A HETEROtic QCD Axion

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**General Remarks**

The **strong CP problem**.

\[ \mathcal{L}_\theta = \frac{\theta}{8\pi^2} tr(F \wedge F)_{QCD} \]

Naively, \( \theta \simeq g_3 \simeq 1 \). Experimental bounds on the electric dipole of the neutron, \( \theta < 10^{-10} \)

Simplest **dynamical solution**: PQ axion.

\[ \mathcal{L}_a = \frac{a}{8\pi^2 f_a} Tr(F \wedge F)_{QCD} \]

Realisation: Goldstone boson of a global \( U(1)_{PQ} \) with mixed anomaly and spontaneously broken at \( \simeq f_a \):

\[ a \rightarrow a + c \]

Astrophysical and cosmological constraints on \( f_a \):

\[
\begin{array}{c}
10^9 \text{GeV} \lesssim f_a \lesssim 10^{12} \text{GeV}
\end{array}
\]

In string models: many axions available; usually \( f_a \simeq 10^{16} \text{GeV} \).

In heterotic compactifications the imaginary part of the dilaton multiplet

\[ S = s + i\sqrt{2} \sigma \]

couples at tree-level to \( F \tilde{F} \). The corresponding \( f_\sigma \simeq M_{\text{GUT}} \).

The axions coming from the \( T^i \) moduli

\[ T^i = t^i + 2i\chi^i \]

couple at one loop level to \( F \tilde{F} \).

Similarly, \( f_{\chi^i} \simeq M_{\text{GUT}} \).
Axions in Heterotic String Theory with Split Bundles

Alternative: consider heterotic CY models with bundles that (somewhere in the allowed Kähler moduli space) split as

\[ V = \bigoplus_{a=1}^{A} V_a \]

Generically, these lead to additional Green-Schwarz anomalous \( U(1) \) symmetries in 4d with D-terms of the form:

\[
D_a = \frac{M_P^2}{\mathcal{V}} \epsilon d_{ijk} k_a t^i t^j t^k - \sum_{I,J} q_{a,I} G_{IJ} C^I \bar{C}^J
\]

where the matter fields \( C^I = h^I e^{i\phi^I} \) transform as

\[
\delta C^I = -i \eta^a q_{a,I} C^I \\
\delta \phi^I = -q_{a,I} \eta^a
\]

We can construct a KSVZ axion (heavy quark axion) if there exists an exotic vector-like quark pair that couples to a singlet fields \( C \),

\[
W = \lambda Q C \bar{Q} + W_{\text{sing}} + \ldots ,
\]

At low energies, \( \phi \) couples to \( F \bar{F} \), with

\[
f = \sqrt{2} h .
\]

The value of \( h \) in the supersymmetric vacuum is controlled by the \( D \)-term equation

\[
D = \frac{M_P^2}{\mathcal{V}} \epsilon d_{ijk} k^i t^j t^k - q h^2 = 0 ,
\]

The FI term vanishes at the split locus and can assume arbitrarily small values close to it.
Heterotic Line Bundle Models with QCD Axions

An explicit example:

Required data: \((X, V)\) and \((\Gamma, \phi)\), \(\mathcal{W}\).

\[
X = \mathbb{C}P^1 \times \mathbb{C}P^1 \times \mathbb{C}P^1 \times \mathbb{C}P^1 \times \mathbb{C}P^1 \times \mathbb{C}P^1
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
0 & 2 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 \\
5 & 45 & -80 & & & \\
\end{bmatrix}
\]

\[
\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2. \text{ Action given by:}
\]

\[
g_1 : x_{m,\alpha} \mapsto (-1)^\alpha x_{m,\alpha}
\]

\[
g_2 : x_{m,\alpha} \mapsto x_{m,\alpha+1}
\]

\[
g_1 : p_\beta \mapsto (-1)^\beta p_\beta
\]

\[
g_2 : p_\beta \mapsto (-1)^{\beta+1} p_\beta
\]

Result: quotient manifold \(\tilde{X} = X/\Gamma\) with \(\pi_1(\tilde{X}) \cong \Gamma\), \(h^{1,1}(\tilde{X}) = 5\) and \(h^{2,1}(\tilde{X}) = 15\).

