



Study of Mueller-Navelet jets at LHC using BFKL + NLO

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LHC FORWARD PHYSICS WG MEETING

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Motivations and Outline

- Motivations

- One of the important longstanding theoretical questions: the behaviour of **QCD** in the **high-energy** (Regge) limit $s \gg -t$
- We expect a **new kind of dynamics** (BFKL dynamics) beyond fixed order perturb. theory; amplitudes $\sim s^\omega$
- For (semi-)hard processes $s \gg -t \gg \Lambda_{\text{QCD}}^2$
P.Th still applicable with all-order resummation of $(\alpha_s \log s)^n$

- Outline

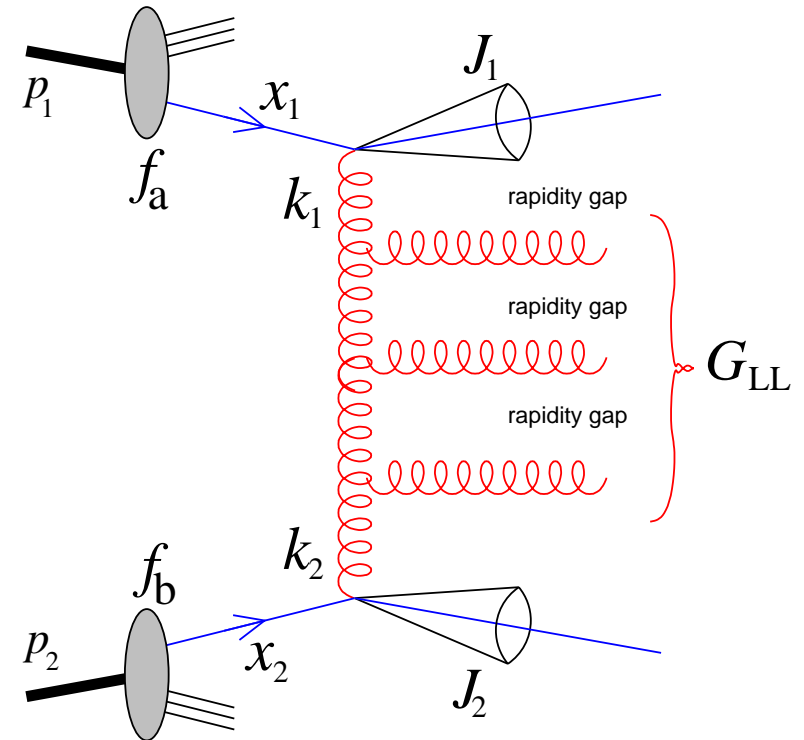
- Process suited for study of high energy QCD: **Mueller-Navelet dijets**
- Review the theoretical description of MN jets within the BFKL approach
- CMS analysis (2012) \rightarrow comparison with BFKL and with MonteCarlo
- Unsatisfactory descriptions \rightsquigarrow ask for improvements
 - **matching BFKL** with fixed **NLO**: method and preliminary results
 - We suggest **exp. data selection** in order to **minimize theoretical uncert.**
 - **jet identification consistent with exp. analysis**

MN Jets in LL approximation

MN jet factorization formula is a convolution of 5 objects

Starting from LL factorization formula [$J \equiv (y, p_T, \phi)$]

$$\begin{aligned} \frac{d\sigma(s)}{dJ_1 dJ_2} &= \sum_{a,b} \int_0^1 dx_1 dx_2 \int d\mathbf{k}_1 d\mathbf{k}_2 \\ &\times f_a(x_1) \\ &\times V_a^{(0)}(x_1, \mathbf{k}_1; J_1) \\ &\times G_{LL}(x_1 x_2 s, \mathbf{k}_1, \mathbf{k}_2) \\ &\times V_b^{(0)}(x_2, \mathbf{k}_2; J_2) \\ &\times f_b(x_2) \end{aligned}$$



where $\frac{\partial}{\partial \log s} G(s, \mathbf{k}_1, \mathbf{k}_2) = \int d\mathbf{k} K(\mathbf{k}_1, \mathbf{k}) G(s, \mathbf{k}, \mathbf{k}_2)$, $K = \alpha_s K_0$

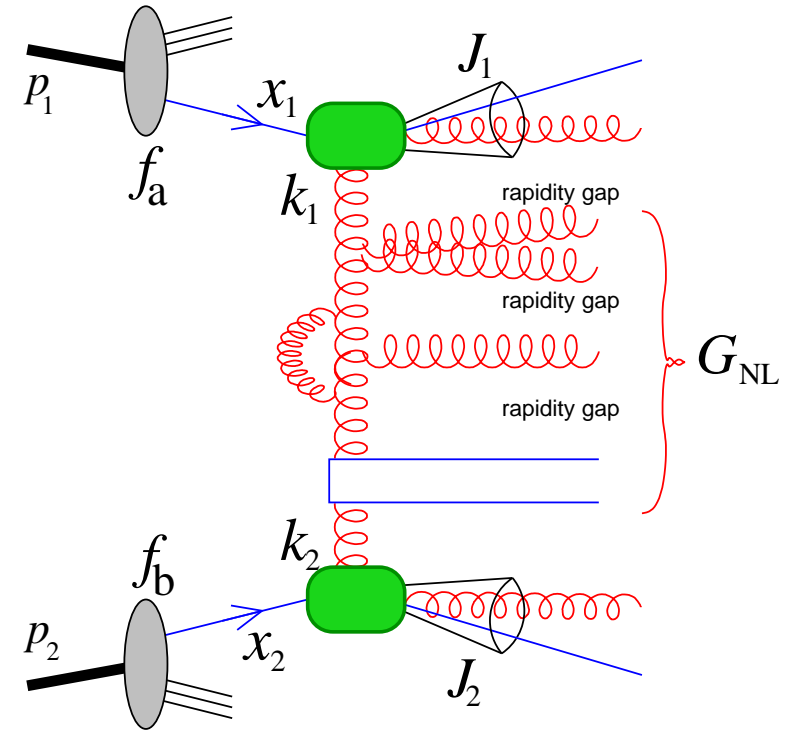
- Kinematics characterized by large rapidity gaps among particles
- At LL level the jet vertex condition is trivial (only 1 parton)

MN Jets in NLL approximation

[Bartels, DC, Vacca '02] computed NLL calculations of impact factors for Mueller-Navelet jets

Proved NLL factorization formula [$J \equiv (y, p_T, \phi)$]

$$\begin{aligned} \frac{d\sigma(s)}{dJ_1 dJ_2} &= \sum_{a,b} \int_0^1 dx_1 dx_2 \int d\mathbf{k}_1 d\mathbf{k}_2 \\ &\times f_a(x_1) \\ &\times \mathbf{V}_a^{(1)}(x_1, \mathbf{k}_1; J_1) \\ &\times G_{\text{NL}}(x_1 x_2 s, \mathbf{k}_1, \mathbf{k}_2) \\ &\times \mathbf{V}_b^{(1)}(x_2, \mathbf{k}_2; J_2) \\ &\times f_b(x_2) \end{aligned}$$



where $\frac{\partial}{\partial \log s} G(s, \mathbf{k}_1, \mathbf{k}_2) = \int d\mathbf{k} K(\mathbf{k}_1, \mathbf{k}) G(s, \mathbf{k}, \mathbf{k}_2)$, $K = \alpha_s K_0 + \alpha_s^2 K_1$

- Pairs of particles can be emitted without rapidity gaps
- At NL level the jet vertex condition is non-trivial (e.g. depends on jet radius R and algorithm)

With LHC we can test these ideas!

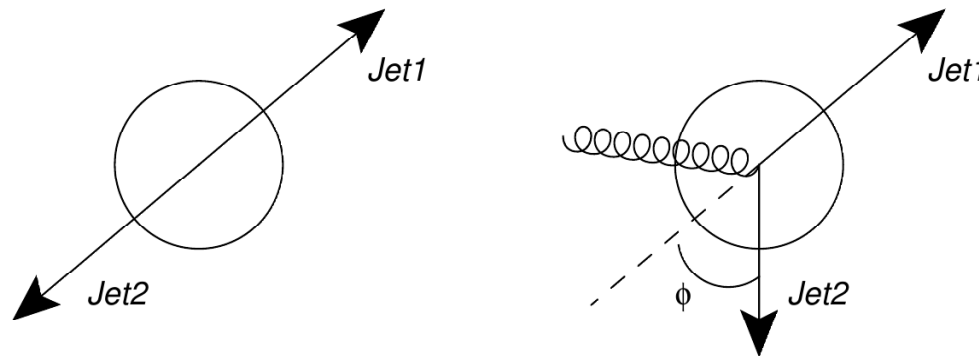
- First NLL analysis for 14 TeV [*DC, Schwensenn, Szymanowski, Wallon '10*] showed sizeable corrections from both GGF and Jet vertices
- NLL prediction definitely different from MC ones
- Mueller-Navelet jets looked promising for finding signals of BFKL dynamics

CMS analysis of MN jets at 7 TeV

Analysis of the **azimuthal decorrelation** of the two jets [*CMS: FSQ-12-002-pas*]

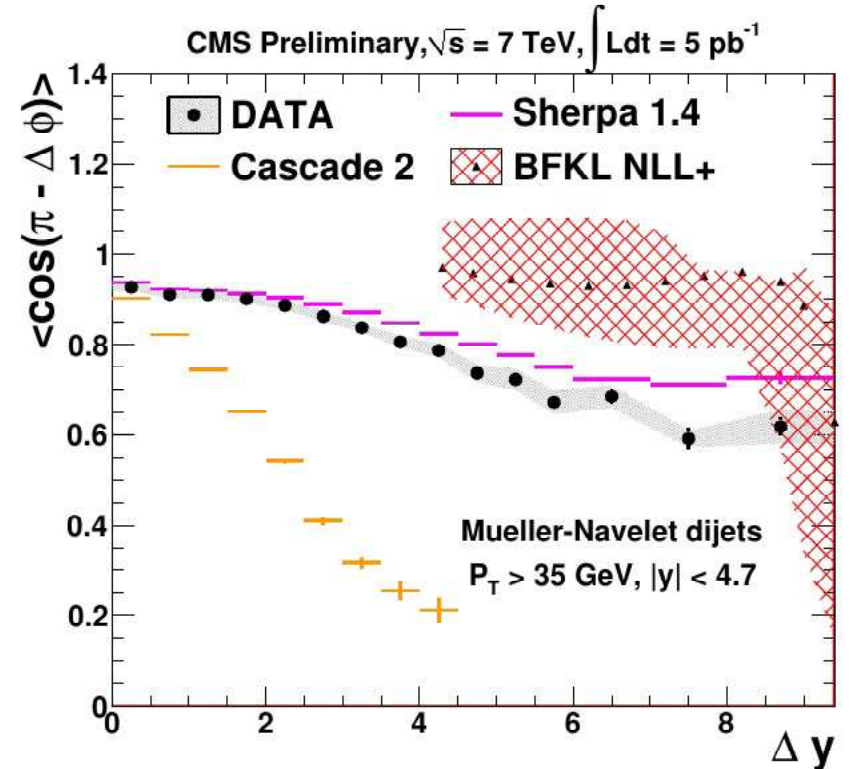
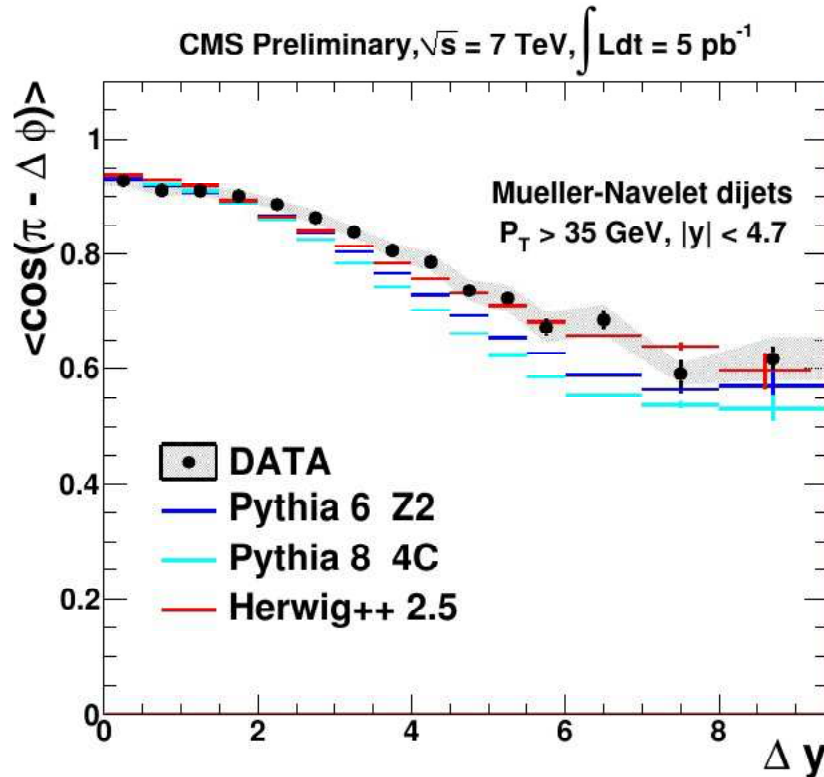
$$\frac{1}{\sigma} \frac{d\sigma}{d\phi} \quad \parallel \quad \langle \cos(m\phi) \rangle = \frac{C_m(Y)}{C_0(Y)} \equiv \frac{\int d\phi \frac{d^2\sigma}{d\phi dY} \cos(m\phi)}{d\sigma/dY}$$

- Distinguishes BFKL dynamics from fixed order one: they provide **different** amount of particle **emissions** between jets, which is responsible for their **decorrelation**
- $\langle \cos(m\phi) \rangle$ has **reduced** theoretical scale **uncertainties** being a ratio of differential cross sections



CMS analysis of MN jets at 7 TeV

Data: $p_{T1,2} > 35\text{GeV}$, $|y_i| < 4.7$ $\Delta y \equiv Y \equiv |y_1 - y_2| < 9.4$ $m = 1$



