

# Hawking radiation from sonic black holes in flowing atom condensates



Fernando Sols  
Universidad Complutense de Madrid

In collaboration with:

M. Albert (LPTMS Orsay - Univ. Geneve)  
J. R. M. de Nova (UCM)  
D. Guéry-Odelin (Toulouse)  
R. Parentani (LPT Orsay)  
I. Zapata (UCM – UAM)



## OUTLINE

- Resonant Hawking radiation
- Violation of Cauchy-Schwarz inequalities by HR
- Birth of quasi-stationary sonic black hole in an outcoupled BEC

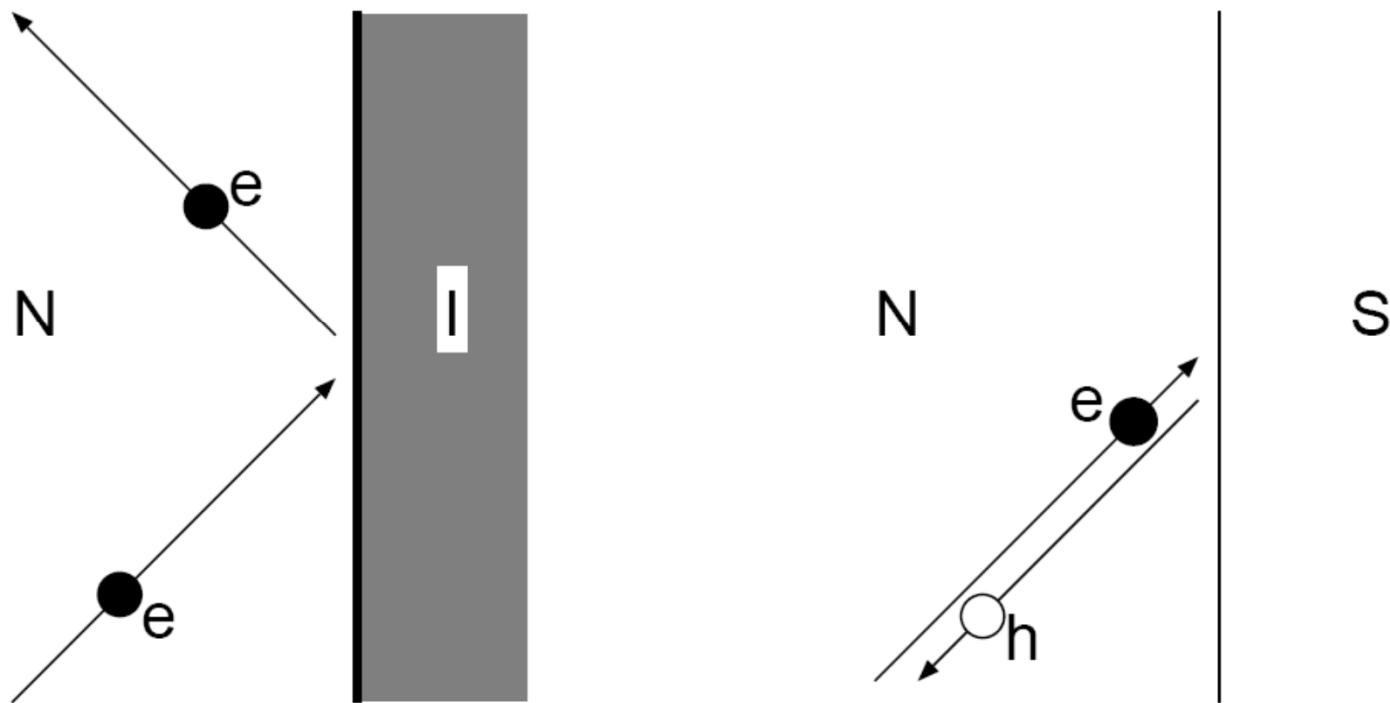
# Hawking radiation from sonic black holes in flowing atom condensates



Zapata, Albert, Parentani, FS  
de Nova, FS, Zapata  
de Nova, Guéry-Odelin, FS, Zapata  
de Nova, FS, Zapata

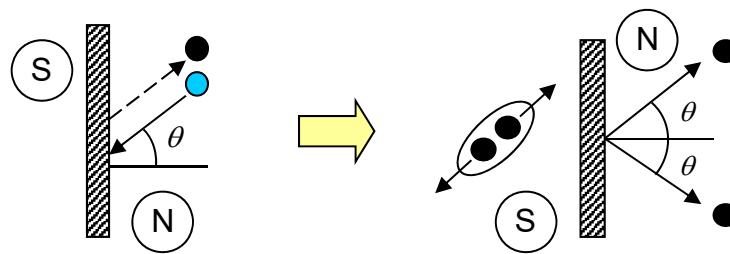
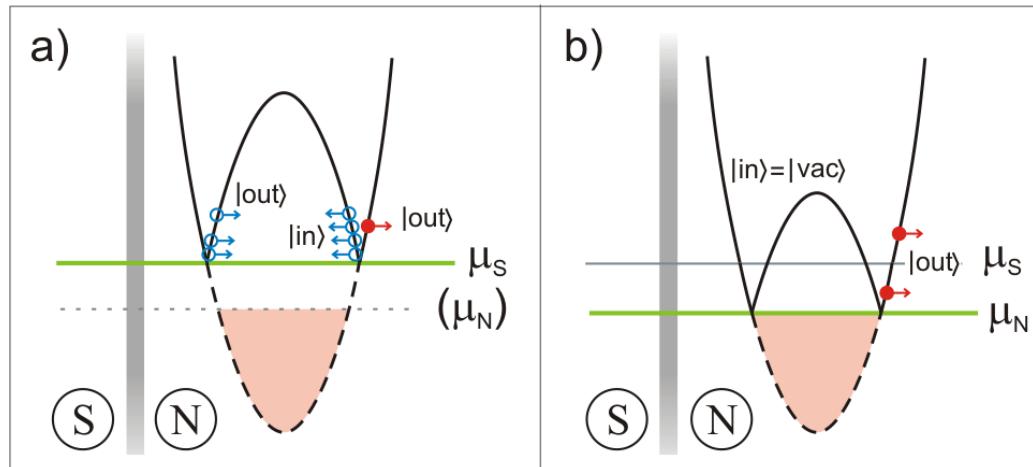
New J. Phys (2011)  
Phys. Rev. A (2014)  
New J. Phys. (2014)  
New J. Phys. (2015)

## Andreev reflection in superconductors



from C. Beenakker (Les Houches, 1994)

## hole Andreev reflection vs. two-electron emission



E. Prada, FS, EPJB (2004)  
also Samuelsson, Sukhorukov, Büttiker, PRL (2003)

## BE Condensate: Gross–Pitaevskii and Bogoliubov – de Gennes equations

$$\hat{\Psi}(x,t) = e^{-i\mu t/\hbar} \left[ \Psi_0(x) + \delta\hat{\Psi}(x,t) \right]$$


---

$$\left[ -\frac{\hbar^2}{2m} \frac{\partial}{\partial x^2} - \mu + V_{\text{ext}}(x) + g_{1D} |\Psi_0(x)|^2 \right] \Psi_0(x) = 0 \quad \begin{array}{l} \text{Gross-Pitaevskii (GP) equation} \\ \text{mean field} \end{array}$$


---

$$\delta\hat{\Psi}(x,t) = \sum_{\substack{\nu_i > 0 \\ \omega_i > 0}} \left[ u_i(x) e^{-i\omega_i t} \hat{\gamma}_i + v_i^*(x) e^{i\omega_i t} \hat{\gamma}_i^\dagger \right] + \sum_{\substack{\nu_i < 0 \\ \omega_i > 0}} \left[ u_i(x) e^{-i\omega_i t} \hat{\gamma}_i^\dagger + v_i^*(x) e^{i\omega_i t} \hat{\gamma}_i \right]$$

Bogoliubov expansion

$$\begin{bmatrix} \hat{H} & g_{1D} \Psi_0(x)^2 \\ -g_{1D} \Psi_0^*(x)^2 & -\hat{H} \end{bmatrix} \begin{bmatrix} u_i(x) \\ v_i(x) \end{bmatrix} = \hbar \omega_i \begin{bmatrix} u_i(x) \\ v_i(x) \end{bmatrix}$$

Bogoliubov-deGennes (BdG) equations  
non-Hermitian!

$$\hat{H} := -\frac{\hbar^2}{2m} \frac{\partial}{\partial x^2} - \mu + V_{\text{ext}}(x) + 2g_{1D} |\Psi_0(x)|^2$$

$$\nu_i := \int dx \left[ |u_i(x)|^2 - |v_i(x)|^2 \right] = \pm 1 \quad \text{normalization}$$

$$[\hat{\gamma}_i, \hat{\gamma}_j^\dagger] = \delta_{ij}$$

*Bogoliubov approximation*  
*= independent quasiparticles*

## BE Condensate: Gross–Pitaevskii - two speeds

$$\hat{H} := -\frac{\hbar^2}{2m} \frac{\partial}{\partial x^2} - \mu + V_{ext}(x) + 2g_{1D} |\Psi_0(x)|^2$$

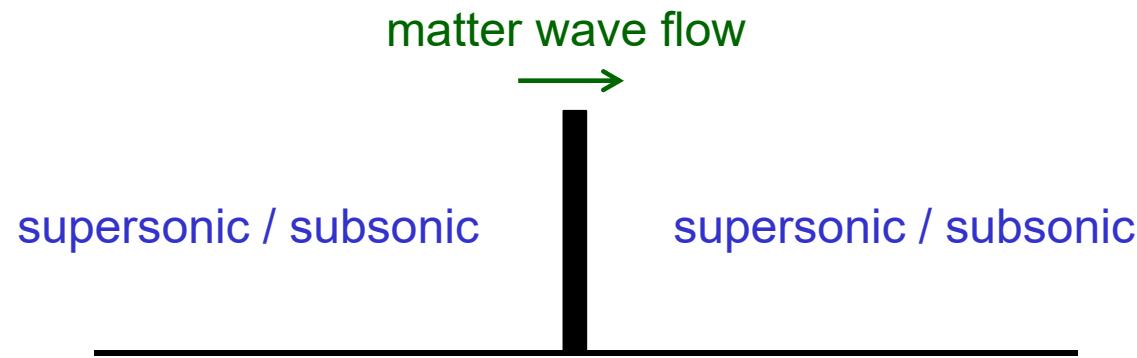
$$c(x) = |\Psi_0(x)| \sqrt{\frac{g_{1D}}{m}} \quad \text{sound speed}$$

$$v(x) = \frac{j}{|\Psi_0(x)|^2} \quad \text{flow speed}$$

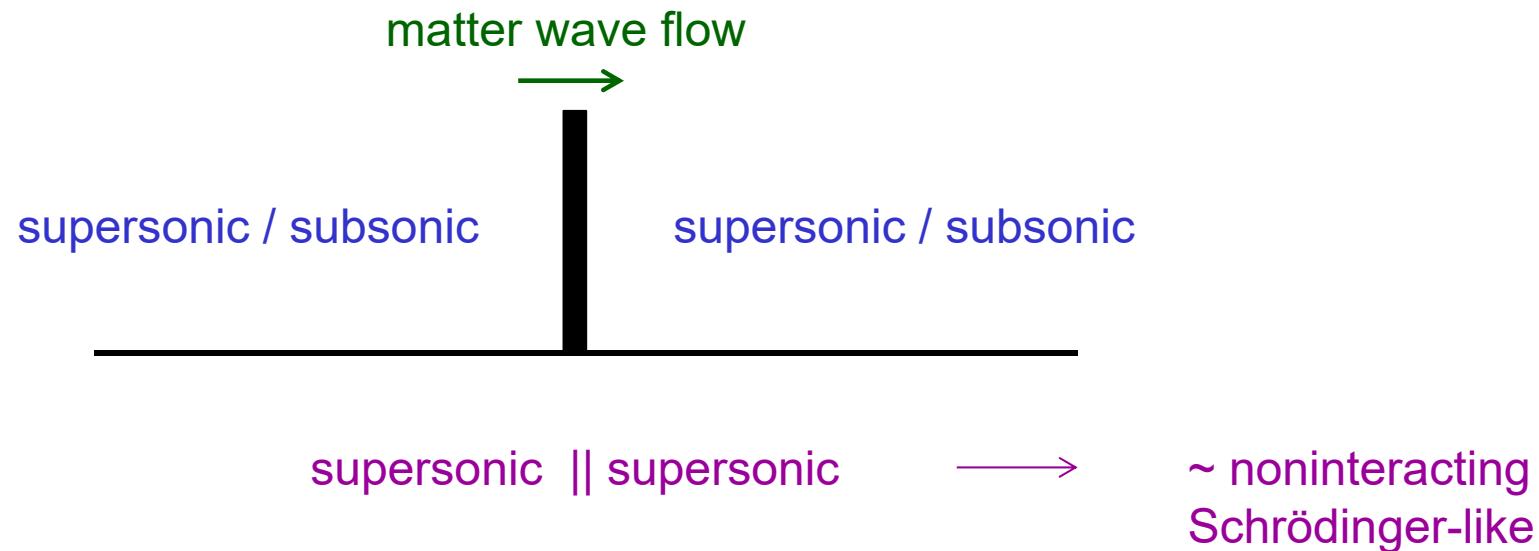
$$v(x) < c(x) \quad \text{subsonic}$$

$$v(x) > c(x) \quad \text{supersonic}$$

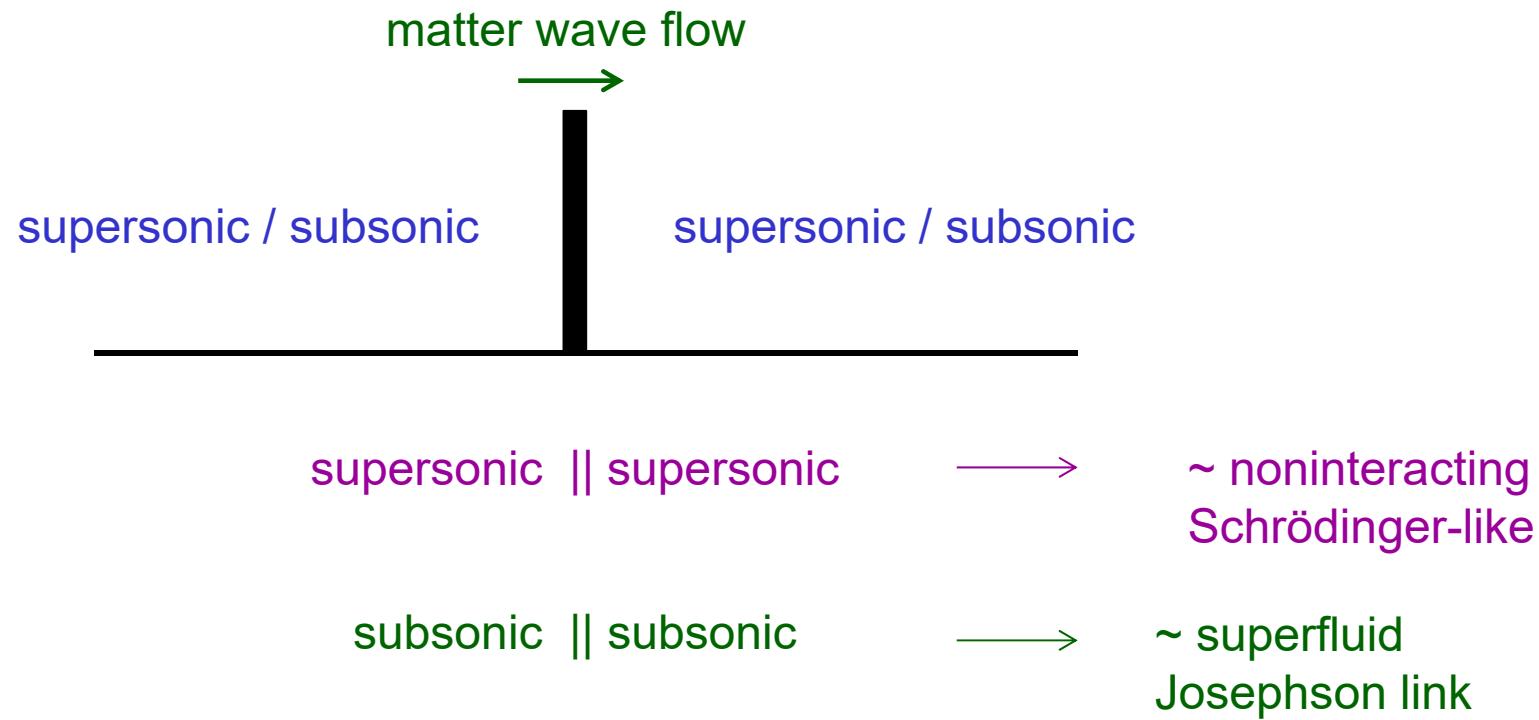
## Three paradigms of quantum transport through a barrier



