Non–Abelian Discrete Gauge Symmetries in F–theory

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based on:
non-Abelian: 1504.06272 with D. Regalado, T. Pugh
Abelian: 1408.6448 with I. García-Etxebarria, J. Keitel
1406.5180 with L. Anderson, I. García-Etxebarria, J. Keitel
Some initial struggle

- in the last years we systematically approached: with Savelli, Pugh, Weissenbacher

Can we derive the N=1 effective action of Type IIB flux compactification with warping?

- challenge: 
  \[ \frac{1}{\alpha^2} \int_{Y_4} G^\text{Flux} \wedge G^\text{Flux} = \frac{\chi(Y_4)}{24} \]
  need to include higher-derivative terms in the action → now known

- important steps:
  - find back-reacted solution with higher-derivative terms [Becker, Becker] [TG, Pugh, Weissenbacher]
  - derive effective action by dimensional reduction Part I: arXiv:1412.5073
  - determine Kähler potential and Kähler coordinates Part II: arXiv:1506.nnnn
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    → Tom Pugh’s talk

Part II: arXiv:1506.nnnn
Goals of this talk

- I like to discuss non-Abelian discrete gauge symmetries in F-theory compactifications on Calabi-Yau fourfolds

- Stepwise introduce:

  (1) Geometrically massive U(1)s and Abelian gauge symmetries

  (2) Discrete non-Abelian gauge symmetries in O7-orientifolds

  (3) Candidate gaugings for discrete non-Abelian gauge symmetries in F-theory → mutually non-local seven-branes

  (4) Derivation via M-theory compactifications on manifolds with cohomological torsion
Geometrically massive U(1)s and Abelian discrete symmetries
Geometrically massive $U(1)$s

- Geometrically massive $U(1)$s arise from $D7 - D7'$ system with Stückelberg coupling:
  \[ S_{St} = \int_{D7^-} C_6 \wedge F_{U(1)} \quad \rightarrow \quad \text{gauging of dual axion: } C_2 = c^a \omega_a \]
  \[ \mathcal{D}c^a = dc^a + m^a A_{U(1)} \]
  [Jockers,Louis]

- Geometrically massive $U(1)$s in F-theory:
  [TG,Weigand] [TG, Kerstan, Palti, Weigand]
  - $U(1)$s arise from M-theory three-form:
    \[ C_M = c^a \alpha_a + A_{U(1)} \wedge \omega_{U(1)} + \ldots \]
  - Non-closed forms induce non-trivial gauging:
    \[ d\omega_{U(1)} = m^a \alpha_a \quad \rightarrow \quad \mathcal{D}c^a = dc^a + m^a A_{U(1)} \]
  - Interpretation as non-Kähler geometry or CY fourfolds with cohomological torsion
    See also [A. Braun, Collinucci, Valandro]
Abelian discrete symmetries

- geometrically massive U(1)s can leave Abelian discrete gauge symmetry

- so far, main examples: geometries with multi-section (no section)

  - examples with $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z}_4$ symmetry studied in
  [Braun, Morrison] [Morrison, Taylor] [Anderson, García-Etxebarria, TG, Keitel] [Klevers et al.]
  [García-Etxebarria, TG, Keitel] [Mayrhofer, Palti, Till, Weigand] [Braun, TG, Keitel] [Cvetic, Donagi, Klevers, et al.]

  - physical interpretation: mixing of KK-vector with massive U(1)s
    importance of the KK-modes
    [Anderson, García-Etxebarria, TG, Keitel]
    [García-Etxebarria, TG, Keitel] [Mayrhofer, Palti, Till, Weigand]

  - selection rules on Yukawa couplings in F-theory
    [García-Etxebarria, TG, Keitel] [Mayrhofer, Palti, Till, Weigand] [Klevers et al.]

