Beyond the Standard Model phenomenology with FeynRules

Benjamin Fuks

LPTHE Paris - UPMC Paris VI

IFT School on Monte Carlo tools
@ IFT Madrid (Spain)

September 07, 2015

Outline

Monte Carlo tools and discoveries at the LHC (I)

Assumption

There is some new physics to be discovered

A BSM LHC story

✦ *A priori* preparation
- ✤ Viable model building (top-down & bottom-up)
- ✤ Phenomenological studies
- ✤ Prospective collider analyses

✦ *A posteriori* reactions to announcements
- ✤ Model building (top-down & bottom-up)
- ✤ Recasting experimental analyses
- ✤ Measurements (precision predictions)

Predictions for the LHC

✦ Option 1: handmade calculations
- ✤ Factorial growth of the number of diagrams
- ✤ Tedious and error prone

✦ Option 2: Monte Carlo simulations
- ✤ Easy to use
- ✤ Can include the full collision environment

Monte Carlo tools and discoveries at the LHC (2)

Predictions for the LHC

~~✦ Option 1: handmade calculations~~
- ~~✤ Factorial growth of the number of diagrams~~
- ~~✤ Tedious and error prone~~

✦ Option 2: Monte Carlo simulations
- ✤ Easy to use
- ✤ Can include the full collision environment

✦ How to implement a new physics model in a Monte Carlo program?
- ★ Model definition: particles, parameters & vertices (≡ Lagrangian)
- ★ To be translated in a programming language, following some conventions, *etc.*
- ★ Tedious, time-consuming, error prone
- ★ Iterations for all considered tools and models
- ★ Beware of the restrictions of each tool (Lorentz structures, color structures)

★ **Highly redundant** (each tool, each model)
★ **No-brainer task** (from Feymman rules to code)

→ FeynRules: **Systematization & automation**

FeynRules in a nutshell

What is FeynRules?

- A framework to develop new physics models
- Automatic export to several Monte Carlo event generators

  ➥ Facilitate phenomenological investigations of BSM models
  ➥ Facilitate the confrontation of BSM models to data

- Validation of an implementation using several of the linked Monte Carlo programs

Main features

- MATHEMATICA package
- Core function: derives Feynman rules from a Lagrangian
- Requirements: locality, Lorentz and gauge invariance
- Supported fields: scalar, (two- and four-component) fermion, vector (and ghost), spin-3/2, tensor, superfield

From FeynRules to Monte Carlo tools...

[Christensen, Duhr (CPC '09); Alloul, Christensen, Degrande, Duhr, BF (CPC'14)]

Model

Particles ^ | Gauge symmetries | Parameters | Lagrangian

TEX output ← FeynRules → Feynman rules

CalcHep

FeynArts

Whizard*

Translation interfaces

ALOHA | GOSAM

Herwig ++ | SHERPA

MadGraph5_aMC@NLO

MadAnalysis 5

The UFO

[Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12)]

Superspace module

[Duhr, BF (CPC '11); BF (IJMPA '12)]

Mass diagonalization

[Alloul, D'Hondt, De Causmaecker, BF, Rausch de Traubenberg (EPJC '13)]

Decay package

[Alwall, Duhr, BF, Mattelaer, Oezturk, Shen (1402.1178)]

NLO module

[Degrande (1406.3030)]

Computational tools

* Whizard interface: Christensen, Duhr, BF, Reuter, Speckner (EPJC '12)

^ Support for spin 3/2: Christensen, de Aquino, Deutschmann, Duhr, BF, Garcia-Cely, Mattelaer, Mawatari, Oexl, Takaesu (EPJC '13)

Outline

(Broken) supersymmetric QCD: the model

Particle content (simplified for the scope of the lecture)

- Two matter supermultiplets in the fundamental representation of $SU(3)_c$
  - ★ One massive Dirac fermion: a quark
  - ★ Two massive scalar fields: a left-handed and a right-handed squark
- One ($SU(3)_c$) gauge supermultiplet
  - ★ One massive Majorana fermion: a gluino
  - ★ One massless gauge boson: the gluon

The dynamics of the model is embedded in the Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{\tilde{g}}\not{D}\tilde{g} + D_\mu\tilde{q}_L^\dagger D^\mu\tilde{q}_L + D_\mu\tilde{q}_R^\dagger D^\mu\tilde{q}_R + i\bar{q}\not{D}q \\ & - m^2_{\tilde{q}_L}\tilde{q}_L^\dagger\tilde{q}_L - m^2_{\tilde{q}_R}\tilde{q}_R^\dagger\tilde{q}_R - m_q\bar{q}q - \frac{1}{2}m_{\tilde{g}}\bar{\tilde{g}}\tilde{g} \\ & - \frac{g_s^2}{2}\Big[-\tilde{q}_L^\dagger T^a\tilde{q}_L + \tilde{q}_R^\dagger T^a\tilde{q}_R\Big]\Big[-\tilde{q}_L^\dagger T^a\tilde{q}_L + \tilde{q}_R^\dagger T^a\tilde{q}_R\Big] \\ & + \sqrt{2}g_s\Big[-\tilde{q}_L^\dagger T^a(\bar{\tilde{g}}^a P_L q) + (\bar{q}P_L\tilde{g}^a)T^a\tilde{q}_R + \text{h.c.}\Big]\end{aligned}$$

- Kinetic terms for all fields (first line)
- Mass terms for the quark, squarks and gluino (second line)
- (supersymmetric) gauge interactions for all fields (the last two lines of the Lagrangian)

How to write a FeynRules model file?

✦ A FeynRules model file is compliant with the Mathematica syntax

✦ It is a .fr file containing:

✤ A preamble
- ★ Author information
- ★ Model information
- ★ Index definitions

✤ The declaration of the gauge group
- ★ Abelian or not
- ★ Representation matrices
- ★ Structure constants
- ★ Coupling constant
- ★ Gauge boson or vector superfield

✤ The declaration of the fields
- ★ Names, spins, PDG codes
- ★ Indices, quantum numbers
- ★ Masses, widths
- ★ Classes and class members

✤ The declaration of the parameters
- ★ External and internal
- ★ Scalar and tensor

✤ A Lagrangian

The preamble of the model file: general information

- An electronic signature for the model implementation
  - Important for traceability, documentation, contact with the model authors, *etc.*
  - Reference publications used can be added
  - Webpage information can be added

```
(* ********************************************************** *)
(* *****                                                ***** *)
(* *****  FeynRules model file: SUSY-QCD                ***** *)
(* *****  Author: B. Fuks                               ***** *)
(* *****                                                ***** *)
(* ********************************************************** *)


(* ************************** *)
(* *****  Information   ***** *)
(* ************************** *)
M$ModelName = "SUSYQCD";

M$Information = {
  Authors      -> {"Benjamin Fuks"},
  Date         ->  “16.06.14”,
  Version      ->  "1.0.0",
  Institutions -> {“CERN / IPHC Strasbourg / U. of Strasbourg"},
  Emails       -> {"benjamin.fuks@iphc.cnrs.fr"}
};
```

The preamble of the model file: indices

The dimension of the indices must be declared

- In our SUSY-QCD model:
  - ★ Fundamental $SU(3)_C$ indices for the squarks and quark: *Colour*, dimension 3
  - ★ Adjoint $SU(3)_C$ indices for the gluon and gluino: *Gluon*, dimension 8
  - ★ Lorentz and spin indices are automatically handled

```
(* ************************** *)
(* *****    Indices     ***** *)
(* ************************** *)
IndexRange[Index[Gluon ]] = NoUnfold[Range[8]];
IndexRange[Index[Colour]] = NoUnfold[Range[3]];
```

- QCD has a special role for Monte Carlo event generators ➢ many special names
  - ★ *Colour* and *Gluon* for the indices
  - ★ *G* for the gluon field
  - ★ *T* for the fundamental representation matrices, *f* and *d* for the structure constants
  - ★ G (gs will be used) and aS for the coupling constants

