The LHC, Cosmology and the Origin of Scales

Alberto Salvio

Universidad Autónoma de Madrid (UAM)

19th of November 2015

IV Postgraduate Meeting on Theoretical Physics (IFT)

Based on

- Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (JHEP) arXiv:1307.3536
- Salvio, Strumia (JHEP) arXiv:1403.4226
- Kannike, Hütsi, Pizza, Racioppi, Raidal, Salvio, Strumia (JHEP) arXiv:1502.01334
Introduction

The stability bound

Inflation and the Standard Model

Models without fundamental masses and inflation

Conclusions
Experimental situation

- *Discovery of a Higgs boson in 2012* at the Large Hadron Collider (LHC). It weighs $M_h \approx 125$ GeV

- So far no significant deviation from the Standard Model (SM) at the electroweak (EW) scale. Although beyond the SM (BSM) physics is not yet excluded!
Experimental situation

- *Discovery of a Higgs boson in 2012* at the Large Hadron Collider (LHC). It weighs $M_h \approx 125$ GeV

- So far no significant deviation from the Standard Model (SM) at the electroweak (EW) scale. Although beyond the SM (BSM) physics is not yet excluded!

But now we can use the SM
- to make predictions up to the Planck mass $M_{\text{Pl}}$
- to test the SM with cosmology as well as with laboratory data ...
Experimental situation

- *Discovery of a Higgs boson in 2012* at the Large Hadron Collider (LHC). It weighs $M_h \approx 125$ GeV

- So far no significant deviation from the Standard Model (SM) at the electroweak (EW) scale. Although beyond the SM (BSM) physics is not yet excluded!

But now we can use the SM
- to make predictions up to the Planck mass $M_{Pl}$
- to test the SM with cosmology as well as with laboratory data ...

Indeed, we are in the era of precision cosmology!
(e.g. Planck, BICEP2/Keck, BICEP3/Keck, SKA, ...)

Can we really extrapolate the SM?

The consistency seems ok: some couplings seem to diverge as a function of the energy $\mu$ (Landau poles), but above $M_{Pl}$

Solutions of the renormalization group equations (RGEs) of the most relevant SM parameters
(defined in the $\overline{MS}$ scheme ...)

Can we really extrapolate the SM?

Still there are unsolved problems:

- Dark matter
- (small) neutrino masses
- Baryon asymmetry
- Understanding of EW symmetry breaking, why $m \ll M_{Pl}$, ...
Can we really extrapolate the SM?

Still there are unsolved problems:

- **Dark matter**
  axions, mirror particles, ...

- **(small) neutrino masses**
  heavy right-handed neutrinos, ...

- **Baryon asymmetry**
  leptogenesis, ...

- **Understanding of EW symmetry breaking, why \( m \ll M_{\text{Pl}} \)?, ...**
  Dynamical explanation: dimensional transmutation, supersymmetry, extra dimensions ...
Can we really extrapolate the SM?

Still there are unsolved problems:

- **Dark matter**
  axions, mirror particles, ...

- **(small) neutrino masses**
  heavy right-handed neutrinos, ...

- **Baryon asymmetry**
  leptogenesis, ...

- **Understanding of EW symmetry breaking, why $m \ll M_{Pl}$?, ...**
  Dynamical explanation: dimensional transmutation, supersymmetry, extra dimensions ...
Can we really extrapolate the SM?

Still there are unsolved problems:

- **Dark matter**
  axions, mirror particles, ...

- **(small) neutrino masses**
  heavy right-handed neutrinos, ...

- **Baryon asymmetry**
  leptogenesis, ...

- **Understanding of EW symmetry breaking, why \( m \ll M_{Pl} \)?, ...**
  Dynamical explanation: dimensional transmutation, supersymmetry, extra dimensions ...

Nevertheless, there are extensions that solve these phenomenological problems and do not invalidate all the SM predictions at very high energies ...
Introduction

The stability bound

Inflation and the Standard Model

Models without fundamental masses and inflation

Conclusions
Origin of the stability bound

In simple terms

$$\lambda h^4$$
Origin of the stability bound

In simple terms

quantum corrections

\[ \lambda h^4 \rightarrow \lambda(h)h^4 \]
Origin of the stability bound

In simple terms, quantum corrections

$$\lambda h^4 \rightarrow \lambda(h)h^4$$

Precise “running” of $\lambda$ and its $\beta$-function

$$\left( \beta_\lambda \equiv \frac{d\lambda}{d \ln \mu} \right)$$

3$\sigma$ bands in

- $M_t = 173.3 \pm 0.8$ GeV (gray)
- $\alpha_3(M_Z) = 0.1184 \pm 0.0007$ (red)
- $M_h = 125.1 \pm 0.2$ GeV (blue)

3$\sigma$ bands in

- $M_t = 171.1$ GeV
- $\alpha_3(M_Z) = 0.1205$
- $\alpha_3(M_Z) = 0.1163$
- $M_h = 125.1 \pm 0.2$ GeV (blue)
Result for the stability bound

\[ M_h > 129.6 \text{ GeV} + 2.0(M_t - 173.34 \text{ GeV}) - 0.5 \text{ GeV} \frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\text{th}} \text{ GeV} \]

Combining in quadrature the experimental and theoretical uncertainties we obtain

\[ M_h > (129.6 \pm 1.5) \text{ GeV} \rightarrow \text{ the stability bound is violated at the 2.8}\sigma \text{ level} \ldots \]
Result for the stability bound

\[ M_h > 129.6 \text{ GeV} + 2.0(M_t - 173.34 \text{ GeV}) - 0.5 \text{ GeV} \frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\text{th}} \text{ GeV} \]

Combining in quadrature the experimental and theoretical uncertainties we obtain

\[ M_h > (129.6 \pm 1.5) \text{ GeV} \rightarrow \text{the stability bound is violated at the 2.8} \sigma \text{ level} \ldots \]

Phase diagram of the SM:
The SM phase diagram in terms of Planck scale couplings

\[ y_t(M_{P1}) \text{ versus } \lambda(M_{P1}) \]

"Planck-scale dominated" corresponds to \( \Lambda_I > 10^{18} \) GeV

"No EW vacuum" corresponds to a situation in which \( \lambda \) is negative at the EW scale
The SM phase diagram in terms of Planck scale couplings

Gauge coupling $g_2$ at $M_{P1}$ versus $\lambda(M_{P1})$

**Left:** $g_1(M_{P1})/g_2(M_{P1}) = 1.22$, $y_t(M_{P1})$ and $g_3(M_{P1})$ are kept to the SM value

**Right:** a common rescaling factor is applied to $g_1, g_2, g_3$. $y_t(M_{P1})$ is kept to the SM value
The SM phase diagram in terms of potential parameters

If $\lambda(M_{Pl}) < 0$ there is an upper bound on $m$ requiring $\langle h \rangle \neq 0$ at the EW scale. This bound is, however, much weaker than the anthropic bound of [Agrawal, Barr, Donoghue, Seckel (1997); Schellekens (2014)].
Meta-stability

Is it worrisome?
Meta-stability

Is it worrisome?

Left: The probability that EW vacuum decay happened in our past light-cone.

Right: The life-time of the EW vacuum, with 2 different assumptions for future cosmology: universes dominated by the cosmological constant ($\Lambda$CDM) or by the dark matter (CDM).
Introduction

The stability bound

**Inflation and the Standard Model**

Models without fundamental masses and inflation

Conclusions
Inflation

[Brout, Englert, Gunzig (1978); Guth (1981); Linde (1982); Albrecht, Steinhardt (1982)]

What it can solve: horizon, flatness, monopole problems

To solve these problems inflation should last enough $\rightarrow$ lower bounds on

$$N \equiv \ln \left( \frac{a(t_{\text{end}})}{a(t_{\text{in}})} \right) \equiv \text{number of e-foldings}$$
Inflation

[Brout, Englert, Gunzig (1978); Guth (1981); Linde (1982); Albrecht, Steinhardt (1982)]

What it can solve: horizon, flatness, monopole problems

To solve these problems inflation should last enough → lower bounds on

\[ N \equiv \ln \left( \frac{a(t_{\text{end}})}{a(t_{\text{in}})} \right) \equiv \text{number of e-foldings} \]

How it is implemented (slow-roll inflation):

- we assume some scalar field \( \varphi \) (the inflaton)
- at some early time the potential \( U(\varphi) \) is large, but flat enough
- → the Hubble \( H \equiv a^{-1} \frac{da}{dt} \) changes slowly → \( a(t) \approx a(t_{\text{in}}) e^{Ht} \)


Inflation
[Brout, Englert, Gunzig (1978); Guth (1981); Linde (1982); Albrecht, Steinhardt (1982)]

What it can solve: horizon, flatness, monopole problems

To solve these problems inflation should last enough → lower bounds on

\[ N \equiv \ln \left( \frac{a(t_{\text{end}})}{a(t_{\text{in}})} \right) \equiv \text{number of e-foldings} \]

How it is implemented (slow-roll inflation):

▶ we assume some scalar field \( \phi \) (the inflaton)
▶ at some early time the potential \( U(\phi) \) is large, but flat enough
▶ → the Hubble \( H \equiv \frac{1}{a} \frac{da}{dt} \) changes slowly → \( a(t) \approx a(t_{\text{in}}) e^{Ht} \)

For each model we can compute parameters that are observable (e.g. by Planck):
▶ the scalar amplitude \( A_s \),
▶ its spectral index \( n_s \)
▶ and the tensor-to-scalar ratio \( r = A_t/A_s \)
Inflation in the SM (the inflaton is \( h \))

[Barvinsky, Kamenshchik, Mishakov (1996); Bezrukov, Shaposhnikov (2008)]

The model:

\[
\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{SM} - \xi |H|^2 R
\]

\( \xi \) is a constant that is typically very large for perturbative unitarity to be violated at an energy close to the inflationary one.

