Ultra-forward particle production from CGC+Lund fragmentation

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in collaboration with
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'LHC Working Group on Forward Physics and Diffraction'
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Outline

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2. **The Monte-Carlo event generator**
   - Perturbative parton production: implementation of DHJ formula
   - Multiple scattering: eikonal model
   - Hadronization: Lund fragmentation model
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   - LHCf: p-p @ 7 TeV
   - LHCf: p-Pb @ 5.02 TeV
   - LHCf: nuclear modification factor $R_{\text{p-Pb}}$ @ 5.02 TeV
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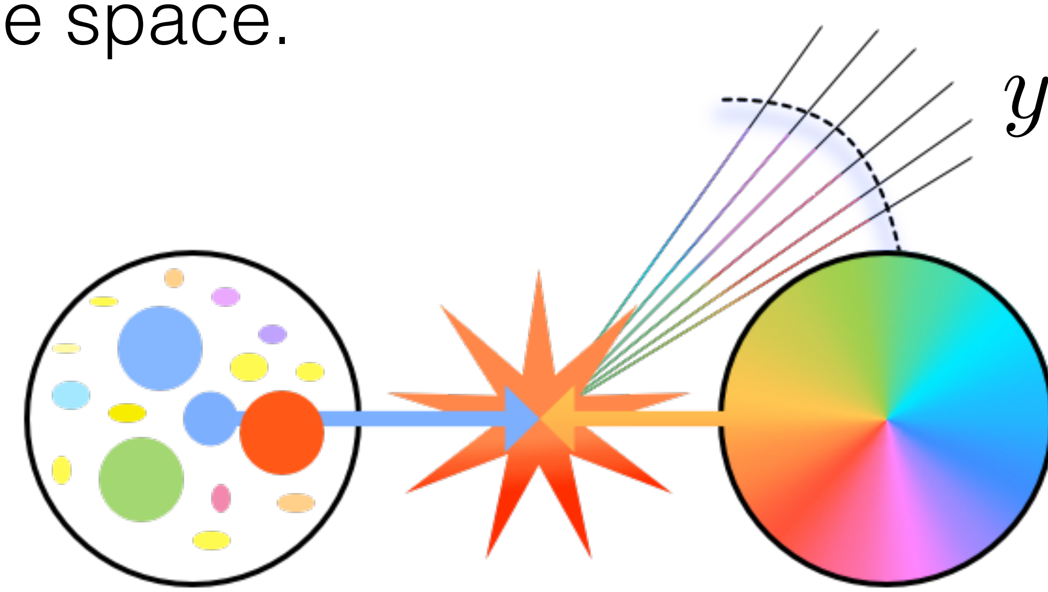
1. Introduction

Forward particle production in the Color Glass Condensate

- The analysis of the very forward region of particle production in high-energy collisions gives us access to the wave functions of colliding objects in the extreme limits of phase space.

