zeta_{D,f}, \mu_f^2\right) ,$$

$$\tilde{R}(b; \zeta_i, \mu_i^2, \zeta_f, \mu_f^2) = \exp\left\{\int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}}\, \gamma\left(\alpha_s(\bar{\mu}), \ln\frac{\zeta_f}{\bar{\mu}^2}\right)\right\} \left(\frac{\zeta_f}{\zeta_i}\right)^{-D(b_T;\mu_i)} ,$$

For consistency the A.D. of the TMD is the opposite of the one of the Hard coefficient

SIDIS:

$$\gamma_H = -\gamma_F\left(\alpha_s(\mu), \ln\frac{\zeta_F}{\mu^2}\right) - \gamma_D\left(\alpha_s(\mu), \ln\frac{\zeta_D}{\mu^2}\right)$$

$$\gamma_{F,D}\left(\alpha_s(\mu), \ln\frac{\zeta_{F,D}}{\mu^2}\right) = -\Gamma_{\text{cusp}}(\alpha_s(\mu)) \ln\frac{\zeta_{F,D}}{\mu^2} - \gamma^V(\alpha_s(\mu))$$

$$\frac{dD}{d\ln\mu} = \Gamma_{cusp}$$

Cusp known at order a^3

D-resummation

$$\frac{dD(b;\mu)}{d\ln\mu} = \Gamma_{cusp}(\alpha_s) \qquad D(b;\mu) = \sum_{n=1}^{\infty} d_n(L_\perp)\left(\frac{\alpha_s}{4\pi}\right)^n$$

LL NLL NNLL

$$\begin{aligned}
d_1(L_\perp) &= d_1^{(1)} L_\perp + d_1^{(0)} \\
d_2(L_\perp) &= d_2^{(2)} L_\perp^2 + d_2^{(1)} L_\perp + d_2^{(0)} \\
d_3(L_\perp) &= d_3^{(3)} L_\perp^3 + d_3^{(2)} L_\perp^2 + d_3^{(1)} L_\perp + d_3^{(0)} \\
d_4(L_\perp) &= d_4^{(4)} L_\perp^4 + d_4^{(3)} L_\perp^3 + d_4^{(2)} L_\perp^2 + d_4^{(1)} L_\perp + d_4^{(0)} \\
d_5(L_\perp) &= \ldots
\end{aligned}$$

$$D(b;Q_i) = D(b;\mu_b) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{\text{cusp}}; \qquad \mu_b = 2e^{-\gamma_E}/b$$

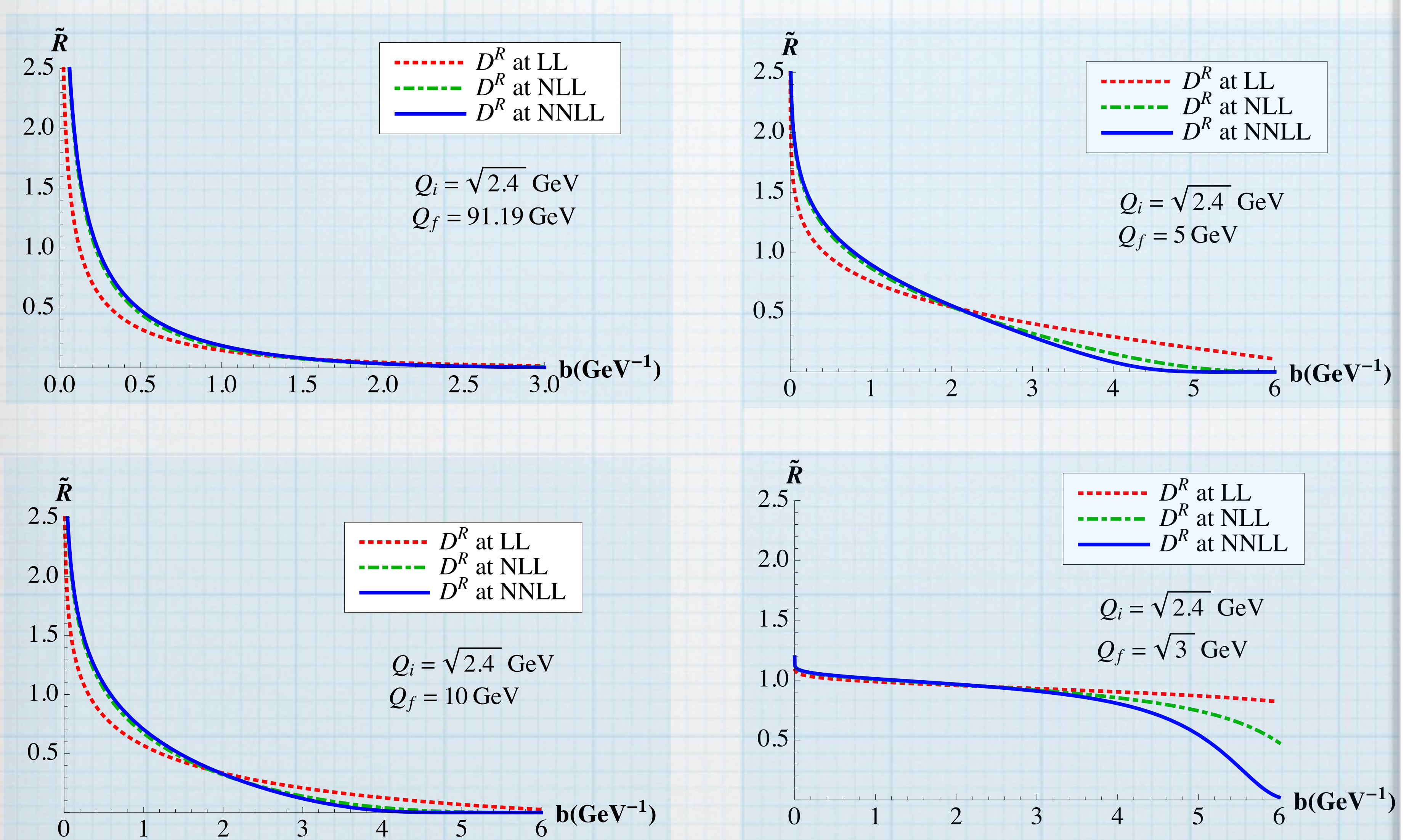
$$D(b;Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln \frac{\alpha_s(Q_i)}{\alpha_s(\mu_b)} \longrightarrow D(b;Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln(1-X)$$

$$\alpha_s(\mu_b) = \alpha_s(Q)/(1-X)$$

Landau pole

The perturbative expansion of the D is valid in limited (but large, using **resummation**) portion of Impact Parameter Space

Plots for evolution kernel (fixed scale example)



- Very good convergence up to b=4-5/GeV in all cases
- The region sensitive to the Landau pole is strongly suppressed b>5/GeV
- For M=Mz we are sensitive only to b<1.5/GeV region
- For M=3-5 GeV we are sensitive only to b<4/GeV region
- For M <2 GeV we can be sensitive to the Landau pole region

