Dispersion Relations and the new TOTEM results at 13 TeV. Can we learn something?

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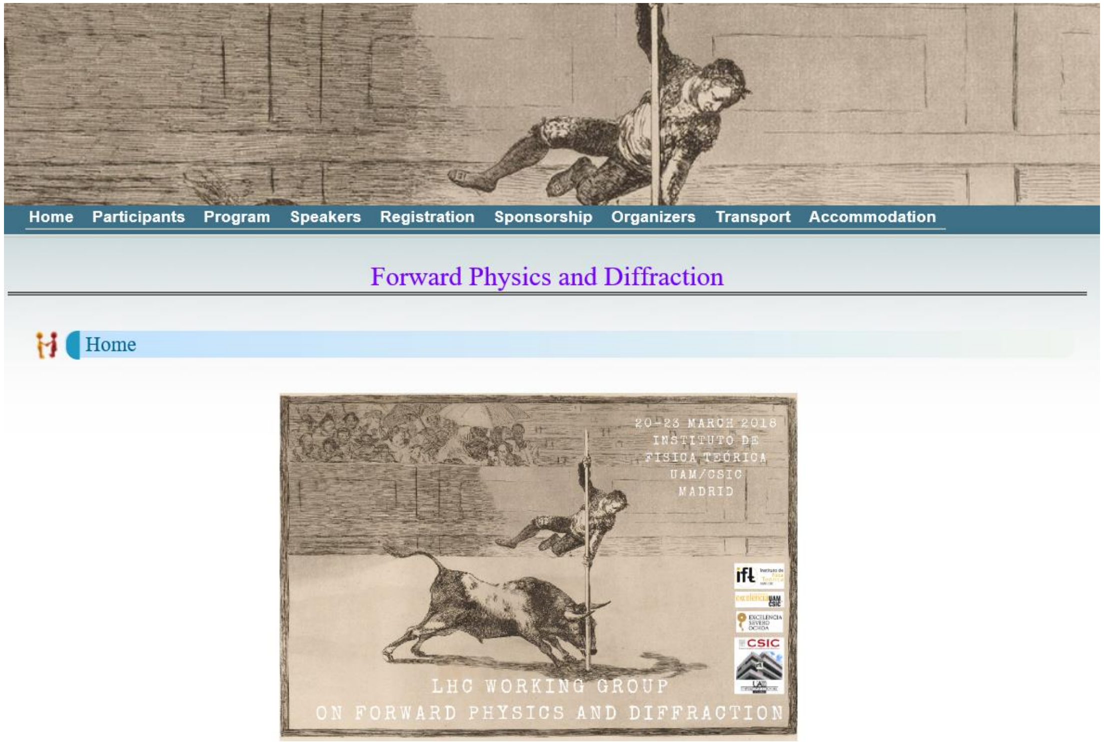


Forward Physics and Diffraction

Work in collaboration
with <u>Grazia Cabras</u>
University of Bologna

But what are Dispersion Relations- short reminder

Dispersion relations are based upon the Kramers-Kronig theorem.

Let $\chi(\omega) = \chi_1(\omega) + i\,\chi_2(\omega)$ be a complex analytic function of the complex variable ω where $\chi_1(\omega)$ and $\chi_2(\omega)$ are real

Then:

$$\chi_1(\omega) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{\chi_2(\omega')}{\omega' - \omega}\, d\omega'$$

derivation based upon the Cauchy integral theorem

Thus the Kramers-Kronig theorem gives a relation between the real and imaginary part of a complex analytic function.

Applied in many different field of physics

Dispersion Relations in Particle Physics

Application in Particle Physics: Suggested by Kronig 1946

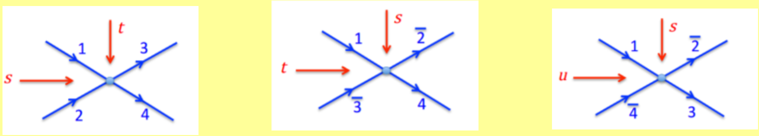
Basics worked out by Gell-Mann, Goldhaber and Thirring 1954

First application in pp scattering P.Soeding 1964

A good example of possible calculations, in the non perturbative domain, based upon very general principles

Corner stones

- Analyticity of the scattering amplitude *Assumes Causality !*
- Crossing symmetry