$$y_{\mathrm{h}} \gg 0$$

$$x_{\mathrm{p}} \approx \frac{p_t}{\sqrt{s}} e^{y_{\mathrm{h}}}$$



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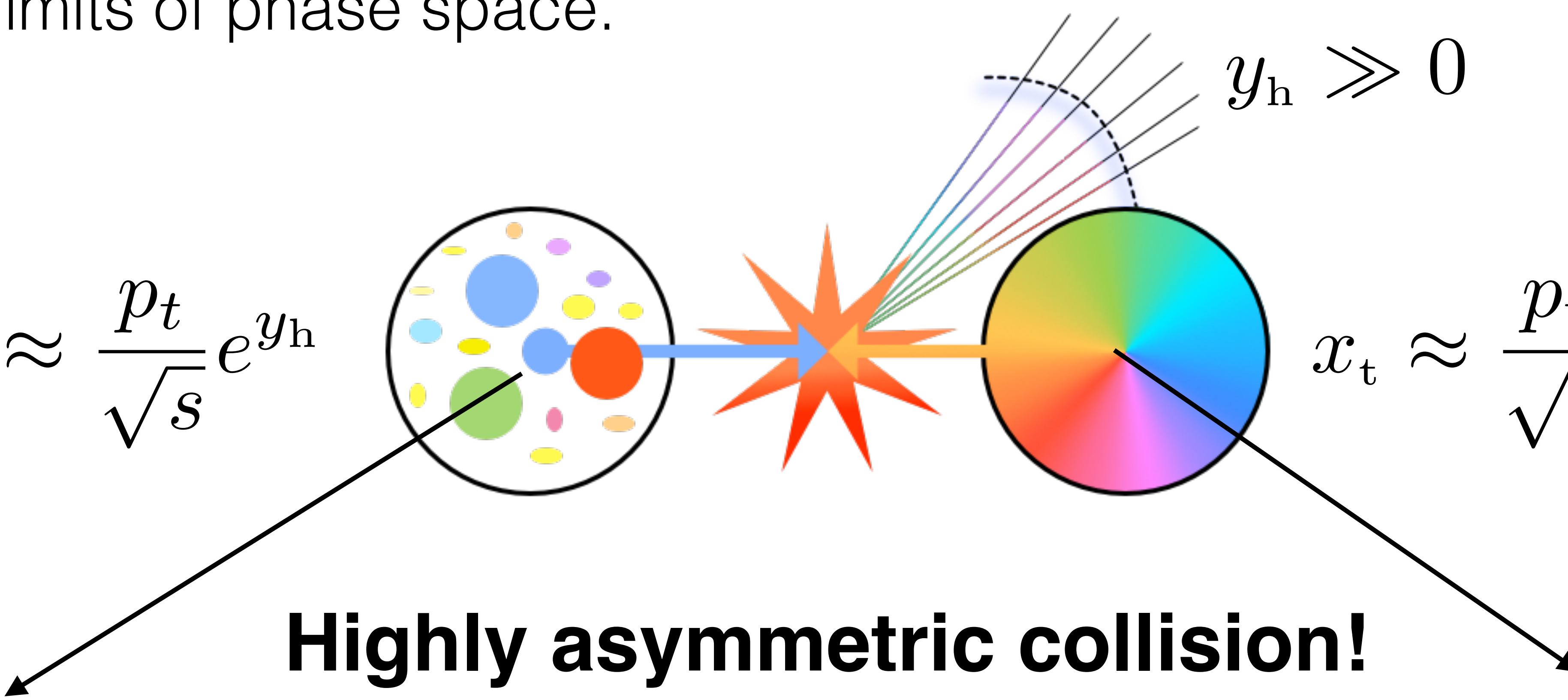

Highly asymmetric collision!

Dilute ensemble of fast valence quarks

Dense pack of ‘slow’ radiated gluons

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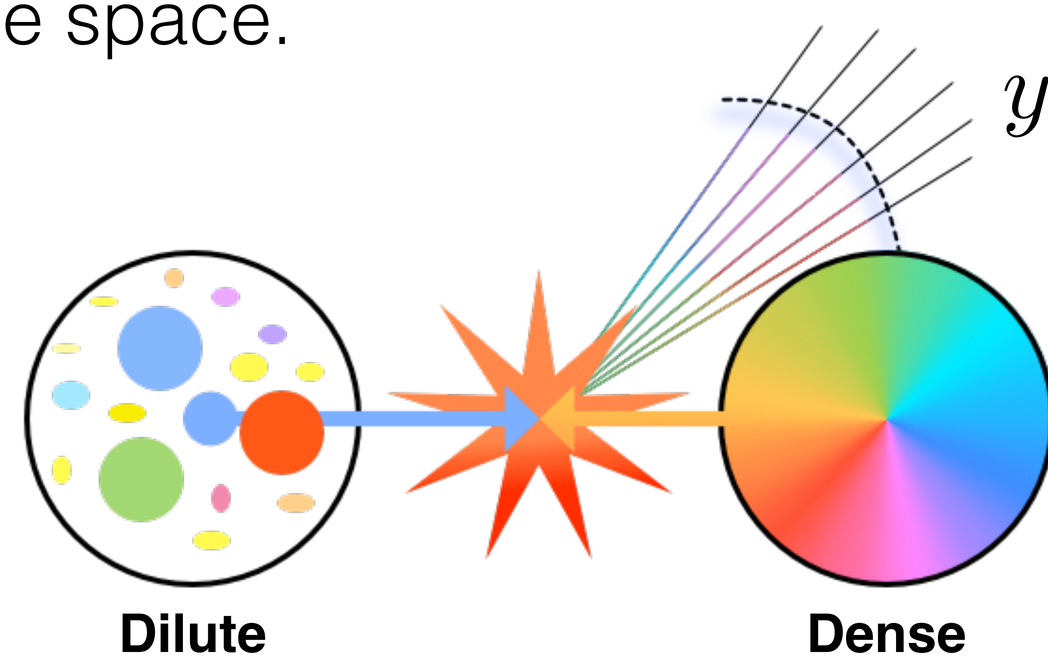