- Studying processes at different energies one explores different regions in IPS
- The Landau pole problems appear there where also the Factorization fails (or starts to fail). **This is a QCD general problem!**

Building a TMD: Inputs from Pert.QCD

The asymptotic limit of TMDs should be included in the construction of TMDs: The Standard TMD form

$$\tilde{F}_{q/N}(x,\vec{b},Q_i,\mu) = \left(\frac{Q_i^2 b^2}{4e^{2\gamma_E}}\right)^{-D_R(b,\mu)} \sum_j \tilde{C}_{q\leftarrow j}(x,\vec{b}_\perp,\mu) \otimes f_{j/N}(x;\mu) \overset{?}{\otimes} M_q(x,\vec{b},Q_i)$$

OPE to PDF, valid for qT>> Λ_{QCD}

PDF

Process independent
Non-perturbative correction

Common to all analysis:
Calculated at 2-loops
Florence (S. Catani et al.'08), Zurich (T. Gehrmann. et al '13-14)

$$\tilde{C}_{q\leftarrow j}(x,\vec{b}_\perp,\mu) \equiv \exp(h_\Gamma - h_\gamma)\hat{C}_{q\leftarrow j}(x,\vec{b}_\perp,\mu)$$

$$\frac{dh_\Gamma}{d\ln\mu} = \Gamma_{cusp} L_\perp$$

$$\frac{dh_\gamma}{d\ln\mu} = \gamma^V$$

$$h_\Gamma^R(b,\mu) = \int_{\alpha_s(1/\hat{b})}^{\alpha_s(\mu)} d\alpha' \frac{\Gamma^F_{cusp}(\alpha')}{\beta(\alpha')} \int_{\alpha_s(1/\hat{b})}^{\alpha'} \frac{d\alpha}{\beta(\alpha)}.$$

- Exponentiation of part of the coefficient (Kodaira, Trentadue 1982, Becher, Neubert Wilhelm 2011)
- Complete resummation of the logs in the exponent, as for the resummed D

Building a TMD: Inputs from Pert.QCD

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OPE to PDF, valid for $q_T \gg \Lambda_{QCD}$

PDF

Process independent Non-perturbative correction

Mq expected to correct resummed D and OPE

The splitting between coeff. and PDF can generate large logs: a wise choice of factorization scale can avoid this problem.

Becher, Neubert, Wilhelm '12

Ideally: $\mu \sim q_T$ In practice: $\mu \sim Q_i = Q_0 + q_T$

Qo is the scale where PDF are better defined: $Q_0 \sim 2$ GeV

Building a TMD: Inputs from Pert.QCD

$$\tilde{M} = H(Q^2/\mu^2)\tilde{F}_n(x_n, b^2; Q^2, \mu^2)\tilde{F}_{\bar{n}}(x_{\bar{n}}, b^2; Q^2, \mu^2)$$

Hard coeff. using π-resummation (Ahrens at al. '08):evolution of H from -Q^2 to Q^2

	$\mu_h^2 = m_Z^2$	$\mu_h^2 = -m_Z^2$
NLL	$1.000^{+0.160}_{-0.060}$	$1.334^{+0.201}_{-0.074}$
NNLL	$1.087^{+0.010}_{-0.001}$	$1.131^{+0.001}_{-0.014}$
N^3LL	$1.119^{+0.006}_{-0.001}$	$1.130^{+0.001}_{-0.001}$

Better convergence of the perturbative series

Experimental Data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
points	32	16	41	9
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
σ	248 ± 11 pb	221 ± 11.2 pb	256 ± 15.2 pb	255.8 ± 16.7 pb

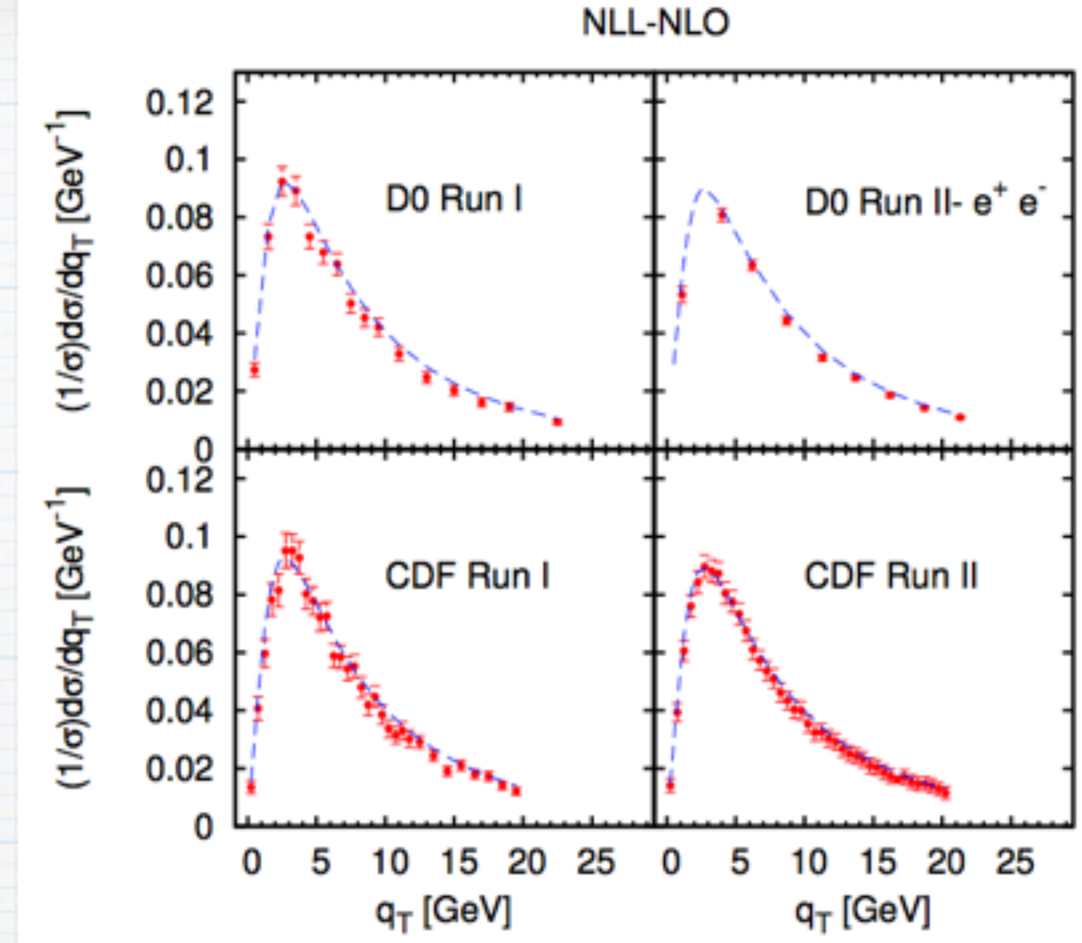
Z, run I: Becher, Neubert, Wilhelm 2011
Catani et al. 2009