The style of the indices can be specified

- In our SUSY-QCD model:
  - ★ Fundamental $SU(3)_C$ indices: starts with the letter *m*
  - ★ Adjoint $SU(3)_C$ indices: starts with the letter *a*

```
IndexStyle[Colour, m];
IndexStyle[Gluon,  a];
```

The declaration of the gauge group

Each direct factor of the group is declared as an element of the *M$GaugeGroups list*

- A declaration ≡ a set of MATHEMATICA replacement rules
- In our SUSY-QCD model:
  - ★ We must only declare $SU(3)_C$: we choose the name *SU3C*

```
(* ************************** *)
(* *****  Gauge groups  ***** *)
(* ************************** *)
M$GaugeGroups = {
  SU3C == {
    Abelian           -> False,
    GaugeBoson        -> G,
    CouplingConstant  -> gs,
    StructureConstant -> f,
    Representations   -> { {T,Colour} }
  }
};
```

- Each rule represents one group property (reminder for QCD: special names exist)
  - ★ *Abelian*: abelian or non-abelian gauge group
  - ★ *GaugeBoson*: the associated gauge boson
  - ★ *CouplingConstant*, *StructureConstant*: coupling and the structure constants
  - ★ *Representation*: list of 2-tuples linking an index (*Colour*) to the symbol of a representation matrix (*T*)

Advantages of a proper gauge group declaration

- Render the writing of the Lagrangian easier:
  - ★ Covariant derivatives (*DC[field, Lorentz index]*)
  - ★ Field strength tensors (*FS[field, Lorentz index 1, Lorentz index 2]*)
  - ★ Useful for Lagrangian building in superspace (very briefly covered in the last part of this lecture)
    - ➢ Duhr, BF [CPC 182 (2011) 2404]; BF [IJMPA 27 (2012) 1230007]

See the manual for more details on gauge groups

Declaring the gluon field

✦ Each field is declared as an element of the *M$ClassesDescription* list

✤ A declaration ≡ a set of MATHEMATICA replacement rules

✤ In our SUSY-QCD model, we first declare the $SU(3)_C$ gauge boson: *G*

```
(* ************************** *)
(* *****     Fields     ***** *)
(* ************************** *)
M$ClassesDescription = {
  V[1] == {
    ClassName       -> G,
    SelfConjugate   -> True,
    Indices         -> {Index[Gluon]},
    Mass            -> 0,
    Width           -> 0,
    PDG             -> 21
  },
  ...
```

✤ Each rule represents a property of the declared field

★ Vector field ➢ the label is V[1] (with V, and not F, S, R, T, *etc.*)

★ Classname: defines the symbol to use in the Lagrangian ➢ *G*

★ SelfConjugate: the gluon is its own antiparticle ➢ *True*

★ Indices: the gluon lies in the adjoint representation of SU(3)c

➢ The gluon has been previously set as the gauge boson of $SU(3)_c$

➢ Its index (*Gluon)* is internally linked to the adjoint representation of the group

★ Other properties: vanishing mass and widths, PDG code set to 21

★ Not used options: *Unphysical, Definitions, PropagatorLabel, PropagatorType, PropagatorArrow, ParticleName, AntiParticleName, QuantumNumbers*

See the manual for more details on field declarations

Declaring the gluino field

✦ A second element in the *M$ClassesDescription* list

```
F[1] == {
  ClassName        -> go,
  SelfConjugate    -> True,
  Indices          -> {Index[Gluon]},
  Mass             -> {Mgo,500},
  Width            -> {Wgo,10},
  PDG              -> 1000021
},
```

✤ Differences with the gluon field declaration

★ Four-component fermionic field ➢ the label is F[1] (with an F)

★ Classname in the fermion case: defines two symbols to use in the Lagrangian ➢ *go* and *gobar*

★ Mass and width: we define two symbols and their associated numerical values

✦ Second option: two-component spinors (Lagrangians are sometimes easier to write)

```
W[1] == {
  ClassName      -> gow,
  Unphysical     -> True,
  Chirality      -> Left,
  SelfConjugate  -> False,
  Indices        -> {Index[Gluon]},
},
```

```
F[1] == {
  ClassName        -> go,
  WeylComponents   -> gow,
  SelfConjugate    -> True,
  Indices          -> {Index[Gluon]},
  Mass             -> {Mgo,500},
  Width            -> {Wgo,10},
  PDG              -> 1000021
},
```

★ Two-component and four-component fermionic fields ➢ the labels are W[1] (with a W) and F[1]

★ Weyl fermions are unphysical and linked to a four-component fermion (WeylComponents)

★ Several symbols are defined ➢ *go* and *gobar; gow* and *gowbar*

★ The chirality of the Weyl fermion can be specified

See the manual for more details on field declarations

Declaring the (top) quark field

✦ A third element in the *M$ClassesDescription* list

```
F[2] == {
  ClassName        -> q,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Mq, 173},
  Width            -> {Wq, 1.50833649},
  PDG              -> 6
},
```

✤ Nothing special compared to the other fields

★ Fundamental QCD indices are specified

✦ Parenthesis: three generations of up-type quarks (*Gen* being the generation index)

```
F[3] == {
  ClassName        -> uq,
  ClassMembers     -> {u, c, t},
  SelfConjugate    -> False,
  Indices          -> {Index[Gen], Index[Colour]},
  FlavorIndex      -> Gen,
  Mass             -> {Mu, {MU,2.55*^-3}, {MC,1.42}, {MT,173}},
  Width            -> {0, 0, {WT, 1.50833649}},
  PDG              -> {2, 4, 6}
},
```

★ ClassMembers: specify all the members of the class

★ FlavorIndex: defines which index is the flavor index (efficiency of the code)

★ Mass, Width, PDG: one for each class member (plus a generic mass symbol)

See the manual for more details on field declarations

Declaring the (top) squark fields

✦ Extra elements in the *M$ClassesDescription* list

```
S[1] == {
  ClassName        -> sqL,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Unphysical       -> True,
  Definitions      -> {sqL[c_] -> Cos[theta] sq1[c] - Sin[theta] sq2[c]}
},
S[2] == {
  ClassName        -> sqR,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Unphysical       -> True,
  Definitions      -> {sqR[c_] -> Sin[theta] sq1[c] + Cos[theta] sq2[c]}
},
```

```
S[3] == {
  ClassName        -> sq1,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Msq1,300},
  Width            -> {Wsq1,10},
  PDG              -> 1000006
},
S[4] == {
  ClassName        -> sq2,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Msq2,800},
  Width            -> {Wsq2,2},
  PDG              -> 2000006
}
```

✤ Squark fields mix:

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

★ Left and right-handed squarks are declared as unphysical

★ Definitions linking gauge- and mass-eigenstates are provided (inversion of the relation above)

➢ The rotations will be performed automatically by FeynRules

➢ The Lagrangian can be written in the gauge basis (easier)

See the manual for more details on field declarations

Parameter declaration

✦ The model contains three parameters:
- ✤ The strong coupling constants: g_s, α_s are both needed (required by the Monte Carlo tools)
- ✤ The squark mixing angle θ
- ✤ Masses and widths are handled automatically

✦ Parameters are declared as elements of the list *M$Parameters*
- ✤ A declaration ≡ a set of MATHEMATICA replacement rules
- ✤ In our SUSY-QCD model, we have:

```
(* ************************** *)
(* *****   Parameters   ***** *)
(* ************************** *)
M$Parameters = {
  aS    == {
    ParameterType    -> External,
    Value            -> 0.1184,
    InteractionOrder -> {QCD, 2}
  },
  gs == {
    ParameterType    -> Internal,
    Value            -> Sqrt[4 Pi aS],
    InteractionOrder -> {QCD, 1},
    ParameterName    -> G
  },
  theta == {
    ParameterType    -> External,
    Value            -> Pi/4.
  }
};
```