The larger Y , the more radiation and decorrelation

BFKL was expected to predict more radiation than fixed order \Rightarrow more decorrelation

Some MC agree with data

NLL BFKL estimate has problems

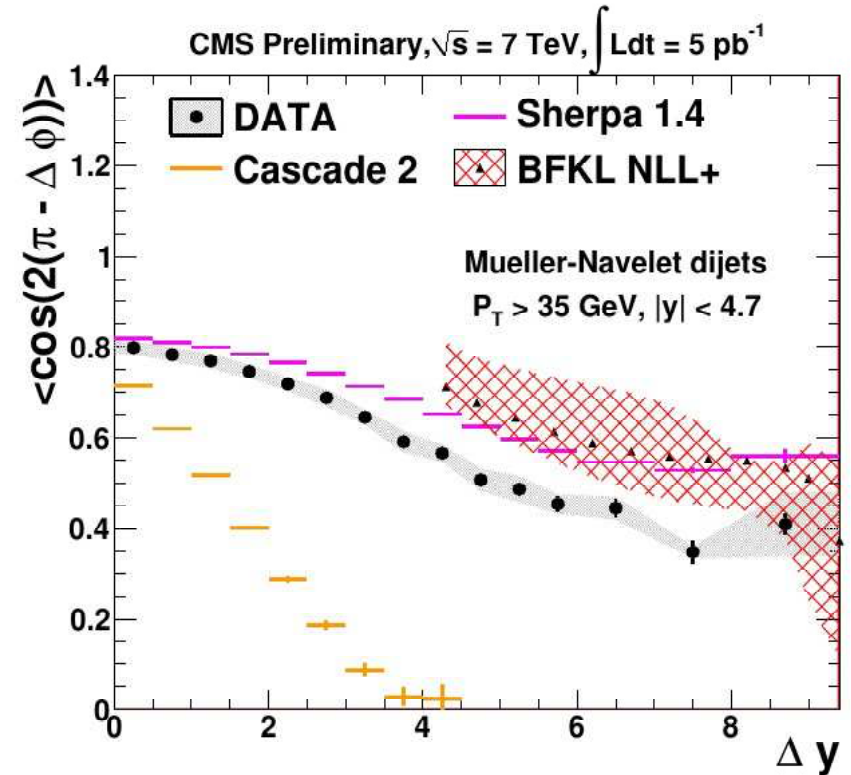
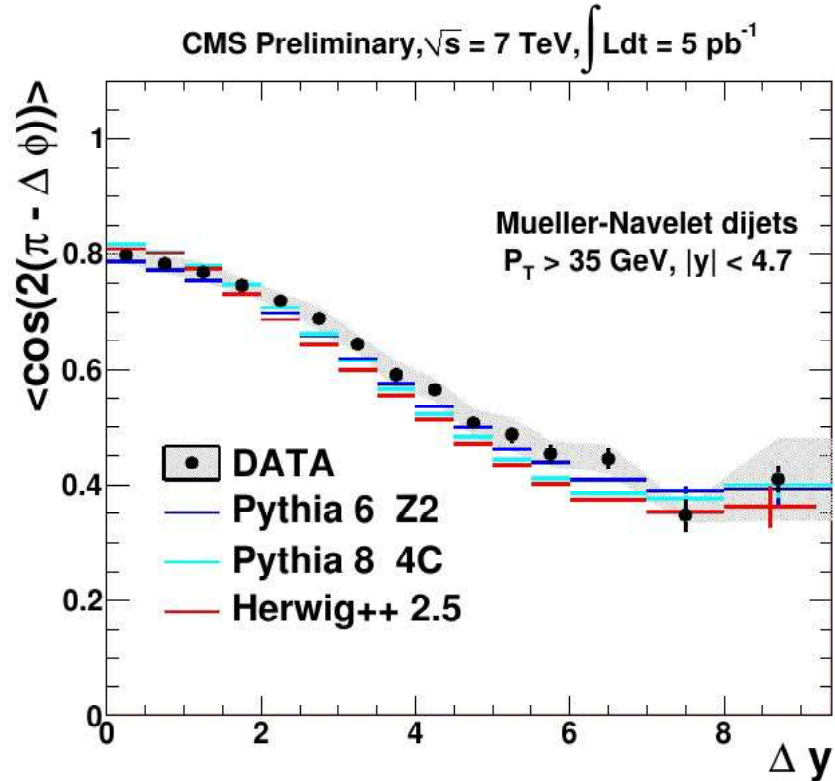
$$\langle \cos \phi \rangle > 1 \text{ for } \mu_R = \mu_F = p_T/2$$

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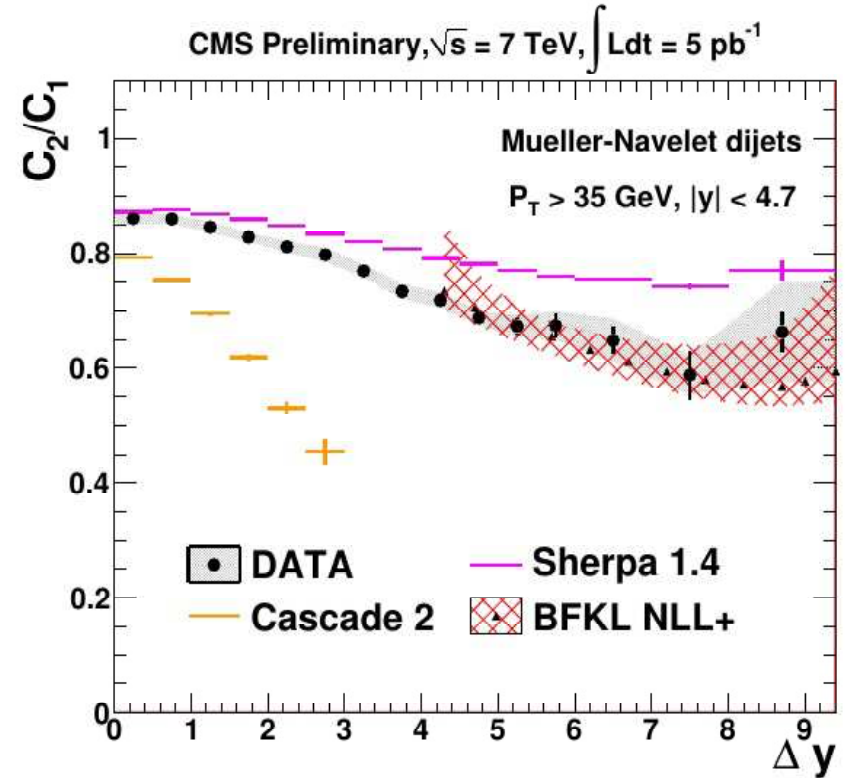
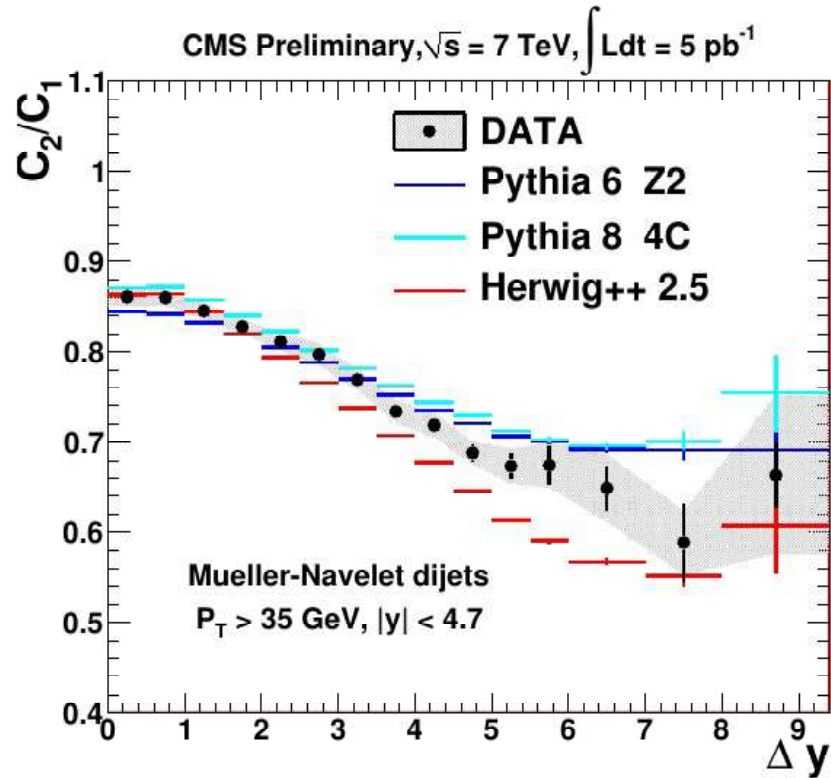
NLL BFKL still unable to reproduce data

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$m = 1, 2$



$$\text{Ratio } \frac{C_2}{C_1} = \frac{\langle \cos(2\phi) \rangle}{\langle \cos \phi \rangle}$$

MCs don't agree well with data

NLL BFKL in perfect agreement with data

- Neither BFKL NLL nor fixed order MC give a satisfactory description of data yet
- BFKL NLL suffers from large scale uncertainties $\sim 10 \div 15\%$

BFKL improvements

- *[Ducloué, Szymanowski, Wallon '13]*
proposed to tame large scale dependence of BFKL
by fixing μ_R with BLM procedure
- *[Ducloué, Szymanowski, Wallon '14]*
try to take into account energy-momentum conservation
by using an effective rapidity Y_{eff} , as suggested by *[Del Duca, Schmidt]*
- *[Caporale, Ivanov, Murdaca, Papa '14]*
consider various representations of the NLL cross section
by fixing energy scales with PMS, FAC, BLM

Underlying idea: to effectively include higher-orders

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Underlying idea: to effectively include higher-orders

- **Why not to include known NLO (+NNLO) calculations?**

Matching BFKL with Fixed NLO

Our aim is to merge fixed NL order and NLL BFKL resummation

- more reliable results \Rightarrow improve description of data
- correctly reproduce not only ratios but absolute values
- \rightsquigarrow can compare different CME \sqrt{s}

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Standard matching procedure:

- add to BFKL the full perturbative NLO result $\mathcal{O}(\alpha_s^3)$
- subtract the $\mathcal{O}(\alpha_s^3)$ part already included in BFKL

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Results for cross section and C_m coefficients

- Results of central values (no scale variation yet)
 \rightsquigarrow important lesson for future analyses

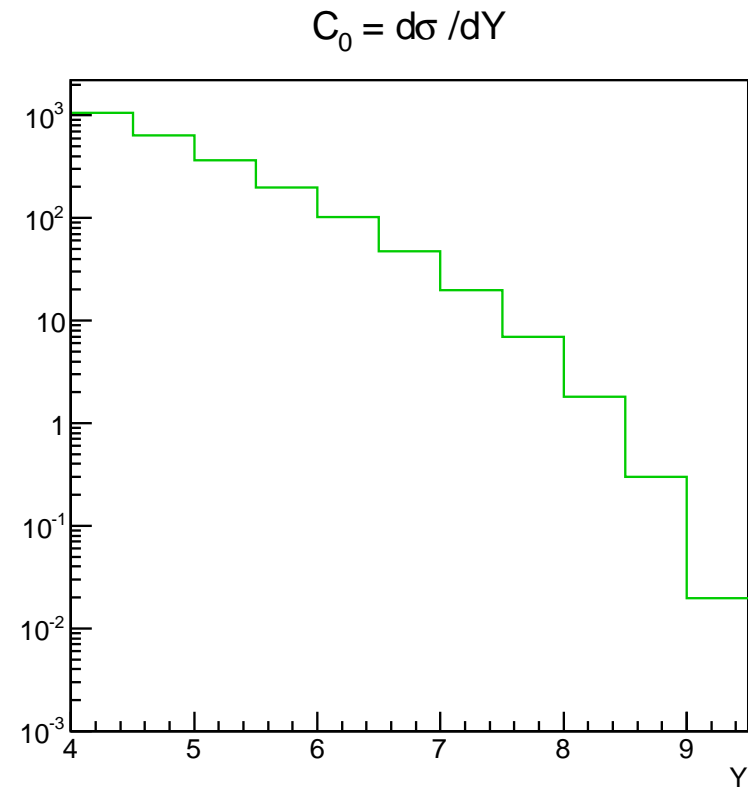
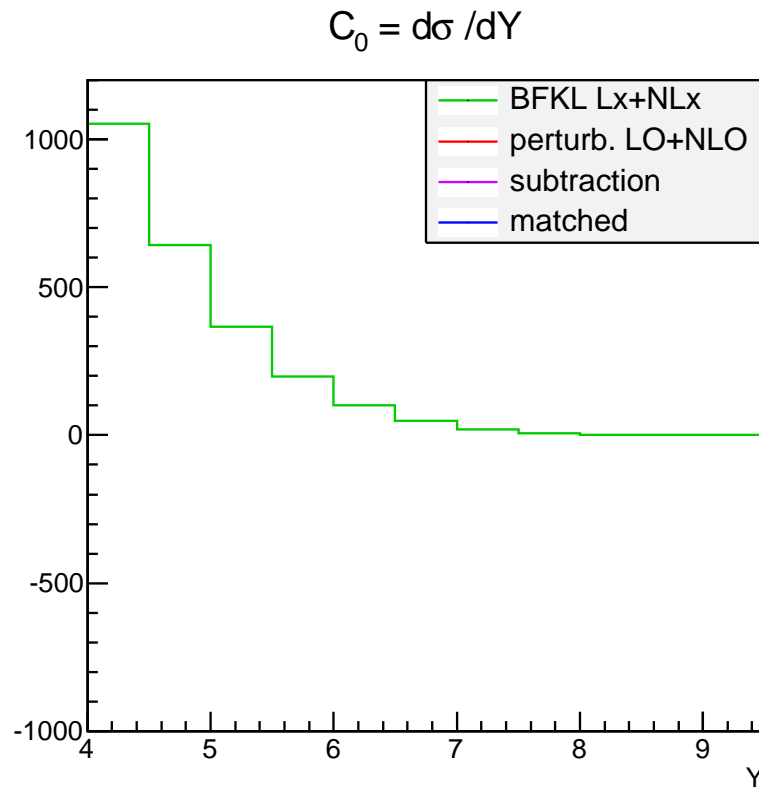
Matching (sym. jets $p_{T1}, p_{T2} > 35\text{GeV}$)

