4D Gauge Theories With Conformal Matter

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Based On

Apruzzi, JJH, Morrison and Tizzano:

hep-th/1803.00582

Also Based On

JJH, Rudelius, Morrison, Vafa: hep-th/1502.05405

Del Zotto, JJH, Tomasiello, Vafa: hep-th/1407.6359

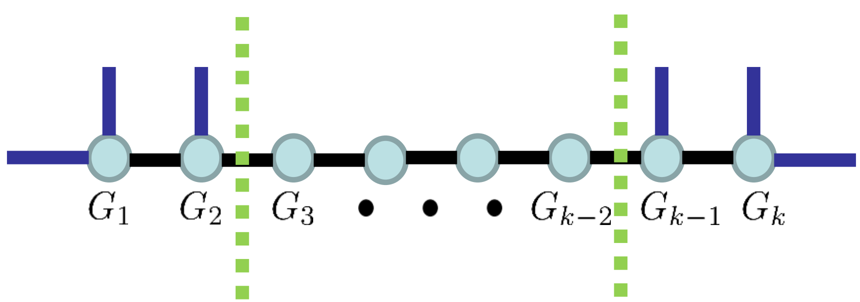
JJH, Morrison, Vafa: hep-th/1312.5746

Background

- Recently, many new 6D SCFTs constructed

- Conjectural F-theory Classification

  (JJH Morrison Vafa '13, JJH Morrison Rudelius Vafa '15)



So What's Next?

- General Expectation:

  Compactification of $\mathcal{N} = (1, 0)$ SCFTs
  will provide insight in 5D/4D/3D/2D/1D.

- Great Success with 6D $\mathcal{N} = (2, 0)$ cpctifications

  c.f. Gaiotto et al. 6D $\rightarrow$ 4D/3D/2D/1D

Focus for Today: 4D Physics

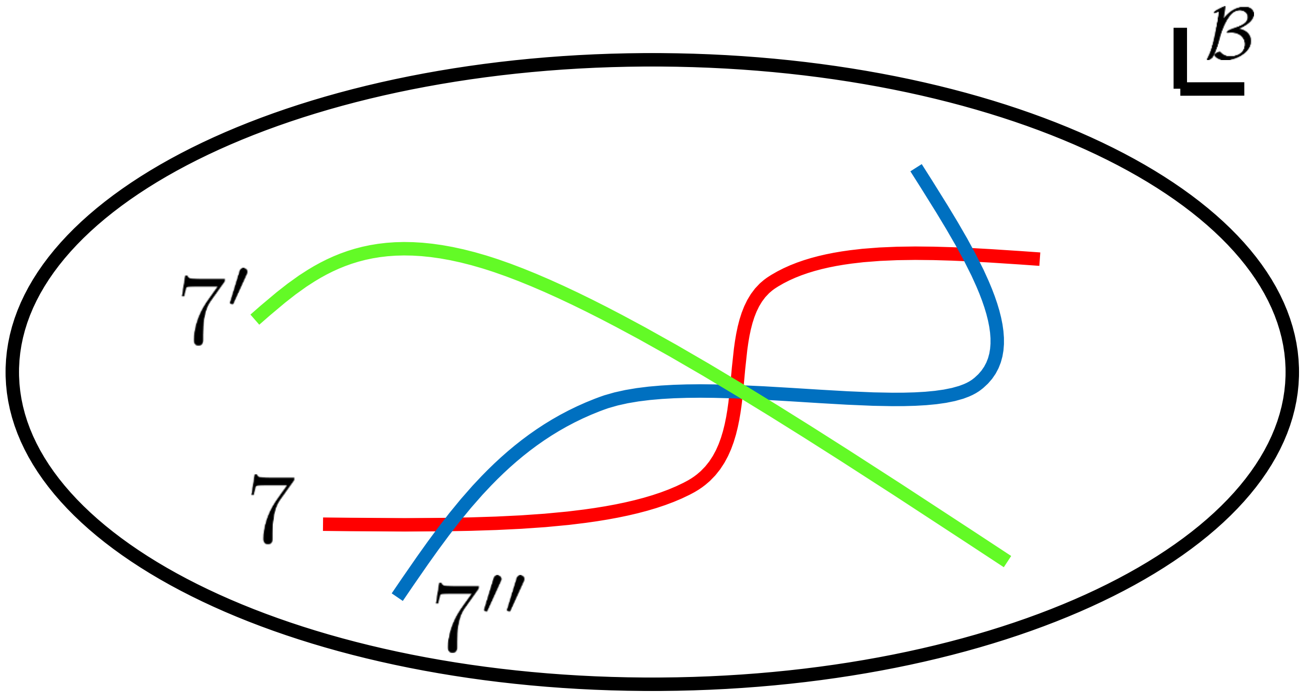
- What 4D QFTs can we realize?

- New F-th Geometric Structures?

- Classical vs Quantum Moduli Spaces

Geometric Setup

Elliptic $CY_4 \to \mathcal{B}$ (threefold base)

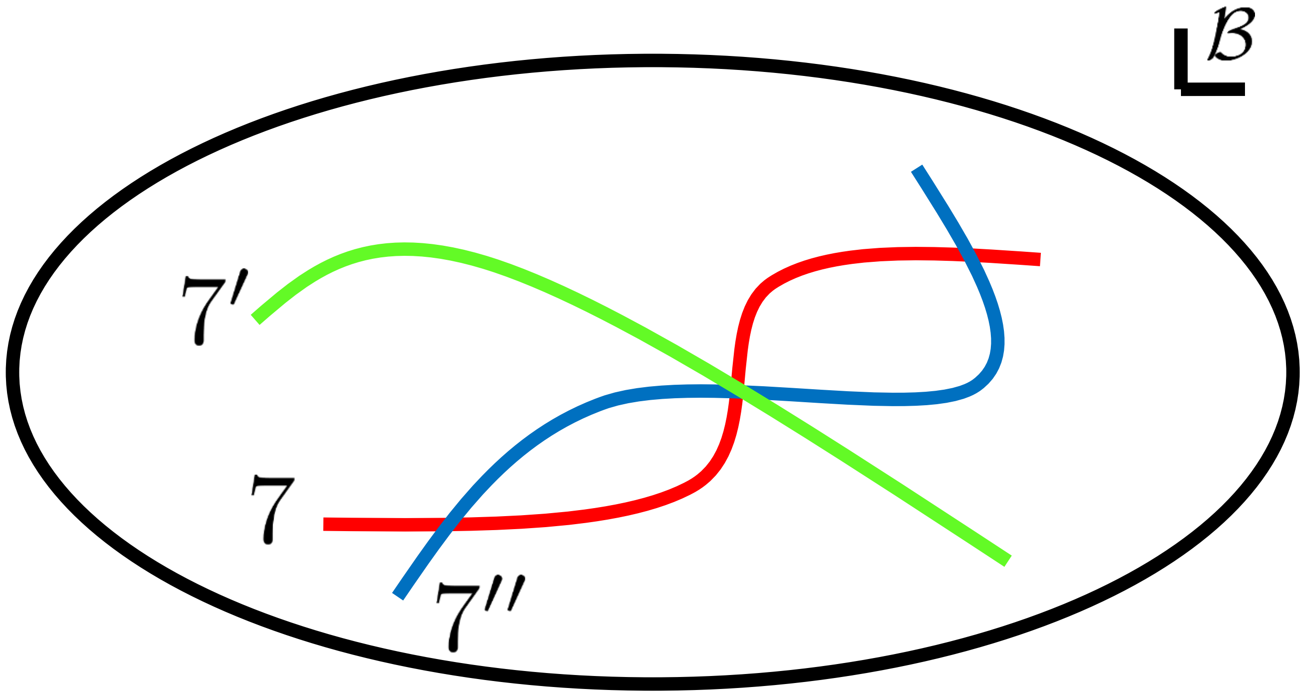


Geometric Localization

(c.f. Beasley JJH Vafa '08, Donagi Wijnholt '08)

Organize by codimension in F-theory base:

- Codim 1: Gauge Groups (Morally 8D)
- Codim 2: Matter (Morally 6D)
- Codim 3: Matter Yukawas (Morally 4D)