\[
\begin{align*}
V &= \bigoplus_{a=1}^5 \mathcal{L}_a = \bigoplus_{a=1}^5 \mathcal{O}_X(k_a) \\
\text{explicitly given by} \\
(k'_a) &= \begin{bmatrix}
-2 & 1 & 1 & 0 & 0 \\
1 & -2 & 0 & 1 & 0 \\
0 & 1 & -2 & 0 & 1 \\
1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 \\
\end{bmatrix}
\end{align*}
\]

Properties: \(c_1(V) = 0\), structure group \(S(U(1)^5 \subset SU(5))\).

\(\chi(V) = -12\). Anomaly cancellation condition and stability also satisfied.

GUT group: \(SU(5) \times S(U(1)^5)\).

Choice for the equivariant structure:

\[
\mathcal{L}_1^{(0,1)} \oplus \mathcal{L}_2^{(0,0)} \oplus \mathcal{L}_3^{(0,0)} \oplus \mathcal{L}_4^{(0,0)} \oplus \mathcal{L}_5^{(0,0)}
\]
## Heterotic Line Bundle Models with QCD Axions

<table>
<thead>
<tr>
<th>multiplet</th>
<th>$S(U(1)^5)$ charge</th>
<th>bundle</th>
<th>cohomology</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10_a$</td>
<td>$e_a$</td>
<td>$V_a$</td>
<td>$H^1(X, V_a)$</td>
</tr>
<tr>
<td>$\bar{10}_a$</td>
<td>$-e_a$</td>
<td>$V_a^*$</td>
<td>$H^1(X, V_a^*)$</td>
</tr>
<tr>
<td>$\bar{5}_{a,b}$</td>
<td>$e_a + e_b$</td>
<td>$V_a \otimes V_b$</td>
<td>$H^1(X, V_a \otimes V_b)$</td>
</tr>
<tr>
<td>$5_{a,b}$</td>
<td>$-e_a - e_b$</td>
<td>$V_a^* \otimes V_b^*$</td>
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</tr>
<tr>
<td>$1_{a,b}$</td>
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</tr>
</tbody>
</table>

After quotienting: MSSM spectrum with a vector-like pair of quarks and singlets:

$$10_1, \ 10_2, \ 10_3, \ 5_{1,3}, \ 5_{1,5}, \ 5_{3,4},$$

$$T_{1,4}, \ \bar{T}_{1,4}, \ H_{4,5}, \ \bar{H}_{4,5}, \ 3\ 1_{1,3}, \ 1_{1,4}, \ 1_{4,1}, \ 1_{1,5}, \ 4\ 1_{2,4}, \ 1_{2,5}, \ 3\ 1_{3,2}, \ 1_{3,4}, \ 3\ 1_{3,5} ,$$
Heterotic Line Bundle Models with QCD Axions

The $U(1)$—charges constrain the allowed superpotential operators. Importantly,

$$ W \supset \bar{T}_{1,4} d_{3,4} S_{1,3} . $$

For $\langle S_{1,3} \rangle \neq 0$, this removes the pair $d_{3,4} - \bar{T}_{1,4}$ from the massless spectrum.

$d_{3,4}, \bar{T}_{1,4}$ play the role of the exotic quark fields $Q$ and $\tilde{Q}$. The “missing” $d$-type quark is replaced by $T_{1,4}$.

If $\langle S_{1,3} \rangle$ can be stabilised at a small value, $10^{-7} \lesssim \langle S_{1,3} \rangle \lesssim 10^{-4}$ in GUT units, the axion coupling parameter will be in the phenomenologically allowed range.
Conclusions

Heterotic CY models with split bundles can accommodate all the required ingredients for a successful axion model:

- Green-Schwarz anomalous $U(1)$ symmetries with associated FI terms that vanish at specify loci in Kähler moduli space;
- SM multiplets, additional singlet matter fields and vector-like pairs of exotic quarks, charged under the extra $U(1)$ symmetries;
- trilinear superpotential coupling $QC\bar{Q}$; $C = he^{i\phi}$
- axion coupling parameter controlled by the FI term can be arbitrarily small close to the split locus.
Thank you!