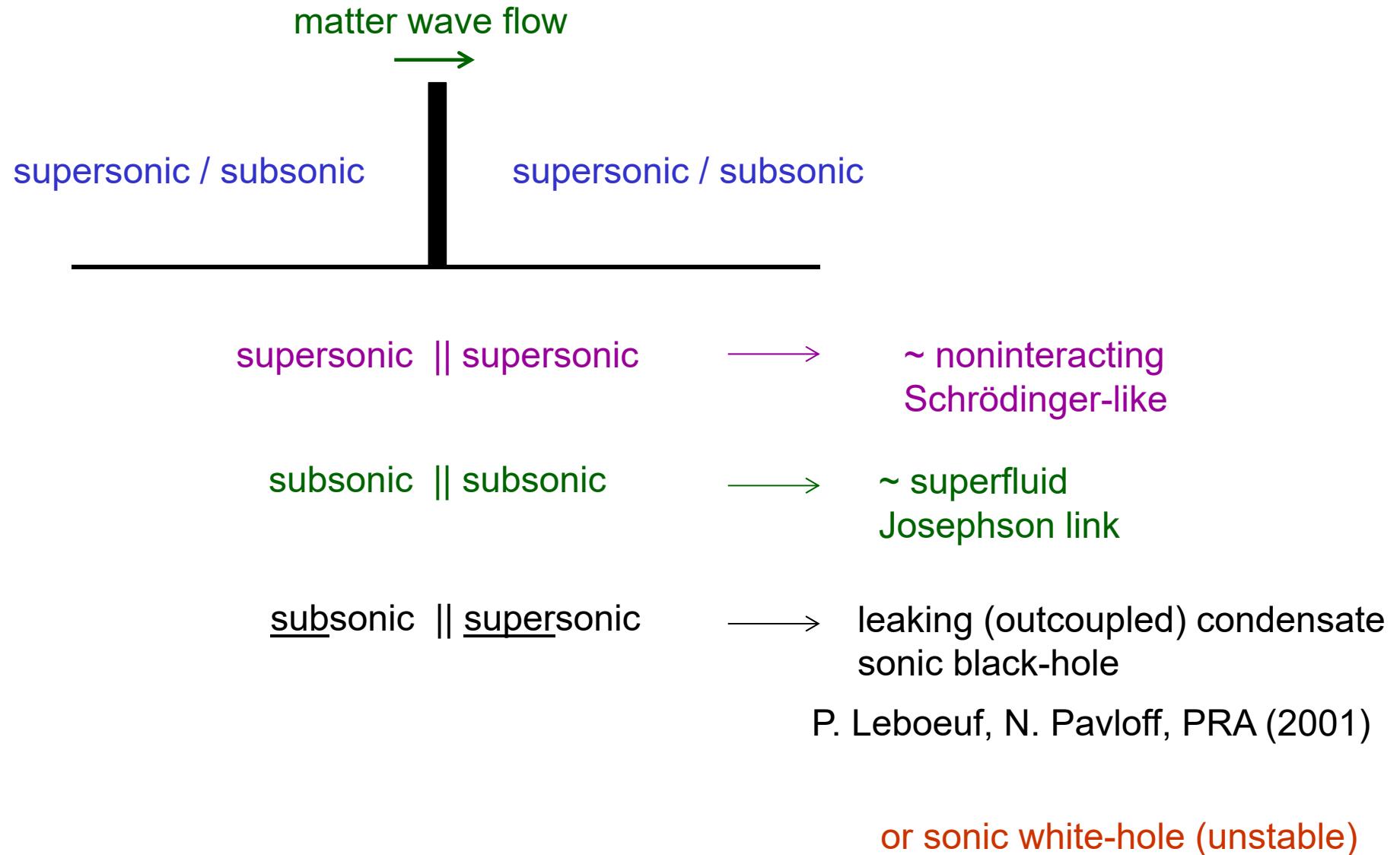
## Three paradigms of quantum transport through a barrier



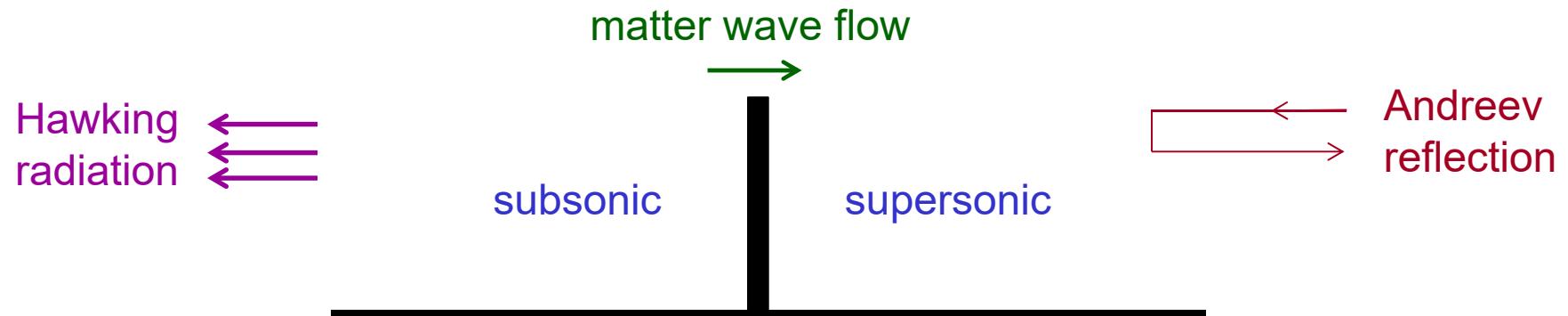
# Three paradigms of quantum transport through a barrier



# Three paradigms of quantum transport through a barrier



## Hawking vs Andreev

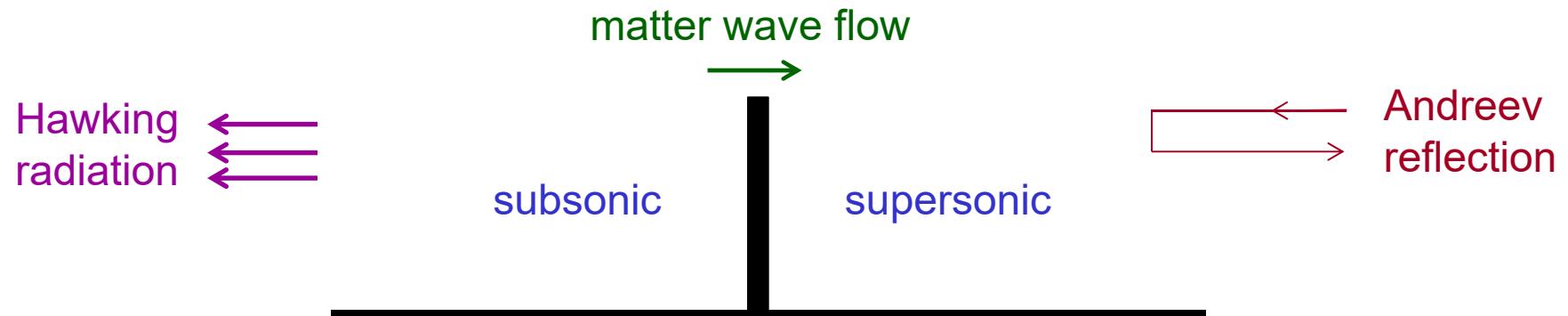


S. W. Hawking,  
Nature (1974);  
Commun. Math.  
Phys. (1975)



A.F. Andreev,  
JETP (1964)

## Hawking-Unruh vs Andreev



S. W. Hawking,  
Nature (1974);  
Commun. Math.  
Phys. (1975)



W. G. Unruh  
PRL (1981)



A.F. Andreev,  
JETP (1964)

## BE Condensate: Gross–Pitaevskii - two speeds

$$\hat{H} := -\frac{\hbar^2}{2m} \frac{\partial}{\partial x^2} - \mu + V_{ext}(x) + 2g_{1D} |\Psi_0(x)|^2$$

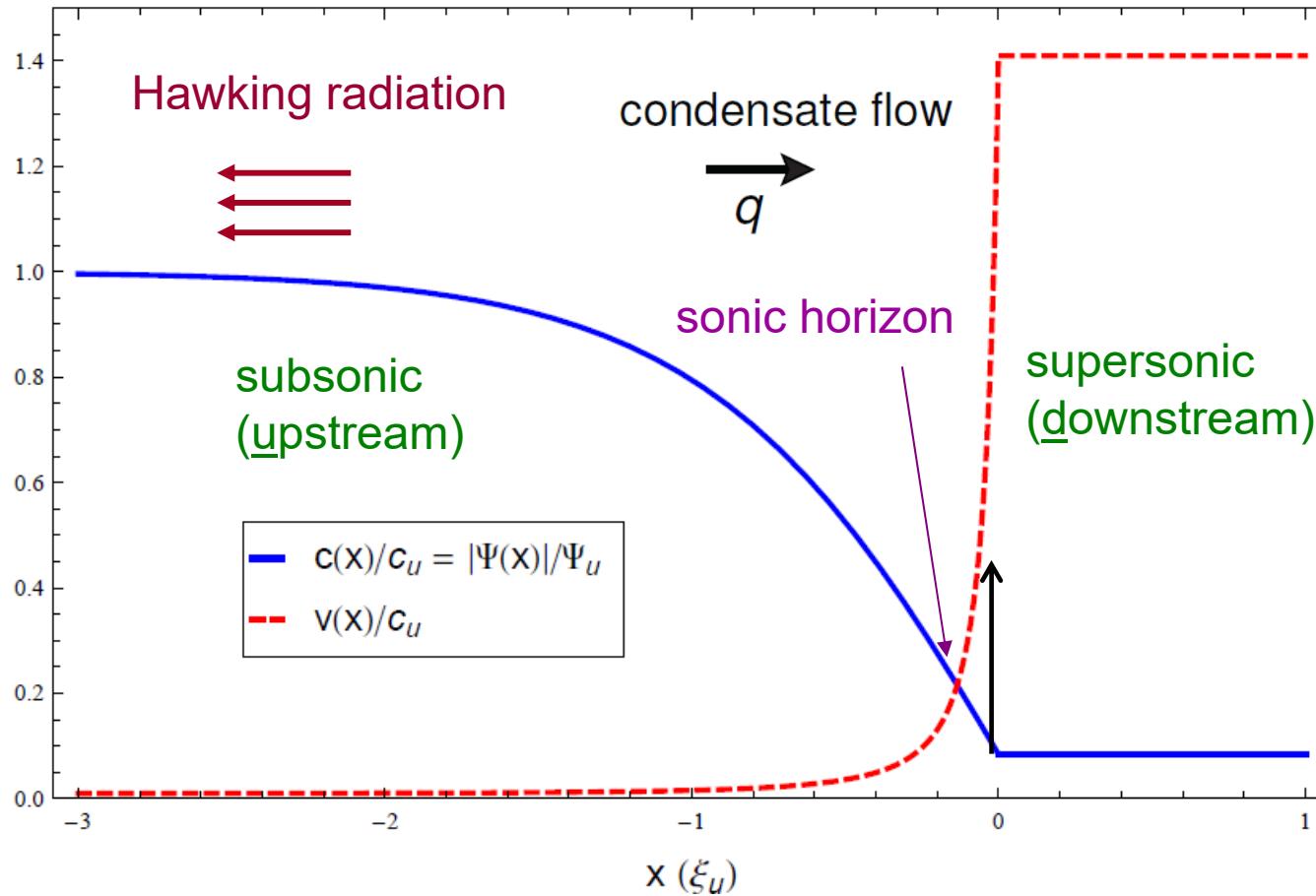
$$c(x) = |\Psi_0(x)| \sqrt{\frac{g_{1D}}{m}} \quad \text{sound speed}$$

$$v(x) = \frac{j}{|\Psi_0(x)|^2} \quad \text{flow speed}$$

$$v(x) < c(x) \quad \text{subsonic}$$

$$v(x) > c(x) \quad \text{supersonic}$$

# Hawking radiation in BECs



$$V(x) = Z \hbar c_u \delta(x)$$

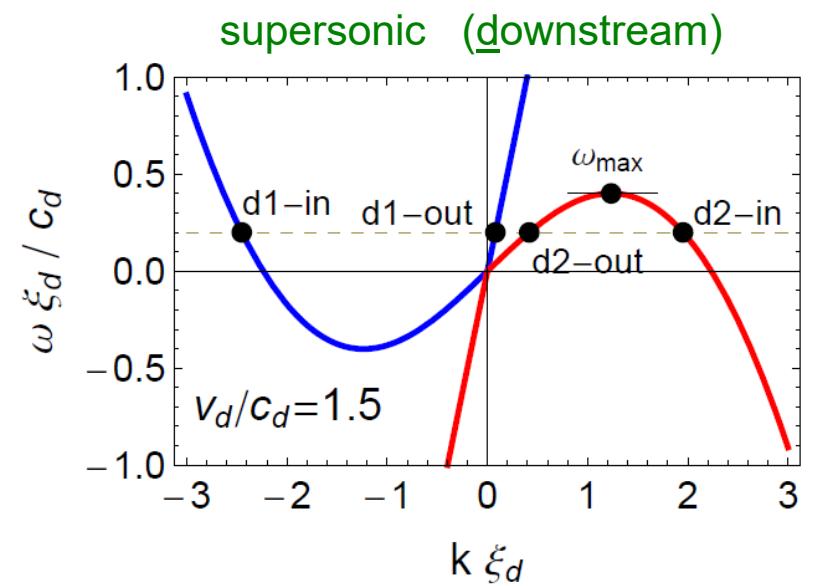
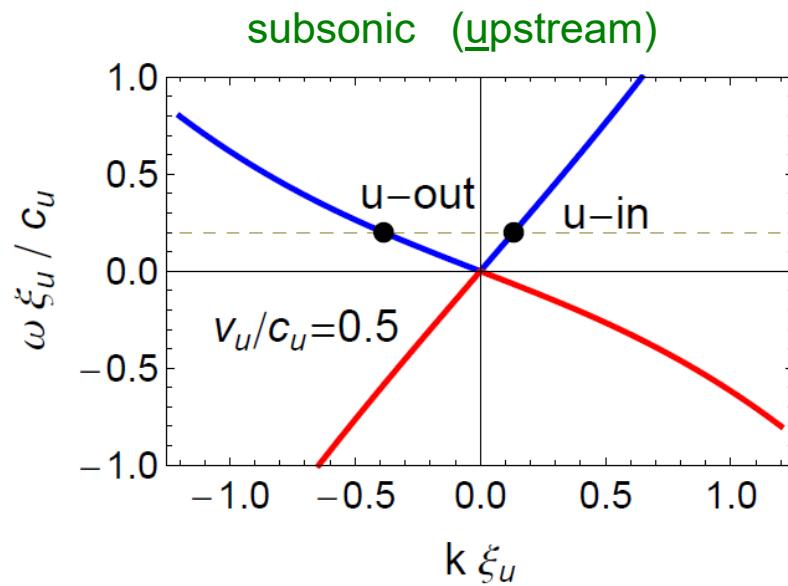
- one (or two) delta barriers
- u: upstream (left, subsonic, “superfluid”)
- d: downstream (right, supersonic, “normal”)
- one black-hole, and several white-hole/black-hole pairs

