F-theory on Quotient Threefolds with (2,0) Discrete Superconformal Matter

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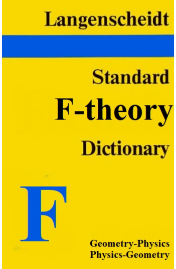
Virginia Polytechnic Institute and State University

Based on
- arXiv:1801.08658 with: L. Anderson, J. Gray and A. Grassi

The Geometry & Physics of F-theory V
Madrid, March 5th 2018

Deutsche Forschungsgemeinschaft

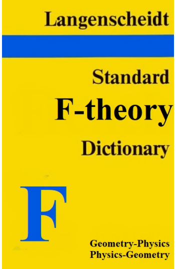
F-theory Dictionary



F-theory Dictionary

- **Geometry** of Torus fibered Calabi-Yau n-folds (Compact)

$\updownarrow$



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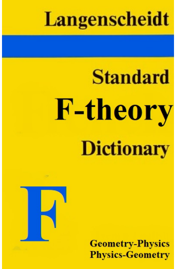

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• **Physics** of $12 - 2n$ Dimensional Supersymmetric Gauge Theories (+Gravity)

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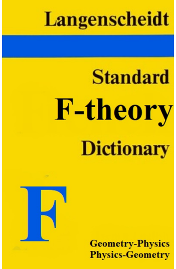


Limits of the Dictionary:

- Physics obtained indirectly via string dualities
- No fundamental 12 D theory formulated

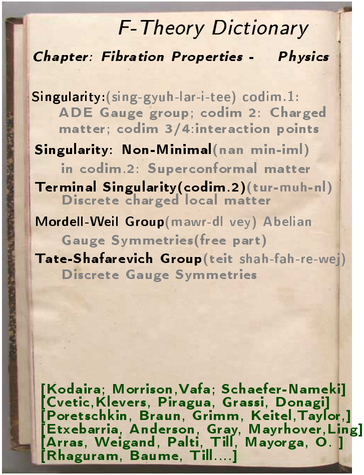
$\rightarrow$ What are the limits of the dictionary?

↕

- **Physics** of $12 - 2n$ Dimensional Supersymmetric Gauge Theories (+Gravity)

The F-Theory Dictionary

Dictionary turned out to be extremely **flexible**



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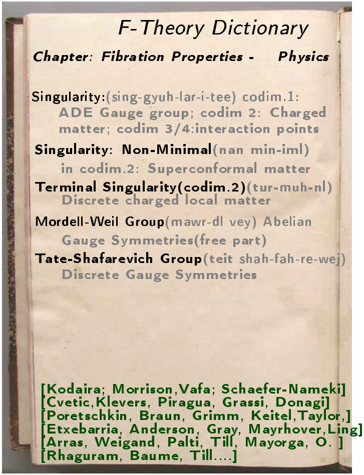

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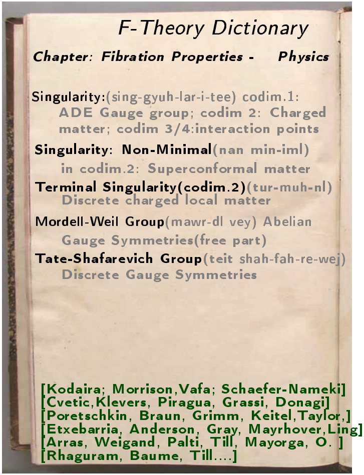

F-Theory Dictionary

Chapter: Fibration Properties - Physics

Singularity:(sing-gyuh-lar-i-tee) codim.1: ADE Gauge group; codim 2: Charged matter; codim 3/4:interaction points

Singularity: Non-Minimal(nan min-iml) in codim.2: Superconformal matter

Terminal Singularity(codim.2)(tur-muh-nl) Discrete charged local matter

Mordell-Weil Group(mawr-dl vey) Abelian Gauge Symmetries(free part)

