Non-exponential behaviour of pp $d\sigma/dt$ a(p)review

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Introduction and motivation
Results before LHC
TOTEM results
Interpretations
Results and cross-checks
Outlook
Summary
Theoretically

\[ f_{\text{c.m.}}(s, t) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos \theta) a_\ell(k), \]

\[ a_\ell(k) = \frac{e^{2i\delta_\ell} - 1}{2i}, \]

\[ a(b, s) = \frac{e^{2i\delta(b,s)} - 1}{2i}, \]

\[ f_{\text{c.m.}}(s, t) = 2k \int_0^{\infty} b \, db J_0(qb) a(b, s) \]
\[ = \frac{k}{\pi} \int a(b, s) e^{i\vec{q} \cdot \vec{b}} \, d^2\vec{b}, \]

\[ \frac{d\sigma_{\text{el}}}{dt} = \pi \left| f_{\text{c.m.}} \right|^2 = 4\pi \left| \int a(b, s) J_0(qb) b \, db \right|^2. \]

\[ \sigma_{\text{el}} = \frac{\pi}{k^2} \int \left| f_{\text{c.m.}} \right|^2 \, dt = \frac{1}{k^2} \int \left| f_{\text{c.m.}} \right|^2 \, d^2\vec{q} \]
\[ = 4 \int |a(b, s)|^2 \, d^2\vec{b}. \]

\[ \sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im} f_{\text{c.m.}}(s, 0) = 4 \int \text{Im} a(b, s) \, d^2\vec{b}. \]

\[ \sigma_{\text{tot}} = 2 \int \text{Im} \left[ i(1 - e^{2i\delta(b,s)}) \right] \, d^2\vec{b}. \]
Gray Disc vs Gray Gaussian

For a Gray Disc, 
\[ a(b,s) = i A/2 \Theta(b-R) \]
\[
\sigma_{el} = \pi R^2 A^2, \\
\sigma_{tot} = 2\pi R^2 A, \\
\frac{\sigma_{el}}{\sigma_{tot}} = \frac{\Sigma_{el}}{\sigma_{tot}} = \frac{A}{2}, \\
B = \frac{R^2}{4}.
\]
\[
\frac{d\sigma_{el}}{dt} = \pi R^4 A^2 \left[ \frac{J_1(qR)}{qR} \right]^2,
\]

For a Gray Gaussian: 
\[ a(b,s) = i A/2 \exp(-2(b/R)^2) \]
\[
\sigma_{el} = \pi R^2 A^2, \\
\sigma_{tot} = 2\pi R^2 A, \\
\frac{\sigma_{el}}{\sigma_{tot}} = \frac{\Sigma_{el}}{\sigma_{tot}} = \frac{A}{2}, \\
B = \frac{R^2}{4}.
\]
\[
\pi R^4 A^2 \exp[-(qR)^2/4] = \left( \frac{d\sigma_{el}}{dt} \right)_{t=0} e^{-B|t|},
\]

Gray Gaussian A(b) \(\rightarrow\)  
Exponential \(d\sigma/dt\).  
Non-exponential \(d\sigma/dt\) \(\rightarrow\)  
a non-Gaussian behaviour of A(b) shadow profile function

For a black disc or Gaussian, A = 1 and \(\sigma_{el}/\sigma_{tot} = 1/2\) in both cases
Possible theory interpretations

from Glauber-Velasco
PLB 147 (1984) 380
Slope is not quite
Exponential:
a non-Gaussian behaviour

ReBB: Quark-Diquark Model
Non-exponential $d\sigma/dt$:
a non-Gaussian $A(b)$
F. Nemes, T.Cs, M. Csanád
Datasets on pp and ppbar, before 2015

FIG. 1. Timeline of proton and antiproton elastic scattering measurements. New accelerators are run first at the maximum available energies; however, at the start of the SpbS accelerator, the pp and the pbar elastic scattering data were measured at the same $\sqrt{s} = 31, 53$ and 62 GeV.