  - connection with cohomological torsion
    [Mayrhofer, Palti, Till, Weigand]

- alternative suggestions for non-Abelian case
  [Karozas, King, Leontaris, Meadowcroft]
  → Leontaris’ talk
Non-Abelian discrete symmetries in O7-orientifolds
Heisenberg symmetries in CY orientifolds

- recall the symmetries of the N=1 orientifold moduli space

\[ G^a = c^a - \tau b^a \quad \text{from orientifold-odd } C_2, B_2 \]

\[ T_\alpha = \rho_\alpha + \frac{1}{2(\tau - \bar{\tau})} \mathcal{K}_{\alpha ab} (G - \bar{G})^b - \frac{1}{2} i \mathcal{K}_{\alpha \beta \gamma} v^\beta v^\gamma \quad \text{from orientifold-even sector: Kähler + } C_4 \]

\[ K = -2 \log \mathcal{V} \]

- Kähler metric admits Heisenberg symmetry (continuous, non-compact, non-semi-simple)

\[ [t_{(1,a)}, t_{(2,b)}] = -\mathcal{K}_{\alpha ab} t^\alpha \]

\[ t_{(1,a)} = \partial G^a \quad t_{(2,a)} = -\tau \partial G^a - \mathcal{K}_{\alpha ab} G^b \partial T_\alpha \quad t^\alpha = \partial T_\alpha \]

⇒ Can this non-Abelian group be gauged? (1) by R-R gauge fields

(2) by seven-brane gauge fields
Non-Abelian gaugings with R-R vectors

- non-Abelian discrete gauge symmetries were suggested to arise in Type IIB orientifolds with torsional cohomology
  [Camara, Ibanez, Marchesano]
  [Berasaluce-Gonzalez, Camara, Marchesano, Regalado, Uranga]

⇒ generalization should be simple ✓
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⇒ generalization should be simple √

- details turn out to be tricky:
  - non-Abelian structure due to $\text{Tor}(H^2(Y_3))$ and $\text{Tor}(H^4(Y_3))$
  - orientifold involution: $\text{Tor}(H^2_-(Y_3))$, $d\gamma_i = k^a_i \omega_a$

$$B_2 = A^1_i \wedge \gamma_i + b^a \omega_a$$
$$C_2 = A^2_i \wedge \gamma_i + c^a \omega_a \quad \Rightarrow \quad DG^a = dG^a + k^a_i (A^2_i - \tau A^1_i)$$

real + imaginary part gauged with different vectors → supersymmetry?

⇒ N=1 orientifold setting more involved then expected supersymmetrization for negative torsion is a challenge ?!
Non-Abelian symmetries in the F-theory effective action
Setting in F-theory

- F-theory avoids this problem in an intriguing way:
  - \( \tau \) is no longer a four-dimensional field
    gets replaced by a holomorphic function \( f_{ab}(z) \):
    \[
    G^a \quad \rightarrow \quad N^a = -i(a^a + if^{ab}b_b)
    \]
    also contains Wilson line moduli of 7-branes

- symmetry algebra in F-theory
  \[
  [t_a, \tilde{t}^b] = -M_{\alpha\alpha}^b t^\alpha
  \]

  \[
  \tilde{t}^b = f^{ab} \partial_{N^a} - N^a (iM_{\alpha\alpha}^b + \frac{1}{2}f^{bc}M_{\alpha\alpha^c}) \partial_{T_\alpha}
  \]

  \[
  t_a = -i \partial_{N^a} - \frac{i}{2}N^b M_{\alpha\alpha b} \partial_{T_\alpha}
  \]

  intersection no. on fourfold:

  \[
  M_{\alpha\alpha b} = \int \omega_\alpha \wedge \alpha_a \wedge \alpha_b,
  \]
  \[
  M_{\alpha\alpha}^b = \int \omega_\alpha \wedge \alpha_a \wedge \beta^b
  \]
Origins of gaugings in F-theory - Part I

- geometric Stückelberg mechanism for D7-branes:
  \[ DG^a = dG^a - \tilde{k}_i^a A_i \]
gauges only the axion arising from R-R two-form \( C_2 \)