Tate-Shafarevich Group(teit shah-fah-re-wej) Discrete Gauge Symmetries

[Kodaira; Morrison,Vafa; Schaefer-Nameki]
[Cvetic,Klevers, Piragua, Grassi, Donagi]
[Poretschkin, Braun, Grimm, Keitel,Taylor,]
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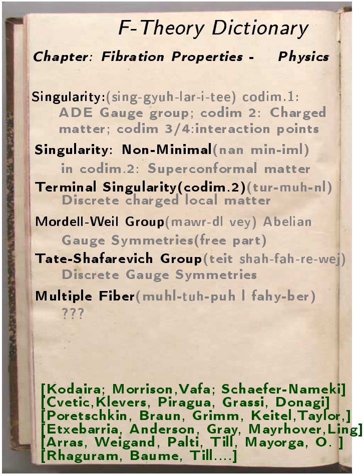

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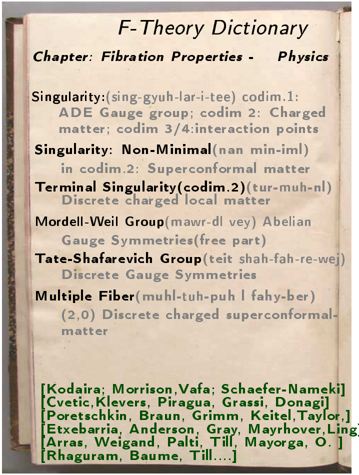

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Multiple Fiber(muhl-tuh-puh l fahy-ber) (2,0) Discrete charged superconformal-matter

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$\rightarrow$ **Discrete (2,0) superconformal matter**
Characterized by a Lens space $L(n,k)$

- Tensor branch: Hyperconifold resolution of $L(n,k)$
- Introducing purely discrete charged singlets

Outline

Quotient Calabi-Yau Geometries

Start with a **Torus fibered** threefold Y_3 **quotiented** by an $\mathbb{Z}_n$ automorphism Γ_n to obtain $\widehat{Y}_3 = Y_3/\Gamma_n$ [Donagi, Ovrut, Pantev, Waldram'99]

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3. Γ_n **respects fibration**/projection π:

$$\begin{array}{ccl} T/\Gamma_{n,f} & \rightarrow & \hat{Y} = Y/\Gamma_n \\ & & \downarrow \pi \\ & & \hat{B} = B/\Gamma_{n,b} \end{array}$$

$\rightarrow \Gamma_n$ **decomposable** into fiber & base action $\Gamma_n = \Gamma_{F,n} \oplus \Gamma_{b,n}$

Fiber and Base must not be mixed!

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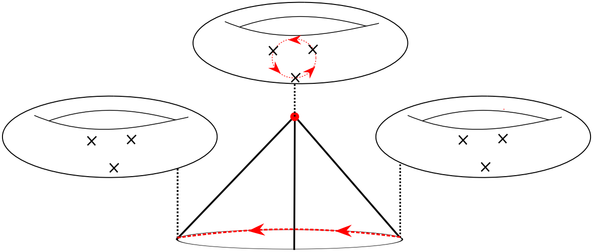
$\rightarrow$ Quotient Base $\hat{B}$ has **orbifold fixed points**

Quotient Fiber Action



Avoid **fixed fibers** over the $\Gamma_{n,b}$ **orbifold** fixed points
$\rightarrow \Gamma_{n,f}$ must act as a **fiber rotation** [Bhardwaj, Del Zotto, Heckman, Morrison, Rudelius, Vafa'15]

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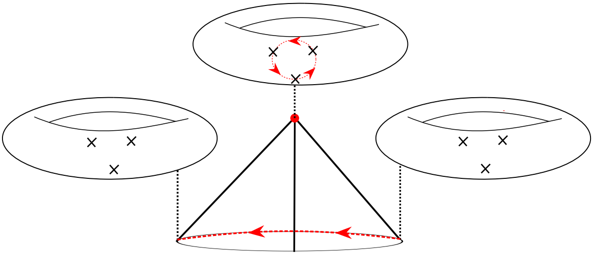