★ We have Internal and External parameters
- ➢ External ≡ free model parameter ⇒ numerical value
- ➢ Internal ≡ dependent model parameter ⇒ formula

★ InteractionOrder: specific to MADGRAPH5_aMC@NLO (more efficient diagram generation)

★ ParameterName: specific to Monte Carlo tools

★ Not used options: *TeX, Definitions, ComplexParameter, Description, BlockName, OrderBlock*

★ Specific to tensor parameters: *Indices, Unitary*

See the manual for more details on parameter declarations

Outline

Implementing the vector Lagrangian

Temporary restriction to the gauge content of the theory

- One ($SU(3)_c$) gauge supermultiplet
  - ★ One massive Majorana fermion: a gluino
  - ★ One massless gauge boson: the gluon

The dynamics of the model is embedded in the vector Lagrangian

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

- Kinetic terms for all the gluino and gluon fields
- Mass terms for the gluino
- Gauge interactions for both fields (embedded into gauge covariant objects)

The implementation in FeynRules is easy (*cf.* predefined functions linked to the gauge group)

- Option 1 (left): all non-Lorentz indices understood (FeynRules takes care of reintroducing them)
- Option 2 (right): all indices explicit

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

```
LVector2 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
             I/2 Ga[mu,s1,s2] gobar[s1,a].DC[go[s2,a],mu] -
             1/2 Mgo gobar[s1,a].go[s1,a];
```

Starting a MATHEMATICA session (1)

Step1: loading FeynRules into MATHEMATICA

- Setting up the FeynRules path
- Loading FeynRules itself

```
In[2]:= $FeynRulesPath = SetDirectory["~/Work/tools/FeynRules/branch/feynrules-current"];
        << FeynRules`

         - FeynRules -

        Version: 2.0.25 ( 22 April 2014).

        Authors: A. Alloul, N. Christensen, C. Degrande, C. Duhr, B. Fuks


        Please cite:
            - arXiv:1310.1921;
            - Comput.Phys.Commun.180:1614-1641,2009 (arXiv:0806.4194).

        http://feynrules.phys.ucl.ac.be

        The FeynRules palette can be opened using the command FRPalette[].
```

The output

- Information on FeynRules, the authors, the version number, references, *etc.*

Starting a MATHEMATICA session (2)

Step2: loading the SUSY-QCD implementation into FeynRules

- Moving to the right directory
- Loading the model itself

```
In[4]:= SetDirectory[NotebookDirectory[]];
        LoadModel["susyqcd.fr"];

        This model implementation was created by

        Benjamin Fuks

        Model Version: 1.0.0

        For more information, type ModelInformation[].

           - Loading particle classes.
           - Loading gauge group classes.
           - Loading parameter classes.

        Model SUSYQCD loaded.
```

The output

- Information of the model fil preamble are printed to the screen

Checking the implementation: comparing with books

Printing the vector Lagrangian and comparing with the textbook expression

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

```
In[8]:= LVector1

Out[8]= -1/2 Mgo gobar.go + 1/2 I gobar.γ^mu.(∂_mu[go] - I gs FSU3C^a$471.go G_mu,a$471) -
        1/4 (-∂_nu[G_mu,a] + ∂_mu[G_nu,a] + gs f_a,bb$469,cc$469 G_mu,bb$469 G_nu,cc$469)
          (-∂_nu[G_mu,a] + ∂_mu[G_nu,a] + gs f_a,bb$470,cc$470 G_mu,bb$470 G_nu,cc$470)
```

The output

- Covariant derivatives and field strength tensors have been automatically evaluated

Restoring all indices automatically

```
In[9]:= ExpandIndices[LVector1]

Out[9]= -1/4 ∂_nu[G_mu,a]^2 + 1/2 ∂_nu[G_mu,a] ∂_mu[G_nu,a] - 1/4 ∂_mu[G_nu,a]^2 - 1/2 Mgo go_i1$478,i2$478 . go_i1$478,i2$478 + 1/4 gs ∂_nu[G_mu,a] f_a,bb$473,cc$473 G_mu,bb$473 G_nu,cc$473 -
        1/4 gs ∂_mu[G_nu,a] f_a,bb$473,cc$473 G_mu,bb$473 G_nu,cc$473 + 1/4 gs ∂_nu[G_mu,a] f_a,bb$474,cc$474 G_mu,bb$474 G_nu,cc$474 -
        1/4 gs ∂_mu[G_nu,a] f_a,bb$474,cc$474 G_mu,bb$474 G_nu,cc$474 - 1/4 gs^2 f_a,bb$473,cc$473 f_a,bb$474,cc$474 G_mu,bb$473 G_mu,bb$474 G_nu,cc$473 G_nu,cc$474 +
        1/4 I go_i$478,i1$480 . ∂_mu[go_j$478,i1$480] γ^mu_i$478,j$478 - 1/4 I gs go_i$481,i$480 . go_j$481,j$480 f_a$475,i$480,j$480 G_mu,a$475 γ^mu_i$481,j$481 -
        1/4 I ∂_mu[go_j$478,i1$480] . go_i$478,i1$480 γ^mu_j$478,i$478 + 1/4 I gs go_j$481,j$480 . go_i$481,i$480 f_a$475,i$480,j$480 G_mu,a$475 γ^mu_j$481,i$481
```

Checking the implementation: hermiticity, normalization

✦ The Lagrangian must be Hermitian

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

```
In[10]:= CheckHermiticity[LVector1];
         Checking for hermiticity by calculating the Feynman rules contained in L-HC[L].
         If the lagrangian is hermitian, then the number of vertices should be zero.
         Starting Feynman rule calculation.
         Expanding the Lagrangian...
         No vertices found.
         0 vertices obtained.
         The lagrangian is hermitian.
```

✦ The Lagrangian must be canonically normalized

```
In[11]:= CheckKineticTermNormalisation[LVector1];
         Neglecting all terms with more than 2 particles.
         All kinetic terms are diagonal.
         All kinetic terms are correctly normalized.
```

✤ Other methods: *CheckDiagonalQuadraticTerms, CheckDiagonalKineticTerms, CheckDiagonalMassTerms*

Checking the implementation: the mass spectrum

- Checks at the level of the masses can be performed
  - The masses can be extracted from the Lagrangian
  - The masses are given when particles are declared
  - They should match

```
In[12]:= CheckMassSpectrum[LVector1]

         Neglecting all terms with more than 2 particles.

         All mass terms are diagonal.

         Getting mass spectrum.

         Checking for less then 0.1% agreement with model file values.

Out[12]//TableForm=
         Particle    Analytic value    Numerical value    Model-file value
         go          Mgo               500.               500.
```

The Feynman rules (1)

✦ Extract all N-point interactions from the Lagrangian (with N>2)

```
In[14]:= FeynmanRules[LVector1];
Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
3 possible non-zero vertices have been found -> starting the computation: 3 / 3.
3 vertices obtained.
(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
Vertex 1
Particle 1 : Vector , G
Particle 2 : Vector , G
Particle 3 : Vector , G
Vertex:
```

$$gs\, f_{a_1,a_2,a_3}\, p_1^{\mu_3}\, \eta_{\mu_1,\mu_2} - gs\, f_{a_1,a_2,a_3}\, p_2^{\mu_3}\, \eta_{\mu_1,\mu_2} - gs\, f_{a_1,a_2,a_3}\, p_1^{\mu_2}\, \eta_{\mu_1,\mu_3} + gs\, f_{a_1,a_2,a_3}\, p_3^{\mu_2}\, \eta_{\mu_1,\mu_3} + gs\, f_{a_1,a_2,a_3}\, p_2^{\mu_1}\, \eta_{\mu_2,\mu_3} - gs\, f_{a_1,a_2,a_3}\, p_3^{\mu_1}\, \eta_{\mu_2,\mu_3}$$