Total cross section improved from run I to run II

	E288 200	E288 300	E288 400	R209
points	35	35	49	6
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p p
M range used	4-9 GeV	4-9 GeV	5-9 and 10.5-14 GeV	5-8 and 11-25 GeV
Other kin. var	y=0.4	y=0.21	y=0.03	
Observable	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$Ed^3\sigma/d^3\mathbf{p}$	$d\sigma/dq_T^2$

Expected to be insensitive to Landau pole region
Factorization hypothesis hold

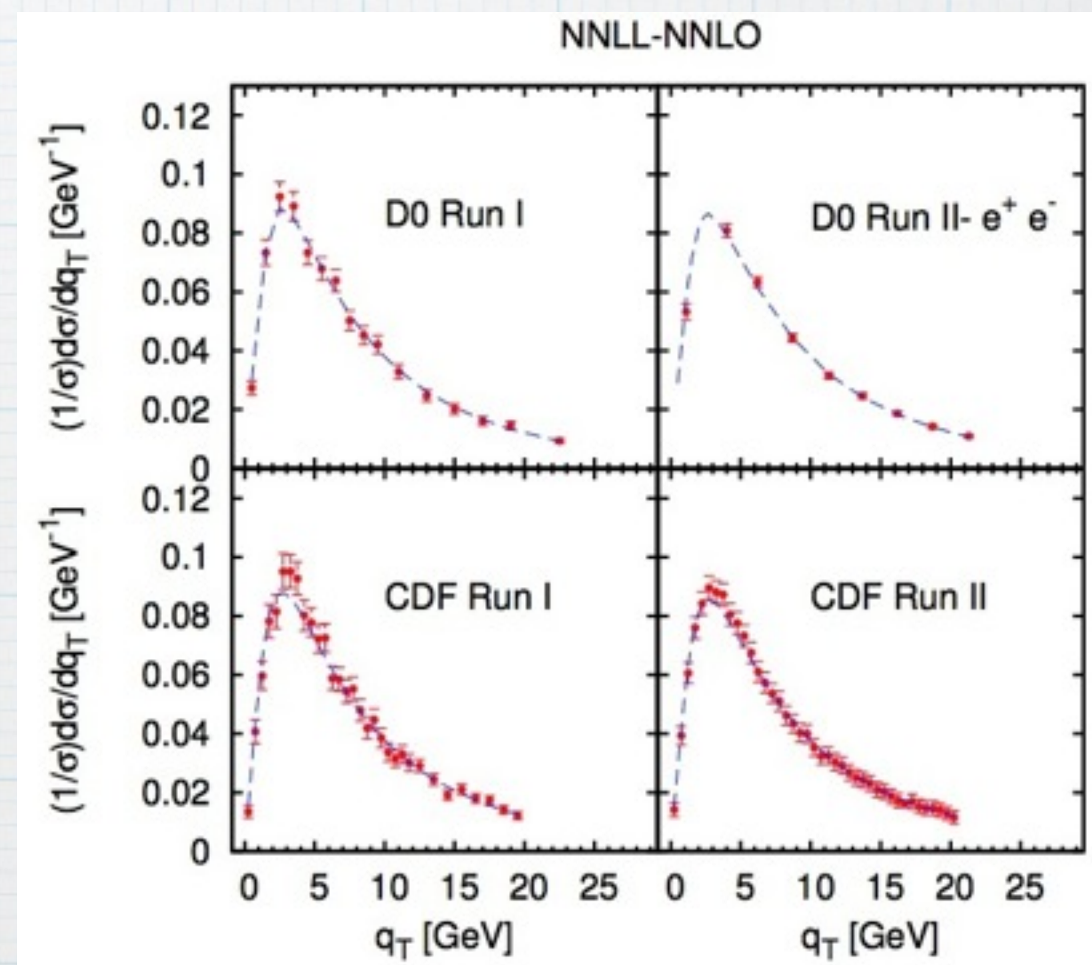
Results



Z-boson data are (fairly) sensitive to functional non-perturbative form (gaussian vs. exponential).
Non-perturbative kinematics poorly caught

Avoiding systematics

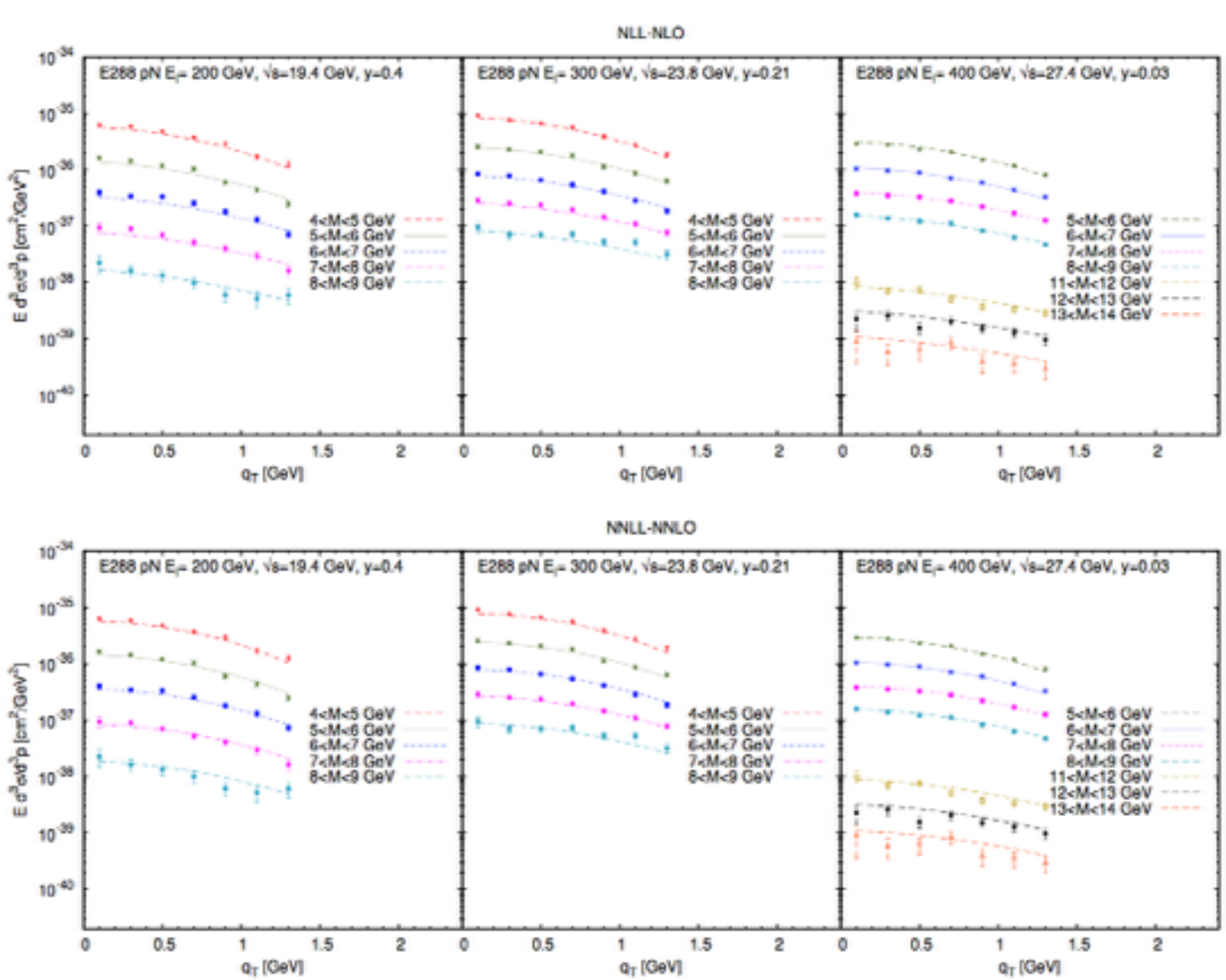
Data: $$\frac{1}{\sigma_{exp}}\left(\frac{d\sigma}{dq_T}\right)_{exp}$$



