5-brane webs and 5d N=1 rank 2 theories

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(Tokai University)

Based on the collaboration with
Sung-Soo Kim (UESTC), Kimyeong Lee (KIAS), Futoshi Yagi (Technion)

[arXiv: 1801.03916] and [arXiv:1803.XXXXX]

6th of March at Physics and Geometry of F-theory 2018 in IFT

- The main focus of this talk is five-dimensional gauge theories with eight supercharges.

- 5-brane web diagrams in type IIB string theory provides a powerful tool to study the five-dimensional gauge theories.

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Aharony, Hanany, Kol 97

- For example, non-perturbative effects and various dualities of five-dimensional gauge theories may be explicitly seen from the construction using 5-brane webs.

- 5-brane web diagrams realize SU(N) gauge theories. Namely, N parallel D5-branes may realize an SU(N) gauge group.

- When we introduce an orientifold 5-plane, we may also realize a USp(2N) or an SO(N) gauge group.

Brunner, Karch 97,
Brandhuber, Itzhaki, Sonnenschein, Theisen, Yankielowizc 97
Zafrir 15

- Then a natural question is:

"Can we construct exceptional gauge groups from 5-brane webs?"

- In this talk, we propose two types of 5-brane web configurations which realize 5d pure G_2 gauge theory.

- Surprisingly, a standard 5-brane web diagram with an O5- or $\widetilde{\text{O5}}$-plane gives rise to the pure G_2 gauge theory.

- The 5-brane web realization can be applied in various ways and we will focus on two applications.

- Recently, a duality involving a G_2 gauge theory has been proposed from a geometric construction.

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Pure G_2 $\leftrightarrow$ SU(3) with Chern-Simons level 7

- The duality may be also seen from the 5-brane web construction.

- We can also calculate the Nekrasov partition function of the pure G_2 gauge theory.

1. Introduction
2. 5d gauge theories from 5-brane webs
3. A 5-brane web for pure G_2 gauge theory
4. Another realization of G_2
5. Conclusion

2. 5d gauge theories from 5-brane webs

- We construct a 5d supersymmetric field theory with eight supercharges as the worldvolume theory on a 5-brane web.

- The 5-brane configuration in type IIB string theory.

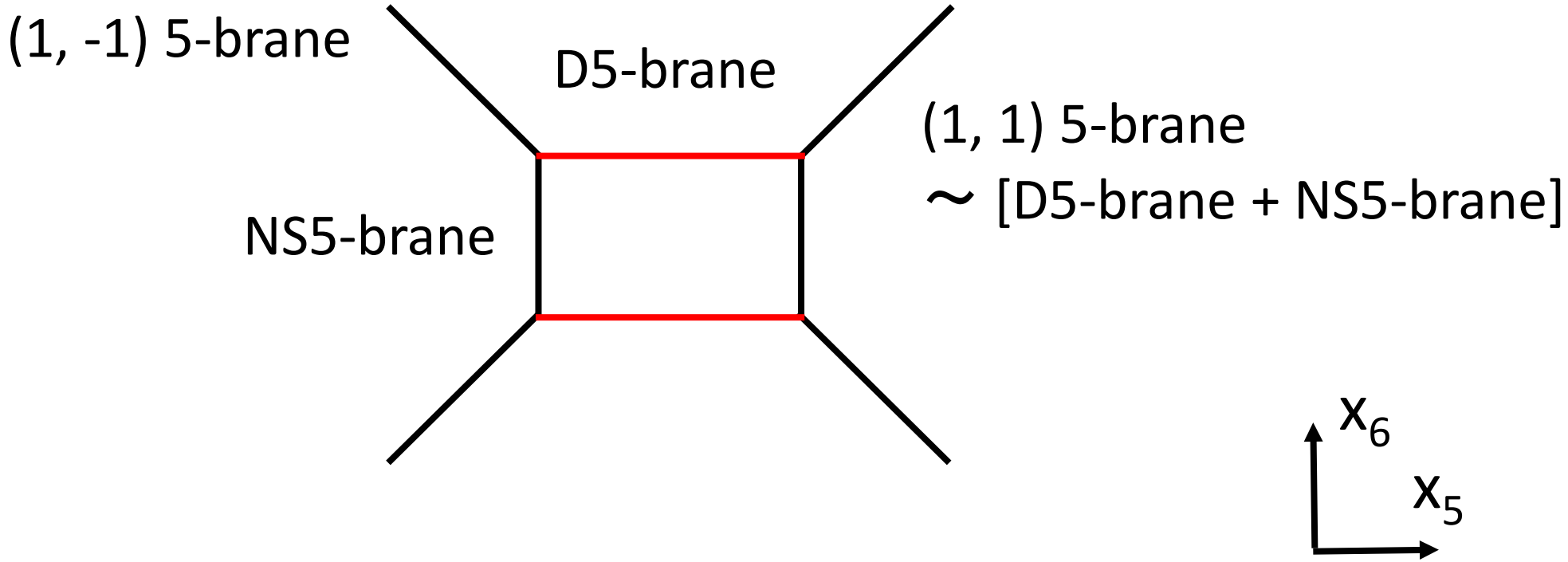
	0	1	2	3	4	5	6	7	8	9
D5-brane	×	×	×	×	×	×				
O5-plane	×	×	×	×	×	×				
NS5-brane	×	×	×	×	×		×			
(p, q) 5-brane	×	×	×	×	×	angle				

5-brane web

Aharony, Hanany 97
Aharony, Hanany, Kol 97

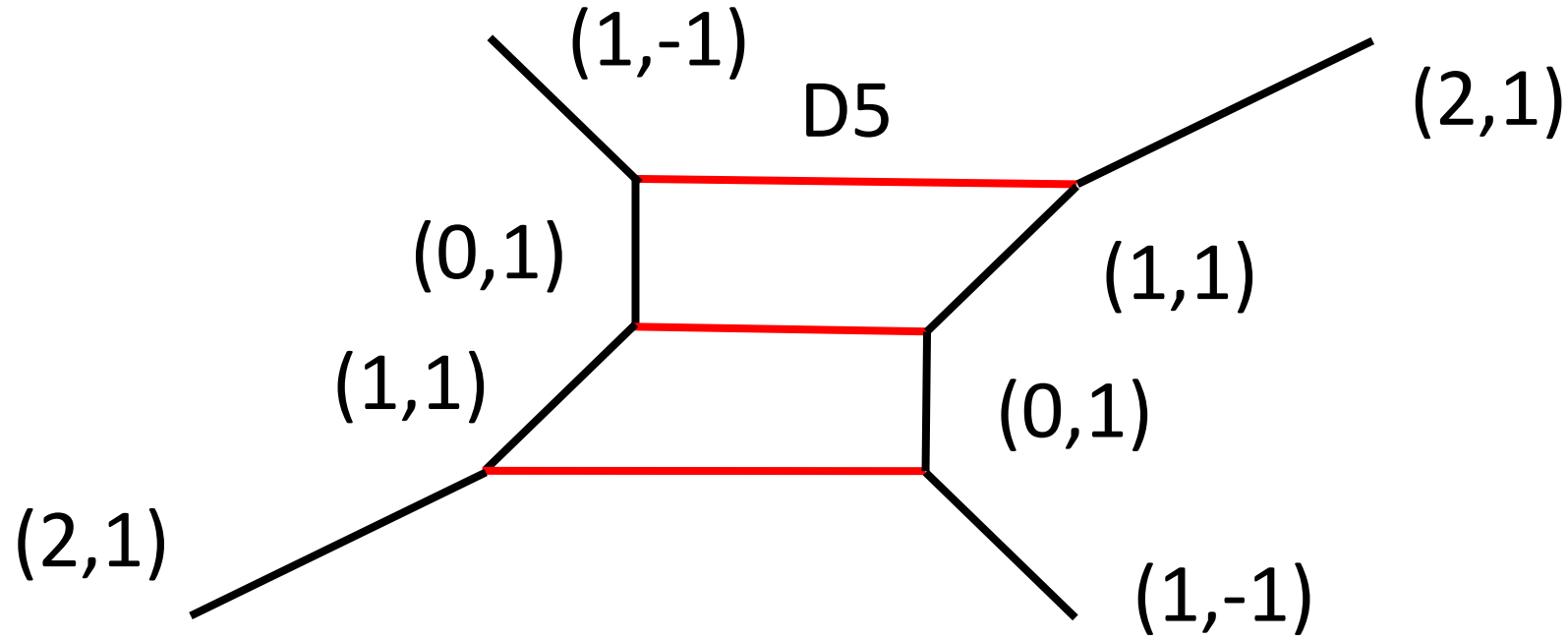
- 5d pure SU(2) gauge theory with $\theta = 0$:

Aharony, Hanany 97
Aharony, Hanany, Kol 97



- A (p, q) 5-brane is a line with slope q/p.

- The web diagram for the SU(3) gauge theory with the zero Chern-Simons level:



- Introducing an orientifold 5-plane may yield a USp(2N) or an SO(N) gauge group.

  - N D5-branes on an $O5^+$- or $\widetilde{O5}^+$-plane: USp(2N)
  - N D5-branes on an $O5^-$-plane: SO(2N)
  - N D5-branes on an $\widetilde{O5}^-$-plane: SO(2N+1)

Brunner, Karch 97,
Brandhuber, Itzhaki, Sonnenschein, Theisen, Yankielowizc 97
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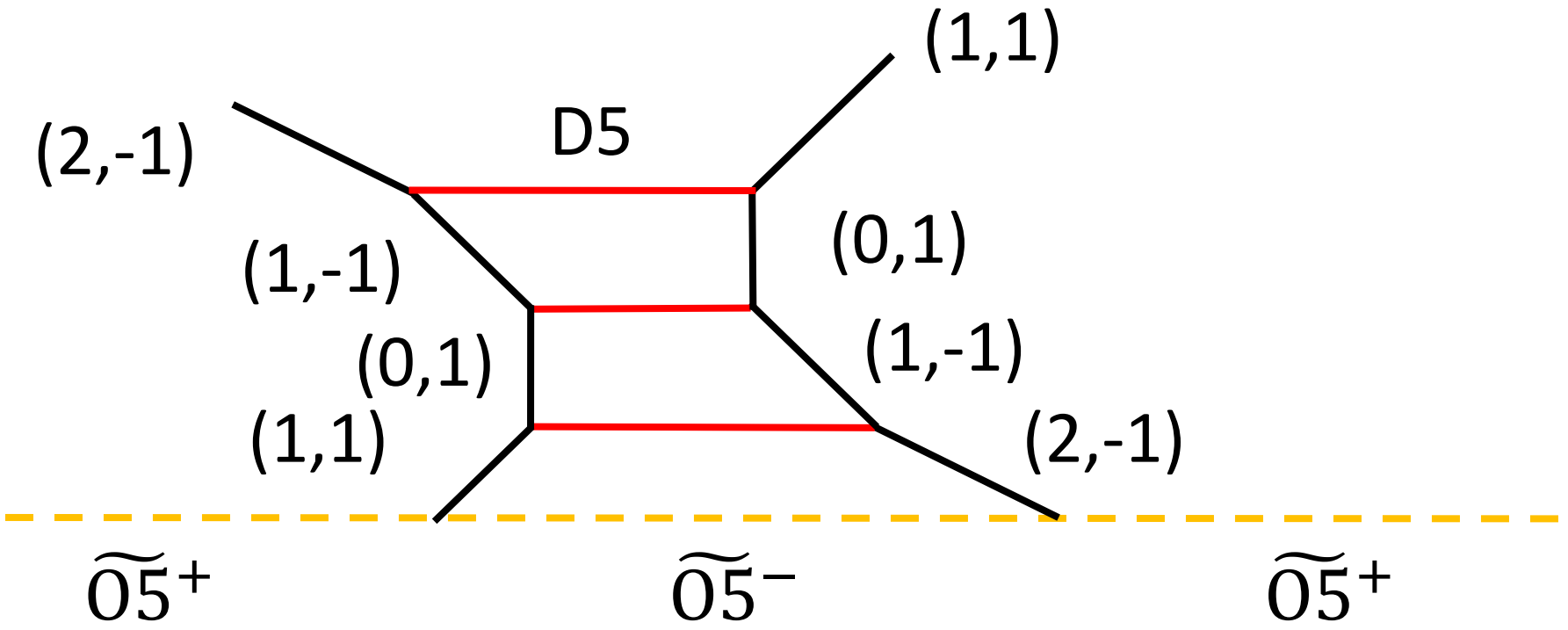
- The pure SO(7) gauge theory:



3. A 5-brane web for pure G_2 gauge theory

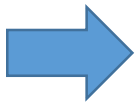
- We then consider a construction of a 5-brane web diagram for the pure G_2 gauge theory.

- An important fact about G_2 gauge theories is that the pure G_2 gauge theory may be obtained by Higgsing the SO(7) gauge theory with a hypermultiplet in the spinor representation.