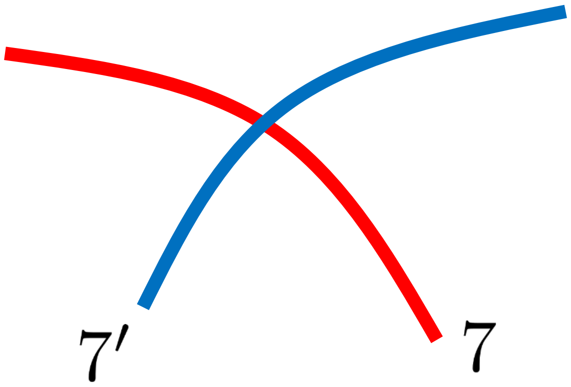
7′
7
7′′
$\mathsf{L}^{\mathcal{B}}$

Matter

6D Matter

$$y^2 = x^3 + fx + g$$

$$\Delta = 4f^3 + 27g^2$$



Weakly Coupled Matter: Kodaira-Tate Form

6D Conformal Matter: $\text{mult}(f, g, \Delta) \geq (4, 6, 12)$

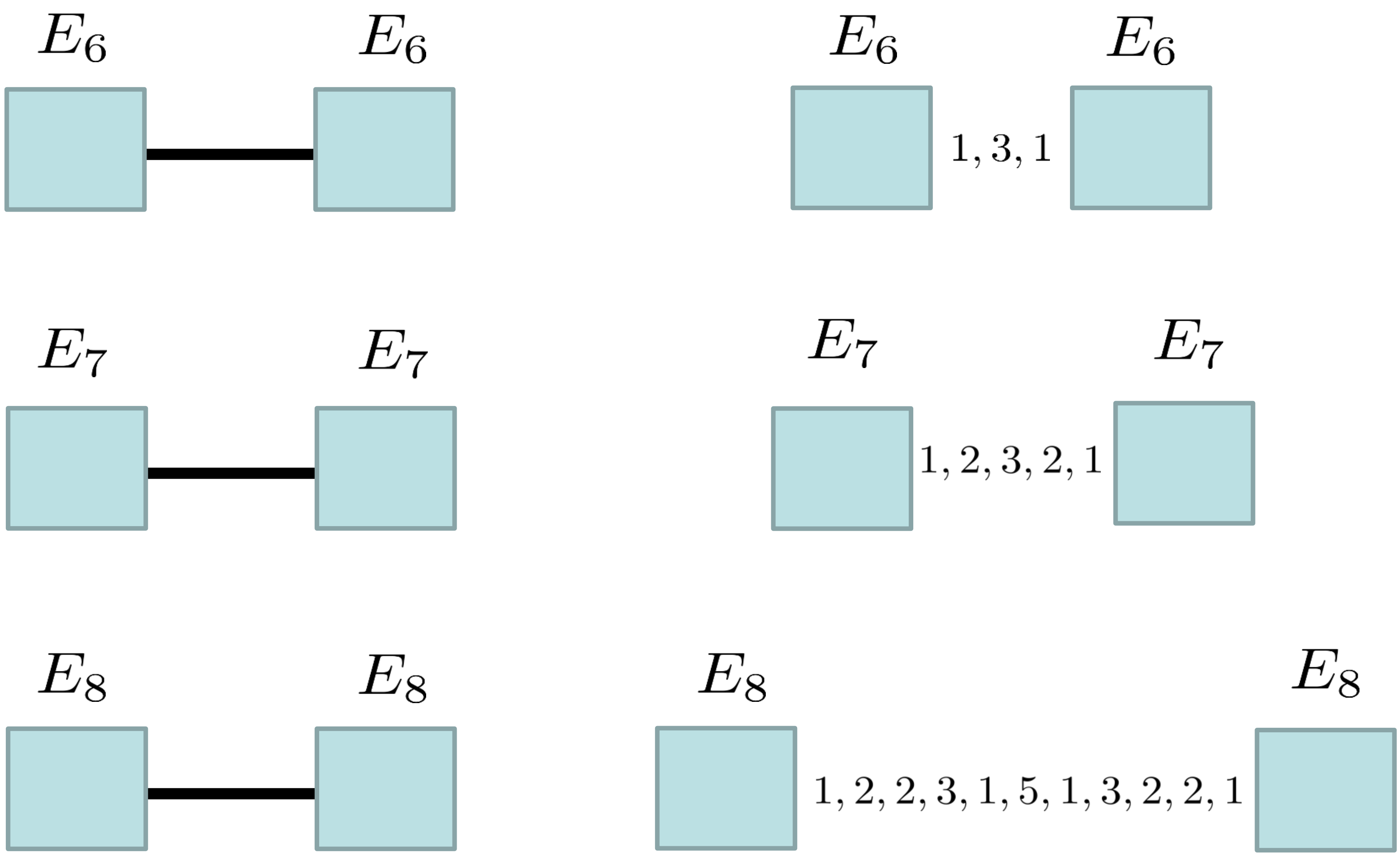
Bershadsky Johansen '96

JJH Morrison Vafa '13

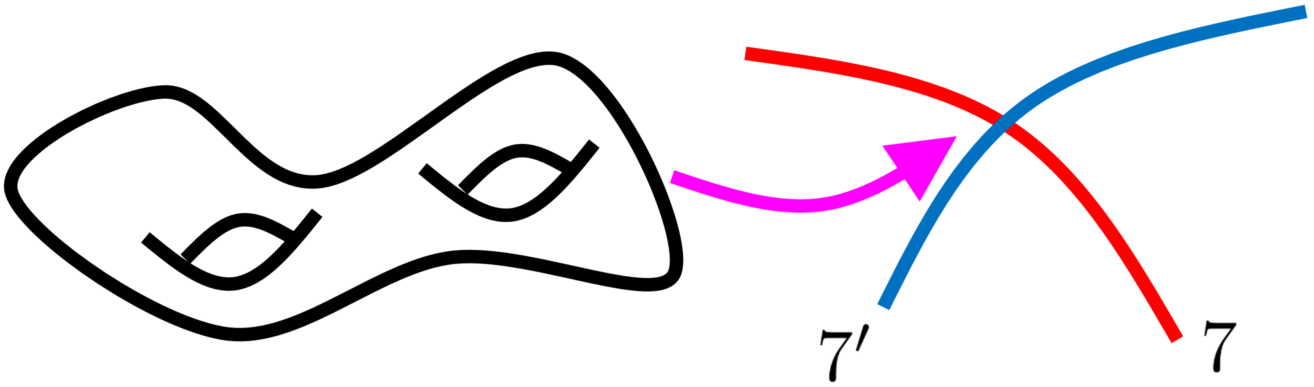
Morrison Taylor '12

Examples

Del Zotto JJH Tomasiello Vafa '14, JJH '14



Matter on a Curve



4D Physics Depends on:

- Choice of $\Sigma_{g,p}$
- 7-brane Fluxes

Anomaly Reduction

6D Anomaly Polynomial is known:

$$I_8 = \alpha c_2(R_{6D})^2 + \beta c_2(R_{6D}) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T)$$

$$+ \text{ terms involving } \mathrm{Tr}F^2 \text{ and } \mathrm{Tr}F^4$$

(c.f. Ohmori Shimizu Tachikawa Yonekura '14)

$\Rightarrow$ 4D Anomaly Polynomial is known!

Integrate over $\Sigma_{g,p}$ with bkgnd curvatures

(c.f. Benini Tachikawa Wecht '09, Razamat Vafa Zafrir '16)

If $\mathcal{N} = 2$ SUSY, different procedure

(c.f. Ohmori Shimizu Tachikawa Yonekura '15)

Anomaly Reduction

$$I_8 = \alpha c_2(R_{6D})^2 + \beta c_2(R_{6D}) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T)$$

$$+ \text{ terms involving } \mathrm{Tr}F^2 \text{ and } \mathrm{Tr}F^4$$

$I_6 = \int_\Sigma I_8$ with:

$$R_{6D} \to (R_{4D} \otimes K_\Sigma^{1/2}) \oplus (R_{4D} \otimes K_\Sigma^{1/2})^\vee \quad \text{(twist)}$$

$$F \to \langle F_{bkgnd} \rangle + F \ \text{(flux)}$$

4D Conformal Matter

Input: •(G_L, G_R) in 6D

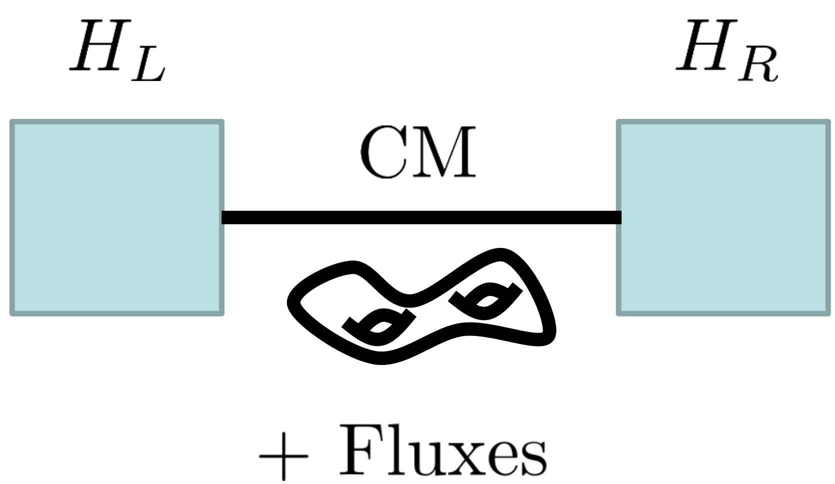
•$\Sigma_{g,p}$ to compactify

• "Flavor Fluxes"

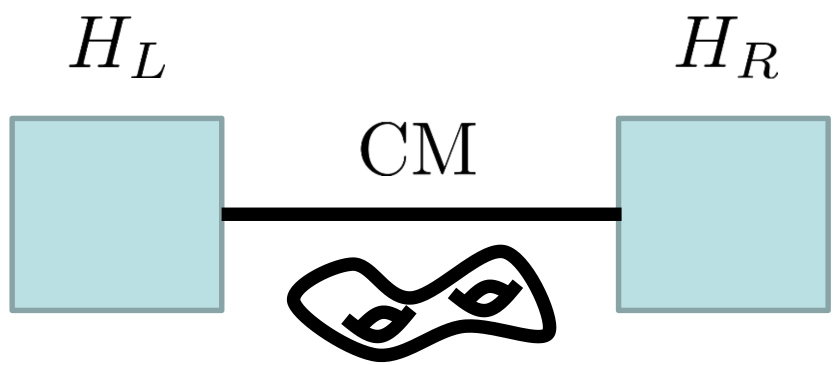
Output: 4D QFT w/ flavor $H_L \times H_R \times U(1)'s$

Often a 4D SCFT (test via a-max / indices)

Starting Point



Starting Point

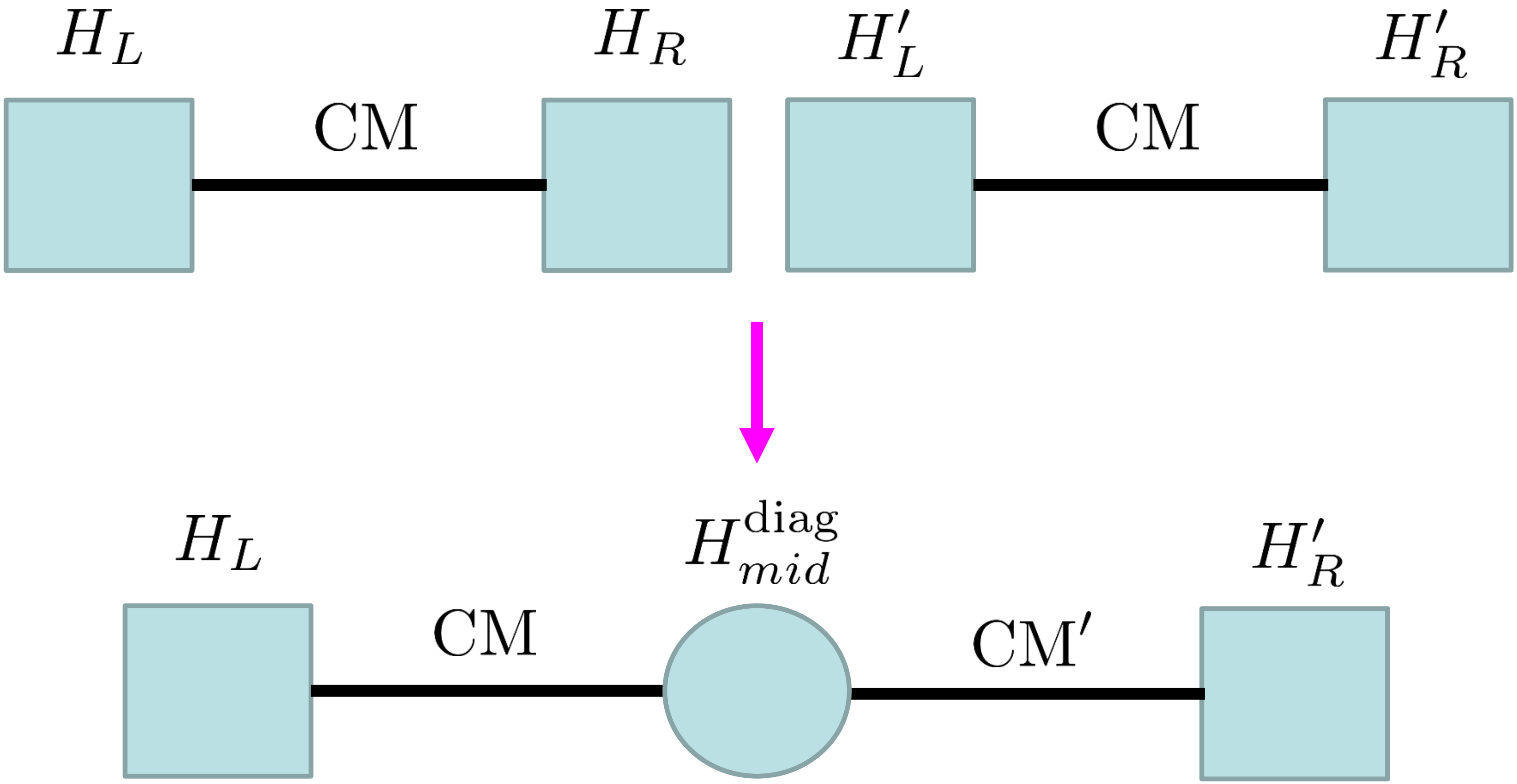


$+$ Fluxes

Note: H's can be any simple Lie group!

$$SU(N), SO(N), Sp(N), G_2, F_4, E_6, E_7, E_8$$

Gauging (via $\mathcal{N}=1$ vector)