\( \bar{M}_{Pl} \approx 2 \times 10^{18} \text{ GeV} \)

\( U \ll \bar{M}_{Pl}^4 \) (such that quantum Einstein gravity effects are small)

\( \xi \) is generically very large.
Inflation in the SM (the inflaton is $h$)

[Barvinsky, Kamenshchik, Mishakov (1996); Bezrukov, Shaposhnikov (2008)]

The model:

$$\mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{SM}} - \xi |H|^2 R$$

The potential is

$$U \equiv \frac{V}{\Omega^4} = \frac{\tilde{M}_{\text{Pl}}^4 \lambda (h^2 - \nu^2)^2}{4(\tilde{M}_{\text{Pl}}^2 + \xi h^2)^2}$$

$\Omega^2$ corresponds, by definition, to what multiplies $R$ in $\mathcal{L}$ and $\tilde{M}_{\text{Pl}} \approx 2.4 \times 10^{18}\text{GeV}$
Inflation in the SM (the inflaton is $h$)

[Barvinsky, Kamenshchik, Mishakov (1996); Bezrukov, Shaposhnikov (2008)]

The model:

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{SM} - \xi |H|^2 R$$

The potential is

$$U \equiv \frac{V}{\Omega^4} = \frac{\bar{M}_{Pl}^4 \lambda (h^2 - v^2)^2}{4(\bar{M}_{Pl}^2 + \xi h^2)^2}$$

$\Omega^2$ corresponds, by definition, to what multiplies $R$ in $\mathcal{L}$ and $\bar{M}_{Pl} \approx 2.4 \times 10^{18}$ GeV

$$U \ll \bar{M}_{Pl}^4$$

(such that quantum Einstein gravity effects are small)

$\xi$ generically is very large

Perturbative unitarity is violated at an energy close to the inflationary one:

[Burgess, Lee, Trott (2009); Barbon, Espinosa (2009); Hertzberg (2010); Burgess, Lee, Trott (2010); Burgess, Patil, Trott (2014)]
Initial conditions for the Higgs field

Another condition to have inflation is that $\Pi = \frac{d\varphi}{dt}(t_{\text{in}})$ is small enough. However, it is not natural: any $|\Pi| \ll \bar{M}^2$ is allowed.

In Higgs inflation one usually requires $\Pi \ll \sqrt{U} \sim 10^{-5} \bar{M}^2$ and $\bar{X} \sim \bar{M}_\text{Pl}$.
Initial conditions for the Higgs field

Another condition to have inflation is that \( \bar{\Pi} = \frac{d\varphi}{dt}(t_{\text{in}}) \) is small enough. However, it is not natural: any \( |\bar{\Pi}| \ll \bar{M}^2_{\text{Pl}} \) is allowed.

In Higgs inflation one usually requires \( \bar{\Pi} \ll \sqrt{U} \sim 10^{-5} \bar{M}^2_{\text{Pl}} \) and \( \bar{\chi} \sim \bar{M}_{\text{Pl}} \).

Classical level

Even if we start from a \( \bar{\Pi}^2 \gg U \) we can compensate with a slightly larger initial value of the Higgs field, \( \bar{\chi} \).
Initial conditions for the Higgs field

Another condition to have inflation is that $\Pi = \frac{d\varphi}{dt}(t_{\text{in}})$ is small enough. However, it is not natural: any $|\Pi| \ll \tilde{M}^2_{\text{Pl}}$ is allowed.

In Higgs inflation one usually requires $\Pi \ll \sqrt{U} \sim 10^{-5} \tilde{M}^2_{\text{Pl}}$ and $\chi \sim \tilde{M}_{\text{Pl}}$.

Classical level

Even if we start from a $\Pi^2 \gg U$ we can compensate with a slightly larger initial value of the Higgs field, $\chi$.

Quantum level

A large value of $\xi$ can generate higher order terms in the Lagrangian: $R^2$, $R^3$, ...

For example this diagram generates $R^2$. 
The absence of fundamental masses is a key to inflation.

For any scalar $s$

$$U(s) = \frac{\lambda s s^4}{(2 \xi s s^2)^2} \tilde{M}_{P1}^4 = \frac{\lambda s}{4 \xi^2} \tilde{M}_{P1}^4$$
Introduction

The stability bound

Inflation and the Standard Model

Models without fundamental masses and inflation

Conclusions
Some motivations for models without fundamental masses (agravity)

Motivation 1: inflation

As we saw, the potential of a scalar \( s \) in agravity is

\[
U(s) = \frac{\lambda_s s^4}{(2\xi_s s^2)^2} \bar{M}_{\text{Pl}}^4 = \frac{\lambda_s}{4\xi_s^2} \bar{M}_{\text{Pl}}^4
\]

The potential is flat at tree-level, but at quantum level \( \lambda_s \) and \( \xi_s \) depend on \( s \). The RGEs give some slope, which is small if the couplings are perturbative.

\( \rightarrow \) inflation!
Some motivations for models without fundamental masses (agravity)

Motivation 1: inflation

As we saw, the potential of a scalar $s$ in agravity is

$$U(s) = \frac{\lambda_s s^4}{(2\xi_s s^2)^2} \tilde{M}_{\text{Pl}}^4 = \frac{\lambda_s}{4\xi_s^2} \tilde{M}_{\text{Pl}}^4$$

The potential is flat at tree-level, but at quantum level $\lambda_s$ and $\xi_s$ depend on $s$. The RGEs give some slope, which is small if the couplings are perturbative.

$\rightarrow$ inflation!

Motivation 2: origin of mass and EW symmetry breaking

Most of the mass of the matter we see has a dynamical origin

Example: the proton mass

Is it possible to generate all the mass dynamically? If yes, with $m \ll M_{\text{Pl}}$?
Agravity scenario

The general agravity Lagrangian:

\[ \mathcal{L} = \frac{R^2}{6f_0^2} + \frac{1}{3} R^2 - \frac{R_{\mu\nu}^2}{f_2^2} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}} \]
Agravity scenario

The general agravity Lagrangian:

\[ \mathcal{L} = \frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R^2_{\mu\nu}}{f_2^2} + \mathcal{L}_{\text{dim SM}} + \mathcal{L}_{\text{BSM}} \]

Non-gravitational sector

- \( \mathcal{L}_{\text{dim SM}} \) is the SM \( \mathcal{L} \) (without \( m^2|H|^2/2 \) plus \(-\xi_H|H|^2R\)):

- \( \mathcal{L}_{\text{BSM}} \) describes BSM physics.
Agravity scenario

The general agravity Lagrangian:

\[ \mathcal{L} = \frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R^2_{\mu\nu}}{f_2^2} + \mathcal{L}_{\text{SM}}^{\text{adim}} + \mathcal{L}_{\text{BSM}}^{\text{adim}} \]

Non-gravitational sector

- \( \mathcal{L}_{\text{SM}}^{\text{adim}} \) is the SM \( \mathcal{L} \) (without \( m^2|H|^2/2 \) plus \( -\xi_H|H|^2R \)):

- \( \mathcal{L}_{\text{BSM}}^{\text{adim}} \) describes BSM physics.

\( \langle s \rangle \) generates the EW scale by adding a scalar \( s \rightarrow \mathcal{L}_{\text{BSM}}^{\text{adim}} = \ldots + \lambda_{HS}s^2|H|^2/2 - \xi_Ss^2R/2 \)
Agravity scenario

The general agravity Lagrangian:

\[ \mathcal{L} = \frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}}{f_2^2} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}} \]

Non-gravitational sector

- \( \mathcal{L}_{\text{SM}} \) is the SM \( \mathcal{L} \) (without \( m^2|H|^2/2 \) plus \( -\xi_H|H|^2R \)):

- \( \mathcal{L}_{\text{BSM}} \) describes BSM physics.

adding a scalar \( s \) \( \rightarrow \mathcal{L}_{\text{BSM}} = \ldots + \lambda_H s^2 |H|^2/2 - \xi_s s^2 R/2 \)

Gravity sector

- \( \langle s \rangle \) generates the EW scale

\[ \langle s \rangle \text{ generates } \bar{M}_{\text{Pl}}: \quad \xi_s s^2 R \rightarrow \bar{M}_{\text{Pl}}^2 = \xi_s \langle s \rangle^2 \]
Agravity scenario

The general agravity Lagrangian:

\[ \mathcal{L} = \frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}}{f_2^2} + \mathcal{L}_{\text{adim}}^{\text{SM}} + \mathcal{L}_{\text{adim}}^{\text{BSM}} \]

Non-gravitational sector

- \( \mathcal{L}_{\text{adim}}^{\text{SM}} \) is the SM \( \mathcal{L} \) (without \( m^2|H|^2/2 + -\xi_H|H|^2R \)):

- \( \mathcal{L}_{\text{adim}}^{\text{BSM}} \) describes BSM physics.

\[ \langle s \rangle \text{ generates the EW scale} \]

adding a scalar \( s \) \( \rightarrow \mathcal{L}_{\text{adim}}^{\text{BSM}} = \ldots + \lambda_{HS}s^2|H|^2/2 - \xi_s s^2 R/2 \]

Gravity sector

\[ \langle s \rangle \text{ generates } \bar{M}_{\text{Pl}}: \quad \xi_s s^2 R \rightarrow \bar{M}_{\text{Pl}}^2 = \xi_s \langle s \rangle^2 \]

One can generate the EW and Planck scales such that their hierarchy is stable under quantum corrections (including gravity effects)! ... This requires \( \lambda_{HS} \ll 1 \) ...
Quantum Agravity is renormalizable
(clear from absence of fundamental scales)

However, looking at the spectrum:

(i) massless graviton
(ii) scalar $z$ with mass $M_0^2 \sim \frac{1}{2} f_0^2 \tilde{M}_{P1}^2$
(iii) massive graviton with mass $M_2^2 = \frac{1}{2} f_2^2 \tilde{M}_{P1}^2$ and negative norm, but with energy bounded from below
Quantum Agravity is renormalizable
(clear from absence of fundamental scales)

However, looking at the spectrum:

(i) massless graviton
(ii) scalar $z$ with mass $M_0^2 \sim \frac{1}{2} f_0^2 \tilde{M}_{P1}^2$
(iii) massive graviton with mass $M_2^2 = \frac{1}{2} f_2^2 \tilde{M}_{P1}^2$ and negative norm, but with energy bounded from below

Is there a dark side of quantum mechanics?

negative norms positive norm
Absence of fundamental scales = “Classical Scale Invariance”

Scale invariance is broken by quantum corrections
Absence of fundamental scales = “Classical Scale Invariance”

Scale invariance is broken by quantum corrections

Previous literature on classical scale invariance ... standing on the shoulder of giants!