- Hence, we will take the following strategy:

1. We construct a 5-brane web for the SO(7) gauge theory with a spinor.

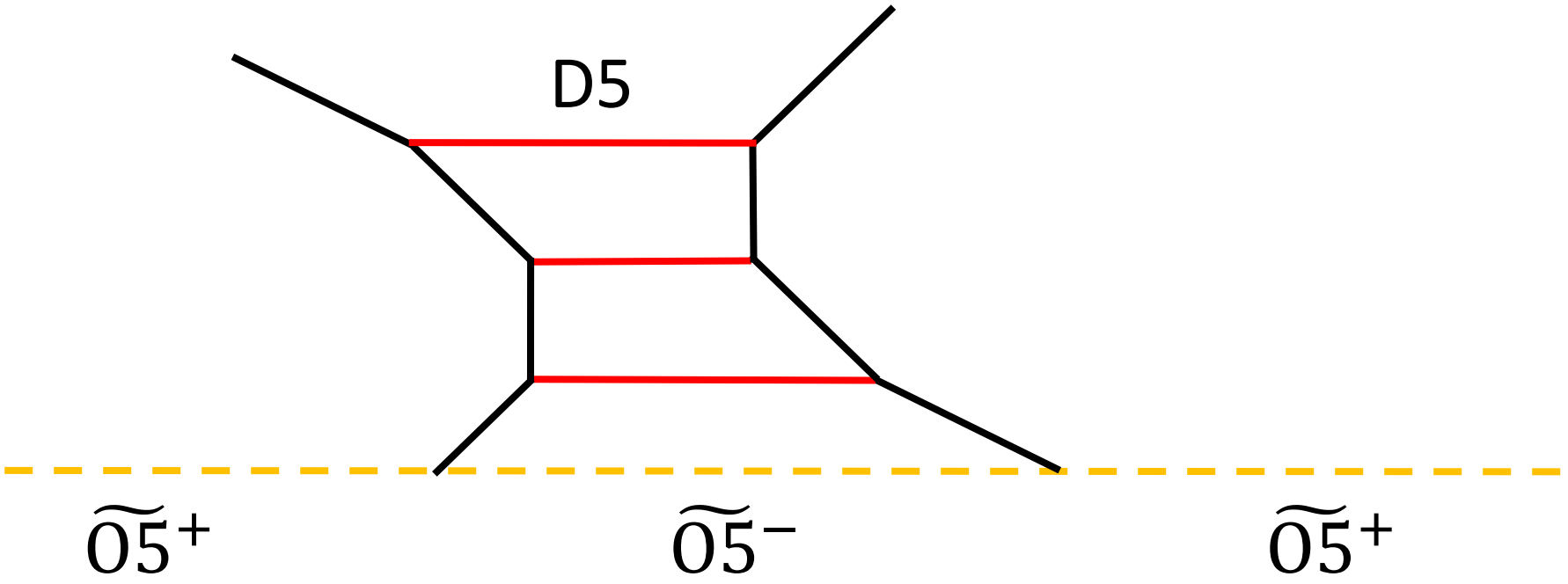
2. We further Higgs the 5-brane web.



a 5-brane web for the pure G_2 gauge theory

1. Incorporating spinor matter :

- The pure SO(7) gauge theory was given by:



- 5d SO(7) gauge theory with a spinor:

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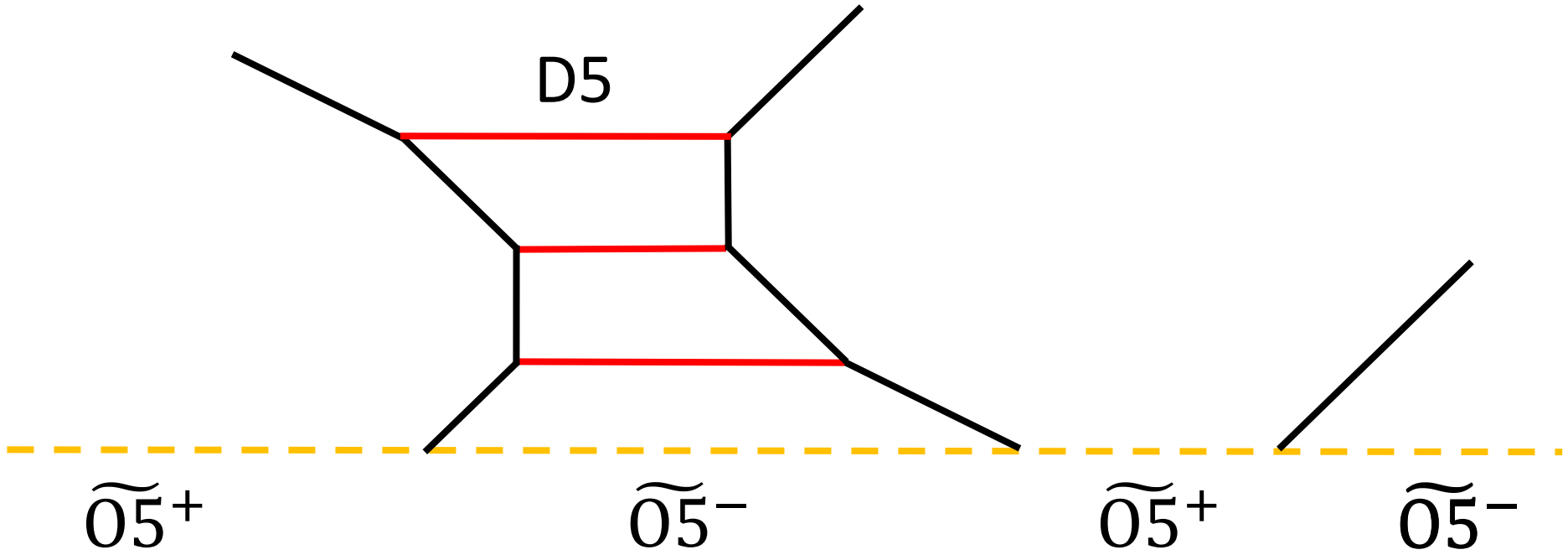

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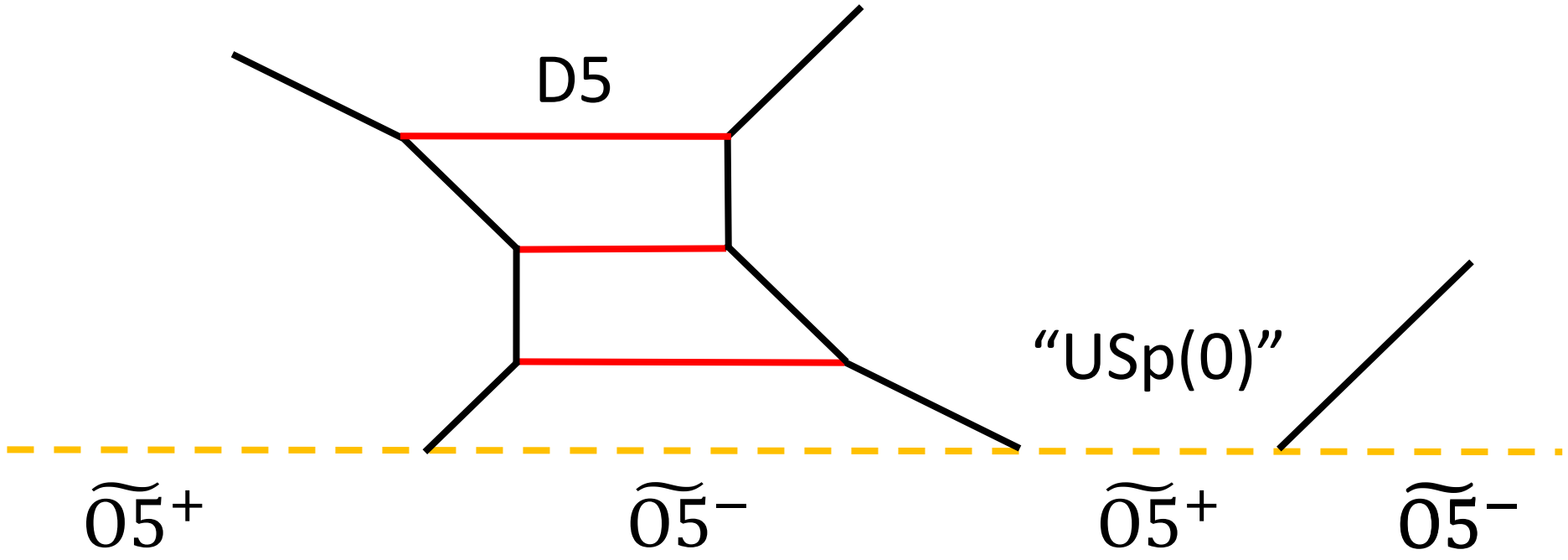
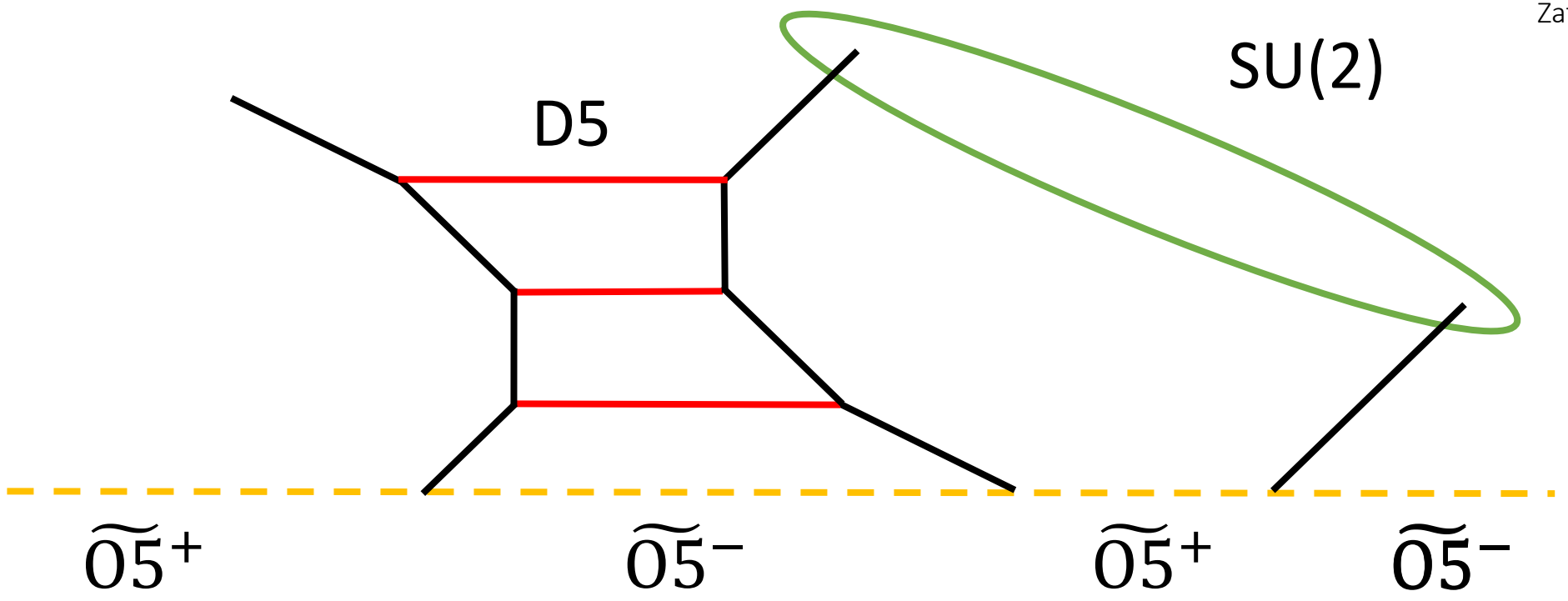
Zafrir 15



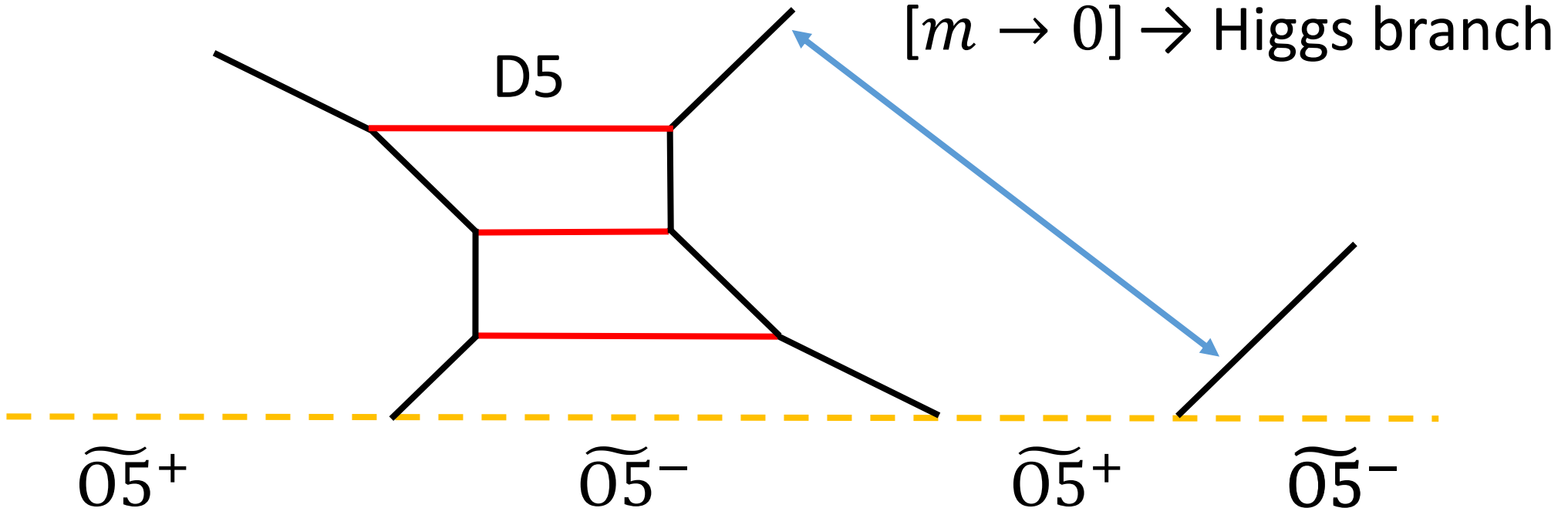
"USp(0) instanton" = spinor matter

- 5d SO(7) gauge theory with a spinor:

Zafrir 15

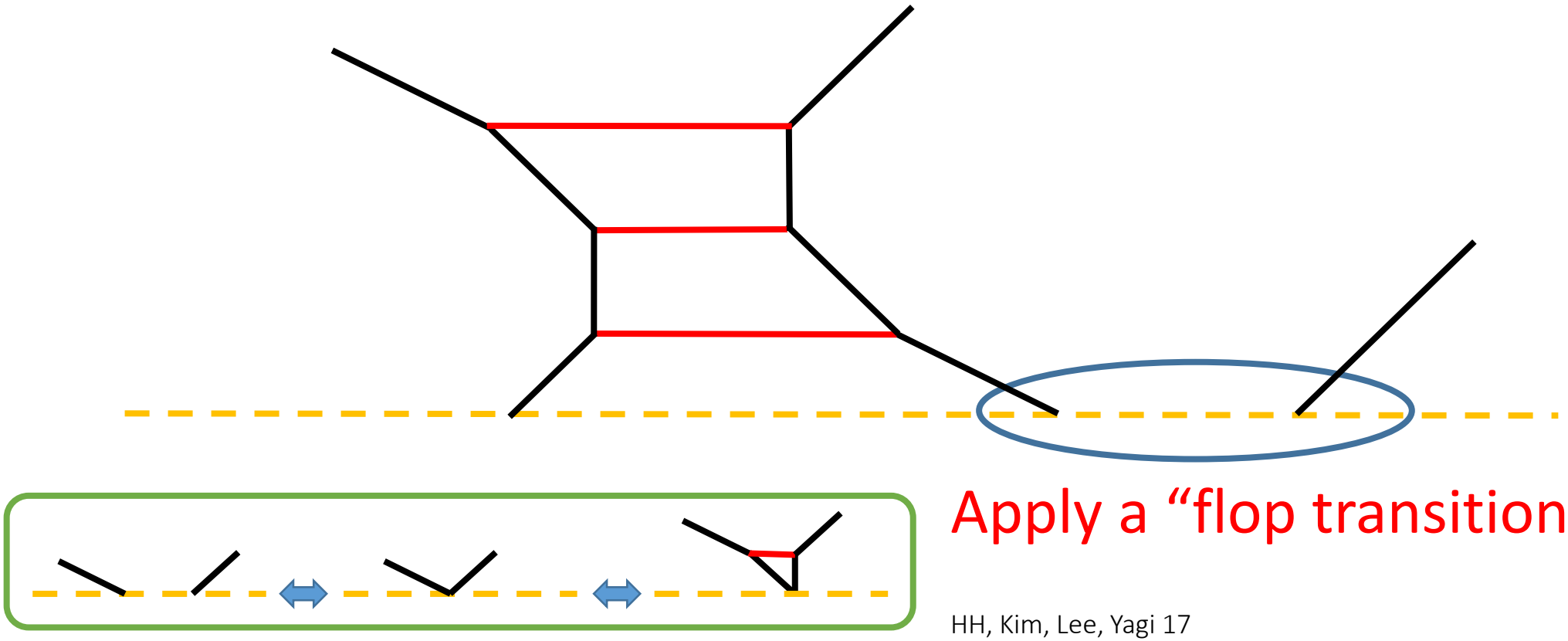


- 5d SO(7) gauge theory with a spinor:



- A Higgsing associated to the SU(2) yields the pure G_2.

2. Higgsing 5d SO(7) gauge theory with a spinor:



Apply a “flop transition”

HH, Kim, Lee, Yagi 17

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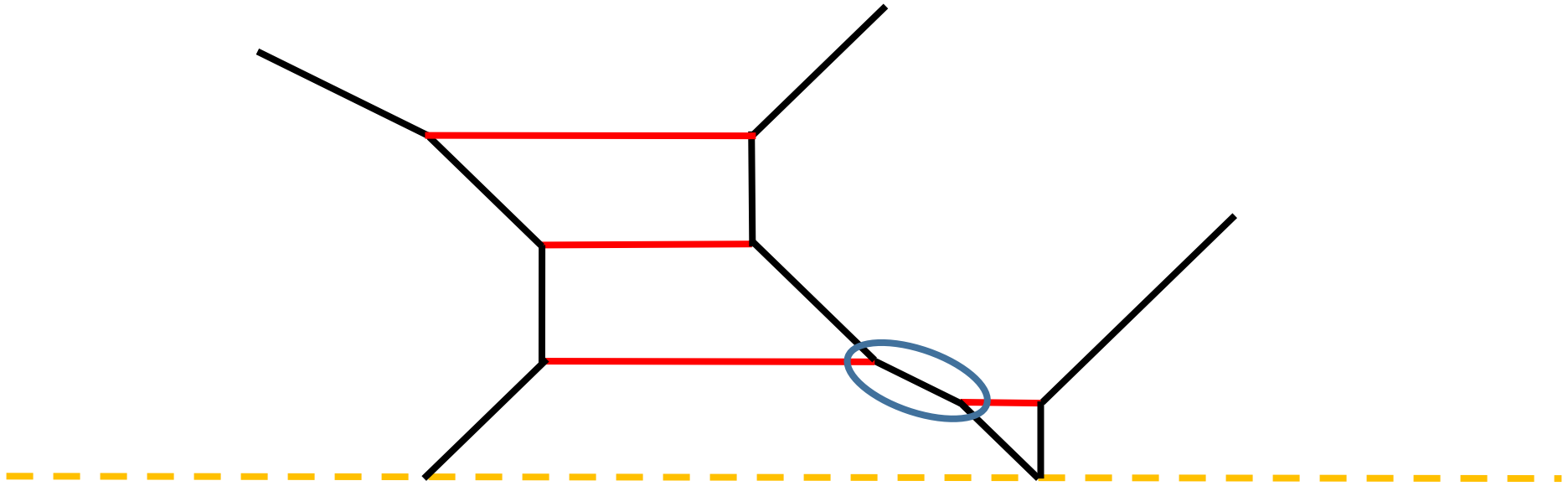

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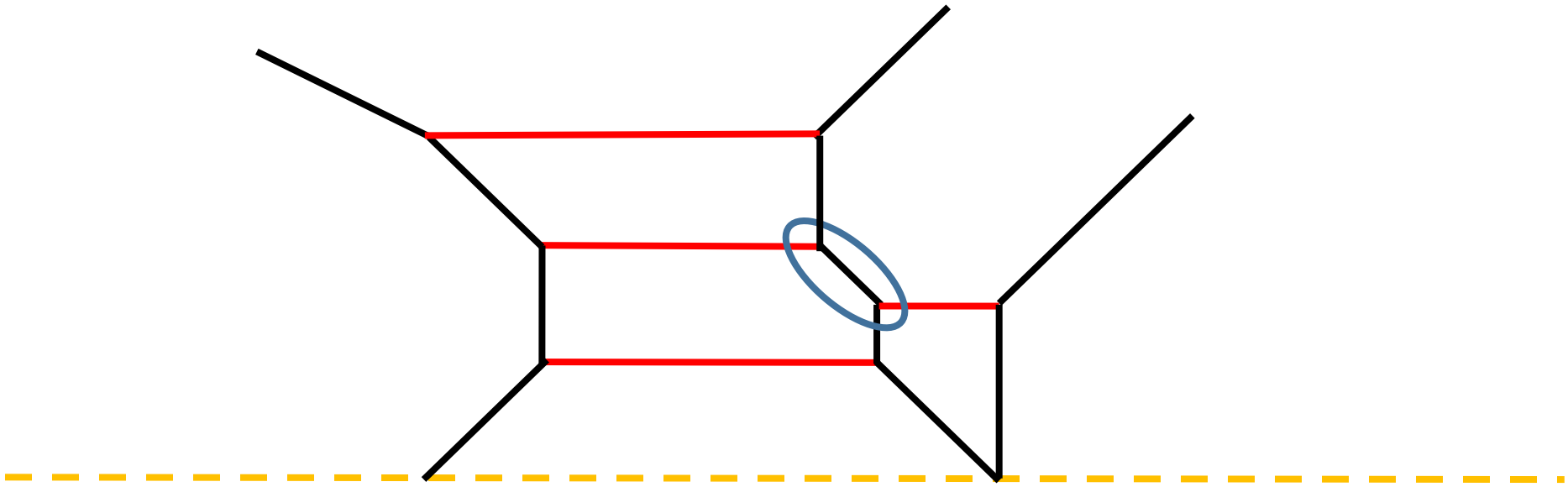

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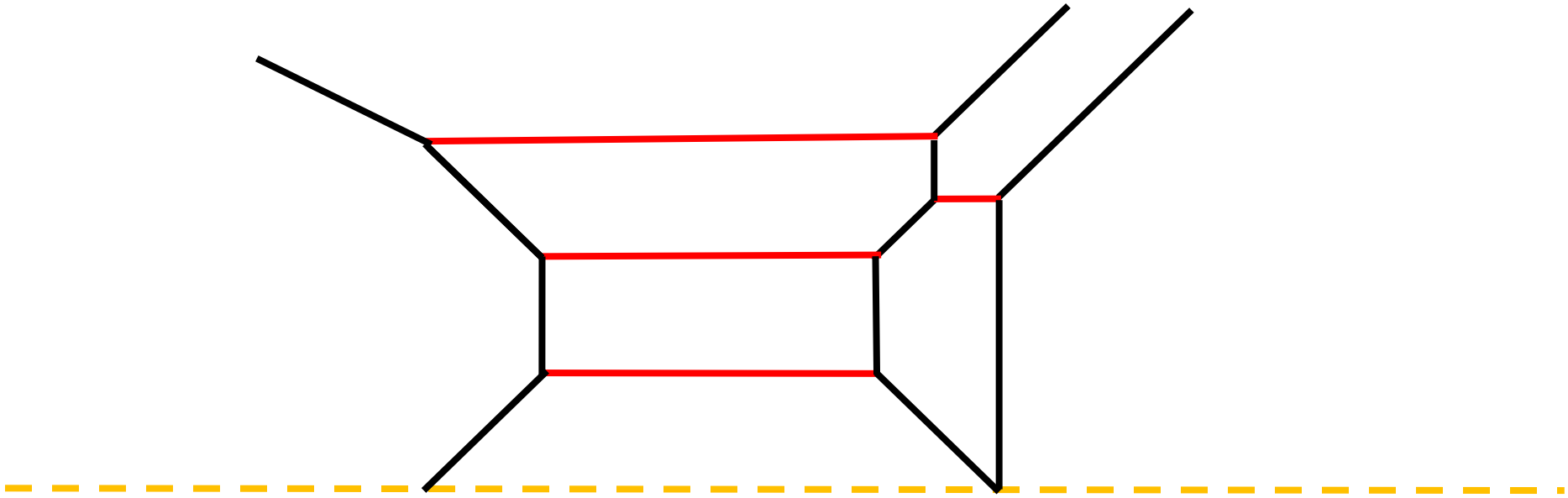