Theory: $$\frac{1}{\sigma_{th}}\left(\frac{d\sigma}{dq_T}\right)_{th}$$

DYNNLO: Catani, Grazzini '07, Catani, Cieri, Ferrera, de Florian, Grazzini '09

Results



Exp. Normalization
N_{E288}, N_{R209}
deduced from the fit.
Total: 4 parameters

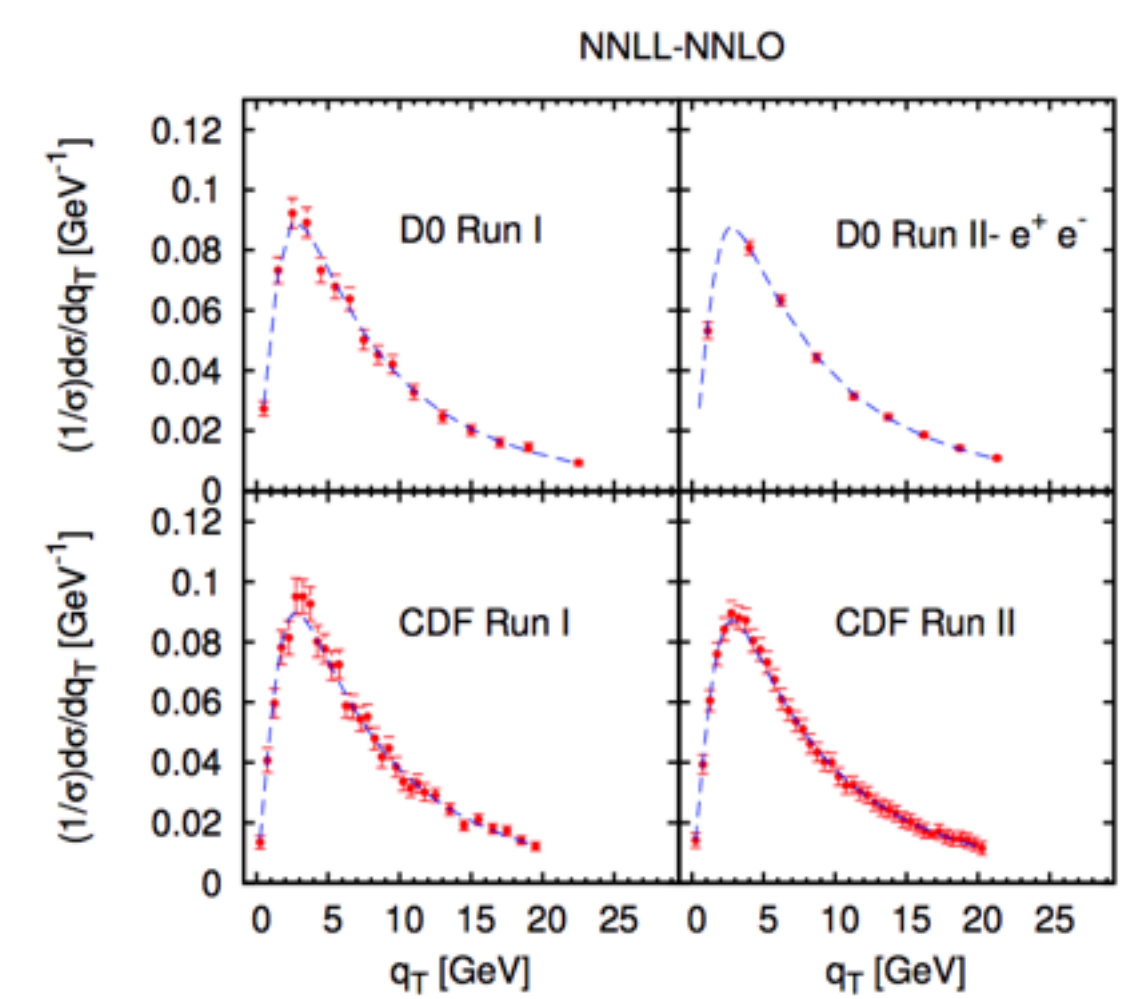
$$\tilde{F}^{NP}_{q/N}(x, b_T; Q) = e^{-\lambda_1 b_T}\left(1 + \lambda_2 b_T^2\right)$$

		NNLL, NNLO	NLL, NLO
	points	χ^2/points	χ^2/points
	223	1.10	1.48
E288 200	35	1.53	2.60
E288 300	35	1.50	1.12
E288 400	49	2.07	1.79
R209	6	0.16	0.25
CDF Run I	32	0.74	1.31
D0 Run I	16	0.43	1.44
CDF Run II	41	0.30	0.62
D0 Run II	9	0.61	2.40

NLL	223 points	$\chi^2/\text{d.o.f.} = 1.51$
	$\lambda_1 = 0.26^{+0.05_{\rm th}}_{-0.02_{\rm th}} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.13 \pm 0.01_{\rm th} \pm 0.03_{\rm stat}$ GeV^2
	$N_{\rm E288} = 0.9^{+0.2_{\rm th}}_{-0.1_{\rm th}} \pm 0.04_{\rm stat}$	$N_{\rm R209} = 1.3 \pm 0.01_{\rm th} \pm 0.2_{\rm stat}$
NNLL	223 points	$\chi^2/\text{d.o.f.} = 1.12$
	$\lambda_1 = 0.33 \pm 0.02_{\rm th} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.13 \pm 0.01_{\rm th} \pm 0.03_{\rm stat}$ GeV^2
	$N_{\rm E288} = 0.85 \pm 0.01_{\rm th} \pm 0.04_{\rm stat}$	$N_{\rm R209} = 1.5 \pm 0.01_{\rm th} \pm 0.2_{\rm stat}$

$$\frac{\chi^2}{points}\Big|_{global} \simeq 1.1$$

Results



Exp. Normalization
NE288, NR209
deduced from the fit.
Total: 4 parameters

Korchemsky, Sterman

$$\tilde{F}^{\rm NP}_{q/N}(x, b_T; Q) = e^{-\lambda_1 b_T}\left(1+\lambda_2 b_T^2\right)\left(\frac{Q^2}{Q_0^2}\right)^{-\frac{\lambda_3}{2}b_T^2}$$