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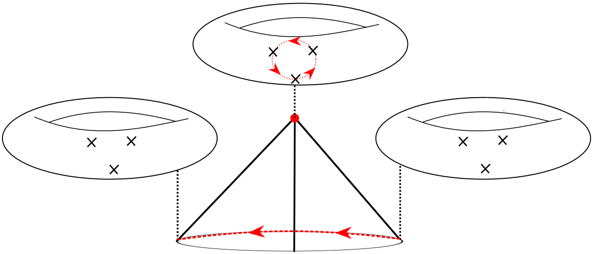

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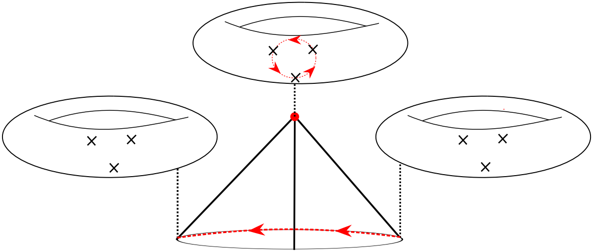

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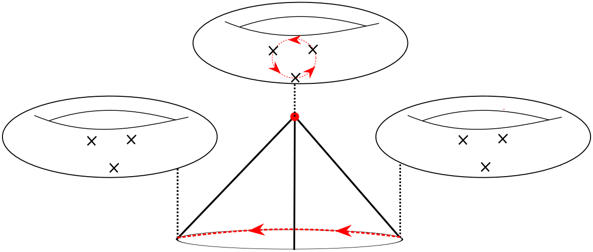

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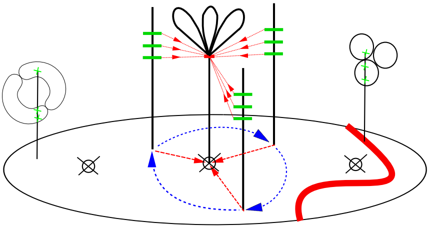
Quotient creates **genus-one fibration with multiple fibers**

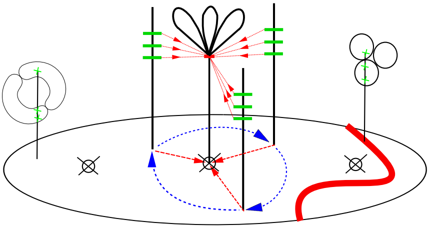
Quotient Geometry



Multiple Fiber

- Over a fixed point in the base $s \in \widehat{B}$ the fiber $\mathcal{C}$ is
$$\mathcal{C}_s = \pi^{-1}(s)$$
- The fiber $\mathcal{C}_s$ is singular **everywhere** $\rightarrow$ multiple fiber [Gross'93]

Quotient Geometry



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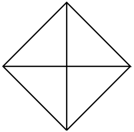
Fibration away from the fixed points

- Regular **genus-one** fibration with $n-$sections
- ADE divisors in the base miss the fix points $\rightarrow$ Cartier in $H_2(\widehat{B}, \mathbb{Z})$.

Tetra Quadric Geometry

Base Space: $\mathbb{F}_0^b$:



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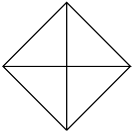

- Start with the **Tetra Quadric** hypersurface P in $\mathbb{F}_0^f \times \mathbb{F}_0^b$
- P is a **generic degree two** hypersurface in $(x_i, y_i | v_i, w_i)$ with $i = 0, 1$
- **Genus-one fibered** CY hypersurface $K_{\mathcal{C}}^{-1} \in \mathcal{O}(2, 2)$

$$P = (b_1 y_0^2 + b_2 y_1 y_0 + b_3 y_1^2) x_0^2 + (b_4 y_0^2 + b_5 y_1 y_0 + b_6 y_1^2) x_0 x_1 + (b_7 y_0^2 + b_8 y_1 y_0 + b_9 y_1^2) x_1^2$$

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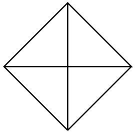