Beta Function

$$b_G = 3h_G^\vee - b_G^{\text{matter}}$$

Beta Function

$$b_G = 3h_G^\vee - b_G^{\text{matter}}$$

Dual Coxeter Number

	$SU(N)$	$SO(2k)$	E_6	E_7	E_8
$h_G^\vee$	N	$2k-2$	12	18	30

Beta Function

$$b_G = b_G^{\text{vec}} - b_G^{\text{matter}}$$

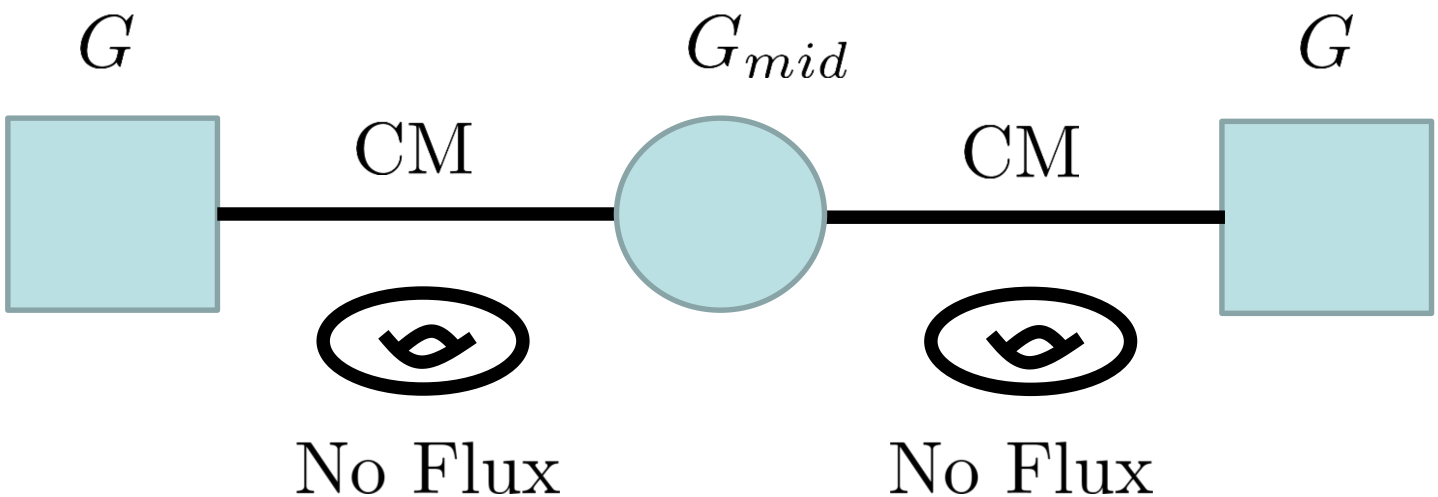
$$b_G^{\mathcal{N}=1\,\text{vec}} = 3h_G^\vee$$

$$b_G^{\mathcal{N}=2\,\text{vec}} = 2h_G^\vee$$

$$b_G^{\text{matter}} = -3\text{Tr}(R_{IR}F_G F_G)\ \text{(calculable anomaly)}$$

Anselmi et al. '97

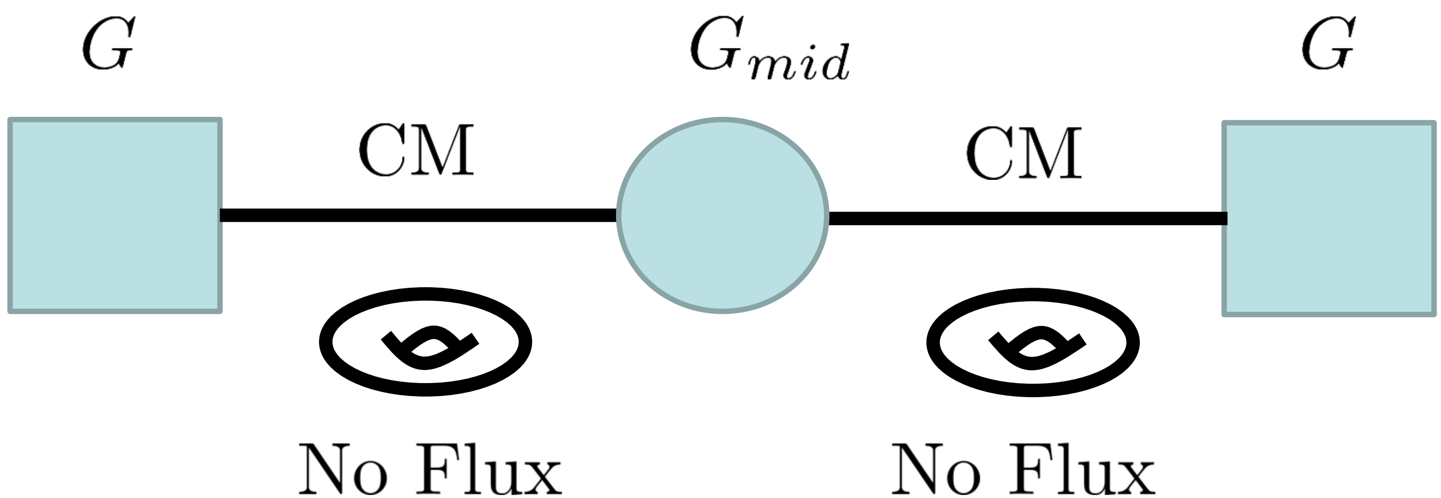
Warmup ($\mathcal{N} = 2$ vector)



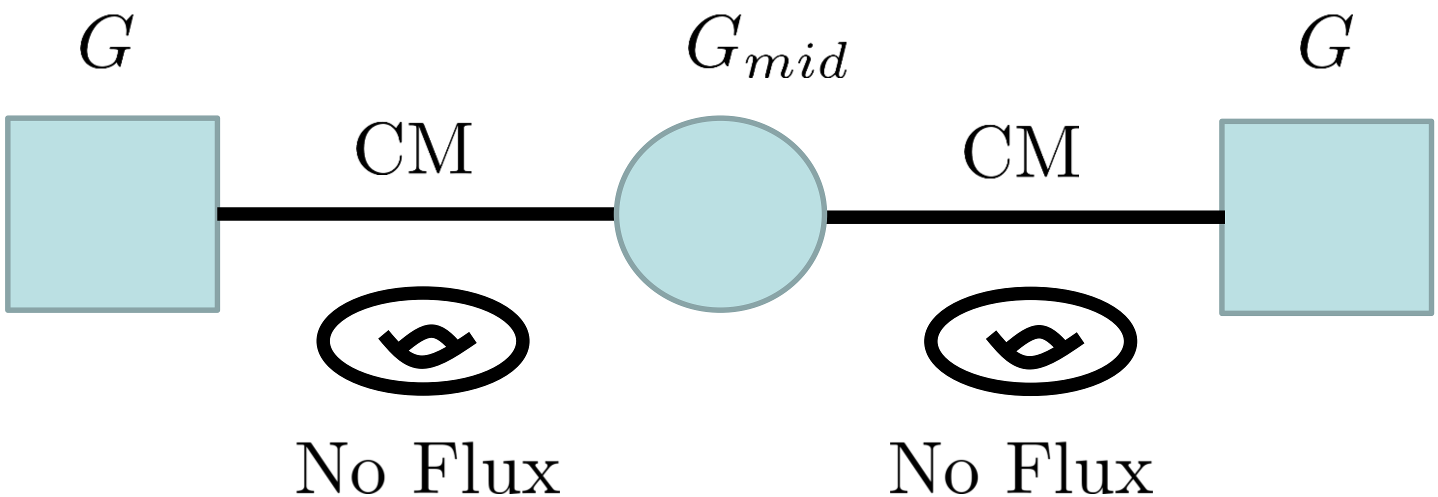
Beta Function

$$b_{G_{mid}} = b_{\mathcal{N}=2\,vec} - b_{\mathrm{CM},L} - b_{\mathrm{CM},R}$$

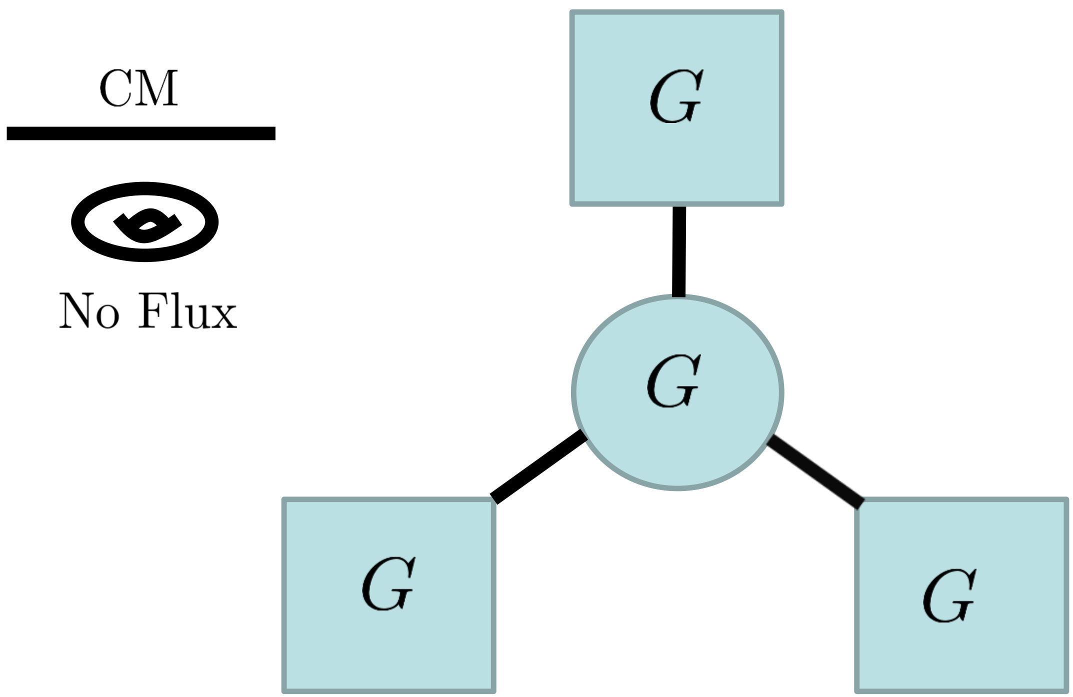
$$= 2h^\vee_G - h^\vee_G - h^\vee_G = 0$$



¡4D $\mathcal{N} = 2$ SCFT!