Suggests to run down LHC to $\sim$ Tevatron energies
A. Ster, L. Jenkovszky and T. Cs, PRD 91 (2015) 074018
Comparison of measurements by several experiments of the logarithmic slope for \( \pi^- p \), \( \pi^+ p \), and \( pp \) elastic scattering.

Fig. 12. Elastic slope parameters versus \( t \) at 200 GeV/c for \( pp \), \( \pi^+ p \) and \( \pi^- p \) scattering (data from ref. [14]). The curves were calculated using eq. (38), as described in the text. [14] A. Schiz et al., Phys. Rev. D24 (1981) 26.

Satisfactory fits with \( \exp(-B(t) |t|) \): non-exponential in 1981!
Hints non-exponential/"break": pp@ISR, $\sqrt{s}=21.5$-52.8 GeV, change of slope, $B=B(t,s)$ $|t|\sim 0.1$ GeV$^2$


$B(t_{\text{low}}) \neq B(t_{\text{higher}})$

At $\sqrt{s}=52.8$ GeV SpbarpS slope is same and breaks the same way in pbarp and in pp

M. Ambrosio et al.


Same slope in pp and pbar-p at $|t|\sim 0.14$ GeV$^2$,
fits with $\exp(-B|t| - C t^2)$
at $\sqrt{s}=31$, 55, 62 GeV

TOTEM results at LHC before 2015

LHC data, pre-2014-15: satisfactory fits with $\exp(-B|t|)$ at low $t$. TOTEM data at 8 TeV, low $|t|$: evidence for non-exponential cone.
TOTEM: LHC Optics for Elastic pp

Precise $\sigma_{\text{tot}}$ and $d\sigma/dt$ determination by TOTEM needs excellent control of LHC optics from data.
TOTEM: Precise control of LHC imperfections with perturbed LHC optics and recalibration from data at IP5: factors of 2 - 10

arXiv:1406.0546
$$t = -p^2 \theta^2; \quad \text{“optimized binning”; almost exponential but if one looks in detail, NOT}$$
Differential cross-section @ 8 TeV

\[ \frac{d\sigma}{dt} = \frac{d\sigma}{dt} \bigg|_{t=0} \exp \left( \sum_{i=1}^{N_b} b_i \ t^i \right), \]

\[ \chi^2 = \Delta^T V^{-1} \Delta, \]

\[ \Delta_i = \left. \frac{d\sigma}{dt} \right|_{\text{bin } i} - \frac{1}{\Delta t_i} \int_{\text{bin } i} f(t) \ dt, \]

\[ V = V_{\text{stat}} + V_{\text{syst}} \]

\( N_b = 1 \) fits excluded. Relative to best exponential, a significant 7.2\( \sigma \) deviation found.
Differential cross-section @ 8 TeV

Table 4: Details of the fits in Figure 11 using parametrisation Eq. (15). The matrices give the correlation factors between the fit parameters.

| \(N_b\) | \(d\sigma/dt|_{t=0}\) [mb/GeV\(^2\)] | \(b_1\) [GeV\(^{-2}\)] | \(b_2\) [GeV\(^{-4}\)] | \(b_3\) [GeV\(^{-6}\)] | \(\chi^2/\text{ndf}\) | p-value | significance |
|---|---|---|---|---|---|---|---|
| 1 | 531 ± 22 | -19.35 ± 0.06 | - | - | 117.5/28 = 4.20 | 6.2 \cdot 10^{-13} | 7.20 \sigma |
| | (+1.00 | -0.11) | -0.11 | +1.00 |
| 2 | 537 ± 22 | -19.89 ± 0.08 | 2.61 ± 0.30 | - | 29.3/27 = 1.09 | 0.35 | 0.94 \sigma |
| | (+1.00 | +0.19 | -0.34 | +1.00 |
| | +0.19 | +1.00 | -0.76 | +1.00 |
| | -0.34 | -0.76 | +1.00 |
| 3 | 541 ± 22 | -20.14 ± 0.15 | 5.95 ± 1.75 | -12.0 ± 6.2 | 25.5/26 = 0.98 | 0.49 | 0.69 \sigma |
| | (+1.00 | +0.08 | -0.04 | -0.02 | +1.00 |
| | +0.08 | +1.00 | -0.90 | +0.85 |
| | -0.04 | -0.90 | +1.00 | -0.99 |
| | -0.02 | +0.85 | -0.99 | +1.00 |

