Semi-hard processes in high-energy perturbative QCD

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Forward Physics and Diffraction

Instituto de Física Teórica UAM/CSIC
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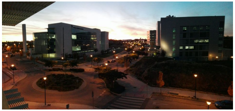
March 20th - 23rd, 2017



Outline

1. Introductory remarks
   - QCD and semi-hard processes
   - BFKL resummation
   - Towards new analyses

2. Phenomenology
   - Mueller–Navelet jet production
   - Inclusive di-hadron and hadron-jet correlations
   - Heavy-quark pair photoproduction

3. Conclusions & Outlook

Outline

The semi-hard sector

High energies reachable at the LHC and at future colliders:

- great opportunity in the search for long-waited signals of New Physics...
- ...faultless chance to test Standard Model in unprecedented kinematic ranges
- only 5% of Universe visible, but 99% of this visible matter described by **QCD**
- duality between non-perturbative and perturbative aspects (**confinement** and **asymptotic freedom** concurrent properties) makes QCD a challenging sector surrounded by a broad and constant interest in its phenomenology

Semi-hard processes

Collision processes with the following **scale hierarchy**: $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$

- Q is the **hard scale** of the process (*e.g.* photon virtuality, heavy quark mass, jet/hadron transverse momentum, t, *etc.*)
- large $Q \implies \alpha_s(Q) \ll 1 \implies$ perturbative QCD
- large $s \implies$ large energy logs $\implies \alpha_s(Q) \log s \sim 1 \implies$ need to **resummation**

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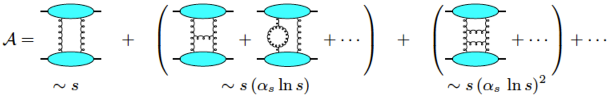
The BFKL resummation

pQCD, **semi-hard processes**: $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$

- **BFKL resummation**: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]; [Y.Y. Balitskii, L.N. Lipatov (1978)]

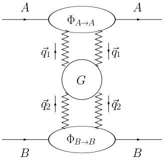
based on → **gluon Reggeization**

leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$



next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$

total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\left(\mathcal{A}_{AB}^{AB}\right)}{s}$ ⇐ ***optical theorem***



- $\mathrm{Im}_s\left(\mathcal{A}_{AB}^{AB}\right)$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

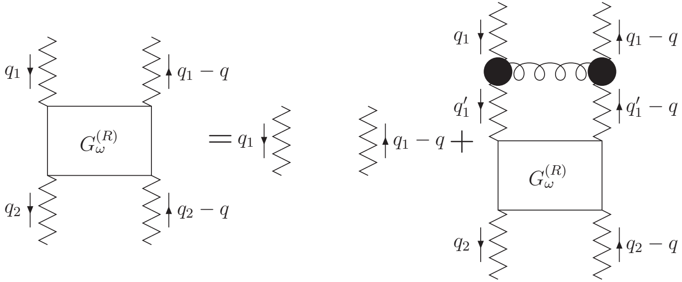
$$\mathrm{Im}_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\Phi_A\left(\vec{q}_1, s_0\right)\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\Phi_B\left(-\vec{q}_2, s_0\right)\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}G_\omega\left(\vec{q}_1, \vec{q}_2\right)$$

- **Green's function** is **process-independent** and takes care of the **energy dependence**

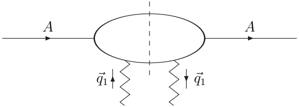
$\longrightarrow$ determined through the **BFKL equation**

[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega\left(\vec{q}_1, \vec{q}_2\right) = \delta^{D-2}\left(\vec{q}_1 - \vec{q}_2\right) + \int d^{D-2}q\, K(\vec{q}_1, \vec{q})\, G_\omega\left(\vec{q}, \vec{q}_1\right) .$$



- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy



$\longrightarrow$ known in the NLA just for few processes

◇ **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni, G. Rodrigo (2000)]

◇ $\gamma^* \longrightarrow V$, with $V = \rho^0,\ \omega,\ \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

◇ forward identified hadron production

[D.Yu. Ivanov, A. Papa (2012)]

◇ $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

BFKL and Mueller–Navelet jets

So far, search for BFKL effects had these general drawbacks:

- too low $\sqrt{s}$ or rapidity intervals among tagged particles in the final state
- too inclusive observables, other approaches can fit them

Advent of LHC:

→ higher energies ↔ larger rapidity intervals
→ unique opportunity to **test pQCD in the high-energy limit**
→ disentangle applicability region of energy-log resummation (**BFKL approach**)

[V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]
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Last years:

Mueller–Navelet jets

- hadroproduction of two jets featuring high transverse momenta and well separed in rapidity
- possibility to define *infrared-safe* observables...
- ...and constrain the PDFs
- theory vs experiment

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

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How could we further and deeply probe BFKL?

1. **Study less inclusive two-body final states...**

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017, 2018)]

Di-hadron production

- inclusive production of a pair of charged light hadrons well separated in rapidity
- much smaller values of the transverse momentum than jets!
- possibility to constrain not only the PDFs, but also the FFs!

Heavy-quark pair photoproduction

- quark masses play the role of hard scale
- e^+e^- at LEP2 and future lepton colliders

2. **Study three- and four-body final-state processes...**

[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]; [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017)]

Multi-jet production

- definition of new, **suitable BFKL observables...**
- ...in order to further investigate the azimuthal distribution of the final state

[talk by David Gordo Gómez]

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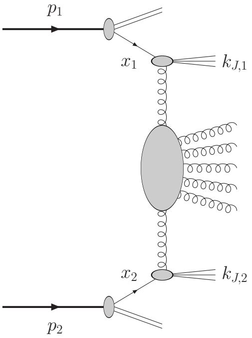
Mueller–Navelet jets

$$\text{proton}(p_1) + \text{proton}(p_2) \to \text{jet}_1(k_{J,1}) + X + \text{jet}_2(k_{J,2})$$

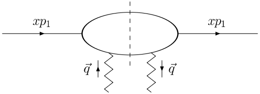


- large jet transverse momenta (hard scales): $\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda^2_{\rm QCD}$
- large rapidity gap between jets, $\Delta y \equiv Y = y_{J_1} - y_{J_2}$,
  which requires large c.m. energy of the proton collisions, $s = 2\, p_1 \cdot p_2 \gg \vec{k}_{1,2}^2$

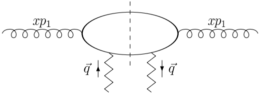
[A.H. Mueller, H. Navelet (1987)]

Forward jet impact factor

- take the impact factors for **colliding partons**
  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

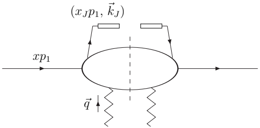


quark vertex

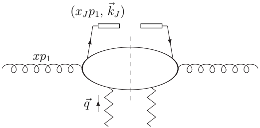


gluon vertex

- “open” one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex

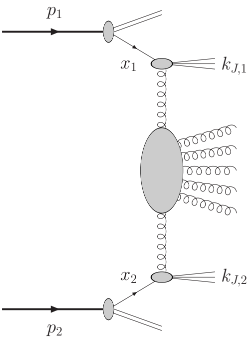


gluon jet vertex

- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes$ [quark vertex] $+ f_g \otimes$ [gluon vertex]

BFKL cross section (Mueller–Navelet jets)...

$$\frac{d\sigma}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}} = \sum_{i,j=q,\bar{q},g}\int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1x_2s,\mu)}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}}$$



- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the (n,ν)-representation
- suitable definition of the **azimuthal coefficients**

$$\frac{d\sigma}{dx_{J_1}dx_{J_2}\,d|\vec{k}_{J_1}|\,d|\vec{k}_{J_2}|d\phi_{J_1}d\phi_{J_2}} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\,\mathcal{C}_n\right]$$

with $\phi = \phi_{J_1} - \phi_{J_2} - \pi$

...useful definitions:

$$Y = \ln\frac{x_{J_1}x_{J_2}s}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}, \qquad Y_0 = \ln\frac{s_0}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$$

On the scale optimization: BLM method

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ...call for some optimization procedure...
- ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

- "Exact" BLM:

  suppress NLO IFs $+$ NLO Kernel β_0-dependent factors

- Partial (approximated) BLM:

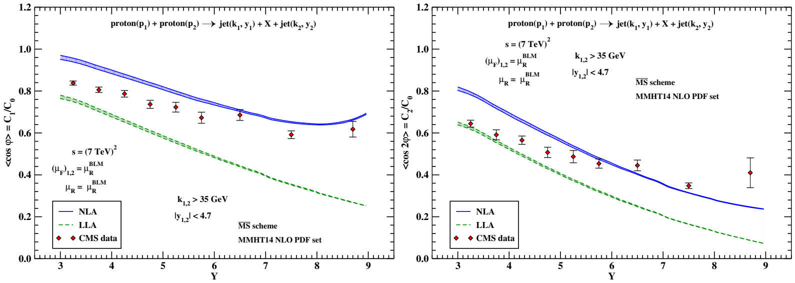
a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - f(\nu) - \frac{5}{3}\right]$ ← NLO IFs $\propto \beta_0$

b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - 2f(\nu) - \frac{5}{3} + \frac{1}{2}\chi(\nu,n)\right]$ ← NLO Kernel $\propto \beta_0$

$f(\nu)$ depends on the process

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

Theory versus experiment



$R_{n0} \equiv C_n / C_0 = \langle \cos[n(\phi_{J_1} - \phi_{J_2} - \pi)] \rangle$

vs $Y = y_{J_1} - y_{J_2}$

small-cone approximation

BLM scale setting

□ **CMS** (7 TeV; $|\vec{k}_1|, |\vec{k}_2| \geqslant 35$ GeV)

(**7 TeV theory vs exp.**) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]
(**7 TeV BFKL vs DGLAP + asym**) [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]
(**13 TeV** predictions + $C_0(Y)$) [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

High-energy DGLAP

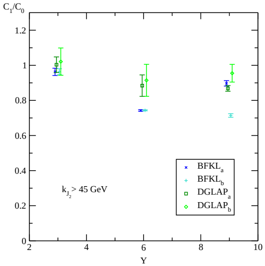
◇ NLA BFKL expressions for the observables truncated to $\mathcal{O}\left(\alpha_s^3\right)$ **!**

Why asymmetric cuts?

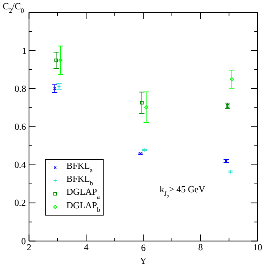
- suspress Born contribution to ϕ-averaged cross section C_0 (back-to-back jets)
  - ◇ avoid instabilities observed in NLO fixed-order calculations
    [J.R. Andersen, V. Del Duca, S. Frixione, C.R. Schmidt, W.J. Stirling (2001)]
    [M. Fontannaz, J.P. Guillet, G. Heinrich (2001)]
  - ◇ **enhance effects of additional hard gluons** $\xrightarrow{\textit{emphasize}}$ **BFKL effects**
- violation of energy-momentum in NLA strongly suppressed respect to LLA
  [B. Ducloué, L. Szymanowski, S. Wallon (2014)]

R_{nm} for $k_{J_1} > 35$ GeV, $k_{J_2} > 45$ GeV at $\sqrt{s} = 7$ TeV

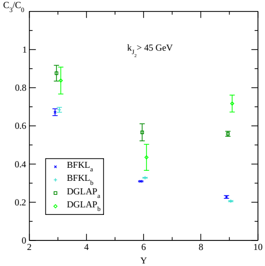
$\langle \cos(\phi) \rangle$



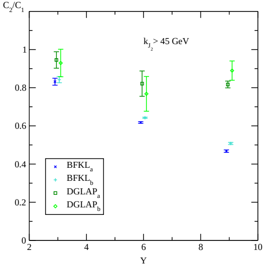
$\langle \cos(2\phi) \rangle$



$\langle \cos(3\phi) \rangle$



$$\frac{\langle \cos(2\phi) \rangle}{\langle \cos(\phi) \rangle}$$

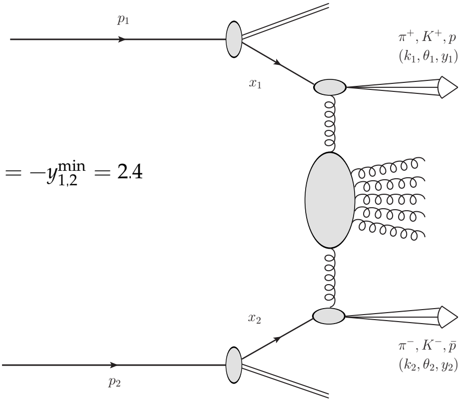


[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

Di-hadron production

Process: $\text{proton}(p_1) + \text{proton}(p_2) \to \text{hadron}(k_1) + X + \text{hadron}(k_2)$

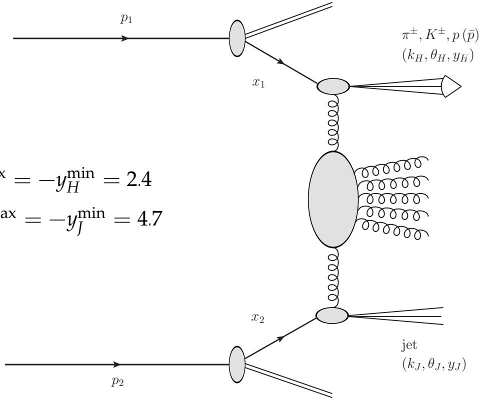
$k_{1,2}^{\text{min}} = 5$ GeV, $y_{1,2}^{\text{max}} = -y_{1,2}^{\text{min}} = 2.4$



(NLO impact factor) [D.Yu. Ivanov, A. Papa (2012)]
[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

A hadron-jet final-state reaction

Process: $\text{proton}(p_1) + \text{proton}(p_2) \rightarrow \text{hadron}(k_H) + X + \text{jet}(k_J)$



$k_H^{\min} = 5$ GeV, $y_H^{\max} = -y_H^{\min} = 2.4$

$k_J^{\min} = 35$ GeV, $y_J^{\max} = -y_J^{\min} = 4.7$

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, M.M. Maher, A. Papa (in progress)]

Observables and kinematics (hadron-jet)

- **Observables:**

  ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos(n\phi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$

  $\langle \cos(2\phi) \rangle / \langle \cos(\phi) \rangle \equiv \mathcal{C}_2/\mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\phi) \rangle / \langle \cos(2\phi) \rangle \equiv \mathcal{C}_3/\mathcal{C}_2 \equiv R_{32}$

◇ *Integrated coefficients:*

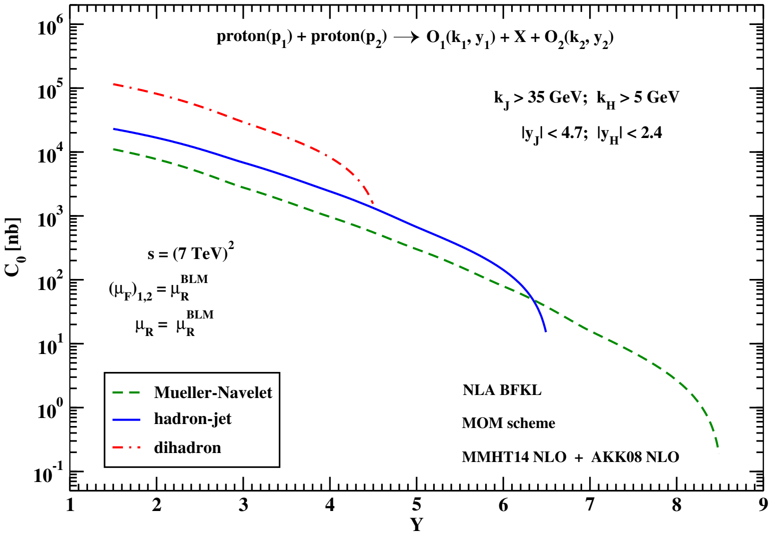
$$C_n = \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \int_{k_{1,\min}}^{k_{1,\max}} dk_1 \int_{k_{2,\min}}^{k_{2,\max}} dk_2 \delta(y_1 - y_2 - Y)\, \mathcal{C}_n(y_1, y_2, k_1, k_2)$$

- **Kinematic settings:**
  - ◇ $\sqrt{s} = 7,\ 13$ TeV
  - ◇ $|y_H| \leqslant 2.4$; $|y_J| \leqslant 4.7$
  - ◇ $k_H \geqslant 5$ GeV; $k_J \geqslant 35$ GeV