One analytic function

- Unitarity

$$\sigma_{tot} = 4\pi \text{ Im } f_{el}(0)$$

Integral Dispersion Relations for Elastic Scattering

Analyticity of the elastic scattering amplitude $f_{el}(s,t)$ implies that Re $f_{el}(s,t)$ is related to Im $f_{el}(s,t)$ via dispersion relations and assuming ***crossing symmetry*** one can derive →

$$\mathrm{Re}\ f_+(E) = C + \frac{E}{\pi}\int_m^\infty dE' \frac{\mathrm{Im}\ f_+(E')}{E'(E'-E)} - \frac{\mathrm{Im}\ f_-(E')}{E'(E'+E)}$$

where C is real constant which can not be determined from theory

and + refers to proton-proton amplitude and - to anti proton-proton amplitude

Unitarity → the optical theorem : $\sigma_{tot} = 4\pi/p\ \ \mathrm{Im}\ \ f_{el}\ (0)$

$$\mathrm{Re}\ f_+(E) = C + \frac{E}{4\pi^2}\int_m^\infty dE'p'\left(\frac{\sigma_+(E')}{E'(E'-E)} - \frac{\sigma_-(E')}{E'(E'+E)}\right) \rightarrow$$

$$\rho_\pm\sigma_\pm = \frac{B}{p} + \frac{E}{\pi p}\mathrm{P}\int_{m_p}^\infty \left[\frac{\sigma_\pm}{E'(E'-E)} - \frac{\sigma_\mp}{E'(E'+E)}\right] p'\, dE'$$

where σ_+ is the proton-proton total cross section and σ_- the anti-proton proton total cross section

$\rho = \mathrm{Re}\ f_{el}(s,t=0)/\mathrm{Im}\ f_{el}\ (s,t=0)$

HOWEVER : Observe- Very important

$$\boxed{\rho_{\pm}\sigma_{\pm} = \frac{B}{p} + \frac{E}{\pi p}\,\mathrm{P}\int\limits_{m_p}^{\infty}\left[\frac{\sigma_{\pm}}{E'(E'-E)} - \frac{\sigma_{\mp}}{E'(E'+E)}\right]p'\,dE'}$$

What we use is the so called "one subtracted " integral dispersion relation which in the derivation assumes :

$$\Delta\sigma = \sigma_{pp}-\sigma_{ppbar} \longrightarrow 0 \text{ at high energies}$$

….if this is not the case one has to use the "two subtracted" integral dispersion relation-more complicated,

…we will come back to this later

$$\rho_\pm \sigma_\pm = \frac{B}{p} + \frac{E}{\pi p} \mathrm{P} \int_{m_p}^{\infty} \left[\frac{\sigma_\pm}{E'(E'-E)} - \frac{\sigma_\mp}{E'(E'+E)} \right] p' \, dE'$$

The integral has an singularity at the energy, E, at which ρ is calculated.
→ the sensitivity to ρ on σ_{tot} is largest at this energy.

In practice: the ρ-value at the energy E is determined by the integral of the total cross section from E/10 to 10 x E….i.e roughly one order of magnitude lower and one order of magnitude higher relative the energy at which you want to calculate ρ.

Also observe that the cross section is in general known in the range from E/10 to E and thus it is the unknown behaviour of the cross section between E and 10 x E that determines the ρ-value at E.

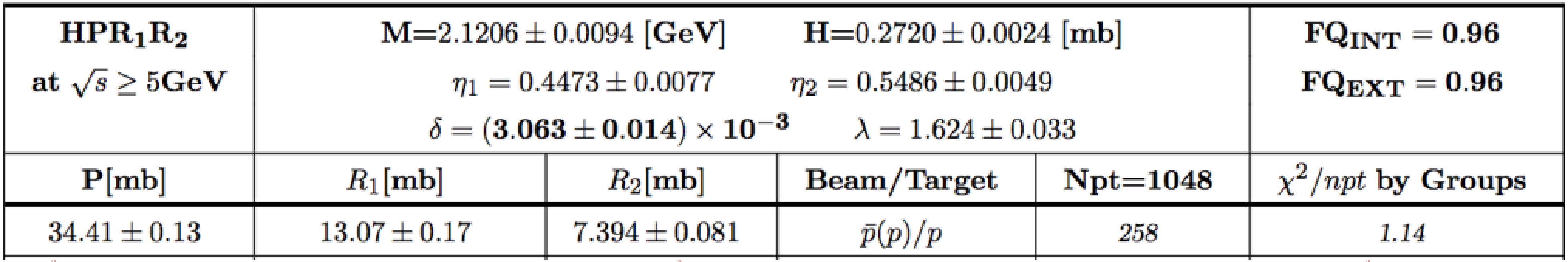
Calculate ρ using the one subtracted dispersion relation

