Abelian gauge symmetries and higher charge states from Matrix Factorization

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Based on work in progress with A. Collinucci, M. Fazzi and D. Morrison

Introduction

- In the M/F-theory geometric engineering, Abelian gauge symmetries emanate from reduction of C_3 along harmonic, normalizable 2-forms.

$$C_3 \sim A_\mu dx^\mu \wedge \omega$$

- The 2-form ω can be described via its Poincaré dual cycle (divisor).
- In F-theory, the elliptically fibered CY has extra sections, which are identified as new divisor classes giving rise to U(1)s. [Morrison,Vafa]

  Techniques expanded and refined over the past few years.

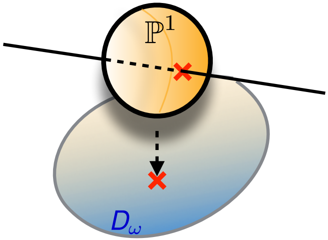
  [Grimm,Weigand; Morrison,Park; (Borchmann),Mayrhofer,Palti,Weigand; Cvetic,(Grassi),Klevers,Piragua,(Song),(Taylor); V.Braun,Grimm,Keitel; Braun,Collinucci,RV]
- In this talk, new way of detecting such divisors in varieties that admit small resolutions. We will focus on CY *three-fold*.

Introduction

Simplest way to get a massless U(1) in F-theory is 'U(1) restriction':
[Grimm,Weigand]

$$(y - a_3)(y + a_3) = x(x^2 + b_2 x + 2b_4)$$

- elliptic fibration has one conifold singularity at $x = y = a_3 = b_4 = 0$;
- small resolution: exceptional $\mathbb{P}^1$ intersects extra divisor D_ω at one point.



M2 couples to the 3-form:

$$\int_{M2} C_3 = \int A_\mu dx^\mu \int_{\mathbb{P}^1} \omega = (\mathbb{P}^1 \cdot D_\omega) \int A_\mu dx^\mu$$

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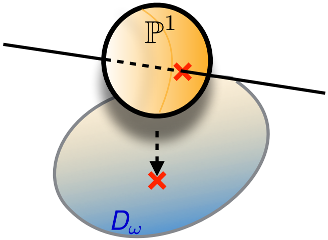

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'U(1) restriction' is so far also the only (compact) case where **Matrix Factorization (MF)** has been applied in F-theory. [Collinucci,Savelli]

This formalism allows to deal with **singular** manifolds without resolution.

- ▷ In particular, a 'line bundle' M on CY arises naturally. $c_1(M) \sim \omega$ related to the U(1) divisor.
- ▷ Identify massless matter charged under this U(1).

Moreover, MF comes naturally with (NCC)Resolution and associated **quiver**. [Aspinwall,Morrison], [Andres' talk]

Apply this formalism to more generic setups with abelian gauge symmetries and matter with different charges. This approach can give new insights.

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Conifold

In U(1) restriction, Weierstrass model factorizes as

$$(y - a_3)(y + a_3) = x(x^2 + b_2 x + 2b_4)$$

$$y_+ y_- = x\, w$$

with $y_\pm \equiv y \pm a_3$, $w \equiv x^2 + b_2 x + 2b_4$

- ※ Non-Cartier divisors $\{y_\pm = x = 0\} \leftrightarrow$ extra section, massless U(1).
- ※ Sing at $C_{q=1}$: $\{a_3 = b_4 = 0\}$ on the base $\leftrightarrow$ charged states

Weak coupling limit: [Sen] CY_3 : $\xi^2 = b_2$, $\Delta_{D7} = (b_4 - \xi a_3)(b_4 + \xi a_3)$

One U(1) brane and its orientifod image,
intersecting at $\{a_3 = b_4 = 0\}$ (matter).