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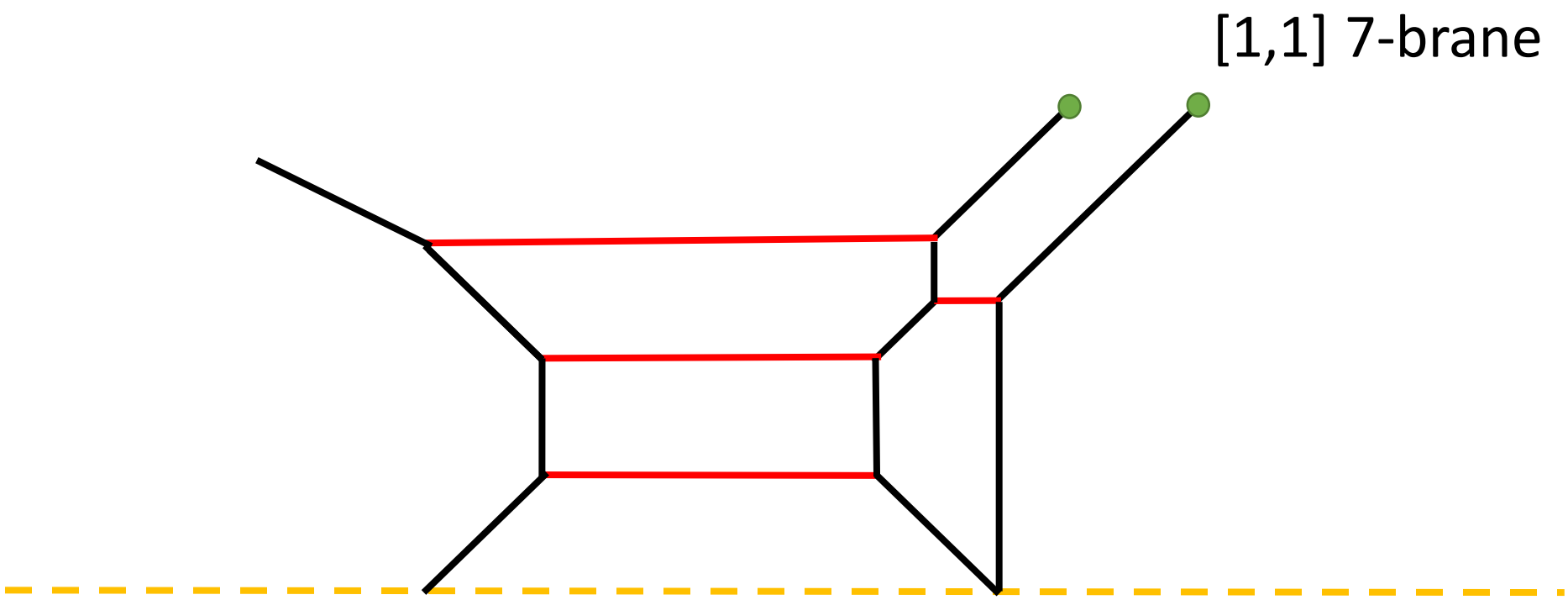

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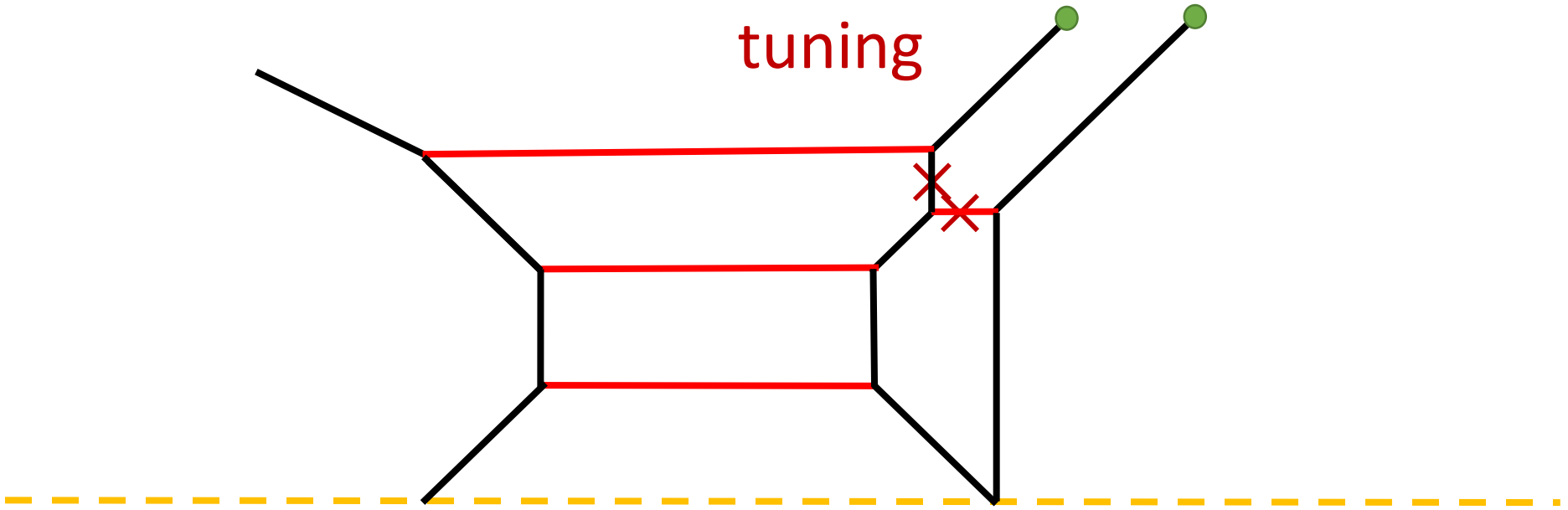

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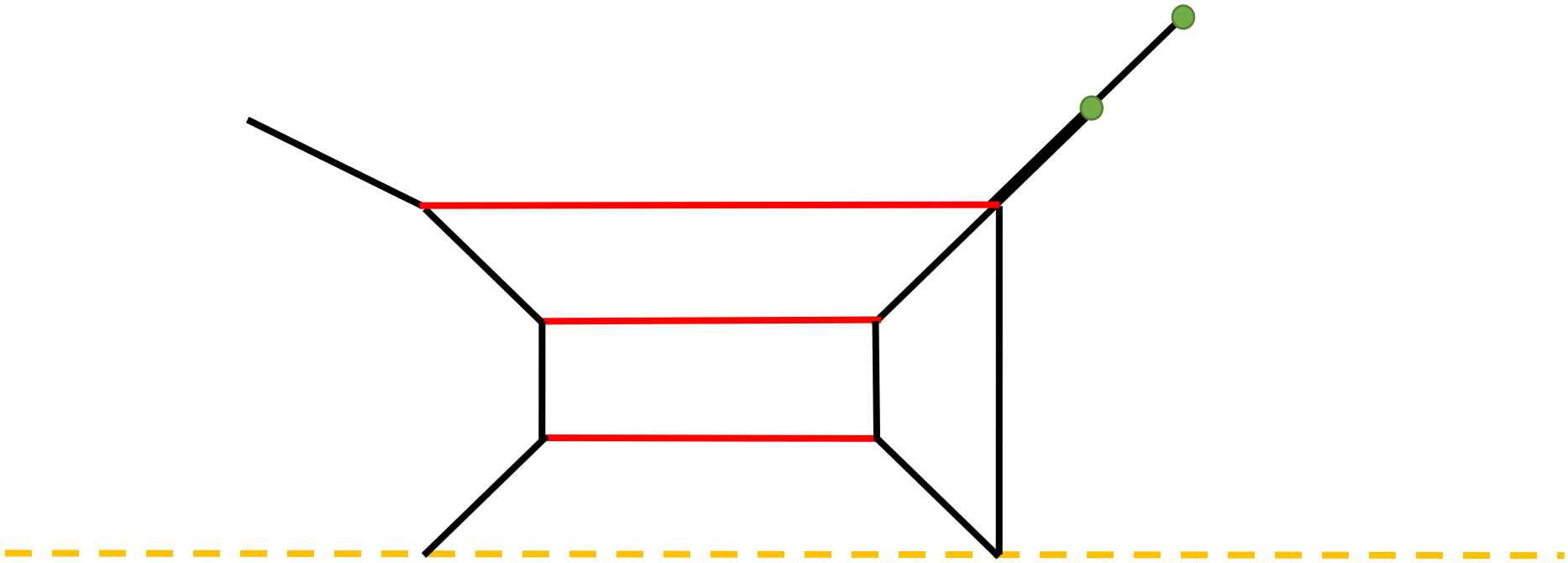

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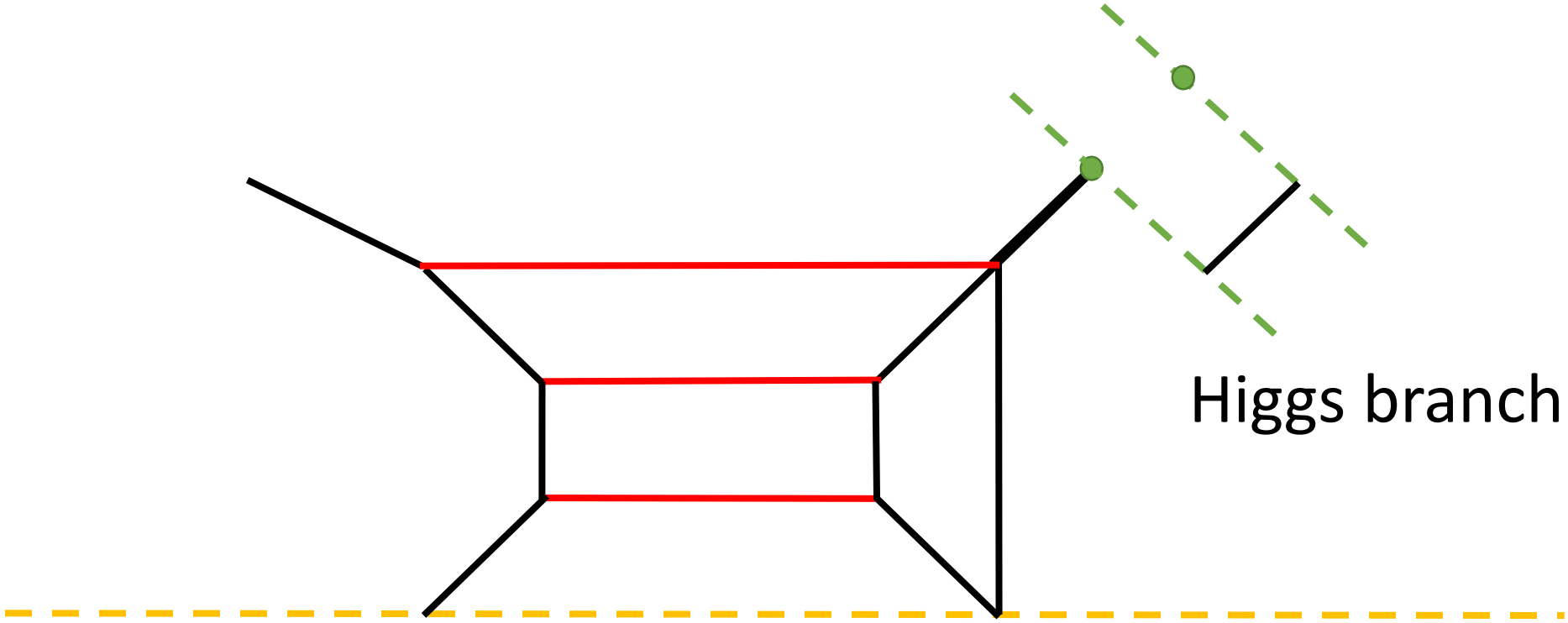
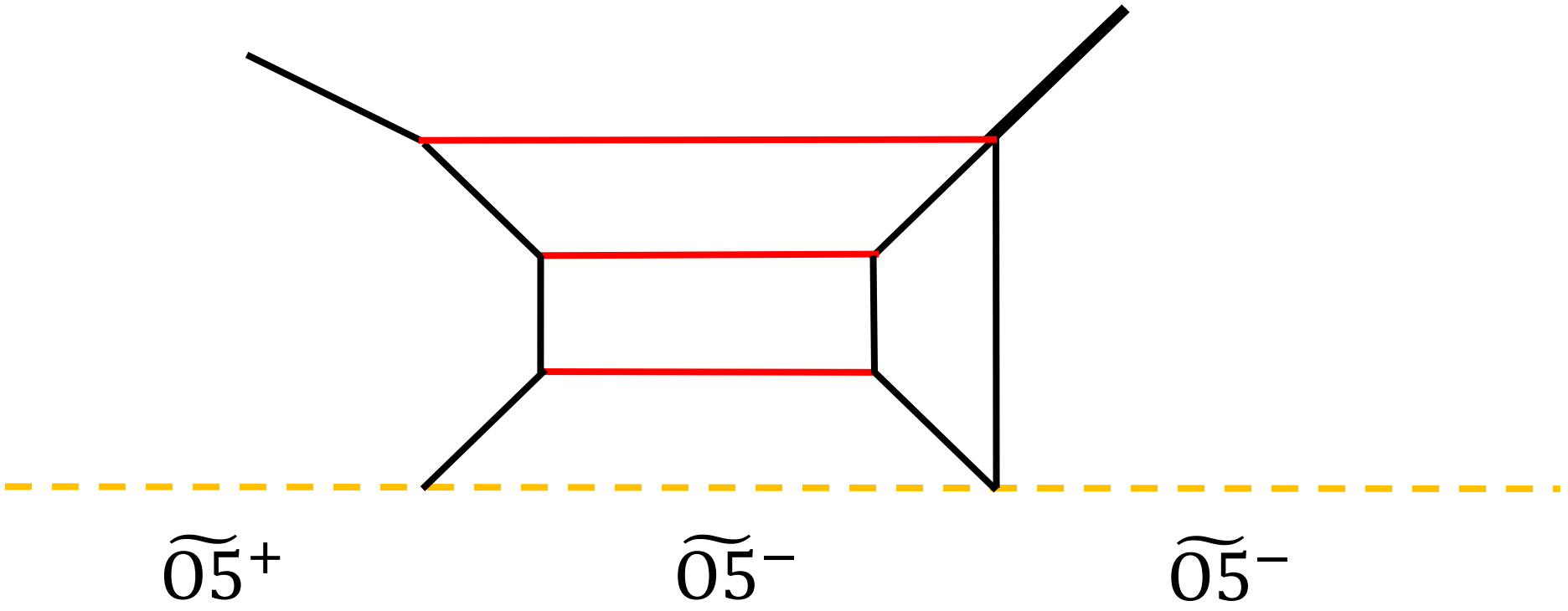