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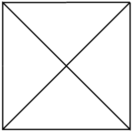
- **Sections of the line bundles** $b_j \in K_b^{-1} = \mathcal{O}(2,2)$ of the base $\mathbb{F}_0^b$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_{\chi} = (4, 68)_{-128}$

Tetra Quadric Quotient Geometry

Base Space: $\mathbb{F}_0^b/\mathbb{Z}_2$: 

- Take **quotient** on the **ambient space** of $P \subset (\mathbb{F}_0^f \times \mathbb{F}_0^b)/\mathbb{Z}_2$ [Batyrev, Kreutzer'05]
- $\mathbb{Z}_2$ **identification:** $(x_i, y_i, v_i, w_i) \sim ((-1)^i x_i, (-1)^i y_i | (-1)^i v_i, (-1)^i w_i)$
- $4_{\text{fiber}} \times 4_{\text{base}}$ ambient fixed points

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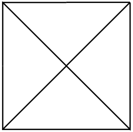
Base Space: $\mathbb{F}_0^b/\mathbb{Z}_2$: 

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- $4_{\text{fiber}} \times 4_{\text{base}}$ ambient fixed points
- **Still smooth genus-one fibered** CY hypersurface $K_{\mathcal{C}}^{-1} \in \mathcal{O}(2,2)$
  $P = (b_1 y_0^2 + b_2 y_1 y_0 + b_3 y_1^2) x_0^2 + (b_4 y_0^2 + b_5 y_1 y_0 + b_6 y_1^2) x_0 x_1 + (b_7 y_0^2 + b_8 y_1 y_0 + b_9 y_1^2) x_1^2$
- Not every monomial is $\mathbb{Z}_2$ **invariant:** $b_1 \ni v_0^2 v_1^2 \checkmark + v_0 v_1 w_0^2 \text{X} + \ldots$

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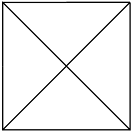

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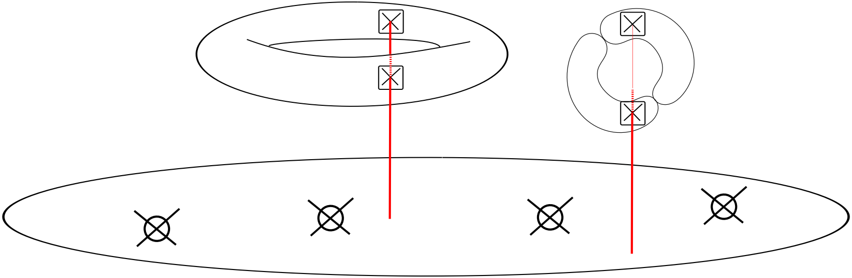
- The base sections must be $\mathbb{Z}_2$ **covariant**: $\Gamma_2 b_j^{(\pm)} \to \pm b_j^{(\pm)}$
- **Hodge numbers:** $(h^{(1,1)}, h^{(2,1)})_\chi = (4, 36)_{-64}$

The multiple fiber geometry



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- **Matter** multiplets **away** from the **fixed points**

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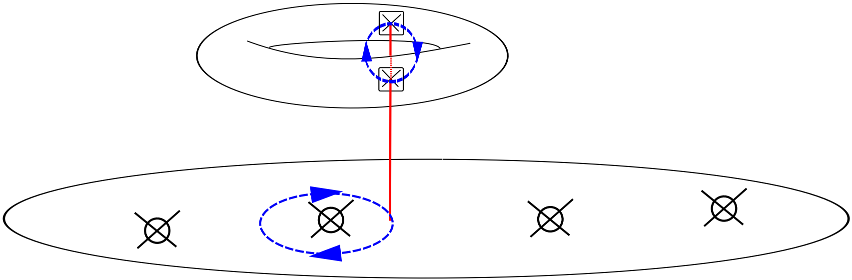