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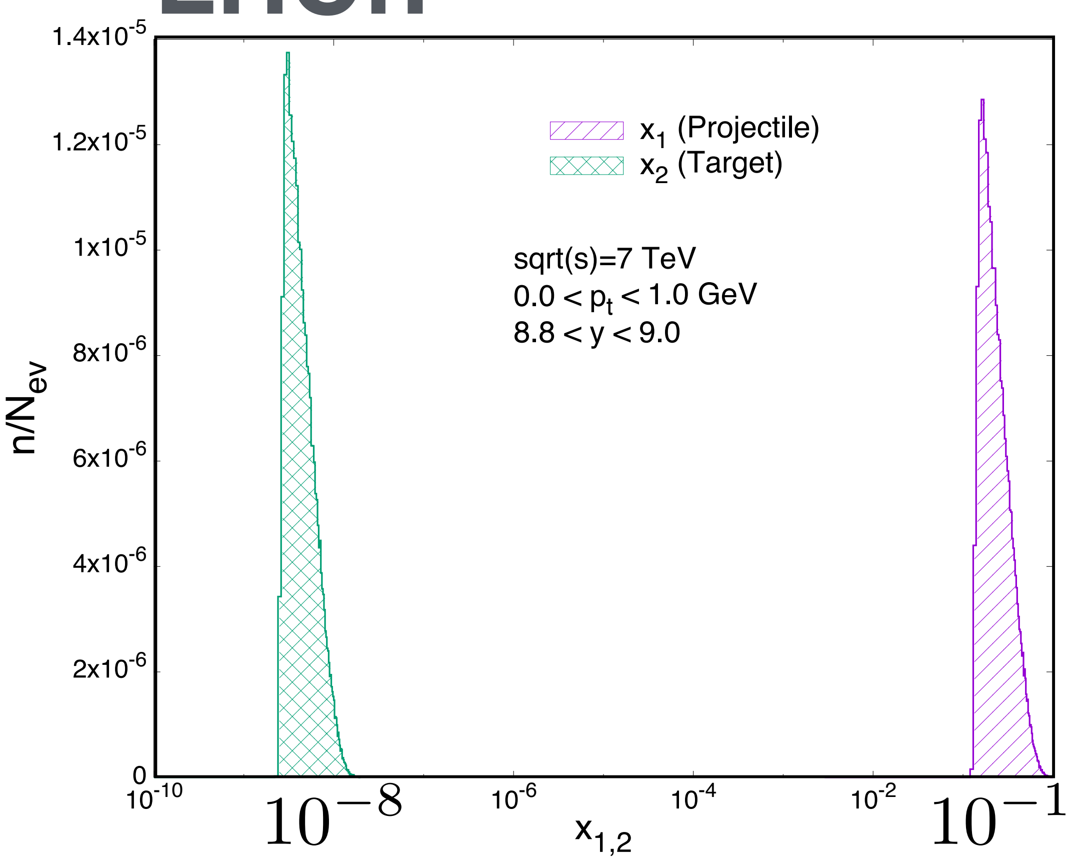
LHCf:



$$\sqrt{s} = 7\ \mathrm{TeV}$$

$$p_t \lesssim 1\ \mathrm{GeV}$$

$$8.8 \leq y \leq 9.0$$

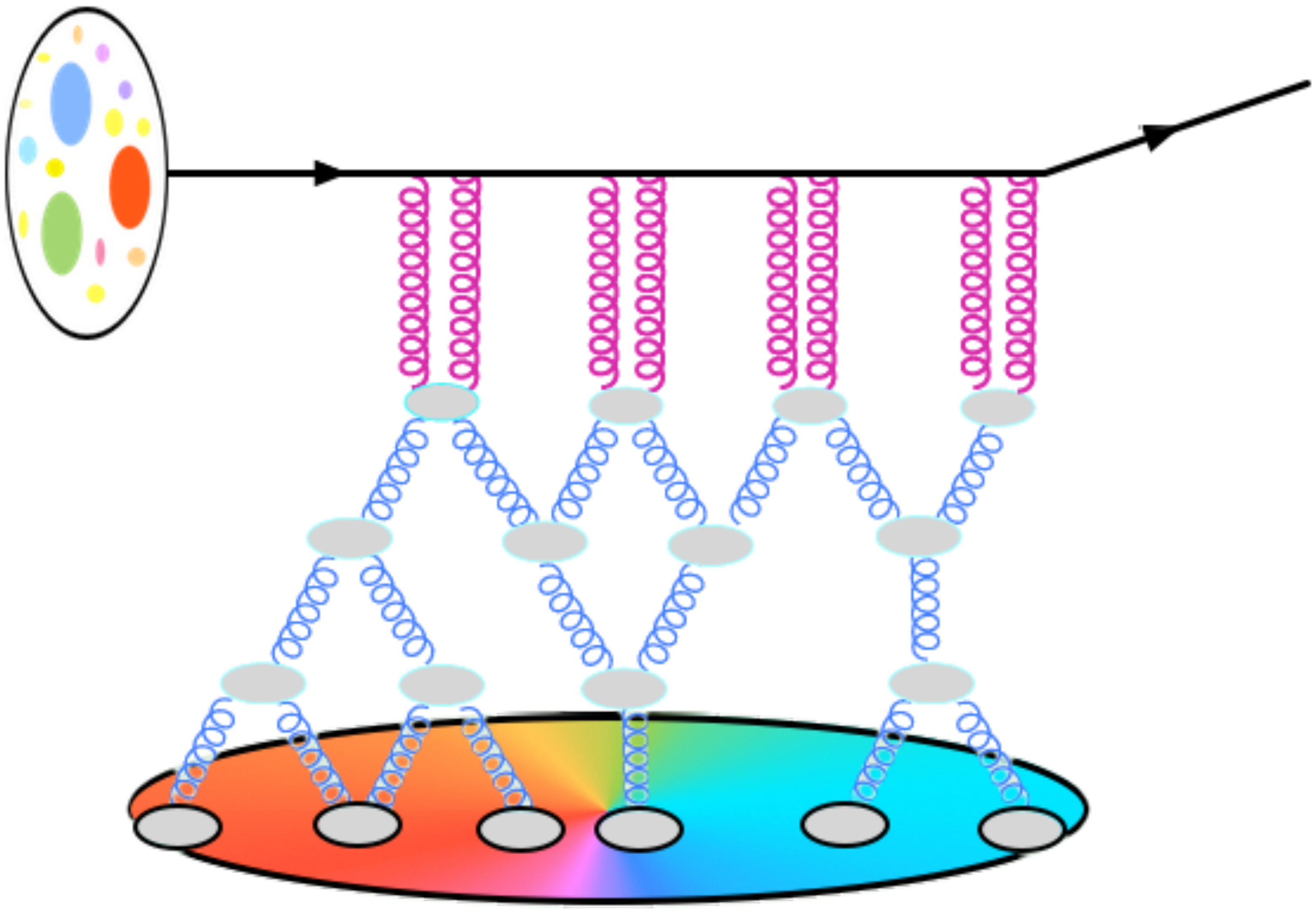
$$\boxed{\begin{array}{l} x_p \sim 10^{-1} \div 1 \\ x_t \sim 10^{-8} \div 10^{-9} \end{array}}$$

Smallest x values observed yet

Forward particle production in the Color Glass Condensate

- Hybrid formalism: the CGC interpretation of dilute-dense interactions (A. Dumitru, A. Hayashigaki and J. Jalilian-Marian, Nucl. Phys. A765 (2006) 464):

$$\frac{d\sigma}{dy d^2 k_\perp} \sim \mathrm{pdf}(x_p, \mu^2) \times \mathrm{uGD}(x_t, k_\perp^2)$$