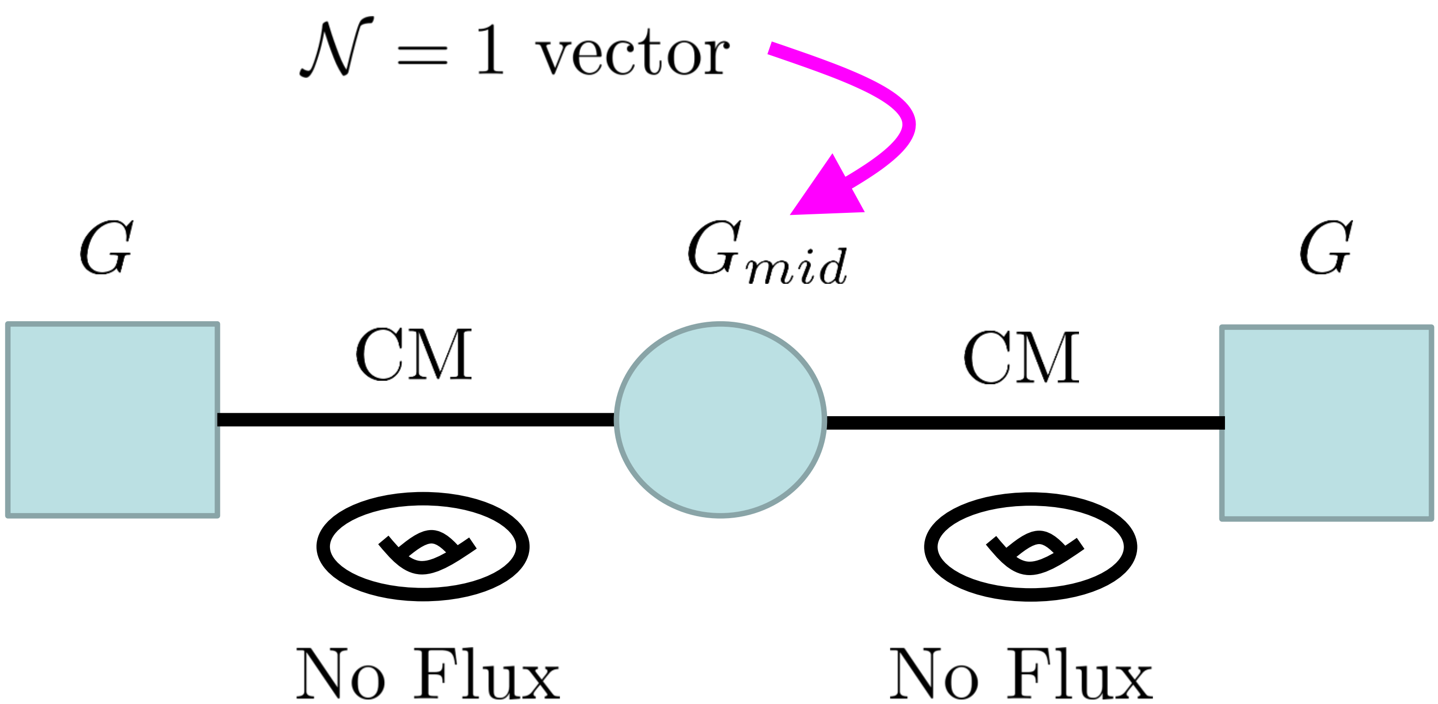


$4D\ \mathcal{N} = 1$ SCFT

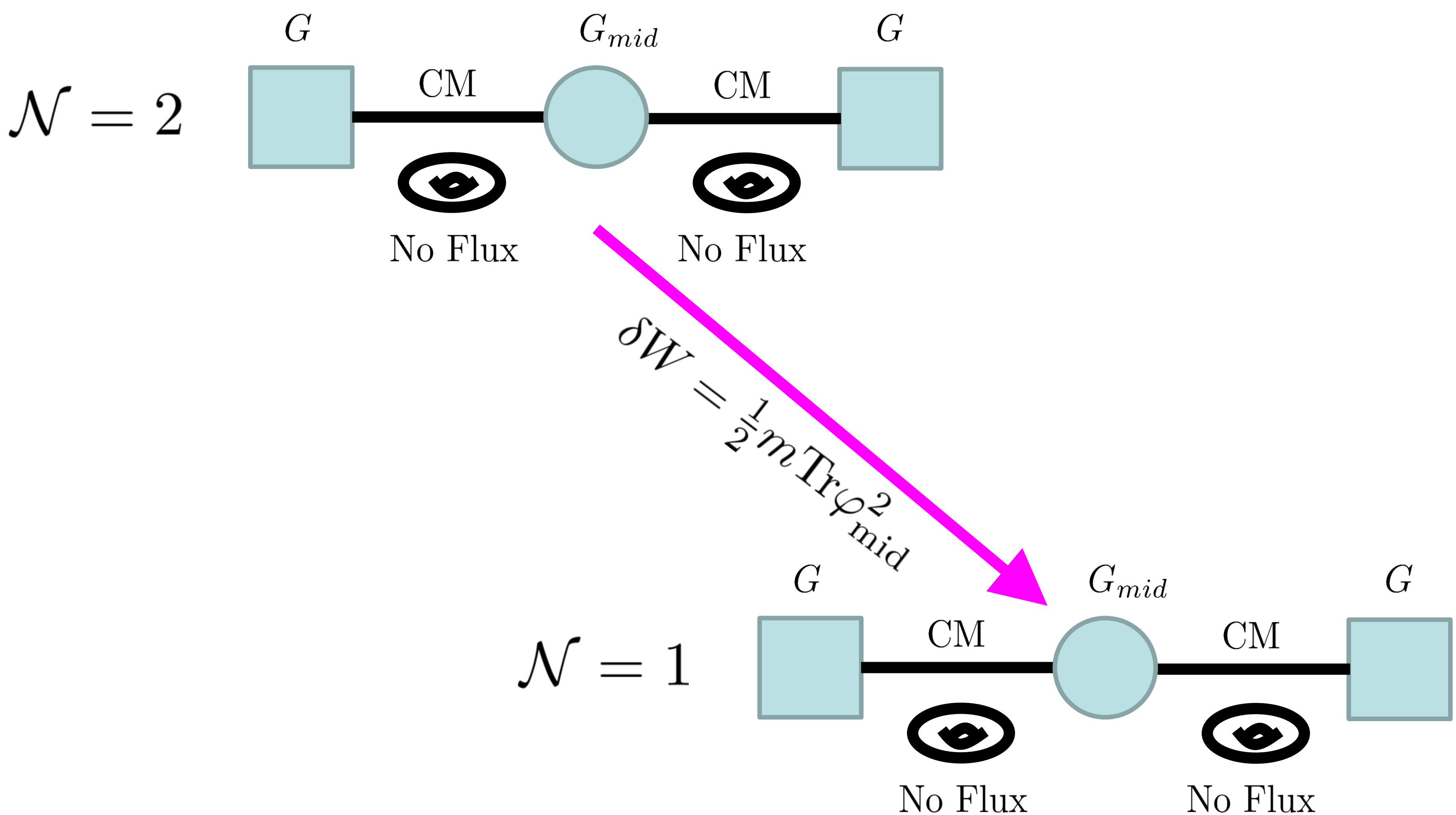


$$b_{G_{mid}} = 3h^{\vee}_{G} - 3b_{\mathrm{CM}} = 0$$

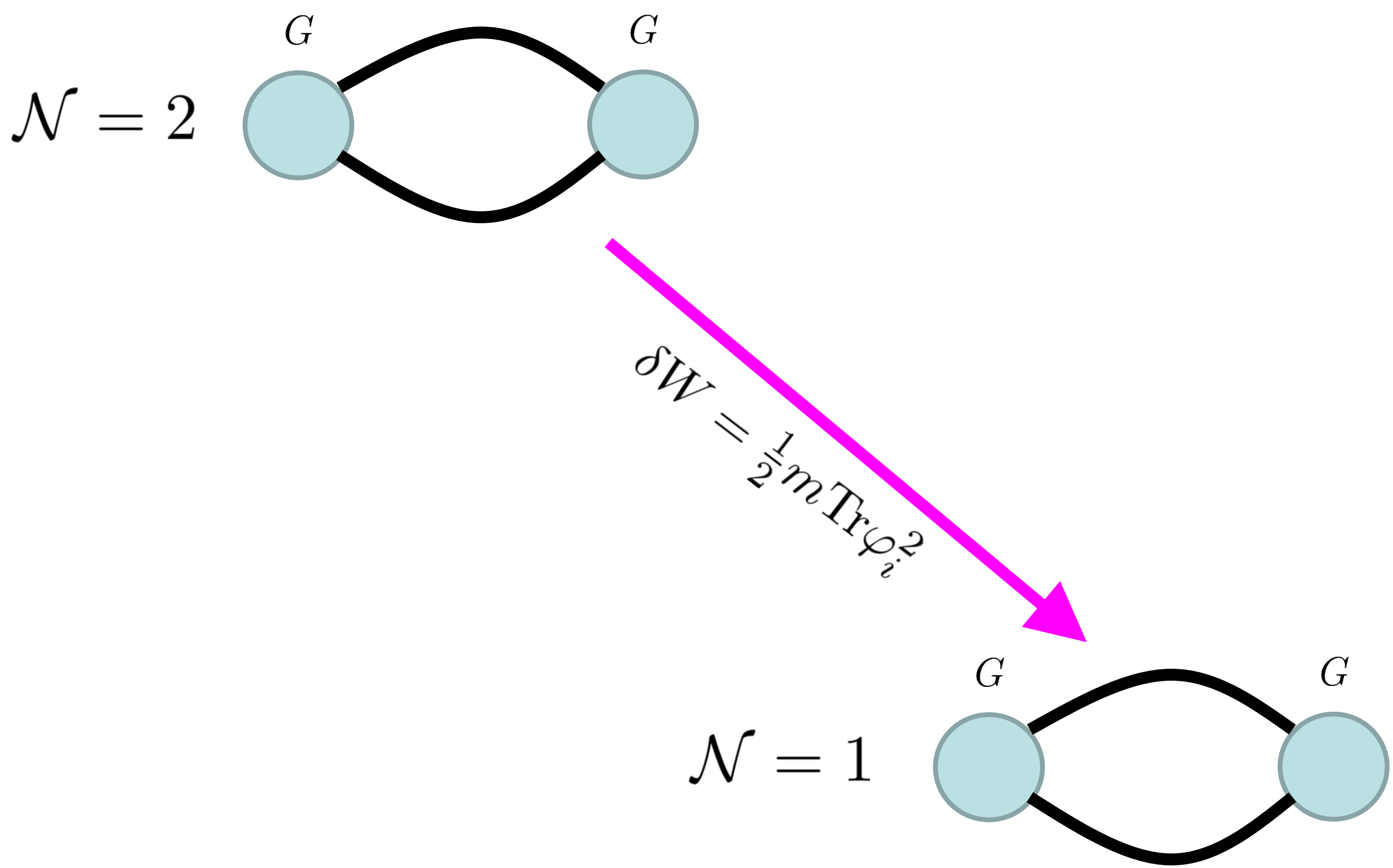
Another 4D $\mathcal{N} = 1$ SCFT



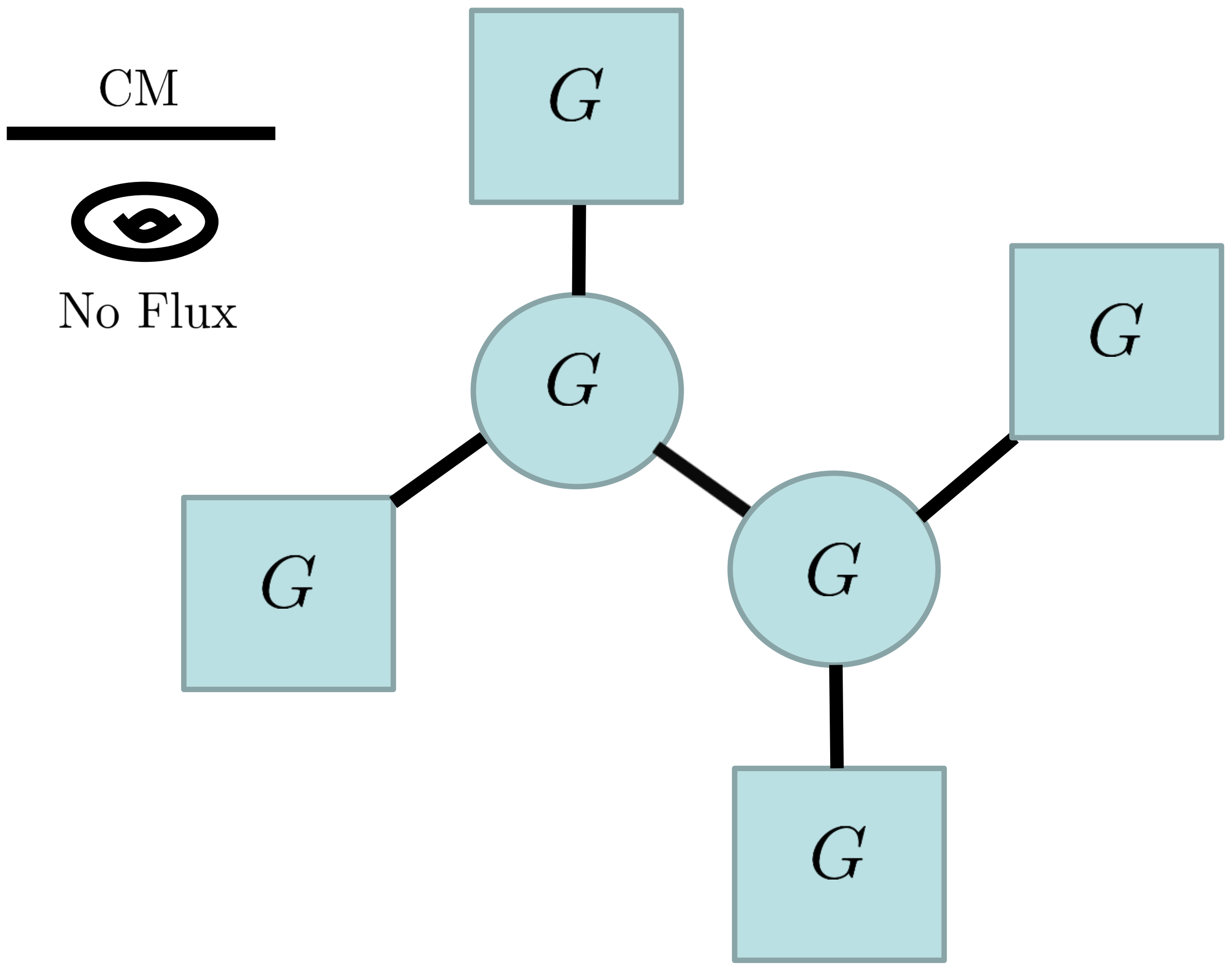
Another 4D $\mathcal{N} = 1$ SCFT



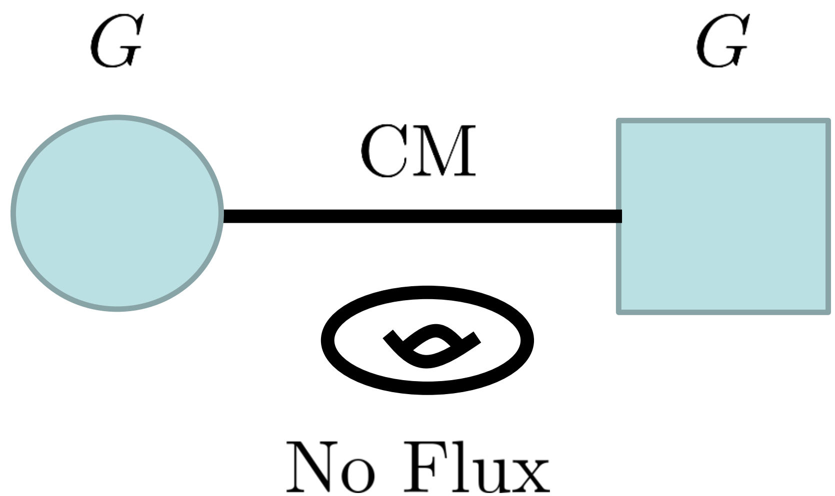
G-Type Conifold



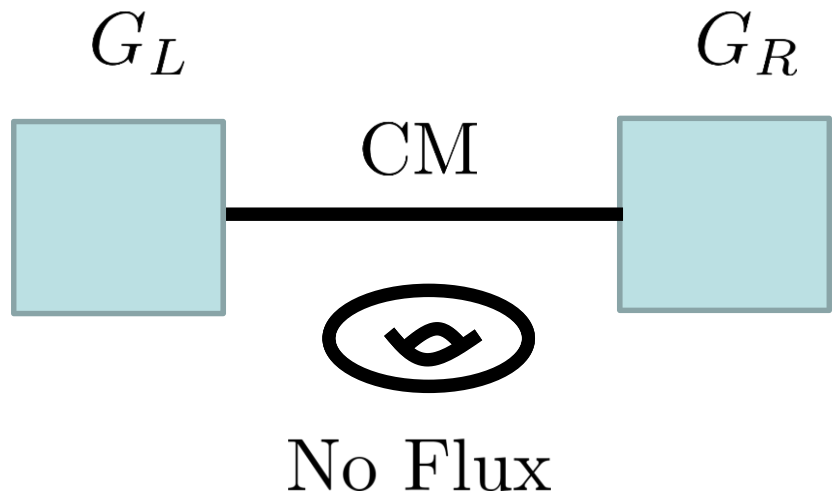
4D $\mathcal{N} = 1$ SCFT Networks



¿What About This Theory?

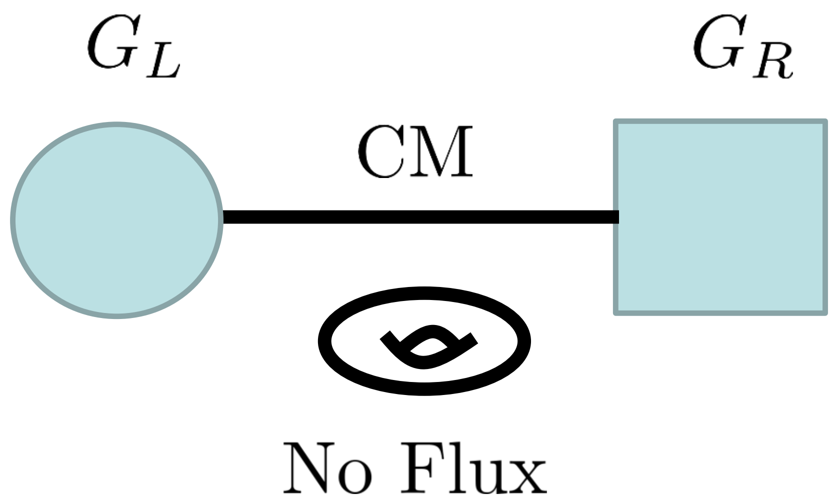


Via $\mathcal{N} = 2$ SUSY...



Dimension 2 Operators M_L and M_R in adjoint reps

¡Confinement!

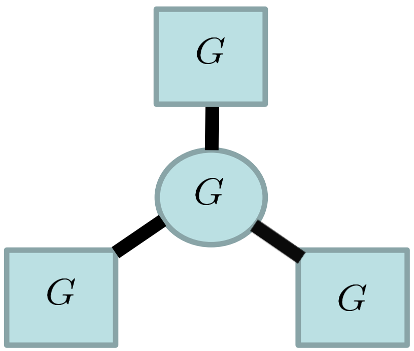