Dynamical generation of $\langle s \rangle$

Two basic conditions:

\[
\begin{align*}
\lambda_S(\langle s \rangle) & \approx 0 \iff \text{nearly vanishing cosmological constant (dark energy)} \\
\frac{d\lambda_S}{ds}(\langle s \rangle) & \approx 0 \iff \text{minimum condition (it fixes $\langle s \rangle$)}
\end{align*}
\]
Dynamical generation of $\langle s \rangle$

Two basic conditions:

\[
\begin{align*}
\lambda_S(\langle s \rangle) & \approx 0 \iff \text{nearly vanishing cosmological constant (dark energy)} \\
\frac{d\lambda_S}{ds}(\langle s \rangle) & \approx 0 \iff \text{minimum condition (it fixes $\langle s \rangle$)}
\end{align*}
\]

It is possible to satisfy these conditions as they are realized in the physics we know (the SM)!

\[\downarrow\]

\[\begin{array}{c}
\text{RGE running of the MS quartic Higgs coupling in the SM}
\end{array}\]
Predictions for inflation

The minimal realistic model has at least 3 scalars:

$h, s$ and “a graviscalar” $z$
(from $f(R) = R^2$)
Predictions for inflation

The minimal realistic model has at least 3 scalars:

$h$, $s$ and “a graviscalar” $z$

(from $f(R) = R^2$)

$M_s =$ mass of $s$
$M_0 =$ mass of $z$

$\xi_S = 1$, $\xi_H = 1$, $M_s/M_0 = 0.10$, $\lambda_H = 0.01$

$\xi_S = 1$, $M_s/M_0 = 1.00$
Predictions for inflation

The minimal realistic model has at least 3 scalars: 

\( h, s \) and “a graviscalar” \( z \) 
from \( f(R) = R^2 \)

\[ M_s = \text{mass of } s \]
\[ M_0 = \text{mass of } z \]

- **left:** when \( M_s \ll (\gg) M_0 \), the inflaton is \( s \) (\( z \))
- **right:** comparison with a global fit of PLANCK and BICEP2/KECK

\[ \xi_s = 1, \xi_H = 1 , M_s/M_0 = 0.10, \lambda_H = 0.01 \]
Other virtues

This scenario also

leads to successful reaheating

has natural dark matter candidates and can account for neutrino masses
Introduction

The stability bound

Inflation and the Standard Model

Models without fundamental masses and inflation

Conclusions
Conclusions

1. The SM is compatible with data (so far), but for sure it has to be extended. There are nevertheless extensions that solve its phenomenological problems and do not invalidate all its predictions at very high energies (See Point 3.)

2. We presented the currently most precise SM stability bound. Data indicate some tension: the EW vacuum is metastable (life-time > than the age of the universe) although absolute stability is still possible. We also discussed inflation in the SM.

3. A rationale for inflation and a dynamical origin of mass can be obtained in models of all interactions (including gravity) where fundamental scales are absent: agravity.

We did not show it here, but in agravity one also has
- dark matter and neutrino masses,
- a hierarchy between the EW and Planck scales that is stable under quantum corrections.
Thank you very much for your attention!
Extra slides
Qualitative origin of the stability bound

\[ V_{\text{eff}} = V + V_1 + V_2 + \ldots \]

\[ V(h) = \frac{\lambda}{4} (h^2 - v^2)^2, \quad V_1(h) = \frac{1}{(4\pi)^2} \sum_i c_i m_i(h)^4 \left( \ln \frac{m_i(h)^2}{\mu^2} + d_i \right), \quad \ldots \]

where \( h^2 \equiv 2|H|^2 \) and \( c_i \) and \( d_i \) are \( \sim 1 \) constants

By substituting bare parameters \( \rightarrow \) renormalized ones

\[ \implies \frac{\partial V_{\text{eff}}}{\partial \mu} = 0 \quad \text{and one is free to choose} \ \mu \text{ to improve perturbation theory} \]

Since at large fields, \( h \gg v \), we have \( m_i(h)^2 \propto h^2 \), we choose \( \mu^2 = h^2 \), then

\[ V_{\text{eff}}(h) = \frac{\lambda(h)}{4} (h^2 - v(h)^2)^2 + \ldots = -\frac{m(h)^2}{2} h^2 + \frac{\lambda(h)}{4} h^4 + \ldots \]
Qualitative origin of the stability bound

\[ V_{\text{eff}} = V + V_1 + V_2 + \ldots \]

\[ V(h) = \frac{\lambda}{4} (h^2 - v^2)^2, \quad V_1(h) = \frac{1}{(4\pi)^2} \sum_i c_i m_i(h)^4 \left( \ln \frac{m_i(h)^2}{\mu^2} + d_i \right), \quad \ldots \]

where \( h^2 \equiv 2|H|^2 \) and \( c_i \) and \( d_i \) are \( \sim 1 \) constants

By substituting bare parameters \( \rightarrow \) renormalized ones

\[ \implies \frac{\partial V_{\text{eff}}}{\partial \mu} = 0 \quad \text{and one is free to choose } \mu \text{ to improve perturbation theory} \]

Since at large fields, \( h \gg v \), we have \( m_i(h)^2 \propto h^2 \), we choose \( \mu^2 = h^2 \), then

\[ V_{\text{eff}}(h) = \frac{\lambda(h)}{4} (h^2 - v(h)^2)^2 + \ldots = -\frac{m(h)^2}{2} h^2 + \frac{\lambda(h)}{4} h^4 + \ldots \]

So for \( h \gg v \)

\[ V_{\text{eff}}(h) \approx \frac{\lambda(h)}{4} h^4 \]

- \( M_h \) contributes positively to \( \lambda \rightarrow \) lower bound on \( M_h \)
- \( y_t \) contributes negatively to the running of \( \lambda \rightarrow \) upper bound on \( M_t \)
**Procedure to extract the stability bound**

**Steps of the procedure**

- $V_{\text{eff}}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called *threshold corrections* or *matching conditions*) at the EW scale (e.g. at $M_t$)...

*Finally impose that $V_{\text{eff}}$ at the EW vacuum is the absolute minimum!*
Procedure to extract the stability bound

Steps of the procedure

- $V_{\text{eff}}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called threshold corrections or matching conditions) at the EW scale (e.g. at $M_t$) ...

Finally impose that $V_{\text{eff}}$ at the EW vacuum is the absolute minimum!

State-of-the-art loop calculation

- Two loop $V_{\text{eff}}$ including the leading couplings = $\{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop RGEs for $\{\lambda, y_t, g_3, g_2, g_1\}$ and one loop RGE for $\{y_b, y_\tau\}$ ...
- Two loop values of $\{\lambda, y_t, g_3, g_2, g_1\}$ at $M_t$ ...

Previous calculations:
- [Cabibbo, Maiani, Parisi, Petronzio (1979); Casas, Espinosa, Quiros (1994, 1996); Bezrukov, Kalmykov, Kniehl, Shaposhnikov (2012); Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia (2012)]
Procedure to extract the stability bound

Steps of the procedure

- $V_{\text{eff}}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called threshold corrections or matching conditions) at the EW scale (e.g. at $M_t$)...

Finally impose that $V_{\text{eff}}$ at the EW vacuum is the absolute minimum!

State-of-the-art loop calculation

- Two loop $V_{\text{eff}}$ including the leading couplings = \{\lambda, y_t, g_3, g_2, g_1\}
- Three loop RGEs for \{\lambda, y_t, g_3, g_2, g_1\} and one loop RGE for \{y_b, y_\tau\}...
- Two loop values of \{\lambda, y_t, g_3, g_2, g_1\} at $M_t$...

Previous calculations

[ Cabibbo, Maiani, Parisi, Petronzio (1979); Casas, Espinosa, Quiros (1994, 1996); Bezrukov, Kalmykov, Kniehl, Shaposhnikov (2012); Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia (2012)]
Input values of the SM observables

(used to fix relevant parameters: $\lambda, y_t, g_1, g_2$)

\[ M_h = 125.15 \pm 0.24 \text{ GeV} \]

[CMS Collaboration (2013, 2014); ATLAS Collaboration (2013, 2014); average from Giardino, Kannike, Masina, Raidal and Strumia (2014)]

\[
\begin{align*}
M_W &= 80.384 \pm 0.014 \text{ GeV} \quad \text{Mass of the } W \text{ boson [1]} \\
M_Z &= 91.1876 \pm 0.0021 \text{ GeV} \quad \text{Mass of the } Z \text{ boson [2]} \\
M_h &= 125.15 \pm 0.24 \text{ GeV} \quad \text{(source already quoted)} \\
M_t &= 173.34 \pm 0.76 \pm 0.3 \text{ GeV} \quad \text{Mass of the top quark [3]} \\
V &\equiv (\sqrt{2} G_{\mu})^{-1/2} = 246.21971 \pm 0.00006 \text{ GeV} \quad \text{Fermi constant [4]} \\
\alpha_3(M_Z) &= 0.1184 \pm 0.0007 \quad \text{SU}(3)_c \text{ coupling (5 flavors) [5]}
\end{align*}
\]


Step 1: effective potential

**RG-improved tree level potential \((V)\)**

Classical potential with couplings replaced by the running ones

**One loop \((V_1)\)**

\(V_{\text{eff}}\) depends mainly on the top, \(W\), \(Z\), \(h\) and Goldstone squared masses in the classical background \(h\): in the Landau gauge ... they are

\[
\begin{align*}
t &\equiv \frac{y_t^2 h^2}{2}, \quad w \equiv \frac{g_2^2 h^2}{4}, \\
z &\equiv \frac{(g_2^2 + \frac{3g_1^2}{5})h^2}{4}, \\
m_h^2 &\equiv 3\lambda h^2 - m^2, \\
g &\equiv \lambda h^2 - m^2
\end{align*}
\]

\(\to (4\pi)^2 V_1\) is (in a suitable renormalization scheme, called \(\overline{\text{MS}}\))

\[
\begin{align*}
\frac{3w^2}{2} \left( \ln \frac{w}{\mu^2} - \frac{5}{6} \right) + \frac{3z^2}{4} \left( \ln \frac{z}{\mu^2} - \frac{5}{6} \right) - 3t^2 \left( \ln \frac{t}{\mu^2} - \frac{3}{2} \right) + \frac{m_h^4}{4} \left( \ln \frac{m_h^2}{\mu^2} - \frac{3}{2} \right) + \frac{3g^2}{4} \left( \ln \frac{g}{\mu^2} - \frac{3}{2} \right)
\end{align*}
\]