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Conifold - Matrix Factorization (MF)

Eq $y_+ y_- = s\,w$ admits a (pair of) **MF**, i.e. a pair of matrices (ϕ, ψ) s.t.

$$\phi \cdot \psi = \psi \cdot \phi = (y_+ y_- - x\,w)\mathbb{1}_2$$

Conifold MF:

$$\phi = \begin{pmatrix} y_- & x \\ w & y_+ \end{pmatrix} \qquad \psi = \begin{pmatrix} y_+ & -x \\ -w & y_- \end{pmatrix}$$

From ϕ, ψ one can define (MCM) modules over R [Eisenbud], e.g.

$$M = \text{coker}(R^{\oplus 2} \xrightarrow{\psi} R^{\oplus 2}) \approx R^{\oplus 2} / \, \text{Im}\psi$$

where $R = \mathbb{C}[y_+, y_-, x, w]/(y_+ y_- - x\,w)$ is the coordinate ring.

- ‘Line bundle’ over conifold (but ‘rank two’ on sing locus)
- defined on sing space

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Conifold - Small resolution

Conifold MF:

$$\phi = \begin{pmatrix} y_- & x \\ w & y_+ \end{pmatrix} \qquad \psi = \begin{pmatrix} y_+ & -x \\ -w & y_- \end{pmatrix}$$

Small resolution:

$$\begin{pmatrix} y_- & x \\ w & y_+ \end{pmatrix} \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix} = 0 \qquad \subset \mathrm{Amb}_4 \times \mathbb{P}^1_{[\ell_1,\ell_2]}$$

- Exceptional locus: $\mathbb{P}^1_{[\ell_1,\ell_2]}$.
- Introduced line bundle $\mathcal{L} = \mathcal{O}(1)$, that is lift of M.
- Associated divisor $c_1(\mathcal{L})$ is locus where a generic section vanishes, i.e.

$$\sigma_1 \ell_2 - \sigma_2 \ell_1 = 0 \qquad \Rightarrow \qquad \sigma_1 y_- + \sigma_2 x = 0\,, \;\; \sigma_1 w + \sigma_2 y_+ = 0$$

- $c_1(\mathcal{L})$ intersects $\mathbb{P}^1_{[\ell_1,\ell_2]}$ at one point.

Conifold - Divisor on singular space

For the conifold:

$$\phi = \begin{pmatrix} y_- & x \\ w & y_+ \end{pmatrix} \qquad\qquad \psi = \begin{pmatrix} y_+ & -x \\ -w & y_- \end{pmatrix}$$

$M = \mathrm{coker}(R^{\otimes 2} \xrightarrow{\psi} R^{\otimes 2}) \sim$ rank-1 (line bundle over resolved conifold)

- ⋇ $c_1(M) \sim$ locus where a generic section vanishes.
- ⋇ coker $\psi \cong \mathrm{Im}\,\phi \;\; \to \;\; c_1 : \begin{pmatrix} y_- & x \\ w & y_+ \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = 0,$

  i.e. $\qquad \sigma_1 y_- + \sigma_2 x = 0\,, \qquad \sigma_1 w + \sigma_2 y_+ = 0$
- ⋇ Family of non-Cartier divisors, among which extra-section of elliptic fibration ($\sigma_1 = 0$, $\sigma_2 = 1$).

Morrison-Park

U(1) restriction is subcase of class of ellitpic fibrations with *one* extra section.

Generic case is described by Morrison-Park (MP)

$$y^2 = x^3 + c_2 x^2 + \left(2c_1 c_3 - b^2 c_0\right) x + c_0 c_3^2 - b^2 c_0 c_2 + b^2 c_1^2$$

- A rational section $\hookrightarrow$ massless U(1);
- two curves of conifold-like sing $\hookrightarrow$ matter with charges $q = 1, 2$.

Morrison-Park — weak coupling limit

$$y^2 = x^3 + c_2 x^2 + \left(2c_1 c_3 - b^2 c_0\right) x + c_0 c_3^2 - b^2 c_0 c_2 + b^2 c_1^2$$

Sen limit realized by rescaling $c_3 \mapsto \epsilon c_3$, $b \mapsto \epsilon b$ and taking $\epsilon \to 0$.