$$\Lambda^{b_{G_L}} = \text{Cas}_{h^\vee_{G_R}}(M_R) - \text{ “Baryons”}$$

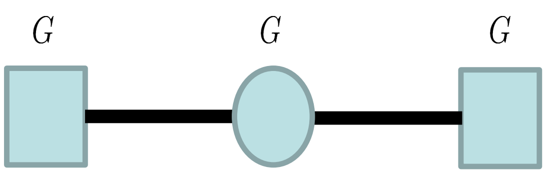
Note: $b_{G_L} = 3h^\vee_G - h^\vee_G = 2h^\vee_G$

Analogy with SQCD

Seiberg '94



$SU(N_c)$ with $N_f = 3N_c$



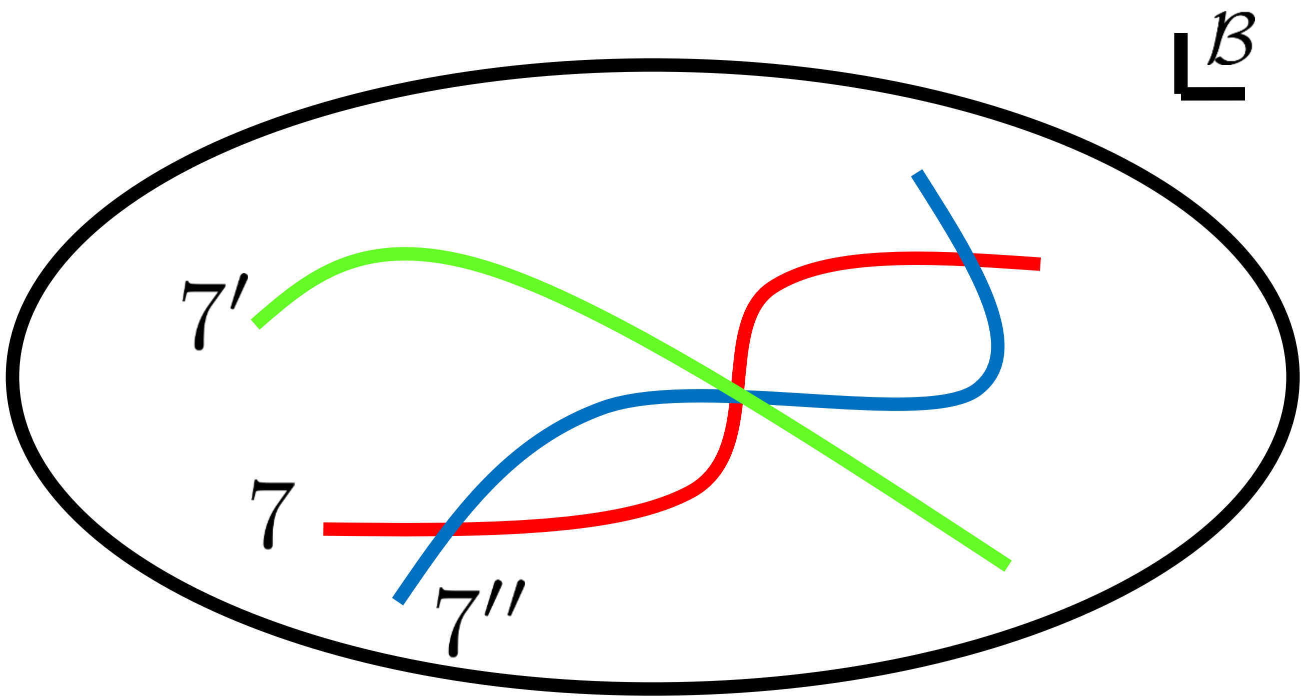
$SU(N_c)$ with $N_f = 2N_c$

$SU(N_c)$ with $N_f = N_c$

¿Conformal Window?

$$\frac{3}{2}h^\vee_G \lesssim b^{\text{matter}}_G \leq 3h^\vee_G$$

Yukawas


$\mathsf{L}^{\mathcal{B}}$
$7'$
7
$7''$

Local Geometry

$$(E_6)^3 : y^3 = x^3 + (uvw)^4$$

$$(E_7)^3 : y^3 = x^3 + (uvw)^3 x$$

$$(E_8)^3 : y^3 = x^3 + (uvw)^5$$

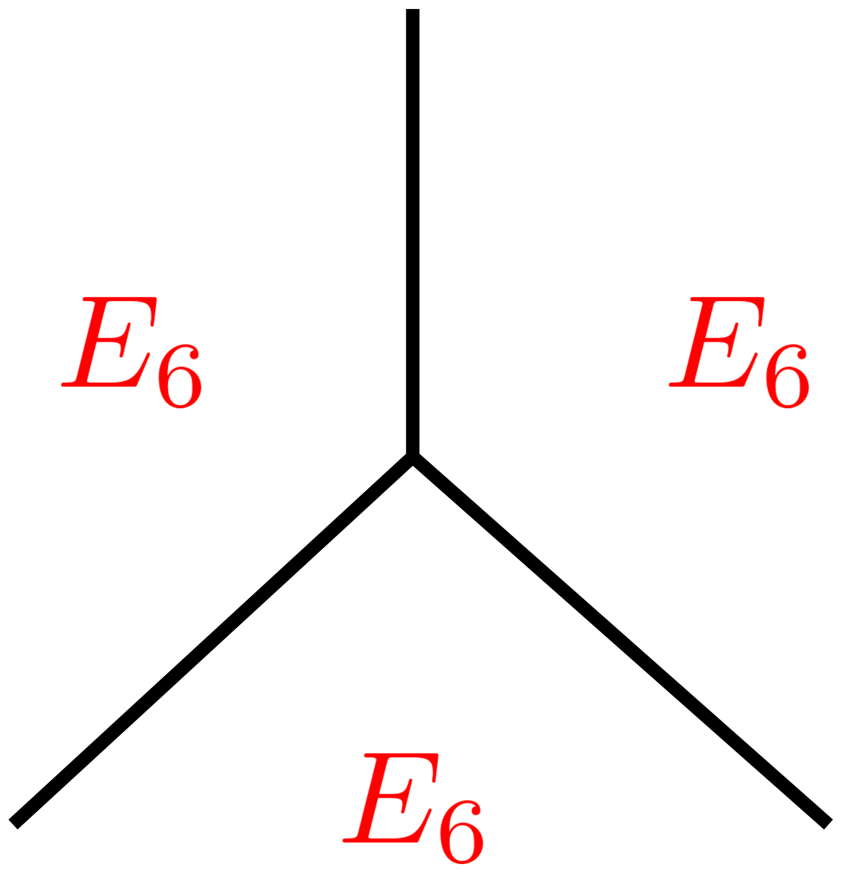
Blowups?