- Three vertices are found
- The expression above consists of the triple gluon vertex
- The index a_i is related to the i^{th} particle
  (reminder: *a* is the index style for the adjoint $SU(3)_C$ indices)
- The index μ_i is the Lorentz index of the i^{th} (vector) particle

The Feynman rules (2)

✦ The second vertex: the quartic gluon interaction

```
Vertex 2
Particle 1 : Vector , G
Particle 2 : Vector , G
Particle 3 : Vector , G
Particle 4 : Vector , G
Vertex:
```

$$\mathrm{i}\,\mathrm{gs}^2\,\mathrm{f}_{a_1,a_3,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_4}\,\eta_{\mu_2,\mu_3} + \mathrm{i}\,\mathrm{gs}^2\,\mathrm{f}_{a_1,a_2,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_3,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_4}\,\eta_{\mu_2,\mu_3} + \mathrm{i}\,\mathrm{gs}^2\,\mathrm{f}_{a_1,a_4,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_3,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_3}\,\eta_{\mu_2,\mu_4} - \mathrm{i}\,\mathrm{gs}^2\,\mathrm{f}_{a_1,a_2,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_3,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_3}\,\eta_{\mu_2,\mu_4} - \mathrm{i}\,\mathrm{gs}^2\,\mathrm{f}_{a_1,a_4,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_3,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_2}\,\eta_{\mu_3,\mu_4} - \mathrm{i}\,\mathrm{gs}^2\,\mathrm{f}_{a_1,a_3,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_2}\,\eta_{\mu_3,\mu_4}$$

- The index a_i is related to the i^{th} particle
  (reminder: *a* is the index style for the adjoint $SU(3)_C$ indices)
- The index *Gluon$1* is repeated ⇒ summed index
- The index μ_i is the Lorentz index of the i^{th} (vector) particle

The Feynman rules (3)

The third vertex: the gluon-gluino interaction

```
Vertex 3
Particle 1 : Majorana , go
Particle 2 : Majorana , go
Particle 3 : Vector , G
Vertex:
```

$$gs\, f_{a_1,a_2,a_3}\, \gamma^{\mu_3}_{s_1,s_2}$$

- The index a_i is related to the i^{th} particle
  (reminder: *a* is the index style for the adjoint $SU(3)_C$ indices)
- The index μ_i is the Lorentz index of the i^{th} (vector) particle
- The index s_i is the spin index of the i^{th} (fermionic) particle

The function FeynmanRules has many options

- Restriction on the interactions (MaxParticles, MaxCanonicalDimension, *etc.*)
- Selection of specific particles (Free, Contains, *etc.*)
- ScreenOutput: displaying the vertices to the screen or not
- FlavorExpand: perform a flavor expansion (otherwise, classes are used)

See the manual for more details on the function *FeynmanRules*

Implementing the matter Lagrangian (I)

Matter fields

- Two matter supermultiplets in the fundamental representation of $SU(3)_c$
  - ★ One massive Dirac fermion: a quark
  - ★ Two mixing massive scalar fields: two squark
  - ★ Gauge interactions with the $SU(3)_c$ gauge supermultiplet

$$\begin{pmatrix}\tilde{q}_1\\ \tilde{q}_2\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\tilde{q}_L\\ \tilde{q}_R\end{pmatrix}$$

The dynamics of the model is embedded in the matter Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{matter}} = {} & D_\mu\tilde{q}_L^\dagger D^\mu\tilde{q}_L + D_\mu\tilde{q}_R^\dagger D^\mu\tilde{q}_R + i\bar{q}\not{D}q - m^2_{\tilde{q}_i}\tilde{q}_i^\dagger\tilde{q}_i - m_q\bar{q}q\\ & - \frac{g_s^2}{2}\Big[-\tilde{q}_L^\dagger T^a\tilde{q}_L + \tilde{q}_R^\dagger T^a\tilde{q}_R\Big]\Big[-\tilde{q}_L^\dagger T^a\tilde{q}_L + \tilde{q}_R^\dagger T^a\tilde{q}_R\Big]\\ & + \sqrt{2}g_s\Big[-\tilde{q}_L^\dagger T^a(\tilde{g}^a P_L q) + (\bar{q}P_L\tilde{g}^a)T^a\tilde{q}_R\Big] + \text{h.c.}\end{aligned}$$

- Kinetic and gauge interaction terms (half of the first line) in the gauge basis
- Mass terms (second half of the first line) in the mass basis (because easier to implement)
- D-terms (second line)
- Supersymmetric gauge quark-squark-gluino interactions (fourth line)

Implementing the matter Lagrangian (2)

✦ Kinetic and gauge interactions

$$\mathcal{L}_{\text{matter}} = D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i\bar{q}\not{D}q - m^2_{\tilde{q}_i}\tilde{q}_i^\dagger \tilde{q}_i - m_q \bar{q} q$$

✦ The implementation in FeynRules is easy (*cf.* predefined functions linked to the gauge group)

```
Lkin := DC[sqLbar[cc],mu] DC[sqL[cc],mu] +
        DC[sqRbar[cc],mu] DC[sqR[cc],mu] +
        I qbar.Ga[mu].DC[q,mu] -
        Mq qbar.q -
        Msq1^2 sq1bar[cc] sq1[cc] -
        Msq2^2 sq2bar[cc] sq2[cc];
```

✤ Repeated indices are summed

✦ We can compute the Feynman rules

```
In[9]:= Simplify[FeynmanRules[Lkin]] // MatrixForm

Starting Feynman rule calculation.

Expanding the Lagrangian...

Collecting the different structures that enter the vertex.

9 possible non-zero vertices have been found -> starting the computation: 9 / 9.

5 vertices obtained.
```

Out[9]//MatrixForm=

$$\begin{pmatrix}
\{\{G,1\},\{sq1,2\},\{sq1^\dagger,3\}\} & i\,gs\left(p_2^{\mu_1}-p_3^{\mu_1}\right)T^{a_1}_{m_3,m_2} \\
\{\{G,1\},\{sq2,2\},\{sq2^\dagger,3\}\} & i\,gs\left(p_2^{\mu_1}-p_3^{\mu_1}\right)T^{a_1}_{m_3,m_2} \\
\{\{G,1\},\{G,2\},\{sq1,3\},\{sq1^\dagger,4\}\} & i\,gs^2\,\eta_{\mu_1,\mu_2}\left(T^{a_1}_{m_4,Colour\$1}T^{a_2}_{Colour\$1,m_3}+T^{a_1}_{Colour\$1,m_3}T^{a_2}_{m_4,Colour\$1}\right) \\
\{\{G,1\},\{G,2\},\{sq2,3\},\{sq2^\dagger,4\}\} & i\,gs^2\,\eta_{\mu_1,\mu_2}\left(T^{a_1}_{m_4,Colour\$1}T^{a_2}_{Colour\$1,m_3}+T^{a_1}_{Colour\$1,m_3}T^{a_2}_{m_4,Colour\$1}\right) \\
\{\{\bar{q},1\},\{q,2\},\{G,3\}\} & i\,gs\,\gamma^{\mu_3}_{s_1,s_2}T^{a_3}_{m_1,m_2}
\end{pmatrix}$$

✤ Field rotations have been performed automatically

Implementing the matter Lagrangian (3)

The D-terms

$$\mathcal{L}_{\text{matter}} = -\frac{g_s^2}{2}\left[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\right]\left[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\right]$$