- **Phenomenological analysis:**
  - ◇ full **NLA** BFKL
  - ◇ JETHAD (CSLIB, F95) + CERNLIB
  - ◇ (MMHT14, CT14, NNPDF3.0) ⊛ (AKK08, DSS07, HKNS07, NNFF1.0)

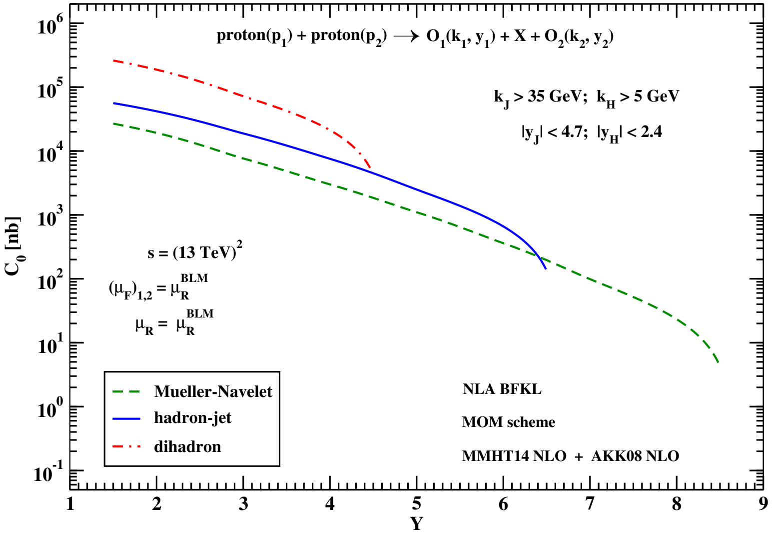
[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (under development)]

MN, hadron-jet and di-hadron C_0 vs Y, $\sqrt{s} = 7$ TeV



preliminary results [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A. Papa (in progress)]

MN, hadron-jet and di-hadron C_0 vs Y, $\sqrt{s} = 13$ TeV



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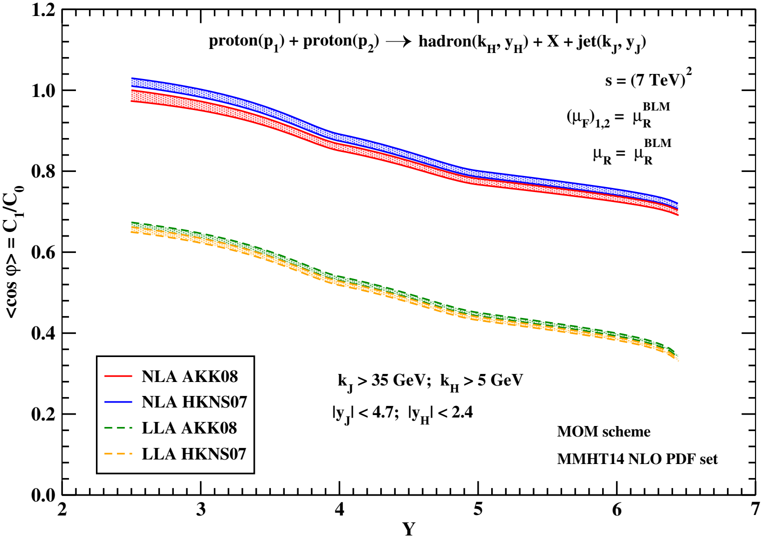
Hadron-jet R_{10} vs Y, $\sqrt{s} = 7$ TeV



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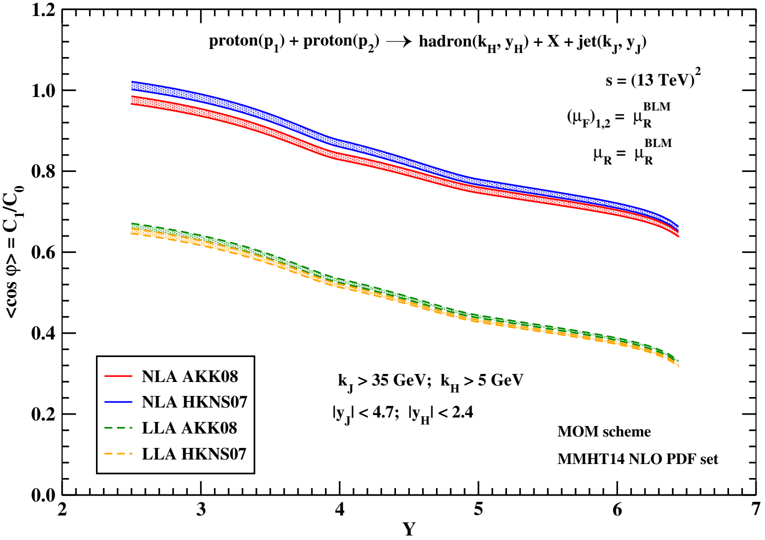

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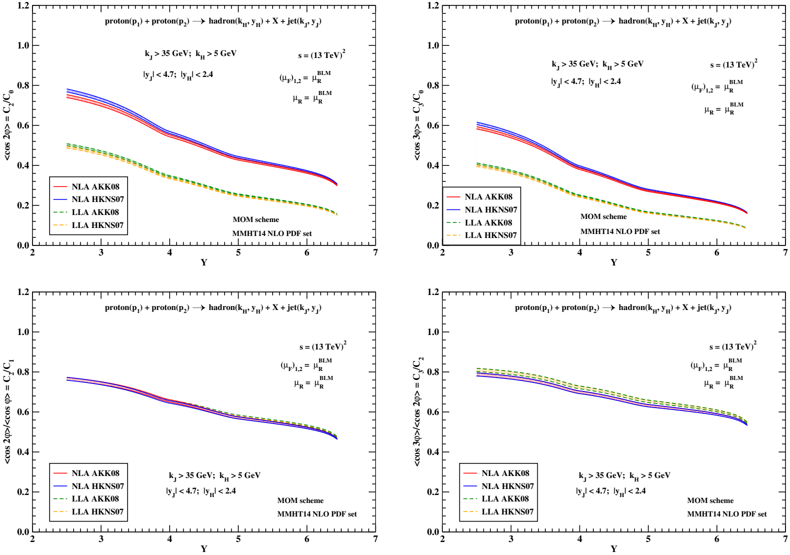
Hadron-jet R_{nm} vs Y, $\sqrt{s} = 13$ TeV

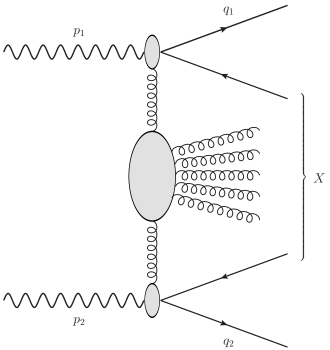


preliminary results [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A. Papa (in progress)]

Heavy-quark pair photoproduction

Process: $\gamma(p_1) + \gamma(p_2) \to Q(q_1) + X + Q(q_2)$

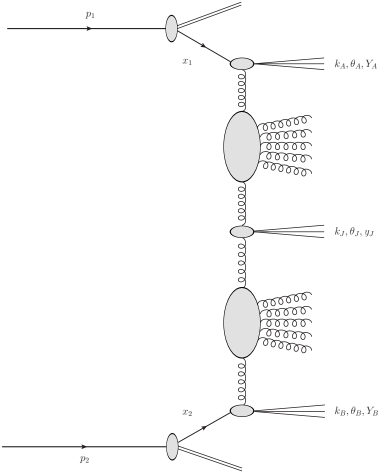
...Q stands for a charm/bottom quark or antiquark

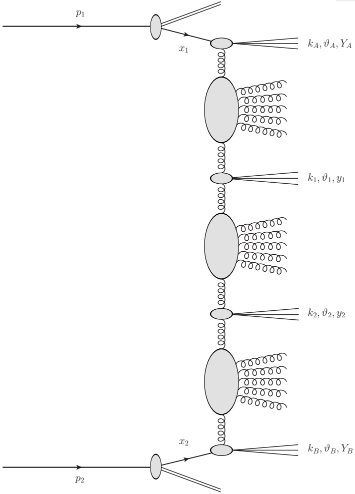


- photoproduction channel
- collision of (quasi-)real photons
- equivalent photon flux approximation
- quark masses play the role of hard scale
- first predictions within partial NLA BFKL (NLA Green's function + LO impact factors)
  - ◇ LEP2 and future e^+e^- colliders

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018)]

Three- and four-jet production





(Three-jets) [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]
(Three-jets) [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017)]
(Four-jets) [F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]
(Four-jets) [F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

Outline

1. Introductory remarks
   - QCD and semi-hard processes
   - BFKL resummation
   - Towards new analyses
2. Phenomenology
   - Mueller–Navelet jet production
   - Inclusive di-hadron and hadron-jet correlations
   - Heavy-quark pair photoproduction
3. Conclusions & Outlook

Conclusions...

- The **BFKL approach** offers a common basis for the description of *semi-hard processes*; it relies on a remarkable property of perturbative QCD, the **gluon Reggeization**

- Physical amplitudes in NLA are written in terms of a universal **Green's function** and of process-dependent **impact factors** of the colliding particles

- The number of reactions which can be investigated within NLA BFKL depends on the list of available NLO impact factors calculated so far

- Successful tests of NLA BFKL in the **Mueller–Navelet** channel with the advent of the LHC; nevertheless, *new BFKL-sensitive observables* as well as *more exclusive final-state reactions* are needed (**(di-)hadron(-jet)**, **heavy-quark pair**, **multi-jet** production processes,...)

...Outlook

- Comparison with: fixed-order DGLAP predictions, Monte Carlo inspired calculations (all processes)
- Comparison with higher-twist predictions: final-state objects stemming from (two) independent gluon ladders (MPI) (all processes)

(Mueller–Navelet jets) [R. Maciula, A. Szczurek (2014)]
(Mueller–Navelet jets) [B. Ducloué, L. Szymanowski, S. Wallon (2015)]
(Four-jets) [K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016, 2016)]

- Rapidity veto effects in Mueller–Navelet jet production

[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (in progress)]

- Inclusion of other resummation effects
- Probe the BFKL dynamics through other processes...
  - **hadron-jet** correlations:
    FF dependence + asymmetric rapidity and transverse momenta ranges
    [A.D. Bolognino, F.G. C., D.Yu. Ivanov, M.M. Maher, A. Papa (in progress)]
  - **heavy-quark pair** production:
    calculation of th NLO $q\bar{q}$ impact factor
    hadroproduction (process initiated by quarks and gluons)
    [A.D. Bolognino, F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (in progress)]

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Gluon Reggeization in perturbative QCD

Elastic scattering process: $A + B \longrightarrow A' + B'$

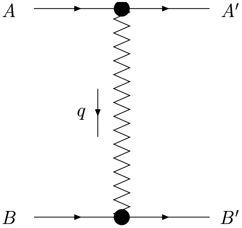
- Gluon quantum numbers in the t-channel: octet color representation, negative signature
- Regge limit: $s \simeq -u \to \infty$, t not growing with s

$\rightarrow$ amplitude governed by **gluon Reggeization**

- all-order resummation:

  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$

  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t) \, , \quad j(0) = 1$$

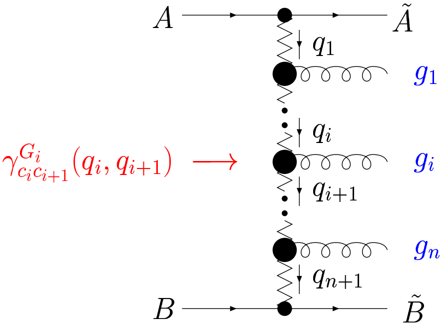
$\omega(t)$ – Reggeized gluon trajectory

$$\Gamma_{A'A}^c = g \langle A' | T^c | A \rangle \Gamma_{A'A}$$

T^c fundamental (quarks) or adjoint (gluons)

BFKL in the LLA (I)

Inelastic scattering process $A + B \;\; \rightarrow \;\; \tilde{A} + \tilde{B} + n$ **in the LLA**



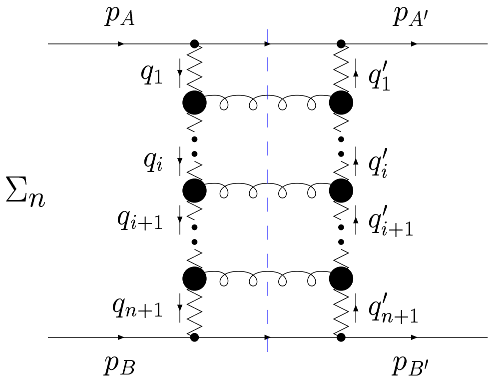
multi-Regge kinematics

$$\text{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

s_R **energy scale, irrelevant in the LLA**

BFKL in the LLA (II)

Elastic amplitude $A + B \longrightarrow A' + B'$ **in the LLA via** s**-channel unitarity**



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \,, \qquad \mathcal{R} = \mathbf{1}\ \textbf{(singlet)},\ \mathbf{8}^-\ \textbf{(octet)}, \dots$$

The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

Mueller–Navelet jets

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...and azimuthal coefficients (MN-jets)

$$
\mathcal{C}_n = \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^2(\mu_R)K^{(1)}(n,\nu)\right]} \alpha_s^2(\mu_R)
$$

$$
\times c_1(n,\nu)\, c_2(n,\nu) \left[1 + \alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right)\right]
$$

where

$$
\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)
$$

$$
K^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu) + \frac{10}{3} + \imath\frac{d}{d\nu}\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right) + 2\ln\left(\mu_R^2\right)\right)
$$

$$
c_1(n,\nu,|\vec{k}|,x) = 2\sqrt{\frac{C_F}{C_A}}(\vec{k}^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x,\mu_F) + \sum_{a=q,\bar{q}} f_a(x,\mu_F)\right)
$$

...several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!

$\longrightarrow$ ...we use the *exponentiated* one

[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa (2014)]

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MN-jets: the BFKL BLM cross section

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - f\left(\nu\right) - \frac{5}{3}\right] \sim 5^2 k_1 k_2$

b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - 2f\left(\nu\right) - \frac{5}{3} + \frac{1}{2}\chi\left(\nu, n\right)\right] < (11.5)^2 k_1 k_2$

$$\mathcal{C}_n^{\text{BFKL}(a)} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu) + \bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(n,\nu) - \frac{T^\beta}{C_A}\chi(n,\nu) - \frac{\beta_0}{8C_A}\chi^2(n,\nu)\right)\right]}$$

$$\times \, \alpha_s^2\left(\mu_R\right) c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})$$

$$\times \left[1 - \frac{2}{\pi}\alpha_s\left(\mu_R\right) T^\beta + \alpha_s\left(\mu_R\right)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|, x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}\right)\right]$$

$$\mathcal{C}_n^{\text{BFKL}(b)} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu) + \bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(n,\nu) - \frac{T^\beta}{C_A}\chi(n,\nu)\right)\right]}$$

$$\times \, \alpha_s^2\left(\mu_R\right) c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})$$

$$\times \left[1 + \alpha_s\left(\mu_R\right)\left(\frac{\beta_0}{4\pi}\chi\left(n,\nu\right) - 2\frac{T^\beta}{\pi}\right) + \alpha_s\left(\mu_R\right)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|, x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}\right)\right]$$

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MN-jets: the DGLAP BLM cross section

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1 + \frac{2}{3}I\right) - f(\nu) - \frac{5}{3}\right] \sim 5^2 k_1 k_2$

b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1 + \frac{2}{3}I\right) - 2f(\nu) - \frac{5}{3} + \frac{1}{2}\chi(\nu, n)\right] < (11.5)^2 k_1 k_2$

$$\mathcal{C}_n^{\text{DGLAP}\,(a)} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \; \alpha_s^2(\mu_R)\, c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$
$$\times \left[1 - \frac{2}{\pi}\alpha_s(\mu_R)\, T^{\beta} + \bar{\alpha}_s(\mu_R)\,(Y - Y_0)\,\chi(n,\nu)\right.$$
$$\left. + \alpha_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}\right)\right]$$

$$\mathcal{C}_n^{\text{DGLAP}\,(b)} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \; \alpha_s^2(\mu_R)\, c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$
$$\times \left[1 + \alpha_s(\mu_R)\left(\frac{\beta_0}{4\pi}\chi(n,\nu) - 2\frac{T^{\beta}}{\pi}\right) + \bar{\alpha}_s(\mu_R)\,(Y - Y_0)\,\chi(n,\nu)\right.$$
$$\left. + \alpha_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}\right)\right]$$

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MN-jets: the “exact" BLM cross section

$$\mathcal{C}_n^{\rm BLM} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\,\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left[\chi(n,\nu)+\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left(\bar{\chi}(n,\nu)+\frac{T^{\rm conf}}{N_c}\chi(n,\nu)\right)\right]}$$

$$\times\,(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}))^2 c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$

$$\times\left[1+\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\left\{\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}+\frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}+\frac{2T^{\rm conf}}{N_c}\right\}\right] ,$$

with the $\mu_R^{\rm BLM}$ scale chosen as the solution of the following integral equation…

$$\mathcal{C}_n^{\beta} \equiv \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\chi(n,\nu)} \left(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\right)^3$$

$$\times\, c_1(n,\nu)c_2(n,\nu)\frac{\beta_0}{2N_c}\left[\frac{5}{3}+\ln\frac{(\mu_R^{\rm BLM})^2}{Q_1Q_2}-2\left(1+\frac{2}{3}I\right)\right.$$

$$\left.+\,\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\ln\frac{s}{s_0}\,\frac{\chi(n,\nu)}{2}\left(-\frac{\chi(n,\nu)}{2}+\frac{5}{3}+\ln\frac{(\mu_R^{\rm BLM})^2}{Q_1Q_2}-2\left(1+\frac{2}{3}I\right)\right)\right] \overset{!}{=} 0$$

...choosing the μ_R^{BLM} scale (MN-jets)

...which represents the condition that terms proportional to β_0 in C_n disappear

$$\alpha^{\text{MOM}} = -\frac{\pi}{2T}\left[1 - \sqrt{1 + 4\alpha_s\left(\mu_R\right)\frac{T}{\pi}}\right] ,$$

with $T = T^{\beta} + T^{\text{conf}}$,

$$T^{\beta} = -\frac{\beta_0}{2}\left(1 + \frac{2}{3}I\right) ,$$

$$T^{\text{conf}} = \frac{C_A}{8}\left[\frac{17}{2}I + \frac{3}{2}\left(I - 1\right)\xi + \left(1 - \frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right] ,$$

where $I = -2\int_0^1 dx\frac{\ln(x)}{x^2-x+1} \simeq 2.3439$ and ξ is a gauge parameter.