$$\rho_\pm \sigma_\pm = \frac{B}{p} + \frac{E}{\pi p} \mathrm{P} \int_{m_p}^{\infty} \left[\frac{\sigma_\pm}{E'(E'-E)} - \frac{\sigma_\mp}{E'(E'+E)} \right] p' \, dE'$$

-use PDG parametrization of σ_+ and σ_-

$$\sigma^{a\mp b} = H \log^2 \left(\frac{s}{s_M^{ab}} \right) + P^{ab} + R_1^{ab} \left(\frac{s}{s_M^{ab}} \right)^{-\eta_1} \pm R_2^{ab} \left(\frac{s}{s_M^{ab}} \right)^{-\eta_2} ;$$

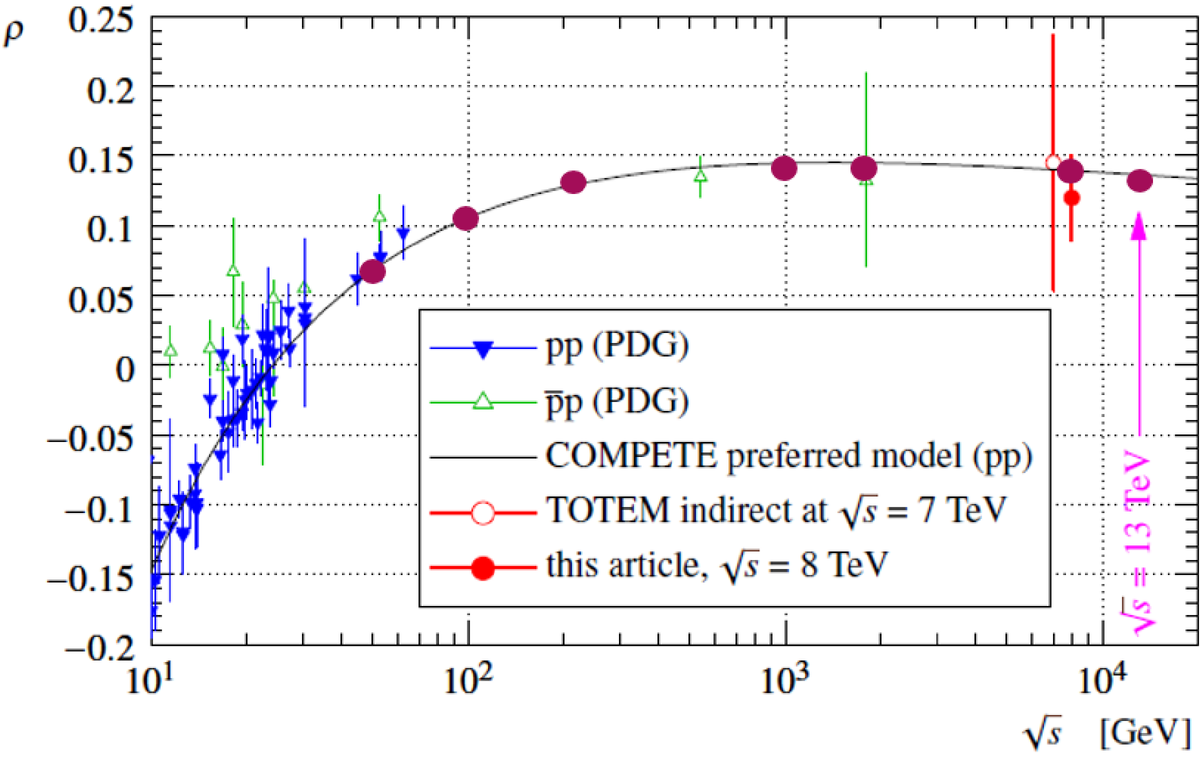
…using the following parameters PDG 2016