		NNLL, NNLO	NLL, NLO
	points	χ^2/points	χ^2/points
	223	0.79	1.40
E288 200	35	1.24	2.27
E288 300	35	0.90	1.20
E288 400	49	1.33	1.69
R209	6	0.24	0.30
CDF Run I	32	0.58	1.26
D0 Run I	16	0.36	1.43
CDF Run II	41	0.15	0.48
D0 Run II	9	0.36	2.26

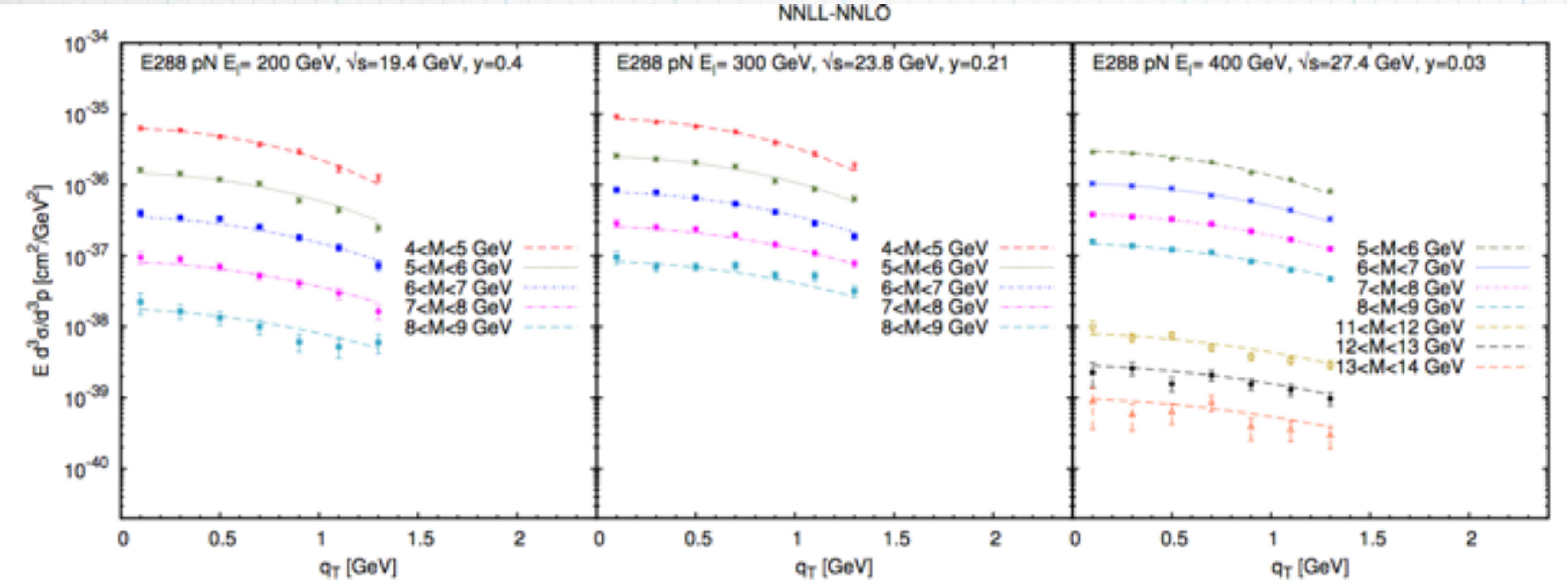
NLL	223 points	χ^2/d.o.f. $= 1.44$
	$\lambda_1 = 0.24^{+0.06_{\rm th}}_{-0.02_{\rm th}} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.17 \pm 0.02_{\rm th} \pm 0.05_{\rm stat}$ GeV^2
	$\lambda_3 = 0.03 \pm 0.02_{\rm th} \pm 0.01_{\rm stat}$ GeV^2	
	$N_{\rm E288} = 0.85^{+0.2_{\rm th}}_{-0.1_{\rm th}} \pm 0.04_{\rm stat}$	$N_{\rm R209} = 1.2 \pm 0.2_{\rm th} \pm 0.2_{\rm stat}$
NNLL	223 points	χ^2/d.o.f. $= 0.81$
	$\lambda_1 = 0.30 \pm 0.02_{\rm th} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.22 \pm 0.01_{\rm th} \pm 0.05_{\rm stat}$ GeV^2
	$\lambda_3 = 0.05 \pm 0.01_{\rm th} \pm 0.02_{\rm stat}$ GeV^2	
	$N_{\rm E288} = 0.78^{+0.08_{\rm th}}_{-0.04_{\rm th}} \pm 0.05_{\rm stat}$	$N_{\rm R209} = 1.3 \pm 0.1_{\rm th} \pm 0.2_{\rm stat}$

$$\chi^2/points|_{global} \simeq 0.8$$

Results

Exp. Normalization
N_{E288}, N_{R209}
deduced from the fit.