In order to keep the logarithms in the effective potential small we choose

\[
\mu = h
\]

Indeed, \(t, w, z, m_h^2, g\) are \(\propto h^2\) for \(h \gg m\)

**Two loop \((V_2)\)**

It is very complicated, but always depend on \(t, w, z, m_h^2, g\) plus \(g_i\)
Step 2: running couplings

For a generic parameter $p$ we write the RGE as

$$\frac{dp}{d \ln \mu^2} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4} + \ldots$$

They were computed before in the literature up to three loops

(very long and not very illuminating expressions at three loops)

One loop RGEs for $\lambda, y_t^2, g_i^2$ and $m^2$

$$\beta^{(1)}_\lambda = \lambda \left( 12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10} \right) - 3y_t^4 + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2g_1^2}{40},$$

$$\beta^{(1)}_{y_t^2} = y_t^2 \left( \frac{9y_t^2}{2} - 8g_3^2 - \frac{9g_2^2}{4} - \frac{17g_1^2}{20} \right),$$

$$\beta^{(1)}_{g_1^2} = \frac{41}{10}g_1^4, \quad \beta^{(1)}_{g_2^2} = -\frac{19}{6}g_2^4, \quad \beta^{(1)}_{g_3^2} = -7g_3^4,$$

$$\beta^{(1)}_{m^2} = m^2 \left( 6\lambda + 3y_t^2 - \frac{9g_2^2}{4} - \frac{9g_1^2}{20} \right)$$
Step 3: threshold corrections

\[
\begin{align*}
\lambda(M_t) &= 0.12604 + 0.00206 \left( \frac{M_h}{\text{GeV}} - 125.15 \right) - 0.00004 \left( \frac{M_t}{\text{GeV}} - 173.34 \right) \pm 0.00030_{\text{th}} \\
\frac{m(M_t)}{\text{GeV}} &= 131.55 + 0.94 \left( \frac{M_h}{\text{GeV}} - 125.15 \right) + 0.17 \left( \frac{M_t}{\text{GeV}} - 173.34 \right) \pm 0.15_{\text{th}} \\
y_t(M_t) &= 0.93690 + 0.00556 \left( \frac{M_t}{\text{GeV}} - 173.34 \right) - 0.00042 \frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.00050_{\text{th}} \\
g_2(M_t) &= 0.64779 + 0.00004 \left( \frac{M_t}{\text{GeV}} - 173.34 \right) + 0.00011 \frac{M_W - 80.384 \text{ GeV}}{0.014 \text{ GeV}} \\
g_Y(M_t) &= 0.35830 + 0.00011 \left( \frac{M_t}{\text{GeV}} - 173.34 \right) - 0.00020 \frac{M_W - 80.384 \text{ GeV}}{0.014 \text{ GeV}} \\
g_3(M_t) &= 1.1666 + 0.00314 \frac{\alpha_3(M_Z) - 0.1184}{0.0007} - 0.00046 \left( \frac{M_t}{\text{GeV}} - 173.34 \right)
\end{align*}
\]

The theoretical uncertainties on these quantities are much lower than those used in previous determinations of the stability bound.
Tunneling probability

The probability of creating a bubble of the absolute minimum in $dV \, dt$ was found by [Kobzarev, Okun, Voloshin (1975); Coleman (1977); Callan, Coleman (1977)]

$$d\varphi = dt \, dV \, \Lambda_B^4 e^{-S(\Lambda_B)}$$

$S(\Lambda_B) \equiv$ the action of the bounce of size $R = \Lambda_B^{-1}$, given by $S(\Lambda_B) = \frac{8\pi^2}{3|\lambda(\Lambda_B)|}$
The part of $S$ that depends on $g_{\mu \nu}$ and $H$ only \[ \rightarrow \] \[ S_{gH} = \int d^4x \sqrt{-g} \left[ -\left( \frac{\bar{M}_{\text{Pl}}^2}{2} + \xi |H|^2 \right) R + |D_{\mu}H|^2 - V(H) \right] \]

The non-minimal coupling can be eliminated through a conformal transformation ... \[ g_{\mu \nu} \rightarrow \hat{g}_{\mu \nu} \equiv \Omega^2 g_{\mu \nu}, \quad \Omega^2 = 1 + \frac{2\xi |H|^2}{\bar{M}_{\text{Pl}}^2} \]

In the unitary gauge, where the only scalar field is the radial mode $h \equiv \sqrt{2|H|^2}$ \[ S_{gH} = \int d^4x \sqrt{-\hat{g}} \left[ -\frac{\bar{M}_{\text{Pl}}^2}{2} \hat{R} + K \frac{(\partial h)^2}{2} - \frac{V}{\Omega^4} \right] \]

where $K \equiv \left( \Omega^2 + 6\xi^2 h^2 / \bar{M}_{\text{Pl}}^2 \right) / \Omega^4$ and we set the gauge fields to zero.

The $h$ kinetic term can be made canonical through $h = h(\chi)$ defined by \[ \frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / \bar{M}_{\text{Pl}}^2}{\Omega^4}} \]
The part of $S$ that depends on $g_{\mu\nu}$ and $H$ only $\rightarrow$

$$S_{gH} = \int d^4x \sqrt{-g} \left[ - \left( \frac{\bar{M}_{P1}^2}{2} + \xi |H|^2 \right) R + |D_\mu H|^2 - V(H) \right]$$

The non-minimal coupling can be eliminated through a *conformal* transformation ...

$$g_{\mu\nu} \rightarrow \hat{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi |H|^2}{\bar{M}_{P1}^2}$$

In the unitary gauge, where the only scalar field is the radial mode $h \equiv \sqrt{2|H|^2}$

$$S_{gH} = \int d^4x \sqrt{-\hat{g}} \left[ - \frac{\bar{M}_{P1}^2}{2} \hat{R} + K \left( \frac{\partial h}{2} \right)^2 - \frac{V}{\Omega^4} \right]$$

where $K \equiv (\Omega^2 + 6\xi^2 h^2 / \bar{M}_{P1}^2) / \Omega^4$ and we set the gauge fields to zero.

The $h$ kinetic term can be made canonical through $h = h(\chi)$ defined by

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2 / \bar{M}_{P1}^2}{\Omega^4}}$$

This is what we want in order to have slow-roll ...

Thus, $\chi$ feels a potential

$$U \equiv \frac{V}{\Omega^4} = \frac{\lambda (h(\chi)^2 - v^2)^2}{4(1 + \xi h(\chi)^2 / \bar{M}_{P1}^2)^2} \approx \frac{\lambda \bar{M}_{P1}^4 / \sqrt{\xi}}{4\xi^2 \bar{M}_{P1}^4}$$

$h > \bar{M}_{P1} / \sqrt{\xi}$
All parameters can be fixed through experiments and observations ...

\( \xi \) can be fixed requiring the normalization of \([\text{Planck Collaboration (2015)}]\)

\[ A_s(h_{\text{in}}) \approx (2.14 \pm 0.05) \times 10^{-9} \]

\( h_{\text{in}} \) is fixed by requiring

\[ N = \int_{h_{\text{end}}}^{h_{\text{in}}} \frac{U}{M^2_{\text{Pl}}} \left( \frac{dU}{dh} \right)^{-1} \left( \frac{d\chi}{dh} \right)^2 dh \approx 59 \]

\([\text{Garcia-Bellido, Figueroa, Rubio (2009); Bezrukov, Gorbunov, Shaposhnikov (2009)}]\)

and \( h_{\text{end}} \) is the field value at the end of inflation:

\[ \epsilon(h_{\text{end}}) \approx 1 \]
All parameters can be fixed through experiments and observations ...

\( \xi \) can be fixed requiring the normalization of \(^ {\text{Planck Collaboration (2015)}}\)

\[ A_s(h_{\text{in}}) \approx (2.14 \pm 0.05) \times 10^{-9} \]

\[ h_{\text{in}} \text{ is fixed by requiring } \int_{h_{\text{end}}}^{h_{\text{in}}} \frac{U}{M_P^2} \left( \frac{dU}{dh} \right)^{-1} \left( \frac{d\chi}{dh} \right)^2 dh \approx 59 \]

\(^ {\text{Garcia-Bellido, Figueroa, Rubio (2009); Bezrukov, Gorbunov, Shaposhnikov (2009)}}\)

and \( h_{\text{end}} \) is the field value at the end of inflation: \( \epsilon(h_{\text{end}}) \approx 1 \)

This leads to \( \xi \approx 4.7 \times 10^4 \sqrt{\lambda} \) and indicates that \( \xi \) has to be large ...
Two regimes

- small fields: $h \ll \tilde{M}_{P1}/\xi$ (the SM is recovered)
- large fields: $h \gg \tilde{M}_{P1}/\xi$ (chiral EW action with VEV set to $h/\Omega \approx \tilde{M}_{P1}/\sqrt{\xi}$) \rightarrow \text{ decoupling of } h \text{ in the inflationary regime}
**$h$ inflation: quantum analysis**

**Two regimes**

- **small fields:** $h \ll \tilde{M}_{P1}/\xi$ (the SM is recovered)

- **large fields:** $h \gg \tilde{M}_{P1}/\xi$ (chiral EW action with VEV set to $h/\Omega \approx \tilde{M}_{P1}/\sqrt{\xi}$) → decoupling of $h$ in the inflationary regime

**State-of-the-art calculation of the bound on $M_h$ to have inflation**

- Two loop effective potential $U_{\text{eff}}$ in the inflationary regime including the effect of $\xi$ and the leading SM couplings = \{\lambda, y_t, g_3, g_2, g_1\}

- Three loop SM RGE from the EW scale up to $\tilde{M}_{P1}/\xi$ for \{\lambda, y_t, g_3, g_2, g_1\} ...