## Hawking radiation in BECs

$$\omega(k) = v k \pm c |k| \underbrace{\sqrt{1 + \frac{(k\xi)^2}{4}}}_{\Omega(k)}$$

Dispersion relation in laboratory frame for a (comoving frame)  
 Bogoliubov dispersion relation (normal/anomalous normalization)

$v > 0$   
 $v < 0$



- u=upstream
- d1=downstream normal
- d2=downstream anomalous → gives rise to Hawking radiation at T=0
- $\omega_{max}$  is maximum possible frequency for Hawking radiation

## Symmetry property of BdG wave functions

For each solution with frequency  $\omega$  and wave function  $(u, v)$ , there exists another (physically identical) solution with frequency  $-\omega^*$ , wave function  $(v^*, u^*)$ , and opposite normalization. And  $\gamma \rightarrow \gamma+$

$$\delta\hat{\Psi}(x, t) = \sum_{\omega_i > 0} \left\{ \begin{array}{l} \sum_{\nu_i > 0} \left\{ u_i(x) e^{-i\omega_i t} \hat{\gamma}_i + v_i^*(x) e^{i\omega_i t} \hat{\gamma}_i^\dagger \right\} + \\ \sum_{\nu_i < 0} \left\{ u_i(x) e^{-i\omega_i t} \hat{\gamma}_i^\dagger + v_i^*(x) e^{i\omega_i t} \hat{\gamma}_i \right\} \end{array} \right.$$

## Symmetry property of BdG wave functions

For each solution with frequency  $\omega$  and wave function  $(u, v)$ , there exists another (physically identical) solution with frequency  $-\omega^*$ , wave function  $(v^*, u^*)$ , and opposite normalization. And  $\gamma \rightarrow \gamma+$

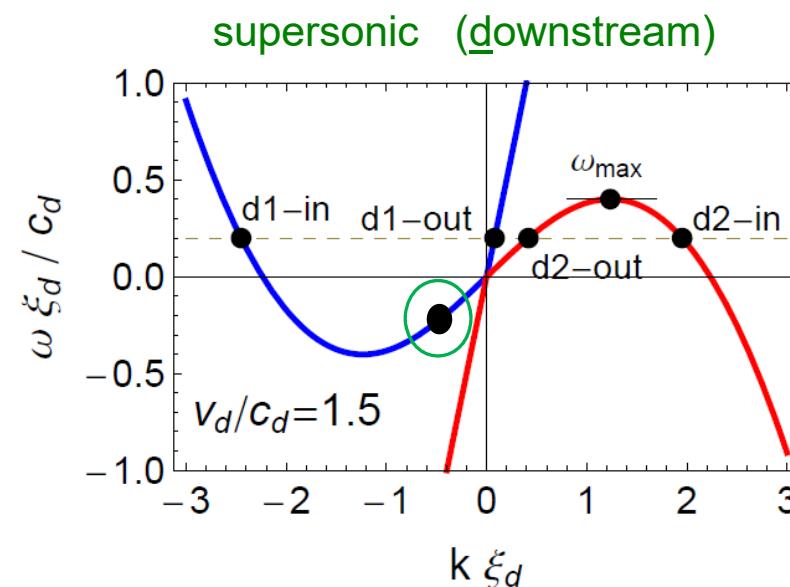
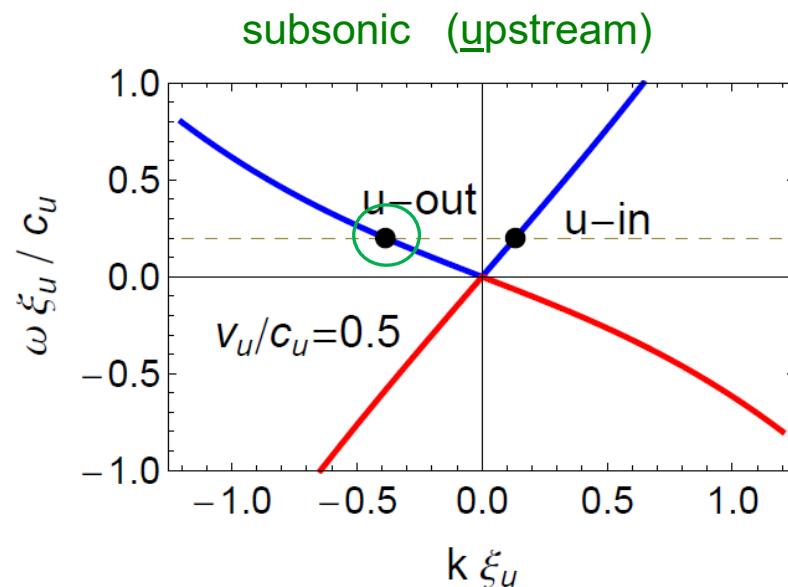
$$\delta\hat{\Psi}(x, t) = \sum_{\omega_i > 0} \left\{ \begin{array}{l} \sum_{\nu_i > 0} \left\{ u_i(x) e^{-i\omega_i t} \hat{\gamma}_i + v_i^*(x) e^{i\omega_i t} \hat{\gamma}_i^\dagger \right\} + \\ \sum_{\nu_i < 0} \left\{ u_i(x) e^{-i\omega_i t} \hat{\gamma}_i^\dagger + v_i^*(x) e^{i\omega_i t} \hat{\gamma}_i \right\} \end{array} \right.$$

We can choose:

all frequencies  $> 0$ , normalizations  $= \pm 1$

frequencies  $< 0$ , all normalizations  $= +1$ ,

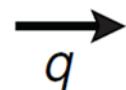
## Hawking radiation (with positive normalization)



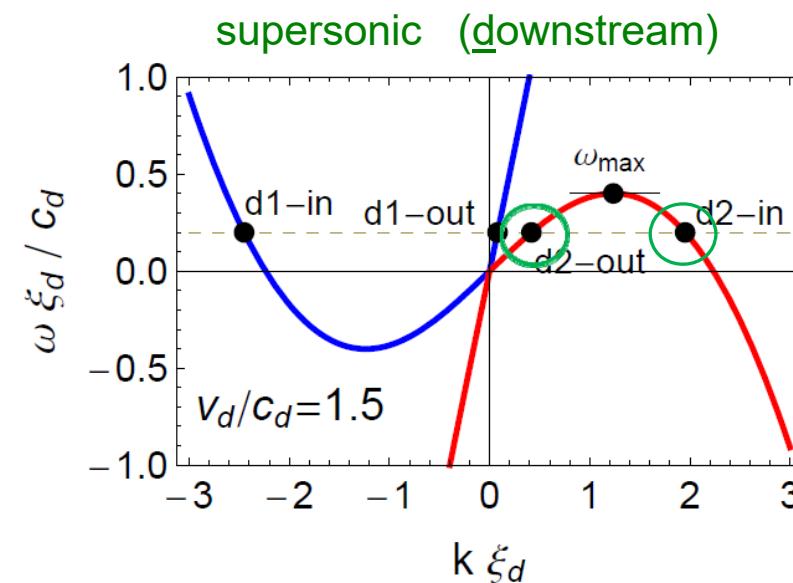
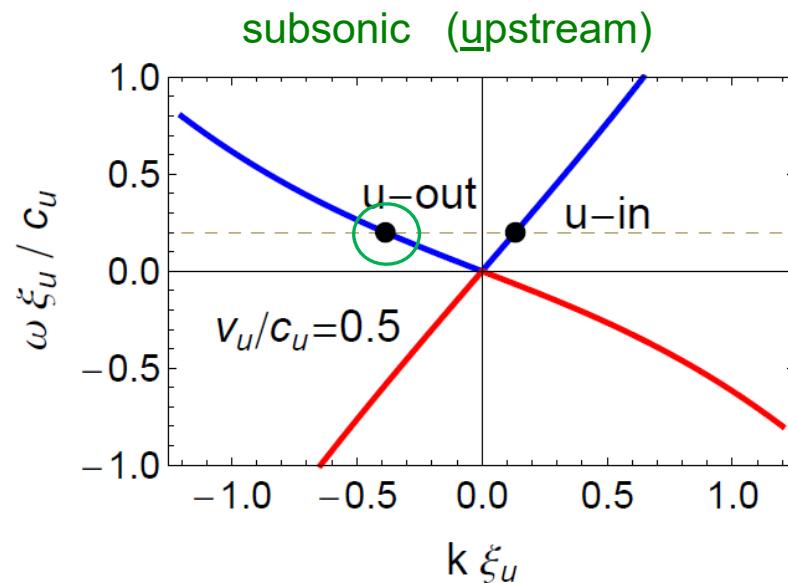
*spontaneous particle-antiparticle pair production*



condensate flow

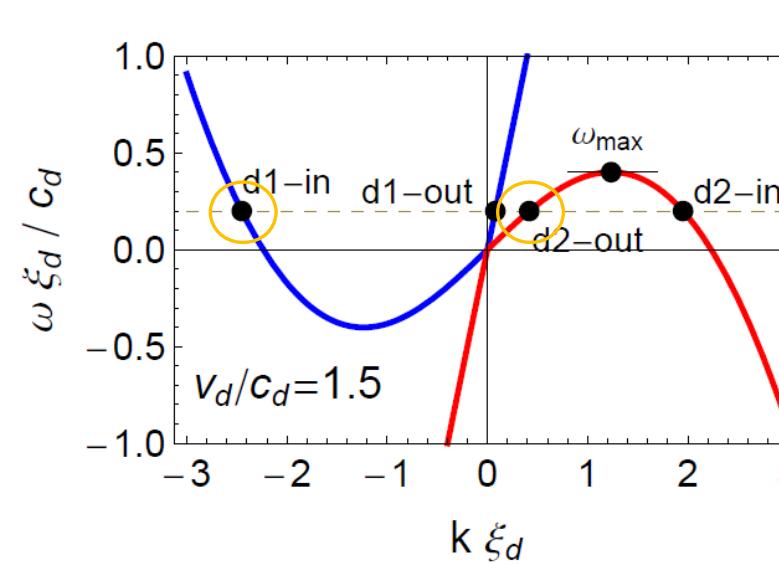
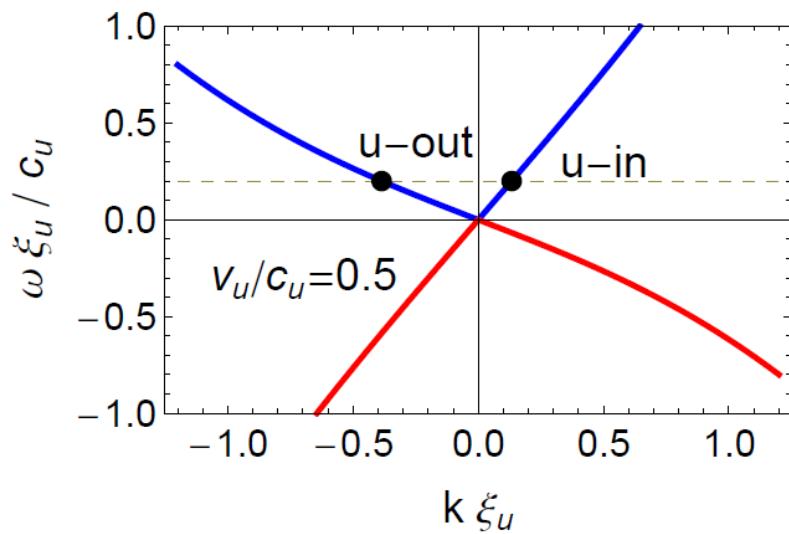


## Hawking radiation (with positive frequency)

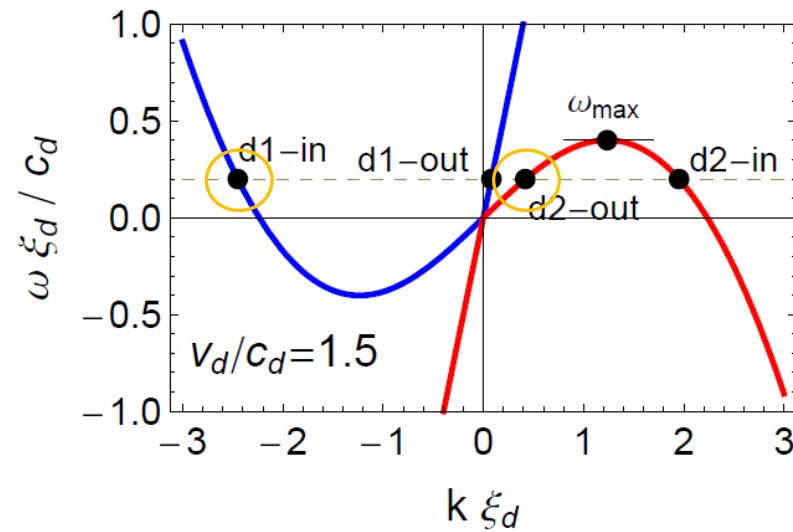
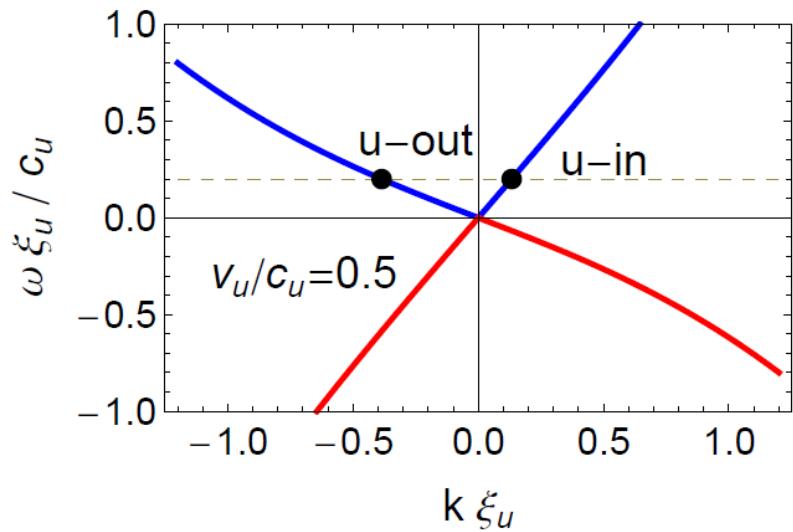
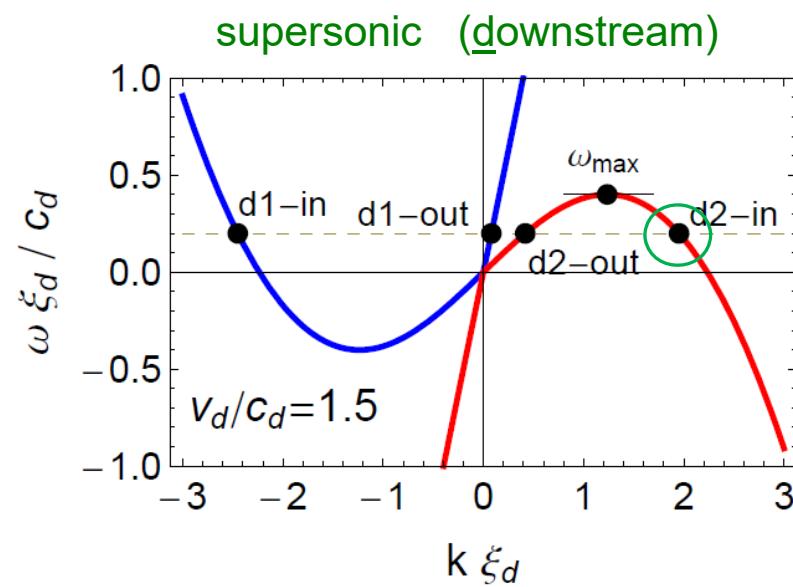
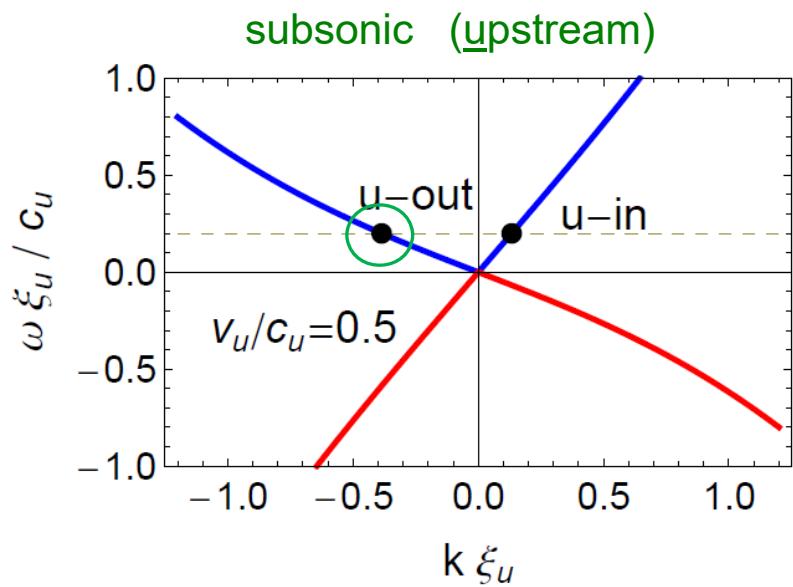