- generalization to mutually non-local (p,q)-seven-branes:
  \( Sl(2, \mathbb{Z}) \) acts on \((B_2, C_2)\) : more general gaugings

\[
DN^a = dN^a + i(\tilde{k}_i^a A^i - i f^{a b} k_{i b} A^i)
\]

(p,q)-generalization of Stückelberg coupling:

\[
S^{(i)} = \int_{\mathbb{M}_4 \times S_i} F^i \wedge (p C_6 + q B_6)
\]

⇒ precisely generalization that is needed to find supersymmetric non-Abelian structure ⇒ leave weak string coupling configurations
- possible non-Abelian completion due to two effects
  - (1) fluxes on seven-branes
    \[ DT_\alpha = dT_\alpha - \Theta_{\alpha i}A^i \]
    \[ \Theta_{\alpha i} = \int_{S_i} \mathcal{F}^i \wedge \omega_\alpha \]
    ⇒ purely open-string (seven-brane) setup

  - (2) non-trivial torsion in base of F-theory \( \text{Tor}(H^4(B_3, \mathbb{Z})) \)
    \[ DT_\alpha = dT_\alpha - k_{\alpha \kappa}A^\kappa \]
    \[ d\omega_\alpha = k_{\alpha \kappa} \beta^\kappa \]
    R-R gauge field from \( C_4 \)

- F-theory settings with mutually non-local \((p,q)\)-seven-branes appear to allow for non-Abelian discrete gauge symmetries
  ⇒ Checks ?!
Gauge coupling function and kinetic mixing

- gauging a Heisenberg group has profound implications:
  - Heisenberg group has no positive definite Killing form
  - kinetic term for the vectors has to involve the gauged scalars:
    \[
    \mathcal{L} = -\frac{1}{2} f^1_{AB} F^A \wedge \ast F^B - \frac{1}{2} f^2_{AB} F^A \wedge F^B - g_{ab} D\phi^a \wedge \ast D\phi^b
    \]
    \[
    \delta f^i_{AB} = \lambda^C (f^D_{CA} f^i_{BD} + f^D_{CB} f^i_{AD})
    \]
    gauge coupling function should depend on \(N^a\) in a very specific way to ensure gauge invariance

- in setting (2) with gaugings arising partly from seven-branes and partly from R-R forms the Heisenberg group dictates the form of the kinetic mixing between brane and bulk gauge fields

  **Check:** result agrees with expectations at weak coupling for Wilson lines
Computing the F-theory effective action via M-theory
Computing the F-theory effective actions

- No twelve-dimensional low-energy effective action for F-theory

**Analyze and define F-theory via M-theory**

(1) **A-cycle:** if small than M-theory becomes Type IIA
(2) **B-cycle:** T-duality ⇒ Type IIA becomes Type IIB,
(3) grow extra dimension: send $T^2$- volume T-dual ⇒ B-cycle becomes large

⇒ **M-theory to F-theory limit connects 4d and 3d effective theories**

- **key insight of recent research:**
  - importance of: circle Kaluza-Klein modes ↔ M2-branes on fiber
  - as of now **non-Abelian gauge symmetries in direct lift from 3d to 4d**
Reduction of M-theory with torsion

- dimensional reduction of eleven-dimensional supergravity starting with two-derivative action \([\text{Cremmer, Julia, Scherk}]\)

- background is taken to be a direct product:

\[
\begin{align*}
    d\tilde{s}^2 &= g_{\mu\nu}dx^\mu dx^\nu + 2(g^0_{m\bar{n}} + i\delta^\Sigma\omega_{\Sigma m\bar{n}})dy^m dy^{\bar{n}} \\
    \hat{C} &= A^\Sigma \wedge \omega + \xi^I \alpha_I + \xi_I \beta^I.
\end{align*}
\]

→ talk of T. Pugh (warping + higher-derivatives)

