Heterotic Cosmic String Vacua and Generalized Half-flat Compactifications

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3 Cosmic strings and Spin(7) structures
   - Cosmic strings
   - Spin(7) and generalized half-flat manifolds
Why the heterotic string?

Some reasons to study heterotic string theory:

- String theory: consistent theory of quantum gravity!
- Comes equipped with an $E_8 \times E_8$ (or SO(32)) gauge group $\Rightarrow$ good framework for grand unified models.
- Many candidate Standard Model compactifications known.
- Calabi–Yau compactification gives $\mathcal{N} = 1$ SUSY in $d = 4$ $\rightarrow$ suitable for MSSM-style models.
- Appealing mathematical framework, reasonably well-studied.
Calabi–Yau compactification

- Superstring theory is self-consistent only in 10 spacetime dimensions.

- Assume the extra 6 spatial dimensions are compactified.

- Lots of supersymmetry in $d = 10$ $\rightarrow$ want to break most of it.

- Amount of broken SUSY $\Rightarrow$ holonomy group of compactification manifold.

- Maximum holonomy is $\text{SO}(6) \cong \text{SU}(4)$ $\Rightarrow$ no SUSY preserved.

- Calabi–Yau manifold: $\text{SU}(3)$ holonomy $\Rightarrow$ 1/4 SUSY preserved
  - e.g. heterotic Calabi-Yau: 4 of 16 supercharges unbroken.
Moduli stabilization: Type IIB example

Compactification gives rise to moduli: flat directions in the potential → need to stabilize.

- Example: moduli stabilization in type IIB string theory.
- Theory contains R-R 3-form flux $F_3$ and NS-NS 3-form flux $H_3$.
- Compactify such that on the manifold, $F_3$ and $H_3$ are non-zero → flux compactification.
- In the right combination, dilaton and all complex structure moduli can be stabilized by these fluxes
- Kähler moduli remain unstabilized, can fix with eg.
  - non-perturbative effects (KKLT),
  - non-perturbative effects and perturbative $\alpha'$ corrections (LVS).
- All moduli stabilized!
Problems with heterotic moduli stabilization

- In heterotic string theory, only have NS-NS flux $H_3$.
- Can stabilize complex structure moduli... then what?
- Dilaton can be stabilized by gaugino condensation.
- No other non-perturbative effects, no other options for flux quantisation.
- In fact, problem is even worse:

**Strominger, 1986**

If a heterotic compactification on a manifold $Y$ has a maximally symmetric (e.g. Poincaré) vacuum and non-vanishing $H_3$, $Y$ is non-Calabi–Yau.

- Hence for a Calabi–Yau compactification, $H_3 = 0!$
A related issue is **mirror symmetry**.

- Type IIA compactified on $Y \leftrightarrow$ type IIB compactified on $\tilde{Y}$.
- Flux compactifications: R-R flux $F_3 \leftrightarrow F_0, F_2, F_4, F_6$.
- Problem: no obvious mirror dual for NS-NS flux $H_3$!
What is an SU(3) structure manifold?

- Mirror dual: manifold with SU(3) structure, but not Calabi-Yau

- **SU(3) structure**: there is a globally-defined spinor $\zeta$ that leaves 1/4 of the SUSY unbroken.

- Calabi–Yau case: $\zeta$ is covariantly constant with respect to the Levi-Civita connection $\nabla$.

- Non-CY case: $\nabla \zeta \sim T^0 \zeta$ (note: $\Gamma$ matrices/indices suppressed).

- $T^0$ is the **intrinsic torsion** of the manifold.

- **SU(3) decomposition**: torsion splits into 5 torsion classes,

$$T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5.$$
**Half-flat manifolds**

Two (not mutually exclusive) ways to satisfy Strominger’s theorem:

**Option 1:**

Study compactifications on SU(3) structure manifolds with torsion.

- Has been studied in eg. hep-th/0408121 (Gurrieri, Lukas, Micu), hep-th/0507173 (de Carlos, Gurrieri, Lukas, Micu).
- Torsion quantization understood for half-flat manifolds.
- Expanding the SU(3) invariant forms on appropriate bases, the only non-closed basis forms in the half-flat case satisfy

  \[ d\omega_i = e_i \beta^0, \quad d\alpha_0 = e_i \tilde{\omega}^i. \]

- For half-flat manifolds, torsion falls into the SU(3) classes

  \[ T^0 \in W_1^+ \oplus W_2^+ \oplus W_3, \]

  where + denotes the real part of \( W \).
Domain wall vacuum

Option 2:

Break maximal symmetry of \( d = 4 \) spacetime.