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Observables and kinematics (MN-jets)

- **Observables:**

ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos [n(\phi_{J_1} - \phi_{J_2} - \pi)] \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}$, with $n = 1, 2, 3$

$\langle \cos(2\phi) \rangle / \langle \cos(\phi) \rangle \equiv \mathcal{C}_2 / \mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\phi) \rangle / \langle \cos(2\phi) \rangle \equiv \mathcal{C}_3 / \mathcal{C}_2 \equiv R_{32}$

◇ *Integrated coefficients:*

$$C_n = \int_{y_{1,\min}}^{y_{1,\max}} dy_1 \int_{y_{2,\min}}^{y_{2,\max}} dy_2 \int_{k_{J_1,\min}}^{\infty} dk_{J_1} \int_{k_{J_2,\min}}^{\infty} dk_{J_2} \delta(y_1 - y_2 - Y) \, \mathcal{C}_n(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2})$$

- **Kinematic settings:**
  - ◇ $R = 0.5$ and $\sqrt{s} = 7, 13$ TeV
  - ◇ $y^{\rm C}_{\max} \leqslant |y_{J_{1,2}}| \leqslant 4.7$
  - ◇ symmetric and asymmetric choices for k_{J_1} and k_{J_2} ranges

- **Numerical tools:** JETHAD (CSLIB, F95) + CERNLIB + NLO MSTW08 PDFs

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (under development)]

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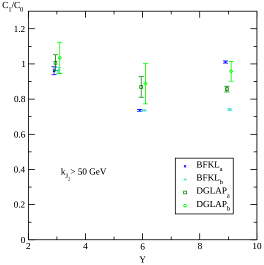
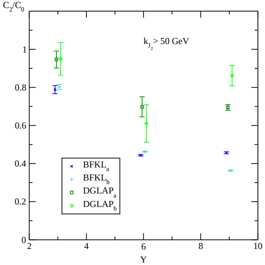
R_{nm} for $k_{J_1} > 35$ GeV, $k_{J_2} > 50$ GeV at $\sqrt{s} = 7$ TeV

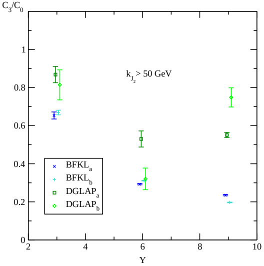
$\langle \cos(\phi) \rangle$



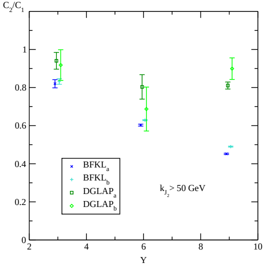
$\langle \cos(2\phi) \rangle$



$\langle \cos(3\phi) \rangle$



$\frac{\langle \cos(2\phi) \rangle}{\langle \cos(\phi) \rangle}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

Exclusion of central jet rapidities (MN-jets)

Motivation...

◇ At given $Y = y_{J_1} - y_{J_2}$ ...

→ $|y_{J_i}|$ could be so small ($\lesssim 2$), that the jet i is actually produced in the central region, rather than in one of the two forward regions

→ longitudinal momentum fractions of the parent partons $x \sim 10^{-3}$

→ for $|y_{J_i}|$ and $|k_{J_i}| < 100$ GeV $\Rightarrow$ increase of C_0 by 25% due to NNLO PDF effects

[J. Currie, A. Gehrmann-De Ridder, E. W. N. Glover, J. Pires (2014)]

! Our BFKL description of the process could be not so accurate...

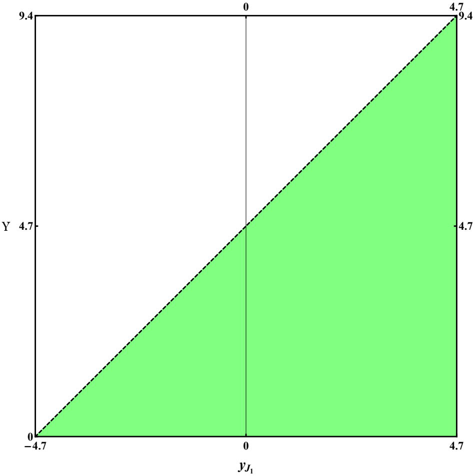
...let's return to the original Mueller–Navelet idea!

◇ remove regions where jets are produced at central rapidities...

→ ...in order to reduce as much as possible theoretical uncertainties

Rapidity range (MN-jets)

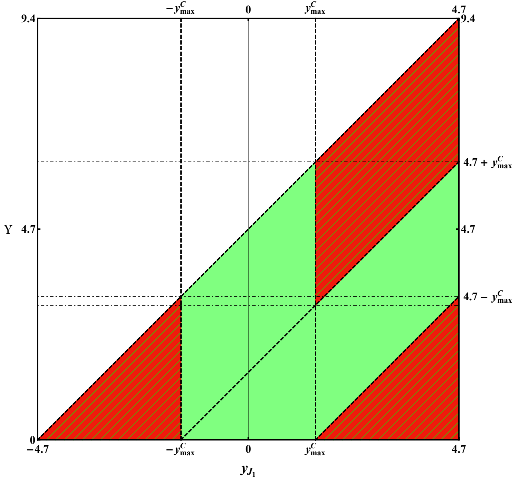
$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2 \, \delta(y_1 - y_2 - Y) \, \theta\left(|y_1| - y_{max}^C\right) \theta\left(|y_2| - y_{max}^C\right) \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$



$Y = y_{J_1} - y_{J_2}$

Rapidity range (MN-jets)

$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2 \; \delta\left(y_1 - y_2 - Y\right) \; \theta\left(|y_1| - y_{max}^C\right) \; \theta\left(|y_2| - y_{max}^C\right) \; \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$



$$Y = y_{J_1} - y_{J_2}$$

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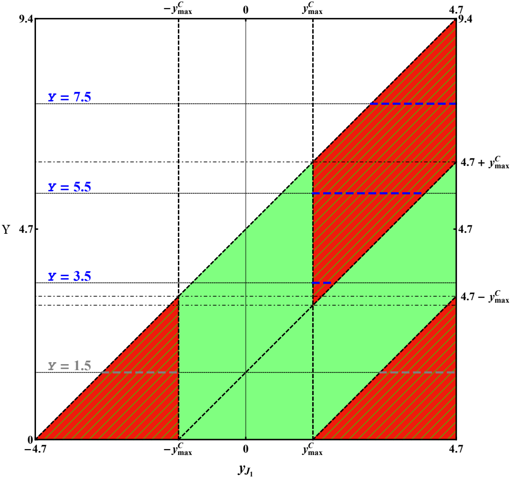
Rapidity range (MN-jets)

$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2 \, \delta(y_1 - y_2 - Y) \, \theta\left(|y_1| - y_{max}^C\right) \theta\left(|y_2| - y_{max}^C\right) \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$



$$Y = y_{J_1} - y_{J_2}$$

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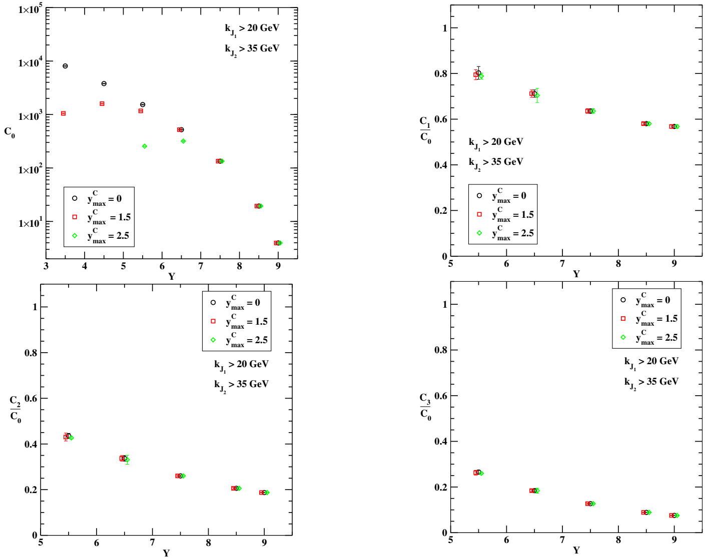
R_{nm} for $k_{J_1} > 20$ GeV, $k_{J_2} > 35$ GeV at $\sqrt{s} = 13$ TeV



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

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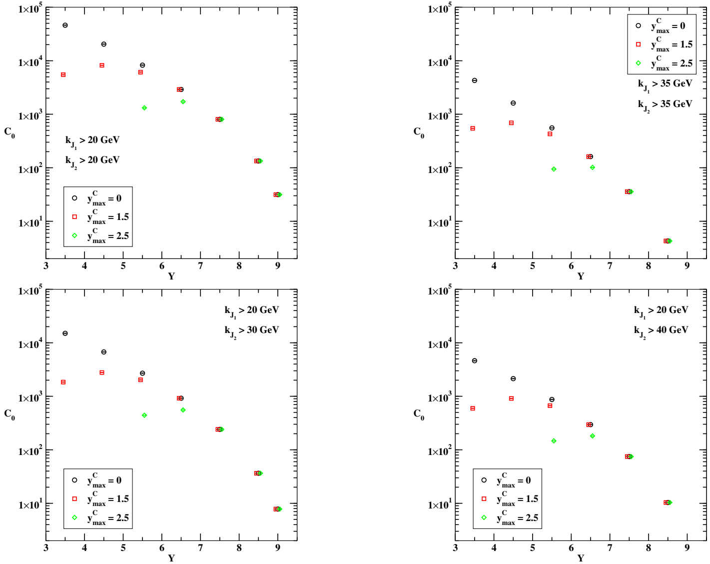
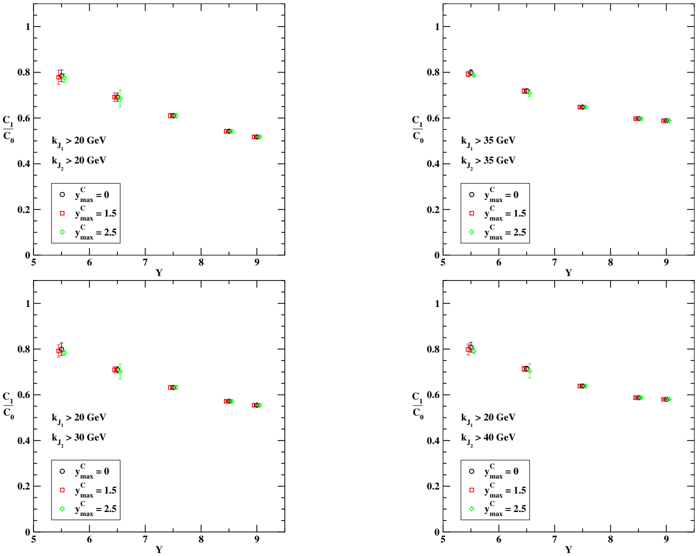
C_0 vs $Y = y_{J_1} - y_{J_2}$ - “exact" MOM BLM method