- 5d pure G_2 gauge theory:



1. $G_2 \leftrightarrow SU(3)_7$ duality:

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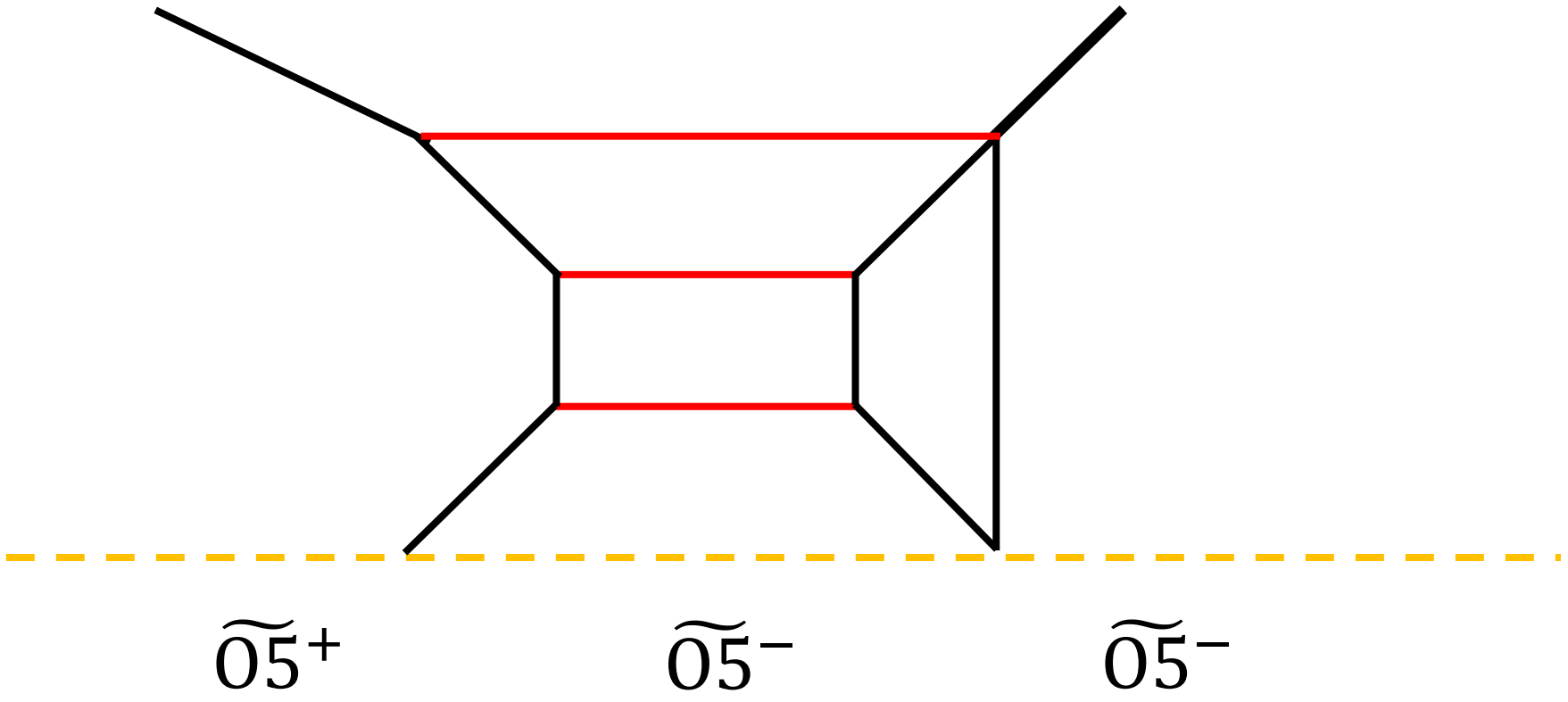
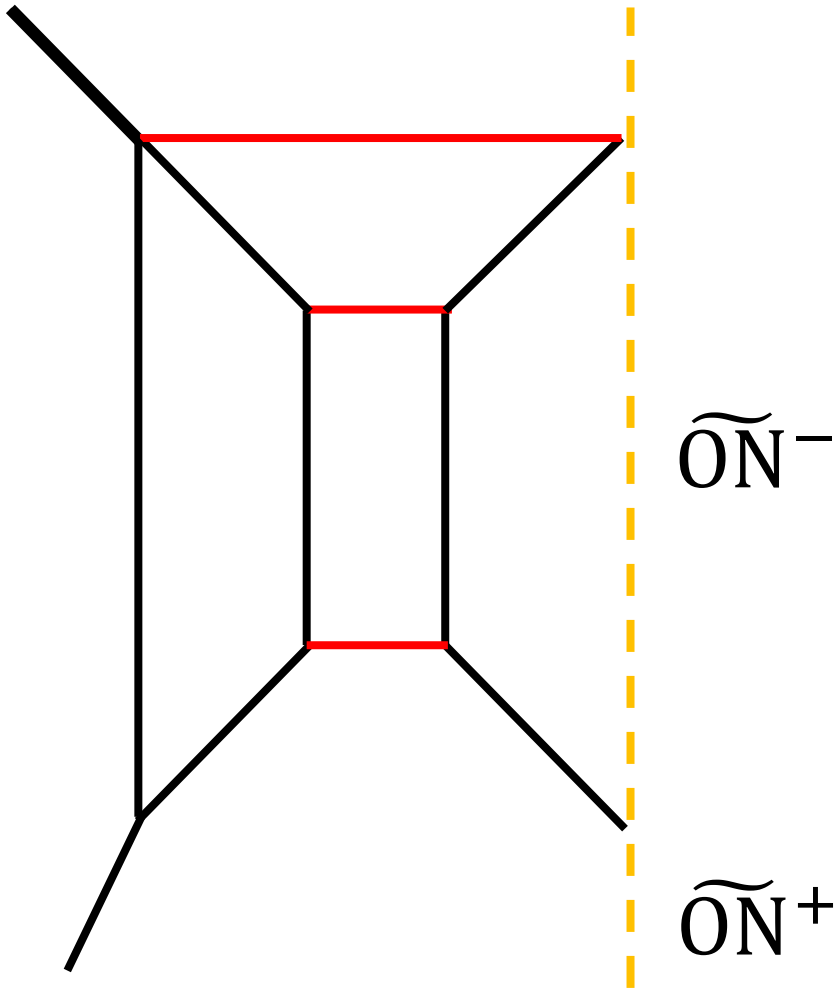
- A 5-brane web for the pure G_2 gauge theory:



- S-dual to the 5-brane web for the pure G_2 gauge theory:



SU(3) gauge theory

- We can confirm the Chern-Simons level by computing the effective prepotential of the 5d theory.

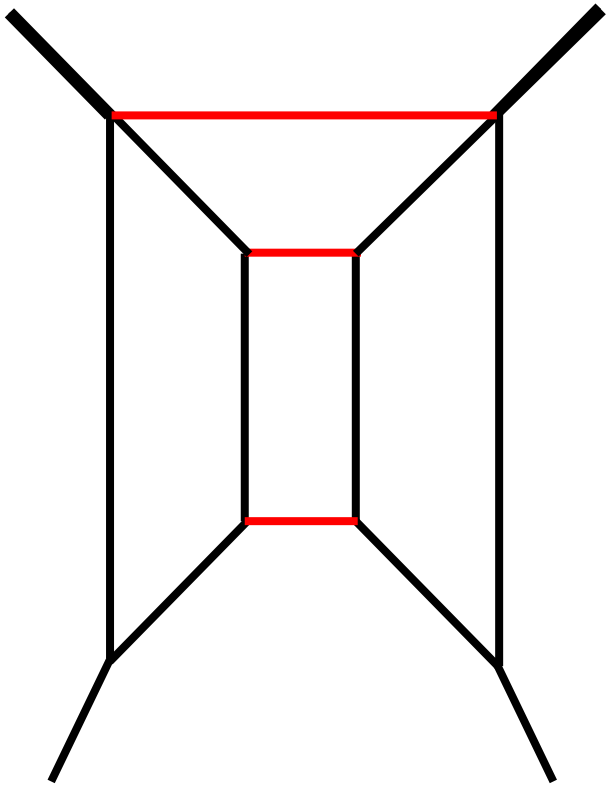
- Or similarly we can compare the tension of a monopole string given by a derivative of the effective prepotential.

- In the 5-brane web diagram, the monopole string tension is given by the area of a face of the diagram.

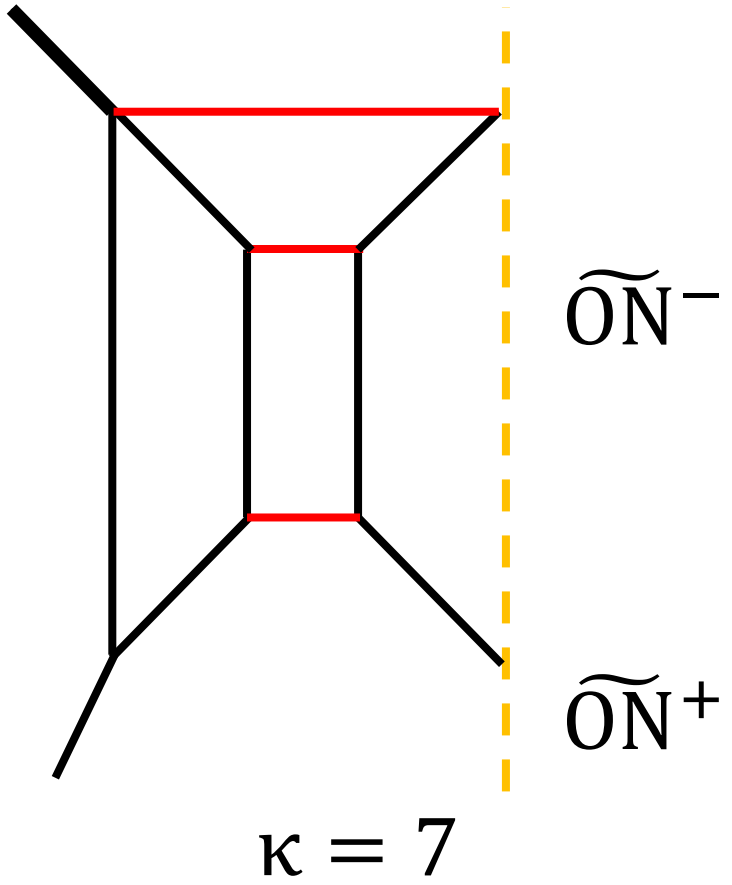
Hanany, Witten 96
Aharony, Hanany 97
Aharony, Hanany, Kol 97

- We confirmed that the SU(3) gauge theory has $\kappa = 7$.

- In fact, we can further increase the Chern-Simons level.

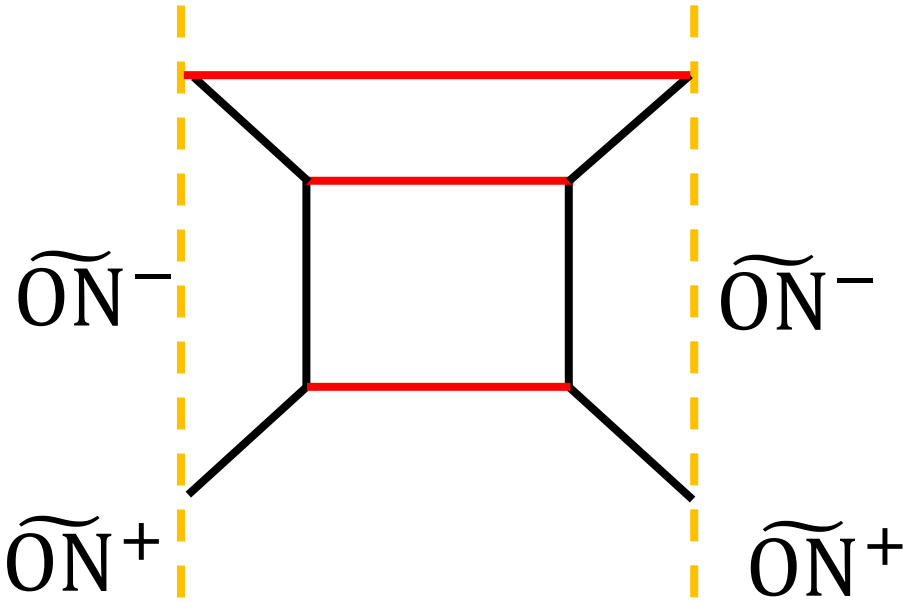


$\kappa = 5$



$\kappa = 7$

- A 5-brane web diagram of the SU(3) gauge theory with Chern-Simons level 9.