- Two loop RGE for the same SM couplings and one loop RGE for $\xi$ in the chiral EW theory

- Two loop threshold corrections at the top mass, for these SM couplings
Bound on $M_h$ to have $h$ inflation

Derivation

1. We fix $\xi$ as in the classical case, but with $U$ replaced by $U_{\text{eff}}$.
   $\ldots$ this already gives $\xi_{\text{inf}} \equiv \xi(\tilde{M}_\text{Pl}/\sqrt{\xi_t})$, where conventionally $\xi_t = \xi(M_t)$

2. If $M_h$ is too small (or $M_t$ is too large) we go from the blue behavior to the red one! When the slope is negative the Higgs cannot roll towards the EW vacuum

We set the th. errors to zero and the input parameters to the central values, except $M_t$:

- **Solid line**: $M_t = 171.43\,\text{GeV}$
  ($\xi$ fixed as described above)

- **Dashed line**: $M_t = 171.437\,\text{GeV}$ ($\xi_t = 300$)
**Bound on $M_h$ to have $h$ inflation**

**Derivation**

1. We fix $\xi$ as in the classical case, but with $U$ replaced by $U_{\text{eff}}$.
   ... this already gives $\xi_{\text{inf}} \equiv \xi(\tilde{M}_\text{Pl}/\sqrt{\xi_t})$, where conventionally $\xi_t = \xi(M_t)$

2. If $M_h$ is too small (or $M_t$ is too large) we go from the blue behavior to the red one! When the slope is negative the Higgs cannot roll towards the EW vacuum

We set the th. errors to zero and the input parameters to the central values, except $M_t$:

- **Solid line:** $M_t = 171.43\, \text{GeV}$
  ($\xi$ fixed as described above)
- **Dashed line:** $M_t = 171.437\, \text{GeV}$ ($\xi_t = 300$)

**Result (bound to have $h$ inflation)**

$$M_h > 129.4 \, \text{GeV} + 2.0(M_t - 173.34 \, \text{GeV}) - 0.5 \, \text{GeV} \frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\text{th}} \, \text{GeV}$$
Violation of perturbative unitarity

Consider the scattering “Higgs Higgs → Higgs Higgs” mediated by a graviton

At tree-level and for large $\xi$ the leading contribution comes from the $-\xi |H|^2 R$ term:

$$-\frac{\xi}{M_{Pl}} |H|^2 (\partial_{\mu} \partial_{\nu} h_{\mu\nu} - \eta_{\mu\nu} \partial^2 h_{\mu\nu})$$

The amplitude in the rest frame is then

$$A(E) = \frac{6\xi^2 E^2}{M_{Pl}^2}$$
Violation of perturbative unitarity

Consider the scattering “Higgs Higgs $\rightarrow$ Higgs Higgs” mediated by a graviton

At tree-level and for large $\xi$ the leading contribution comes from the $-\xi|H|^2 R$ term:

$$-\frac{\xi}{\tilde{M}_{P1}} |H|^2 \left( \partial_\mu \partial_\nu h_{\mu\nu} - \eta_{\mu\nu} \partial^2 h_{\mu\nu} \right)$$

The amplitude in the rest frame is then

$$A(E) = \frac{6\xi^2 E^2}{\tilde{M}_{P1}^2}$$

$\rightarrow$ unitarity is violated at

$$E \sim \frac{\tilde{M}_{P1}}{\xi}$$
Violation of perturbative unitarity

Consider the scattering “Higgs Higgs → Higgs Higgs” mediated by a graviton

At tree-level and for large $\xi$ the leading contribution comes from the $-\xi|H|^2 R$ term:

$$-\frac{\xi}{\tilde{M}_{Pl}} |H|^2 (\partial_\mu \partial_\nu h_{\mu\nu} - \eta_{\mu\nu} \partial^2 h_{\mu\nu})$$

The amplitude in the rest frame is then

$$A(E) = \frac{6\xi^2 E^2}{\tilde{M}_{Pl}^2}$$

→ unitarity is violated at

$$E \sim \frac{\tilde{M}_{Pl}}{\xi}$$

This is typically smaller than the energy during inflation

$$E_{\text{inf}} \sim \left(\frac{\lambda}{4\xi^2 \tilde{M}_{Pl}^4}\right)^{1/4}$$
Quantum effects are mostly encoded in the RGEs ...

They are important to obtain $n_s$ and $r$ and to dynamically generate $\bar{M}_{Pl}$ and $m$
Quantum gravity

Quantum effects are mostly encoded in the RGEs ...

They are important to obtain $n_s$ and $r$ and to dynamically generate $\bar{M}_{P_L}$ and $m$

The most general agravity can be parameterized by the following $\mathcal{L}$

$$
\frac{R^2}{6f^2_0} + \frac{1}{3} \frac{R^2 - R^2_{\mu\nu}}{f^2_2} - \frac{(F^A_{\mu\nu})^2}{4} + \frac{(D_{\mu} \phi_a)^2}{2} - \xi_{ab} \phi_a \phi_b R - \frac{\lambda_{abcd}}{4!} \phi_a \phi_b \phi_c \phi_d + \bar{\psi}_j i D_j \psi - Y_{ij} \psi_i \psi_j \phi_a + \text{h.c.}
$$

We obtain the RGEs of this renormalizable quantum field theory:

$$
\beta_p \equiv \frac{dp}{d \ln \mu} \quad (\text{of all parameters } p)
$$

Without gravity this was done before [Machacek and Vaughn (1983,1984,1985)]

We include gravity
Results for RGEs

Gauge couplings

Their contributions to the RGEs cancel!

This was previously noticed in [Narain, Anishetty (2013)]

Possible explanation:
the graviton is not charged

Yukawa couplings

We find the one-loop RGE (where $C_{2F} \equiv t^A t^A$ and $t^A \equiv \text{“fermion gauge generators”}$):

$$
(4\pi)^2 \frac{dY^a}{d\ln \mu} = \frac{1}{2} (Y^{b^b} Y^a Y^a + Y^a Y^{b^b} Y^b) + 2 Y^b Y^{a^b} Y^b + Y^{b^b} \text{Tr}(Y^{b^b} Y^a) - 3 \{C_{2F}, Y^a\} + \frac{15}{8} f_2^2 Y^a
$$

All remaining RGEs

We also computed the RGEs for $\lambda_{abcd}$, $\xi_{ab}$, $f_0$ and $f_2$. 
Tens of Feynman diagrams contribute to these RGEs ... we obtain

\[
(4\pi)^2 \frac{d\lambda_{abcd}}{d \ln \mu} = \sum_{\text{perms}} \left[ \frac{1}{8} \lambda_{abef} \lambda_{efcd} + \frac{3}{8} \{\theta^A, \theta^B\}_{ab} \{\theta^A, \theta^B\}_{cd} - \text{Tr} \ Y^a Y^\dagger b Y^c Y^\dagger d + \right.
\]

\[
+ \frac{5}{8} f_4 \xi_{ab} \xi_{cd} + \frac{f_0^4}{8} \xi_{ae} \xi_{cf} (\delta_{eb} + 6 \xi_{eb})(\delta_{fd} + 6 \xi_{fd})
\]

\[
+ \frac{f_0^2}{4!} (\delta_{ae} + 6 \xi_{ae})(\delta_{bf} + 6 \xi_{bf}) \lambda_{efcd} \left] + \lambda_{abcd} \left[ \sum_k (Y^k_2 - 3 C^k_{2S}) + 5 f_2^2 \right] \right.,
\]

where the first sum runs over the 4! permutations of \( abcd \) and the second sum over \( k = \{a, b, c, d\} \), with \( Y^k_2 \) and \( C^k_{2S} \) defined by

\[
\text{Tr}(Y^\dagger a Y^b) = Y^a_2 \delta^{ab}, \quad \theta^A_{ac} \theta^A_{cb} = C^a_{2S} \delta_{ab}
\]

(\( \theta^A \) are the scalar gauge generators)
**RGEs for the quartic couplings: SM case**

For the SM $H$ plus a complex scalar singlet $S$ the RGEs become:

\[
(4\pi)^2 \frac{d\lambda_S}{d \ln \mu} = 20\lambda_S^2 + 2\lambda_{HS}^2 + \frac{\xi_S^2}{2} \left[5f_2^4 + f_0^4(1 + 6\xi_S)^2\right] + \lambda_S \left[5f_2^2 + f_0^2(1 + 6\xi_S)^2\right]
\]

\[
(4\pi)^2 \frac{d\lambda_{HS}}{d \ln \mu} = -\xi_H\xi_S \left[5f_2^4 + f_0^4(6\xi_S + 1)(6\xi_H + 1)\right] - 4\lambda_{HS}^2 + \lambda_{HS} \left\{8\lambda_S + 12\lambda_H + 6y_t^2 + 5f_2^2 + \frac{f_0^2}{6} \left[(6\xi_S + 1)^2 + (6\xi_H + 1)^2 + 4(6\xi_S + 1)(6\xi_H + 1)\right]\right\}
\]

\[
(4\pi)^2 \frac{d\lambda_H}{d \ln \mu} = \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2g_2^2 + \frac{27}{200}g_1^4 - 6y_t^4 + 24\lambda_H^2 + \lambda_{HS}^2 + \frac{\xi_H^2}{2} \left[5f_2^4 + f_0^4(1 + 6\xi_H)^2\right]
\]

\[+ \lambda_H \left(5f_2^2 + f_0^2(1 + 6\xi_H)^2 + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2\right).\]
RGEs for the scalar/graviton couplings

Complicated calculation (but computer algebra helps!)

\[
(4\pi)^2 \frac{d\xi_{ab}}{d\ln \mu} = \frac{1}{6} \lambda_{abcd} (6\xi_{cd} + \delta_{cd}) + (6\xi_{ab} + \delta_{ab}) \sum_k \left[ \frac{Y_k^2}{3} - \frac{C_{2S}^k}{2} \right] + \\
- \frac{5f_2^4}{3f_0^2} \xi_{ab} + f_0^2 \xi_{ac} \left( \xi_{cd} + \frac{2}{3} \delta_{cd} \right) (6\xi_{db} + \delta_{db})
\]

For the SM $H$ plus a complex scalar singlet $S$ the RGEs become:

\[
(4\pi)^2 \frac{d\xi_S}{d\ln \mu} = \left(1 + 6\xi_S \right) \frac{4}{3} \lambda_S - \frac{2\lambda_{HS}}{3} (1 + 6\xi_H) + \frac{f_0^2}{3} \xi_S (1 + 6\xi_S)(2 + 3\xi_S) - \frac{5}{3} \frac{f_2^4}{f_0^2} \xi_S
\]

\[
(4\pi)^2 \frac{d\xi_H}{d\ln \mu} = \left(1 + 6\xi_H \right) (2y_t^2 - \frac{3}{4} g_2^2 - \frac{3}{20} g_1^2 + 2\lambda_H) - \frac{\lambda_{HS}}{3} (1 + 6\xi_S) + \\
+ \frac{f_0^2}{3} \xi_H (1 + 6\xi_H)(2 + 3\xi_H) - \frac{5}{3} \frac{f_2^4}{f_0^2} \xi_H
\]
RGE for the gravitational couplings

Huge calculation ... (computer algebra practically needed!!)

\[
(4\pi)^2 \frac{df_2^2}{d \ln \mu} = -f_2^4 \left( \frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60} \right)
\]

\[
(4\pi)^2 \frac{df_0^2}{d \ln \mu} = \frac{5}{3} f_2^4 + 5 f_2^2 f_0^2 + \frac{5}{6} f_0^4 + \frac{f_0^4}{12} (\delta_{ab} + 6 \xi_{ab}) (\delta_{ab} + 6 \xi_{ab})
\]

Here \(N_V, N_f, N_s\) are the number of vectors, Weyl fermions and real scalars.