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

BACKUP slides

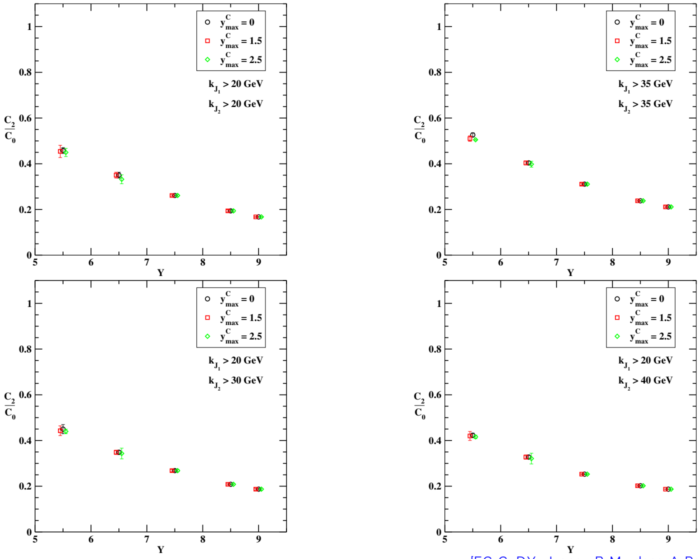
C_1/C_0 vs Y - "exact" BLM method



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

BACKUP slides

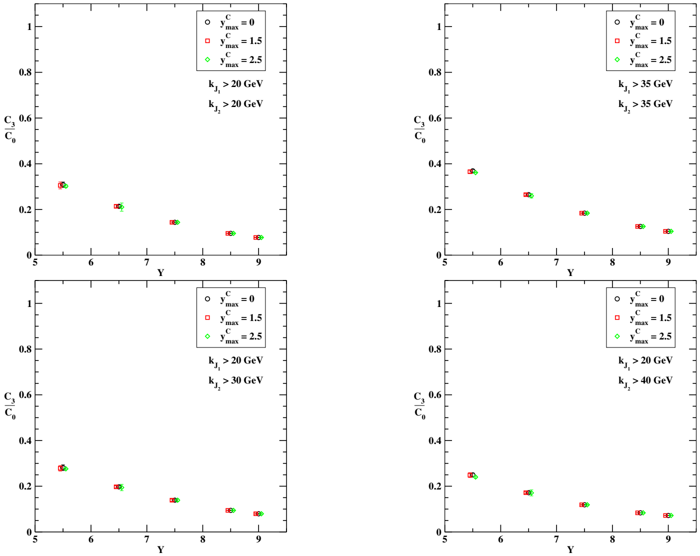
C_2/C_0 vs Y - “exact" BLM method



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

BACKUP slides

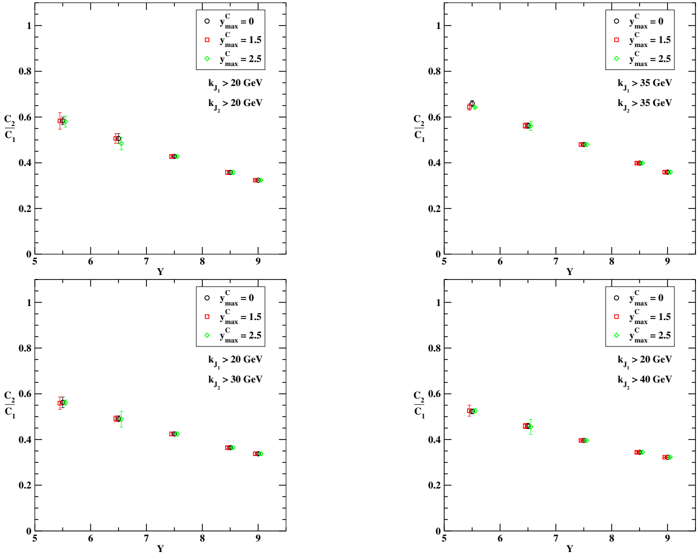
C_3/C_0 vs Y - "exact" BLM method



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

BACKUP slides

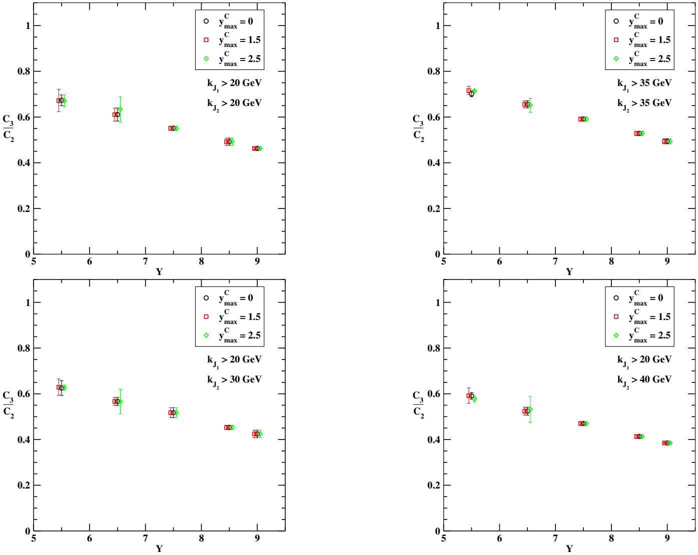
C_2/C_1 vs Y - "exact" BLM method



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

BACKUP slides

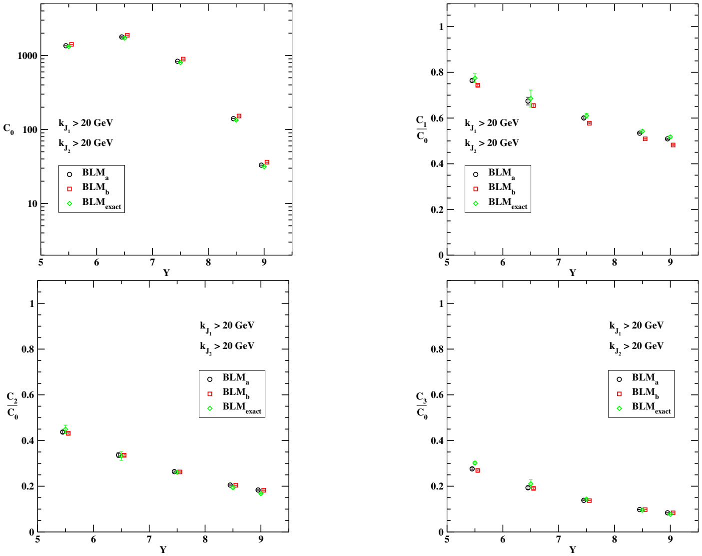
C_3/C_2 vs Y - “exact" BLM method



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

BACKUP slides

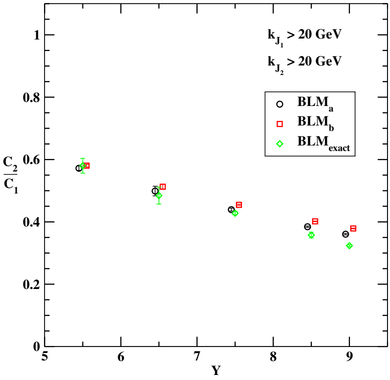
BLM comparisons of C_0 and R_{n0} vs Y - $y^C_{\max} = 2.5$

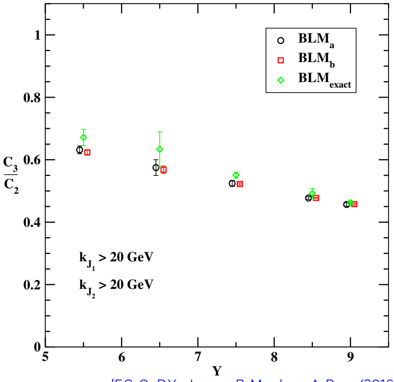


[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

BACKUP slides

BLM comparisons of C_2/C_1 and C_3/C_2 vs Y - $y^C_{max} = 2.5$





[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016)]

di-hadron production

Di-hadron production

Process: $\text{proton}(\text{p}_1) + \text{proton}(\text{p}_2) \to \text{hadron}_1(\text{k}_1) + \text{X} + \text{hadron}_2(\text{k}_2)$

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{i,j=q,g} \int_0^1 \int_0^1 dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}(x_1 x_2 s, \mu)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$

- large hadron transverse momenta: $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda^2_{\rm QCD}$ $\Rightarrow$ *pQCD allowed*
- QCD collinear factorization
- large rapidity intervals between hadrons (high energies) $\Rightarrow$ $\Delta y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}$

  $\Rightarrow$ BFKL resummation: $\sum_n \left(a_n^{(0)}\ \alpha_s^n \ln^n s + a_n^{(1)}\ \alpha_s^n \ln^{n-1} s \right)$

- Collinear fragmentation of the parton i into a hadron h
  $\Rightarrow$ convolution of D_i^h with a coefficient function C_i^h

$$d\sigma_i = C_i^h(z) dz \to d\sigma^h = d\alpha_h \int_{\alpha_h}^1 \frac{dz}{z} D_i^h \left(\frac{\alpha_h}{z}, \mu\right) C_i^h(z,\mu)$$

where α_h is the momentum fraction carried by the hadron

BACKUP slides

The BFKL BLM cross section (di-hadrons)

$$
\begin{aligned}
C_n^{\rm BLM} = \frac{e^Y}{s} \int_{y_{\rm min}}^{y_{\rm max}} dy_1 \int_{k_{1,\rm min}}^{\infty} dk_1 \int_{k_{2,\rm min}}^{\infty} dk_2 \int_{-\infty}^{+\infty} d\nu \, \exp\left[(Y - Y_0)\, \bar\alpha_s^{\rm MOM}(\mu_R^*) \left\{ \chi(n,\nu) \right.\right. \\
\left.\left. + \, \bar\alpha_s^{\rm MOM}(\mu_R^*) \left(\bar\chi(n,\nu) + \frac{T^{\rm conf}}{C_A} \chi(n,\nu) \right) \right\} \right] 4 (\alpha_s^{\rm MOM}(\mu_R^*))^2 \frac{C_F}{C_A} \frac{1}{|\vec k_1||\vec k_2|} \left(\frac{\vec k_1^2}{\vec k_2^2} \right)^{i\nu} \\
\times \int_{\alpha_1}^{1} \frac{dx}{x} \left(\frac{x}{\alpha_1} \right)^{2i\nu - 1} \left[\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_1}{x} \right) + \sum_{a=q,\bar q} f_a(x) D_a^h \left(\frac{\alpha_1}{x} \right) \right] \\
\times \int_{\alpha_2}^{1} \frac{dz}{z} \left(\frac{z}{\alpha_2} \right)^{-2i\nu - 1} \left[\frac{C_A}{C_F} f_g(z) D_g^h\left(\frac{\alpha_2}{z} \right) + \sum_{a=q,\bar q} f_a(z) D_a^h \left(\frac{\alpha_2}{z} \right) \right] \\
\times \left[1 + \bar\alpha_s^{\rm MOM}(\mu_R^*) \left(\frac{\bar c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{\bar c_2^{(1)}(n,\nu)}{c_2(n,\nu)} + 2 \frac{T^{\rm conf}}{C_A} \right) \right] ,
\end{aligned}
$$

with the μ_R^* scale chosen as the solution of the following integral equation...

Observables and kinematics (di-hadrons)

- **Observables:**

  ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos(n\phi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$

  $\langle \cos(2\phi) \rangle / \langle \cos(\phi) \rangle \equiv \mathcal{C}_2/\mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\phi) \rangle / \langle \cos(2\phi) \rangle \equiv \mathcal{C}_3/\mathcal{C}_2 \equiv R_{32}$

◇ *Integrated coefficients:*

$$C_n = \int_{y_{1,\rm min}}^{y_{1,\rm max}} dy_1 \int_{y_{2,\rm min}}^{y_{2,\rm max}} dy_2 \int_{k_{1,\rm min}}^{k_{1,\rm max}} dk_1 \int_{k_{2,\rm min}}^{k_{2,\rm max}} dk_2 \, \delta(y_1 - y_2 - Y)\, \mathcal{C}_n(y_1, y_2, k_1, k_2)$$

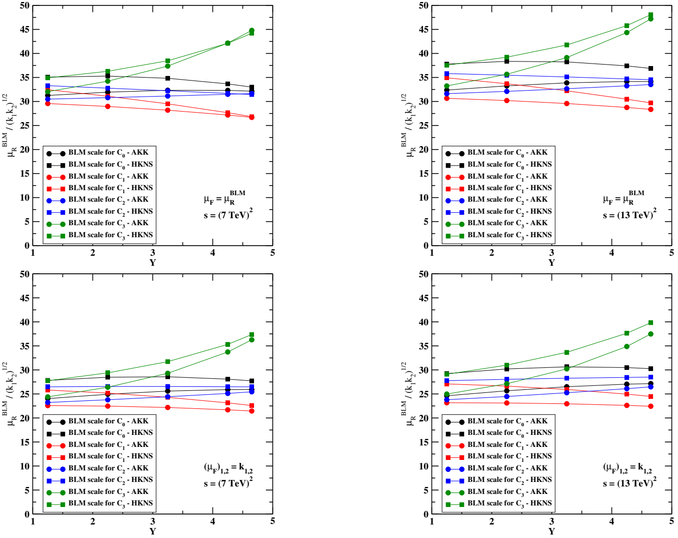
- **Kinematic settings:**
  - ◇ $\sqrt{s} = 7,\ 13$ TeV
  - ◇ $|y_i| \leqslant 2.4, 4.7$, with $i = 1, 2$
  - ◇ $k_{1,2} \geqslant$ **5 GeV** ...vs $k_{J_{1,2}}^{\rm MN-jets} \geqslant 35$ GeV! → more secondary gluon emissions!

- **Phenomenological analysis:**
  - ◇ full **NLA** BFKL
  - ◇ JETHAD (CSLIB, F95) + CERNLIB
  - ◇ (MSTW08, MMHT14, CT14) PDFs ⊛ (AKK08, DSS07, HKNS07) FFs

[F.G. C., D.Yu Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

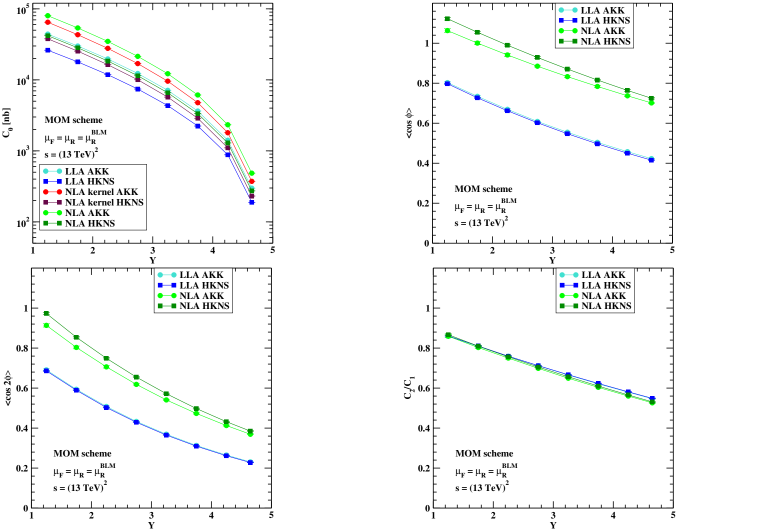
BLM values for μ_R (di-hadrons)



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

C_0 and R_{nm} at $\sqrt{s} = 13$ TeV, $Y \leqslant 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

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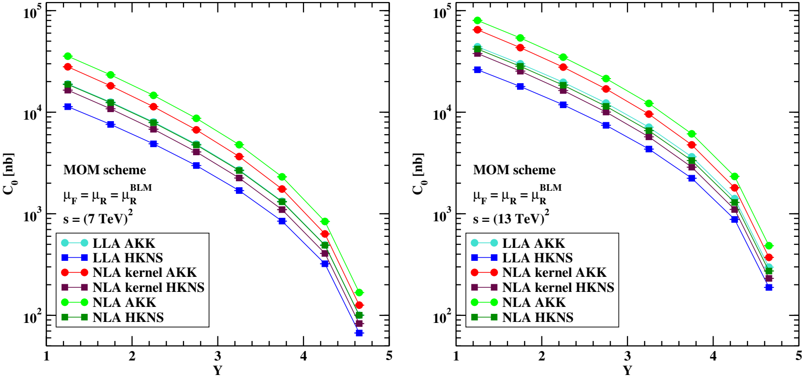

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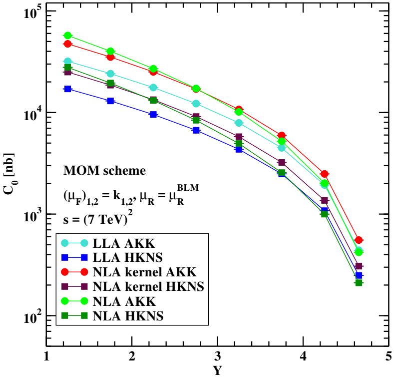
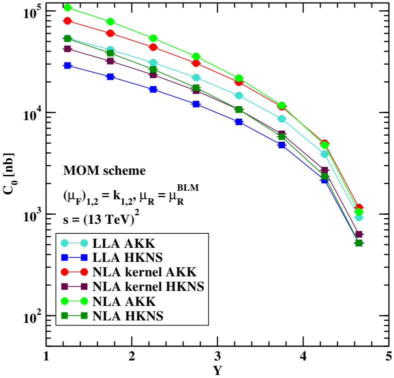
BACKUP slides

C_0 at $\sqrt{s} = 7, 13$ TeV, $Y \leqslant 4.8$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$

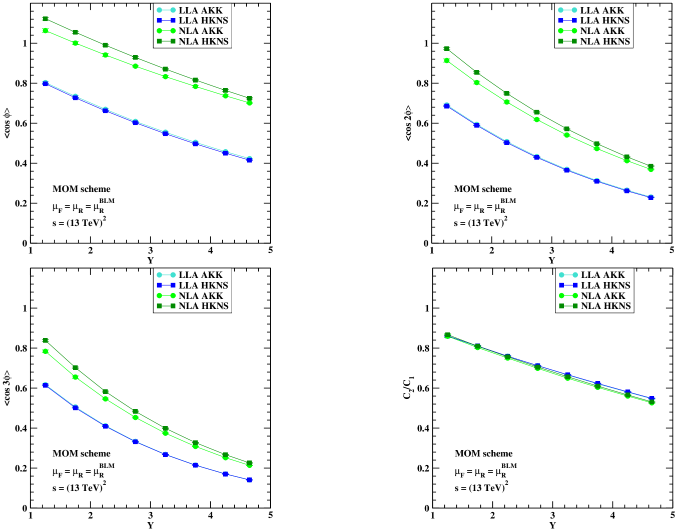




[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

R_{nm} at $\sqrt{s} = 13$ TeV, $Y \leqslant 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

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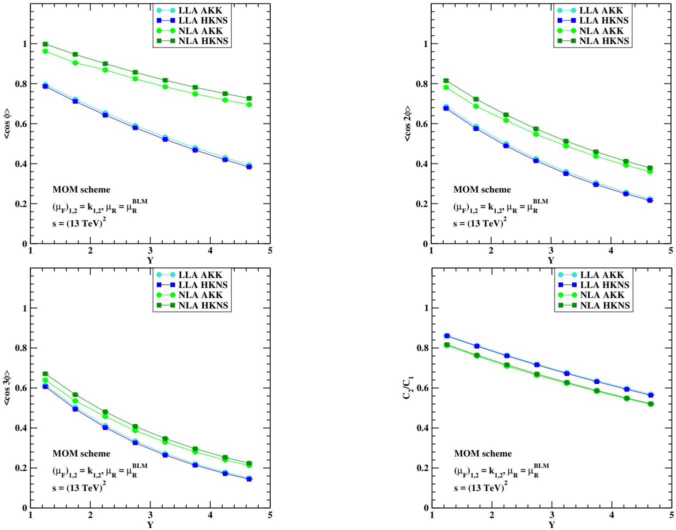
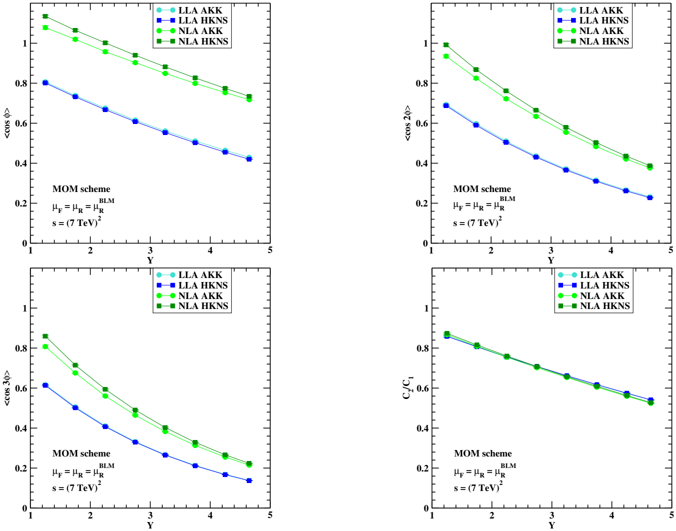