<table>
<tr><td>$\mathbf{HPR_1R_2}$
at $\sqrt{s} \geq 5$GeV</td><td colspan="4">M=2.1206 ± 0.0094 [GeV] H=0.2720 ± 0.0024 [mb]
$\eta_1 = 0.4473 \pm 0.0077$ $\eta_2 = 0.5486 \pm 0.0049$
$\delta = (\mathbf{3.063 \pm 0.014}) \times \mathbf{10^{-3}}$ $\lambda = 1.624 \pm 0.033$</td><td>$\mathbf{FQ_{INT} = 0.96}$
$\mathbf{FQ_{EXT} = 0.96}$</td></tr>
<tr><td>P[mb]</td><td>R_1[mb]</td><td>R_2[mb]</td><td>Beam/Target</td><td>Npt=1048</td><td>χ^2/npt by Groups</td></tr>
<tr><td>34.41 ± 0.13</td><td>13.07 ± 0.17</td><td>7.394 ± 0.081</td><td>$\bar{p}(p)/p$</td><td><i>258</i></td><td><i>1.14</i></td></tr>
</table>

For what we discuss here we have seen that using the PDG 2014 or 2016 parametrization does not change anything significantly.
We have also seen that the Regge terms are negligible in the LHC energy range

Result of our calculation



To start with…check that we can calculate ρ via Dispersion Relations Use PDG parametrisation of σ_{tot} as input

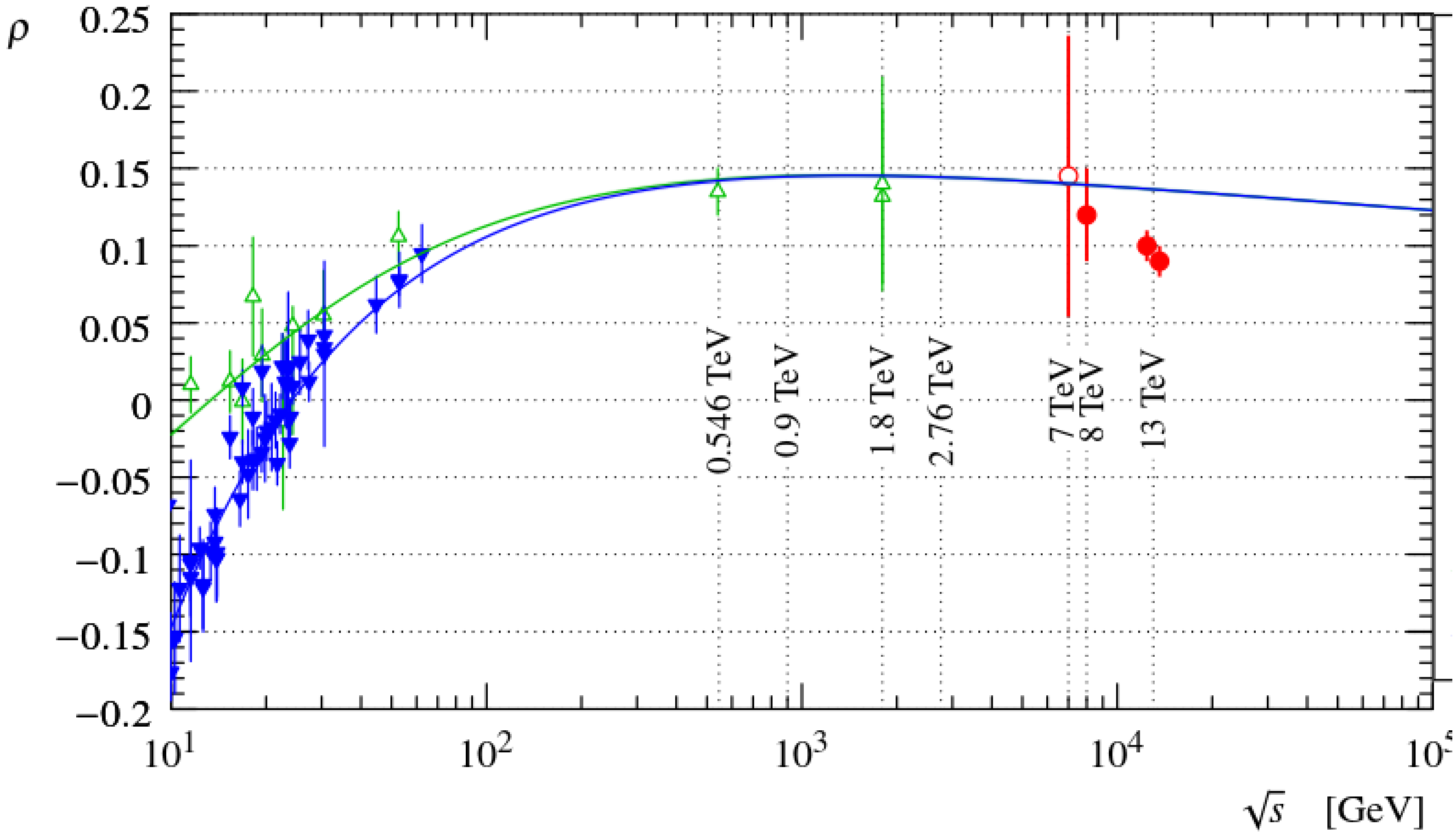
Encouraging agreement between our calculation and the COMPETE parametrization

