Transplanckian axions and the weak gravity conjecture

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Suddenly a very popular topic!

- *Large-Field Inflation with Multiple Axions and the Weak Gravity Conjecture*, Daniel Junghans, 1504.03566.
Why transplanckian axions?

- Inflation: Transplanckian field range for sizeable $r$.
- Need to control Planck-suppressed terms in the potential
  \[ V(\phi) \sim \left( \frac{\phi}{M_P} \right)^n \]
- Good idea: Use axions with shift symmetry $\phi \to \phi + c$
  broken to $\phi \to \phi + 2\pi f$ by nonperturbative effects:
  \[ V(\phi) \sim \cos(\phi/f) \]
- These EFT models might be in the Swampland.
Weak Gravity Conjecture [Arkani-Hamet et al. ’06]: For any abelian $p$-form field, there must be a charged $p$-dimensional object with tension

$$T \lesssim \frac{g}{\sqrt{G_N}}.$$ 

- For an axion in 4d, $g = 1/f$ and $T = S$, the object is an instanton. These instantons can generate a potential which prevents transplanckian field excursions.
- Idea: Try to understand WGC-like constraints from studying small black holes/instantons in EFT.
We consider EFT of a single axion coupled to Einsteinian gravity

\[
\int \left( \frac{-1}{16\pi G} R dV + \frac{f^2}{2} d\phi \wedge *d\phi - V(\phi) dV \right)
\]

- The potential \( V(\phi) \) does not include gravitational instanton contributions.
- We are given an inflationary model and study gravitational instanton effects on top of that.
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In the dilute-gas approximation, summing over all instantons with same charge yields a contribution to the path integral of the form

$$S_{\text{inst.}} = \int d^4x \sqrt{-g} \mathcal{P} e^{-S_E} \cos(n\phi), \quad S_E = \frac{\sqrt{3\pi}}{16} \frac{M_{\text{Pl}} n}{f}$$

$\mathcal{P}$ is a prefactor for which we assume $\mathcal{P} \sim M_{\text{Pl}}^4$. This is essential. Computation of $\mathcal{P}$ in only a few cases [Gross, Perry & Yaffe ’82, Volkov & Wipf ’00].
Consequences for a transplanckian axion

The instanton generates a huge potential, cutting down the effective field range to $f/n$ as long as the action is small. $S_E \sim 1$ means $f \sim M_P n$, so the effective range is approximately subplanckian.
Do gravitational instantons pose a threat to multiple-axion inflationary models as well?

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \phi) G(\partial_\nu \phi) - \sum_{i=1}^{N} \Lambda_i^4 (1 - \cos(\phi_i)) \]

Here, the axions are periodic \( \phi_i \rightarrow \phi_i + 2\pi \). This is the lattice basis.

With canonical kinetic term

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \hat{\zeta}_i)^2 - \sum_{i=1}^{N} \Lambda_i^4 (1 - \cos(R_{ji}\hat{\zeta}_i/f_i)), \quad \hat{\zeta}_i = f_i R_{ij} \phi_j. \]

\{\hat{\zeta}_i\} parametrices the axions in the kinetic basis.
Three classes of models

- N-flation [Dimopoulos et al. ’08]: $\sqrt{Nf}$ enhancement
- Kinetic alignment [Bachlechner ’15]: $\sqrt{Nf_{\text{max}}}$ enhancement
- Lattice alignment [Kim, Nilles, Peloso ’05]: parametrically flat direction:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \theta_1)^2 + \frac{1}{2}(\partial_\mu \theta_2)^2 - \Lambda_1^4 \left(1 - \cos \left(\frac{\theta_1}{f} + \frac{\theta_2}{g_1}\right)\right) - \Lambda_2^4 \left(1 - \cos \left(\frac{\theta_1}{f} + \frac{\theta_2}{g_2}\right)\right)$$

As $\epsilon \equiv g_1 - g_2 \to 0$ we get a parametrically flat direction in the potential.
The action of the gravitational instanton becomes

$$S_E = \frac{\sqrt{3\pi}}{16} M_P \sqrt{\vec{n}^T G^{-1} \vec{n}}$$

and induces a potential for the system of axions given by

$$V = \mathcal{P}' e^{-S_E} \cos(n_i \phi_i)$$
If $S_E < c$ is smaller than a prescribed value, the instantons will not hinder inflation. These define ellipses in the dual lattice:

So these models evade the effects of gravitational instantons!
Lots of instantons get vanishing action in the KNP limit.
The resulting contributions ruin the parametric flatness of the potential.
Similar to single axion.
String theory is full of axions! A simple choice:

\[ \phi \rightarrow \int_S C_p, \quad \text{Instanton} \rightarrow \text{Euclidean } D_p\text{-brane on } p\text{-cycle}. \]

For \( p = 4 \), and reduction to \( 4d \), we have

\[ \frac{M_P}{f} \sim \frac{1}{g_s} \quad \Rightarrow \quad S_E \sim \frac{M_P}{f} n \sim \frac{n}{g_s}, \]

same parametric dependence as a stack of \( n \) euclidean D3’s. One can also extend this to the multi-axion setup (in some particular models) to get the right dependence

\[ S_{D3} \sim \sqrt{\tilde{n}^T \tilde{G}^{-1} \tilde{n}} \]
Taking into account gravitational effects is essential for a successful inflationary model. For N-flation and kinetic alignment these effects are suppressed. For KNP the effects ruin parametric flatness. Some instantons are absent only if strong form of WGC does not hold.

Outlook:
- Can we embed models with suppressed effects in ST?
- What about monodromy?
Summary

- Taking into account gravitational effects is essential for a successful inflationary model.
- For $N$-flation and kinetic alignment these effects are suppressed.
- For KNP the effects ruin parametric flatness.
- Some instantons are absent only if strong form of WGC does not hold.

Outlook:
- Can we embed models with suppressed effects in ST?
- What about monodromy?
Thank you very much!
When to trust these computations?

\( a^{1/4} \) controls size & curvature of the solution. It should be larger than inverse cuttoff and lower than \( V(\phi) \) scale

\[
\Lambda^{-1} < a^{1/4} < V_0^{-1} \quad \Rightarrow \quad \left( \frac{M_P}{\Lambda} \right)^2 \lesssim S_E < \left( \frac{M_P}{V_0} \right)^2.
\]

- Presumably instantons with \( a^{1/4} < \Lambda^{-1} \) still exist (typically, euclidean D-branes).
- Above bounds mean that the terms reducing the field range can only be trusted to do so if \( \Lambda \) is close to \( M_P \).
Let us rewrite the KNP model in the lattice basis. The metric is

\[ G = \frac{1}{(g_1 - g_2)^2} \begin{pmatrix} g_1^2 (f^2 + g_2^2)^2 & -g_1 g_2 (f^2 + g_1 g_2) \\ -g_1 g_2 (f^2 + g_1 g_2) & g_2^2 (f^2 + g_2^2)^2 \end{pmatrix} \]
In UV: approximate $\mathbb{Z}_2$ symmetry

\[
\left( \frac{f}{\sqrt{2}}, \frac{g}{\sqrt{2}} \right), \quad \left( \frac{f}{\sqrt{2}}, -\frac{g}{\sqrt{2}} \right), \quad \left( \sqrt{2}f, 0 \right)
\]
In other words, when $g \to 0$ we have two sets of gravitational instantons

- Those whose action depends on $g$ and are suppressed
- Those who are not

and the second set obeys periodicities in a more refined lattice.

This violates the *strong* form of the weak gravity conjecture [Brown, Cottrell, Shiu, Soler 1503.04783]. (instanton of lowest charge should have smallest action).