[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

R_{nm} at $\sqrt{s} = 7$ TeV, $Y \leqslant 4.8$, $\mu_F = \mu_R^{\rm BLM}$



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BACKUP slides

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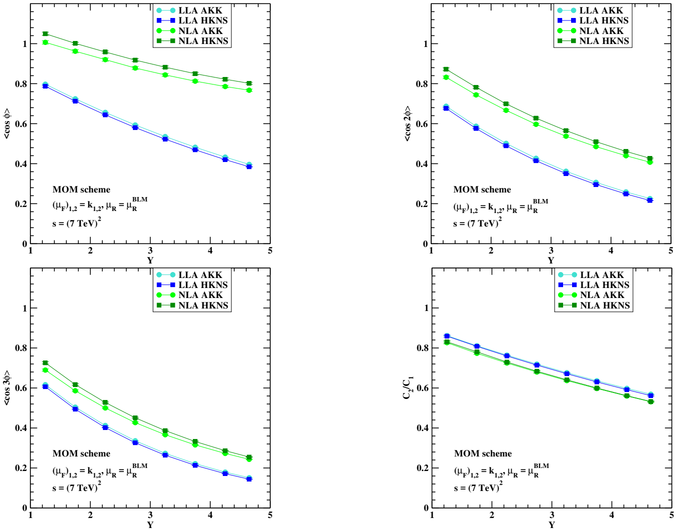
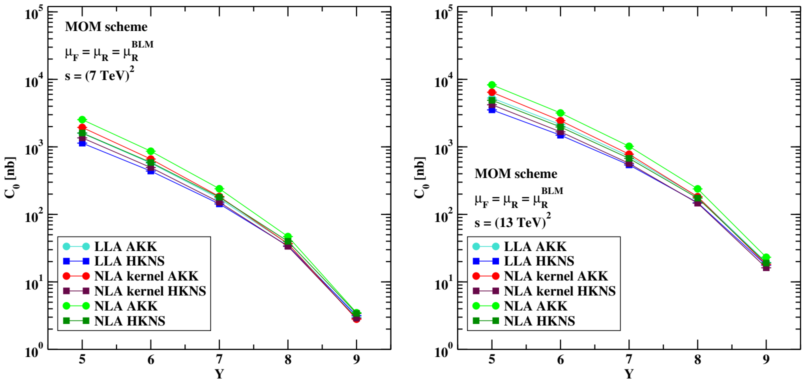


[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

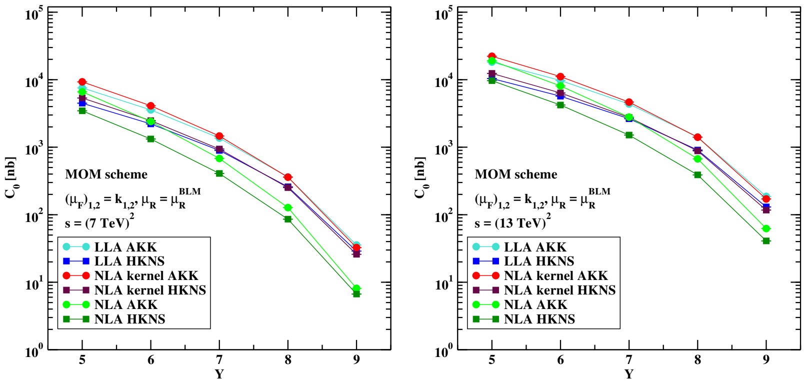
C_0 at $\sqrt{s} = 7, 13$ TeV, $Y \leqslant 9.4$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

C_0 at $\sqrt{s} = 7, 13$ TeV, $Y \leqslant 9.4$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$



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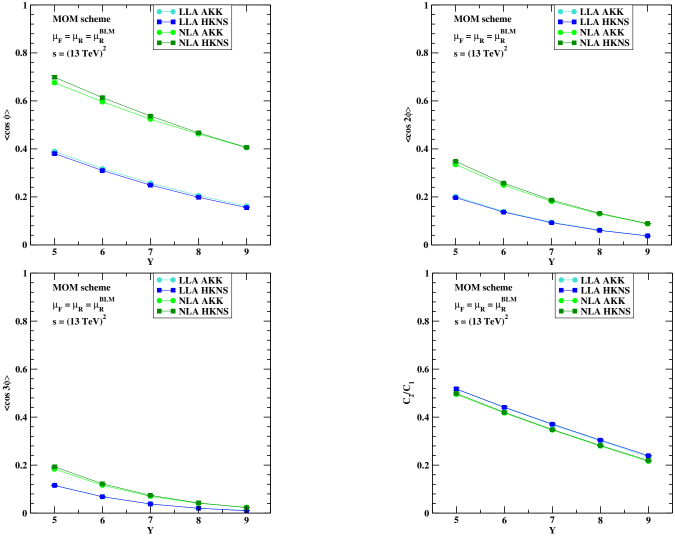
BACKUP slides

R_{nm} at $\sqrt{s} = 13$ TeV, $Y \leqslant 9.4$, $\mu_F = \mu_R^{\text{BLM}}$



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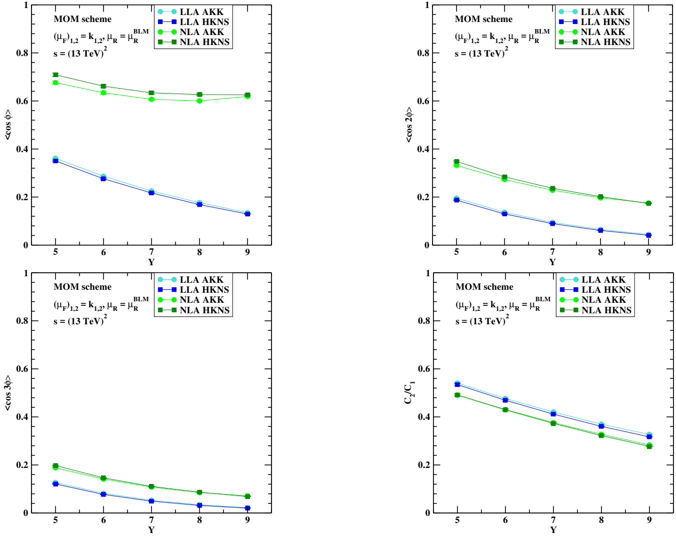

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

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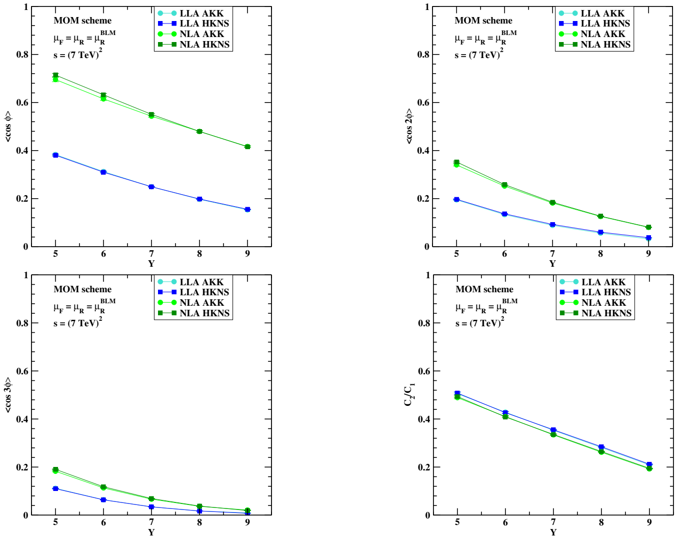

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BACKUP slides

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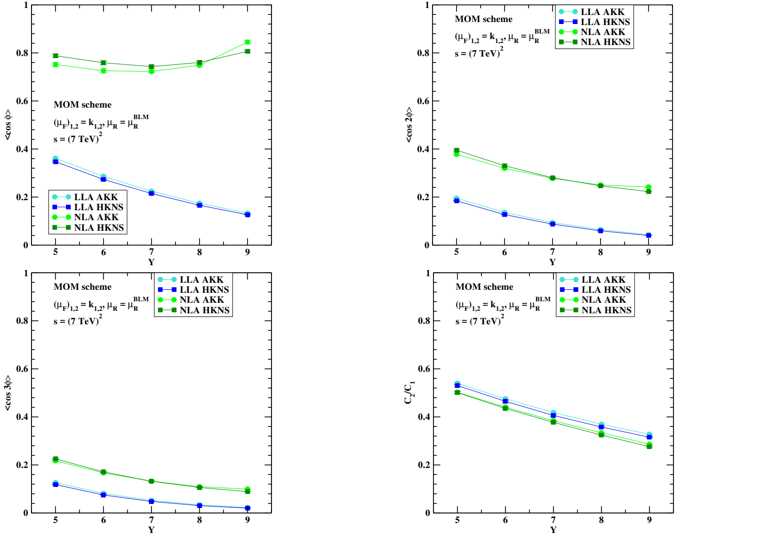
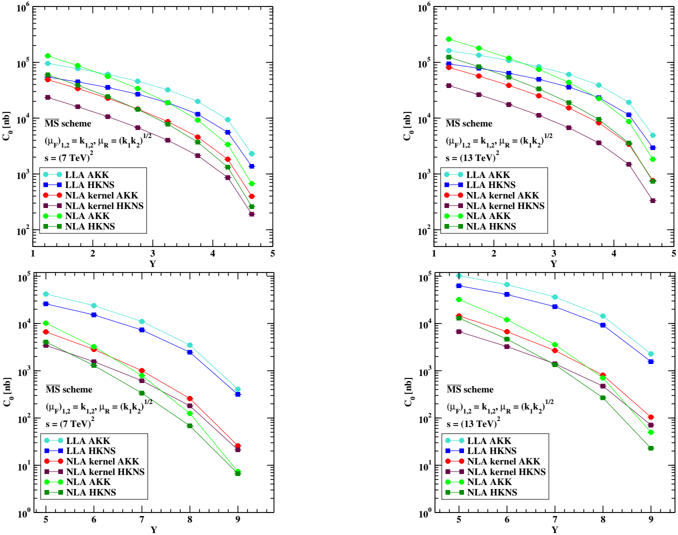


[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

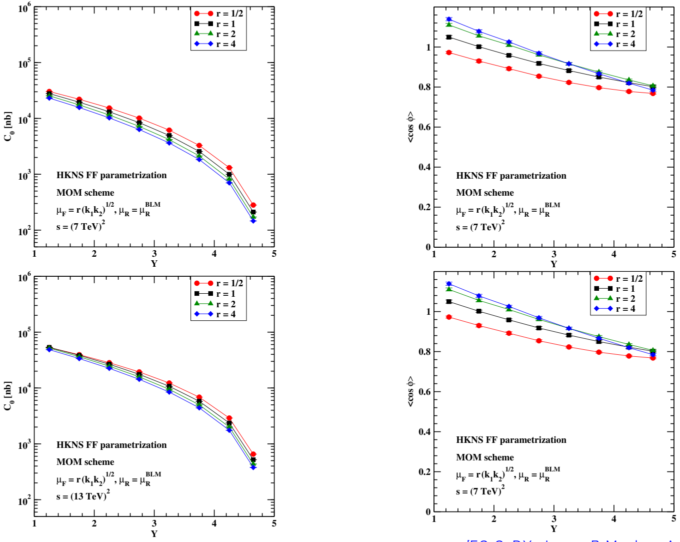
C_0 at $\sqrt{s} = 7, 13$ TeV, $\mu_R = \sqrt{|\vec{k}_1||\vec{k}_2|}$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

C_0, R_{10} at $\sqrt{s} = 7, 13$ TeV, $Y \leqslant 4.8$, $\mu_F = r\sqrt{|\vec{k}_1||\vec{k}_2|}$

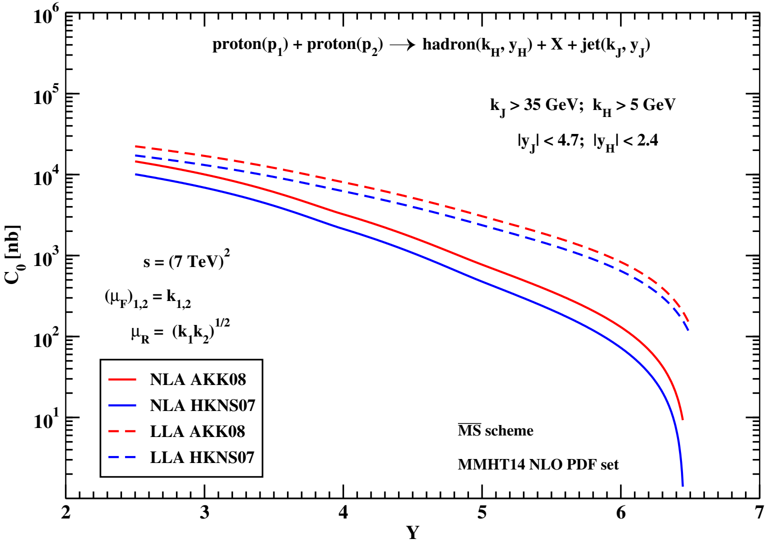


[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

hadron-jet correlations

BACKUP slides

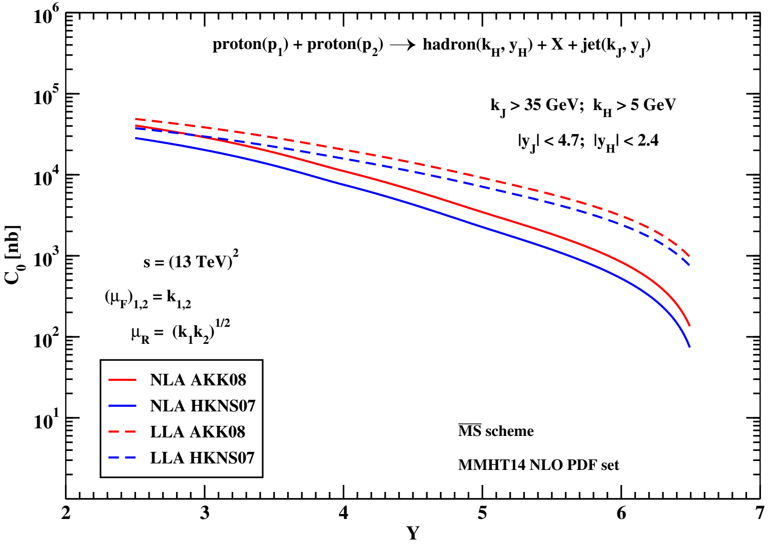
Hadron-jet C_0 vs Y, $\sqrt{s} = 7$ TeV, NS $\overline{\text{MS}}$



preliminary results [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A. Papa (in progress)]