- CY 3-fold: $\xi^2 = c_2$. Orientifold involution: $\xi \mapsto -\xi$.
- D7-brane locus:

$$\Delta_{D7} = \left(c_3^2 - c_2 b^2\right)\left(c_1^2 - c_2 c_0\right) = (c_3 - \xi b)(c_3 + \xi b)\left(c_1^2 - \xi^2 c_0\right)$$

  Pair of brane-imagebrane and invariant brane: one massless $U(1)$.
- Matter curves:

$$C_{q=2} = \{\ c_3 = b = 0\ \}\,, \qquad C_{q=1} = \{\ c_3^2 - c_2 b^2 = c_1^2 - c_2 c_0 = 0\ \}$$

Morrison-Park from Universal Flop of length 2

Is there a 2×2 MF ?

The answer is NO ! But there is a 4×4:

MP-threefold can be seen as a submanifold inside the universal flop of length 2 (that is a six-fold inside $\mathbb{C}^7_{[x,y,z,t,u,v,w]}$): [Curto,Morrison; Aspinwall,Morrison]

$$y^2 = u\,x^2 + 2v\,x\,z + w\,z^2 - (u\,w - v^2)t^2 \,.$$

MP given by:

$$u = c_2 + x\,, \qquad t = b\,, \qquad w = c_0\,, \qquad v = c_1\,, \qquad z = c_3$$

4×4 MF of universal flop of length 2 is valid for MP as well.

MP — Matrix Factorization

$$P_{MP} \equiv -y^2 + x^3 + c_2 x^2 + \left(2c_1 c_3 - b^2 c_0\right) x + c_0 c_3^2 - b^2 c_0 c_2 + b^2 c_1^2$$

There exists a 4×4 MF, i.e. (Ψ, Φ) such that

$$\Psi \cdot \Phi = \Phi \cdot \Psi = P_{MP} \mathbb{1}_4$$

with

$$\Psi = \begin{pmatrix} y + c_1 b & x & -c_3 & -b \\ 2c_1 c_3 + x(x + c_2) & y - c_1 b & -b(x + c_2) & -c_3 \\ -c_0 c_3 & c_0 b & y + c_1 b & -x \\ c_0 b(x + c_2) & -c_0 c_3 & -2c_1 c_3 - x(x + c_2) & y - c_1 b \end{pmatrix}$$

and $\Phi = 2y \mathbb{1}_4 - \Psi$.

How can we extract massless U(1) and charged matter?

U(1) divisor from MF

$M = \text{coker}(R^{\otimes 4} \xrightarrow{\Psi} R^{\otimes 4})$ is now rank 2.

with $$\Psi = \begin{pmatrix} y + c_1 b & x & -c_3 & -b \\ 2c_1 c_3 + x(x+c_2) & y - c_1 b & -b(x+c_2) & -c_3 \\ -c_0 c_3 & c_0 b & y + c_1 b & -x \\ c_0 b(x+c_2) & -c_0 c_3 & -2c_1 c_3 - x(x+c_2) & y - c_1 b \end{pmatrix}$$

- Again $c_1(M)$ is U(1) div $\rightarrow$ locus where two sections become parallel.

$$\begin{pmatrix} c_3^2 - b^2(x+c_2) & c_3 x - b(y - c_1 b) & -c_3(y + c_1 b) - b\,x(x+c_2) \\ c_3 x + b(y + c_1 b) & x^2 - c_0 b^2 & x(y + c_1 b) + c_0 c_3 b \\ c_3(y - c_1 b) - b\,x(x+c_2) & -x(y - c_1 b) + c_0 c_3 b & c_0 c_3(y + c_1 b) \end{pmatrix} \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} = 0$$

- In general, family of non-Cartier divisors. With specific choice of σ_i:

$$c_3^2 - b^2(x + c_2) = 0, \quad c_3 x - b(y - c_1 b) = 0, \quad c_3(y + c_1 b) + b\,x(x + c_2) = 0\,,$$

intersected with MP-equation.