Cross section: **NLL BFKL** + **NLO pert. $\mathcal{O}(\alpha_s)^3$** - **BFKL $\mathcal{O}(\alpha_s^3)$**

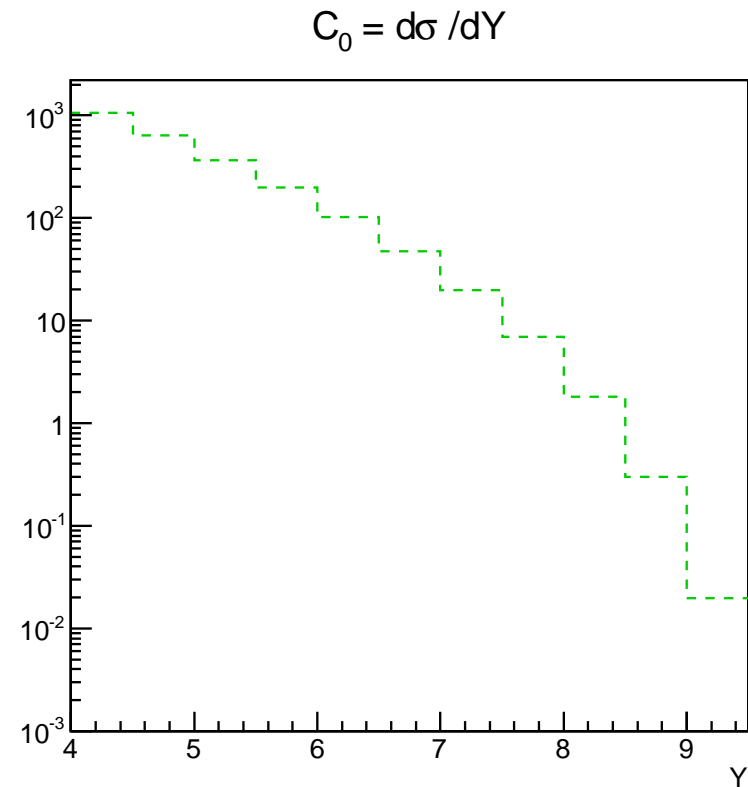
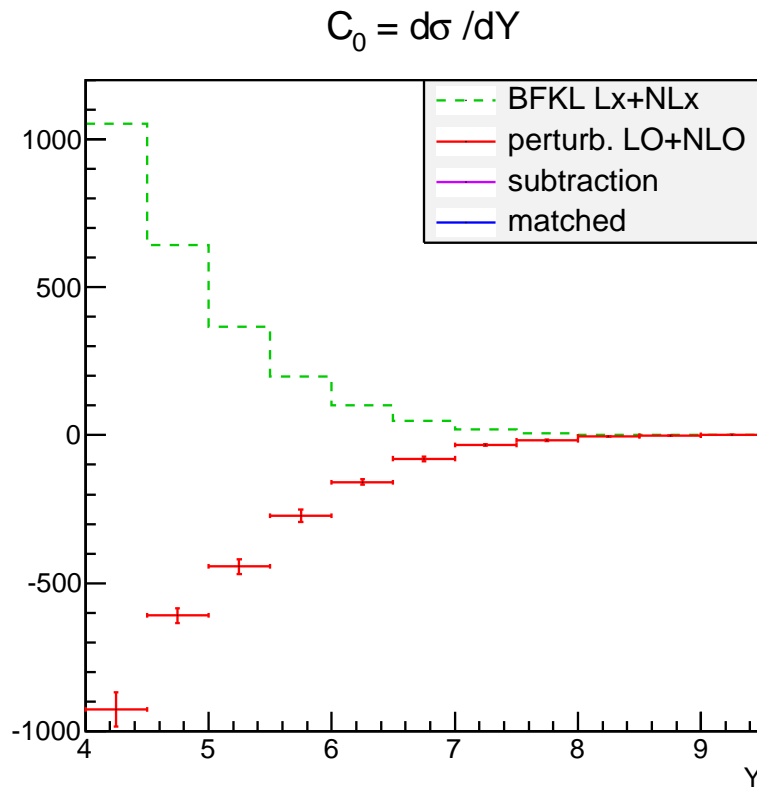
$$\begin{aligned} \frac{d\sigma(s)}{dJ_1 dJ_2} = & \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1) f_b(x_2) \left\{ \right. \\ & \int d\mathbf{k}_1 d\mathbf{k}_2 \left[V_a^{(0+1)}(x_1, \mathbf{k}_1; J_1) G_{\text{NLL}}(x_1 x_2 s, \mathbf{k}_1, \mathbf{k}_2) V_b^{(0+1)}(x_2, \mathbf{k}_2; J_2) \right] \\ & + \frac{d\hat{\sigma}^{(NLO)}(x_1, x_2)}{dJ_1 dJ_2} \\ & - \int d\mathbf{k}_1 d\mathbf{k}_2 \left[V_a^{(0)}(x_1, \mathbf{k}_1; J_1) \delta^2(\mathbf{k}_1 - \mathbf{k}_2) V_b^{(0)}(x_2, \mathbf{k}_2; J_2) \right] \\ & - \int d\mathbf{k}_1 d\mathbf{k}_2 \left[V_a^{(1)}(x_1, \mathbf{k}_1; J_1) \delta^2(\mathbf{k}_1 - \mathbf{k}_2) V_b^{(0)}(x_2, \mathbf{k}_2; J_2) \right] \\ & - \int d\mathbf{k}_1 d\mathbf{k}_2 \left[V_a^{(0)}(x_1, \mathbf{k}_1; J_1) \delta^2(\mathbf{k}_1 - \mathbf{k}_2) V_b^{(1)}(x_2, \mathbf{k}_2; J_2) \right] \\ & \left. - \int d\mathbf{k}_1 d\mathbf{k}_2 \left[V_a^{(0)}(x_1, \mathbf{k}_1; J_1) \alpha_s \log \frac{\hat{s}}{s_0} K_0(\mathbf{k}_1, \mathbf{k}_2) V_b^{(0)}(x_2, \mathbf{k}_2; J_2) \right] \right\} \end{aligned}$$

(same colours in plots)

Matching (sym. jets $p_{T1}, p_{T2} > 35\text{GeV}$)



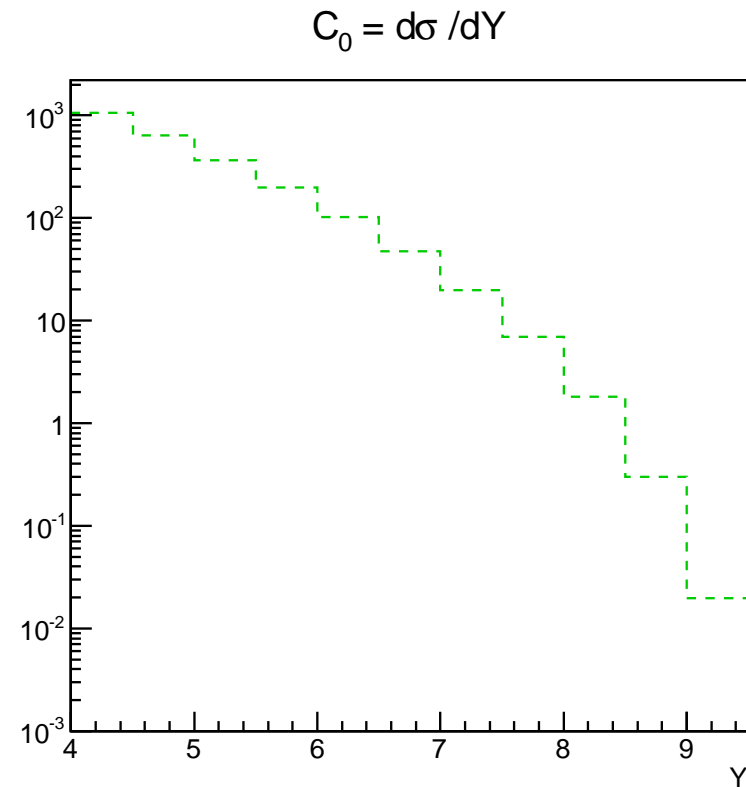
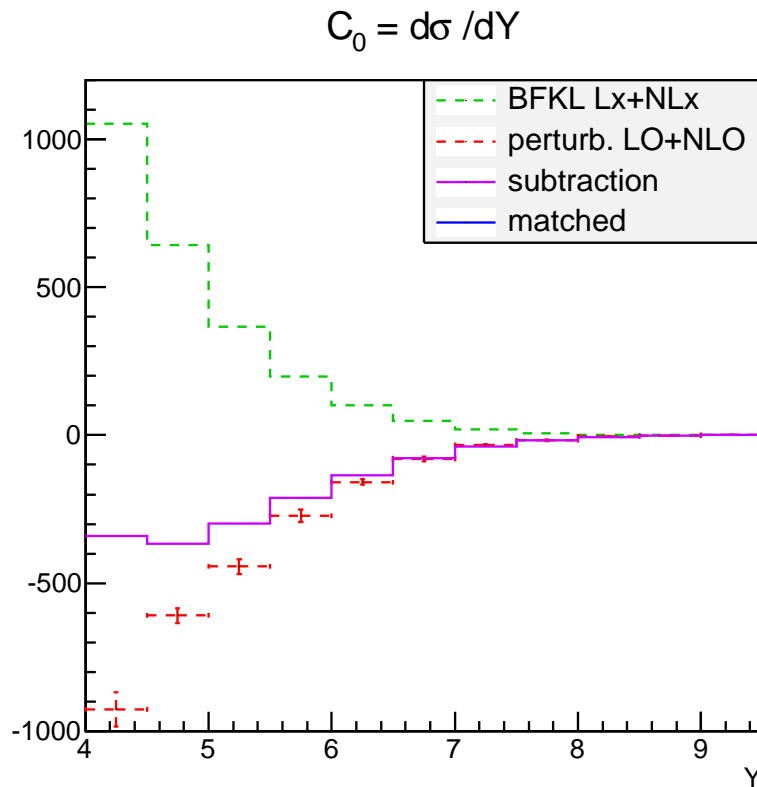
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LO+NLO cross section obtained with NLOJET++ [*Nagy*] is negative!

Large **errors** due to very slow convergence in **MC** integration

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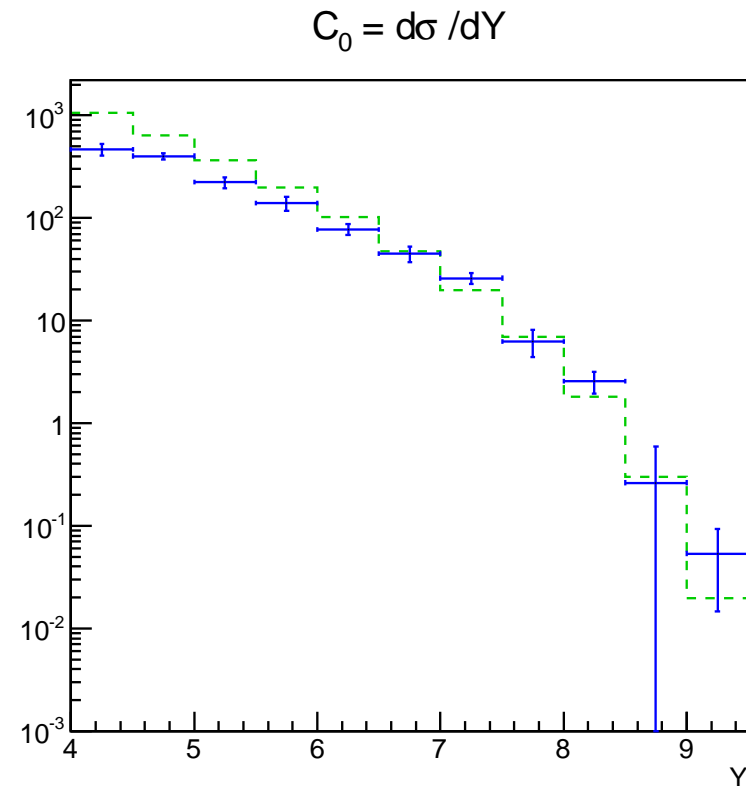
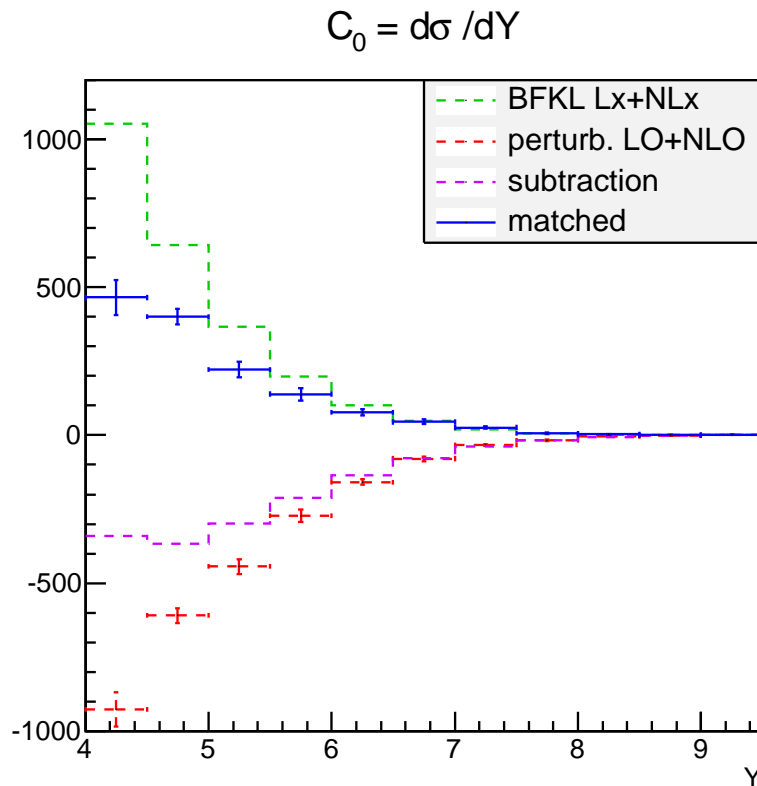
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However, also the subtraction is negative