● = present calculation

(agreement worse below √s= 50 GeV-I attribute this to the importance of Regge terms at lower energy)

What does data tell us today?

Red points …TOTEM published



It seems we are confronted with at least one of the 5 possibilities

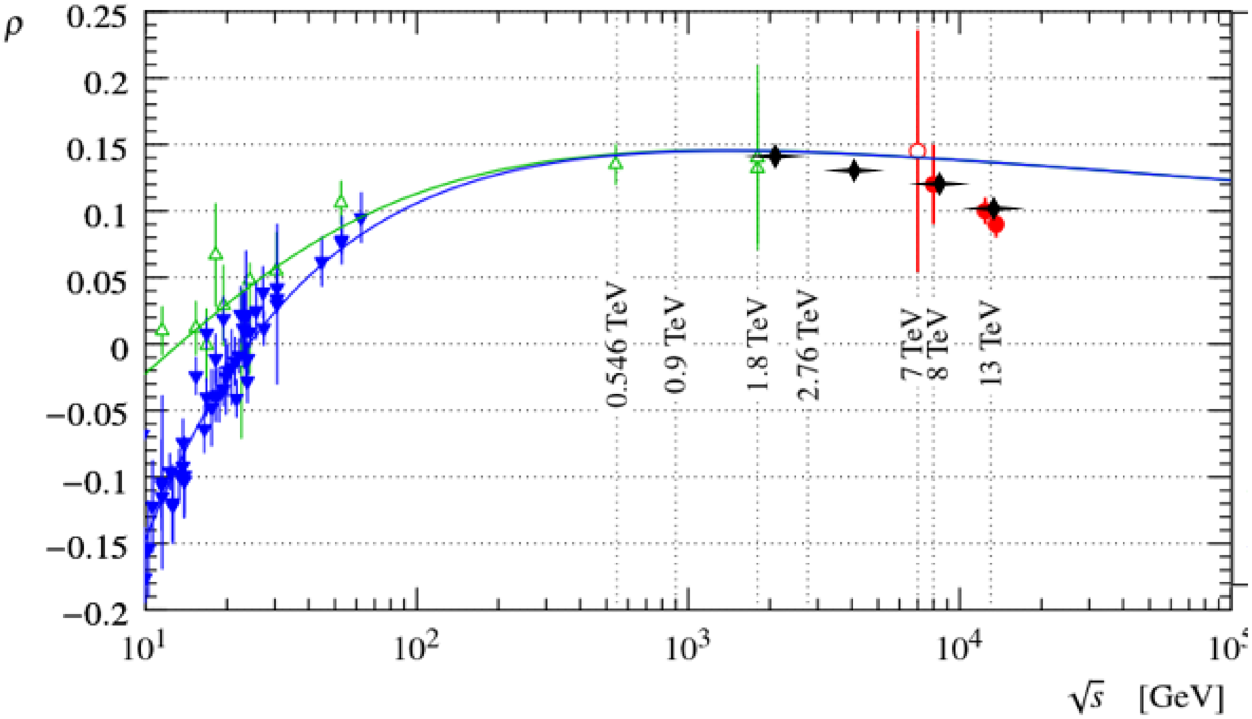
- (1) Crossing symmetry is violated ?
- (2) Unitarity breaks down?
- (3) Analyticity of the scattering amplitude (causality ?) breaks down
- (4) $\Delta\sigma = \sigma_{pp} - \sigma_{ppbar}$ does NOT go to 0 at high energies pointing to the existence of an Odderon
- (5) The cross section between 13 TeV and 100 TeV does not follow anymore the $\ln^2 s$ rule