- We recognize the extra (rational) section of elliptic fibration, i.e.

$$y = c_1 b - \frac{c_2 c_3}{b} + \frac{c_3^3}{b^3}\,, \qquad x = -c_2 + \frac{c_3^2}{b^2}$$

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Matter

$$y^2 = x^3 + c_2 x^2 + \left(2c_1c_3 - b^2 c_0\right) x + c_0 c_3^2 - b^2 c_0 c_2 + b^2 c_1^2$$

$$\Psi = \begin{pmatrix} y + c_1 b & x & -c_3 & -b \\ 2c_1c_3 + x(x+c_2) & y - c_1 b & -b(x+c_2) & -c_3 \\ -c_0c_3 & c_0 b & y + c_1 b & -x \\ c_0 b(x+c_2) & -c_0c_3 & -2c_1c_3 - x(x+c_2) & y - c_1 b \end{pmatrix}$$

- Matter is at codimension-2 singular loci: $\{b(c_1^2 - c_2c_0 - c_0 x) = 0, ...\}$
- MP is a determinantal variety; singular loci where matrix changes rank.

Matter where Ψ becomes rank lower than 2:

* charge two matter at rk$=0$:

$$C_{q=2} : \qquad \{y = x = c_3 = b = 0\}$$

* charge one matter at rk$=1$.

$$C_{q=1} : \{c_3^2 - c_2 b^2 - x b^2 = c_1^2 - c_0 c_2 - c_0 x = c_1 x + c_0 c_3 = y = ... = 0\}$$

Grassmann blowup

Resolved space: following eq in $\mathrm{Amb}_4 \times$ Gr(2, 4) [Curto,Morrison]

$$\begin{pmatrix} y + c_1 b & x & -c_3 & -b \\ 2c_1 c_3 + x(x + c_2) & y - c_1 b & -b(x + c_2) & -c_3 \\ -c_0 c_3 & c_0 b & y + c_1 b & -x \\ c_0 b(x + c_2) & -c_0 c_3 & -2c_1 c_3 - x(x + c_2) & y - c_1 b \end{pmatrix} \begin{pmatrix} s_1 & \ell_1 \\ s_2 & \ell_2 \\ s_3 & \ell_3 \\ s_4 & \ell_4 \end{pmatrix} = 0$$

with $\begin{pmatrix} s_1 & \ell_1 \\ s_2 & \ell_2 \\ s_3 & \ell_3 \\ s_4 & \ell_4 \end{pmatrix} \in \mathrm{Gr}(2, 4).$

- Gr(2, 4) is a quadric in $\mathbb{P}^5$: $X_{12}X_{34} + X_{13}X_{24} + X_{14}X_{23}$.
- Exceptional fiber is given by 3 eqn's in Gr(2, 4), linear in $\mathbb{P}^5$.
- Hence, exceptional fiber is a quadratic $\mathbb{P}^1$ in $\mathbb{P}^2$.

Matter with charge 1

Take $b \neq 0$ (open patch away from charge two locus) $\rightarrow$ one can bring MF to the form

$$\Psi = \begin{pmatrix} 1 & 0 & & \\ 0 & P_{MP} & & \\ & & y - c_1 b - \frac{c_3 x}{b} & x + c_2 + \frac{c_3^2}{b^2} \\ & & -c_0 b^2 - x^2 & y + c_1 b + \frac{c_3 x}{b} \end{pmatrix} = \left(\begin{array}{c|c} \psi_{\rm triv} & \\ \hline & \psi \end{array} \right)$$