The implementation in FeynRules is easy

```
LD := -1/2 gs^2 *
      (sqRbar[cc1] T[a,cc1,cc2] sqR[cc2] - sqLbar[cc1] T[a,cc1,cc2] sqL[cc2]) *
      (sqRbar[cc3] T[a,cc3,cc4] sqR[cc4] - sqLbar[cc3] T[a,cc3,cc4] sqL[cc4]);
```

- Repeated indices (*cc1*, *cc2*, *cc3*, *cc4*) are summed
- A single index can only be used twice

We can compute the Feynman rules

{{sq1, 1}, {sq1, 2}, {sq1†, 3}, {sq1†, 4}}	$-i\,gs^2\,\text{Cos}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)$
{{sq1, 1}, {sq1†, 2}, {sq1†, 3}, {sq2, 4}}	$\frac{1}{2}\,i\,gs^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_2,m_4}T^{\text{Gluon\$1}}_{m_3,m_1}+T^{\text{Gluon\$1}}_{m_2,m_1}T^{\text{Gluon\$1}}_{m_3,m_4}\right)$
{{sq1†, 1}, {sq1†, 2}, {sq2, 3}, {sq2, 4}}	$-i\,gs^2\,\text{Sin}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_1,m_4}T^{\text{Gluon\$1}}_{m_2,m_3}+T^{\text{Gluon\$1}}_{m_1,m_3}T^{\text{Gluon\$1}}_{m_2,m_4}\right)$
{{sq1, 1}, {sq1, 2}, {sq1†, 3}, {sq2†, 4}}	$\frac{1}{2}\,i\,gs^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)$
{{sq1, 1}, {sq1†, 2}, {sq2, 3}, {sq2†, 4}}	$-i\,gs^2\left(\text{Sin}[2\,\text{theta}]^2\,T^{\text{Gluon\$1}}_{m_2,m_3}T^{\text{Gluon\$1}}_{m_4,m_1}-\text{Cos}[2\,\text{theta}]^2\,T^{\text{Gluon\$1}}_{m_2,m_1}T^{\text{Gluon\$1}}_{m_4,m_3}\right)$
{{sq1†, 1}, {sq2, 2}, {sq2, 3}, {sq2†, 4}}	$-\frac{1}{2}\,i\,gs^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_1,m_3}T^{\text{Gluon\$1}}_{m_4,m_2}+T^{\text{Gluon\$1}}_{m_1,m_2}T^{\text{Gluon\$1}}_{m_4,m_3}\right)$
{{sq1, 1}, {sq1, 2}, {sq2†, 3}, {sq2†, 4}}	$-i\,gs^2\,\text{Sin}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)$
{{sq1, 1}, {sq2, 2}, {sq2†, 3}, {sq2†, 4}}	$-\frac{1}{2}\,i\,gs^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)$
{{sq2, 1}, {sq2, 2}, {sq2†, 3}, {sq2†, 4}}	$-i\,gs^2\,\text{Cos}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)$

- All nine vertices automatically derived from the (very compact) Lagrangian

Implementing the matter Lagrangian (4)

✦ The squark-quark-gluino interactions

$$\mathcal{L}_{\text{matter}} = \sqrt{2} g_s \Big[-\tilde{q}_L^\dagger T^a (\tilde{g}^a P_L q) + (\bar{q} P_L \tilde{g}^a) T^a \tilde{q}_R \Big] + \text{h.c.}$$

✦ The implementation in FeynRules is again easy

```
Lgosqq :=  Sqrt[2] gs ProjM[s1,s2] * (
    - sqLbar[cc1] T[a,cc1,cc2] gobar[s1,a].q[s2,cc2] +
      qbar[s1,cc1].go[s2,a] T[a,cc1,cc2] sqR[cc2]);
```

- ✤ All indices must this time be explicit (the scalars cannot be included in a fermion chain)
- ✤ *ProjM* denotes the left-handed chirality projector (*ProjP* is the right-handed one)

✦ We can compute the Feynman rules (including the Hermitian conjugate pieces, with HC)

```
In[37]:= Simplify[FeynmanRules[Lgosqq + HC[Lgosqq]]] // MatrixForm

Starting Feynman rule calculation.

Expanding the Lagrangian...

Collecting the different structures that enter the vertex.

4 possible non-zero vertices have been found -> starting the computation: 4 / 4.

4 vertices obtained.
```

Out[37]//MatrixForm=

$$\begin{pmatrix}
\{\{\bar{q}, 1\}, \{go, 2\}, \{sq1, 3\}\} & -i\sqrt{2}\, gs \left(\text{Cos}[\text{theta}]\, P_{+s_1,s_2} - P_{-s_1,s_2}\, \text{Sin}[\text{theta}]\right) T^{a_2}_{m_1,m_3} \\
\{\{go, 1\}, \{q, 2\}, \{sq1^\dagger, 3\}\} & -i\sqrt{2}\, gs \left(\text{Cos}[\text{theta}]\, P_{-s_1,s_2} - P_{+s_1,s_2}\, \text{Sin}[\text{theta}]\right) T^{a_1}_{m_3,m_2} \\
\{\{\bar{q}, 1\}, \{go, 2\}, \{sq2, 3\}\} & i\sqrt{2}\, gs \left(\text{Cos}[\text{theta}]\, P_{-s_1,s_2} + P_{+s_1,s_2}\, \text{Sin}[\text{theta}]\right) T^{a_2}_{m_1,m_3} \\
\{\{go, 1\}, \{q, 2\}, \{sq2^\dagger, 3\}\} & i\sqrt{2}\, gs \left(\text{Cos}[\text{theta}]\, P_{+s_1,s_2} + P_{-s_1,s_2}\, \text{Sin}[\text{theta}]\right) T^{a_1}_{m_3,m_2}
\end{pmatrix}$$

From FeynRules to phenomenology

- The SUSY-QCD model has been implemented into FeynRules

```
LMatter := Lkin + LD + Lgosqq + HC[Lgosqq];

LSUSYQCD := LVector1 + LMatter;
```

- The Feynman rules have been extracted (and checked)

- We are now ready to export the model to one or several Monte Carlo tools
  - CalcHep / CompHep
  - FeynArts / FormCalc
  - UFO ➢ MadGraph5_aMC@NLO / Sherpa / Herwig++
  - Whizard / Omega

```
WriteCHOutput[LSUSYQCD]
WriteFeynArtsOutput[LSUSYQCD]
WriteUFO[LSUSYQCD]
WriteWOOutput[LSUSYQCD]
```

Limitations and fine prints

Particle / parameter names

- The strong interaction has a special role
  - ★ Name for the gluon field, the coupling constant, *etc.*
  - ★ Which parameter is internal/external
  - ★ The numerical value of α_s at the Z-pole
- For some generators, the electroweak interaction has also a special role
  - ★ Name for the Fermi coupling, the Z-boson mass
  - ★ Which parameter is external/internal
  - ★ At which scale must the numerical values be given

Color structures: not supported in full generality

- The interfaces discard the non-supported vertices
- Representations handled by the FeynRules interfaces:
  - ★ CalcHep: 1, 3, 8 (but limited)
  - ★ FeynArts: all
  - ★ MadGraph5_aMC@NLO: 1, 3, 6, 8
  - ★ Sherpa: 1, 3, 8
  - ★ Whizard: 1, 3, 8

Lorentz structures and spins: not supported in full generality

- The interfaces discard the non-supported vertices
- Representations handled by the FeynRules interfaces:
  - ★ CalcHep: MSSM-like Lorentz structures; spin 0,1/2,1,3/2, 2
  - ★ FeynArts: all Lorentz structures; spin 0,1/2,1
  - ★ MadGraph5_aMC@NLO: all Lorentz structures; spin 0,1/2,1,3/2,2
  - ★ Sherpa: all Lorentz structures; spin 0,1/2,1
  - ★ Whizard: MSSM-like Lorentz structures; spin 0,1/2,1,2