- Compactification with \( H \)-flux on a half-flat or Calabi-Yau manifold.
- There exist 1/2-BPS domain wall solutions 1305.0594 (Klaput, Lukas, Svanes).
- 1/2-BPS: 2 of the 4 SUSY generators in \( d = 4, \mathcal{N} = 1 \) unbroken.
- \( d = (2 + 1) \) Poincaré symmetry preserved; DW breaks symmetry in transverse \( y \) direction.
- Moduli satisfy flow equations in the \( y \) coordinate.
- 10d perspective: SU(3) fibred over \( y \rightarrow G_2 \) structure arXiv:1005.5302 (Lukas, Matti).
Cosmic strings and Spin(7) structures

Extension: cosmic strings

The following is based on research in progress, in collaboration with Cyril Matti (City University, London) and Eirik Svanes (LPTHE, Paris).

- The 1/2-BPS domain wall breaks maximal symmetry along one noncompact direction.

**Proposition:**

Fibre the SU(3) structure over an interval that is not a Cartesian coordinate direction.

- Cylindrical polar coordinates $(\rho, \phi, z)$: fibre along $\rho$  
  $\implies$ cosmic string.
- 4d metric:

$$ds_4^2 = e^{-2B(\rho)} \left( \eta_{\alpha\beta} dx^\alpha dx^\beta + d\rho^2 + \rho^2 d\phi^2 \right).$$
Cosmic string flow equations

The Killing spinor equations reduce to

\[ \partial_\rho A^I = -ie^{-B} e^K e^{K/2} K^I J^* D_J W^* , \]
\[ \partial_\rho B = i e^{-B} e^K e^{K/2} W , \]
\[ 0 = \text{Im}(K I \partial_\rho A^I) , \]
\[ 2 \partial_\rho \zeta = -\partial_\rho B \zeta , \]

with the 1/2-BPS constraint \( \bar{\zeta} = \sigma^r \zeta \), where

\[ \sigma^\rho = \frac{\hat{x}}{\rho} \sigma^1 + \frac{\hat{y}}{\rho} \sigma^2 = \begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix} . \]

- **Note:** same structure as in the domain wall case (with \( \rho \leftrightarrow y \)).
- Related to DW by conformal transformation: \( \rho = e^y \); \( x = -\phi \).
Spin(7): two transverse coordinates

- The cosmic string solution is a special case of the metric
  \[ ds_4^2 = e^{-2B(x_a)} \left( \eta_{\alpha\beta} d\tilde{x}^\alpha d\tilde{x}^\beta + g_{ab} dx^a dx^b \right) \]

- We can consider more general codimension-2 topological defects.
- 10d perspective: looks like an 8-dimensional Spin(7) structure.
- For the corresponding 6d compact SU(3) structure manifold, consider a generalized half-flat manifold, which satisfies
  \[ d\omega_i = p_{Ai}^B \beta^A - q_i^A \alpha_A, \quad d\alpha_A = p_{Ai} \tilde{\omega}^i, \quad d\beta^A = q_i^A \tilde{\omega}^i, \quad d\tilde{\omega}^i = 0, \]
  where \( \omega_i \) and \((\alpha_A, \beta^B)\) are basis 2- and 3-forms, respectively.
- Relevant SU(3) torsion classes are now
  \[ T^0 \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3. \]
Simple example

- 1/4-BPS ansatz: $\bar{\zeta} = \sigma^1 \zeta = -i \sigma^2 \zeta$ gives Killing spinor equations
  
  \[
  (\partial_x - i \partial_y) A^I = -ie^{-B} e^{K/2} K^{IJ} D_J W^*, \\
  (\partial_x + i \partial_y) B = ie^{-B} e^{K/2} W, \\
  0 = \text{Im}(K_l \partial_a A^l), \\
  2 \partial_a \zeta = -\partial_a B \zeta.
  \]

- Simple case: $p_{0i} = q_i^A = 0$, $H = 0$, vanishing axions.
- In this simple example, 4d and 10d equations match when $x$-dependence vanishes
  $\Rightarrow$ alternative 1/2-BPS DW solution, $G_2$ sublocus of Spin(7).

- KSEs satisfy
  \[
  \frac{K'}{K} = \frac{\tilde{K}'}{\tilde{K}} = -2\phi' = \frac{3}{\sqrt{KK}} p_{ai} v^i w^a,
  \]

  where $(v^i, w^a) = \text{Im}(T^i, Z^a)$, $K = K_{ijk} v^i v^j v^k$, $\tilde{K} = \tilde{K}_{abc} w^a w^b w^c$. 
String compactifications generate moduli, which must be stabilized. This can be done using fluxes.

For the heterotic string, only $H_3$ present. One can compactify on SU(3) structure manifolds which are not Calabi–Yau, and/or sacrifice maximal symmetry in $d = 4$. Domain wall solutions have been considered.

We found a cosmic string solution, which is a 1/2-BPS $G_2$ fibration. It is related to the domain wall case by a conformal transformation.

In the more general codimension-2 case, for vanishing H-flux we found a new type of domain wall solution at a $G_2$ locus of Spin(7).

Goal: include flux; 1/4-BPS solutions (e.g. intersecting domain walls $\rightarrow$ extra warp factor?); more general Spin(7) structure ... work in progress!