## Andreev reflection

I. Zapata, FS – PRL (2009)



## Andreev-Hawking (anomalous) processes



## Hawking radiation in BEC

$$\text{if } \omega < \omega_H \rightarrow \begin{bmatrix} \hat{a}_u^{\text{out}} \\ \hat{a}_{d1}^{\text{out}} \\ \hat{a}_{d2}^{\dagger\text{out}} \end{bmatrix} = S(\omega) \begin{bmatrix} \hat{a}_u^{\text{in}} \\ \hat{a}_{d1}^{\text{in}} \\ \hat{a}_{d2}^{\dagger\text{in}} \end{bmatrix}, \quad S^\dagger \eta S = \eta, \quad \eta := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow |S_{d2d2}|^2 - |S_{ud2}|^2 - |S_{d1d2}|^2 = 1, \text{ etc... (non-Hermitian)}$$

$$\Rightarrow \frac{dI_u^{\text{out}}}{d\omega} = |S_{uu}|^2 \frac{dI_u^{\text{in}}}{d\omega} + |S_{ud1}|^2 \frac{dI_{d1}^{\text{in}}}{d\omega} + |S_{ud2}|^2 \left( \frac{dI_{d2}^{\text{in}}}{d\omega} + 1 \right),$$

$$\frac{dI_j^{\text{in}}}{d\omega} = \frac{1}{e^{\frac{\hbar\Omega_j}{k_B T}} - 1}, \quad \Omega_j = \text{comoving frequency of mode } j$$

in-vacuum  $\neq$  out-vacuum

Stationary transport:

A. Recatti, N. Pavloff and I. Carusotto, Phys. Rev. A **80**, 043603 (2009)

J. Macher and R. Parentani, Phys. Rev. A **80**, 043601 (2009)

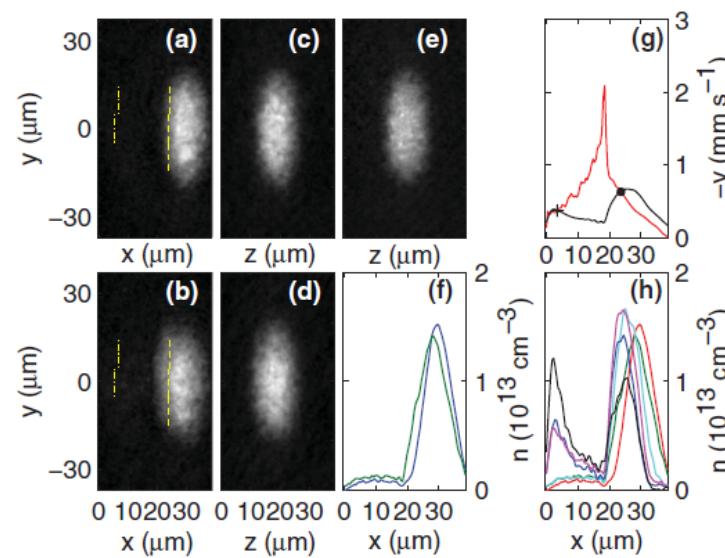
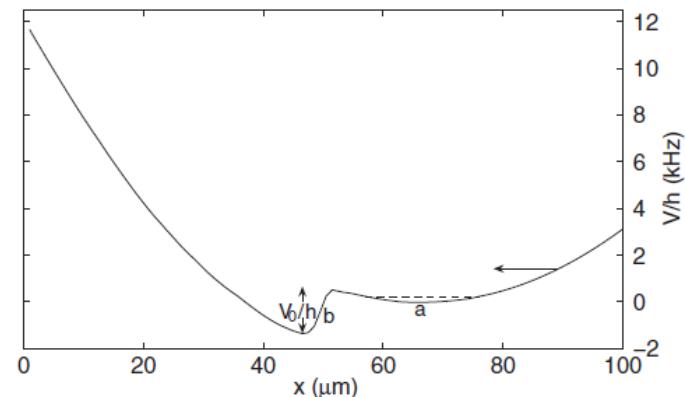
X.-J. CHen, Z.-D. CHen and N.-N. Huang, J. Phys. A: Math. Gen. **31**, 6929 (1998)

...

## Realization of a Sonic Black Hole Analog in a Bose-Einstein Condensate

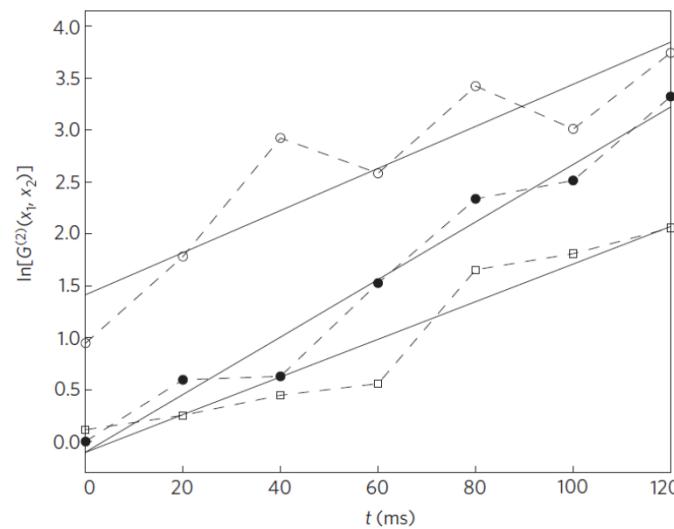
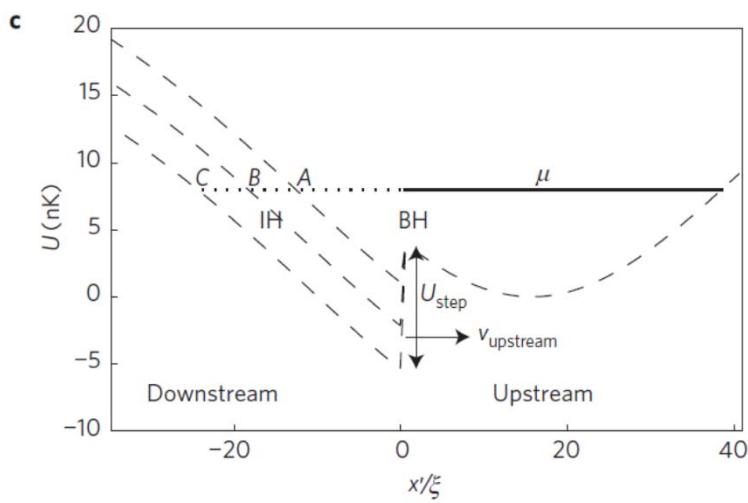
Oren Lahav, Amir Itah, Alex Blumkin, Carmit Gordon, Shahar Rinott, Alona Zayats, and Jeff Steinhauer

*Technion—Israel Institute of Technology, Haifa, Israel*



# Observation of self-amplifying Hawking radiation in an analogue black-hole laser

Jeff Steinhauer

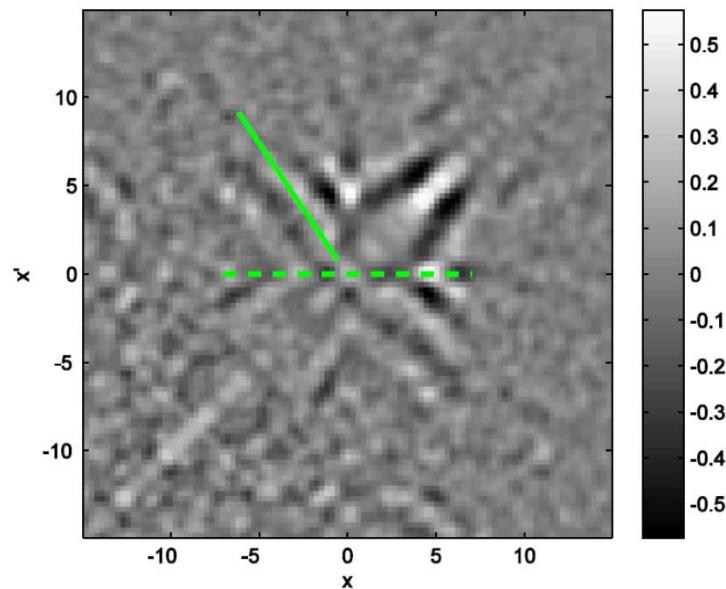


The exponential growth of the lasing mode.

# Observation of thermal Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer

*Department of Physics, Technion—Israel Institute of Technology, Technion City, Haifa 32000, Israel*



arXiv:1510.00621

**Fig. 5. Wave motion near the horizon.** A preliminary experiment is shown in which the step potential at the horizon is caused to oscillate at 50 Hz with an amplitude of 1  $\mu\text{m}$ . The solid line is drawn parallel to the feature marking equal times on opposite sides of the horizon.

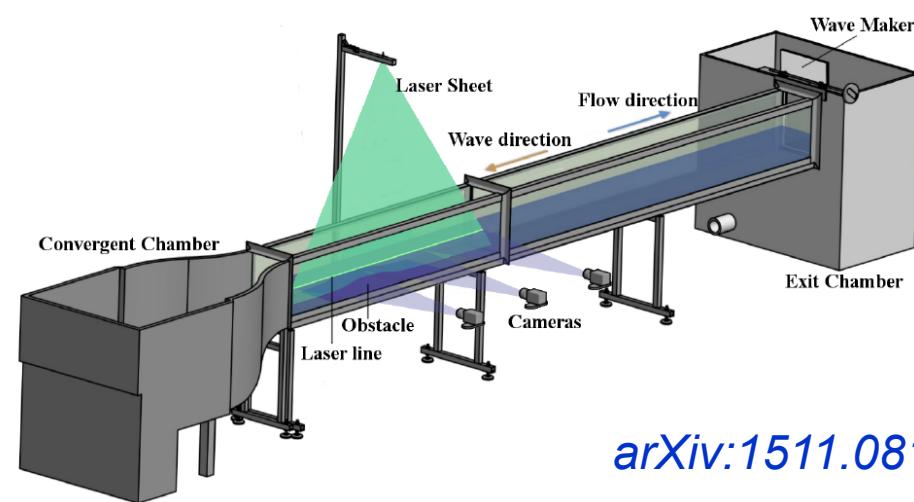
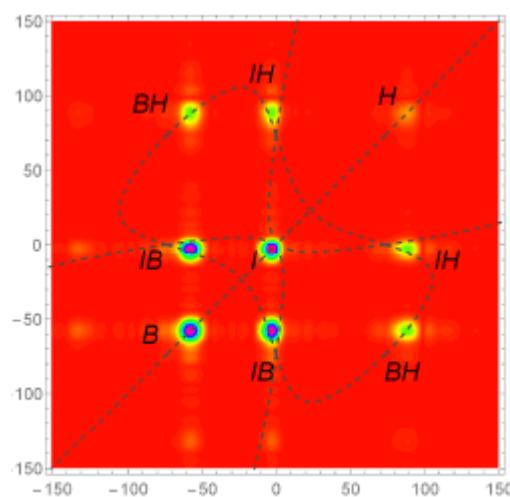
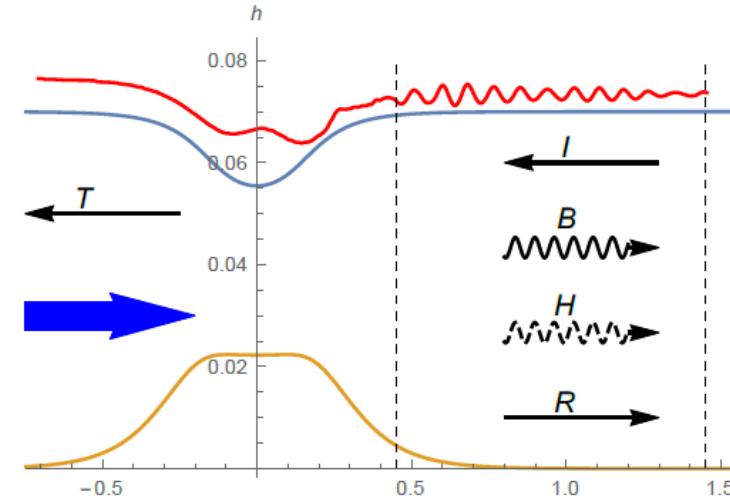
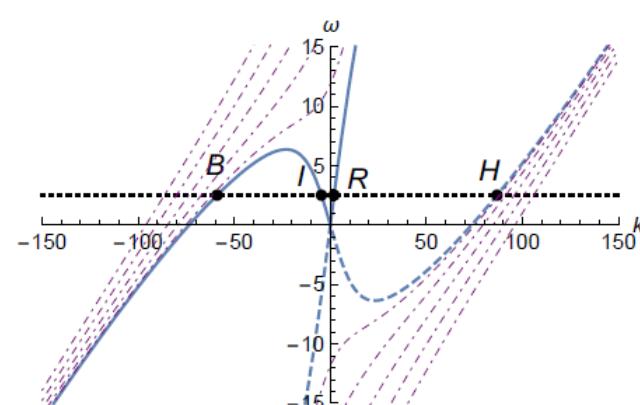
# Observation of noise correlated by the Hawking effect in a water tank

L.-P. Euvé,<sup>1</sup> F. Michel,<sup>2</sup> R. Parentani,<sup>2</sup> T. G. Philbin,<sup>3</sup> and G. Rousseaux<sup>1</sup>

<sup>1</sup>*Institut Pprime, UPR 3346, CNRS-Université de Poitiers-ISAE ENSMA 11 Boulevard Marie et Pierre Curie-Téléport 2, BP 30179, 86962 Futuroscope Cedex, France*