Their difference is moderate

Matching (sym. jets $p_{T1}, p_{T2} > 35\text{GeV}$)



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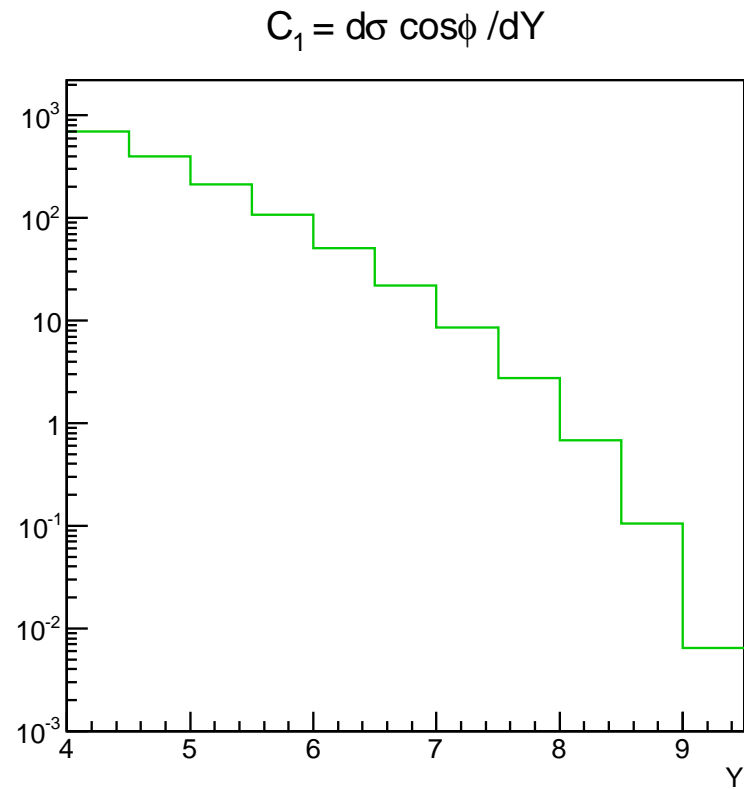
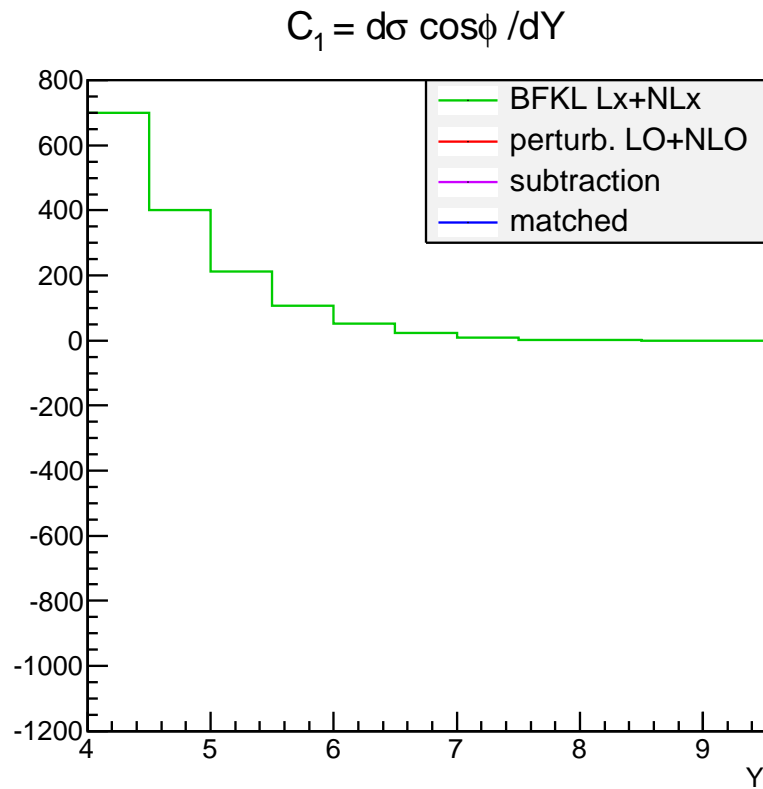
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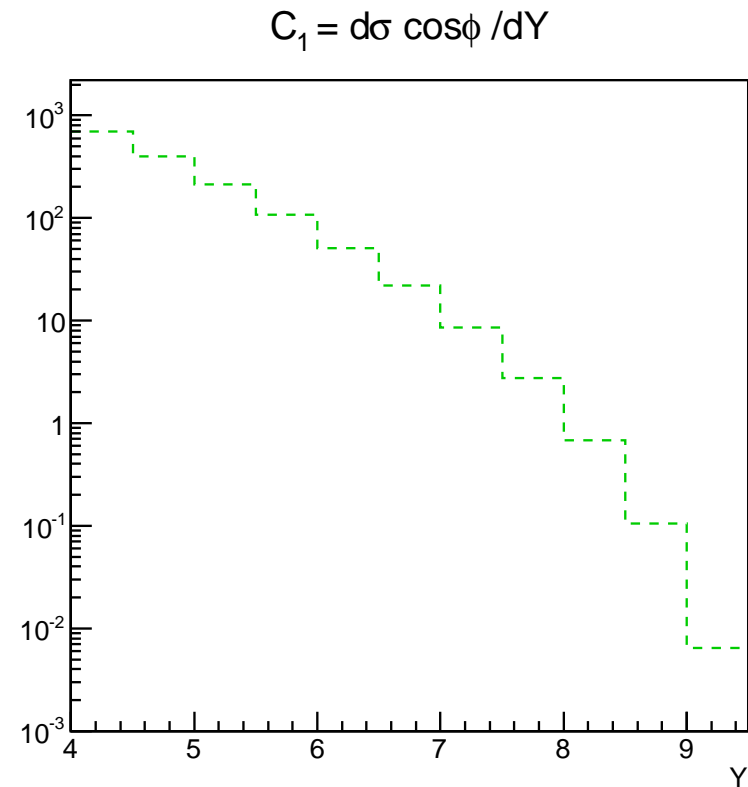
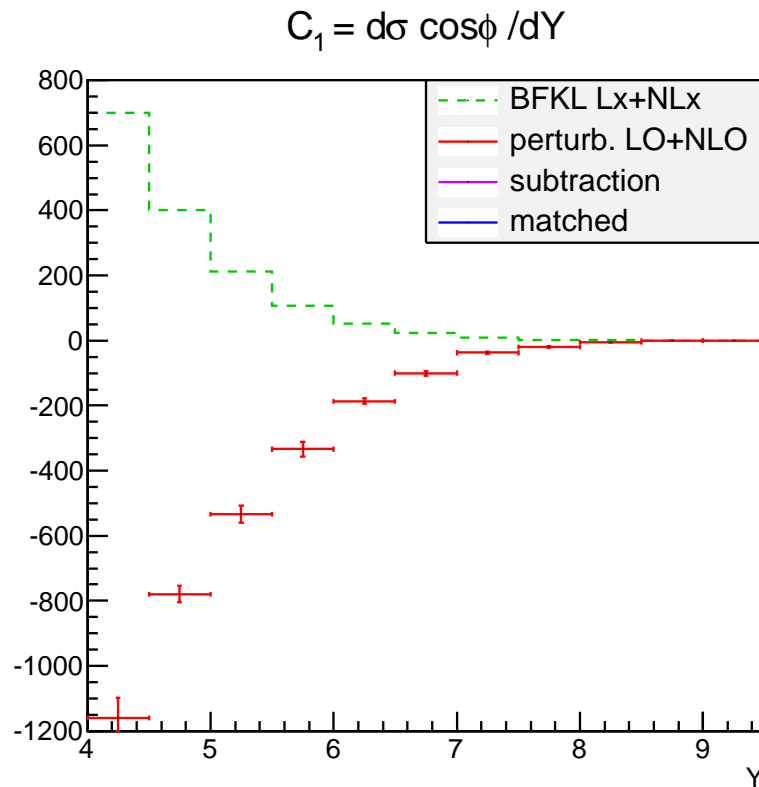
Their difference is moderate

Matched cross section is positive, of the same magnitude of NLL BFKL prediction

Matching (azimuthal coeff. C_1)

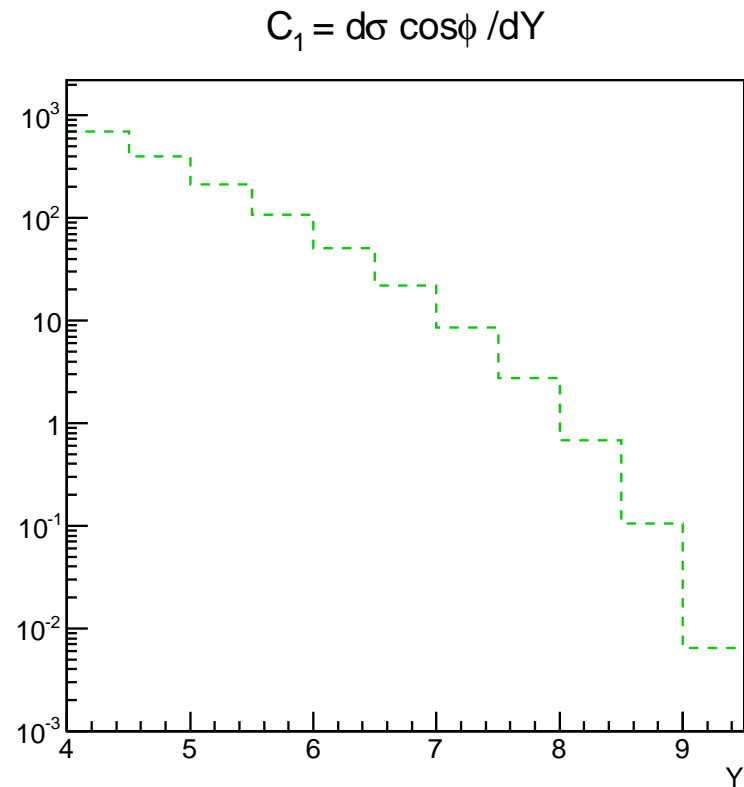
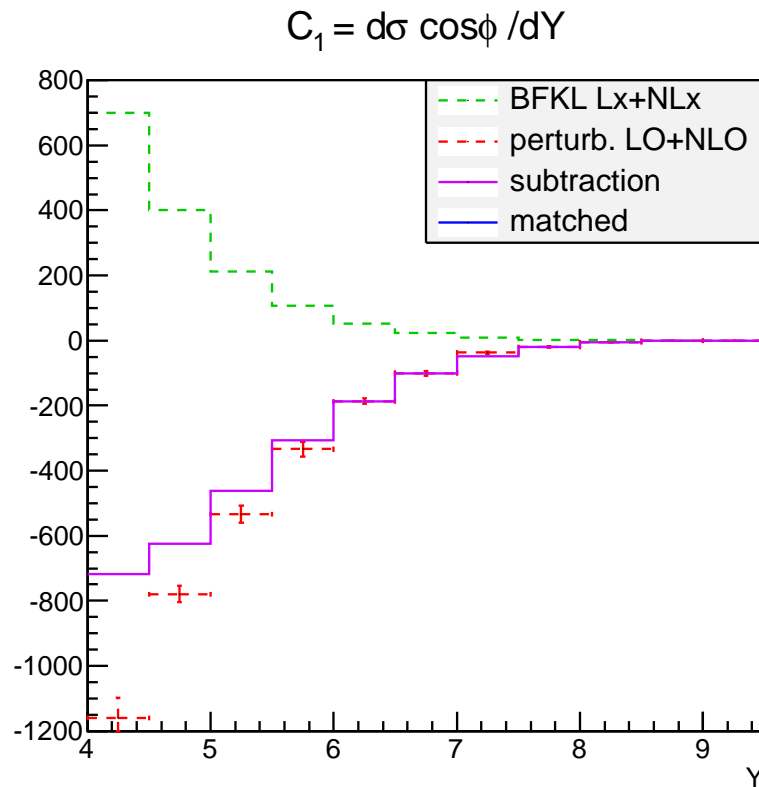


Matching (azimuthal coeff. C_1)