From FeynRules to Monte Carlo tools: the UFO

✦ The UFO in a nutshell

- ✤ UFO ≡ Universal FeynRules output (not tied to any Monte Carlo tool)
- ✤ Allows the model to contain generic color and Lorentz structures
- ✤ The FeynRules interface creates a Python module to be linked to any code
- ✤ This module contains all the model information
- ✤ More information ➢ Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter [CPC 183 (2012) 1201]

✦ The UFO version of the model can be used by several programs

- ✤ ALOHA
- ✤ GOSAM
- ✤ HERWIG++
- ✤ MADANALYSIS 5
- ✤ MADGRAPH5_AMC@NLO
- ✤ SHERPA

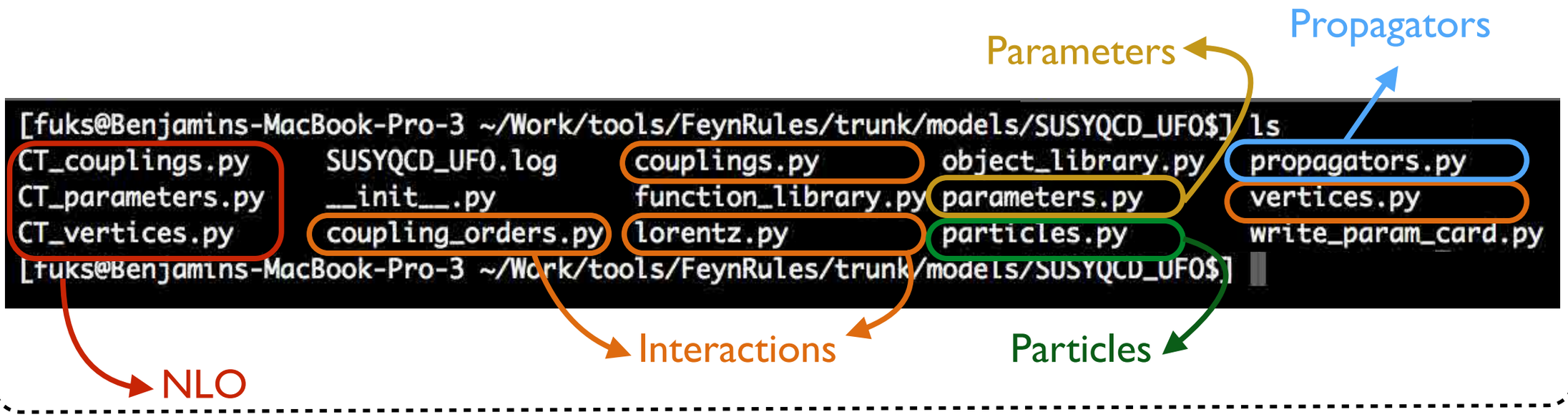
✦ Used with copy-pasting

```
In[6]:= WriteUFO[LSUSYQCD]

      --- Universal FeynRules Output (UFO) v 1.1 ---

Warning: no electric charge defined. Putting all electric charges to zero.
Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
25 possible non-zero vertices have been found -> starting the computation: 25 / 25.
21 vertices obtained.
Flavor expansion of the vertices: 21 / 21
   - Saved vertices in InterfaceRun[ 1 ].
Computing the squared matrix elements relevant for the 1->2 decays:
4 / 4
Squared matrix elent compute in 0.112652 seconds.
Decay widths computed in 0.002791 seconds.
Preparing Python output.
    - Splitting vertices into building blocks.
    - Optimizing: 21/21 .
    - Writing files.
Done!
```

UFOs in details

- The UFO is a set of Python files
  - Particle information (particles.py)
  - Interaction information (vertices.py, couplings.py, lorentz.py, couplings_orders.py)
  - Parameter information (parameters.py)
  - Propagator information (propagators.py)
  - Tools (function_library.py, object_library.py, write_param_card.py, decays.py)
  - NLO counterterms (CT_couplings.py, CT_parameters.py, CT_vertices.py)

- Example: a UFO for supersymmetric QCD
  - All UFOs have a similar structure
  - CT_xxx.py files optional: no official standards for NLO (yet)



Particle implementations in UFOs

Particles

- Similar to the FeynRules model format, but with different names for the attributes (masses, widths, PDG codes, *etc.*)
- Antiparticles are automatically derived from the knowledge of the corresponding particle
- Special keyword for the (numerical value) zero
- Note: the embedding of the color and spin representation

```
G = Particle(pdg_code = 21,
             name = 'G',
             antiname = 'G',
             spin = 3,
             color = 8,
             mass = Param.ZERO,
             width = Param.ZERO,
             texname = 'G',
             antitexname = 'G',
             charge = 0)

go = Particle(pdg_code = 1000021,
              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0)
```

```
sq1 = Particle(pdg_code = 1000006,
               name = 'sq1',
               antiname = 'sq1~',
               spin = 1,
               color = 3,
               mass = Param.Msq1,
               width = Param.Wsq1,
               texname = 'sq1',
               antitexname = 'sq1~',
               charge = 0)

sq1__tilde__ = sq1.anti()

sq2 = Particle(pdg_code = 2000006,
               name = 'sq2',
               antiname = 'sq2~',
               spin = 1,
               color = 3,
               mass = Param.Msq2,
               width = Param.Wsq2,
               texname = 'sq2',
               antitexname = 'sq2~',
               charge = 0)

sq2__tilde__ = sq2.anti()
```

```
q = Particle(pdg_code = 6,
             name = 'q',
             antiname = 'q~',
             spin = 2,
             color = 3,
             mass = Param.Mq,
             width = Param.Wq,
             texname = 'q',
             antitexname = 'q~',
             charge = 0)

q__tilde__ = q.anti()
```

Parameter implementations in UFOs

Parameters

- Similar to the FeynRules model format, but with different names
- Note: the automated generation of a LesHouches structure for the external parameters (in particular for the masses and widths)
- Python-compliant formula for the internal parameters

```
aS = Parameter(name = 'aS',
               nature = 'external',
               type = 'real',
               value = 0.1184,
               texname = '\\text{aS}',
               lhablock = 'FRBlock',
               lhacode = [ 1 ])

theta = Parameter(name = 'theta',
                  nature = 'external',
                  type = 'real',
                  value = 0.7853981633974483,
                  texname = '\\theta',
                  lhablock = 'FRBlock',
                  lhacode = [ 2 ])
```

```
Mgo = Parameter(name = 'Mgo',
                nature = 'external',
                type = 'real',
                value = 500,
                texname = '\\text{Mgo}',
                lhablock = 'MASS',
                lhacode = [ 1000021 ])

Wq = Parameter(name = 'Wq',
               nature = 'external',
               type = 'real',
               value = 1.50833649,
               texname = '\\text{Wq}',
               lhablock = 'DECAY',
               lhacode = [ 6 ])
```

```
G = Parameter(name = 'G',
              nature = 'internal',
              type = 'real',
              value = '2*cmath.sqrt(aS)*cmath.sqrt(cmath.pi)',
              texname = 'G')
```

Strategy for implementing interactions in UFO

✦ Vertices are decomposed in a spin x color basis, coupling strengths being coordinates

✤ Example: the quartic gluon vertex can be written as

$$\begin{aligned} & ig_s^2\, f^{a_1a_2b} f^{ba_3a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}) \\ & \quad + ig_s^2\, f^{a_1a_3b} f^{ba_2a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \\ & \quad + ig_s^2\, f^{a_1a_4b} f^{ba_2a_3}\, (\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \end{aligned} \Rightarrow \begin{aligned} & \left(f^{a_1a_2b} f^{ba_3a_4},\ f^{a_1a_3b} f^{ba_2a_4}, f^{a_1a_4b} f^{ba_2a_3}\right) \\ & \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix} \end{aligned}$$