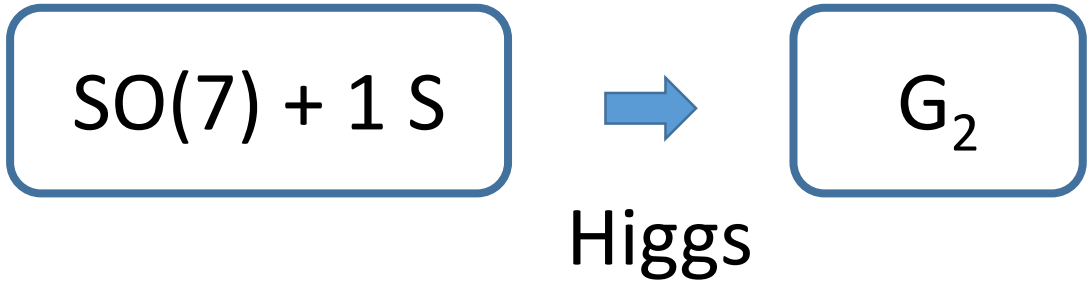
- The $SU(3)_9$ theory has been also constructed from geometry.

Jefferson, Katz, Kim, Vafa 18

4. Another realization of G_2

- In fact, there is another way to obtain a 5-brane web for the pure G_2 gauge theory.

- What we have done:



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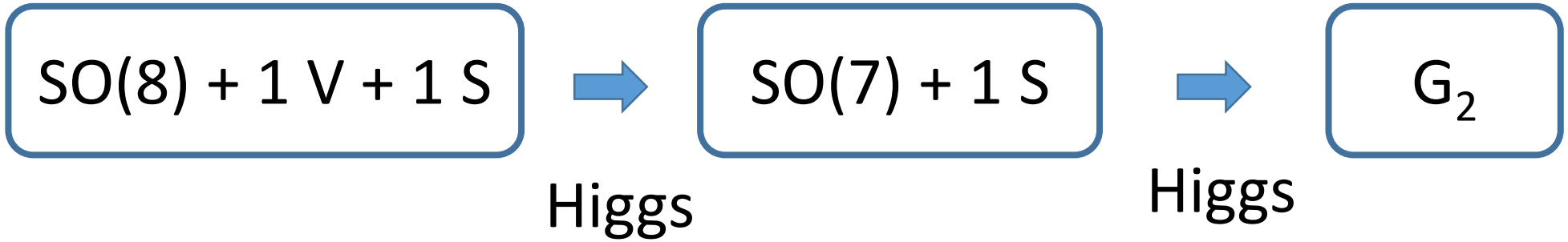

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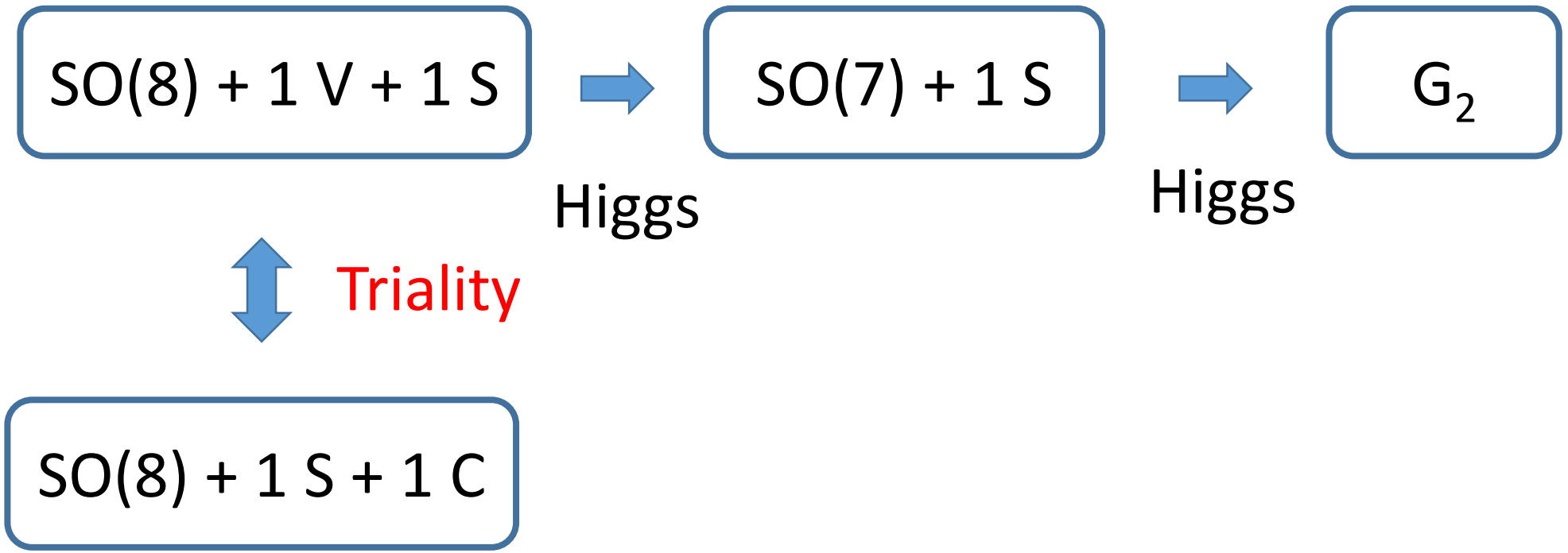

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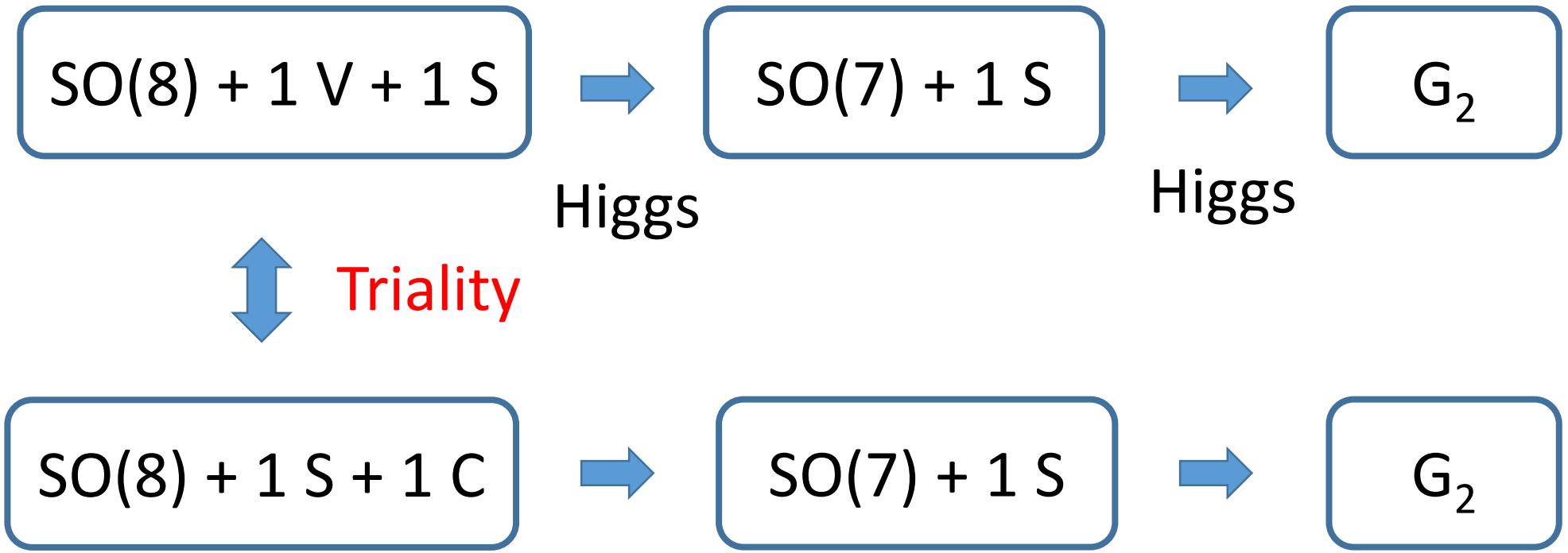
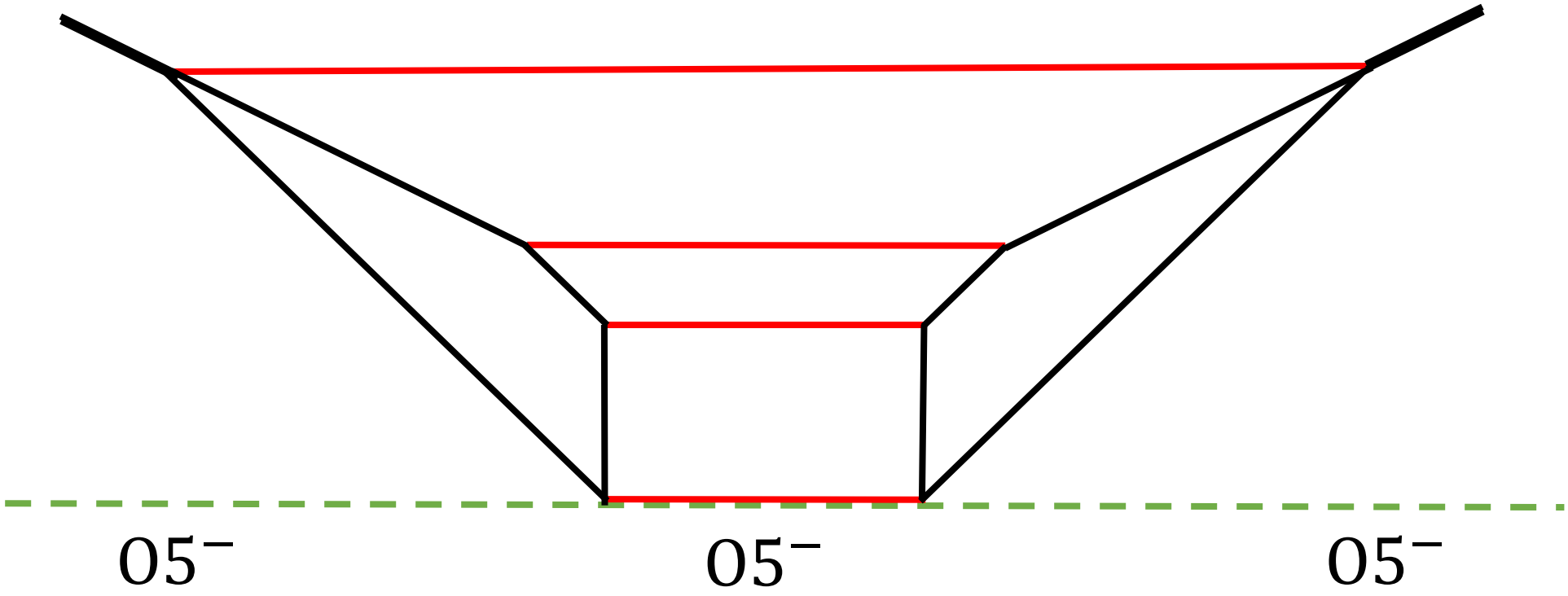


- The pure G_2 gauge theory:



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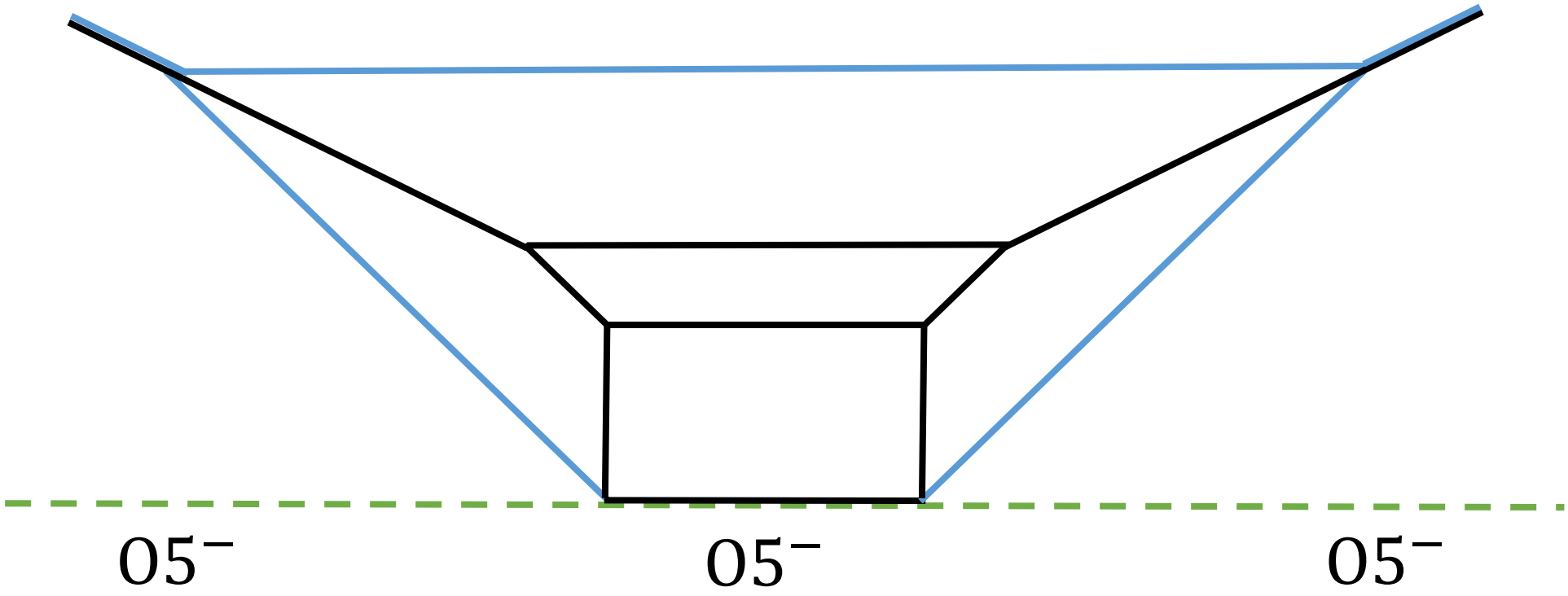
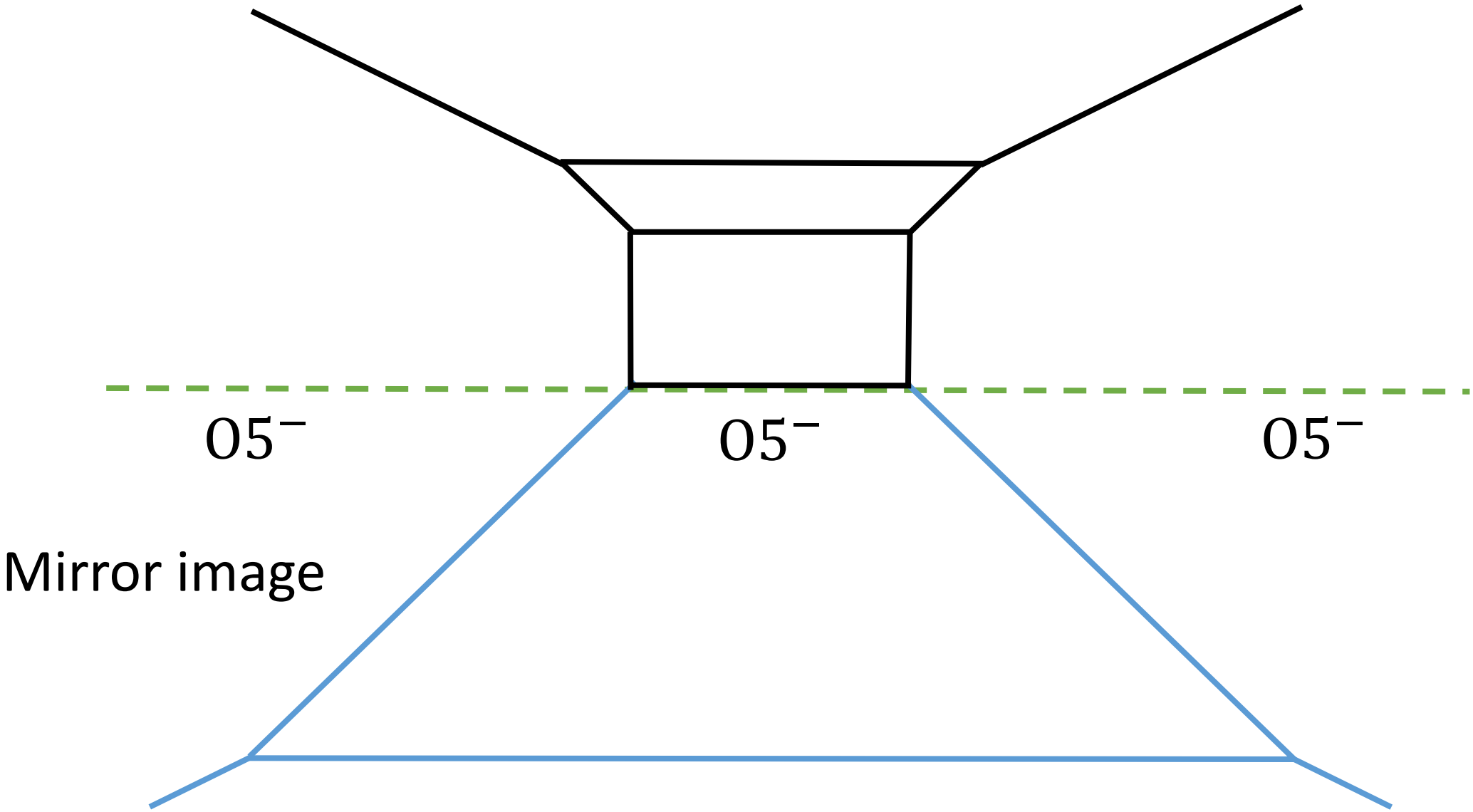
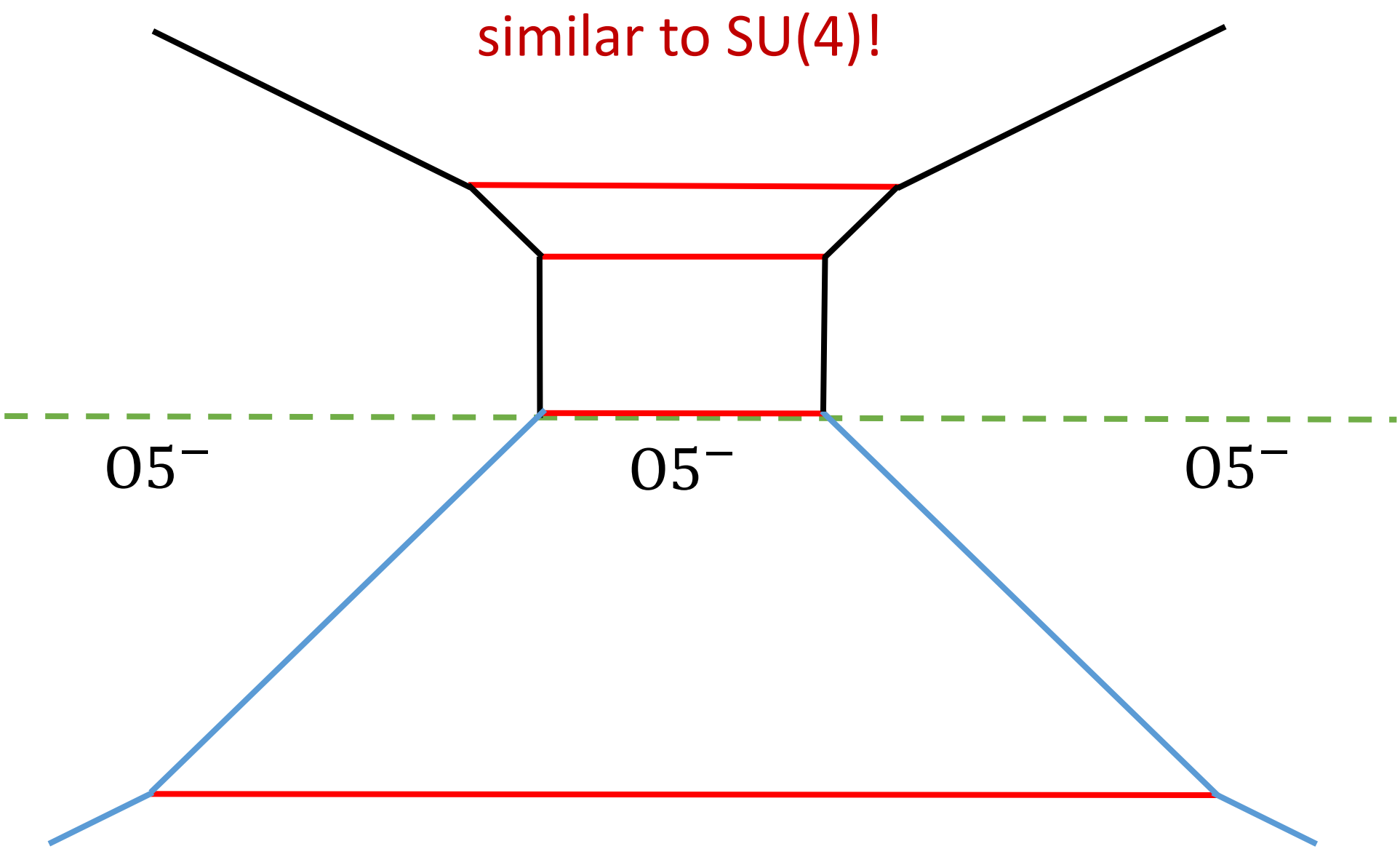



$O5^-$
$O5^-$
$O5^-$
Mirror image


similar to SU(4)!
$O5^-$
$O5^-$
$O5^-$

2. Nekrasov partition function of the pure G_2 gauge theory

- The application of the topological vertex to a 5-brane web computes the Nekrasov partition function of a 5d theory realized on the web.

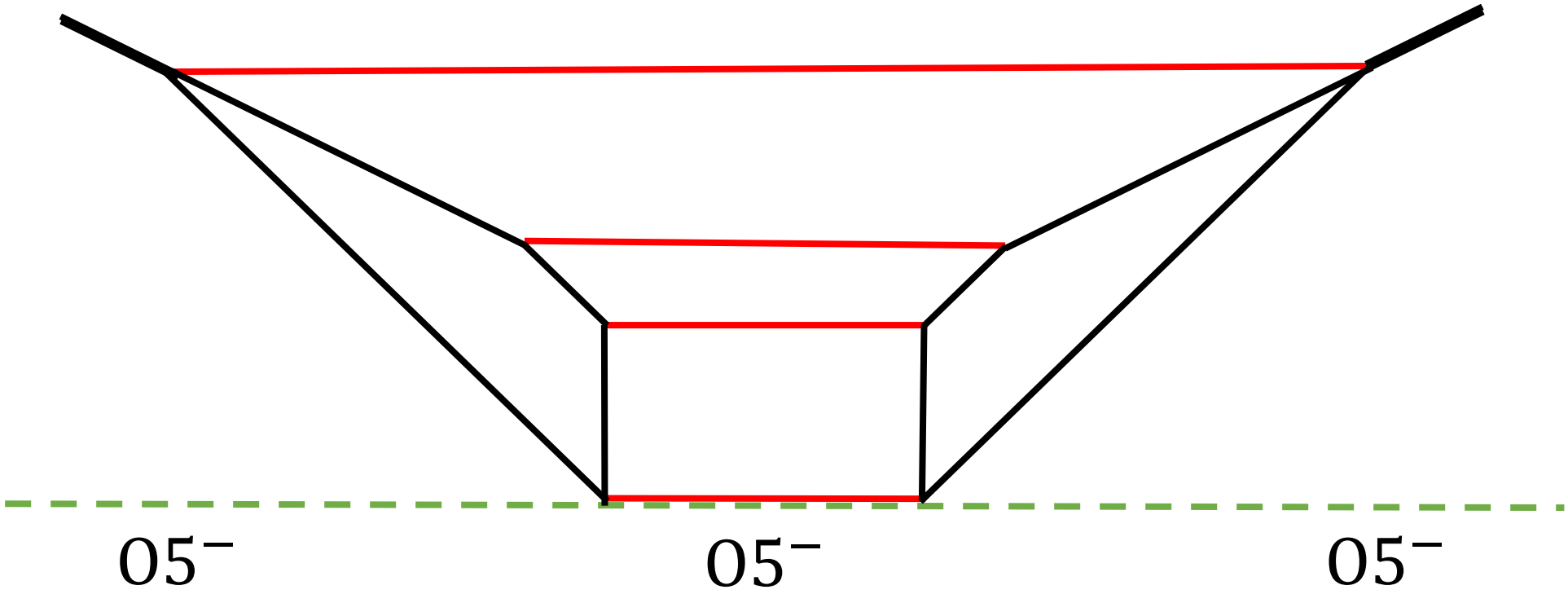
Aganagic, Klemm, Marino, Vafa 03
Awata, Kanno 05, Iqbal, Kozcaz Vafa 07