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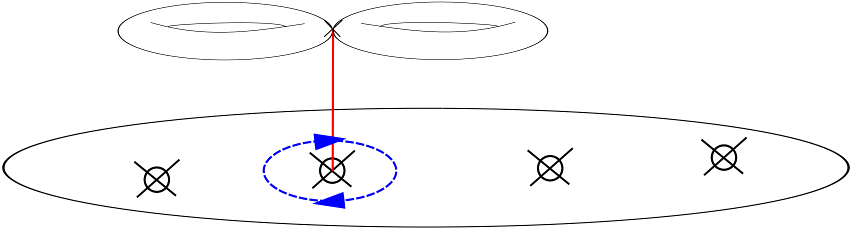

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F-theory Physics of the Quotient

- **On the Covering geometry** we have
- $U(1) \times \mathbb{Z}_2$ 6d SUGRA theory [Klevers, Mayorga, O., Piragua, Reuter'14]

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$$
\begin{array}{lll}
H: & \mathbf{1}_{(0,-)}: & = 80 \\
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\hline
 & \mathbf{1}_{(0,+)} & = 69 \\
T: & & = 1 \\
V: & & = 1
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$$T: \quad = 1$$

$$V: \quad = 1$$

- With **satisfied** anomalies

$$\text{Grav}^4 \qquad H - V - 29T - 273 = 0\,, \qquad 9 - T - \underbrace{(K_b^{-1})^2}_{=8} = 0 \checkmark$$

$$U(1)^2\text{Grav}^2 \qquad -\frac{1}{6}\sum_q H_q q^2 = -2b_{11} \cdot \mathcal{K}_b^{-1} \checkmark$$

F-theory Physics of the Quotient

- **On the Quotient Geometry** we find
- $U(1) \times \mathbb{Z}_2$ gauged 6d SUGRA theory
- **Massless spectrum changes**

$$
\begin{array}{ll}
H: & \boxed{\begin{array}{ll} \mathbf{1}_{(0,-)} & = 80 \to 40 \\ \mathbf{1}_{(1,-)} & = 48 \to 24 \\ \mathbf{1}_{(1,+)} & = 48 \to 24 \\ \hline \mathbf{1}_{(0,+)} & = 69 \to 37 \end{array}} \\
T: & = 1 \\
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\end{array}
$$

- With **satisfied** anomalies

$$\text{Grav}^4 \qquad H - V - 29T - 273 = 0\,, \qquad 9 - T - \underbrace{(K_b^{-1})^2}_{=8} = 0 \checkmark$$

$$U(1)^2\text{Grav}^2 \qquad -\frac{1}{6}\sum_q H_q q^2 = -2b_{11} \cdot \mathcal{K}_b^{-1} \checkmark$$

F-theory Physics of the Quotient

- **On the Quotient Geometry** we find
- $U(1) \times \mathbb{Z}_2$ gauged 6d SUGRA theory
- **Massless spectrum changes**

$$
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- With **unsatisfied** gravitational anomalies

$$\text{Grav}^4 \qquad H - V - 29T - 273 = -4 \cdot 30\,, \quad 9 - T - \underbrace{(K_b^{-1})^2}_{=8 \to 4} = -4 \quad X$$

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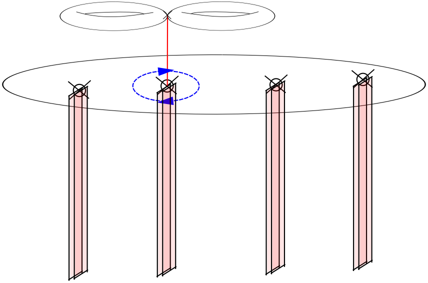
(2,0) Subsectors



- **Spacetime filling M5 branes** probe local $\mathbb{C}^2/\mathbb{Z}_2$ singularities [Harvey, Menasian, Moore' 98]
- Each stack **contributes a free** A_1 (2,0) theory strongly coupled theory
  - $A_1^{(2,0)} \sim T_{(1,0)} \oplus H_{1_0}$