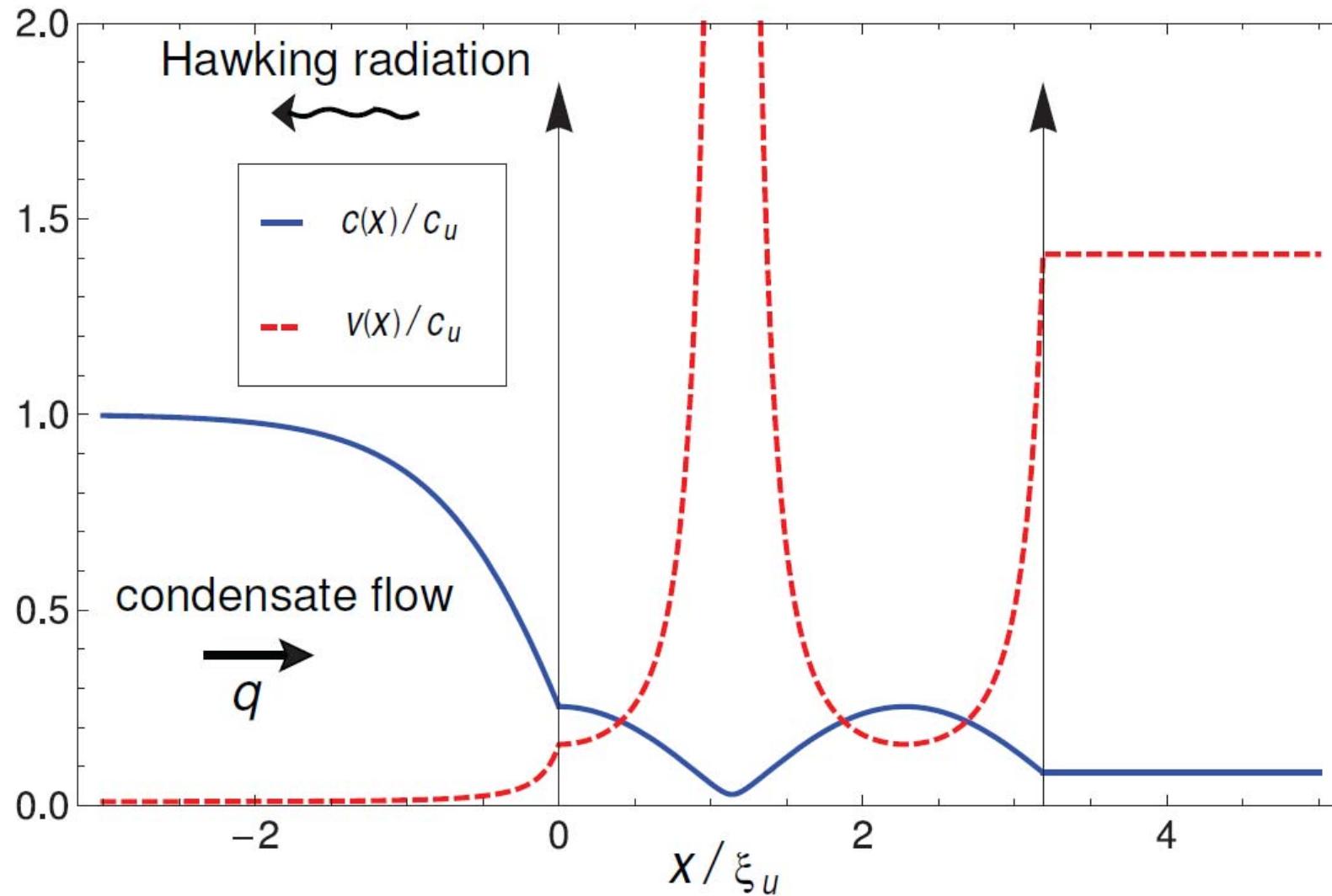
<sup>2</sup>*Laboratoire de Physique Théorique, CNRS, Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France*

<sup>3</sup>*Physics and Astronomy Department, University of Exeter, Stocker Road, Exeter EX4 4OL, UK*



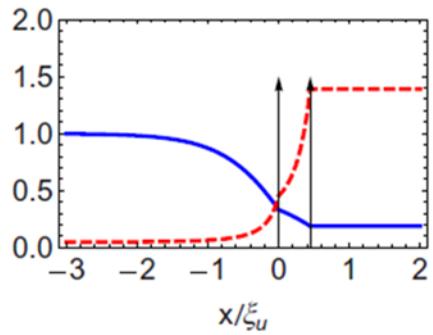
arXiv:1511.08145

## Resonant Hawking radiation



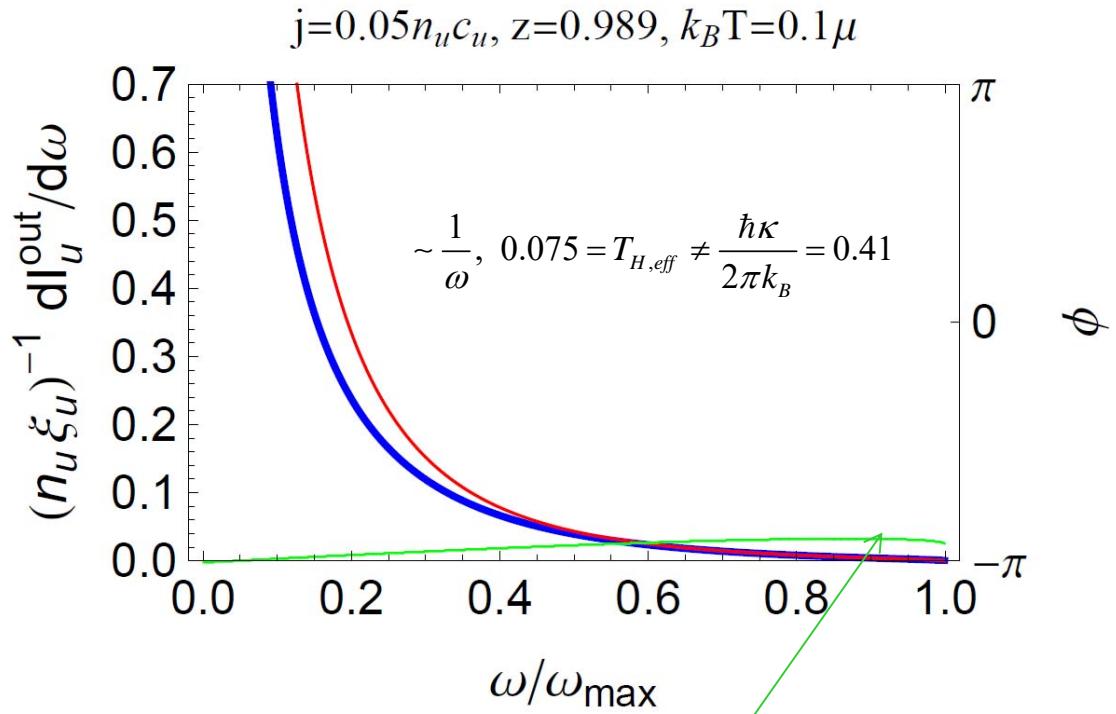
$$V(x) = z\hbar c_u [\delta(x) + \delta(x-d)]$$

# Resonant Hawking radiation: Hawking spectrum



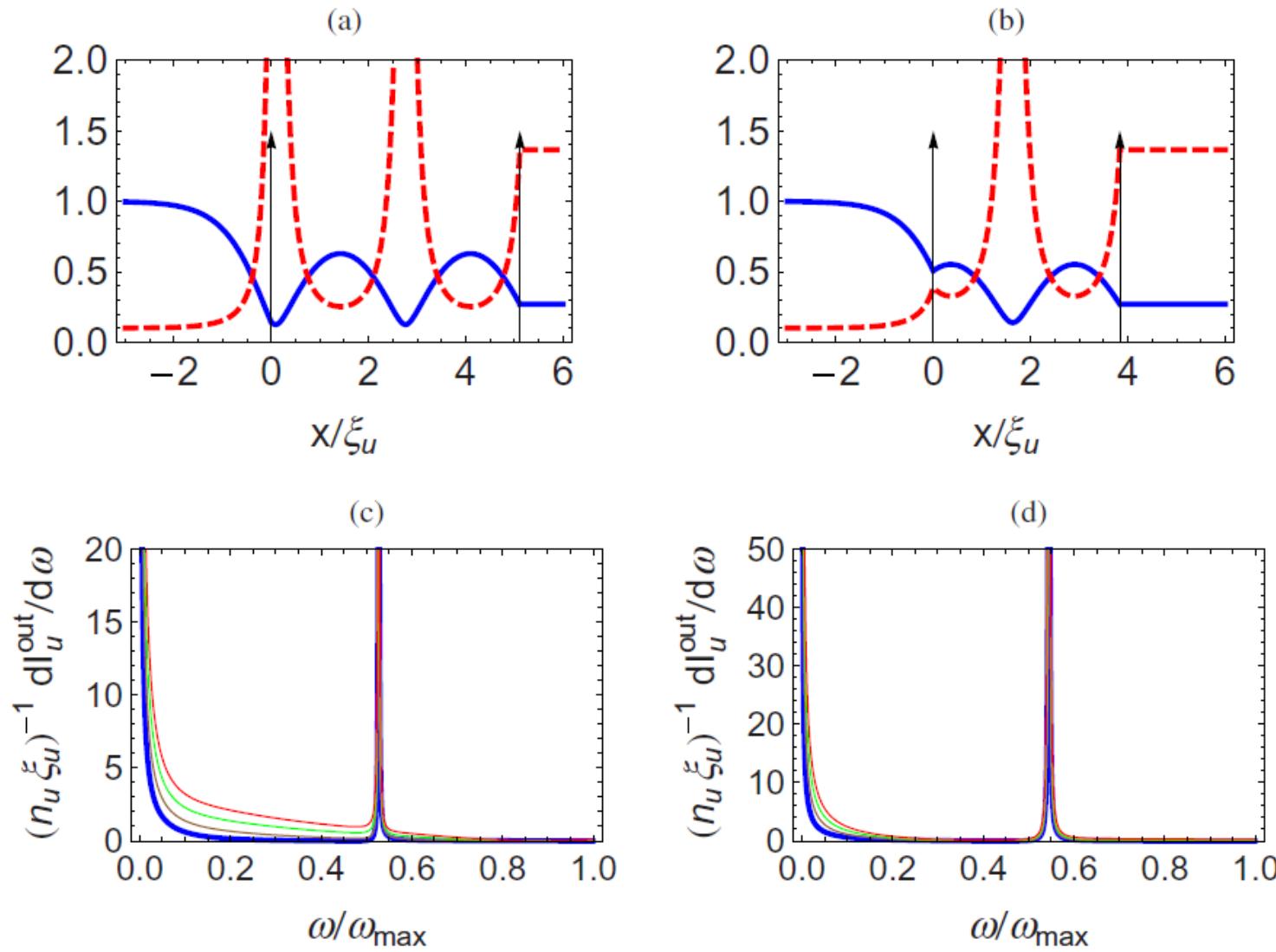
Upstream current:

Zero temperature (Hawking radiation)  
Finite temperature



Phase shift of determinant of S-matrix

# Resonant Hawking radiation: Spectrum



$$k_B T / \mu = 0, 0.3, 0.6, 0.9$$

## Bogoliubov vacuum: depletion cloud

mean field

$$\left(c_0^\dagger\right)^N |\text{vac}\rangle$$

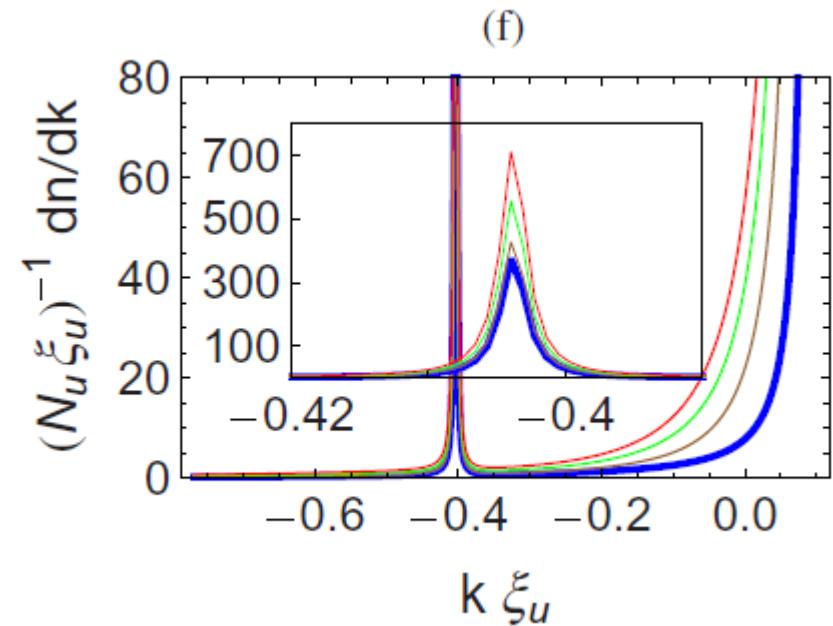
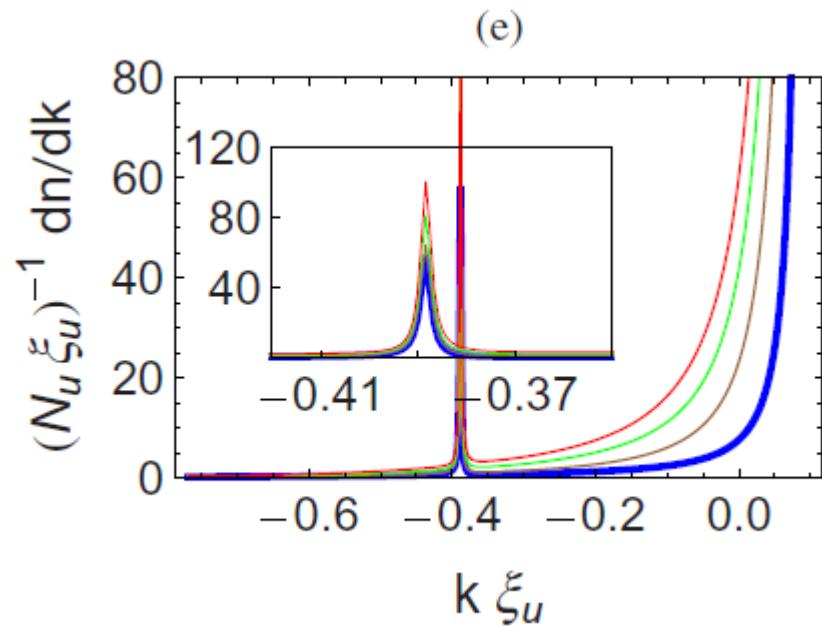
Bogoliubov vacuum

$$\left(c_0^\dagger c_0^\dagger + \sum_k \lambda_k c_k^\dagger c_{-k}^\dagger\right)^{N/2} |\text{vac}\rangle$$

due to interactions

e.g. A J Leggett – RMP (2001)

## Resonant Hawking radiation: Time of flight



I. Zapata, M. Albert, R. Parentani, FS  
New J. Phys. (2011)

# Classical vs Quantum coherence (I)

$$g^{(2)}(\tau) := \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t) \rangle}{\langle E^*(t)E(t) \rangle^2};$$

$$g_{a,b}^{(2)}(\tau) := \frac{\langle E_a^*(t)E_b^*(t+\tau)E_b(t+\tau)E_a(t) \rangle}{\langle E_a^*(t)E_a(t) \rangle \langle E_b^*(t+\tau)E_b(t+\tau) \rangle};$$

Classical inequalities:

$$g^{(2)}(\tau) \leq g^{(2)}(0);$$

$$1 \leq g^{(2)}(0);$$

$$\left[ g_{a,b}^{(2)}(\tau) \right]^2 \leq g_{a,a}^{(2)}(0)g_{b,b}^{(2)}(0)$$

Quantum violations:

$$g^{(2)}(\tau) > g^{(2)}(0)$$

⇒ Anti-bunching

$$1 > g^{(2)}(0)$$

⇒ Sub-Poissonian statistics

$$\left[ g_{a,b}^{(2)}(\tau) \right]^2 > g_{a,a}^{(2)}(0)g_{b,b}^{(2)}(0)$$

⇒ Cauchy-Schwarz inequality violation

$$\langle a^\dagger b^\dagger b a \rangle^2 > \langle a^\dagger a^\dagger a a \rangle \langle b^\dagger b^\dagger b b \rangle$$

(two-mode squeezed light)

## Classical vs Quantum coherence (II)

$$\text{if } \omega < \omega_{\max} \rightarrow \begin{pmatrix} \hat{\gamma}_u^{\text{out}} \\ \hat{\gamma}_{d1}^{\text{out}} \\ \hat{\gamma}_{d2}^{\dagger \text{out}} \end{pmatrix} = S(\omega) \begin{pmatrix} \hat{\gamma}_u^{\text{in}} \\ \hat{\gamma}_{d1}^{\text{in}} \\ \hat{\gamma}_{d2}^{\dagger \text{in}} \end{pmatrix}, \quad S(\omega)^\dagger \eta S(\omega) = \eta, \quad \eta := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\left\langle \hat{\gamma}_u^{\dagger \text{out}} \hat{\gamma}_{d2}^{\dagger \text{out}} \hat{\gamma}_{d2}^{\text{out}} \hat{\gamma}_u^{\text{out}} \right\rangle^2 \leq \left\langle \hat{\gamma}_u^{\dagger \text{out}} \hat{\gamma}_u^{\dagger \text{out}} \hat{\gamma}_u^{\text{out}} \hat{\gamma}_u^{\text{out}} \right\rangle \left\langle \hat{\gamma}_{d2}^{\dagger \text{out}} \hat{\gamma}_{d2}^{\dagger \text{out}} \hat{\gamma}_{d2}^{\text{out}} \hat{\gamma}_{d2}^{\text{out}} \right\rangle$$

$$T=0 \Rightarrow 4|S_{ud2}|^4 \left\{ |S_{d2d2}|^2 - \frac{1}{2} \right\}^2 \leq 4|S_{ud2}|^4 \left\{ |S_{d2d2}|^2 - 1 \right\}^2$$