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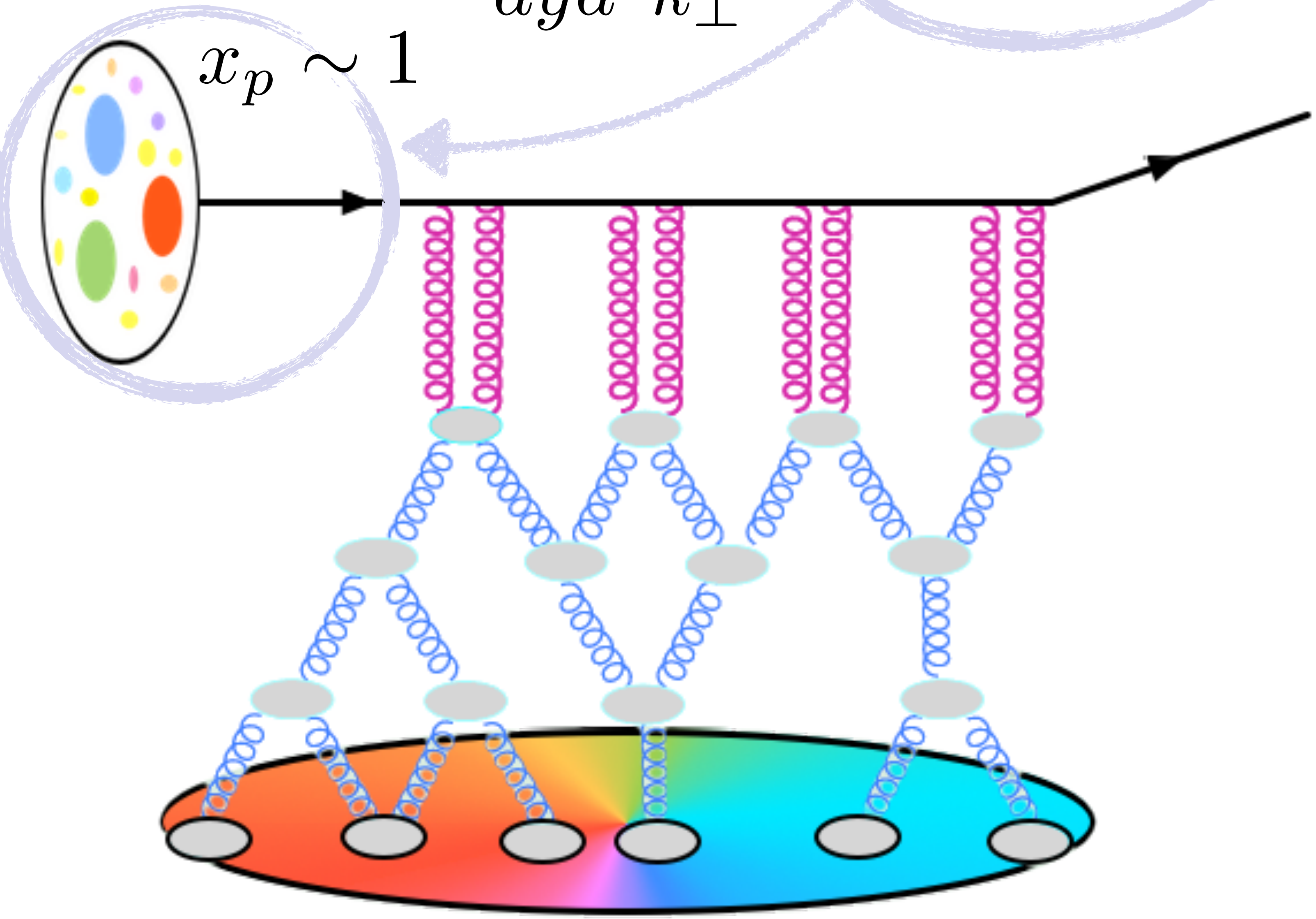

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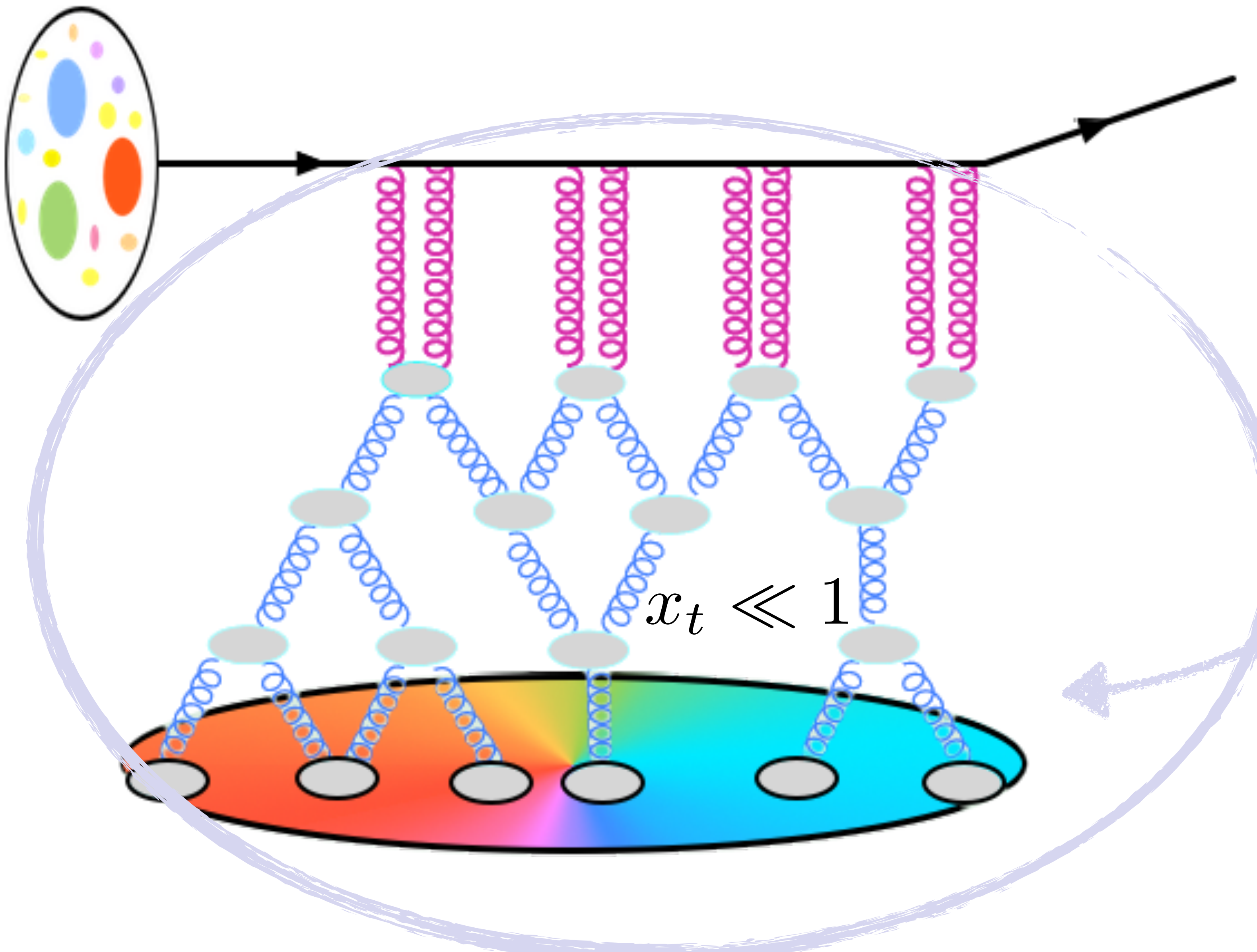