Total: 4 parameters



		NNLL, NNLO	NLL, NLO
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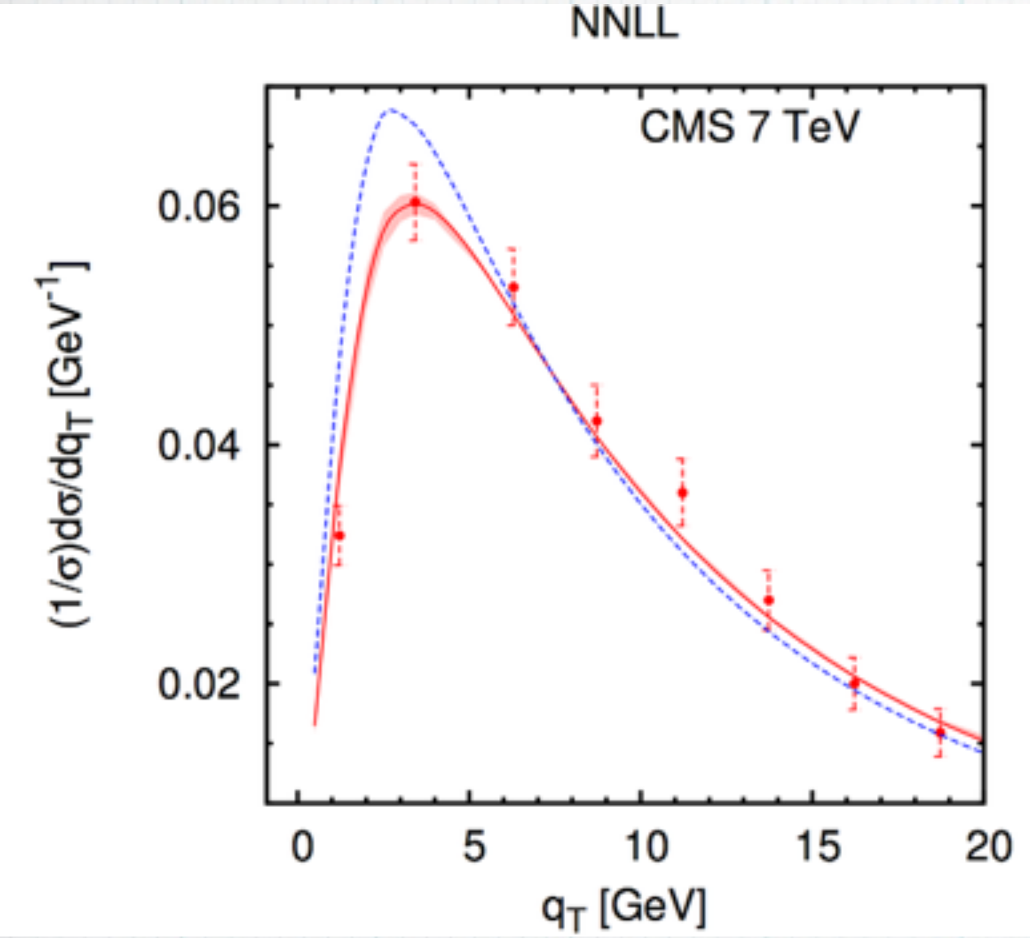
NLL	223 points	$\chi^2/\text{d.o.f.} = 1.44$
	$\lambda_1 = 0.24^{+0.06_{\rm th}}_{-0.02_{\rm th}} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.17 \pm 0.02_{\rm th} \pm 0.05_{\rm stat}$ GeV^2
	$\lambda_3 = 0.03 \pm 0.02_{\rm th} \pm 0.01_{\rm stat}$ GeV^2	
	$N_{\rm E288} = 0.85^{+0.2_{\rm th}}_{-0.1_{\rm th}} \pm 0.04_{\rm stat}$	$N_{\rm R209} = 1.2 \pm 0.2_{\rm th} \pm 0.2_{\rm stat}$
NNLL	223 points	$\chi^2/\text{d.o.f.} = 0.81$
	$\lambda_1 = 0.30 \pm 0.02_{\rm th} \pm 0.05_{\rm stat}$ GeV	$\lambda_2 = 0.22 \pm 0.01_{\rm th} \pm 0.05_{\rm stat}$ GeV^2
	$\lambda_3 = 0.05 \pm 0.01_{\rm th} \pm 0.02_{\rm stat}$ GeV^2	
	$N_{\rm E288} = 0.78^{+0.08_{\rm th}}_{-0.04_{\rm th}} \pm 0.05_{\rm stat}$	$N_{\rm R209} = 1.3 \pm 0.1_{\rm th} \pm 0.2_{\rm stat}$

$$\chi^2/points|_{global} \simeq 0.8$$

LHC



Prediction for Z production with CMS data: Peak region

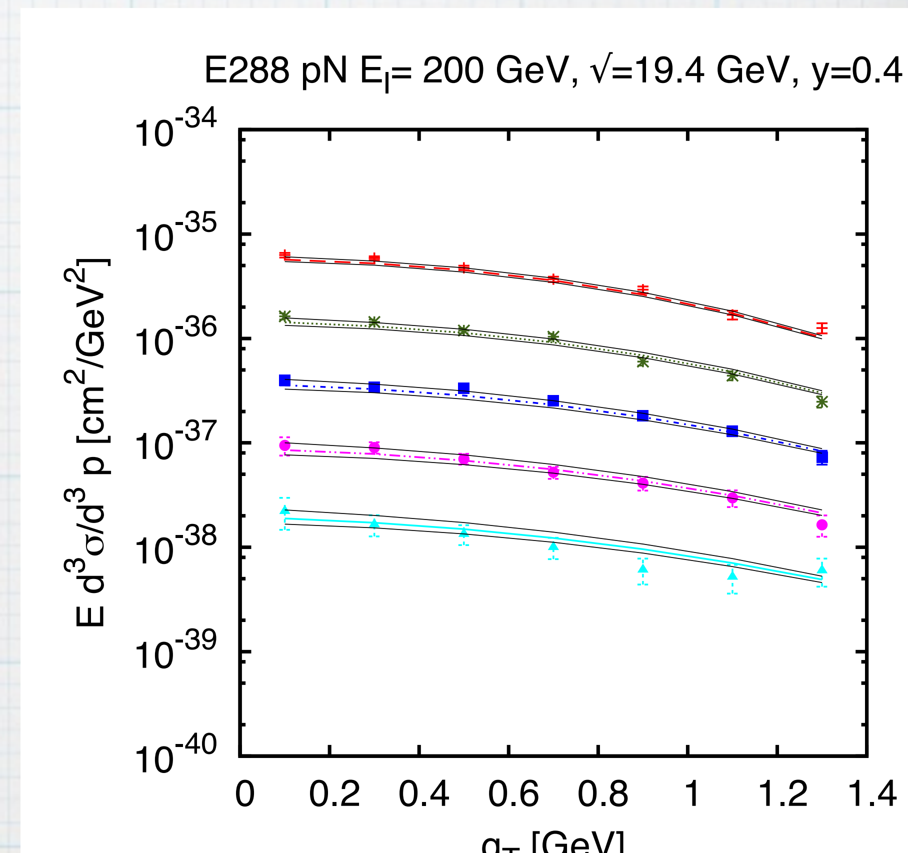
Bin width not shown

$$\frac{\chi^2}{points} \simeq 0.9$$

Scale dependence

$$m_c \sim 1.3\ \text{GeV} < Q_0 < 2.7\ \text{GeV}$$

1st bin energy



* Data are not sensitive to: the Landau pole region in IPS, non-perturbative Q-dependence. The study of flavor dependence needs to include also W-production data....

Conclusions

The analysis of current data coming from experiments run at different energy require the use of evolution of TMD's and full resummation techniques:

Reduction of model dependence
Recovery of the complete perturbative limit (we have to connect to pQCD results)
Improved Convergence of the perturbative series
We have tested all this in DY and Z-boson qT spectra: agreement with data

* Fits for unpolarized TMDPDF in DY and Z-production, performed with data which fulfill fundamental conditions for factorization, allow to fix the non-perturbative parameters. More data and more processes are welcome (SIDIS, ee-> jj,....)
* TMDs can be used to fix the final precision at LHC, fixing the amount of non-perturbative QCD effects: all knowledge from PQCD studies must be used.
* Features of TMD's:

  The evolution for TMDPDF's and TMDFF's is the same and spin independent
  TMD's are universal (they can be extracted from DY, SIDIS, ee->2 jets,....)
* QCD non perturbative effects can be taken under control with TMD formalism and (not too extremely) low energy data.
* A new era for precision physics ?