Obviously (4) and (5) are the most natural candidates

Let's look at possibility (5) "The cross section between 13 TeV and 100 TeV does not follow anymore the $\ln^2 s$ rule "

As an example calculate the energy dependence of ρ for a scenario where the exponent of the ln (s) term change from 2 to 1.6 at 15 TeV

—✦— = $\sigma \propto \ln^{1.6}(s)$ above 15 TeV

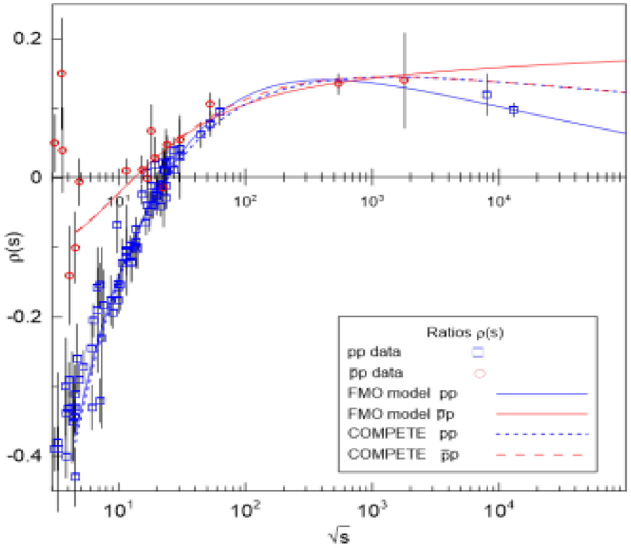


The black points describe well the data

Observe :This is not a model but just an example of a functional form that can describe the data ….or an indication of what a possible slowing down of the rise of σ_{tot} does to ρ

Let us now look at possibility (4)
“$\Delta\sigma = \sigma_{pp}-\sigma_{ppbar}$ does not go to 0 at high energies “

If the Odderon exist $\Delta\sigma$ does not need to go to 0.



Many possibilities that opens up….
depending on what type of odderon is assumed….
Not the topic of this talk.
To the left just an example
the so called “maximal” odderon

Just one comment:
$\Delta\sigma = \sigma_{pp}-\sigma_{ppbar}$ does not go to 0 does not necessarily violate the Pomeranchuk theorem

The general Pomeranchuk theorem states:
The ratio σ_{pp} /σ_{ppbar} goes to 1 the but the difference does not necessarily need to go to 0.

What can we learn from this?

The published TOTEM data points to two possibilities.

- (4) $\Delta\sigma = \sigma_{pp} - \sigma_{ppbar}$ does NOT go to 0 at high energies pointing to the existence of an Odderon
- (5) The cross section between 13 TeV and 100 TeV does not follow anymore the $\ln^2 s$ rule

Can we separate the two possibilities?

Might be difficult !

This little exercise using dispersion relations demonstrates that it is not too hard to find a functional form describing a slowing down of the rise of the total cross section and which at the same time will describe the measured rho values

Obviously it is another story to justify theoretically such a functional form .
HOWEVER the $\ln^2(s)$ form is not justfied theoretically either!

What can we learn from this?

Can we separate the two possibilities?

Might be difficult !

HOWEVER a low energy point might help

Remember
In practice: the ρ-value at the energy E is determined by the integral of the total cross section from E/10 to 10 x E….i.e roughly one order of magnitude lower and one order of magnitude higher relative the energy at which you want to calculate ρ.

The behaviour of the total cross between 1-2 and 13 TeV is known and thus the rho value at 1-2 TeV is not affected by uncertainties of the energy dependencies of the total cross section

THUS a deviation from the one subtracted integral dispersion relation at 1-2 TeV can only be due to an Odderon contribution !!
(disregarding possibilities of violation of analyticity, crossing symmetery or unitarity)

Back up

Example of early application…

Optics:

Propagation of light through matter.

Complex index of refraction: $\kappa(\omega) = n(\omega) + ik(\omega)$; ω=wavelength of the photon

dispersion absorption

…The real part represents the dispersion while absorption is modelled by the imaginary part…connected via dispersion relation

$$n(\omega) = 1 + P \int_{-\infty}^{+\infty} \frac{d\omega'}{\pi} \frac{k(\omega')}{\omega' - \omega}$$