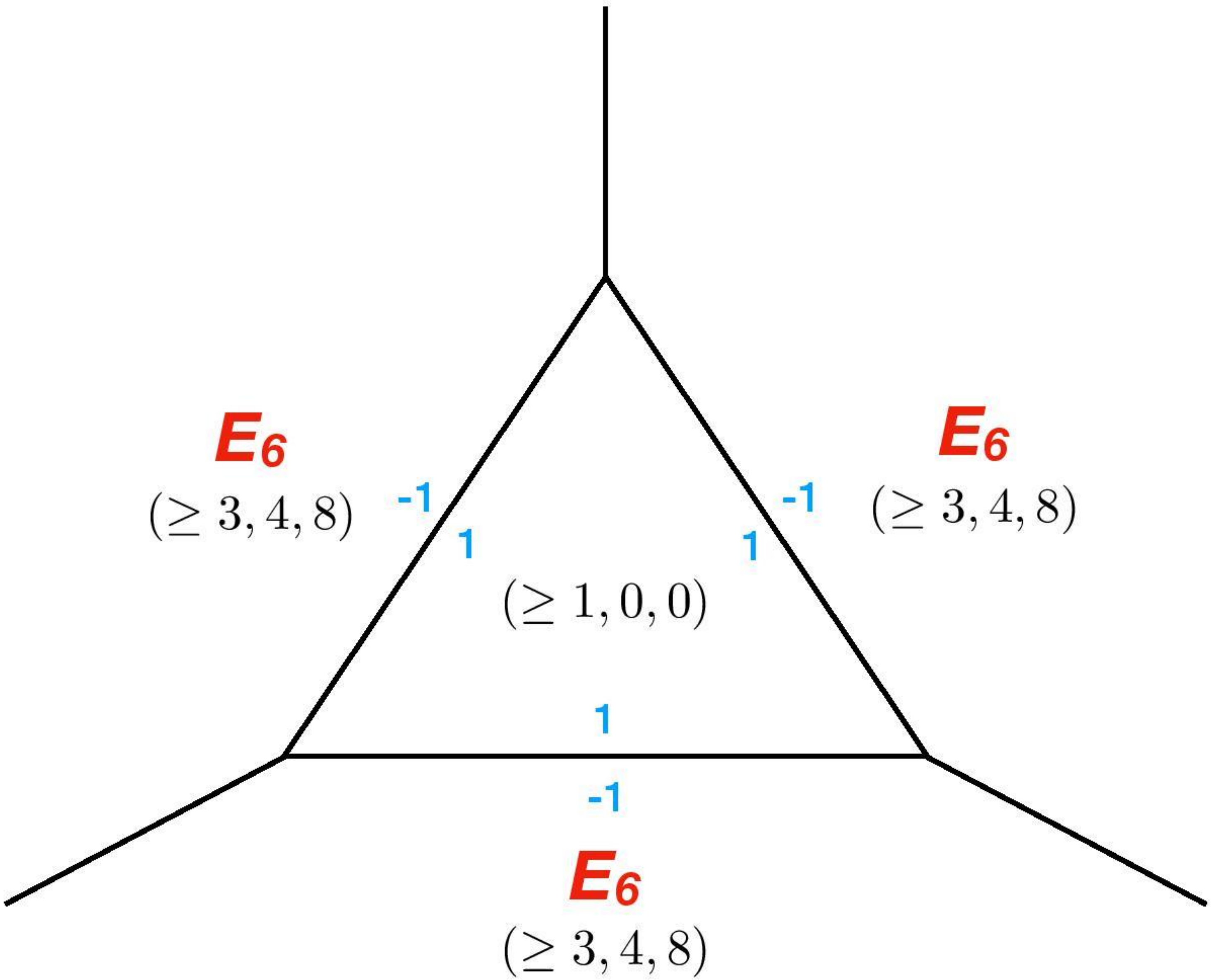
Codim 2: $\text{mult}(f, g, \Delta) \geq (4, 6, 12)$

Codim 3: $\text{mult}(f, g, \Delta) \geq (8, 12, 24)$

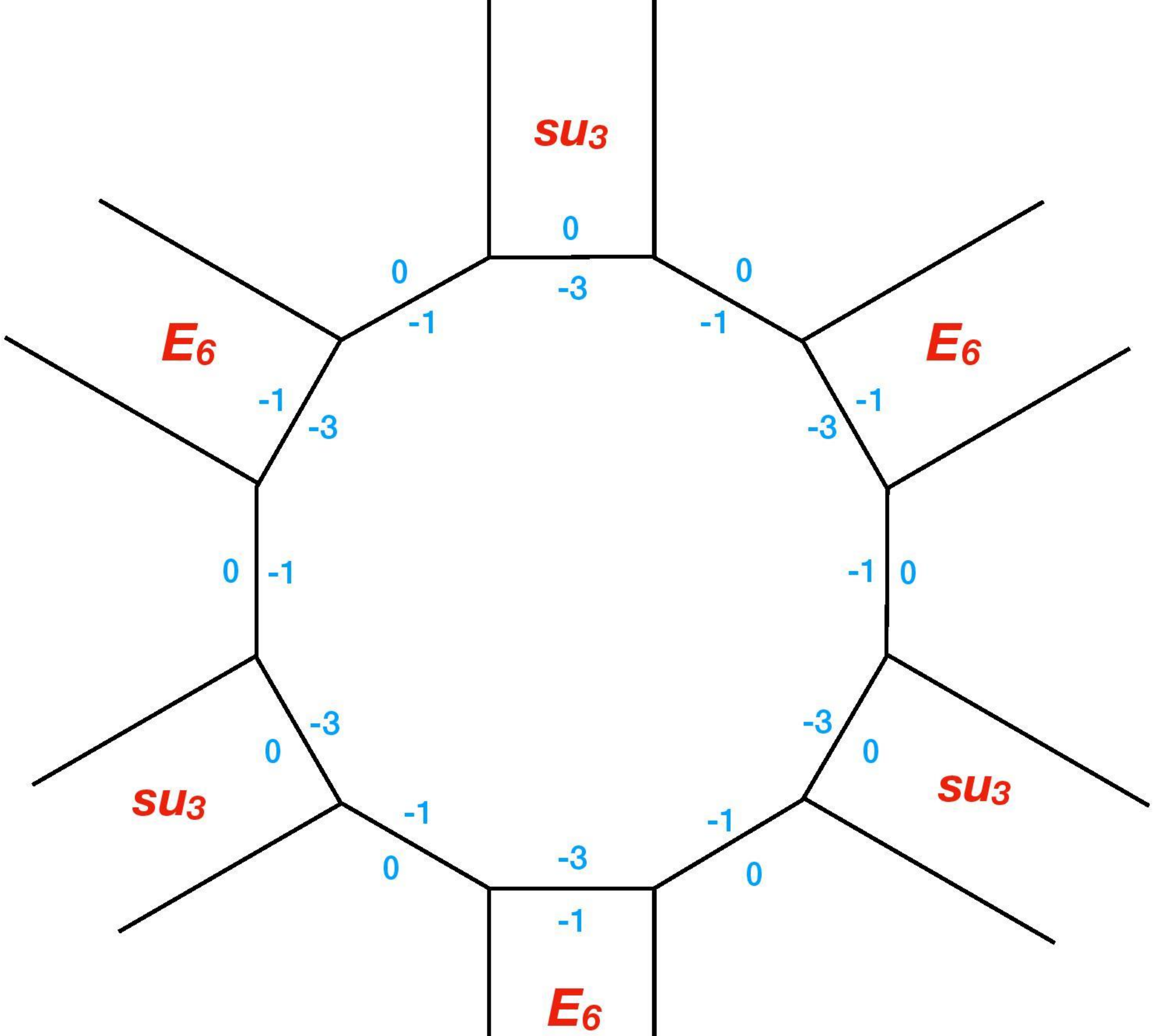
Codim 4: $\text{mult}(f, g, \Delta) \geq (12, 18, 36)$
(for 2D models)