Thanks.....

Back up:

Evolution Kernel: EIS vs CSS

Modes in EFT

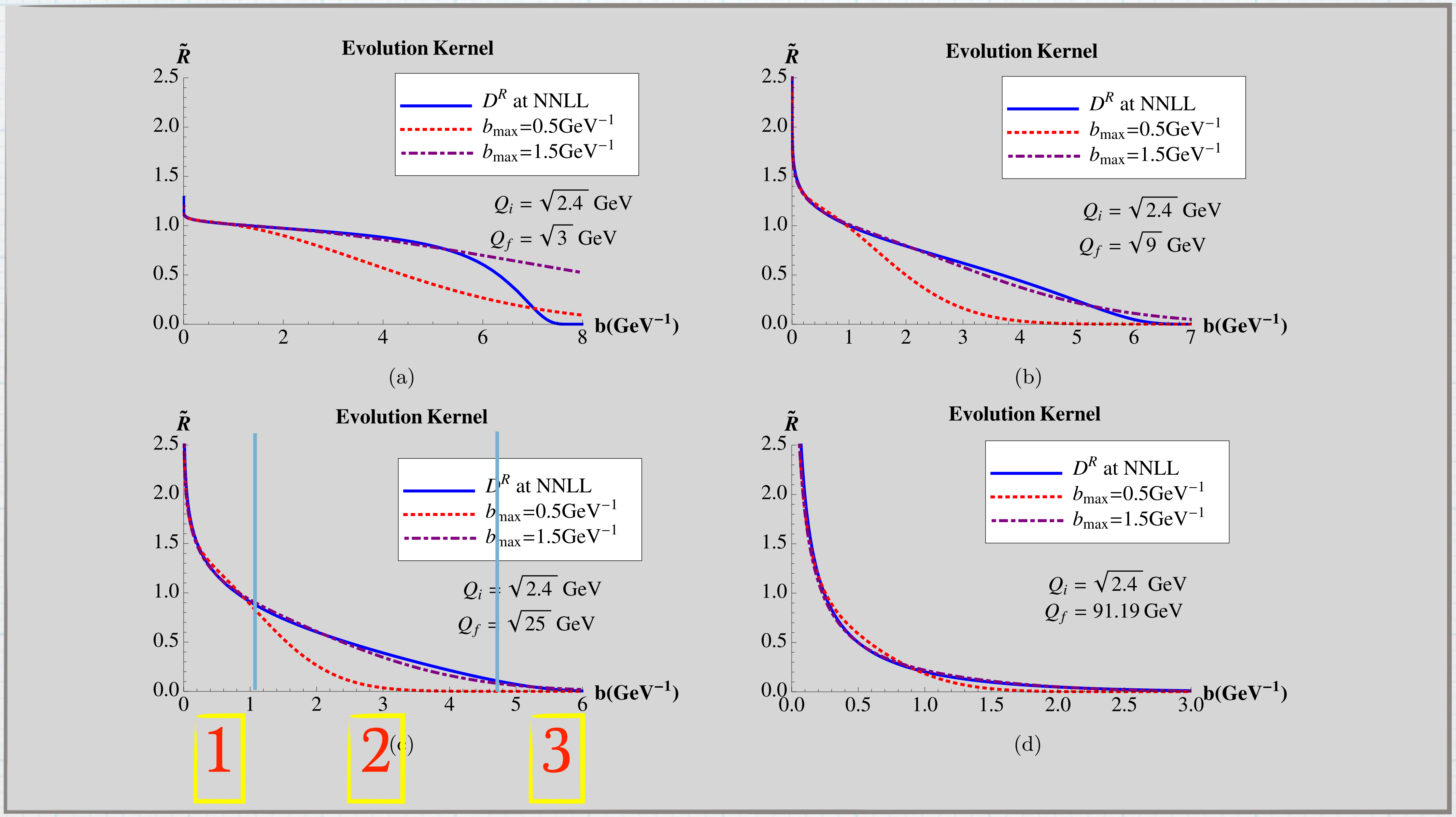
Rapidity divergences

Definition of TMDs

Regulators

Soft function

EIS vs CSS

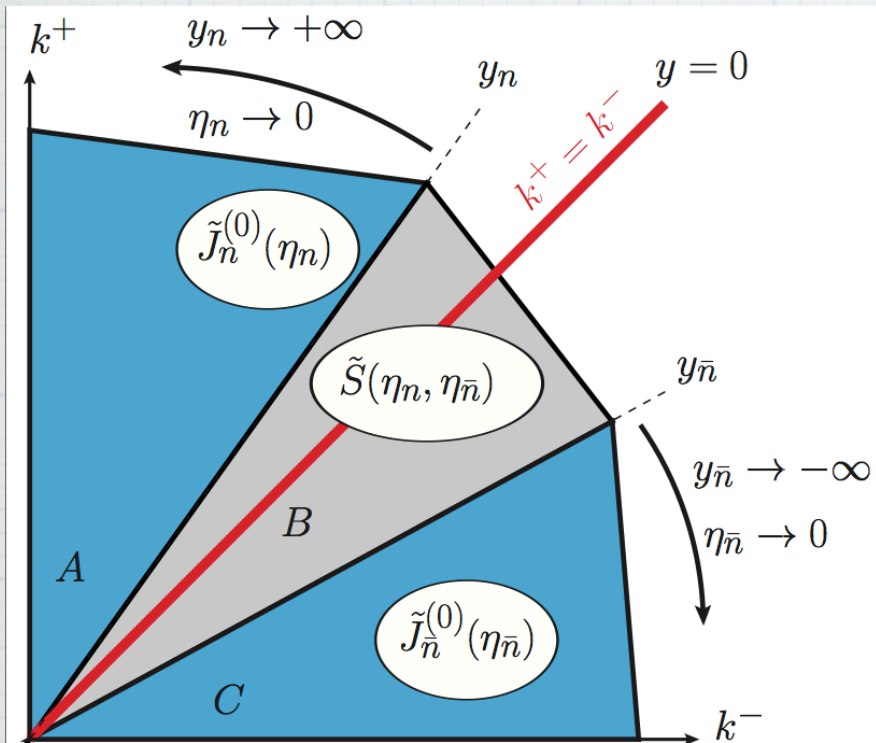


1: EIS and CSS agree, complete perturbative region. Model for TMD bulk

2:EIS: Model Independent Evolution (Completely Resummed Kernel). Model for TMD bulk
CSS:Model Dependent Evolution (bmax and g2). Model for TMD bulk+more "cooking" (see Stefano talk)