- non-closed forms:

\[
d\omega = \tilde{k}_I^I \alpha_I + k_{I\Sigma} \beta^I
\]

can be interpreted as cohomological torsion: \(\text{Tor}(H^3(Y_4, \mathbb{Z}))\)

includes:
(1) M-theory dual of geom. massive U(1)s
(2) non-trivial torsion in the base

⇒ should find the non-Abelian structure suggest by F-theory configuration
performing dimensional reduction:

- covariant derivatives found

\[ \hat{G} = dA^\Sigma \wedge \omega_\Sigma + D\tilde{\xi}^I \wedge \alpha_I + D\xi_I \wedge \beta^I + \tilde{\xi}^I d\alpha_I + \xi_I d\beta^I \]

\[ D\tilde{\xi}^I = d\tilde{\xi}^I - A^\Sigma \tilde{k}_\Sigma^I \quad D\xi_I = d\tilde{\xi}^I - A^\Sigma k_\Sigma^I \]

- checking the gauged symmetry: purely Abelian gauging
Slide of hope

- performing dimensional reduction:
  - covariant derivatives found
    \[ \hat{G} = dA^\Sigma \wedge \omega_\Sigma + D\tilde{\xi}^I \wedge \alpha_I + D\xi_I \wedge \beta^I + \tilde{\xi}^I d\alpha_I + \xi_I d\beta^I \]
    \[ D\tilde{\xi}^I = d\tilde{\xi}^I - A^\Sigma \tilde{k}^I_\Sigma \]
    \[ D\xi_I = d\tilde{\xi}^I - A^\Sigma k^I_\Sigma \]
  - checking the gauged symmetry: purely Abelian gauging

- However: We are not yet in the correct duality frame to perform the F-theory limit!
  - split the 3d fields and dualize:
    example: \[ A^\Sigma \rightarrow A^i, \quad A^\alpha \] has to be dualize into scalar \Rightarrow axion in 4d complex field
Discovering the Non-Abelian structure

- Dualization of $A^\alpha$ into axion $\rho_\alpha$, and $\tilde{\xi}_\kappa$ into R-R vector $A^\kappa$

\[
D\rho_\alpha = d\rho_\alpha - k_{\alpha\kappa} A^\kappa + \frac{1}{2} M_{\alpha a}^\ k (k_{ib}a^a - \tilde{k}_{i}^a b_b) A^i
\]

\[
F^\kappa = dA^\kappa + \frac{1}{2} (\tilde{k}_{j}^a M_{ia}^\kappa + k_{ja} M_{i}^{a\kappa}) A^i \wedge A^j
\]

\* dual frame features non-Abelian gauge symmetry as suggested

\[
\tilde{k}_{j}^a M_{ia}^\kappa + k_{ja} M_{i}^{a\kappa}
\]

generically massive
seven-brane gauge fields
\rightarrow non-closed forms / torsion

control kinetic mixing of R-R and seven-brane U(1)s
\rightarrow necessary for gauged Heisenberg group

\[
M_{\Sigma I}^J = \int_{\hat{Y}_4} \omega_\Sigma \wedge \alpha_I \wedge \beta^J
\]
Conclusions

- Discrete Abelian gauge symmetries in F-theory
  - much recent progress: analyzing F-theory geometries with multi-sections

- Discrete non-Abelian gauge symmetries at weak string coupling
  - pure R-R gaugings from cohomological torsion in CY orientifolds
    ⇒ in apparent conflict with supersymmetry
    ⇒ precise reason is mysterious (torsion in orientifold-odd cohomology?)

- Discrete non-Abelian gauge symmetries in F-theory
  - mutually non-local seven-branes required ⇒ genuinely F-theoretic
  - gauging of a generalization of a Heisenberg group
    ⇒ non-trivial insights about the gauge-coupling function
    ⇒ generally true in string theory? insights about kinetic mixing?
  - purely open string settings with fluxes on seven-branes
    ⇒ dual description via Higgsing?
    ⇒ selection rules on Yukawa couplings
  - discovered Abelian to non-Abelian duality (any dimension) [TG, Regalado, Pugh]