Large errors of NLO calculation due to very slow convergence in MC integration

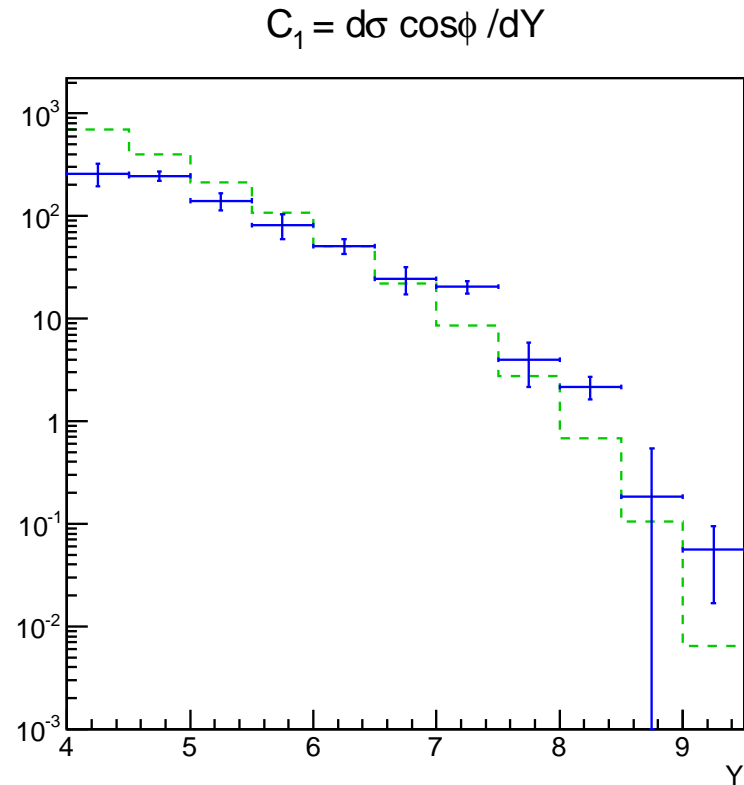
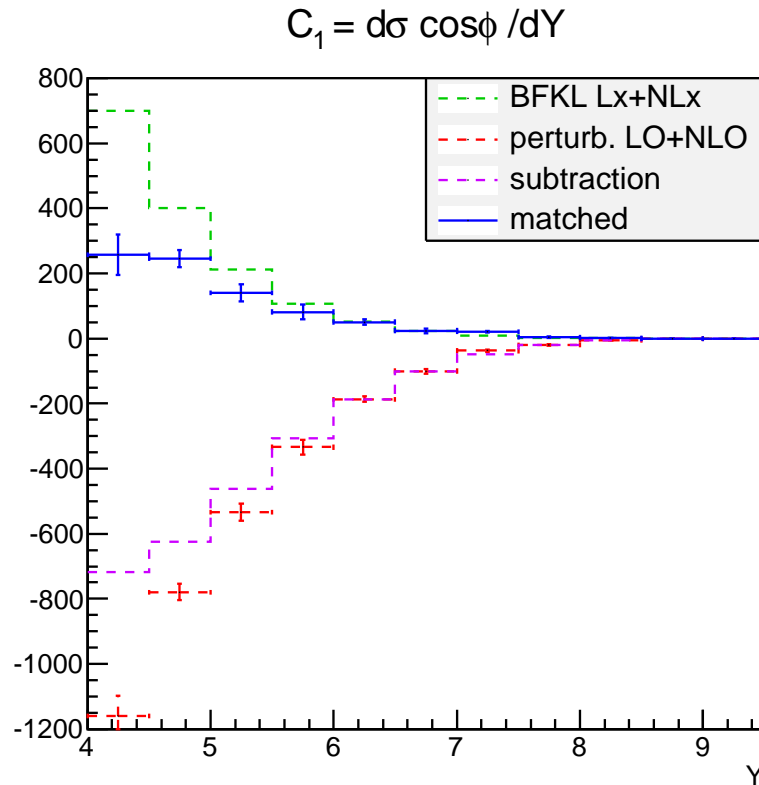
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Moderate difference between NLO and subtraction

Matching (azimuthal coeff. C_1)



Large errors of NLO calculation due to very slow convergence in MC integration

Moderate difference between NLO and subtraction

Matched C_1 of the same magnitude of NLL BFKL prediction

but definitely different at intermediate $Y \simeq 4 \div 6$

PT instability of symmetric jets

It is well known that cross section of jets at NLO is very sensitive to the asymmetry parameter $\Delta = p_{T1} - p_{T2}$ [Frixione, Ridolfi '97]

The leading collinear singularity for real emission is given by

$$\begin{aligned}\sigma^{(r)} &\propto \int d\mathbf{k}_1 d\mathbf{k}_2 \Theta(|\mathbf{k}_1| - p_T) \Theta(|\mathbf{k}_2| - (p_T + \Delta)) \frac{1}{(\mathbf{k}_1 + \mathbf{k}_2)^2 + \epsilon^2} \\ &= A(\Delta, \epsilon) + B \log(\epsilon) - C (\Delta + \epsilon) \log(\Delta + \epsilon)\end{aligned}$$

thus fixed order PTh is not reliable in this case (finite, but infinite deriv at $\Delta = 0$)

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An analogous singularity occurs in the PT expansion of LL BFKL [Andersen, Del Duca et al. '01]

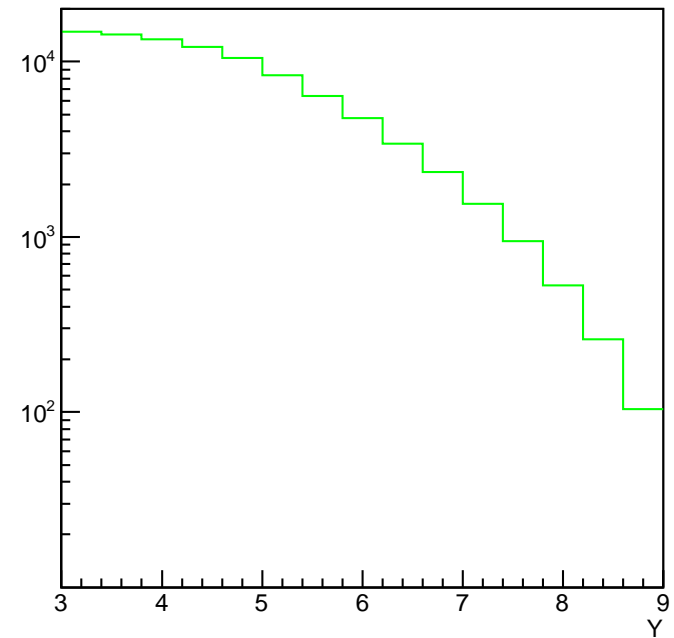
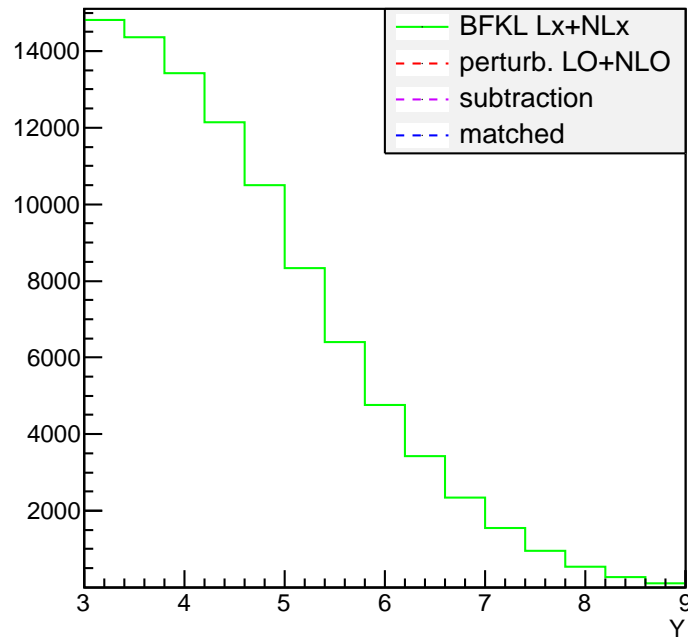
$$\sigma_{gg} \propto \frac{1}{(p_T + \Delta)^2} \left[1 - \alpha_s Y \left(\frac{2p_T \Delta + \Delta^2}{p_T^2} \log \frac{2p_T \Delta + \Delta^2}{(p_T + \Delta)^2} + 2 \log \frac{p_T}{p_T + \Delta} \right) \right]$$

In the matching procedure such collinear $\Delta \log(\Delta)$ cancels out to a large extent, therefore the matching procedure should be safe

$\langle p_T \rangle$ cut: $\frac{1}{2}(p_{T1} + p_{T2}) > 35\text{GeV}$

$C_0 = d\sigma / dY$ (nb)

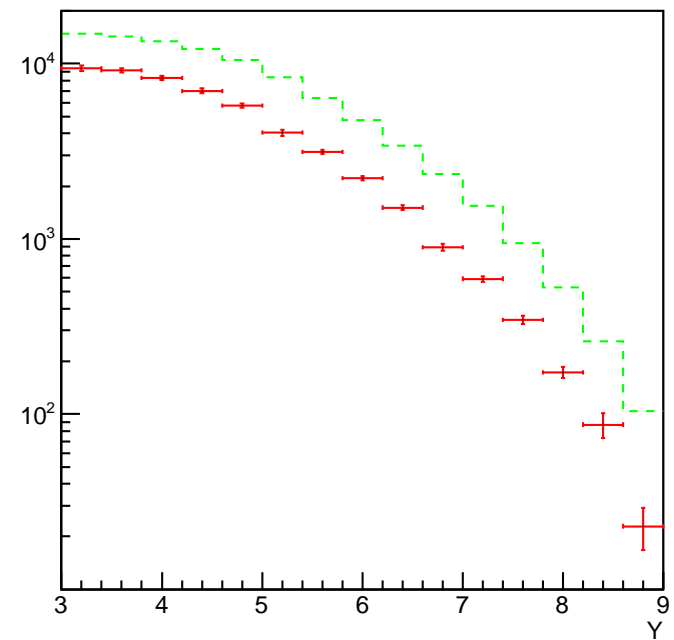
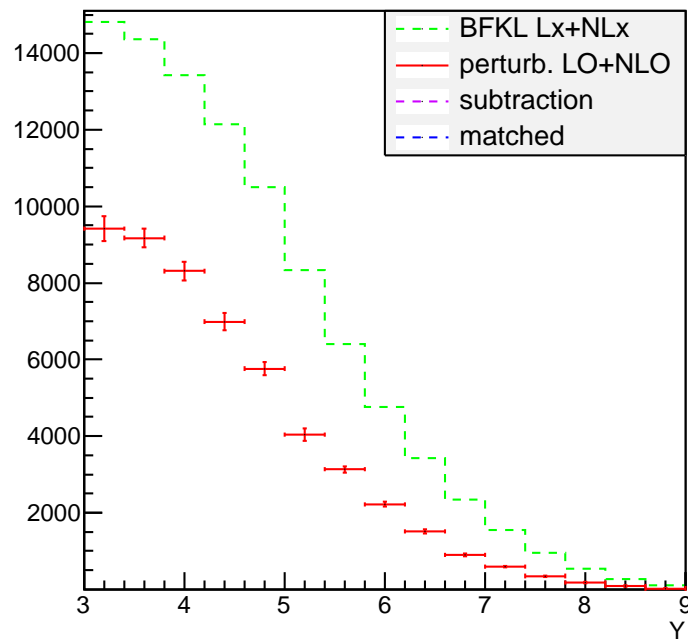
same in log scale