- 2×2 MF $\psi \longleftrightarrow$ small resolution $\mathbb{P}^1_{s.r.}$, s.t. $\int_{\mathbb{P}^1_{s.r.}} c_1(\psi) = 1$.
- Grassmann blowup now reduces to

$$\left(\begin{array}{c|c} \psi_{\rm triv} & \\ \hline & \psi \end{array} \right) \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ s_3 & 0 \\ s_4 & 0 \end{pmatrix} = 0 \qquad \rightarrow \qquad \mathbb{P}^1_{Gr} \sim \mathbb{P}^1_{s.r.}$$

- Rank-two module splits: $M = M_{\rm triv} \oplus M_\psi$; hence

$$c_1(\Psi) = c_1(\psi_{\rm triv}) + c_1(\psi) = c_1(\psi)$$

and then

$$q = \int_{\mathbb{P}^1_{Gr}} c_1(\Psi) = \int_{\mathbb{P}^1_{s.r.}} c_1(\psi) = 1.$$

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Matter with charge 2

Take $c_1^2 - c_2 c_0 - c_0 x \neq 0$ (open patch away from charge one locus) $\rightarrow$ one can bring MF to the form

$$\Psi = \left(\begin{array}{c|c} \chi & \\ \hline & \chi \end{array} \right)$$

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  Exceptional locus $\chi = 0$ and $\ell_1 s_4 - \ell_2 s_3$, i.e. $\mathbb{P}^1_{Gr} \subset \mathbb{P}^{1,\ell}_{s.r.} \times \mathbb{P}^{1,s}_{s.r.}$
- Rank-two module splits: $M = M_\chi \oplus M_\chi$; hence

$$c_1(\Psi) = c_1(\chi) + c_1(\chi)$$

  and then

$$q = \int_{\mathbb{P}^1_{Gr}} c_1(\Psi) = 2 \int_{\mathbb{P}^1_{s.r.}} c_1(\chi) = 2.$$

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Higher charge models

Extrapolate:

If variety comes from universal flop of length ℓ with MF that is $2\ell \times 2\ell$,

then it will have $q = 1, 2, ..., \ell$ states when rank goes from ℓ to $\ell - 1, ..., 0$.

Example $\ell = 3$:

$$\begin{pmatrix} \psi_{tr} & & \\ & \psi_{tr} & \\ & & \psi_1 \end{pmatrix} \longleftrightarrow \text{charge 1 states}$$

$$\begin{pmatrix} \psi_{tr} & & \\ & \psi_2 & \\ & & \psi_2 \end{pmatrix} \longleftrightarrow \text{charge 2 states}$$

$$\begin{pmatrix} \psi_3 & & \\ & \psi_3 & \\ & & \psi_3 \end{pmatrix} \longleftrightarrow \text{charge 3 states}$$

Since $\ell \leq 6$ [Katz,Morrison], we expect an upper bound on the charge $\mathbf{q \leq 6}$.

[Wati's talk]

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[Wati's talk]

Conclusions

- Matrix factorization for threefold with massless U(1).
- Naturally encode extra U(1) divisor (already in the singular limit): family of representatives that includes extra section of elliptic fibration.
- Charge 1 and 2 matter on loci where rank goes from 2 to 1 and 0. Charge given by intersection of exceptional $\mathbb{P}^1$ with extra divisor: encoded into how matrix splits around sing. Bound $q \leq 6$?

Open issues:

- More complicated geometries.
- Check that known q-charge models admit a $2q \times 2q$ MF and descend from flop of length q.
- Quiver of MP and NCCR.

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- Matrix factorization for threefold with massless U(1).
- Naturally encode extra U(1) divisor (already in the singular limit): family of representatives that includes extra section of elliptic fibration.
- Charge 1 and 2 matter on loci where rank goes from 2 to 1 and 0. Charge given by intersection of exceptional $\mathbb{P}^1$ with extra divisor: encoded into how matrix splits around sing. Bound $q \leq 6$?

Open issues:

- More complicated geometries.
- Check that known q-charge models admit a $2q \times 2q$ MF and descend from flop of length q.
- Quiver of MP and NCCR.

Thank you!