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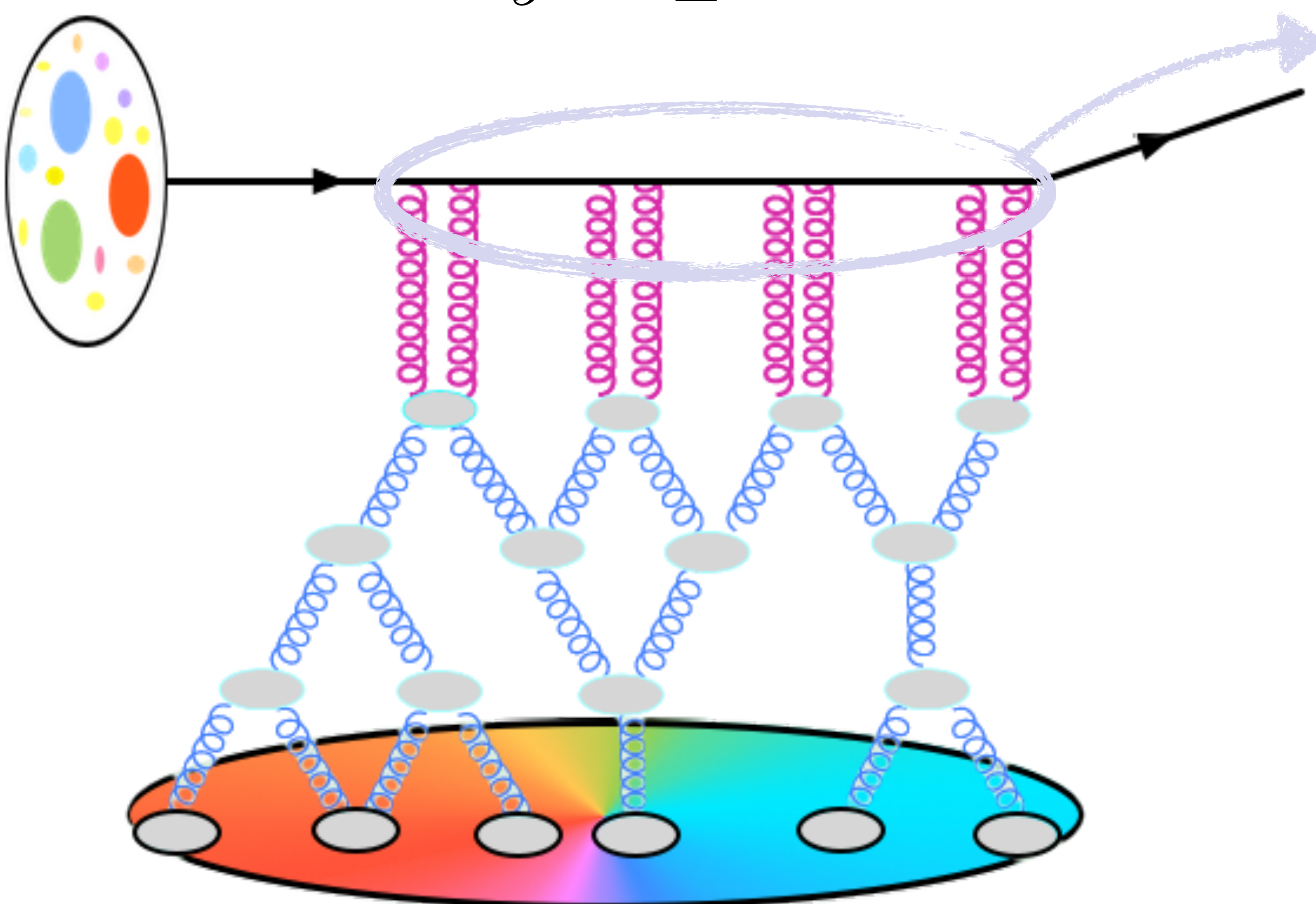