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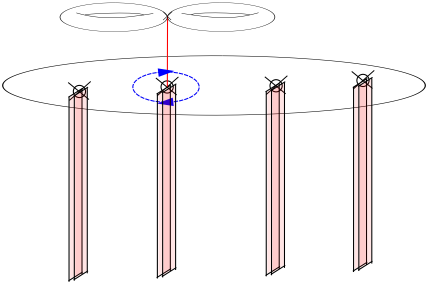

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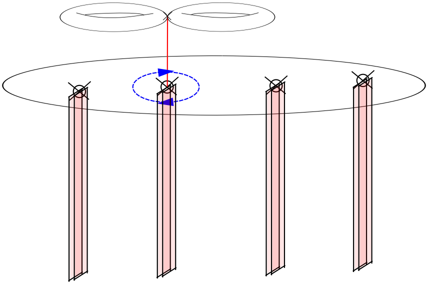

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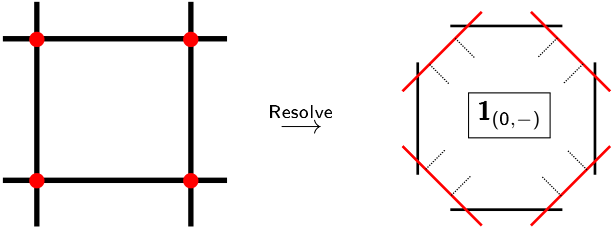
- Does the multiple fiber **gauge** the (2,0) subsectors ?

(2,0) Tensor branch



Tensor Branch Physics

- **Tune in** $\mathbb{Z}_2$ ambient fixed points onto $\widehat{Y_3}$ and resolve

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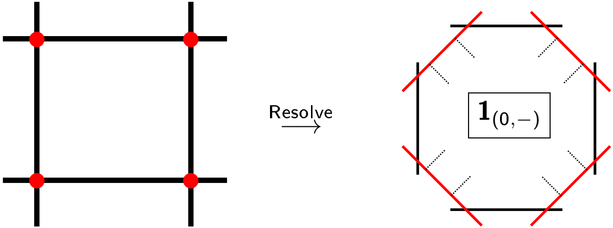

Tensor Branch Physics

- **Tune in** $\mathbb{Z}_2$ ambient fixed points onto $\widehat{Y_3}$ and resolve
- Obtain **smooth base, no multiple fibers**, change in the spectrum:

$$4 \times A_1^{(2,0)} \to 4 \cdot \left(T_{(1,0)} - 1 \cdot H_{\mathbf{1}_{(\mathbf{0},+)}} + 2 \cdot H_{\mathbf{1}_{(\mathbf{0},-)}} \right)$$

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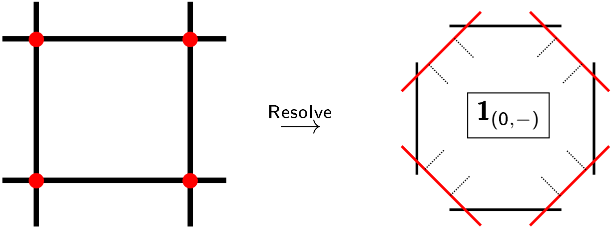

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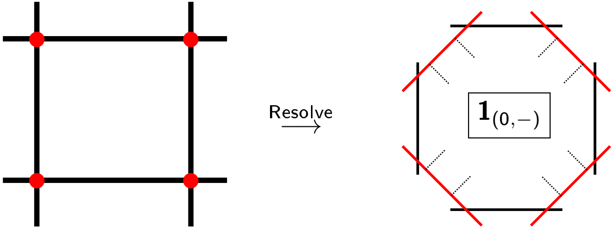

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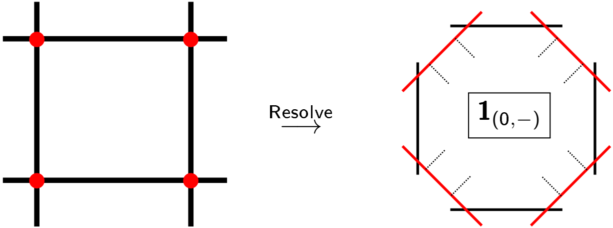