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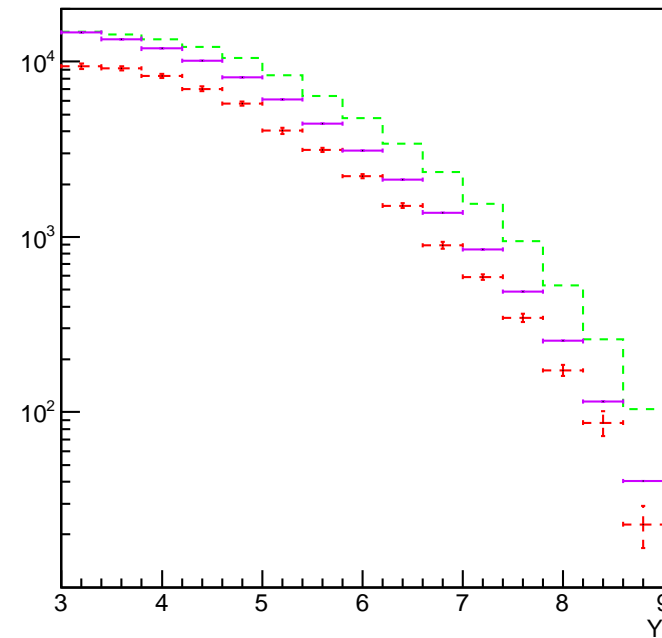
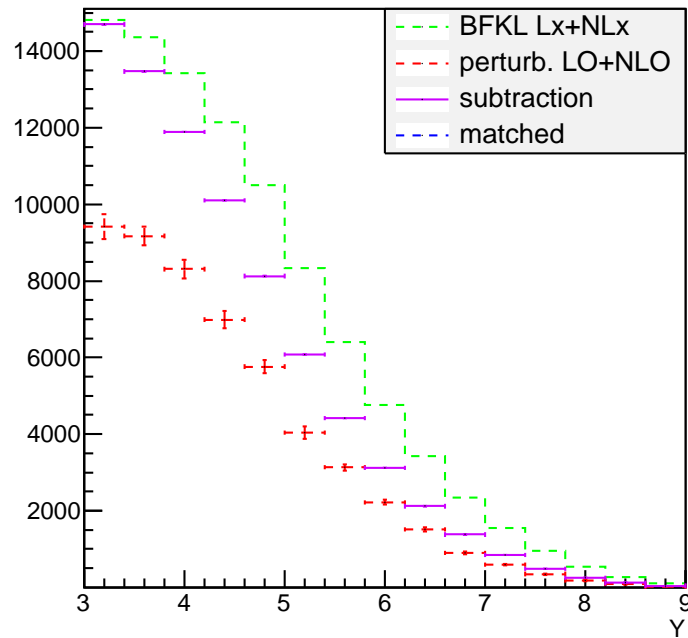
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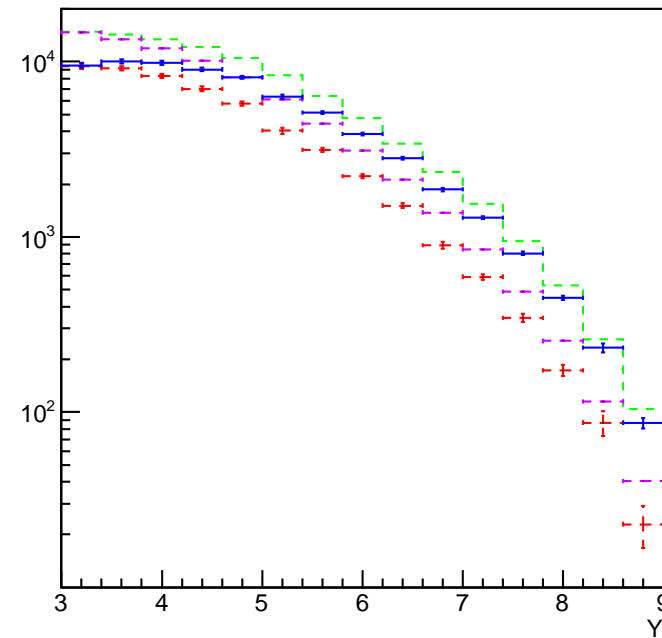
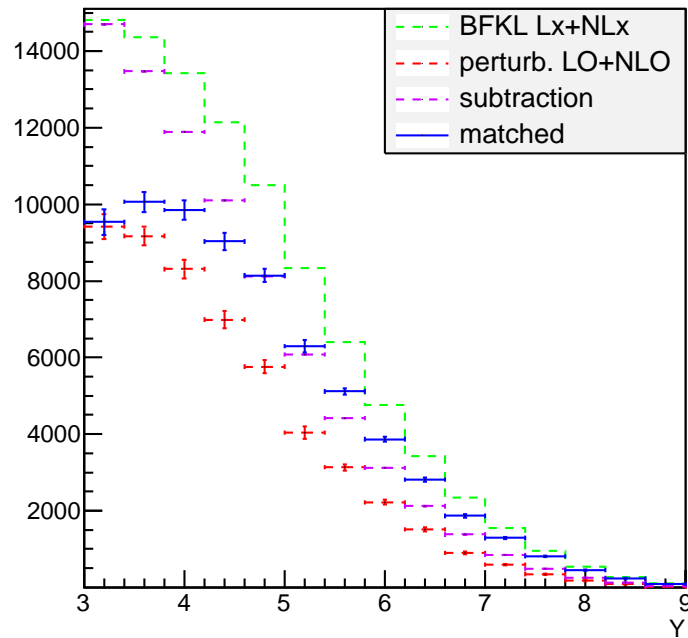
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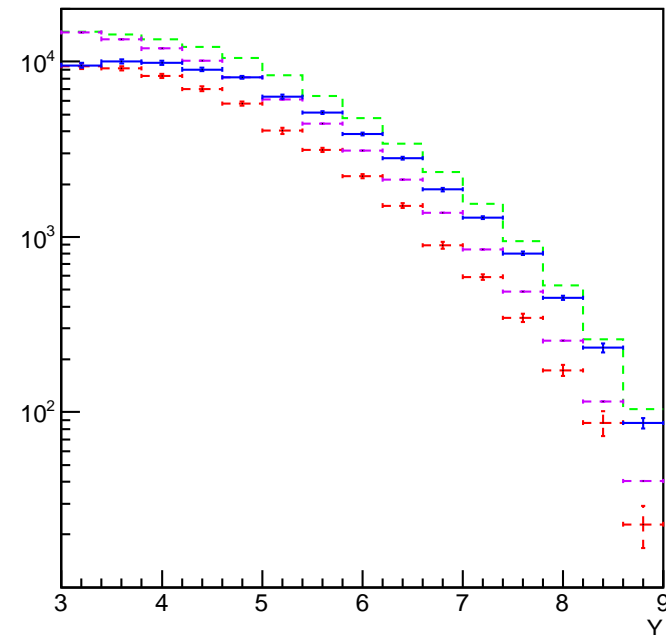
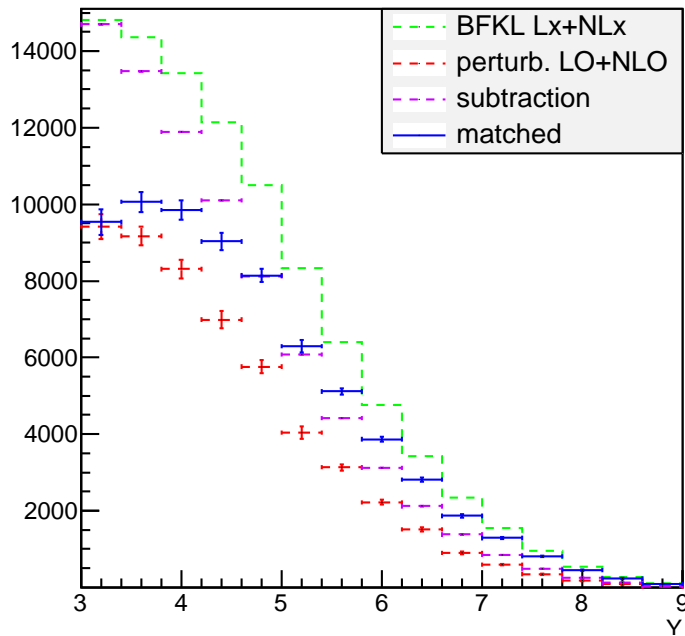


Procedure is more stable than the previous one

$\langle p_T \rangle$ cut: $\frac{1}{2}(p_{T1} + p_{T2}) > 35\text{GeV}$

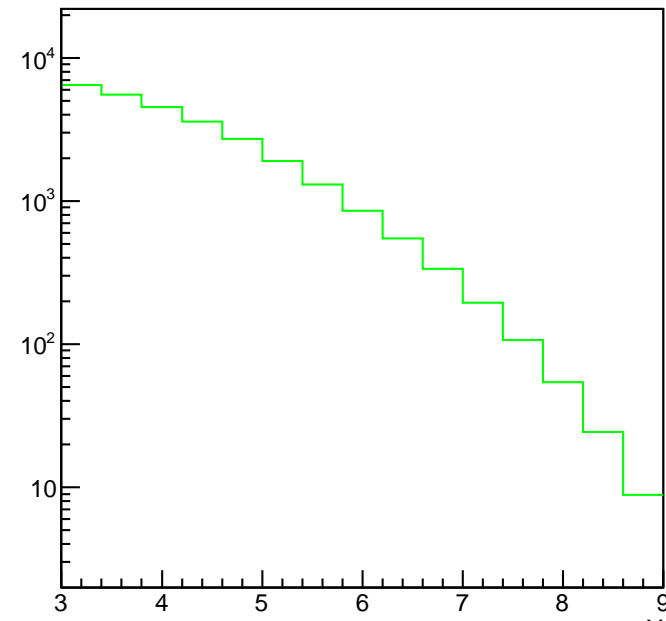
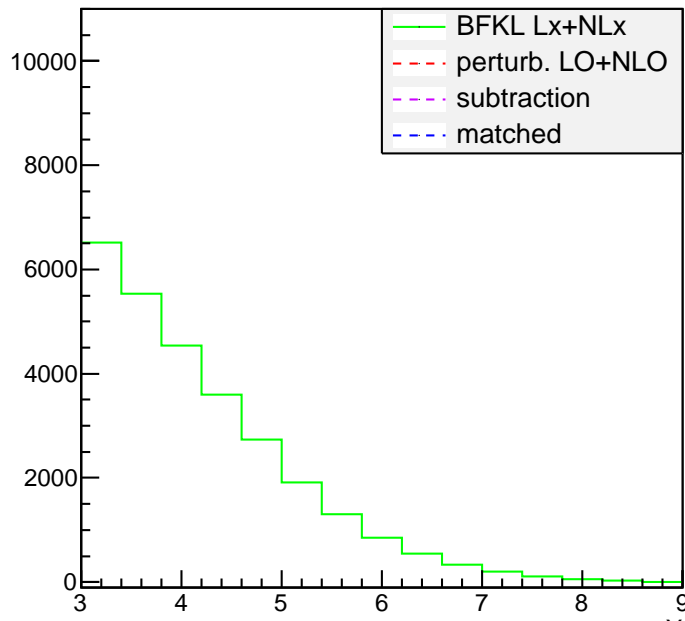
$C_0 = d\sigma / dY$ (nb)

same in log scale



$C_1 = d\sigma \cdot \cos(\Delta\phi) / dY$ (nb)

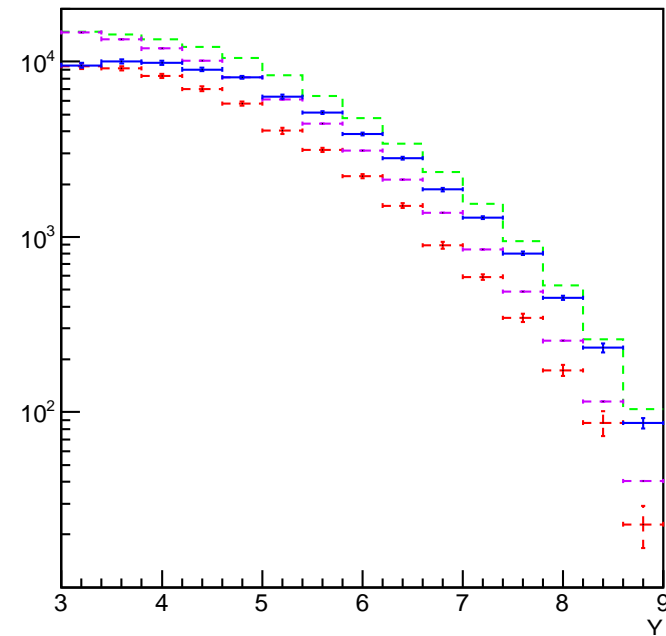
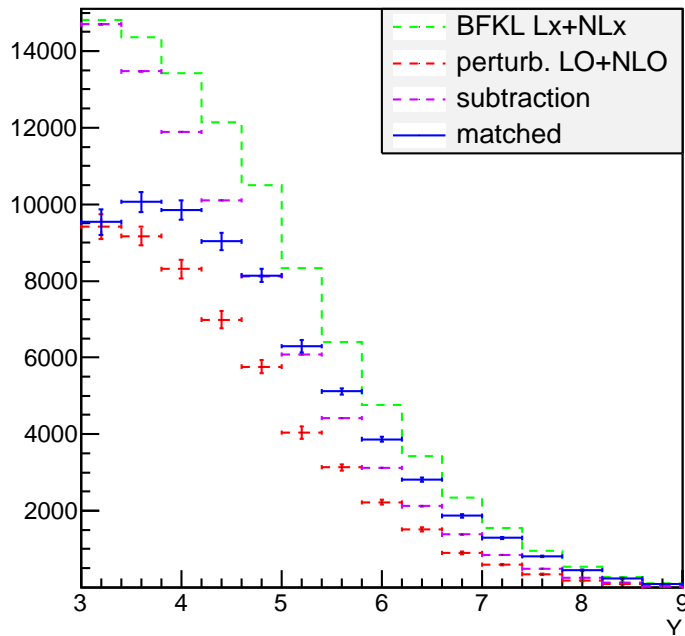
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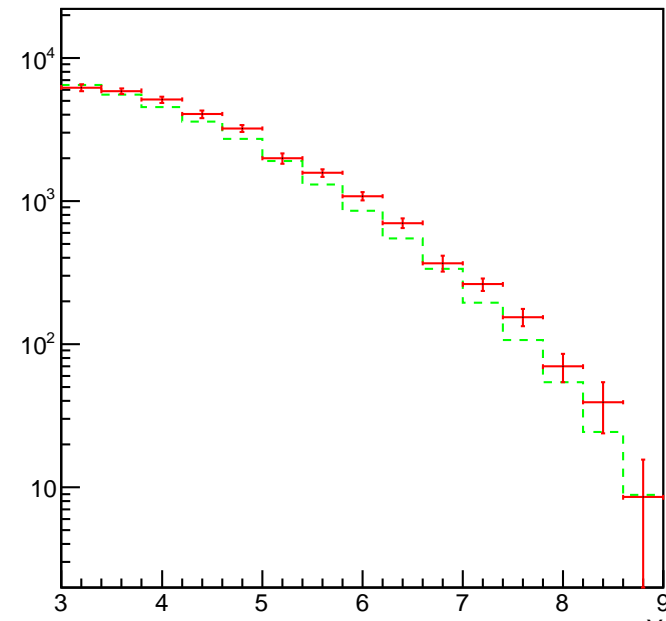
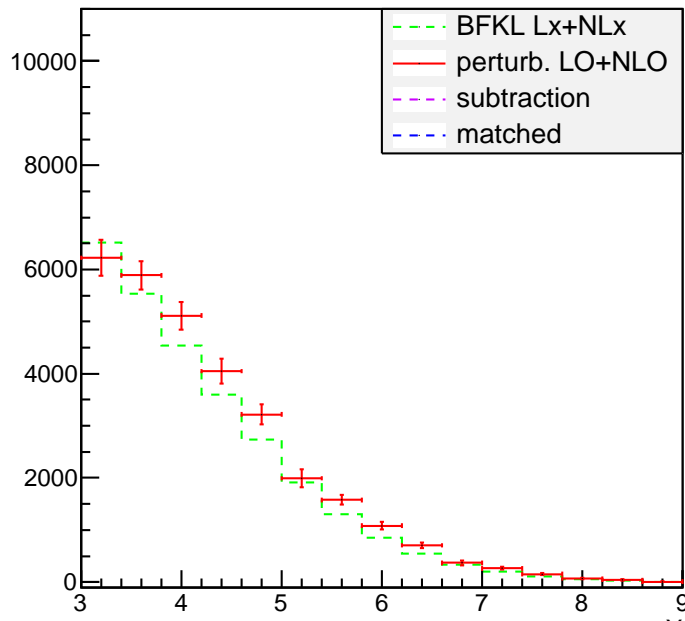
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$C_1 = d\sigma \cdot \cos(\Delta\phi) / dY$ (nb)