\[
\frac{d\sigma}{dt}(t) = \left. \frac{d\sigma}{dt} \right|_{t=0} \exp\left(\sum_{i=1}^{N_b} b_i t^i\right),
\]

\[
\chi^2 = \Delta^T V^{-1} \Delta,
\]

\[
\Delta_i = \left. \frac{d\sigma}{dt} \right|_{\text{bin }i} - \frac{1}{\Delta t_i} \int_{\text{bin }i} f(t) \, dt,
\]

\[V = V_{\text{stat}} + V_{\text{syst}}\]

\(N_b = 1\) fits excluded. Relative to best exponential, a significant 7.2\(\sigma\) deviation found.
Cross-check: „per-mille” binnings

Figure 12: Differential cross-section using the “per-mille” binning and plotted as relative difference from the reference exponential (see vertical axis). The black dots represent data points with statistical uncertainty bars. The red line shows pure exponential fits in regions below and above $|t| = 0.07$ GeV$^2$, see Eq. (19). The yellow band corresponds to the full systematic uncertainty, the brown-hatched one includes all systematic contributions except the normalisation. Both bands are centred around the fit curve.

$$\frac{d\sigma}{dt}(t) = \begin{cases} a_1 e^{b_1|t|} & |t| < 0.07 \text{ GeV}^2 \\ a_2 e^{b_2|t|} & |t| > 0.07 \text{ GeV}^2 \end{cases}$$

$$\chi_p^2 = \Delta_p^T V_p^{-1} \Delta_p , \quad \Delta_p = \begin{pmatrix} a_1 - a_2 \\ b_1 - b_2 \end{pmatrix}$$

Simple exp fits excluded. Different binnings show the same effect. Here 7.8 $\sigma$ significance.
Fig. 10: Differential cross-section from Table 3 with statistical (bars) and systematic uncertainties (bands). The yellow band represents all systematic uncertainties, the green one all but normalisation. The bands are centred around a data fit including both nuclear and Coulomb components (the fit shown in Figure 14). INSET: a low-|t| zoom of cross-section rise due to the Coulomb interaction.
TOTEM: non-exponential behavior is seen clearly also at 13 TeV, but emphasis on $\rho$
TOTEM results on non-exponential behavior and indications of Odderon: triggered theoretical interpretations.

60+ theory models so far

Work in progress on reviewing these models trying to find the common, important part if possible model-independently

Only a few of all the possible models will be highlighted or summarized here

BEL or BnotEL effect?
Prediction: Black disc limit is reached at LHC, but 7 TeV data not yet fitted.
Conclusions:
The fitted data satisfy the black disc limit within errors but the new TOTEM data at 13 TeV data challenges this interpretation.
Summary

Nucl. Phys. B899 (2015) 297 by TOTEM:
low-|t| dσ/dt for elastic pp at √s = 8 TeV
with unprecedented precision

Significantly non-exponential behaviour
first at FNAL-0069 already,
TOTEM confirmed at 13 TeV

Theoretical interpretations:
Work in progress on reviewing them

Common picture:
Non-Gaussian shadow profile of protons
Opening of a new channel likely
Between 2.76 and 7 TeV,
New trends seen also at 13 TeV
Thank you for your attention

Questions and Comments?
Backup slides – Questions?
LHC optics and proton acceptance

\[ t = -p^2 \theta^2 \]: four-momentum transfer squared; \[ \xi = \Delta p/p \]: fractional momentum loss

\[ \beta^* = 90 \text{ m} \] (special development for RP runs)

\[ \beta^* = 90 \text{ m} \] MC simulation shown
Parallel to point focussing, \( v_y \sim 0 \)
Large effective length \( L_y \)
Elastic scattering events: in vertical RPs

Diffraction:
all \( \xi \) if \( |t| \geq 10^{-2} \text{ GeV}^2 \),
soft & semi-hard diffraction
Elastic: low to mid \( |t| \)