$$y^2 = x^3 + (uvw)^4$$



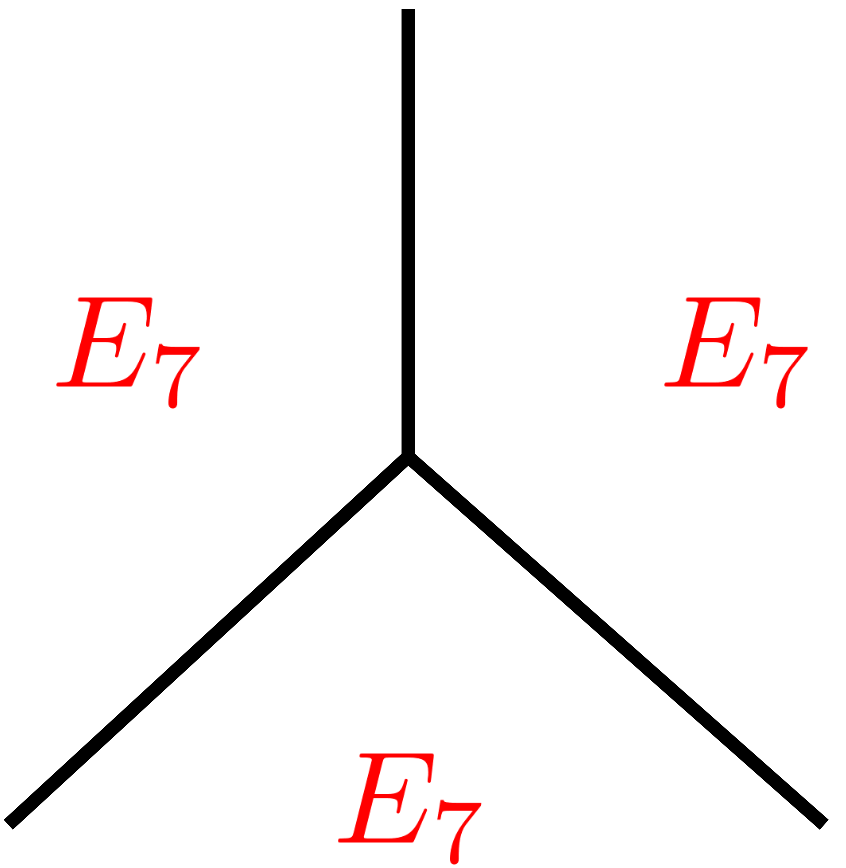


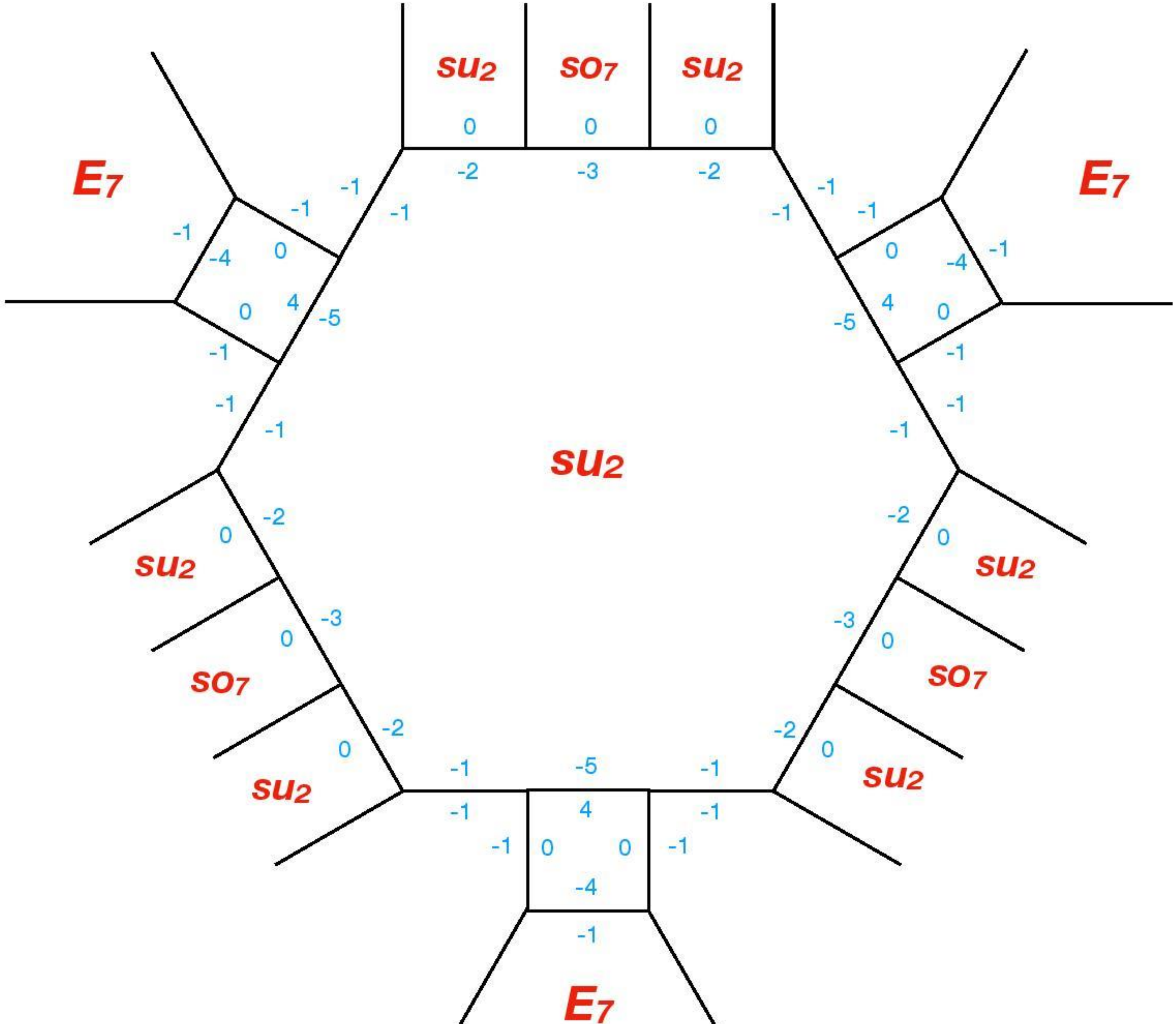
E6
(≥ 3, 4, 8)
-1
1
E6
(≥ 3, 4, 8)
-1
1
(≥ 1, 0, 0)
1
-1
E6
(≥ 3, 4, 8)



su3
0
0
-3
0
E6
-1
-1
E6
-1
-3
-3
-1
0
-1
-1
0
-3
-3
0
0
su3
su3
-1
-1
-3
0
0
-1
E6

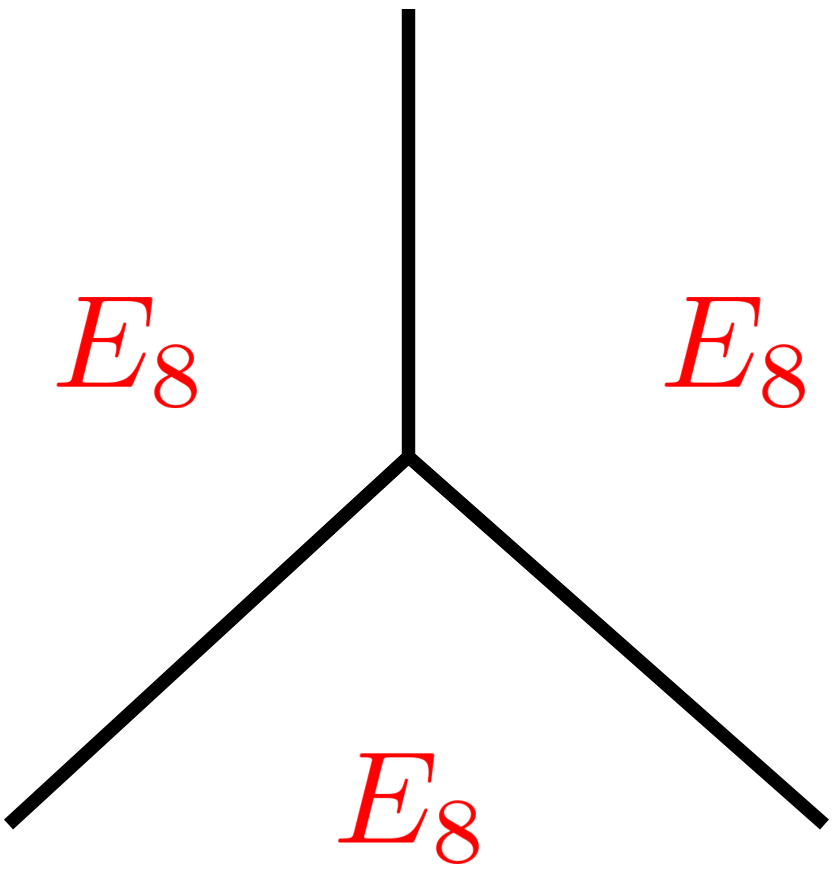
$$y^2 = x^3 + (uvw)^3 x$$

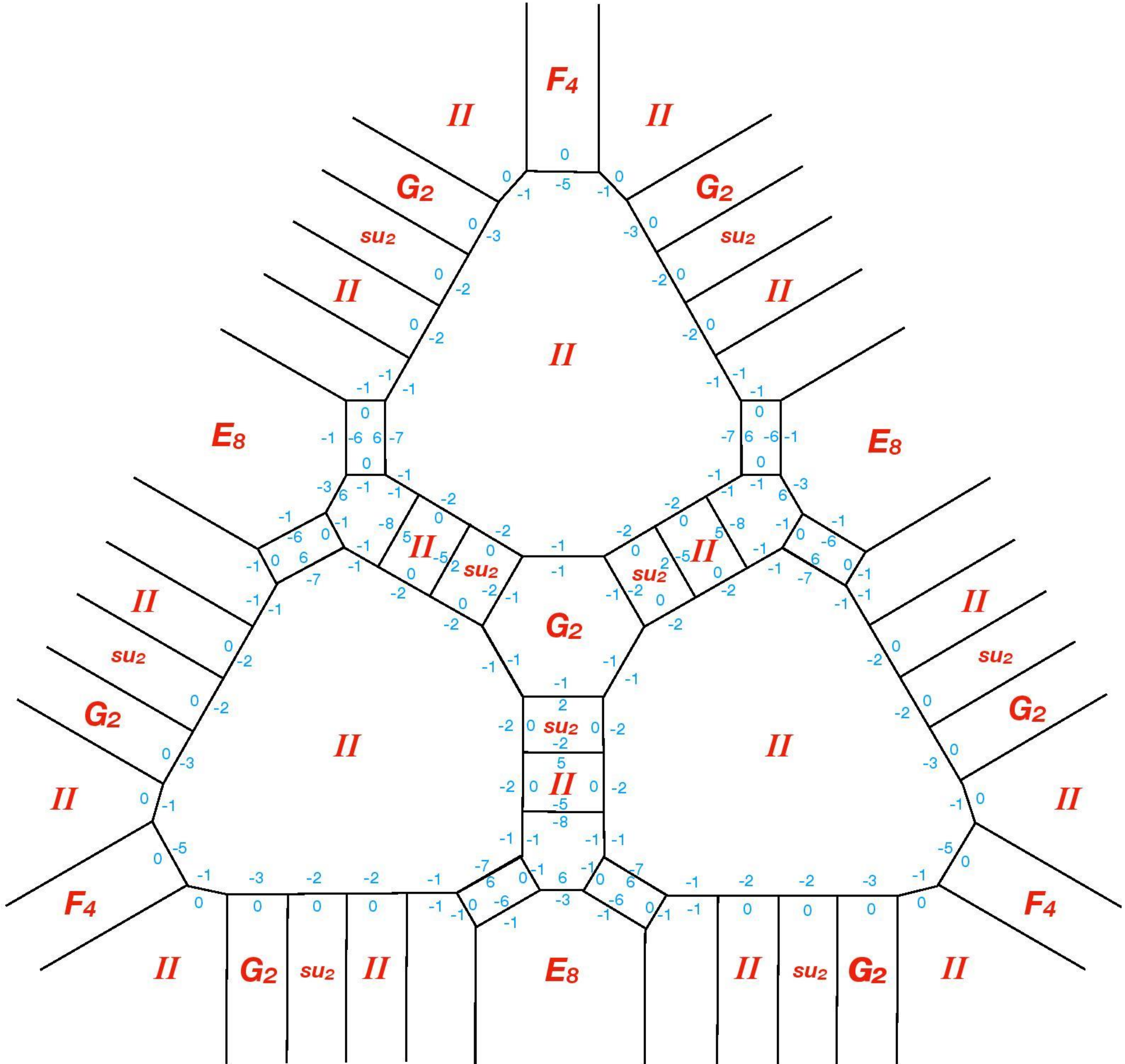




su2
so7
su2
0
0
0
-2
-3
-2
E7
E7
-1
-1
-1
-1
-1
-1
-1
-4
0
-4
0
-1
0
4
-5
-5
4
0
-1
-1
-1
-1
-1
-1
su2
-2
-2
0
0
su2
su2
-3
-3
0
0
so7
so7
-2
-2
0
0
su2
su2
-1
-5
-1
4
-1
-1
-1
0
0
-1
-4
-1
E7

$$y^2 = x^3 + (uvw)^5$$

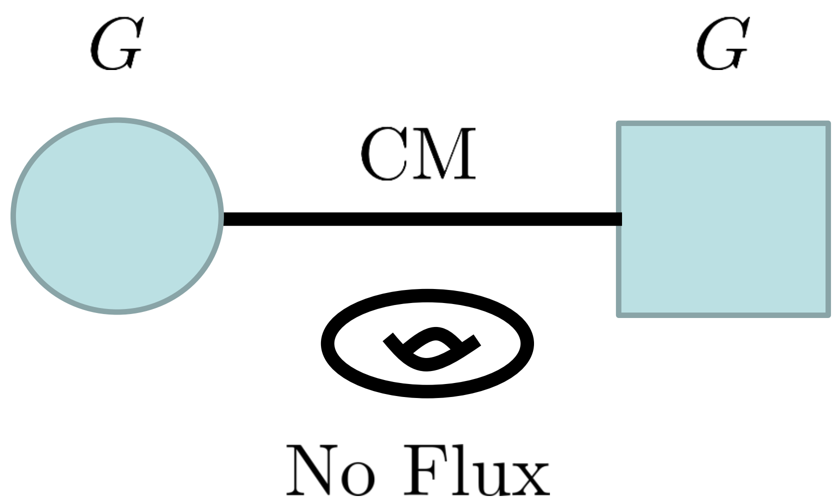



F4
II
II
G2
G2
su2
su2
II
II
II
E8
E8
II
II
su2
su2
G2
II
II
su2
su2
G2
G2
II
II
II
II
II
F4
F4
II
G2
su2
II
E8
II
su2
G2
II