BACKUP slides

Hadron-jet C_0 vs Y, $\sqrt{s} = 13$ TeV, NS $\overline{\rm MS}$



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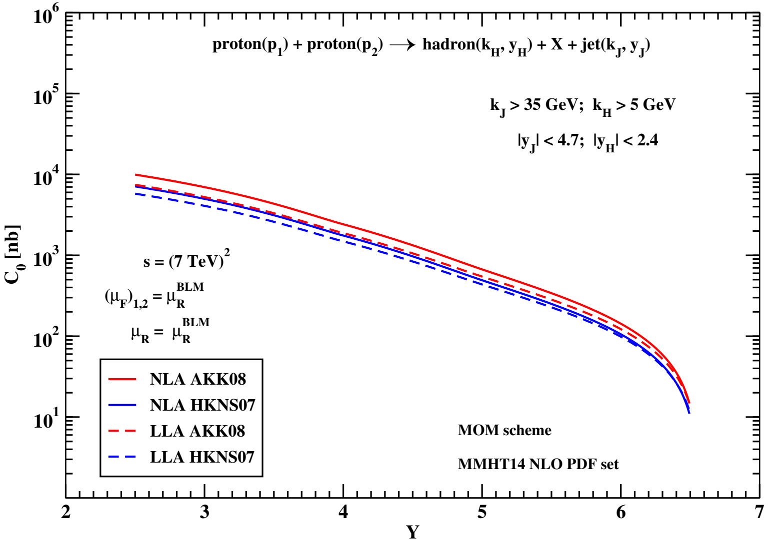
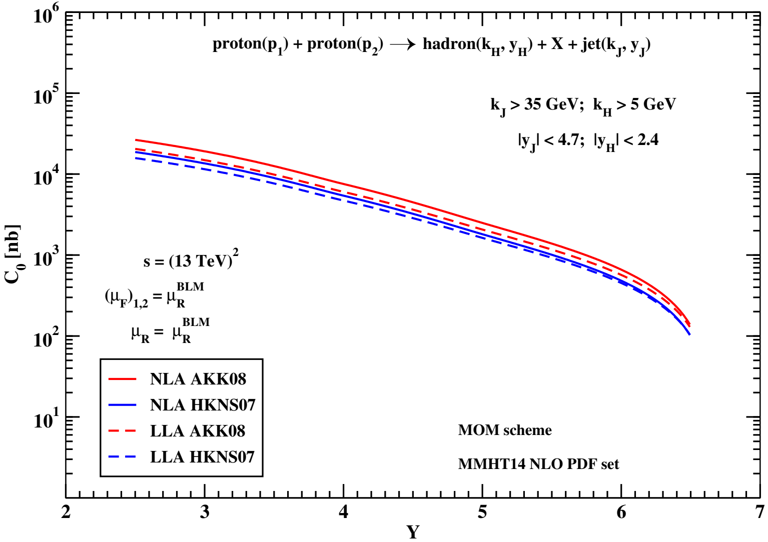
Hadron-jet C_0 vs Y, $\sqrt{s} = 7$ TeV



preliminary results [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A. Papa (in progress)]

BACKUP slides

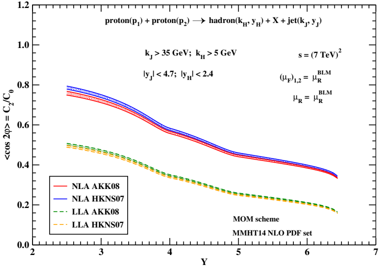
Hadron-jet C_0 vs Y, $\sqrt{s} = 13$ TeV

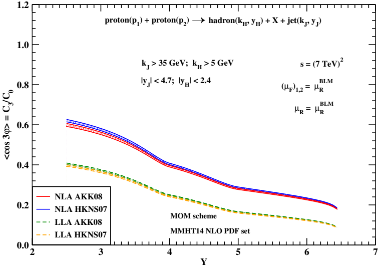


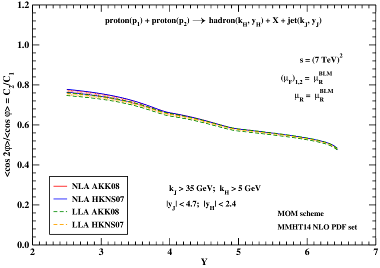
preliminary results [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A. Papa (in progress)]

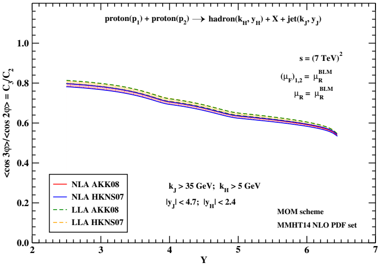
BACKUP slides

Hadron-jet R_{nm} vs Y, $\sqrt{s} = 7$ TeV







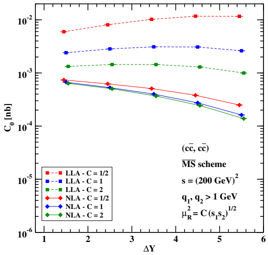


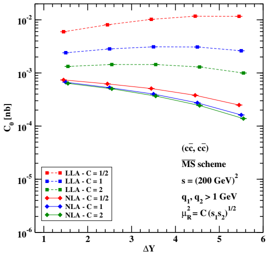
preliminary results [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, A. Papa (in progress)]

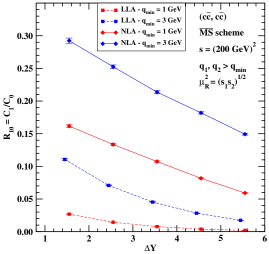
heavy-quark pair photoproduction

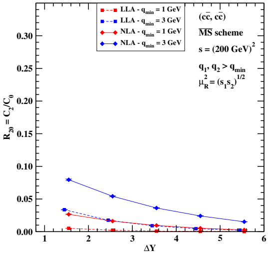
BACKUP slides

C_0 and R_{n0} vs Y at LEP2 (heavy quarks)







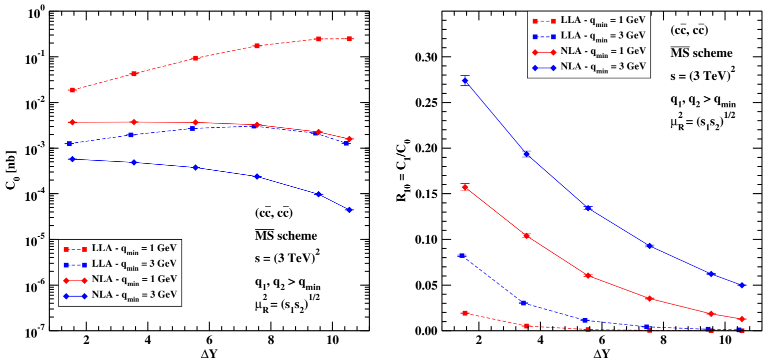


$$s_{1,2} = m_{1,2}^2 + q_{1,2}^2$$

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018) **arXiv:1709.10032 [hep-ph]**]

BACKUP slides

C_0 and R_{10} vs Y at e^+e^- future colliders (heavy quarks)



$$s_{1,2} = m_{1,2}^2 + q_{1,2}^2$$

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018) **arXiv:1709.10032 [hep-ph]**]

Looking for new observables

- BFKL feature: factorization between transverse and longitudinal (rapidities) degrees of freedom
- Usual "growth with energy" signal mainly probes the longitudinal degrees of freedom
- Mueller–Navelet correlation momenta mainly probe one of the transverse components, the azimuthal angles

! We would like to study observables for which the p_T (any p_T along the BFKL ladder) enters the game...
  - ◇ ...to probe not only the general properties of the BFKL ladder, but also "to peek into the interior"...
  - ◇ ...by studying azimuthal decorrelations where the p_T of extra particles introduces a new dependence...

...multi-jet production!

[R. Maciula, A. Szczurek (2014, 2015)]
[K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016)]

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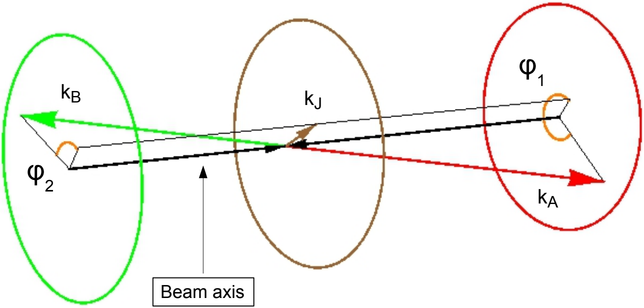
...multi-jet production!

[R. Maciula, A. Szczurek (2014, 2015)]
[K. Kutak, R. Maciula, M. Serino, A. Szczurek, A. van Hameren (2016)]

three-jet production

An event with three tagged jets



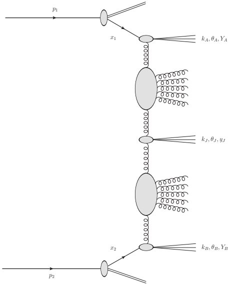
$$Y_B < y_J < Y_A$$

BACKUP slides

The three-jet partonic cross section

Starting point: differential partonic cross-section (no PDFs)

$$\frac{d^3\hat{\sigma}^{3-\text{jet}}}{dk_J d\theta_J dy_J} = \frac{\bar{\alpha}_s}{\pi k_J}\int d^2\vec{p}_A \int d^2\vec{p}_B\, \delta^{(2)}\left(\vec{p}_A + \vec{k}_J - \vec{p}_B\right)$$
$$\times \quad \varphi\left(\vec{k}_A, \vec{p}_A, Y_A - y_J\right)\varphi\left(\vec{p}_B, \vec{k}_B, y_J - Y_B\right)$$



- *Multi-Regge kinematics* Rapidity ordering: $Y_B < y_J < Y_A$
- k_J lie above the experimental resolution scale
- φ is the LO BFKL gluon Green function
- $\bar{\alpha}_s = \alpha_s N_c/\pi$

BACKUP slides

Three-jets: generalized azimuthal correlations

Prescription: integrate over all angles after using the projections on the two azimuthal angle differences indicated below...to define:

$$\int_0^{2\pi} d\theta_A \int_0^{2\pi} d\theta_B \int_0^{2\pi} d\theta_J \cos\left(M\left(\theta_A - \theta_J - \pi\right)\right) \cos\left(N\left(\theta_J - \theta_B - \pi\right)\right) \frac{d^3\hat{\sigma}^{3-\text{jet}}}{dk_J d\theta_J dy_J}$$

$$= \bar{\alpha}_s \sum_{L=0}^{N} \binom{N}{L} \left(k_J^2\right)^{\frac{L-1}{2}} \int_0^{\infty} dp^2 \left(p^2\right)^{\frac{N-L}{2}} \int_0^{2\pi} d\theta \frac{(-1)^{M+N} \cos\left(M\theta\right) \cos\left((N-L)\theta\right)}{\sqrt{\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2} \cos\theta\right)^N}}$$

$$\times\ \phi_M\left(k_A^2, p^2, Y_A - y_J\right) \phi_N\left(p^2 + k_J^2 + 2\sqrt{p^2 k_J^2}\cos\theta, k_B^2, y_J - Y_B\right)$$

Main observables: **generalized azimuthal correlation momenta**

$$\mathcal{R}_{PQ}^{MN} = \frac{\mathcal{C}_{MN}}{\mathcal{C}_{PR}} = \frac{\langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle}{\langle \cos(P(\theta_A - \theta_J - \pi)) \cos(Q(\theta_J - \theta_B - \pi)) \rangle}$$

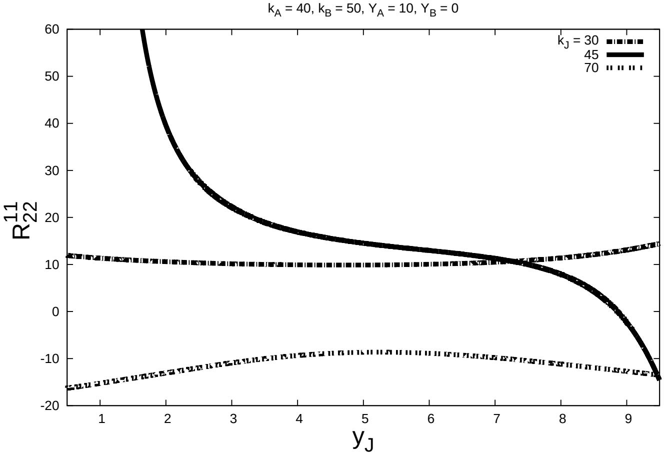
◇ Remove the contribution from the zero conformal spin

$\xrightarrow{\text{to}}$ drastically reduce the dependence on collinear configurations

study $\mathcal{R}_{PQ}^{MN}$ with integer $M, N, P, Q > 0$

BACKUP slides

Partonic prediction of $\mathcal{R}_{22}^{11}$ for $k_J = 30, 45, 70$ GeV

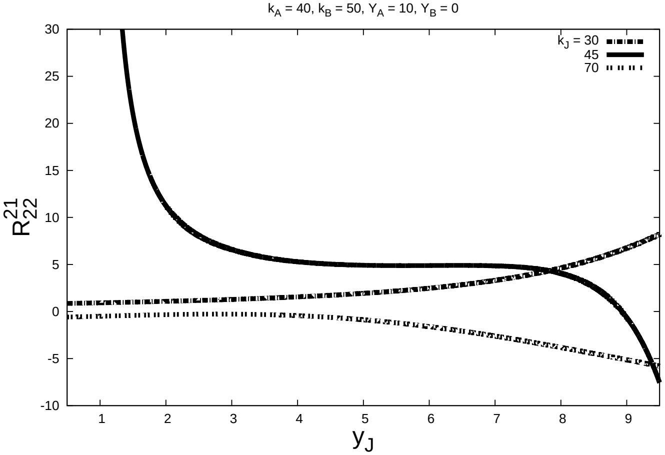


[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

$Y_A - Y_B$ is fixed to 10; y_J varies beetwen 0.5 and 9.5.

BACKUP slides

Partonic prediction of $\mathcal{R}^{21}_{22}$ for $k_J = 30, 45, 70$ GeV



[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]

$Y_A - Y_B$ is fixed to 10; y_J varies beetwen 0.5 and 9.5.

Next step: hadronic level predictions (3-jets)

- Introduce PDFs and running of the strong coupling:

$$\frac{d\sigma^{3-\text{jet}}}{dk_A\,dY_A\,d\theta_A\,dk_B\,dY_B\,d\theta_B\,dk_J\,dy_J d\theta_J} =$$

$$\frac{8\pi^3\,C_F\,\bar{\alpha}_s\,(\mu_R)^3}{N_C^3}\,\frac{x_{J_A}\,x_{J_B}}{k_A\,k_B\,k_J}\int d^2\vec{p}_A\int d^2\vec{p}_B\,\delta^{(2)}\left(\vec{p}_A+\vec{k}_J-\vec{p}_B\right)$$

$$\times\left(\frac{N_C}{C_F}f_g(x_{J_A},\mu_F)+\sum_{r=q,\bar{q}}f_r(x_{J_A},\mu_F)\right)$$

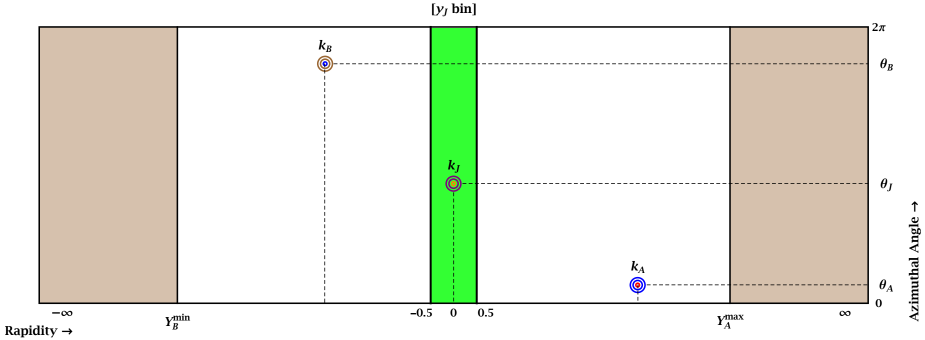
$$\times\left(\frac{N_C}{C_F}f_g(x_{J_B},\mu_F)+\sum_{s=q,\bar{q}}f_s(x_{J_B},\mu_F)\right)$$

$$\times\,\varphi\left(\vec{k}_A,\vec{p}_A,Y_A-y_J\right)\,\varphi\left(\vec{p}_B,\vec{k}_B,y_J-Y_B\right)$$

- Match the LHC kinematical cuts (integrate $d\sigma^{3-\text{jet}}$ on k_T and rapidities):
  - ◇ **1.** $k_A \geqslant 35$ GeV; $k_B \geqslant 35$ GeV; symmetric cuts
    2. $k_A \geqslant 35$ GeV; $k_B \geqslant 50$ GeV; asymmetric cuts
  - ◇ **a)** Y_A and Y_B integrated on windows
    b) $Y_A - Y_B \equiv Y$ fixed
  - ◇ binning on y_J