Always violated because  $|S_{d2d2}|^2 \geq 1$  (unless  $|S_{ud2}|=0$ )

$T > 0$  & general case

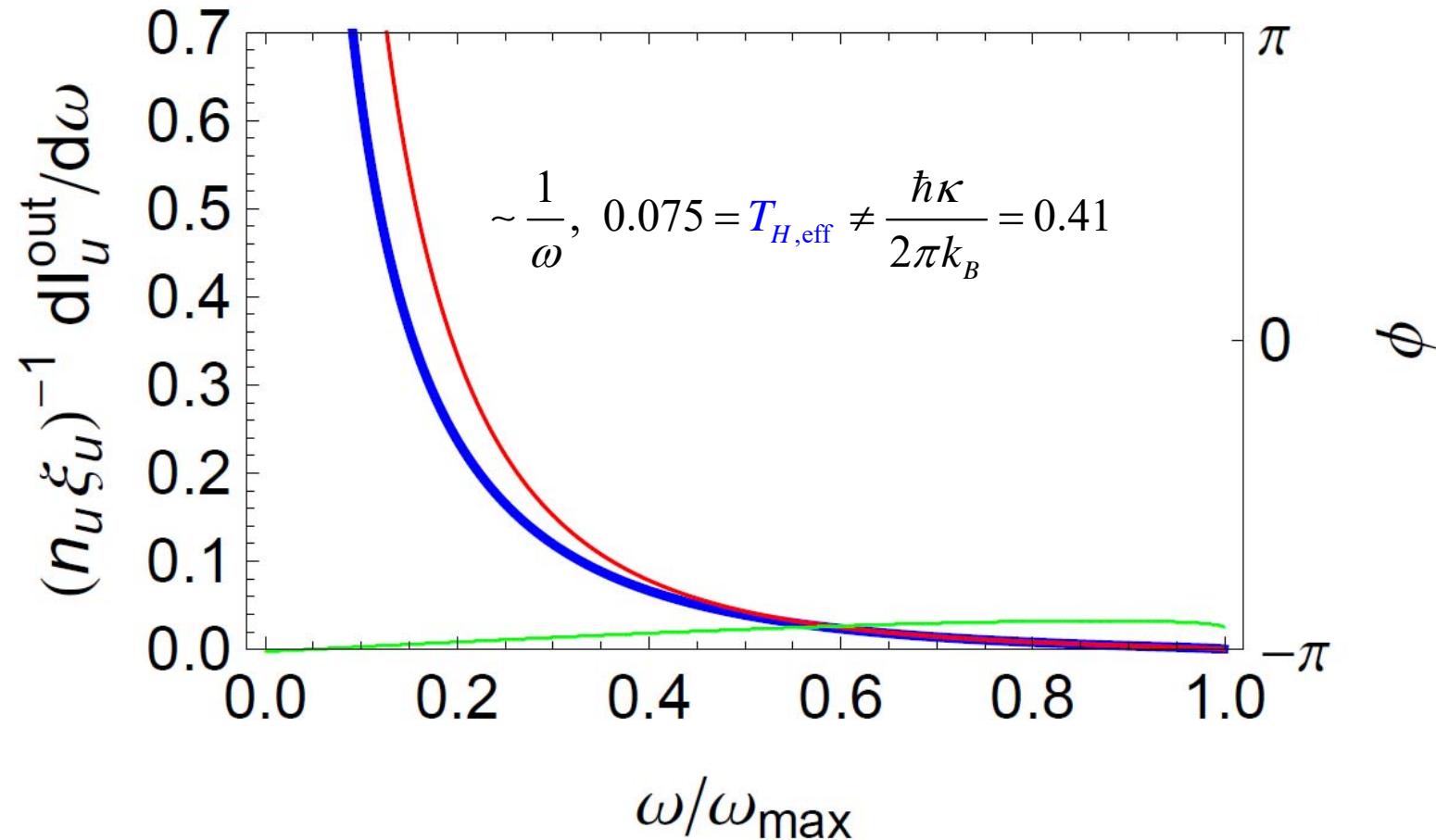
$$\Rightarrow |S_{ud2}|^2 (1 + n_u + n_{d1} + n_{d2}) \leq |S_{d1u}|^2 n_{d1} n_{d2} + |S_{d1d1}|^2 n_u n_{d2} + |S_{d1d2}|^2 n_u n_{d1} + |S_{d2d1}|^2 n_u + |S_{d2u}|^2 n_{d1}$$

$$n_j := \langle \hat{\gamma}_j^{\dagger \text{in}} \hat{\gamma}_j^{\text{in}} \rangle = \frac{1}{e^{\frac{\hbar \Omega_j}{k_B T}} - 1}$$

CS never violated at  $\omega=0$

## One barrier - Hawking spectrum (non-resonant)

$j=0.05n_u c_u, z=0.989, k_B T=0.1\mu$



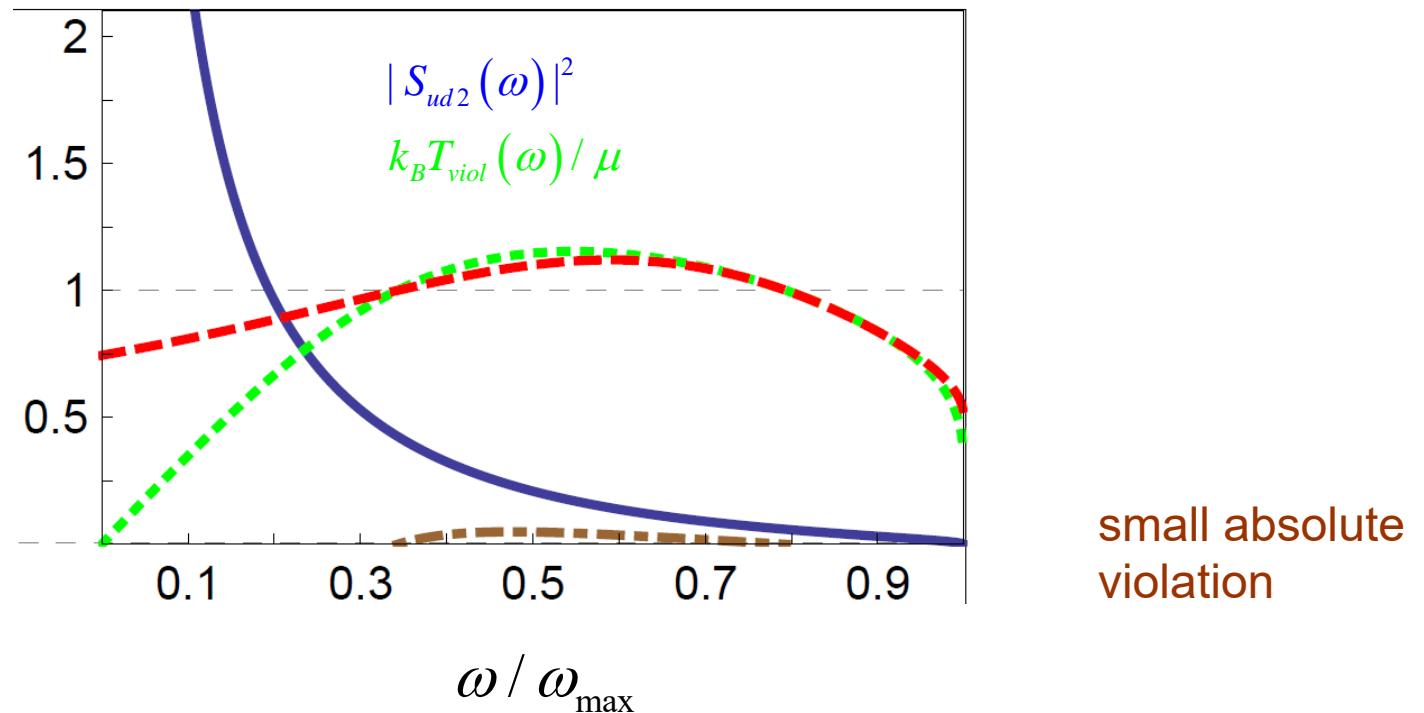
Upstream current:

Zero temperature (Hawking radiation,  $T_{H,eff}$ )

Finite temperature

Phase shift of the determinant of the S-matrix

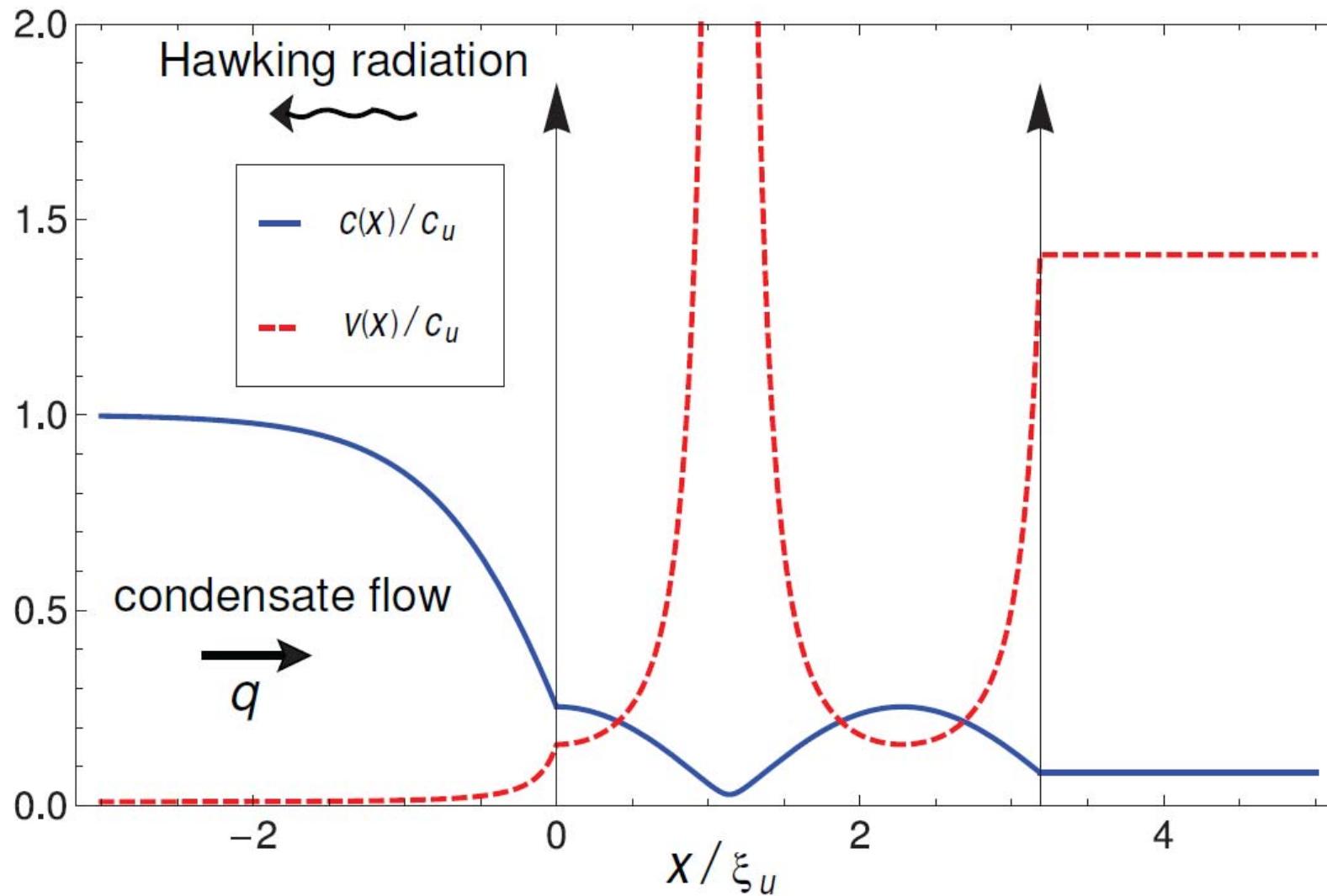
## One barrier - CS violation (non-resonant)



$$\frac{\left\langle \gamma_{d2,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}{\sqrt{\left\langle \gamma_{u,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{u,out} \right\rangle_{k_B T=\mu} \left\langle \gamma_{d2,out}^\dagger \gamma_{d2,out}^\dagger \gamma_{d2,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}} \\ \frac{\left\langle \gamma_{d2,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu} - \sqrt{\left\langle \gamma_{u,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{u,out} \right\rangle_{k_B T=\mu} \left\langle \gamma_{d2,out}^\dagger \gamma_{d2,out}^\dagger \gamma_{d2,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}}{\sqrt{\left\langle \gamma_{u,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{u,out} \right\rangle_{k_B T=\mu} \left\langle \gamma_{d2,out}^\dagger \gamma_{d2,out}^\dagger \gamma_{d2,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}}}$$

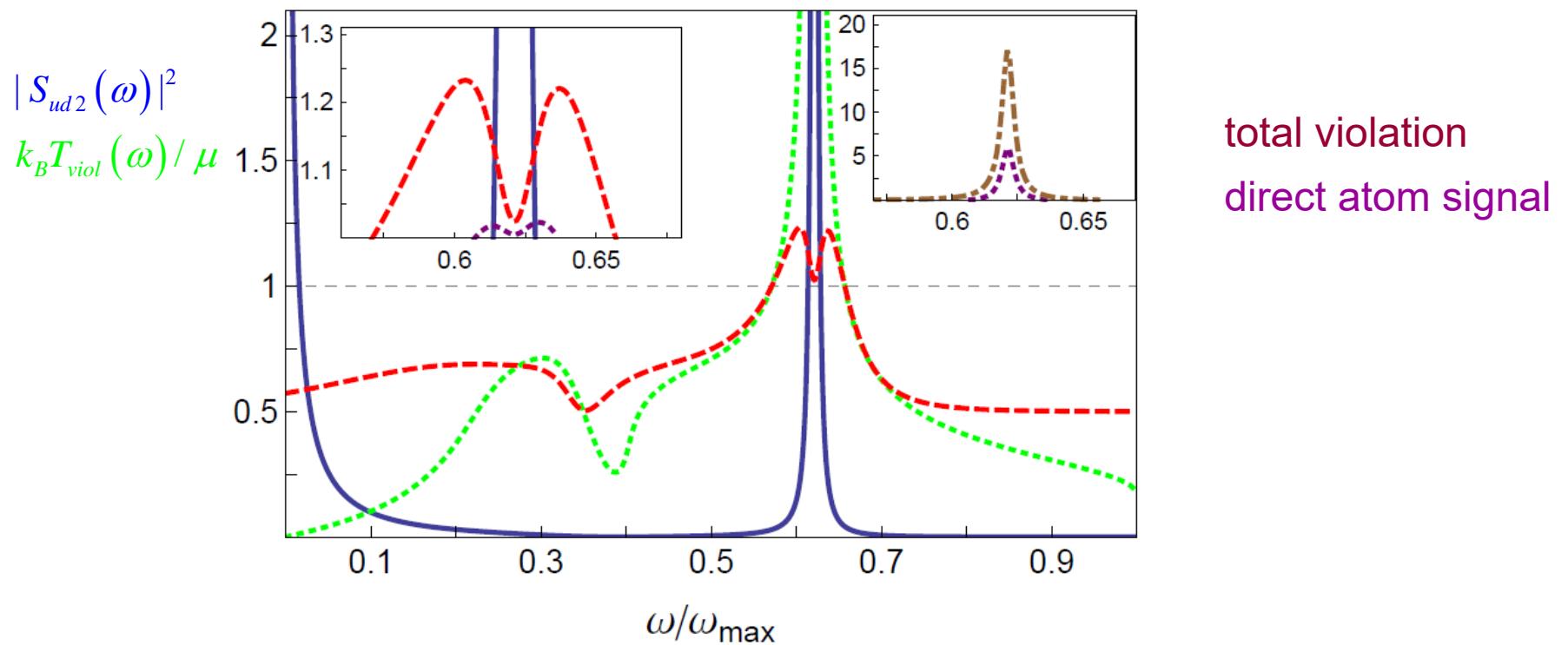
No CS violation near the conventional  $\omega \rightarrow 0$  peak