Quantum Corrections

Instanton Effects

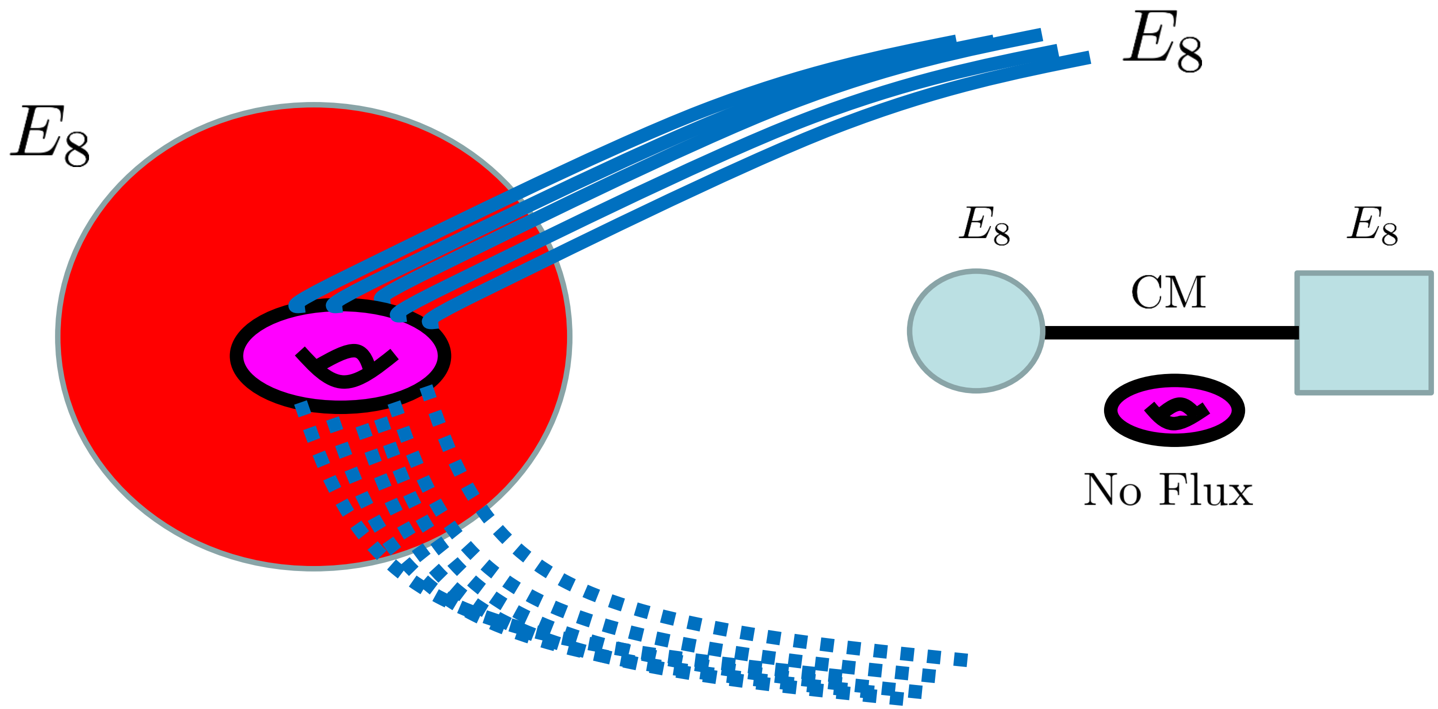
Generically, expect instantons / corrections:



$$\Lambda^{b_{G_L}} = \mathrm{Cas}_{h^\vee_{G_R}}(M_R) - \text{“Baryons”}$$

Example

$$y^2 = x^3 + v^5 (g_{[-K_S]})^5 \text{ with } (v = 0) \text{ a del Pezzo}$$



Kähler and Cplx Mixing

$$y^2 = x^3 + v^5 (g_{[-K_S]})^5 \to y^2 = x^3 + (vg - r)^5$$

$$(\Lambda_{UV} \times e^{-\text{Vol}_{\mathbb{C}}})^{b_{G_L}} = \text{Cas}_{h^\vee_{G_R}}(M_R) - \text{ ``Baryons''}$$

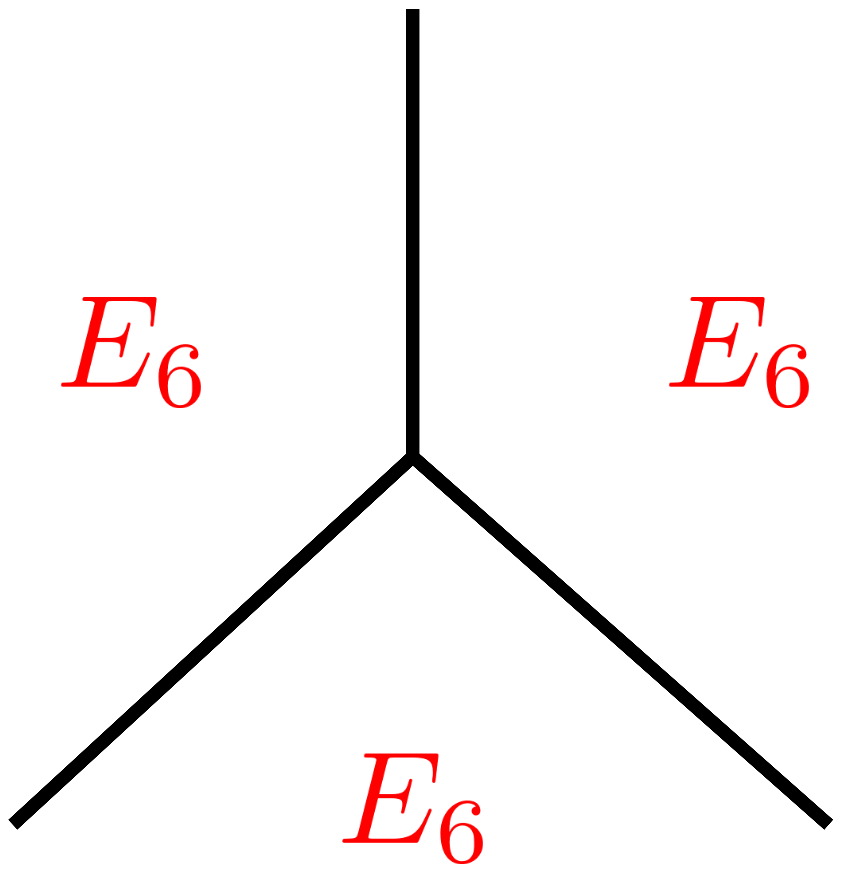
From Euclidean D3-brane

Deformation Types

"Mesonic Branch": $y^2 = x^3 + (f + \delta f)x + (g + \delta g)$
(Complex Structure)

"Baryonic Branch": $(y^2 = x^3 + fx + g) + \delta(\text{Deligne})$
(T-Branes)

Classical Geometry

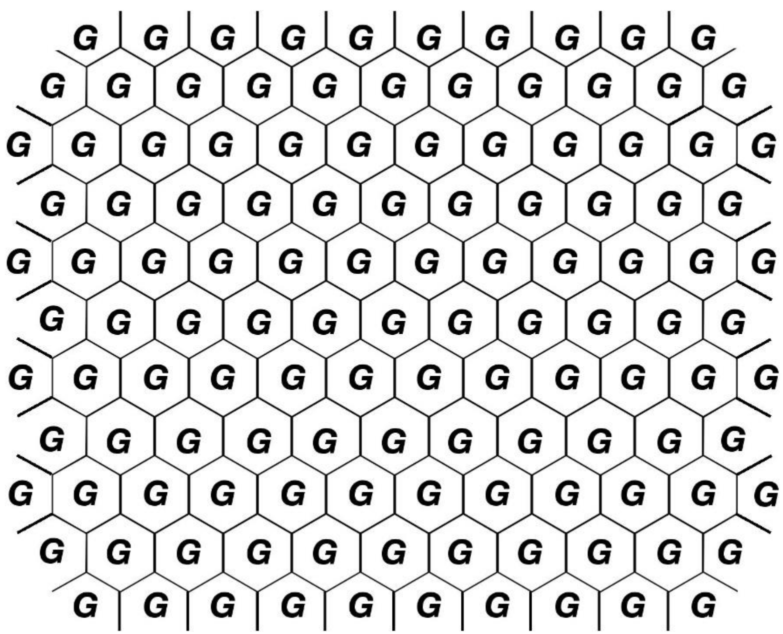


Quantum Corrections

$$y^3 = x^3 + (uvw)^4 \to y^2 = x^3 + (g(u,v,w))^4$$

$$(E_6)^3_{\text{classical}} \to (E_6)_{\text{quantum}}$$

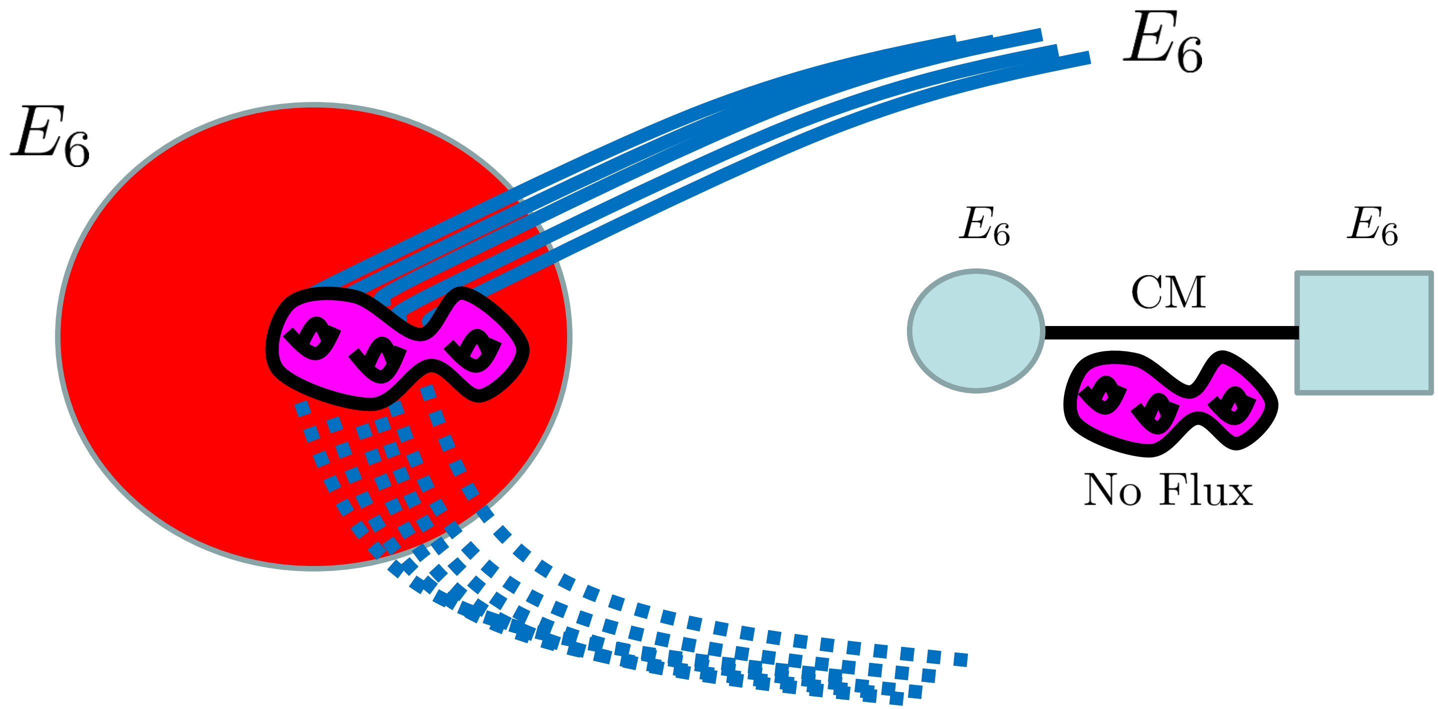
More Involved Example



→ All G's Confine

SCFT at Top of Window

$y^2 = x^3 + v^4 (g_{4H})^4$ (genus 3 curve in $\mathbb{P}^2$)



Conclusions / Future

Conclusions

- Cpctified 6D conf. matter $\rightarrow$ 4D conf. matter

- Gauging / Yukawas all embed in F-theory

- Quantum Corrections to Classical Moduli

Future

- ¿E-type Seiberg Dualities?

- ¿Superconformal Indices via Theory Gluing?

- ¿Spectral Cover Description?

- ¿2D QFTs and CY_5's?