In the SM \(N_V = 12, N_f = 45, N_s = 4\).

We confirmed the calculations of [Avramidi (1995)]
rather than those of [Fradkin and Tseytlin (1981,1982)]
Natural dynamical generation of the electroweak scale

1) **Low energies** \((\mu < M_{0,2})\): agavity can be neglected and the SM RGE apply:

\[
(4\pi)^2 \frac{dm^2}{d \ln \mu} = m^2 \beta^\text{SM}_m, \quad \beta^\text{SM}_m = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}
\]
Natural dynamical generation of the electroweak scale

1) Low energies ($\mu < M_{0,2}$): agravity can be neglected and the SM RGE apply:
\[
(4\pi)^2 \frac{d m^2}{d \ln \mu} = m^2 \beta_m^{\text{SM}}, \quad \beta_m^{\text{SM}} = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}
\]

2) Intermediate energies ($M_{0,2} < \mu < \bar{M}_{\text{Pl}}$): Both $m$ and $\bar{M}_{\text{Pl}}$ appear and we find
\[
(4\pi)^2 \frac{d}{d \ln \mu} \frac{m^2}{\bar{M}_{\text{Pl}}^2} = -\xi_H [5f_2^4 + f_0^4 (1 + 6\xi_H)] + ...
\]

The red term is a non-multiplicative potentially dangerous correction to $m$

\[
m^2 \sim \bar{M}_{\text{Pl}}^2 g^2, \quad \text{naturalness} \rightarrow f_2, f_0 (1 + 6\xi_H)^{1/4} \sim \sqrt{\frac{4\pi m}{\bar{M}_{\text{Pl}}}} \sim 10^{-8}
\]

These ultraweak couplings are preserved by the RGE even for $f_0 \gtrsim 10^{-5}$ if $\xi \approx -1/6$
**Natural dynamical generation of the electroweak scale**

1) **Low energies** ($\mu < M_{0,2}$): agravity can be neglected and the SM RGE apply:

\[
(4\pi)^2 \frac{d m^2}{d \ln \mu} = m^2 \beta^\text{SM}_m, \quad \beta^\text{SM}_m = 12\lambda_H + 6y^2_t - \frac{9g^2_2}{2} - \frac{9g^2_1}{10}
\]

2) **Intermediate energies** ($M_{0,2} < \mu < \bar{M}_\text{Pl}$): Both $m$ and $\bar{M}_\text{Pl}$ appear and we find

\[
(4\pi)^2 \frac{d}{d \ln \mu} \frac{m^2}{\bar{M}_{\text{Pl}}^2} = -\xi_H [5f^4_2 + f^4_0 (1 + 6\xi_H)] + ...
\]

The red term is a non-multiplicative potentially dangerous correction to $m$

\[
m^2 \sim \bar{M}_{\text{Pl}}^2 g^2, \quad \text{naturalness} \rightarrow f_2, f_0 (1 + 6\xi_H)^{1/4} \sim \sqrt{\frac{4\pi m}{\bar{M}_{\text{Pl}}}} \sim 10^{-8}
\]

These *ultraweak couplings* are preserved by the RGE even for $f_0 \gtrsim 10^{-5}$ if $\xi \approx -1/6$

3) **Large energies** ($\mu > \bar{M}_{\text{Pl}}$):

\[
\lambda_{HS} |H|^2 s^2 \rightarrow m^2 = \lambda_{HS} \langle s \rangle^2
\]

$\lambda_{HS}$ can be naturally small (looking at the RGE of $\lambda_{HS}$):

\[
\rightarrow \quad \lambda_{HS} \sim f^4_{0,2}
\]
Agravity inflation: inflaton identified with $s$

We identify inflaton $= s$ by taking the other scalar fields heavy ...

Then we can easily convert $s$ into a scalar $s_E$ with canonical kinetic term and find

$$
\epsilon \equiv \frac{\bar{M}_{P1}^2}{2} \left( \frac{1}{U} \frac{\partial U}{\partial s_E} \right)^2 = \frac{1}{2} \frac{\xi_s}{1 + 6\xi_s} \left( \frac{\beta \lambda_s}{\lambda_s} - 2 \frac{\beta \xi_s}{\xi_s} \right)^2
$$

$$
\eta \equiv \frac{\bar{M}_{P1}^2}{U} \frac{1}{\partial^2 s_E} = \frac{\xi_s}{1 + 6\xi_s} \left( \frac{\beta (\beta \lambda_s)}{\lambda_s} - 2 \frac{\beta (\beta \xi_s)}{\xi_s} + \frac{5 + 36\xi_s}{1 + 6\xi_s} \frac{\beta^2 \xi_s}{\xi_s^2} - \frac{7 + 48\xi_s}{1 + 6\xi_s} \frac{\beta \lambda_s \beta \xi_s}{2\lambda_s \xi_s} \right)
$$

The slow-roll parameters are given by the $\beta$-functions ...
Agravity inflation: inflaton identified with $s$

We identify inflaton $s = s$ by taking the other scalar fields heavy ...

Then we can easily convert $s$ into a scalar $s_E$ with canonical kinetic term and find

\[
\epsilon \equiv \frac{1}{2} \frac{\bar{M}_P^2}{\epsilon} \left( \frac{1}{U} \frac{\partial U}{\partial s_E} \right)^2 = \frac{1}{2} \frac{\xi_s}{1 + 6 \xi_s} \left( \frac{\beta \lambda_s}{\lambda_s} - 2 \frac{\beta \xi_s}{\xi_s} \right)^2
\]

\[
\eta \equiv \frac{\bar{M}_P^2}{\epsilon} \frac{1}{U} \frac{\partial^2 U}{\partial s_E^2} = \frac{\xi_s}{1 + 6 \xi_s} \left( \frac{\beta (\beta \lambda_s)}{\lambda_s} - 2 \frac{\beta (\beta \xi_s)}{\xi_s} \right) + \frac{5}{1 + 6 \xi_s} \frac{\beta^2 \xi_s}{\xi_s^2} - \frac{7}{1 + 6 \xi_s} \frac{\beta \lambda_s \beta \xi_s}{2 \lambda_s \xi_s}
\]

The slow-roll parameters are given by the $\beta$-functions ...

We can insert them in the formulae for the observable parameters $A_s$, $n_s$ and $r$:

\[
n_s = 1 - 6 \epsilon + 2 \eta, \quad A_s = \frac{U/\epsilon}{24 \pi^2 \bar{M}_P^4}, \quad r = 16 \epsilon
\]

where everything is evaluated at about $N \approx 60$ e-foldings when the inflaton $s_E(N)$ was

\[
N = \frac{1}{\bar{M}_P^2} \int_0^{s_E(N)} \frac{U(s_E)}{U'(s_E)} ds_E
\]
Agravity inflation: inflaton identified with $s$ (analytic approximation)

\[
\begin{align*}
\lambda_{S}(s) & \approx 0 \\
\beta \lambda_{S}(s) & \approx 0 \\
\xi_{S}(s)s^2 &= \tilde{M}_{P1}^2
\end{align*}
\]

$$\rightarrow \lambda_{S}(\mu \approx s) \approx \frac{b}{2} \ln^2 \frac{s}{\langle s \rangle}, \quad \xi_{S}(\mu) \approx \xi_{S}$$

for simplicity

$b \equiv g^4/(4\pi)^4$ can be computed in any given model ...
Agravity inflation: inflaton identified with $s$ (analytic approximation)

\[
\begin{cases}
\lambda S(s) & \approx 0 \\
\beta \lambda S(s) & \approx 0 \\
\xi S(s)s^2 & = \tilde{M}_{\text{Pl}}^2
\end{cases} \quad \rightarrow \quad \lambda S(\mu \approx s) \approx \frac{b}{2} \ln^2 \frac{s}{\langle s \rangle}, \quad \xi S(\mu) \approx \xi S
\]

for simplicity

$b \equiv g^4/(4\pi)^4$ can be computed in any given model ...

\[
\rightarrow \quad \epsilon \approx \eta \approx \frac{2\xi S}{1 + 6\xi S} \frac{1}{\ln^2 s/\langle s \rangle} = \frac{2\tilde{M}_{\text{Pl}}^2}{s_E^2}
\]

The Einstein-frame potential is nearly quadratic around its minimum:

\[
U = \frac{\tilde{M}_{\text{Pl}}^4}{4} \frac{\lambda S}{\xi_S^2} \approx \frac{M_s^2}{2} s_E^2 \quad \text{with} \quad M_s = \frac{g^2 \tilde{M}_{\text{Pl}}}{2(4\pi)^2} \frac{1}{\sqrt{\xi_S(1 + 6\xi_S)}}
\]
Agravity inflation: inflaton identified with $s$ (analytic approximation)

\[
\begin{align*}
\lambda_s(s) & \approx 0 \\
\beta \lambda_s(s) & \approx 0 \\
\xi_s(s)s^2 & = \tilde{M}_{P1}^2
\end{align*}
\]

$\Rightarrow \lambda_s(\mu \approx s) \approx \frac{b}{2} \ln^2 \frac{s}{\langle s \rangle}$, \hspace{1cm} $\xi_s(\mu) \approx \xi_s$ for simplicity

$b \equiv g^4/(4\pi)^4$ can be computed in any given model ...