## Two barriers - Hawking spectrum (resonant)



double barrier:  $V(x) = z\hbar c_u [\delta(x) + \delta(x-d)]$

## Two barriers - CS violation (resonant)

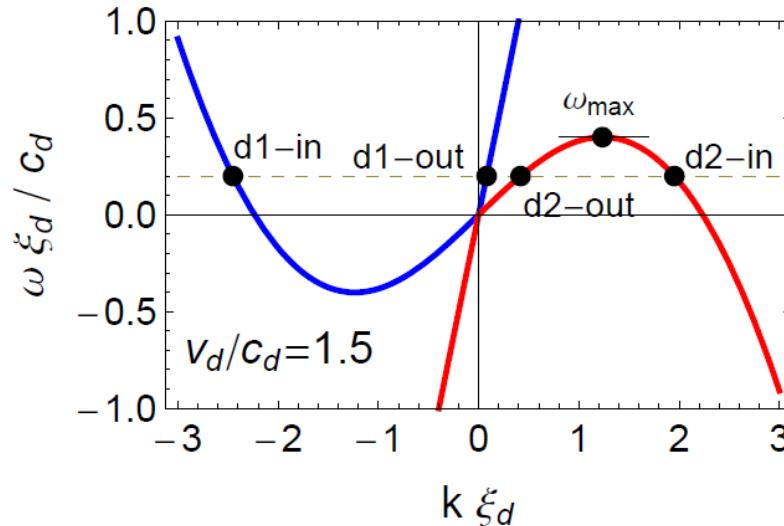


$$\frac{\left\langle \gamma_{d2,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}{\sqrt{\left\langle \gamma_{u,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{u,out} \right\rangle_{k_B T=\mu} \left\langle \gamma_{d2,out}^\dagger \gamma_{d2,out}^\dagger \gamma_{d2,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}} \\ \frac{\left\langle \gamma_{d2,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu} - \sqrt{\left\langle \gamma_{u,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{u,out} \right\rangle_{k_B T=\mu} \left\langle \gamma_{d2,out}^\dagger \gamma_{d2,out}^\dagger \gamma_{d2,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}}{\sqrt{\left\langle \gamma_{u,out}^\dagger \gamma_{u,out}^\dagger \gamma_{u,out} \gamma_{u,out} \right\rangle_{k_B T=\mu} \left\langle \gamma_{d2,out}^\dagger \gamma_{d2,out}^\dagger \gamma_{d2,out} \gamma_{d2,out} \right\rangle_{k_B T=\mu}}}$$

Large CS violation near resonant peaks

# Resonant Hawking radiation

## Time-of-flight CS violation by atoms



$$\hat{c}_{k_{d1}(\omega_{\text{peak}})} \sim \hat{\gamma}_{d1}^{\text{out}}(\omega_{\text{peak}})$$

$$\hat{c}_{k_{d2}(\omega_{\text{peak}})} \sim \hat{\gamma}_{d2}^{\text{out}}(\omega_{\text{peak}})$$

$$\hat{c}_{k_u(\omega_{\text{peak}})} \sim \hat{\gamma}_u^{\text{out}}(\omega_{\text{peak}})$$

ω-peak high  
particle-like

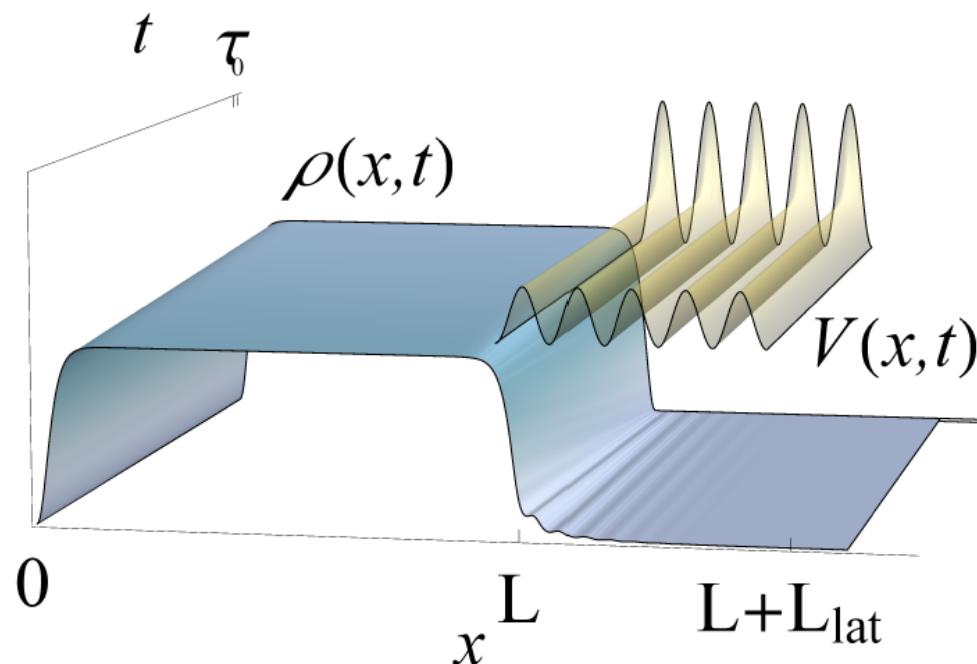
## Gradual outcoupling of a BEC

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \left[ -\frac{\hbar^2}{2m} \partial_x^2 + V(x,t) + g|\Psi(x,t)|^2 \right] \Psi(x,t)$$

$$V(x,t) = V(t) \cos^2 [k_L(x - L)]$$

$$V(t) = V_\infty + (V_0 - V_\infty)e^{-t/\tau}$$

Gradual lowering of the  
(initially) confining optical  
lattice

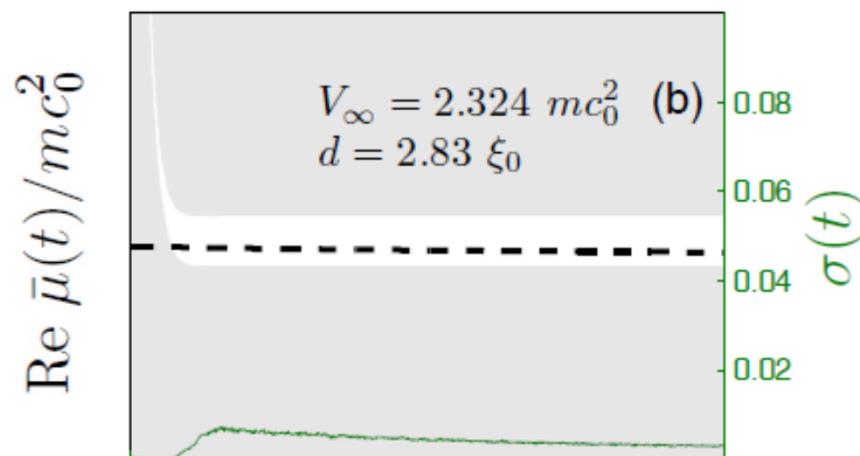


## measure of flatness $\sigma(t)$

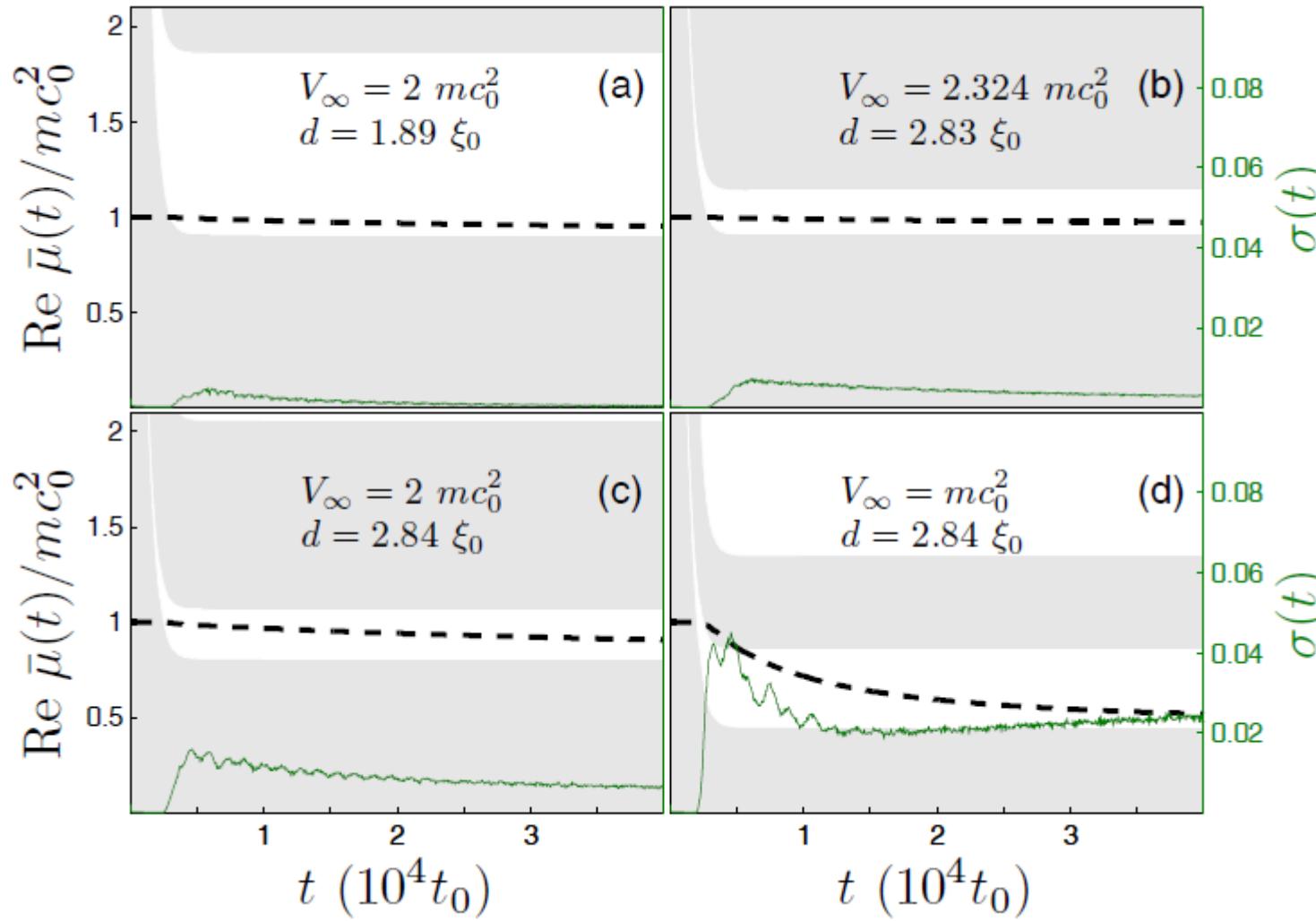
$$\mu(x, t) \equiv -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)/\partial x^2}{\Psi(x, t)} + V(x, t) + g|\Psi(x, t)|^2$$

$$\bar{\mu}(t) \equiv \frac{\int_0^{L_t} dx \rho(x, t) \mu(x, t)}{\int_0^{L_t} dx \rho(x, t)}$$

$$\sigma(t) \equiv \frac{1}{\bar{\mu}(t)} \left[ \frac{\int_0^{L_t} dx \rho(x, t) |\mu(x, t) - \bar{\mu}(t)|^2}{\int_0^{L_t} dx \rho(x, t)} \right]^{\frac{1}{2}}$$

