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HEALPIX EXERCISES

1. **Aperture photometry from a Healpix map**. Write a code to extract the flux of a compact (point-like) source from a Healpix map. Apply the code to compute the flux of the CRAB nebulae at 30GHz using the WMAP maps. Compare the result with that obtained with the Planck 30GHz map.

2. **Two point correlation function.**
   a. Starting from the harmonic decomposition of the temperature map ($\Delta T(n)=\Sigma\, a_{lm}\, Y_{lm}(n)$), derive the analytic equation for the two-point correlation function, $C(\theta)$, defined as $C(\theta) = \langle \Delta T(n_1)\, \Delta T(n_2) \rangle$, where θ is the angle defined by the two directions n_1 and n_2. Hint: you will need to use the Spherical Harmonic Addition Theorem. The final solution should be:

   $$C(\theta) = \sum_{\ell} \frac{2\ell+1}{4\pi} C_\ell P_\ell(cos\theta)$$

   b. Plot this function $C(\theta)$ for two different cosmological models generated with CAMB. Model1 and model2 should have identical cosmological parameters except for the baryon density: Ω_b(model 1)=0.04 and Ω_b(model 2)=0.08. What angular scales are different between those two plots?

3. **Cosmic Variance.**
   a. Use Monte Carlo simulations (nsim=1000) to demonstrate that the cosmic variance is given by:

   $$Var(C_\ell) = \frac{2}{2\ell+1} C_\ell^2$$

   b. Impact of white (uncorrelated) noise. Add white noise to the simulations and estimate the contribution to the variance. Demonstrate analytically that the power spectrum of a map of white noise is given by $C_l = 1/w$, with $w^{-1}=\sigma^2\Omega_{pix}$.

4. **Pseudo power spectra and the effect of a mask.** Incomplete sky coverage affects our estimation of the angular power spectrum of a given map. In the incomplete sky, the spherical harmonics are no longer a orthonormal base function, and the inversion of the temperature map into a_{lm} produces an artificial coupling of modes.
   a. Demonstrate the effect of a mask on the power spectrum estimation by using a CMB simulation and a galactic mask excluding the region |b|<10º.
   b. Demonstrate that, at first order, the power spectrum can be corrected by the approximate factor 1/fsky, where fsky is the fraction of the total sky observed. Show that this approximation fails for more restrictive masks.
   c. The effect of the mask can be written in terms of a mixing matrix, as shown below. Use simulations to compute this mixing matrix numerically.

$$\tilde{a}_{lm} = \int W(\hat{\mathbf{n}}) x(\hat{\mathbf{n}}) Y^*_{lm}(\hat{\mathbf{n}}) d\Omega_n \qquad \tilde{C}_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |\tilde{a}_{lm}|^2$$

$$\langle \tilde{C}_l \rangle = \sum_{l'} M_{ll'} C_{l'}$$

   d. MASTER code. Hivon et al. (2002) provide analytical expressions for those mixing matrices. Compare the analytical result with the simulations.

$$M_{l_1 l_2} = \frac{2l_2+1}{4\pi} \sum_{l_3} (2l_3+1) \mathcal{W}_{l_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}^2$$

5. **Aberration.** Simulate the effect of the aberration due to the movement of the observer in a CMB map with Healpix. Hint: the basic equations can be found in Chluba (2011), MNRAS, 415, 3227 (see eq.3 in the paper).