$\Rightarrow \epsilon \approx \eta \approx \frac{2\xi_s}{1 + 6\xi_s} \frac{1}{\ln^2 \frac{s}{\langle s \rangle}} = \frac{2\tilde{M}_{P1}^2}{s_E^2}$

The Einstein-frame potential is nearly quadratic around its minimum:

\[U = \frac{\tilde{M}_{P1}^4}{4} \frac{\lambda_s}{\xi_s^2} \approx \frac{M_s^2}{2} s_e^2 \quad \text{with} \quad M_s = g^2 \tilde{M}_{P1} \frac{1}{2(4\pi)^2} \frac{1}{\sqrt{\xi_s(1 + 6\xi_s)}}\]

Inserting $s_e$ at $N \approx 60$ e-foldings, $s_e(N) \approx 2\sqrt{N}\tilde{M}_{P1}$, ... we obtain the predictions

\[n_s \approx 1 - \frac{2}{N} \approx 0.967, \quad r \approx \frac{8}{N} \approx 0.13, \quad A_s \approx \frac{g^4 N^2}{24\pi^2 \xi_s(1 + 6\xi_s)}\]

(remember inflaton $= s$). Such predictions are typical of quadratic potentials
Agravity inflation: inflaton identified with $s$ (analytic approximation)

\[
\begin{align*}
\lambda_S(s) & \approx 0 \\
\beta \lambda_S(s) & \approx 0 \\
\xi_S(s)s^2 & = \bar{M}_{\text{Pl}}^2
\end{align*}
\rightarrow \lambda_S(\mu \approx s) & \approx \frac{b}{2} \ln^2 \frac{s}{\langle s \rangle},
\xi_S(\mu) & \approx \xi_S
\]

for simplicity

$b \equiv g^4/(4\pi)^4$ can be computed in any given model ...

\[
\rightarrow \epsilon \approx \eta \approx \frac{2\xi_S}{1 + 6\xi_S} \frac{1}{\ln^2 s/\langle s \rangle} \approx \frac{2\bar{M}_{\text{Pl}}^2}{s_E^2}
\]

The Einstein-frame potential is nearly quadratic around its minimum:

\[
U = \frac{\bar{M}_{\text{Pl}}^4}{4} \frac{\lambda_S}{\xi_S^2} \approx \frac{M_s^2}{2} s_E^2
\quad \text{with} \quad M_s = \frac{g^2 \bar{M}_{\text{Pl}}}{2(4\pi)^2 \sqrt{\xi_S(1 + 6\xi_S)}}
\]

Inserting $s_E$ at $N \approx 60$ e-foldings, $s_E(N) \approx 2\sqrt{N} \bar{M}_{\text{Pl}}$, ... we obtain the predictions

\[
n_s \approx 1 - \frac{2}{N} \approx 0.967, \quad r \approx \frac{8}{N} \approx 0.13, \quad A_s \approx \frac{g^4 N^2}{24\pi^2 \xi_S(1 + 6\xi_S)}
\]

(remember inflaton $= s$). Such predictions are typical of quadratic potentials

VEVs above $\bar{M}_{\text{Pl}}$, $s_E \approx 2\sqrt{N} \bar{M}_{\text{Pl}}$, are needed for a quadratic potential

Agravity predicts physics above $\bar{M}_{\text{Pl}}$, and a quadratic potential is a good approximation, even at $s_E > \bar{M}_{\text{Pl}}$, because coefficients of higher order terms are suppressed by extra powers of the loop expansion parameters, which are small at weak coupling
Tricks to bring the theory in a more standard form

\[
\frac{\mathcal{L}}{\sqrt{g}} = \frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}}{f_2^2} - \frac{\xi}{2} \varphi^2 R + \mathcal{L}_{\text{matter}}
\]
Tricks to bring the theory in a more standard form

First, we can trade the $R^2$ term with a scalar field: consider an auxiliary field $\chi$

\[
\frac{\mathcal{L}}{\sqrt{g}} = \frac{R^2}{6f_0^2} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}^2}{f_2^2} - \frac{\xi \varphi^2}{2} R + \mathcal{L}_{\text{matter}} - \frac{(R + 3f_0^2 \chi/2)^2}{6f_0^2}
\]

where \( f = \chi + \xi \varphi^2 \) and \( \mathcal{L}_{\text{matter}} = \frac{(D_{\mu}\varphi)^2}{2} - \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} D\psi + (y \varphi \psi \psi + \text{h.c.}) - V(\varphi) \)

Second, perform a conformal transformation of the metric and the other fields:

\[
g_E^{\mu\nu} = g^{\mu\nu} \times \frac{f}{\bar{\cal M}_2} \quad \bar{\cal M}_2 \quad \phi_E = \phi \times \left( \frac{\bar{\cal M}_2}{f} \right)^{1/2} \quad \psi_E = \psi \times \left( \frac{\bar{\cal M}_2}{f} \right)^{3/4} \quad A_E^{\mu} = A^{\mu}
\]
Tricks to bring the theory in a more standard form

First, we can trade the $R^2$ term with a scalar field: consider an auxiliary field $\chi$

$$\frac{L}{\sqrt{g}} = \frac{R^2}{6f_0} + \frac{1}{3} \frac{R^2 - R_{\mu\nu}}{f_2^2} - \frac{\xi \varphi^2}{2} R + L_{\text{matter}} - \frac{(R + 3f_0^2 \chi/2)^2}{6f_0^2}$$

zero on-shell

$$= \frac{1}{3} \frac{R^2 - R_{\mu\nu}}{f_2^2} - \frac{f}{2} R - \frac{3f_0^2}{8} \chi^2 + L_{\text{matter}}$$

where $f = \chi + \xi \varphi^2$ and $L_{\text{matter}} = \frac{(D_\mu \varphi)^2}{2} - \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} i D\psi + (y \varphi \psi \psi + \text{h.c.}) - V(\varphi)$

Second, perform a conformal transformation of the metric and the other fields:

$$g^{E}_{\mu\nu} = g_{\mu\nu} \times f / \tilde{M}^2_{\text{Pl}} \quad \varphi_E = \varphi \times (\tilde{M}^2_{\text{Pl}}/f)^{1/2} \quad \psi_E = \psi \times (\tilde{M}^2_{\text{Pl}}/f)^{3/4} \quad A_{\mu E} = A_\mu$$

$$\frac{L}{\sqrt{g^{E}}} = \frac{1}{3} \frac{R^2_{E} - R_{E\mu\nu}}{f_2^2} - \frac{1}{4} F_{E\mu\nu}^2 + \bar{\psi}_E i D\psi_E + (y \varphi_E \psi_E \psi_E + \text{h.c.}) - \frac{\tilde{M}^2_{\text{Pl}}}{2} R_E + L_\varphi - U$$

where $L_\varphi = \tilde{M}^2_{\text{Pl}} \left[ \frac{(D_\mu \varphi)^2}{2f} + \frac{3(\partial_\mu f)^2}{4f^2} \right]$ and $U = \frac{\tilde{M}^4_{\text{Pl}}}{f^2} \left[ V(\varphi) + \frac{3f_0^2}{8} \chi^2 \right]$ - $\hat{z}$

By redefining $z = \sqrt{6f}$,

$$L_\varphi = \frac{6 \tilde{M}^2_{\text{Pl}}}{z^2} \left( \frac{D_\mu \varphi)^2 + (\partial_\mu z)^2}{2} \right) \quad U(z, \varphi) = \frac{36 \tilde{M}^4_{\text{Pl}}}{z^4} \left[ V(\varphi) + \frac{3f_0^2}{8} \left( \frac{z^2}{6} - \xi \varphi^2 \right)^2 \right]$$
Matching the scalar amplitude

1. Planckian inflation ($M_0 \gg M_s$)

$$n_s \approx 1 - \frac{2}{N} \approx 0.967, \quad r \approx \frac{8}{N} \approx 0.13$$

The scalar amplitude $A_s = M_s^2 N^2 / 6\pi^2 \tilde{M}_{pl}^2$ is reproduced for $M_s \approx 1.4 \times 10^{13}$GeV
Matching the scalar amplitude

1. Planckon inflation \((M_0 \gg M_s)\)

\[
ns \approx 1 - \frac{2}{N} \frac{N \approx 60}{\tilde{N}} \approx 0.967, \quad r \approx \frac{8}{N} \frac{N \approx 60}{\tilde{N}} \approx 0.13
\]

The scalar amplitude \(A_s = M_s^2 N^2 / 6\pi^2 \tilde{M}_P^2\) is reproduced for \(M_s \approx 1.4 \times 10^{13}\) GeV

2. Graviscalar inflation (Starobinsky inflation) \((M_0 \ll M_s)\)

\[
ns \approx 1 - \frac{2}{N} \frac{N \approx 60}{\tilde{N}} \approx 0.967, \quad r \approx \frac{12}{N^2} \frac{N \approx 60}{\tilde{N}} \approx 0.003
\]

The scalar amplitude \(A_s = f_0^2 N^2 / 48\pi^2\) is reproduced for \(f_0 \approx 1.8 \times 10^{-5}\)
Matching the scalar amplitude

1. **Planckion inflation** \((M_0 \gg M_s)\)

\[
ns \approx 1 - \frac{2}{N} N \approx 60 \approx 0.967, \quad r \approx \frac{8}{N} N \approx 60 \approx 0.13
\]

The scalar amplitude \(A_s = M_s^2 N^2 / 6\pi^2 \bar{M}_{Pl}^2\) is reproduced for \(M_s \approx 1.4 \times 10^{13}\) GeV

2. **Graviscalar inflation (Starobinsky inflation)** \((M_0 \ll M_s)\)

\[
ns \approx 1 - \frac{2}{N} N \approx 60 \approx 0.967, \quad r \approx \frac{12}{N^2} N \approx 60 \approx 0.003
\]

The scalar amplitude \(A_s = f_0^2 N^2 / 48\pi^2\) is reproduced for \(f_0 \approx 1.8 \times 10^{-5}\)

So generically we have

\[
f_0 \gtrsim 10^{-5}
\]
The decay of $I$ with mass $M_I$ and width $\Gamma_I$ reheats the universe up to a temperature

$$T_{\text{RH}} = \left[ \frac{90}{\pi^2 g_*} \frac{\Gamma_I^2 \bar{M}_{\text{Pl}}^2}{\bar{M}_{\text{Pl}}} \right]^{1/4},$$

where $g_* \sim 100$ is the number of relativistic degrees of freedom at $T_{\text{RH}}$.