BACKUP slides

a) Integrate over a central rapidity bin

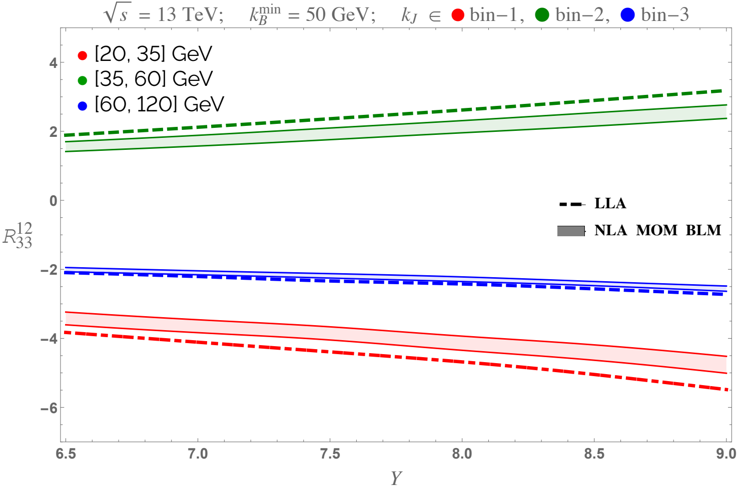


$$Y_A^{\max} = -Y_B^{\min} = 4.7$$

BACKUP slides

a) R_{33}^{12} vs Y at 13 TeV

$k_A^{\min} = 35$ GeV, $k_B^{\min} = 50$ GeV, $k_A^{\max} = k_B^{\max} = 60$ GeV (asymmetric)

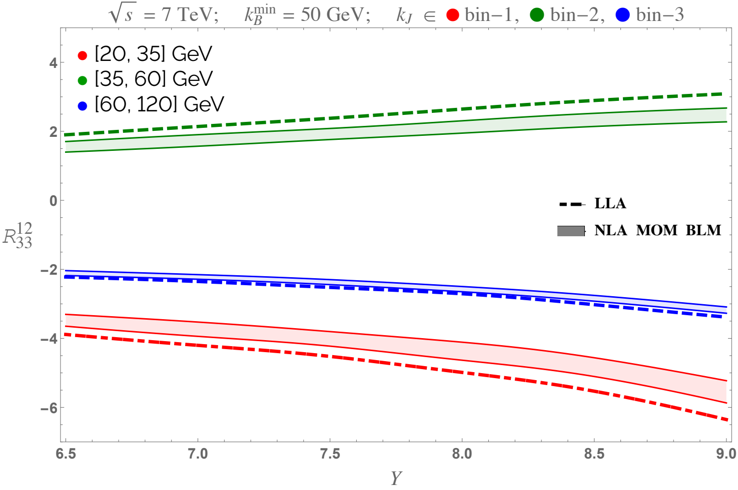


[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

BACKUP slides

a) R^{12}_{33} vs Y at 7 TeV

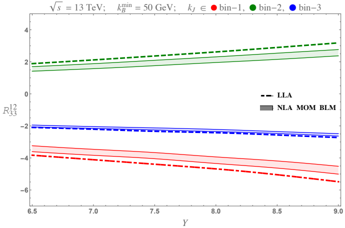
$k_A^{\min} = 35$ GeV, $k_B^{\min} = 50$ GeV, $k_A^{\max} = k_B^{\max} = 60$ GeV (asymmetric)

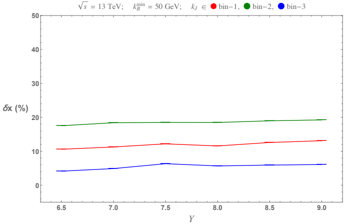


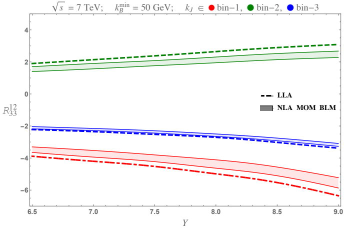
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

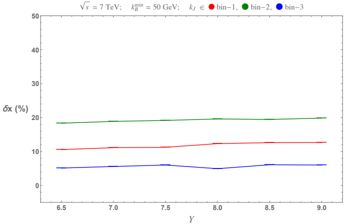
BACKUP slides

a) R^{12}_{33} vs Y at 13 and 7 TeV





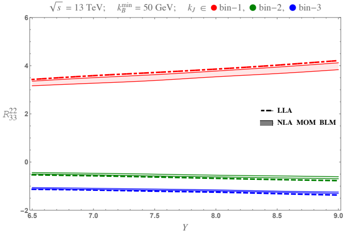


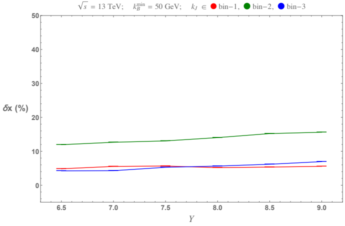


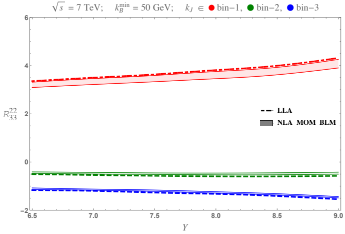
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

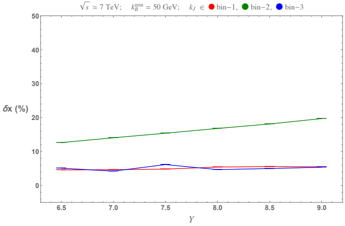
BACKUP slides

a) R_{33}^{22} vs Y at 13 and 7 TeV





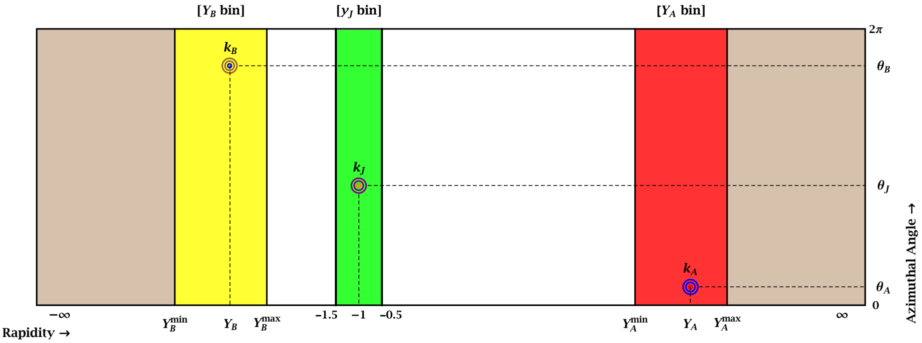




[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

BACKUP slides

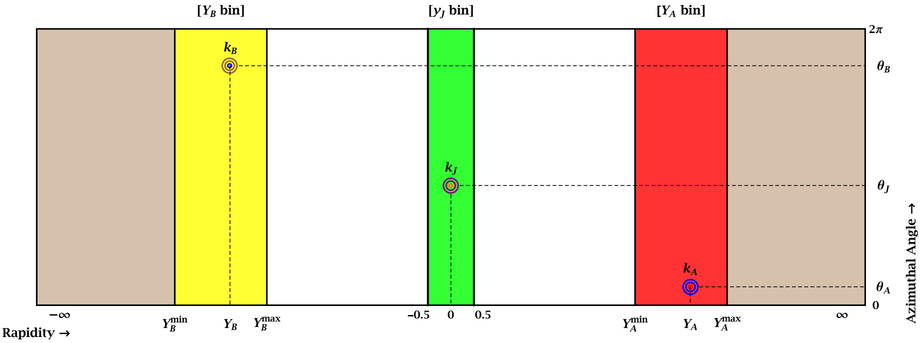
b) Integrate over a forward, backward and central rapidity bin



$$Y_A^{max} = -Y_B^{min} = 4.7$$
$$Y_A^{min} = -Y_B^{max} = 3$$

BACKUP slides

b) Integrate over a forward, backward and central rapidity bin



$$Y_A^{\rm max} = -Y_B^{\rm min} = 4.7$$
$$Y_A^{\rm min} = -Y_B^{\rm max} = 3$$

BACKUP slides

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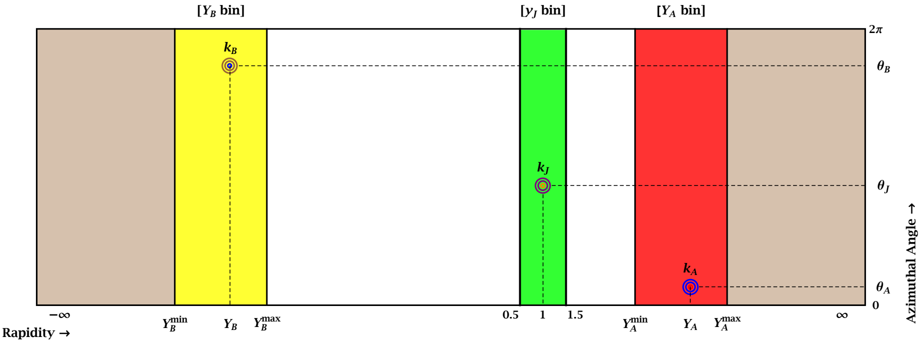
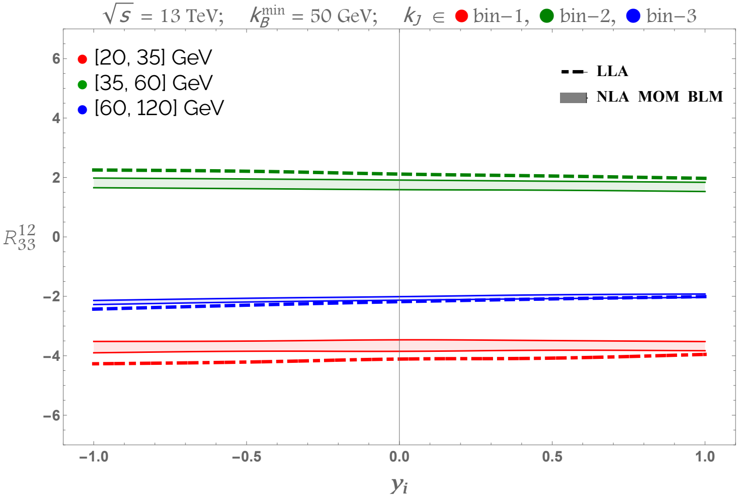


$$Y_A^{\rm max} = -Y_B^{\rm min} = 4.7$$
$$Y_A^{\rm min} = -Y_B^{\rm max} = 3$$

BACKUP slides

b) $R^{12}_{33}(y_i)$ at 13 TeV

$k_A^{\min} = 35$ GeV, $k_B^{\min} = 50$ GeV, $k_A^{\max} = k_B^{\max} = 60$ GeV (asymmetric)



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

b) $R_{33}^{12}(y_i)$ at 7 TeV

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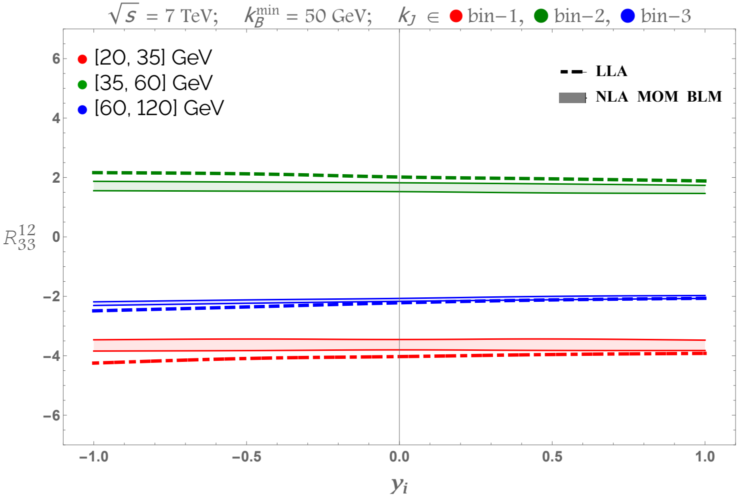
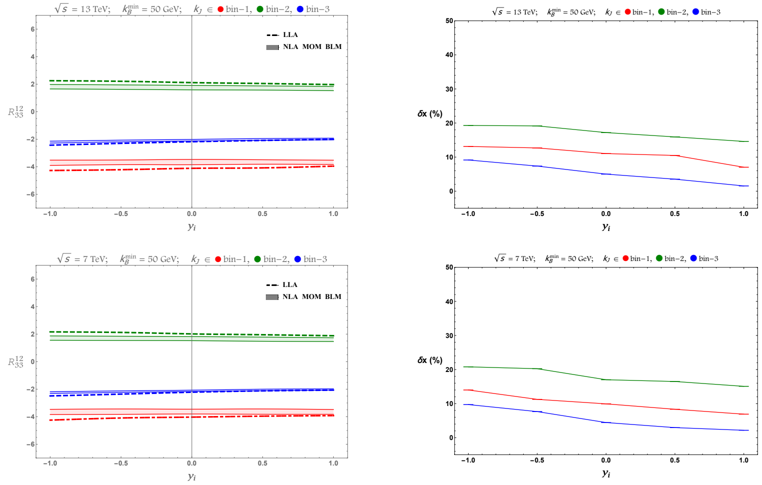


[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

BACKUP slides

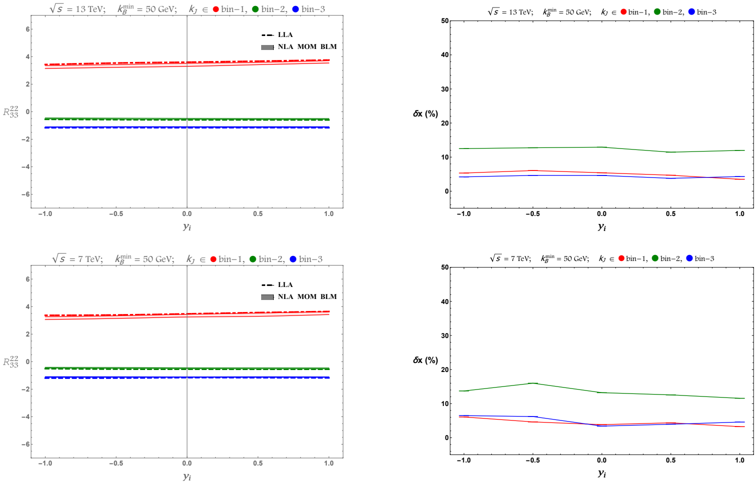
b) $R^{12}_{33}(y_i)$ at 13 and 7 TeV



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

BACKUP slides

b) $R^{22}_{33}(y_i)$ at 13 and 7 TeV

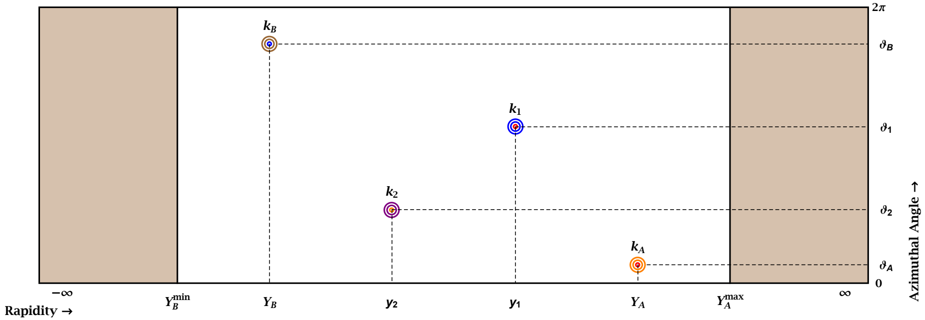


[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

four-jet production

BACKUP slides

A four-jet primitive lego-plot



$$Y_A^{\max} = -Y_B^{\min} = 4.7$$

Four-jets: generalized azimuthal coefficients - partonic level

$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \, \cos\left(M\left(\vartheta_A - \vartheta_1 - \pi\right)\right)$$

$$\cos\left(N\left(\vartheta_1 - \vartheta_2 - \pi\right)\right)\cos\left(L\left(\vartheta_2 - \vartheta_B - \pi\right)\right) \frac{d^6\sigma^{4-\text{jet}}\left(\vec{k_A}, \vec{k_B}, Y_A - Y_B\right)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

$$= \frac{2\pi^2 \bar{\alpha}_s\left(\mu_R\right)^2}{k_1 k_2}\left(-1\right)^{M+N+L}\left(\tilde{\Omega}_{M,N,L} + \tilde{\Omega}_{M,N,-L} + \tilde{\Omega}_{M,-N,L}\right.$$

$$\left. + \tilde{\Omega}_{M,-N,-L} + \tilde{\Omega}_{-M,N,L} + \tilde{\Omega}_{-M,N,-L} + \tilde{\Omega}_{-M,-N,L} + \tilde{\Omega}_{-M,-N,-L}\right)$$

with

$$\tilde{\Omega}_{m,n,l} = \int_0^{+\infty} dp_A \, p_A \int_0^{+\infty} dp_B \, p_B \int_0^{2\pi} d\phi_A \int_0^{2\pi} d\phi_B$$

$$\frac{e^{-im\phi_A} \, e^{il\phi_B} \, \left(p_A e^{i\phi_A} + k_1\right)^n \, \left(p_B e^{-i\phi_B} - k_2\right)^n}{\sqrt{\left(p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A\right)^n} \sqrt{\left(p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B\right)^n}}$$

$$\varphi_m\left(|\vec{k_A}|, |\vec{p_A}|, Y_A - y_1\right) \varphi_l\left(|\vec{p_B}|, |\vec{k_B}|, y_2 - Y_B\right)$$

$$\varphi_n\left(\sqrt{p_A^2 + k_1^2 + 2p_A k_1 \cos\phi_A}, \sqrt{p_B^2 + k_2^2 - 2p_B k_2 \cos\phi_B}, y_1 - y_2\right)$$