✤ Each element of this decomposition is stored separately in the vertex.py file

★ vertices.py: defines all model decompositions
★ lorentz ≡ the spin basis (stored in lorentz.py; reused across vertices for economical reasons)
★ color ≡ the color basis (directly defined in the file)
★ couplings ≡ the coordinates (stored in couplings.py; reused across vertices for economical reasons)

```
V_2 = Vertex(name = 'V_2',
             particles = [ P.G, P.G, P.G, P.G ],
             color = [ 'f(-1,1,2)*f(3,4,-1)', 'f(-1,1,3)*f(2,4,-1)', 'f(-1,1,4)*f(2,3,-1)' ],
             lorentz = [ L.VVVV1, L.VVVV2, L.VVVV3 ],
             couplings = {(1,1):C.GC_4,(0,0):C.GC_4,(2,2):C.GC_4})
```

★ lorentz.py

```
VVVV1 = Lorentz(name = 'VVVV1',
                spins = [ 3, 3, 3, 3 ],
                structure = 'Metric(1,4)*Metric(2,3) - Metric(1,3)*Metric(2,4)')
```

★ couplings.py

```
GC_4 = Coupling(name = 'GC_4',
                value = 'complex(0,1)*G**2',
                order = {'QCD':2})
```

Coupling orders: information that may be used for selecting diagrams

Outline

Advanced techniques for implementing models in FeynRules

- Extension / restriction of existing models
- The superspace module of FeynRules
- Mass diagonalization
- Two-body decays
- Next-to-leading order module

Merging and extending model implementations

- Many BSM models of interest are simple extensions of another model
  - FeynRules allows one to start from a given model
    - ★ Add new particles, parameters, Lagrangian terms
    - ★ Modify existing particles, parameters, Lagrangian terms
    - ★ Remove some particles, parameters, Lagrangian terms
  - Special cases very relevant for LHC physics: Simplified Model Spectra
    - ★ The Standard Model + one or two new particles
    - ★ Often inspired by the MSSM
    - ★ Example: the SM + lightest stop and neutralino + the relevant subset of MSSM interactions

- The merged FeynRules model contains two .fr files
  - The parent model implementation
  - One extra file with the modifications
  - They must be loaded together (the parent model first)

```
LoadModel["SM.fr", "stops.fr"];
```

    - ★ No need to re-implement what is common (gauge groups, *etc.*)

- One can start from any of the models available on the FeynRules model database
  http://feynrules.irmp.ucl.ac.be

The FeynRules model database

- O(50) models are available online
  - Simple extensions of the Standard Model
    - ★ Simplified model spectra
    - ★ Four generation models
    - ★ Vector-like quarks
    - ★ Two-Higgs-Doublet Models, Hidden Abelian Higgs
    - ★ *etc.*
  - Supersymmetric models
    - ★ MSSM with and without R-parity
    - ★ The NMSSM
    - ★ R-symmetric supersymmetric models
    - ★ Left-right supersymmetric models
  - Extra-dimensional models
    - ★ Universal extra-dimensions
    - ★ Large extra-dimensions
    - ★ Heidi, Minimal Higgsless models
    - ★ Randall-Sundrum
  - Strongly coupled and effective field theories
    - ★ Technicolor
    - ★ Models with dimension-six and dimension-eight operators
    - ★ *etc.*

Restricting model implementations

- Many BSM models of interest are subset of other models
  - Equivalent to the parent model, but with some parameters set to specific values (0 or 1)
    - ★ Example 1: the massless version of a model (massless light quarks in the Standard Model)
    - ★ Example 2: a mixing matrix set to the identity (no-CKM matrix in the Standard Model)
  - FeynRules allows one to start from a given model
    - ★ Write the restrictions under the form of a list of Mathematica replacement rules (M$Restrictions)
    - ★ Read them into FeynRules
    - ★ Apply them before the computation of any Feynman rule

- The output Feynman rules (and this Monte Carlo model files)
  - Are free from the restricted parameters
  - Smaller files, more efficiency at the Monte Carlo level
    - ★ Example: the general MSSM has more than 10000 vertices; its flavor-conserving version only ~1000

- One practical example: the Standard Model without CKM-mixing

```
M$Restrictions = {
                CKM[i_,i_] -> 1,
                CKM[i_?NumericQ, j_?NumericQ] :> 0 /; (i =!= j),
                cabi -> 0
}
```

```
LoadRestriction["DiagonalCKM.rst"]
```

The supersymmetry module

- A module dedicated to calculations in superspace
  - Superfield declaration and links to the component fields
  - Series expansion in terms of component fields
  - Automatic derivation of supersymmetric Lagrangians
  - Solving the equations of motion of the unphysical fields
  - Many extra built-in functions
  - See: Duhr, BF [CPC 182 (2011) 2404]; BF [IJMPA 27 (2012) 1230007]

A left-handed squark superfield

```
CSF[1] == {
  ClassName       -> QL,
  Chirality       -> Left,
  Weyl            -> qLw,
  Scalar          -> sqL,
  Indices         -> {Index[Colour]}
}
```

- Supersymmetric model implementation
  - Declaration of the model gauge group
  - Declaration of all fields and superfields
  - Declaration of all model parameters
  - Can the writing of the Lagrangian be simplified?
- Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi_{|_{\theta^2\bar{\theta}^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(W^\alpha W_\alpha)_{|_{\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}})_{|_{\bar{\theta}^2}}$$
$$+ W(\Phi)_{|_{\theta^2}} + W^\star(\Phi^\dagger)_{|_{\bar{\theta}^2}} + \mathcal{L}_{\mathrm{soft}}$$

➢ Model independent ⇒ can be automated

➢ Model dependent ⇒ to be provided

- First line: kinetic and gauge interaction terms for all fields
- Second line: model dependent superpotential and supersymmetry breaking Lagrangian

See the manual for more details on the superspace module

Implementing supersymmetric Lagrangians

Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi_{|_{\theta^2\bar{\theta}^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(W^\alpha W_\alpha)_{|_{\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}})_{|_{\bar{\theta}^2}}$$
$$+ W(\Phi)_{|_{\theta^2}} + W^\star(\Phi^\dagger)_{|_{\bar{\theta}^2}} + \mathcal{L}_{\mathrm{soft}}$$

- First line: kinetic and gauge interaction terms for all fields
  - ★ Model independent ⇒ can be automated (CSFKineticTerms and VSFKineticTerms functions)
  - ★ Dedicated methods to access the components of a superfield (Theta2Component, *etc.*)

```
Lag = Theta2Thetabar2Component[ CSFKineticTerms[] ] +
   Theta2Component[ VSFKineticTerms[] ] +
   Thetabar2Component[ VSFKineticTerms[] ];
```

- Second line: superpotential and supersymmetry-breaking terms
  - ★ Model dependent ⇒ to be provided by the user (*SuperW* and *LSoft*)

```
Lag2 = LSoft + Theta2Component[ SuperW ] +  Thetabar2Component[ SuperW ];
```

The Lagrangian above must be post-processed

- Solving the equation of motion for the auxiliary fields and inserting the solution back into the Lagrangian
  - ★ Automated (SolveEqMotionF and SolveEqMotionD)
- Replacement of Weyl spinors in terms of Majorana and Dirac spinors
  - ★Automated (WeylToDirac)
- Rotation to the mass basis
  - ★Standard FeynRules function (ExpandIndices)

See the manual for more details on the superspace module

Mass matrices and their diagonalization

The problematics of the mass matrices

- Lagrangians are usually easily written in the gauge basis
- The included mass matrices are thus in general non-diagonal ➢ diagonalization required
- The gauge basis of fields must be rotated to the mass basis where the mass matrices are diagonal
- This diagonalization cannot in general be achieved analytically