Multiple scattering:
All terms of order $g\mathcal{A}(x) \sim \mathcal{O}(1)$ must be resummed.

Strong color field: $\mathcal{A}(x) \sim \dfrac{1}{g}$

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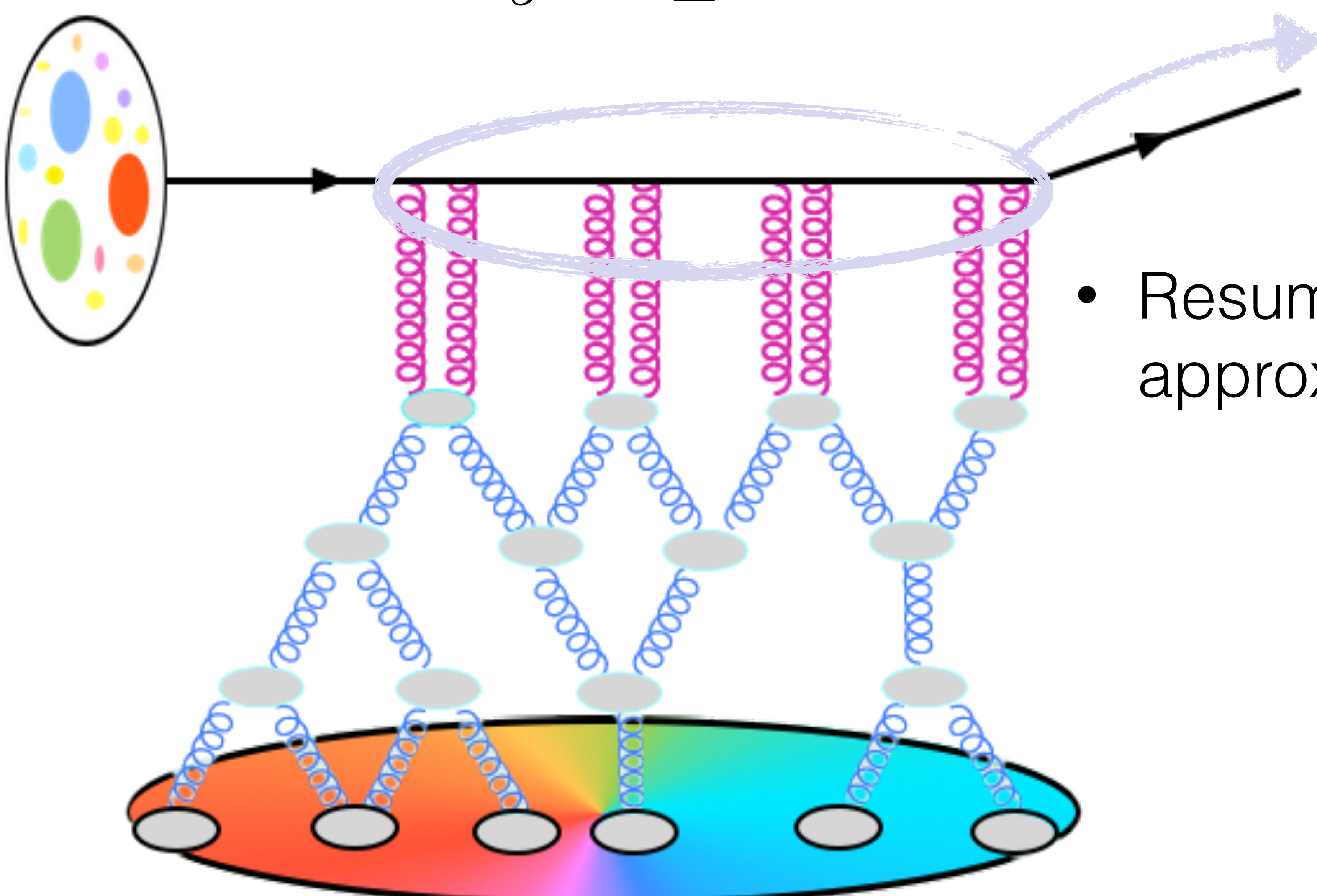