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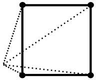
- **Unhiggs** $\mathbb{Z}_2 \to U(1)$: **height pairing** b_{22} **non-Cartier!**
  $\to b_{22}$ *fractional divisor*: **Strongly coupled** U(1) [Del Zotto, Heckman, Morrison, Park'14]

Tensor Branches and Hyperconifolds

Understand **tensor branch** as **hyperconifold** resolution of $\widehat{Y_3} = Y_3/\mathbb{Z}_n^k$ [Davis'11]

Tensor Branches and Hyperconifolds

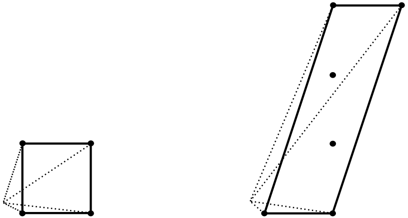


Understand **tensor branch** as **hyperconifold** resolution of $\widehat{Y_3} = Y_3/\mathbb{Z}_n^k$ [Davis'11]

Conifold Transition on the covering space Y

- **Toric Fan** $\Sigma_1 : \{(1,0,0)\,,(1,1,0)\,,(1,0,1)\,,(1,1,1)\,\}$
- **Deformation Phase**: three-sphere $\mathbb{S}^3$
- **Resolution Phase**: two-sphere $\mathbb{P}^1$

Tensor Branches and Hyperconifolds



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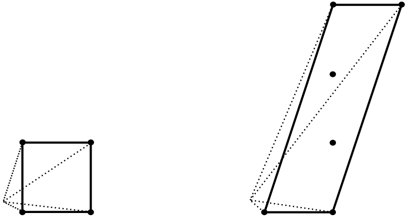
Hyperconifold: $\mathbb{Z}_n$ quotient of conifold on Y

- $\mathbb{Z}_n$ Refined Lattice: $\Sigma_1' = \{(1,0,0),(1,1,0),(1,k,n),(1,k+1,n)\}$
- **Deformation Phase**: Lens space $L(n,k)$ (twisted $\mathbb{S}^3$)
- **Resolution Phase**: Chain of $(n-1)\cdot\mathbb{P}^1$'s
- **Change** in Hodge numbers: $(\Delta h^{1,1},\Delta h^{2,1})_{\Delta\chi} = (n-1,-1)_{2n}$ [Davis'13]

Tensor Branches and Hyperconifolds



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Change in the 6d Spectrum

- $\mathbf{L(n,k)}$ **restricts** to a $\mathbb{Z}_n$ singularity in the base with **n-multiple fiber**
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$$\mathcal{A}_{n-1}^{(2,0)} \to \left((n-1)\cdot T_{(1,0)} - H_{\mathbf{1}_{(\mathbf{0})}} \qquad\qquad \right)$$

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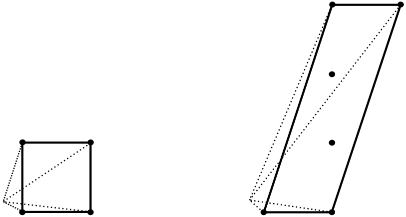

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- **Gravitational Anomaly**: Additional n-discrete $\mathbb{Z}_n$ charged singlets **necessary**

Summary and ...

Taken **freely acting quotients** Γ_n of genus-one fibered CY three-folds
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The Physics of the Multiple Fiber

- Contributes the d.o.f. of a free A_{n-1} (2,0) **superconformal point**
- **Tensor branch** obtained by resolution of a **Lens space**
- **Over the Tensor branch**, n purely $\mathbb{Z}_n$ charged hypermultiplets appear $\to$ **New type of discrete charged superconformal matter**

…More

- For $h^{1,1}$ preserving quotients: **general anomaly cancellation proven**
- **Coupling** the (2,0) theory **to** the **un-Higgsed U(1)**
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Thank You Very Much!

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