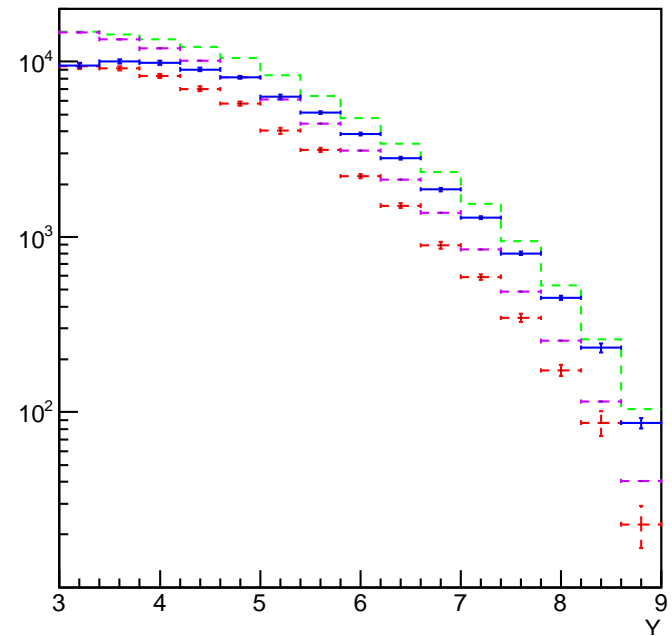
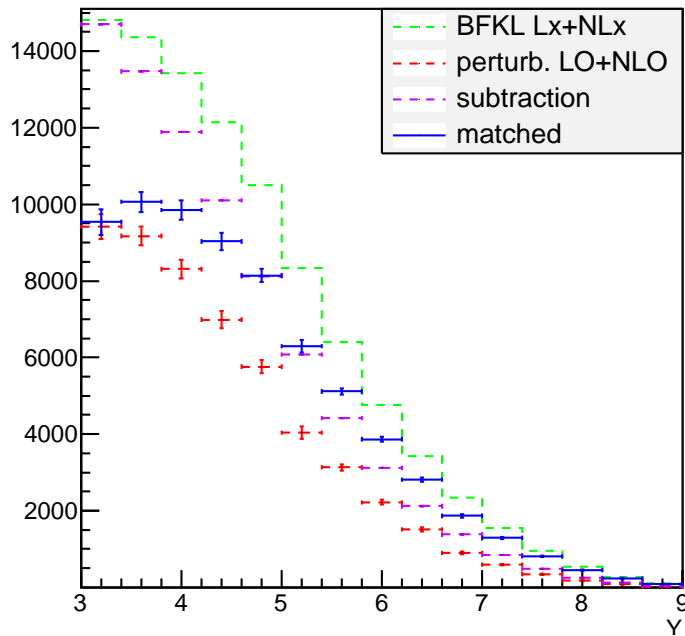
same in log scale



$\langle p_T \rangle$ cut: $\frac{1}{2}(p_{T1} + p_{T2}) > 35\text{GeV}$

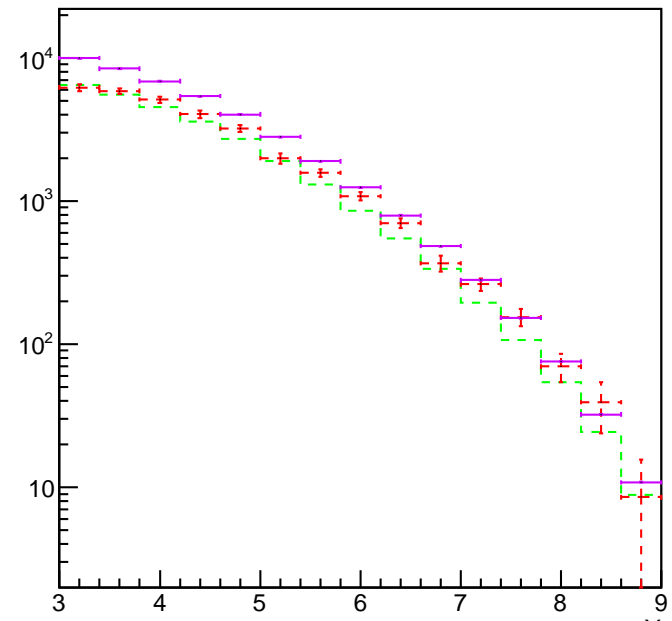
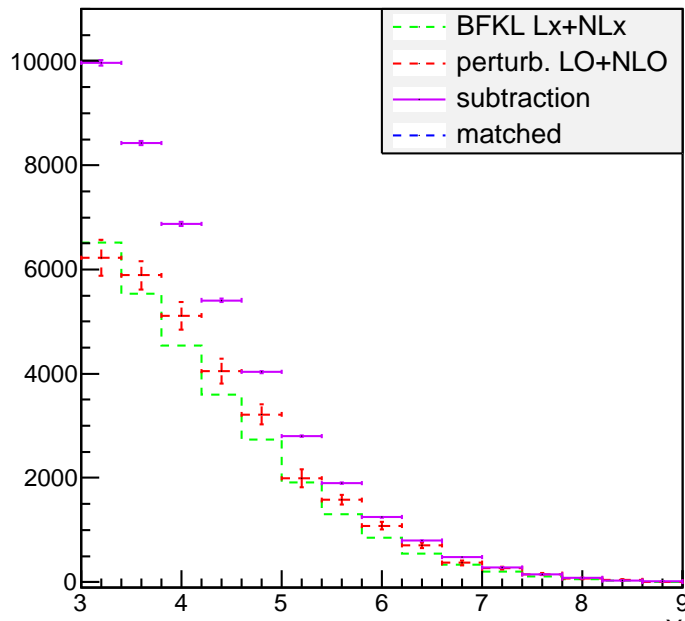
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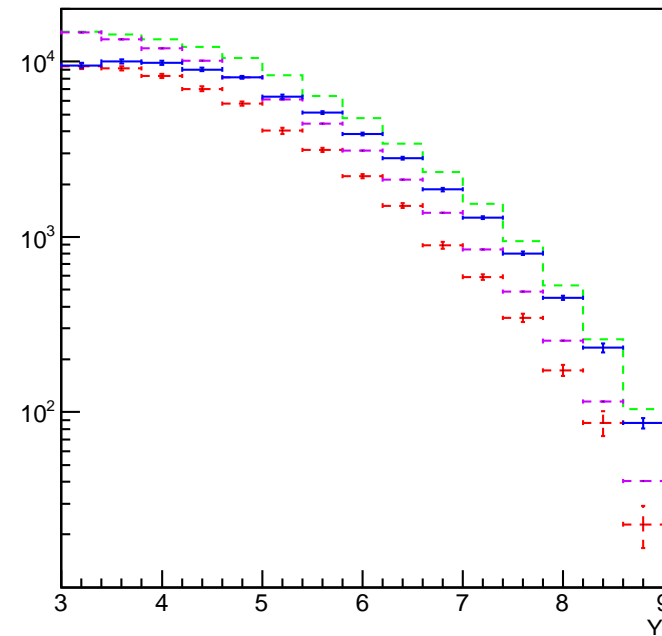
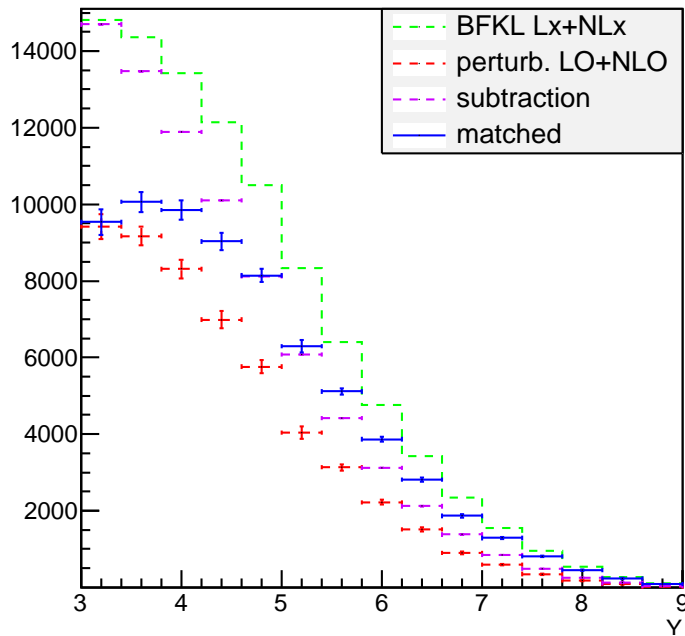
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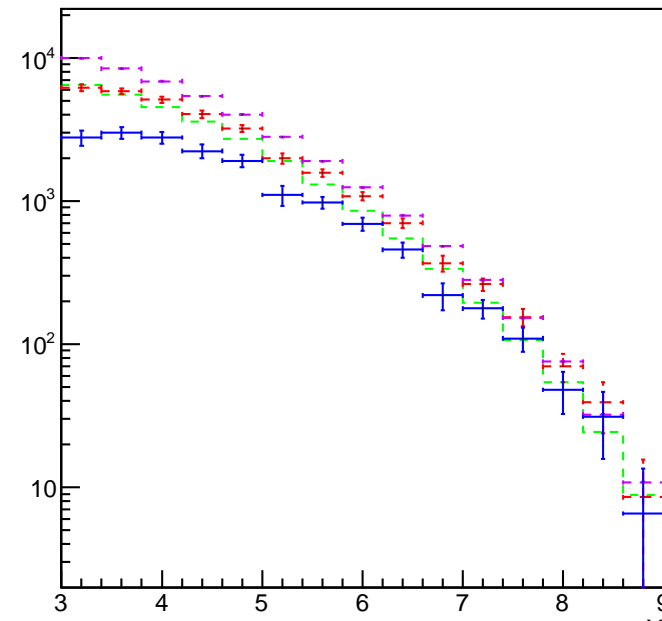
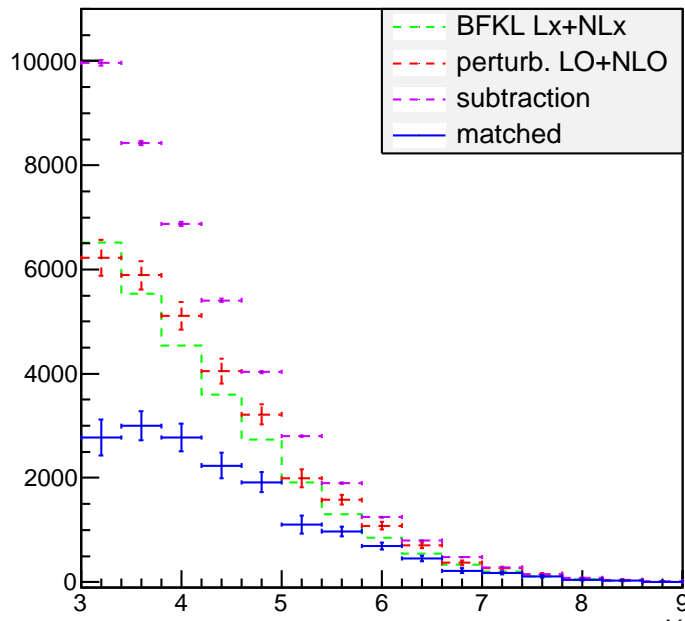
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Advice for future analysis

We *strongly suggest* experimentalists to perform MN jet analysis with **average p_T cut**: $\frac{1}{2}(p_{T1} + p_{T2}) > p_{\text{cut}}$ in order to avoid perturbative sensitivity to phase space corner $p_{T1} = p_{T2} = p_{\text{cut}}$

- **Smaller theoretical uncertainties**
- MNJ better tool for finding evidence of BFKL dynamics still competing with fixed-order contributions, even at LHC

On the definition of MN Jets

Mismatch between

- theoretical MN jet definition at NLO of [*Bartels, DC, Vacca '02*]
(checked by [*Caporale, Ivanov et al '11*])
- event selection of experimental CMS analysis

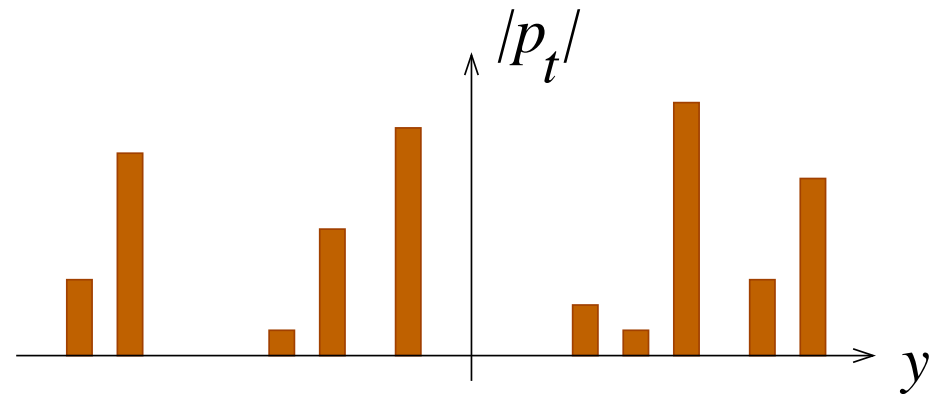
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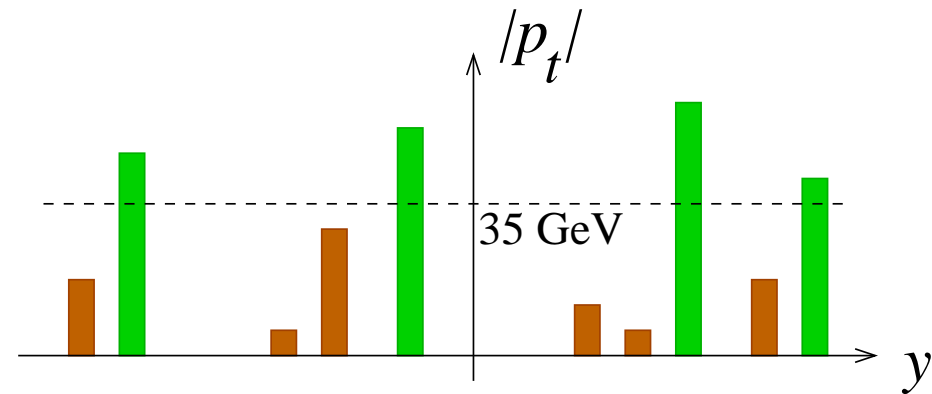
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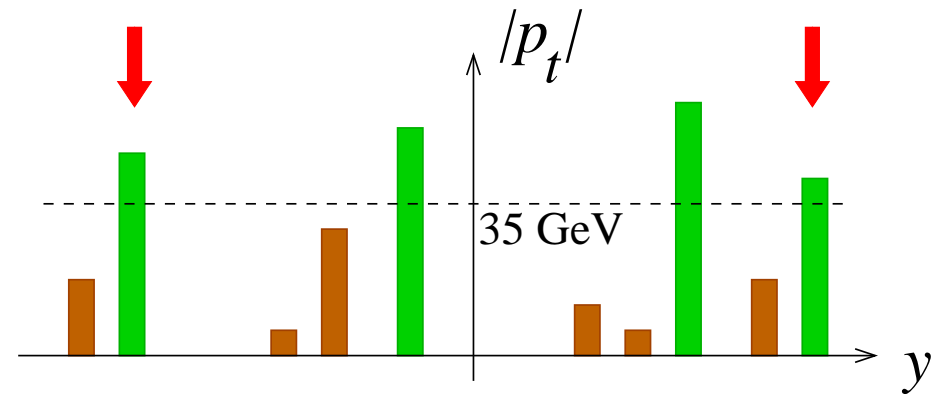
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Experimental analysis:

- Cluster particles into jets
- Consider jets with $p_t > 35\text{GeV}$
- Tag jets with largest rapidity difference (**MN jets**)

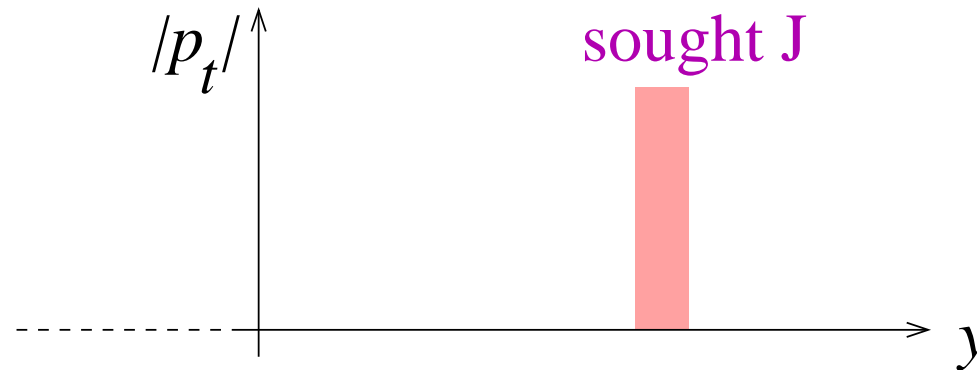


On the definition of MN Jets

Theoretical prescription

A different definition of jet vertices was adopted in NL BFKL approximation

$$\frac{d\sigma}{dJ_1 dJ_2} = f_b \otimes V_b \otimes G \otimes \left(V_a^{(0)} + \alpha_s V_a^{(1)} \right) \otimes \left(f_a^{(0)} + \frac{\alpha_s}{\varepsilon} f_a^{(1)} \right)$$

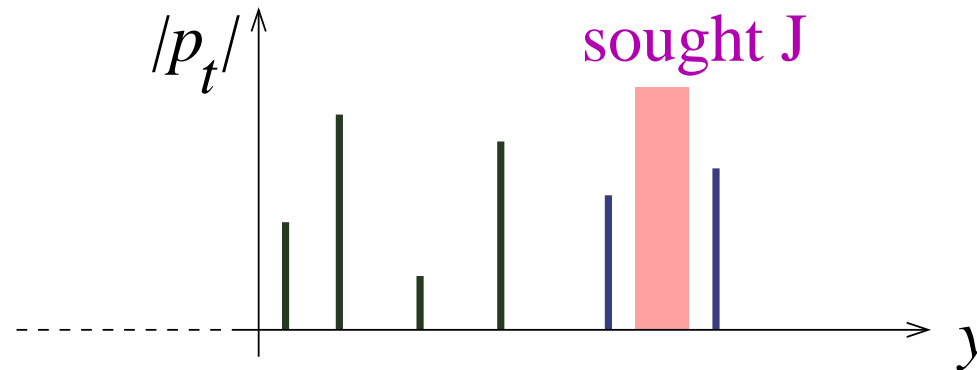


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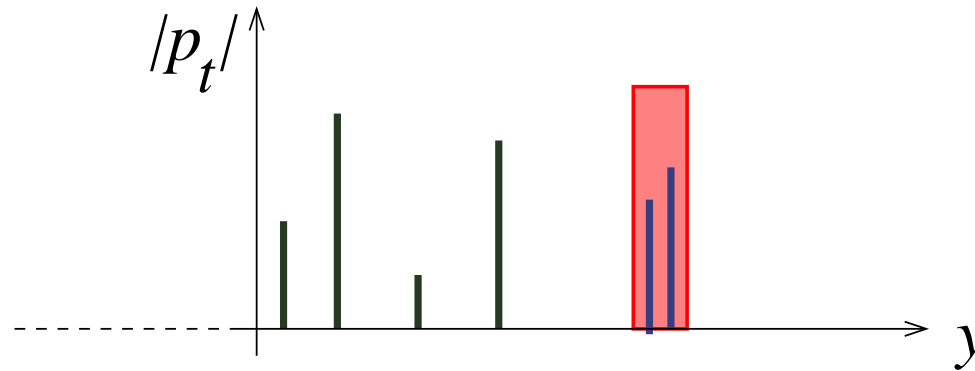


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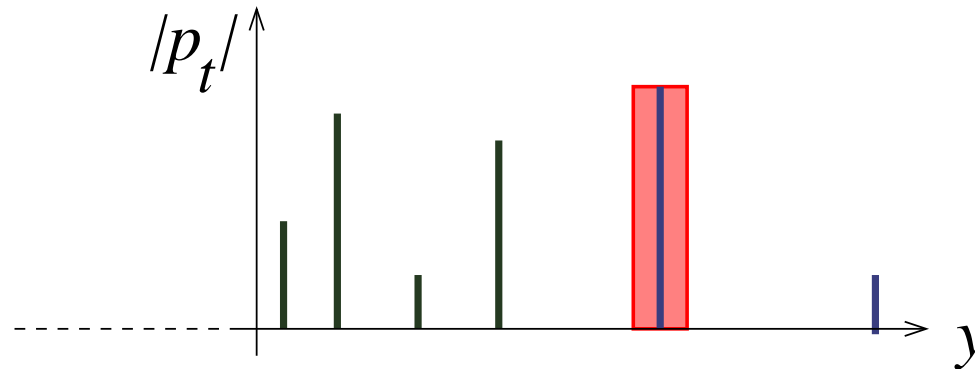


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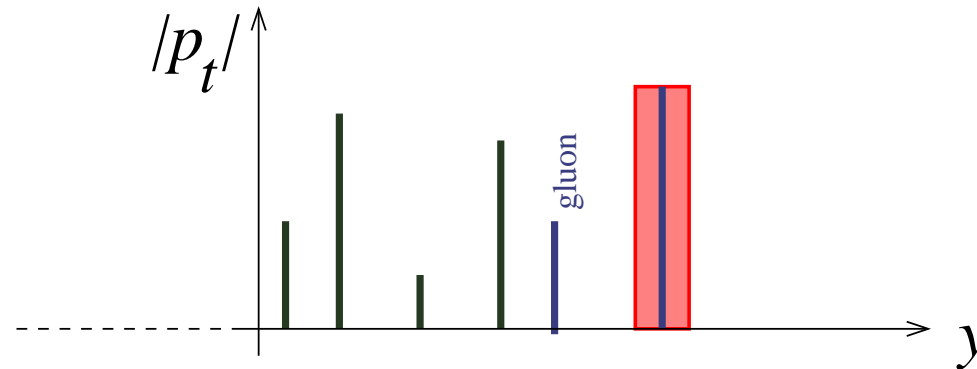


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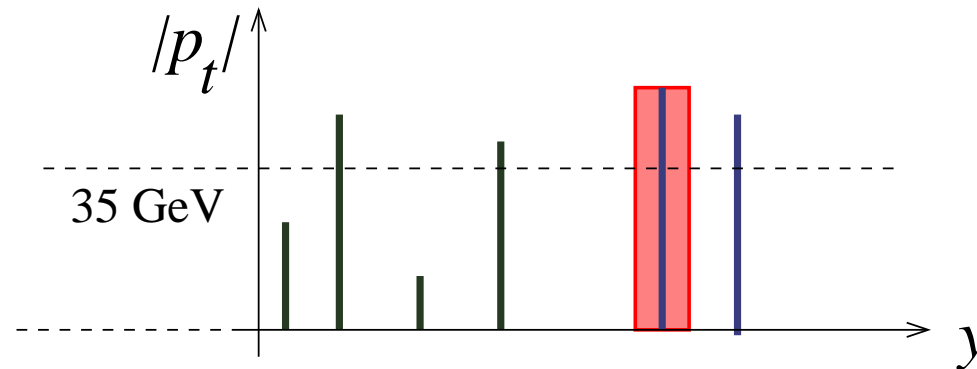


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A hard parton (\rightarrow jet at hadron level)
can be emitted at rapidity $y > y_J$

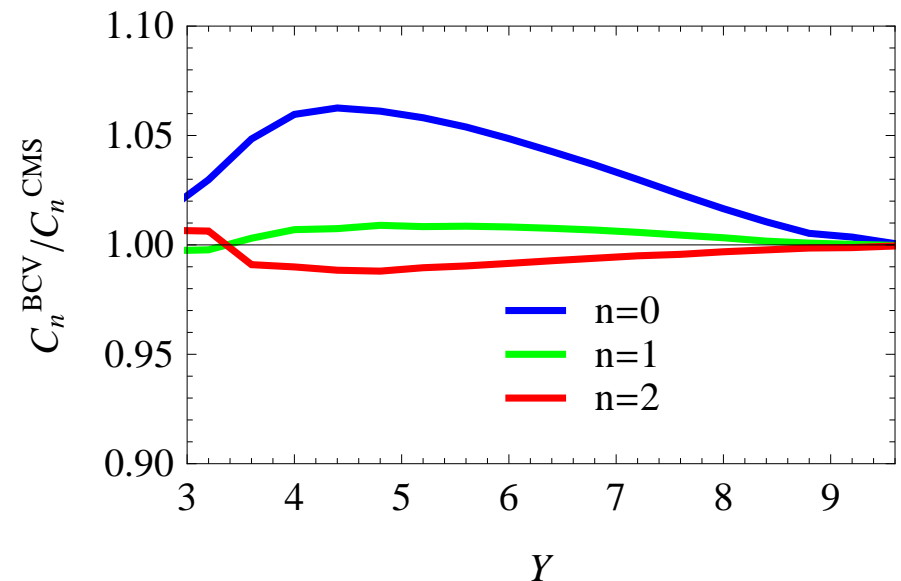
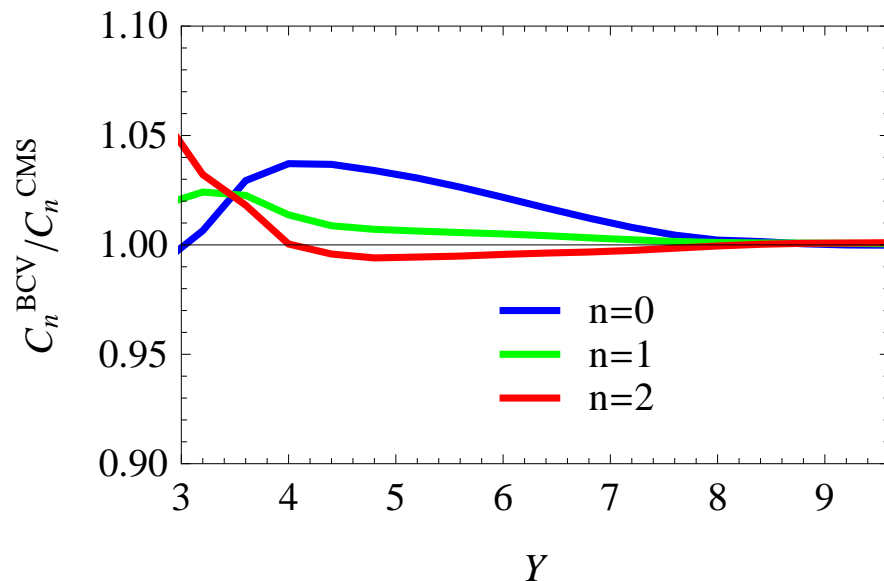
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- In practice, since $Y \equiv y_{J1} - y_{J2} \gg 1$, it is rather unlikely to emit additional partons with $y > y_{J1}$ or $y < y_{J2}$

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- Conceptually, the 2 prescriptions are quite different
- In practice, since $Y \equiv y_{J1} - y_{J2} \gg 1$, it is rather unlikely to emit additional partons with $y > y_{J1}$ or $y < y_{J2}$
- Largest difference at $\sqrt{s} = 7$ TeV is $\simeq 4\%$ at $Y \simeq 4$; **at 13 TeV $\simeq 7\%$**

[DC, Deganutti '16]



Better (and easy) to modify the theoretical prescription for $V^{(1)}$ by requiring the absence of partons/jets with $p_t > p_{t,\text{min}}$ and $y > y_J$

Conclusions and outlook

- Mueller-Navelet jets are a good observable for demonstrating presence of BFKL dynamics at high energy. Yet there is room for improving theoretical description
- Original **jet vertices** have to be **modified** in order to comply with experimental analysis
- We propose an improved theoretical description by **matching** BFKL with NLO.
 - Preliminary results of various observables are encouraging
 - ... in particular with $\langle p_T \rangle$ **cut**
 - Full analysis with error is under way
- **Experimental analysis of MNJ at 13 TeV very valuable**
 - In particular for comparing different \sqrt{s} values