Total cross-section

<table>
<thead>
<tr>
<th>RP unit</th>
<th>( L_x )</th>
<th>( v_x )</th>
<th>( L_y )</th>
<th>( v_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>near</td>
<td>2.45 m</td>
<td>-2.17 m</td>
<td>239 m</td>
<td>0.040 m</td>
</tr>
<tr>
<td>far</td>
<td>-0.37 m</td>
<td>-1.87 m</td>
<td>264 m</td>
<td>0.021 m</td>
</tr>
</tbody>
</table>
Kinematic cuts: selection of elastics

Precise control of LHC optics and elastic scattering:
- Kinematics reconstruction
- Alignment
- Optics recalibration
- Resolution unfolding
- Acceptance correction
- Background subtraction
- Detection & efficiency
- Angular resolution
- Normalization
- Binning

Table 2: The elastic selection cuts. The superscripts R and L refer to the right and left arm, N and F correspond to the near and far units, respectively. The constant $\alpha = L_N^L / L_F^N - 1 \approx 0.11$. The right-most column gives a typical RMS of the cut distribution.

<table>
<thead>
<tr>
<th>discriminator</th>
<th>cut quantity</th>
<th>RMS ($= 1\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\theta_R^r - \theta_L^l$</td>
<td>9.5 $\mu$rad</td>
</tr>
<tr>
<td>2</td>
<td>$\theta_R^r - \theta_L^l$</td>
<td>3.3 $\mu$rad</td>
</tr>
<tr>
<td>3</td>
<td>$\alpha y_R^N - (y_R^F - y_R^N)$</td>
<td>18 $\mu$m</td>
</tr>
<tr>
<td>4</td>
<td>$\alpha y_L^N - (y_L^F - y_L^N)$</td>
<td>18 $\mu$m</td>
</tr>
<tr>
<td>5</td>
<td>$x_R^L - x_L^l$</td>
<td>8.5 $\mu$m</td>
</tr>
</tbody>
</table>
Table 3: The elastic differential cross-section as determined in this analysis using the “optimised” binning. The three left-most columns describe the bins in $t$. The representative point gives the $t$ value suitable for fitting [23]. The other columns are related to the differential cross-section. The four right-most columns give the leading systematic biases in $d\sigma/dt$ for 1-$\sigma$-shifts in the respective quantities, $\delta x_q$, see Eqs. (13) and (14). The two contributions due to optics correspond to the two vectors in Eq. (8).