BACKUP slides

Four-jets: generalized azimuthal coefficients - partonic level

$$\mathcal{C}_{MNL} = \int_0^{2\pi} d\vartheta_A \int_0^{2\pi} d\vartheta_B \int_0^{2\pi} d\vartheta_1 \int_0^{2\pi} d\vartheta_2 \; \cos\left(M\left(\vartheta_A - \vartheta_1 - \pi\right)\right)$$

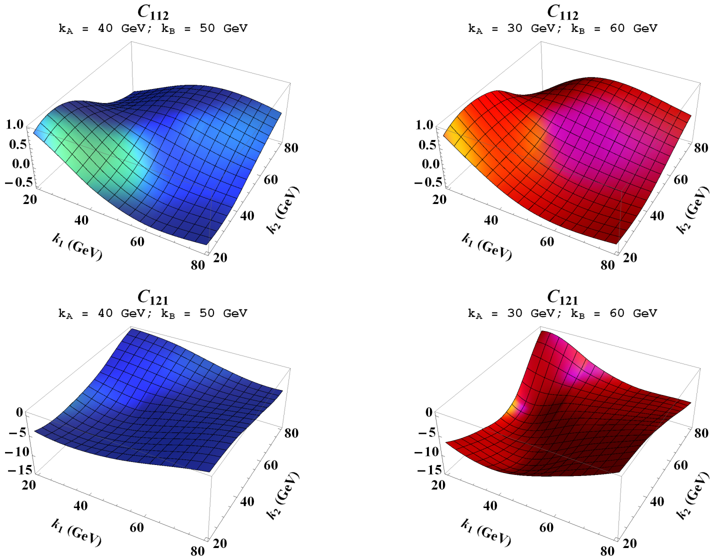
$$\cos\left(N\left(\vartheta_1 - \vartheta_2 - \pi\right)\right) \cos\left(L\left(\vartheta_2 - \vartheta_B - \pi\right)\right) \frac{d^6\sigma^{4-\text{jet}}\left(\vec{k_A}, \vec{k_B}, Y_A - Y_B\right)}{dk_1 dy_1 d\vartheta_1 dk_2 d\vartheta_2 dy_2}$$

Main observables: **generalized azimuthal correlation momenta**

$$\mathcal{R}^{MNL}_{PQR} = \frac{C_{MNL}}{C_{PRQ}} = \frac{\langle \cos(M(\vartheta_A - \vartheta_1 - \pi)) \cos(N(\vartheta_1 - \vartheta_2 - \pi)) \cos(L(\vartheta_2 - \vartheta_B - \pi)) \rangle}{\langle \cos(P(\vartheta_A - \vartheta_1 - \pi)) \cos(Q(\vartheta_1 - \vartheta_2 - \pi)) \cos(R(\vartheta_2 - \vartheta_B - \pi)) \rangle}$$

BACKUP slides

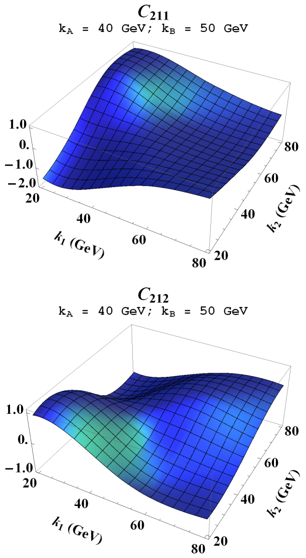
Partonic prediction of $\mathcal{C}_{MNL}$ vs $k_{1,2}$ (4-jets)



[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]

BACKUP slides

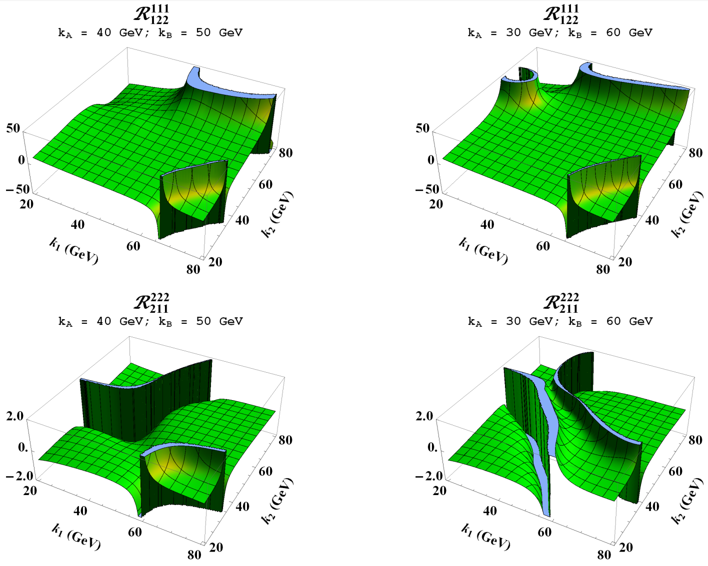
Partonic prediction of $\mathcal{C}_{MNL}$ vs $k_{1,2}$ (4-jets)



[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]

BACKUP slides

Partonic prediction of $\mathcal{R}^{MNL}_{PQR}$ vs $k_{1,2}$ (4-jets)



[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2017)]

BACKUP slides

Next step: hadronic level predictions (4-jets)

- Introduce PDFs and running of the strong coupling

- Use realistic LHC kinematical cuts:

◇ 1. $k_A^{min} = 35$ GeV, $k_A^{max} = 60$ GeV
$k_B^{min} = 45$ GeV, $k_B^{max} = 60$ GeV
$k_1^{min} = 20$ GeV, $k_1^{max} = 35$ GeV
$k_2^{min} = 60$ GeV, $k_2^{max} = 90$ GeV

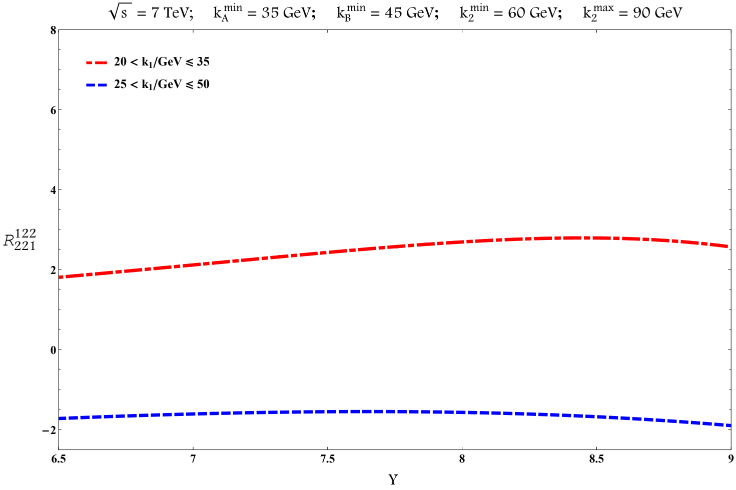
2. $k_A^{min} = 35$ GeV, $k_A^{max} = 60$ GeV
$k_B^{min} = 45$ GeV, $k_B^{max} = 60$ GeV
$k_1^{min} = 25$ GeV, $k_1^{max} = 50$ GeV
$k_2^{min} = 60$ GeV, $k_2^{max} = 90$ GeV

◇ $Y = Y_A - Y_B$ fixed;
$Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y/3$

◇ $\sqrt{s} = 7, 13$ TeV

BACKUP slides

R_{221}^{122} at $\sqrt{s} = 7$ TeV vs $Y = Y_A - Y_B$ for two k_1 bins



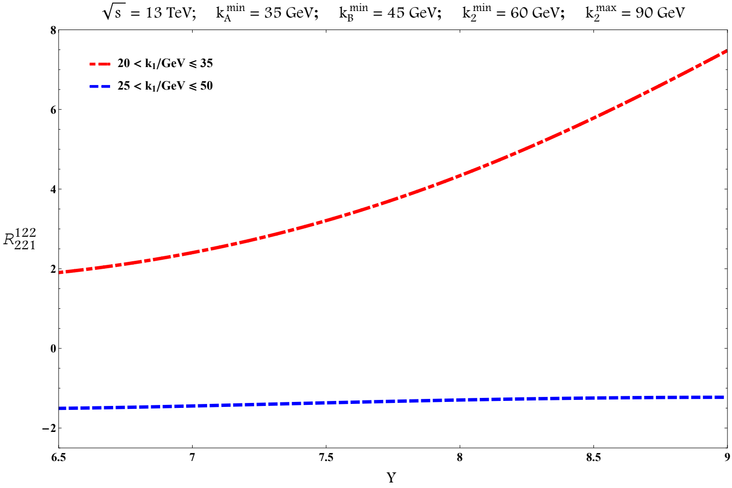
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016)]

Y is the rapidity difference between the most forward/backward jet;

$$Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y/3.$$

BACKUP slides

R^{122}_{221} at $\sqrt{s} = 13$ TeV vs $Y = Y_A - Y_B$ for two k_1 bins



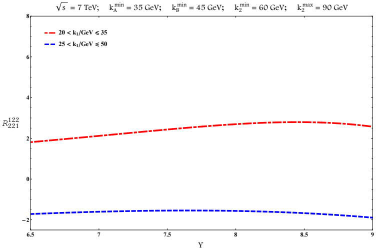
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

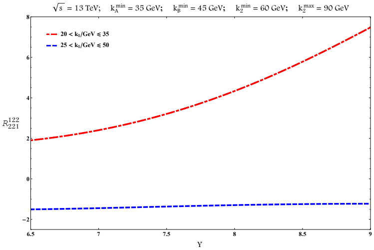
Y is the rapidity difference between the most forward/backward jet;

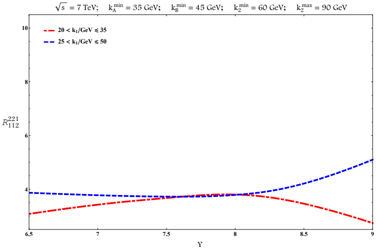
$$Y_A - y_1 = y_1 - y_2 = y_2 - Y_B = Y/3.$$

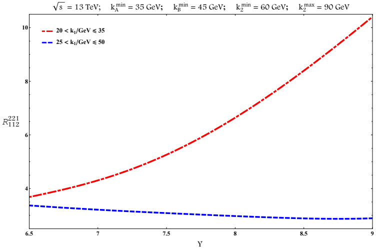
BACKUP slides

R^{122}_{221} and R^{221}_{112} vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two k_1 bins





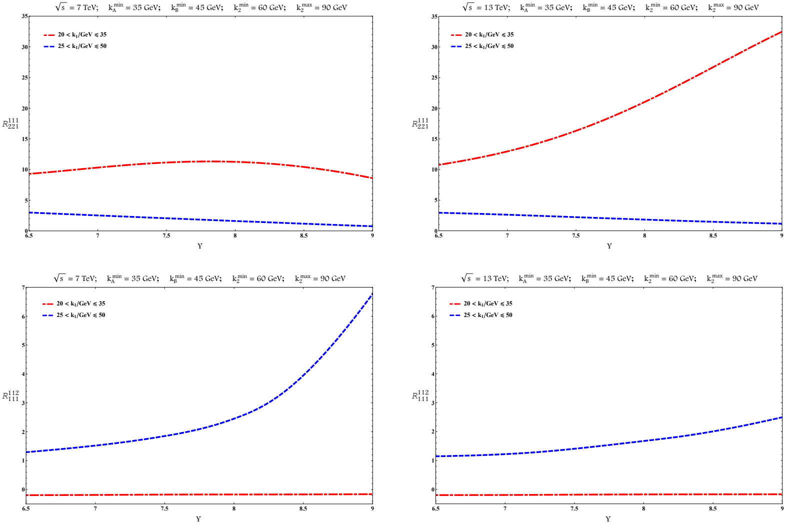




[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

BACKUP slides

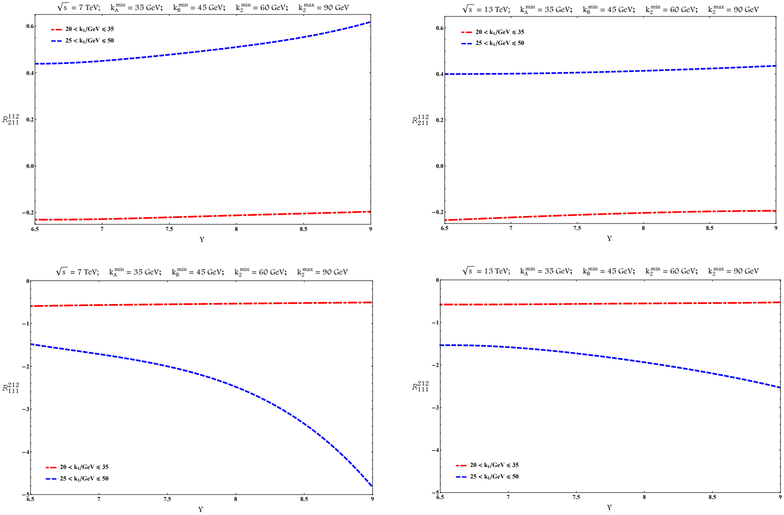
R^{111}_{221} and R^{112}_{111} vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two k_1 bins



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

BACKUP slides

R^{112}_{211} and R^{212}_{111} vs $Y = Y_A - Y_B$ and $\sqrt{s}$ for two k_1 bins



[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2017)]

ρ mesons and the UGD

BACKUP slides

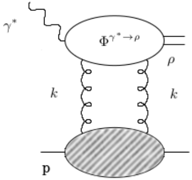
Electroproduction of ρ mesons and UGDs

Process: $\gamma^* + \text{proton} \to \rho + \text{proton}$ *...exclusive process!*

◇ leading **helicity amplitudes** are known (Wandzura-Wilczek)

→ process solved in helicity

$$T_{\lambda_\rho \lambda_\gamma}(s; Q^2) = is \int \frac{d^2\mathbf{k}}{(\mathbf{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}(\mathbf{k}^2, Q^2)\, \mathcal{F}(x, \mathbf{k}^2)\,, \quad x = \frac{Q^2}{s}$$



Interesting transitions:

- $\gamma_L^* \to \rho_L \xrightarrow{\text{encoded by}} \Phi^{\gamma_L^* \to \rho_L}$
- $\gamma_T^* \to \rho_T \xrightarrow{\text{encoded by}} \Phi^{\gamma_T^* \to \rho_T}$

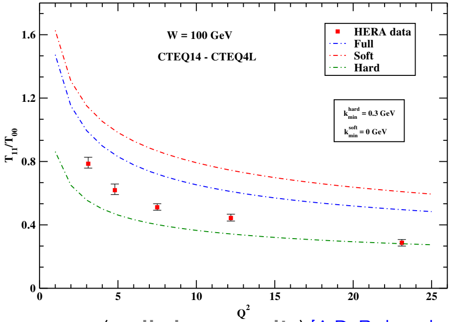
◇ HERA data available for $\mathbf{T}_{11}/\mathbf{T}_{00}$ [H1 Collaboration (2010)]

▶ ideal testing ground to probe and constrain the proton UGD!

BACKUP slides

Electroproduction of ρ mesons - T_{11}/T_{00} (preliminary)

- ◇ Different models of UGDs need to be tested...
- ◇ ...and then compared with the standard definition (*à la* BFKL)
- ● example: unpolarized model [I.P. Ivanov and N.N. Nikolaev (2002)]



(**preliminary results**) [A.D. Bolognino, MD thesis (2017)]

$\Longrightarrow$ Further, dedicated investigation is underway (...a new *Ansatz*?)
[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (in progress)]