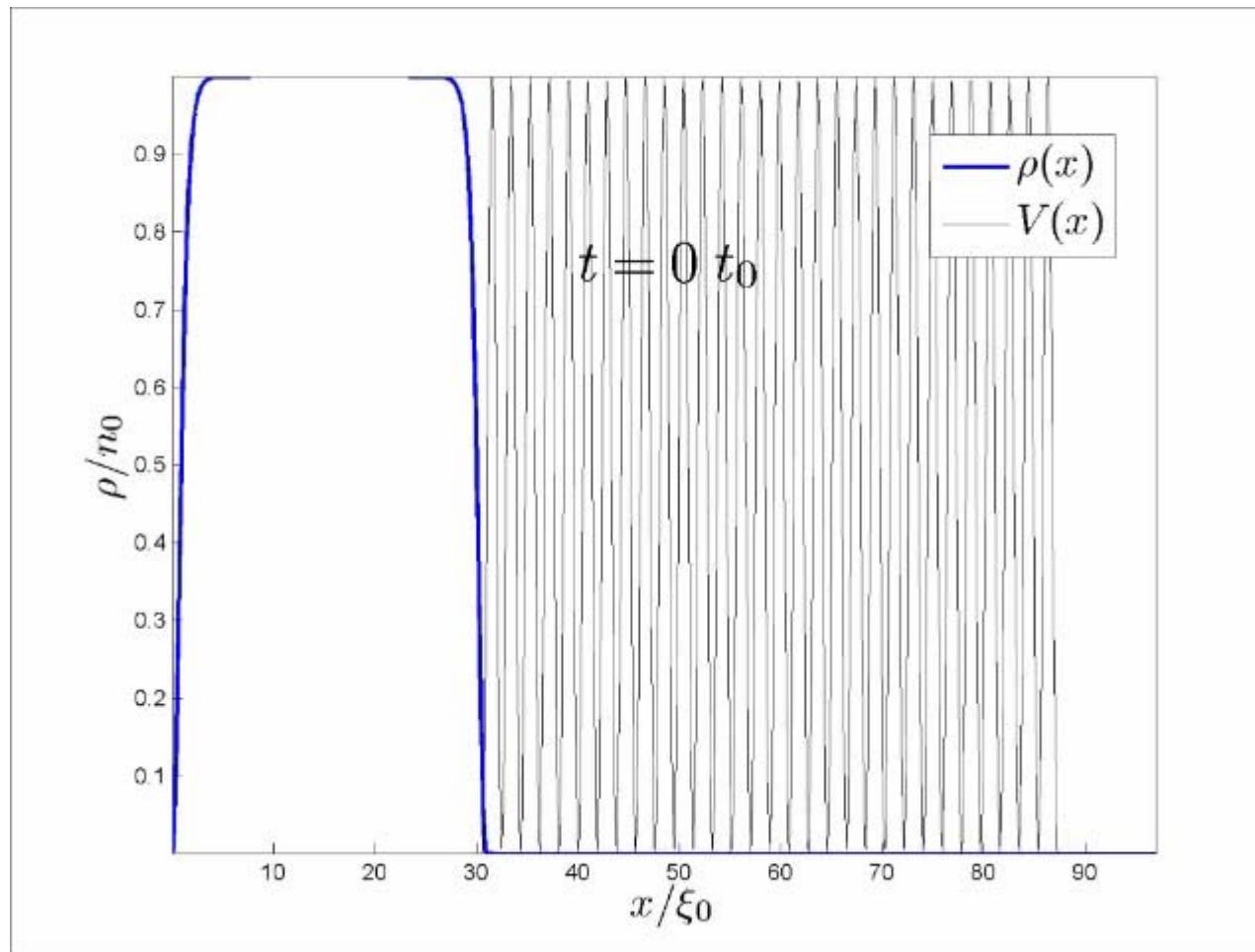


## Evolution of the condensate



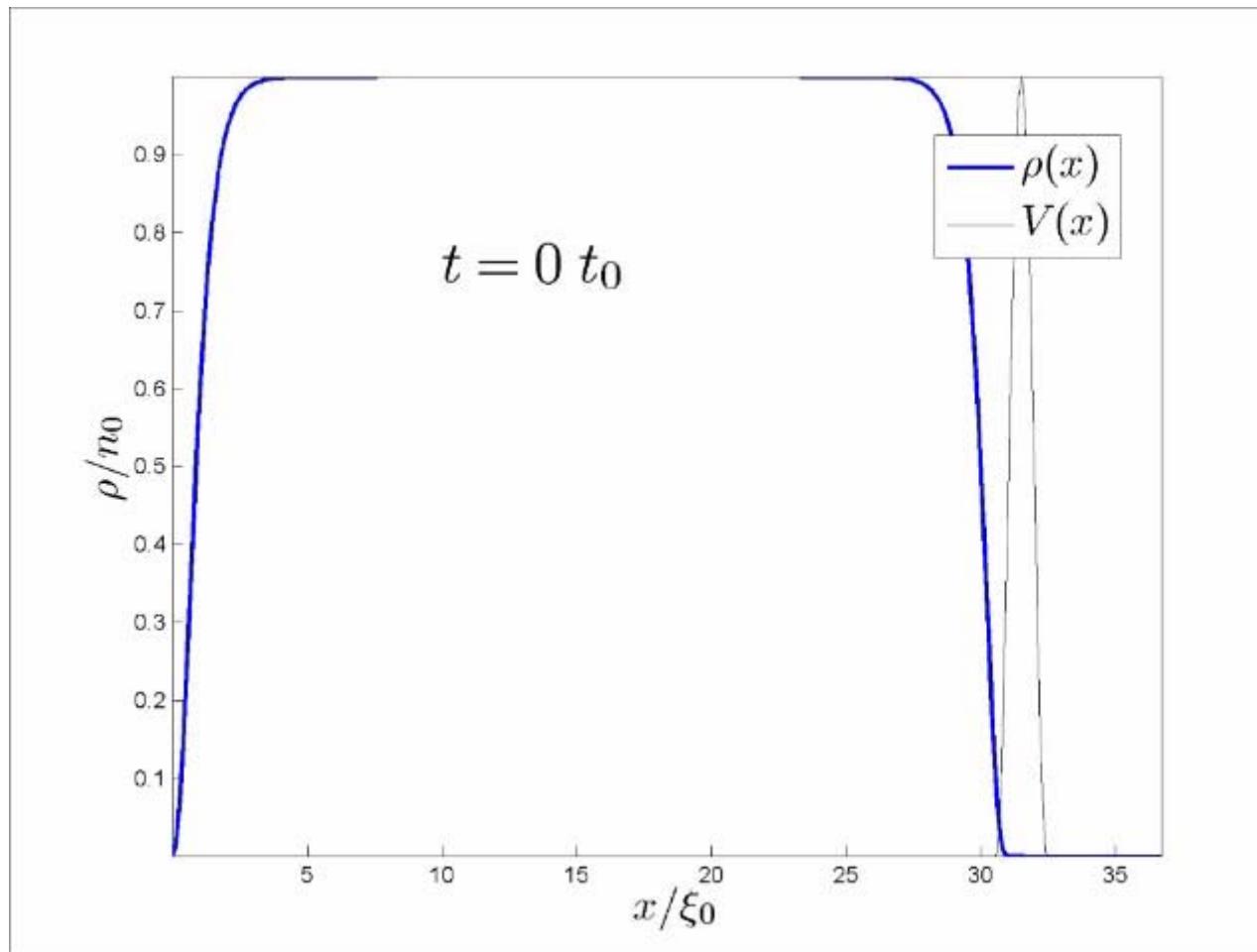
initial  
chemical  
potential  
vs  
final  
conduction  
band

Time for a movie!



flat optical lattice - 30 barriers

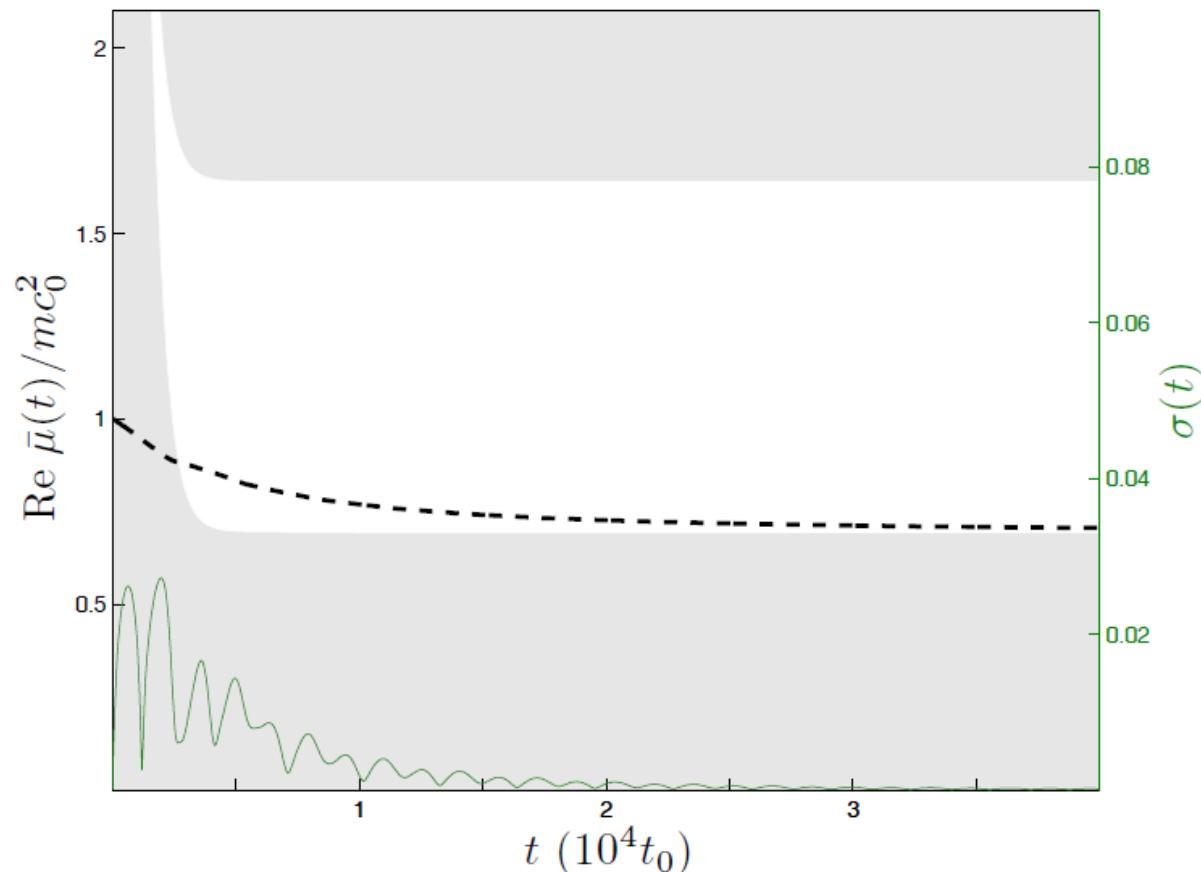
one more movie...



single barrier

## Gaussian-shaped (realistic) optical lattice

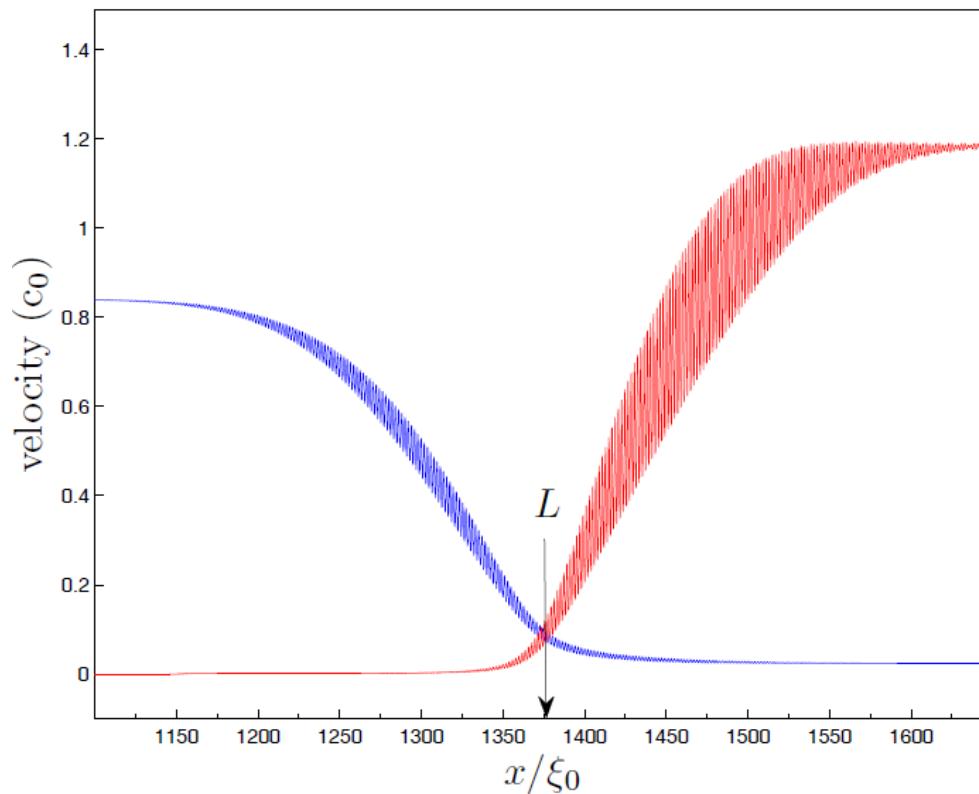
$$V(x, t) = V(t) \cos^2 [k_L(x - L)] \exp \left[ -2 \left( \frac{x - L}{\tilde{w}} \right)^2 \right]$$



similar  
behavior  
as for  
flat lattice

## Gaussian – local velocities

$$V(x, t) = V(t) \cos^2 [k_L(x - L)] \exp \left[ -2 \left( \frac{x - L}{\tilde{w}} \right)^2 \right]$$



## Event horizon at the center of the Gaussian envelope (simplified proof, $m$ cnst.)

$$\mu \simeq \bar{V}(x) + \frac{1}{2}m\bar{v}^2(x) + m\bar{c}^2(x)$$

*potential + kinetic + interaction = constant*

$\bar{c}^2(x)\bar{v}(x)$  constant because of uniform current (stationary regime)

$\frac{1}{2}m\bar{v}^2(x) + m\bar{c}^2(x)$  constant at  $V$  maximum ( $x=L$ )

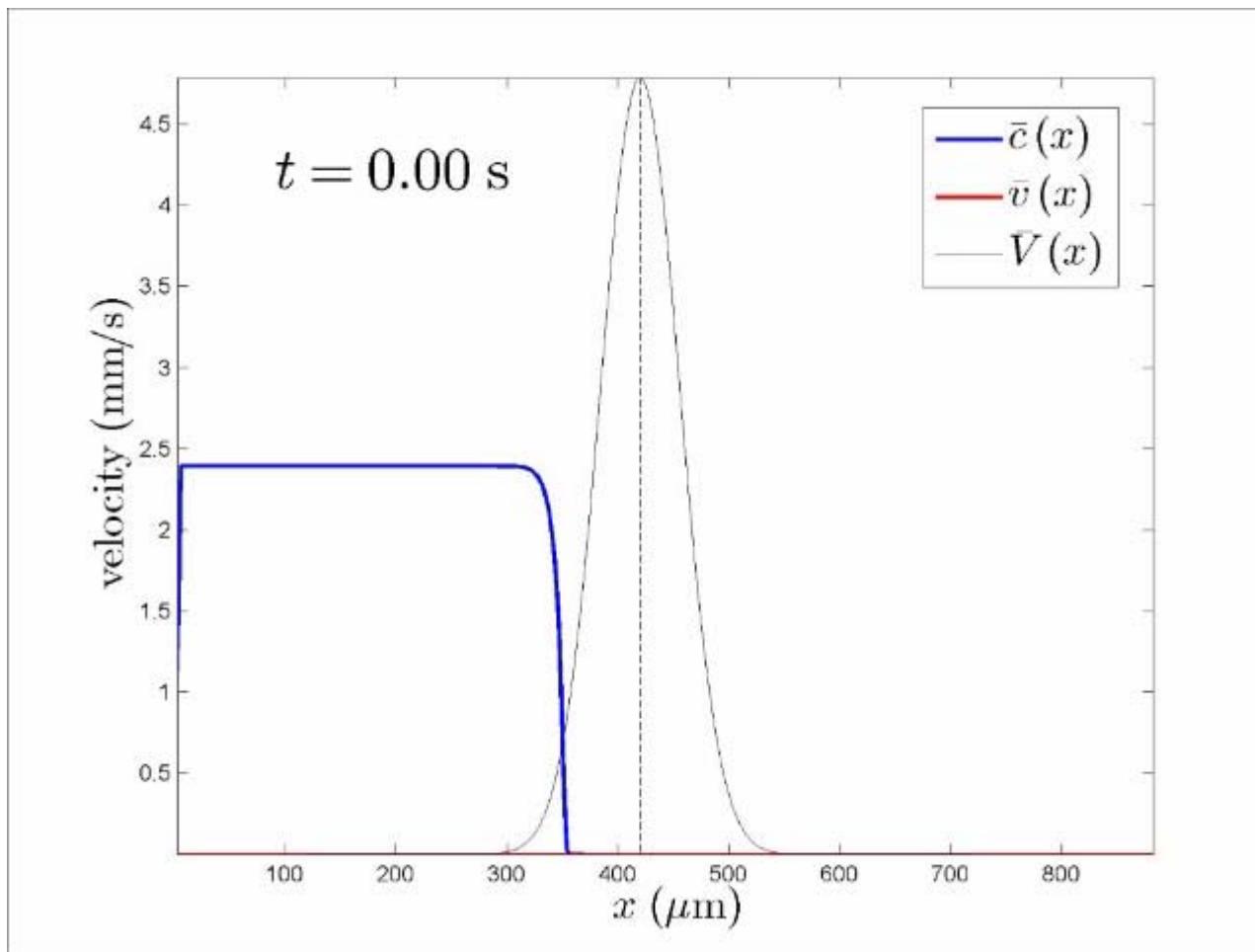
$$\bar{v}'\bar{c} + 2\bar{c}'\bar{v} = 0$$



$$\bar{v}\bar{v}' + 2\bar{c}\bar{c}' = 0$$

$$\boxed{\bar{c}(L) = \bar{v}(L)}$$

## The movie



Gaussian-shaped (realistic) optical lattice

sonic horizon right at the center of  
the Gaussian lattice!  
(can be explained theoretically)

## CONCLUSIONS – Hawking radiation

- Highly non-thermal Hawking (zero-point) radiation is generated on the subsonic (superfluid) side of a condensate leaking through a double barrier interface
- Many of the interesting solutions are stable, i.e. true resonances
- Cauchy-Schwarz violation in the u-d2 channel occurs if and only if spontaneous (zero-point) Hawking radiation is present.
- It can be observed near the peak in the Hawking spectrum of a resonant double-barrier structure.
- In some setups, CS violation can be directly observed in the atom signal of a time-of-flight experiment.
- The decisive CS violation cannot occur near the conventional, zero-frequency peak universally shown by one-dimensional sonic black-hole structures.

## CONCLUSIONS – birth of a sonic black hole

- Lowering of an extended optical lattice favors the emergence of well controlled quasi-stationary subsonic -> supersonic condensate flow.
- Lowering of a Gaussian-envelope (realistic) optical lattice is likely to give birth to a robust quasi-stationary sonic black hole.
- This scenario offers hopes for the observation of spontaneous Hawking radiation.



J. R. Muñoz de Nova  
UCM - Haifa



Ivar Zapata  
UCM



Mathias Albert  
Univ. Geneva - Nice



David Guery-Odelin  
Univ. Toulouse



Renaud Parentani  
Univ. Paris-Sud, Orsay

- END