| $|t|$ bin [GeV$^2$] | $d\sigma/dt$ [mb/GeV$^2$] |
|------------------|-----------------------------|
| left  | right  | represent. | value | statistical uncertainty | systematic uncertainty | normalisation $N$ | optics mode 1 | optics mode 2 | beam momentum |
| 0.02697 | 0.03005 | 0.02850 | 305.09 | 0.527 | 12.85 | +12.83 | -0.479 | -0.263 | +0.257 |
| 0.03005 | 0.03325 | 0.03164 | 287.95 | 0.478 | 12.08 | +12.06 | -0.502 | -0.217 | +0.206 |
| 0.03325 | 0.03658 | 0.03491 | 269.24 | 0.436 | 11.32 | +11.31 | -0.491 | -0.174 | +0.159 |
| 0.03658 | 0.04005 | 0.03831 | 251.31 | 0.401 | 10.59 | +10.57 | -0.478 | -0.135 | +0.115 |
| 0.04005 | 0.04365 | 0.04184 | 235.15 | 0.371 | 9.874 | +9.861 | -0.465 | -0.0981 | +0.0750 |
| 0.04365 | 0.04740 | 0.04551 | 218.32 | 0.343 | 9.185 | +9.172 | -0.451 | -0.0647 | +0.0383 |
| 0.04740 | 0.05129 | 0.04933 | 202.64 | 0.318 | 8.521 | +8.509 | -0.437 | -0.0343 | +0.0052 |
| 0.05129 | 0.05534 | 0.05330 | 187.10 | 0.295 | 7.882 | +7.870 | -0.421 | -0.0070 | -0.0244 |
| 0.05534 | 0.05956 | 0.05743 | 173.06 | 0.274 | 7.270 | +7.257 | -0.405 | +0.0172 | -0.0504 |
| 0.05956 | 0.06394 | 0.06173 | 158.77 | 0.255 | 6.685 | +6.672 | -0.388 | +0.0385 | -0.0731 |
| 0.06394 | 0.06850 | 0.06620 | 144.93 | 0.236 | 6.127 | +6.114 | -0.370 | +0.0569 | -0.0925 |
| 0.06850 | 0.07324 | 0.07085 | 133.12 | 0.219 | 5.597 | +5.584 | -0.352 | +0.0724 | -0.109 |
| 0.07324 | 0.07817 | 0.07568 | 121.24 | 0.203 | 5.096 | +5.082 | -0.334 | +0.0853 | -0.122 |
| 0.07817 | 0.08329 | 0.08071 | 109.77 | 0.188 | 4.623 | +4.609 | -0.316 | +0.0957 | -0.132 |
| 0.08329 | 0.08862 | 0.08593 | 99.077 | 0.174 | 4.179 | +4.164 | -0.297 | +0.104 | -0.140 |
| 0.08862 | 0.09417 | 0.09137 | 89.126 | 0.161 | 3.762 | +3.747 | -0.279 | +0.109 | -0.145 |
| 0.09417 | 0.09994 | 0.09702 | 79.951 | 0.148 | 3.374 | +3.359 | -0.260 | +0.113 | -0.147 |
| 0.09994 | 0.10593 | 0.10290 | 71.614 | 0.137 | 3.014 | +2.998 | -0.242 | +0.115 | -0.148 |
| 0.10593 | 0.11217 | 0.10902 | 63.340 | 0.125 | 2.680 | +2.664 | -0.224 | +0.115 | -0.147 |
| 0.11217 | 0.11866 | 0.11538 | 56.218 | 0.115 | 2.373 | +2.357 | -0.206 | +0.114 | -0.144 |
| 0.11866 | 0.12540 | 0.12199 | 49.404 | 0.105 | 2.092 | +2.075 | -0.189 | +0.111 | -0.139 |
| 0.12540 | 0.13242 | 0.12887 | 43.300 | 0.0961 | 1.835 | +1.818 | -0.173 | +0.107 | -0.134 |
| 0.13242 | 0.13972 | 0.13602 | 37.790 | 0.0876 | 1.601 | +1.585 | -0.157 | +0.102 | -0.127 |
| 0.13972 | 0.14730 | 0.14436 | 32.650 | 0.0795 | 1.391 | +1.374 | -0.142 | -0.0974 | -0.120 |
| 0.14730 | 0.15520 | 0.15220 | 28.113 | 0.0720 | 1.201 | +1.185 | -0.127 | +0.0924 | -0.112 |
| 0.15520 | 0.16340 | 0.15925 | 24.155 | 0.0659 | 1.030 | +1.016 | -0.0955 | +0.0864 | -0.106 |
| 0.16340 | 0.17194 | 0.16761 | 20.645 | 0.0616 | 0.877 | +0.866 | -0.0590 | +0.0804 | -0.0951 |
| 0.17194 | 0.18082 | 0.17632 | 17.486 | 0.0574 | 0.743 | +0.733 | -0.0302 | +0.0739 | -0.0865 |
| 0.18082 | 0.19005 | 0.18537 | 14.679 | 0.0543 | 0.626 | +0.617 | -0.0081 | +0.0673 | -0.0780 |
| 0.19005 | 0.19965 | 0.19478 | 12.291 | 0.0504 | 0.524 | +0.515 | +0.0052 | +0.0606 | -0.0697 |
No significant effect found on the total pp cross-section, $\sigma_{\text{tot}}$. 

Figure 9: Impact of $t$-dependent systematic effects on the differential cross-section. Each curve corresponds to a systematic error at 1 $\sigma$, cf. Eq. (13). The two contributions due to optics correspond to the two vectors in Eq. (8). The envelope is determined by summing all shown contributions in quadrature for each $|t|$ value. The right-hand plot provides a vertical zoom; note that the envelope is out of scale.
Elastic and diffractive scattering: colorless exchange