3:Landau pole region: Modeled both in EIS and CSS

Modes in EFT



(+,-,perp)

$$k_n \sim Q(1, \lambda^2, \lambda) \quad \to \quad y \gg 0$$
$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda) \quad \to \quad y \ll 0$$
$$k_s \sim Q(\lambda, \lambda, \lambda) \quad \to \quad y \approx 0$$
$$\lambda \sim \frac{q_T}{Q}$$

$$n = (1, 0, 0, 1)$$
$$\bar{n} = (1, 0, 0, -1)$$

$$\chi_n = W_n^\dagger \xi_n$$

Using power counting we have collinear, anti-collinear, and soft sectors

$$H(Q^2)\, \tilde{J}_n^{(0)}(\eta_n)\, \tilde{S}(\eta_n, \eta_{\bar{n}})\, \tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})$$

In EFT each mode belongs to a Hilbert space separate from the others.
To each mode correspond a different Lagrangian
Boosts mix soft and collinear modes (same invariant mass)

$$J_n^{(0)}(0^+, y^-, y_\perp) = \frac{1}{2}\sum_{\sigma_1} \langle N_1(P,\sigma_1)\,|\bar{\chi}_n(0^+, y^-, y_\perp)\frac{\bar{\not{n}}}{2}\chi_n(0)|\, N_1(P,\sigma_1)\rangle$$

$$J^{(0)}_{\bar{n}}(y^+, 0^-, y_\perp) = \frac{1}{2}\sum_{\sigma_1} \langle N_2(\bar{P},\sigma_2)\,|\bar{\chi}_{\bar{n}}(0)\frac{\not{n}}{2}\chi_{\bar{n}}(y^+, 0^-, y_\perp)|\, N_2(\bar{P},\sigma_2)\rangle$$

$$S(0^+, 0^-, y_\perp) = \langle 0\,|\,\mathrm{Tr}\, \bar{\mathbf{T}}\big[S_n^{T\dagger} S_{\bar{n}}^T\big](0^+, 0^-, y_\perp)\mathbf{T}\big[S_{\bar{n}}^{T\dagger} S_n^T\big](0)\,|\,0\rangle, \qquad \chi = W^{T\dagger}\xi$$

multipole expansion fixes arguments

Definition of TMD's

Positive and negative rapidity quanta can be collected into 2 different TMDs because of the splitting of the soft function

$$\tilde{S}(b_T; \frac{Q^2\mu^2}{\Delta^+\Delta^-}, \mu^2) = \tilde{S}_-\left(b_T; \zeta_F, \mu^2; \Delta^-\right) \tilde{S}_+\left(b_T; \zeta_D, \mu^2; \Delta^+\right) ,$$

$$\tilde{S}_-\left(b_T; \zeta_F, \mu^2; \Delta^-\right) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \alpha \frac{\Delta^-}{\bar{p}^-}\right)} ,$$

$$\tilde{S}_+\left(b_T; \zeta_D, \mu^2; \Delta^+\right) = \sqrt{\tilde{S}\left(\frac{1}{\alpha}\frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\zeta_F = Q^2/\alpha$$

$$\zeta_D = \alpha\ Q^2$$

Pure collinear

TMDPDF

$$\ln F_{ij}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2; \Delta^-) = \ln \tilde{\Phi}_{ij}^{(0)}(x, \mathbf{b}, S; \mu^2; \Delta^-) + \ln \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-)$$

TMDFF

$$\ln D_{ij}(x, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2; \Delta^+) = \ln \tilde{\Delta}_{ij}^{(0)}(x, \mathbf{b}, S_h; \mu^2; \Delta^+) + \ln \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^-)$$

Soft function: structure and properties

In the high qT limit: $Q \gg q_T \gg \Lambda_{QCD}$ the hadronic tensor is

$$\tilde{M} = H(Q^2/\mu^2)\tilde{C}(x_n, z_{\bar{n}}, L_\perp, Q^2/\mu^2) f_n(x_n, \Delta^-/\mu^2) d_{\bar{n}}(z_{\bar{n}}, \Delta^+/\mu^2)$$

and PDF, f_n, and FF, d_n, have single log dependence on UV/IR cutoff (Korchemsky, Radyushkin 1987)

$$\ln f_n = \mathcal{R}_{f1}(x_n, \alpha_s) + \mathcal{R}_{f2}(x_n, \alpha_s) \ln \frac{\Delta^-}{\mu^2}$$

$$\ln d_{\bar{n}} = \mathcal{R}_{f1}(z_{\bar{n}}, \alpha_s) + \mathcal{R}_{f2}(z_{\bar{n}}, \alpha_s) \ln \frac{\Delta^+}{\mu^2}$$

Splitting of the soft function

$$\ln \tilde{S} = \mathcal{R}_s(b_T, \alpha_s) + 2D(b_T, \alpha_s)\, \ln\left(\frac{\Delta^+\Delta^-}{Q^2\mu^2}\right)$$

$$\ln \tilde{S}_- = \frac{1}{2}\mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln\left(\frac{(\Delta^-)^2}{\zeta_F \mu^2}\right),$$

$$\ln \tilde{S}_+ = \frac{1}{2}\mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln\left(\frac{(\Delta^+)^2}{\zeta_D \mu^2}\right)$$

Using single log dependence

Q-dependence of TMD's

$$\zeta_F = Q^2/\alpha$$
$$\zeta_D = \alpha\ Q^2$$

$$\frac{d}{d\ln\zeta_F} \ln \tilde{F}^{[\Gamma]}_{f/N}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) = -D(b_T; \mu^2)\,,$$

$$\frac{d}{d\ln\zeta_D} \ln \tilde{D}^{[\Gamma]}_{h/f}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) = -D(b_T; \mu^2)\,.$$

The Q-dependence of the TMD is dictated by the soft function: spin independent

Soft function: structure and properties

All this implies that each **pure** collinear and soft sectors are of the form

$$\ln \tilde{J}_n^{(0)} = \mathcal{R}_{n1}(x_n, \alpha_s, L_\perp) + \mathcal{R}_{n2}(x_n, \alpha_s, L_\perp)\ \ln \frac{\Delta^-}{\mu^2}\ ,$$

$$\ln \tilde{J}_{\bar{n}}^{(0)} = \mathcal{R}_{\bar{n}1}(x_{\bar{n}}, \alpha_s, L_\perp) + \mathcal{R}_{\bar{n}2}(x_{\bar{n}}, \alpha_s, L_\perp)\ \ln \frac{\Delta^+}{\mu^2}\ ,$$

$$\ln \tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + 2D(\alpha_s, L_\perp)\ \ln \frac{\Delta^-\Delta^+}{Q^2\mu^2}\ .$$

* Each sector is linear in logs of the rapidity cut
* Each collinear sector depends just on 1 IR/rapidity regulator
* Each collinear sector depends solely on the corresponding collinear momentum. It is not possible that a Q dependence arise in a pure collinear sector, Q dependence arises only in the soft sector.
* The soft function is linear in the logs of rapidity regulator to cancel the corresponding logs in collinear sectors (In QCD there are no rapidity divergences)
* **The soft function is Hermitian, so it is the same for DIS, DY and ee to 2 jets**