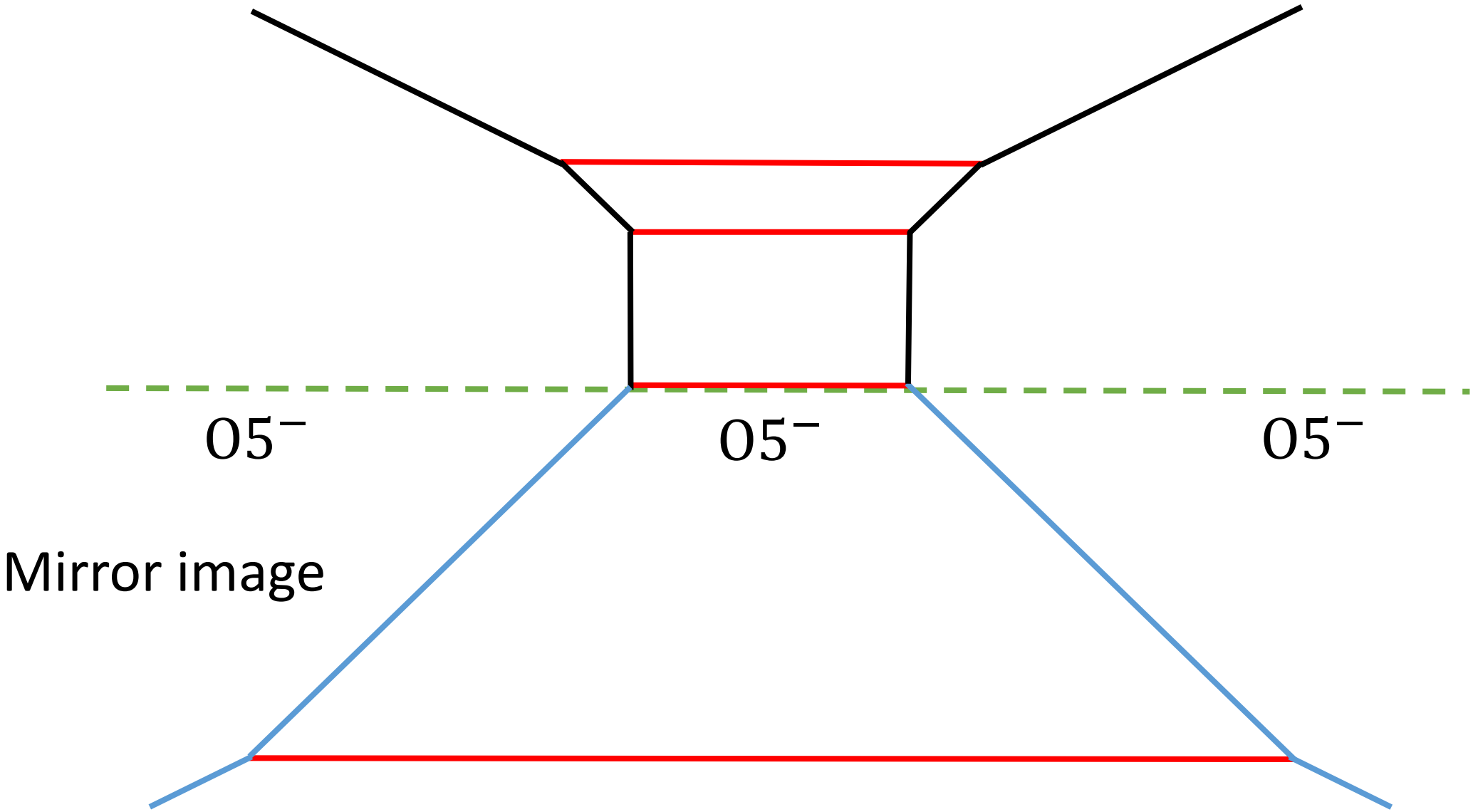
- Recently, the formalism is extended to certain configurations with an O5-plane.

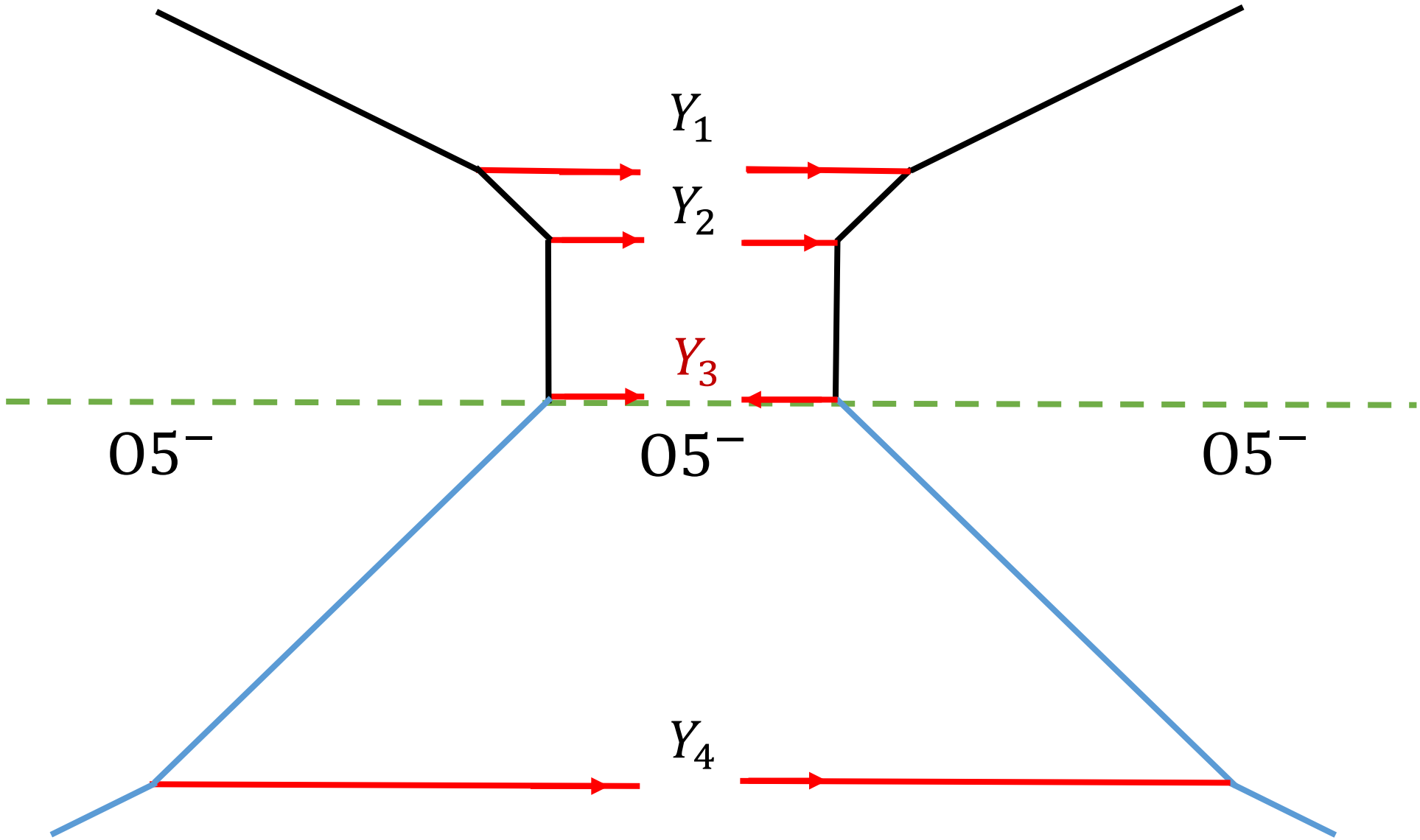
Kim, Yagi 17

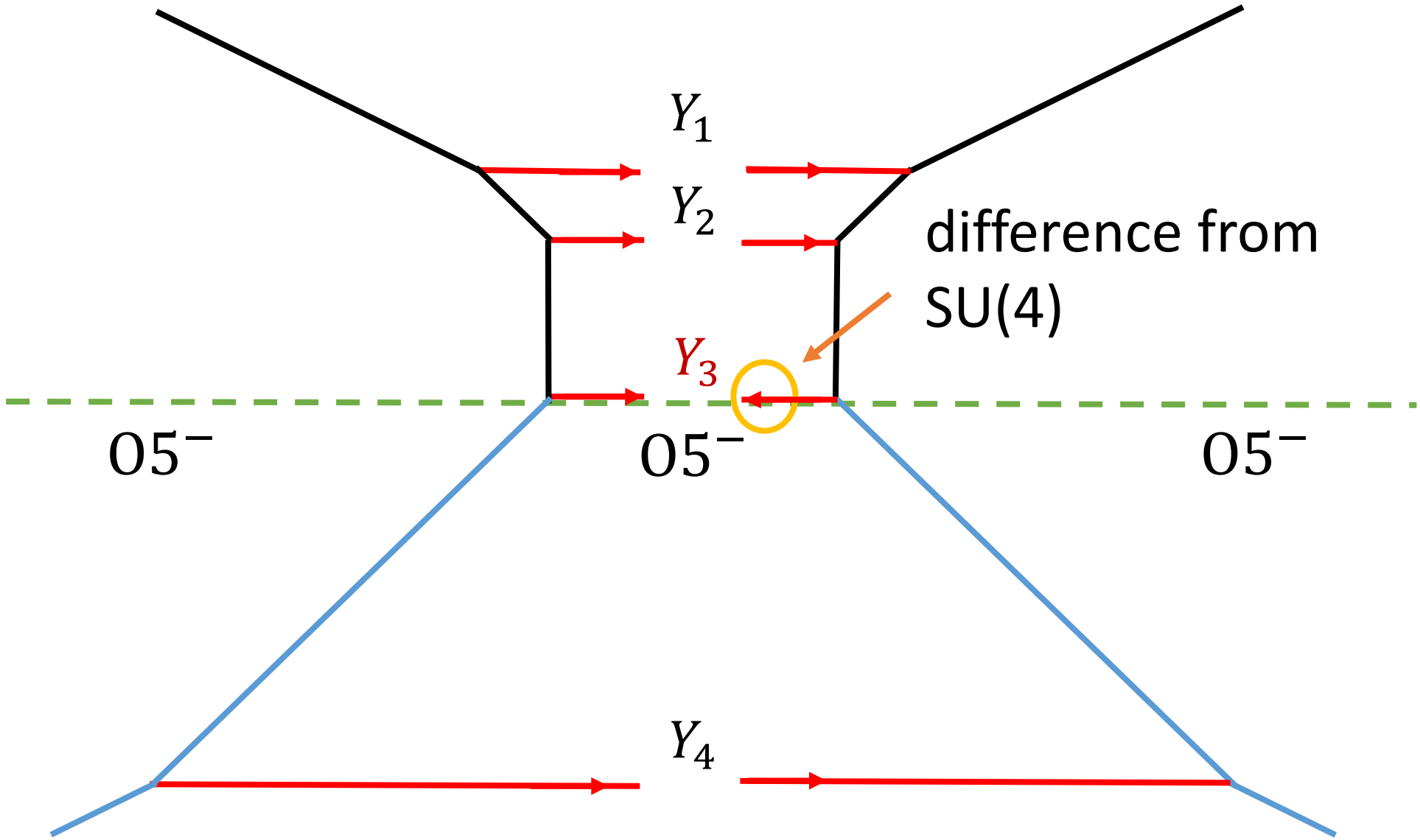
- The method makes use of the mirror image with a careful assignment of Young diagrams and the framings.

- The pure G_2 gauge theory:




$O5^-$
$O5^-$
$O5^-$
Mirror image


Y_1
Y_2
Y_3
$O5^-$
$O5^-$
$O5^-$
Y_4


Y_1
Y_2
Y_3
Y_4
difference from SU(4)
$O5^-$
$O5^-$
$O5^-$

$$Z_{G_2} = \sum_{\{Y_i\}} (-1)^{|Y_3|} {f_{Y_1}}^3 {f_{Y_2}}^1 {f_{Y_3}}^1 {f_{Y_4}}^{-3} \prod_{i=1}^{4} \left(-Q_{B_i}\right)^{|Y_i|}$$
$$Z_L\left(\{Q_{ij}\}: Y_1, Y_2, Y_3, Y_4\right) Z_R\left(\{Q_{ij}\}: Y_1, Y_2, {Y_3}^t, Y_4\right)$$

- We compared the result with the known result from field theory and found agreement until the two-instanton order.

Hanany, Mekareeya, Razamat 12
Keller, Song 12
Cremonesi, Ferlito, Hanany, Mekareeya 14

5. Conclusion

- We constructed two types of 5-brane web diagrams for the pure 5d G_2 gauge theory.

- Surprisingly, the web diagrams are simple in a sense that they are given by standard 5-brane webs with an orientifold 5-plane.

- The diagrams can be applied in various ways and we saw the $G_2 \leftrightarrow SU(3)_7$ duality and also computed the Nekrasov partition function of the pure G_2 gauge theory.

- It is straightforward to add flavors to the diagrams of the pure G_2 gauge theory.

- We can compute the Nekrasov partition function of the G_2 gauge theories with flavors.

HH, Kim, Lee, Yagi 18

- It would be interesting to compare it with the result computed by different methods recently.

Kim, Kim, Kim, Lee, Park 18
Del Zotto Lockhart

- For the case with the maximal number of flavors, there are also dualities among G_2 + 6 F, Sp(2) + 2 AS + 4 F, $SU(3)_4$ + 6 F.

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- We can see the dualities also from 5-brane webs.

HH, Kim, Lee, Yagi to appear