The ASperGe package of FeynRules

- A Mathematica module allowing one to extract the mass matrices from the Lagrangian
- A generator of C++ code ➢ numerical diagonalization of all the model mass matrices (the generated code can be used in a standalone way)
- See: Alloul, D'Hondt, De Causmaecker, BF, Rausch de Traubenberg [EPJC 73 (2013) 2325]

Example: the Z-boson and photon in the Standard Model

- Each mixing is declared as a set of replacement rules (in *M$MixingsDescription)*
- Each rule represent a property of the mixing relation

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = U_w \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \Rightarrow$$

```
Mix["AZ"] == {
  MassBasis    -> {A, Z},
  GaugeBasis   -> {B, Wi[3]},
  MixingMatrix -> UW,
  BlockName    -> WEAKMIX
}
```

- ASperGe can compute the mass matrices:
- ASperGe can generate its standalone C++ version:

```
ComputeMassMatrix[Lag];
WriteASperGe[Lag];
```

See the manual for more details on the ASperGe module

Two-body decays

✦ The problematics of the decay widths and branching ratios

- ✤ Some Monte Carlo tools need the decay table (widths and branching ratios) to decay particles
- ✤ Widths and branching ratios are not independent quantities ➢ need to be calculated
- ✤ Some Monte Carlo tools compute these quantities on the fly
  - ➢ the procedure is repeated each time it is needed
- ✤ FeynRules offers a way to include analytical information on the two-body decay

✦ The decay module of FeynRules

- ✤ Two body decays can be directly read from the three-point vertices ($\mathcal{V}$) of the model

$$\Gamma_{1\to 2} = \frac{1}{2|M|S}\int \mathrm{d}\Phi_N \,|\mathcal{M}_{1\to 2}|^2 = \frac{\sqrt{\lambda(M^2,m_1^2,m_2^2)}}{16\,\pi\, S\,|M|^3}\mathcal{V}^{a_1a_2a_3}_{\ell_1\ell_2\ell_3}\,\mathcal{P}_1^{\ell_1\ell_1'}\,\mathcal{P}_2^{\ell_2\ell_2'}\,\mathcal{P}_3^{\ell_3\ell_3'}\,(\mathcal{V}^*)^{a_1a_2a_3}_{\ell_1'\ell_2'\ell_3'}$$

  - ★ Partial width for the decay of a particle of mass *M* to two particles of masses m_1 and m_2
  - ★ Includes a symmetry factor *S* and $\mathcal{P}$ denotes the polarization tensor of each particle
- ✤ FeynRules makes use of Mathematica to compute all partial widths of the model
  - ★ Ignores open and closed channels ➢ benchmark independent
  - ★ The information is exported to the UFO (already used by MadWidth, the decay module of MadGraph5)

The decay module of FeynRules

✦Automatic decay width computations

- ✤ All two-body decay widths can be easily computed from the Lagrangian

```
verts      = FeynmanRules[Lag];
vertsexp   = FlavorExpansion[verts];
results    = ComputeWidths[vertsexp];
```

- ✤ Many functions available for analytical calculations (PartialWidth, TotWidth, BranchingRatio)
- ✤ The numerical value provided for the particle widths can be updated accordingly (UpdateWidths)
- ✤ See: Alwall, Duhr, BF, Mattelaer, Öztürk, Shen [CPC (in press)]

✦The information is (by default) employed at the UFO interface level

- ✤ Can be turned of: (AddDecays → False)
- ✤ The UFO contains an extra file *decays.py*
- ✤ Example of the Standard Model UFO: the top quark

```
Decay_t = Decay(name = 'Decay_t',
                particle = P.t,
                partial_widths = {(P.W__plus__,P.d):'((MT**2 - MW**2)*((3*CKM3x1*ee**2*MT**2*complexconjugate(CKM3x1))/(2.*sw**2) +
(3*CKM3x1*ee**2*MT**4*complexconjugate(CKM3x1))/(2.*MW**2*sw**2) - (3*CKM3x1*ee**2*MW**2*complexconjugate(CKM3x1))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                                (P.W__plus__,P.s):'((MT**2 - MW**2)*((3*CKM3x2*ee**2*MT**2*complexconjugate(CKM3x2))/(2.*sw**2) +
(3*CKM3x2*ee**2*MT**4*complexconjugate(CKM3x2))/(2.*MW**2*sw**2) - (3*CKM3x2*ee**2*MW**2*complexconjugate(CKM3x2))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                                (P.W__plus__,P.b):'(((3*CKM3x3*ee**2*MB**2*complexconjugate(CKM3x3))/(2.*sw**2) +
(3*CKM3x3*ee**2*MT**2*complexconjugate(CKM3x3))/(2.*sw**2) + (3*CKM3x3*ee**2*MB**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MB**2*MT**2*complexconjugate(CKM3x3))/(MW**2*sw**2) + (3*CKM3x3*ee**2*MT**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MW**2*complexconjugate(CKM3x3))/sw**2)*cmath.sqrt(MB**4 - 2*MB**2*MT**2 + MT**4 - 2*MB**2*MW**2 - 2*MT**2*MW**2 +
MW**4))/(96.*cmath.pi*abs(MT)**3)'})
```

See the manual for more details on the decay module

FeynRules @ NLO

✦ Ingredients for a next-to-leading order calculation
- ✤ Tree-level vertices
- ✤ UV counterterms
- ✤ R_2 counterterms (depending on the NLO event generator)

✦ The R_2 counterterms
- ✤ One-loop modules are based on unitarity approaches
- ✤ This requires the computation of the rational parts of the loops (R_1 and R_2)
- ✤ The R_2 counterterm can be calculated once and for all for each model
  - ★ From the knowledge of the tree-level Lagrangian
  - ★ Using special Feynman rules (numerators in ε dimensions; denominators in D dimensions)

✦ Technical details for the NLO module of FeynRules
- ✤ Automatic renormalization of the Lagrangian
- ✤ Use of the FeynArts interface of FeynRules
- ✤ Generation of a script for NLO diagram generation
  - ☑ R_2 and UV counterterms
- ✤ Export to the UFO
- ✤ See: Degrande [arXiv:1406.3030] and Degrande, BF, Hirschi, Proudom & Shao [PRD 91 (2015) 094005]

Outline

Summary

✦ The quest for new physics at the LHC has started
- ✤ Rely on Monte Carlo event generators for background and signal modeling
- ✤ FeynRules facilitates the implementation of new physics models in those tools

✦ FeynRules: http://feynrules.irmp.ucl.ac.be
- ✤ Straightforward implementation of new physics model in Monte Carlo tools
  - ★ Interfaces to many programs
- ✤ FeynRules is shipped with its own computational modules
  - ★ A superspace module
  - ★ A decay package
  - ★ A mass diagonalization module (ASperGe)
  - ★ A brand new NLO module

Try it on with your favorite model!

The tutorial model

✦ We consider a simple extension of the Standard Model

✤ Two extra scalar fields that mix

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

✤ Kinetic Lagrangian in the gauge basis:

$$\mathcal{L}_{kin,scalar} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \frac{m_1^2}{2}\phi_1^2 - \frac{m_2^2}{2}\phi_2^2 - m_{12}^2\phi_1\phi_2$$

✤ Two extra Dirac fermions U and E

★ Whose quantum numbers are those of the right-handed up-quark and electron

★ Massive fields

$$\mathcal{L}_{dirac,mass} = M_U\overline{U}U + M_E\overline{E}E$$

✤ New Yukawa interactions

$$\mathcal{L}_{FFS} = \lambda_{1,i}\,\phi_1\,\overline{U}P_R u_i + \lambda_{2,i}\,\phi_2\,\overline{U}P_R u_i + \lambda'_{1,i}\,\phi_1\,\overline{E}P_R l_i + \lambda'_2\,\phi_2\,\overline{E}P_R l_i + \text{h.c.}$$

✦ All FeynRules files are available for download (ask, nicely, the school organizers)