The inflaton $I$ is in the hidden sector. How can it reheat the universe?
The decay of $I$ with mass $M_I$ and width $\Gamma_I$ reheats the universe up to a temperature

$$T_{\text{RH}} = \left[ \frac{90}{\pi^2 g_*} \frac{\Gamma_I^2}{{\bar{M}}_{\text{Pl}}^2} \right]^{1/4},$$

where $g_* \sim 100$ is the number of relativistic degrees of freedom at $T_{\text{RH}}$.

The inflaton $I$ is in the hidden sector. How can it reheat the universe?

The Planckion $s$ and the graviscalar $z$ respectively couple to

$$\frac{\partial_\mu \mathcal{D}_\mu}{M_{\text{Pl}}/\sqrt{\xi_s}}$$

and

$$\frac{T_{\mu\mu}}{{\bar{M}}_{\text{Pl}}},$$

($T_{\mu\mu}$ is the trace of the energy-momentum tensor and $\mathcal{D}_\mu$ is the dilatation current)

The theory is classically scale-invariant $\rightarrow$ a non-zero $\partial_\mu \mathcal{D}_\mu$ arises only at loop level:

$$\partial_\mu \mathcal{D}_\mu = \frac{\beta_{g_1}}{2g_1} Y_{\mu\nu}^2 + \frac{\beta_{g_2}}{2g_2} W_{\mu\nu}^2 + \frac{\beta_{g_3}}{2g_3} G_{\mu\nu}^2 + \beta_{y_t} H Q_3 U_3 + \beta_{\lambda_H} |H|^4 + \ldots,$$

where $\ldots$ are BSM terms (they are relevant for DM production as we will see)
The decay of $I$ with mass $M_I$ and width $\Gamma_I$ reheats the universe up to a temperature

$$T_{\text{RH}} = \left[ \frac{90}{\pi^2 g_*} \frac{\Gamma_I^2 \bar{M}_{\text{Pl}}^2}{M_I} \right]^{1/4},$$

where $g_* \sim 100$ is the number of relativistic degrees of freedom at $T_{\text{RH}}$.

The inflaton $I$ is in the hidden sector. How can it reheat the universe?

The Planckion $s$ and the graviscalar $z$ respectively couple to

$$\frac{\partial_\mu D_\mu}{\bar{M}_{\text{Pl}}/\sqrt{\xi_S}} \quad \text{and} \quad \frac{T_{\mu\mu}}{\bar{M}_{\text{Pl}}},$$

($T_{\mu\mu}$ is the trace of the energy-momentum tensor and $D_\mu$ is the dilatation current)

The theory is classically scale-invariant $\rightarrow$ a non-zero $\partial_\mu D_\mu$ arises only at loop level:

$$\partial_\mu D_\mu = \frac{\beta_{g_1}}{2g_1} Y_{\mu\nu}^2 + \frac{\beta_{g_2}}{2g_2} W_{\mu\nu}^2 + \frac{\beta_{g_3}}{2g_3} G_{\mu\nu}^2 + \beta_{Y_t} H Q_3 U_3 + \beta_{\lambda_H} |H|^4 + \ldots,$$

where $\ldots$ are BSM terms (they are relevant for DM production as we will see)

leading decay from $\partial_\mu D_\mu$: $\Gamma(I \to gg) \approx \frac{|\xi_S| g_3^4 M_s^3}{(4\pi)^5 \bar{M}_{\text{Pl}}^2} \rightarrow T_{\text{RH}} \xi_S \sim 10^7 \text{ GeV} \left( \frac{M_s}{10^{13} \text{ GeV}} \right)^{3/2}$
The decay of $I$ with mass $M_I$ and width $\Gamma_I$ reheats the universe up to a temperature

$$T_{\text{RH}} = \left[ \frac{90}{\pi^2 g_*} \frac{\Gamma_I^2 \bar{M}_{\text{Pl}}^2}{\Gamma_I} \right]^{1/4},$$

where $g_* \sim 100$ is the number of relativistic degrees of freedom at $T_{\text{RH}}$.

The inflaton $I$ is in the hidden sector. How can it reheat the universe?

The Planckion $s$ and the graviscalar $z$ respectively couple to

$$\frac{\partial_\mu D_\mu}{\bar{M}_{\text{Pl}}/\sqrt{\xi_S}} \quad \text{and} \quad \frac{T_{\mu\mu}}{\bar{M}_{\text{Pl}}},$$

($T_{\mu\mu}$ is the trace of the energy-momentum tensor and $D_\mu$ is the dilatation current)

The theory is classically scale-invariant $\rightarrow$ a non-zero $\partial_\mu D_\mu$ arises only at loop level:

$$\partial_\mu D_\mu = \frac{\beta_{g_1}}{2 g_1} Y_{\mu\nu}^2 + \frac{\beta_{g_2}}{2 g_2} W_{\mu\nu}^2 + \frac{\beta_{g_3}}{2 g_3} G_{\mu\nu}^2 + \beta_{y_t} H Q_3 U_3 + \beta_{\lambda_H} |H|^4 + \ldots,$$

where $\ldots$ are BSM terms (they are relevant for DM production as we will see)

leading decay from $\partial_\mu D_\mu$ : $\Gamma(I \rightarrow gg) \approx \frac{|\xi_S| g_3^4 M_s^3}{(4\pi)^5 \bar{M}_{\text{Pl}}^2} \rightarrow T_{\text{RH}} \sim 10^7 \text{ GeV} \left( \frac{M_s}{10^{13} \text{ GeV}} \right)^{3/2}$

leading decay from $T_{\mu\mu}$ : $\Gamma(I \rightarrow h_E h_E) \approx \frac{(1 + 6\xi_H)^2 |\xi_S|}{|1 + 6\xi_S|} \frac{M_I^3}{64\pi \bar{M}_{\text{Pl}}^2} \rightarrow T_{\text{RH}} \sim 10^9 \text{ GeV}$
Dark matter

There should be fermions in the $s$-sector. Two types of candidates come to mind.
Dark matter, neutrino masses and leptogenesis

There should be fermions in the $s$-sector. Two types of candidates come to mind.
Dark matter, neutrino masses and leptogenesis

There should be fermions in the $s$-sector. Two types of candidates come to mind

**Fermions in the $s$-sector with no gauge interactions**

- They can couple to the SM behaving as right-handed neutrinos $N$ and generate the observed neutrino masses via $NLH$ couplings. The right-handed neutrino masses can be generated by $sN^2$ terms. [Alexander-Nunneley, Pilaftsis (2010)]
- They can provide baryogenesis via leptogenesis.
- [Dodelson, Widrow (1993)] claimed that $N$ may also account for DM in special cases
Dark matter, neutrino masses and leptogenesis

There should be fermions in the $s$-sector. Two types of candidates come to mind.

**Fermions in the $s$-sector with no gauge interactions**

- They can couple to the SM behaving as right-handed neutrinos $N$ and generate the observed neutrino masses via $NLH$ couplings. The right-handed neutrino masses can be generated by $sN^2$ terms. [Alexander-Nunneley, Pilaftsis (2010)]
- They can provide baryogenesis via leptogenesis.
- [Dodelson, Widrow (1993)] claimed that $N$ may also account for DM in special cases.

**Fermions in the $s$-sector which cannot couple to the SM**

The lightest fermion in the $s$-sector is a stable DM candidate if it cannot couple to the SM sector (for example because it has gauge interactions under the inflaton sector).
A predictive model (no extra parameters)

Take a 2nd copy of the SM and impose a $Z_2$ symmetry, spontaneously broken by the fact that the mirror Higgs field ($S$) has

$$\langle S \rangle \sim \bar{M}_{P1} \quad \text{while} \quad \langle H \rangle \sim M_W$$

*Mirror SM particles (e.g. a mirror neutrino or electron) may be DM ...*

Interactions between these candidates and the SM are suppressed by $\lambda_{HS} ...$
Terms in $\partial_\mu D_\mu$ and $T_{\mu\mu}$ lead to decays of the inflaton $I$ into DM (along the lines of the reheating calculation)

More in general the DM fermions can also get a mass $M$ from another source. Then such fermion masses would contribute to $\partial_\mu D_\mu$ and to $T_{\mu\mu}$ as $M\bar{\Psi}\Psi$ (we are considering, for example, a Dirac mass term)

$\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_{cr}} \approx 0.110 \times 10^2 M_0$.40 eV $\Gamma(I \rightarrow DM) / \Gamma(I \rightarrow SM)$ having inserted the present Hubble constant $H_0 = h \times 100 \text{ km/sec Mpc}$
Dark matter abundance

Terms in $\partial_\mu D_\mu$ and $T_{\mu\mu}$ lead to decays of the inflaton $I$ into DM (along the lines of the reheating calculation)

More in general the DM fermions can also get a mass $M$ from another source. Then such fermion masses would contribute to $\partial_\mu D_\mu$ and to $T_{\mu\mu}$ as $M\bar{\Psi}\Psi$ (we are considering, for example, a Dirac mass term)

By identifying the fermion $\Psi$ with DM, its abundance is

$$\Omega_{DM} \equiv \frac{\rho_{DM}}{\rho_{cr}} \approx 0.110 \frac{M}{h^2} \times \frac{\Gamma(I \to DM)}{0.40\text{eV}} \frac{\Gamma(I \to SM)}{\Gamma(I \to SM)}$$

having inserted the present Hubble constant $H_0 = h \times 100 \text{km/sec Mpc}$
The observed DM abundance is reproduced for

\[ M \approx (10 - 200) \text{TeV} \left( \frac{M_I}{10^{13} \text{GeV}} \right)^{2/3} \]

where the lower (higher) estimate applies if \( \Gamma(I \rightarrow gg) \) (\( \Gamma(I \rightarrow h_E h_E) \)) dominates.