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- Resummation to all orders + eikonal approximation: Wilson line $U(z_\perp)$

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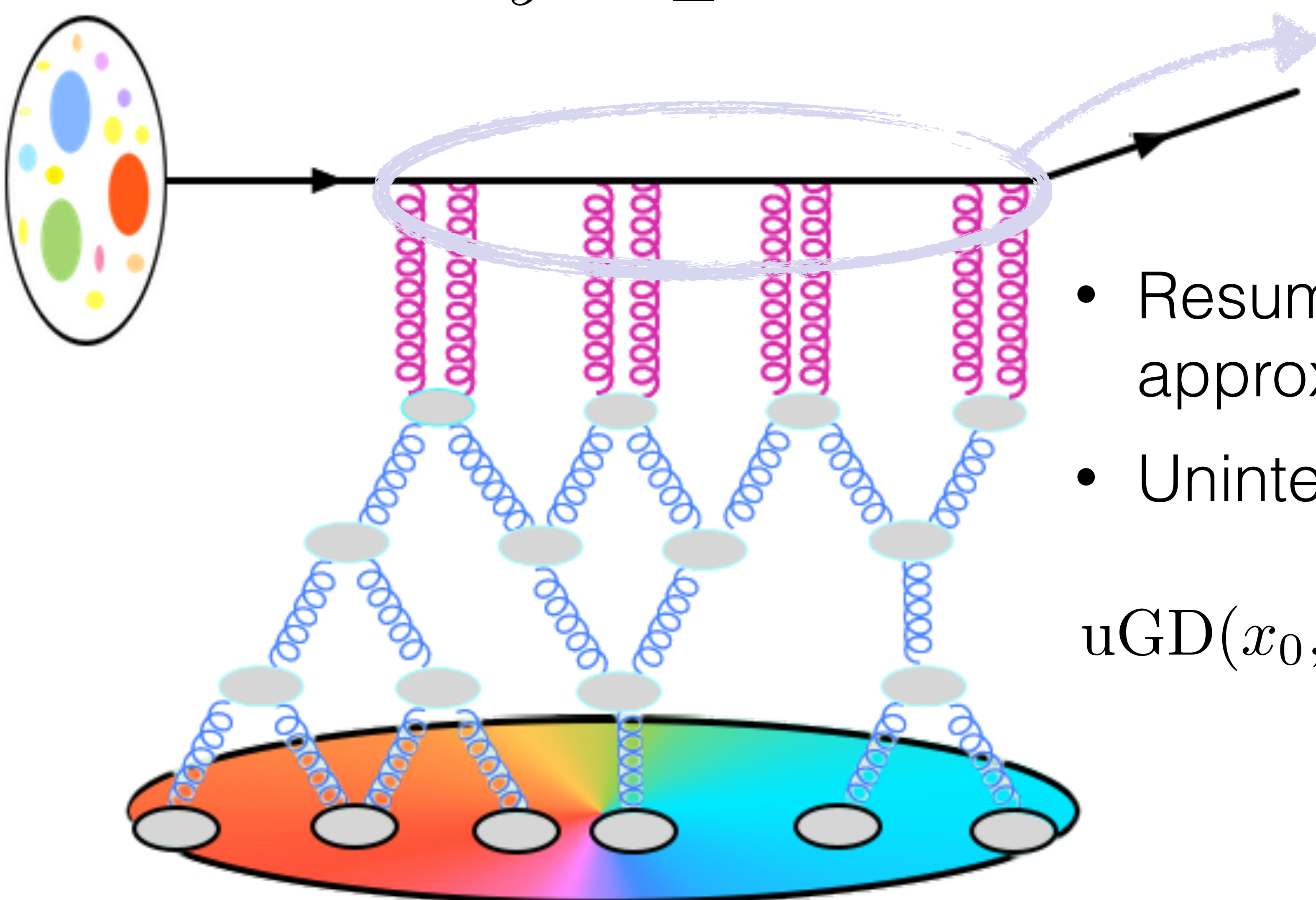

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- Resummation to all orders + eikonal approximation: Wilson line $U(z_\perp)$
- Unintegrated gluon distribution:

$$\mathrm{uGD}(x_0, k_t) = \mathrm{FT}\left[1 - \frac{1}{N_c}\langle \underbrace{\mathrm{tr}(UU^\dagger)}\rangle_{x_0}\right]$$

Dipole scattering amplitude

Strong color field: $\mathcal{A}(x) \sim \frac{1}{g}$

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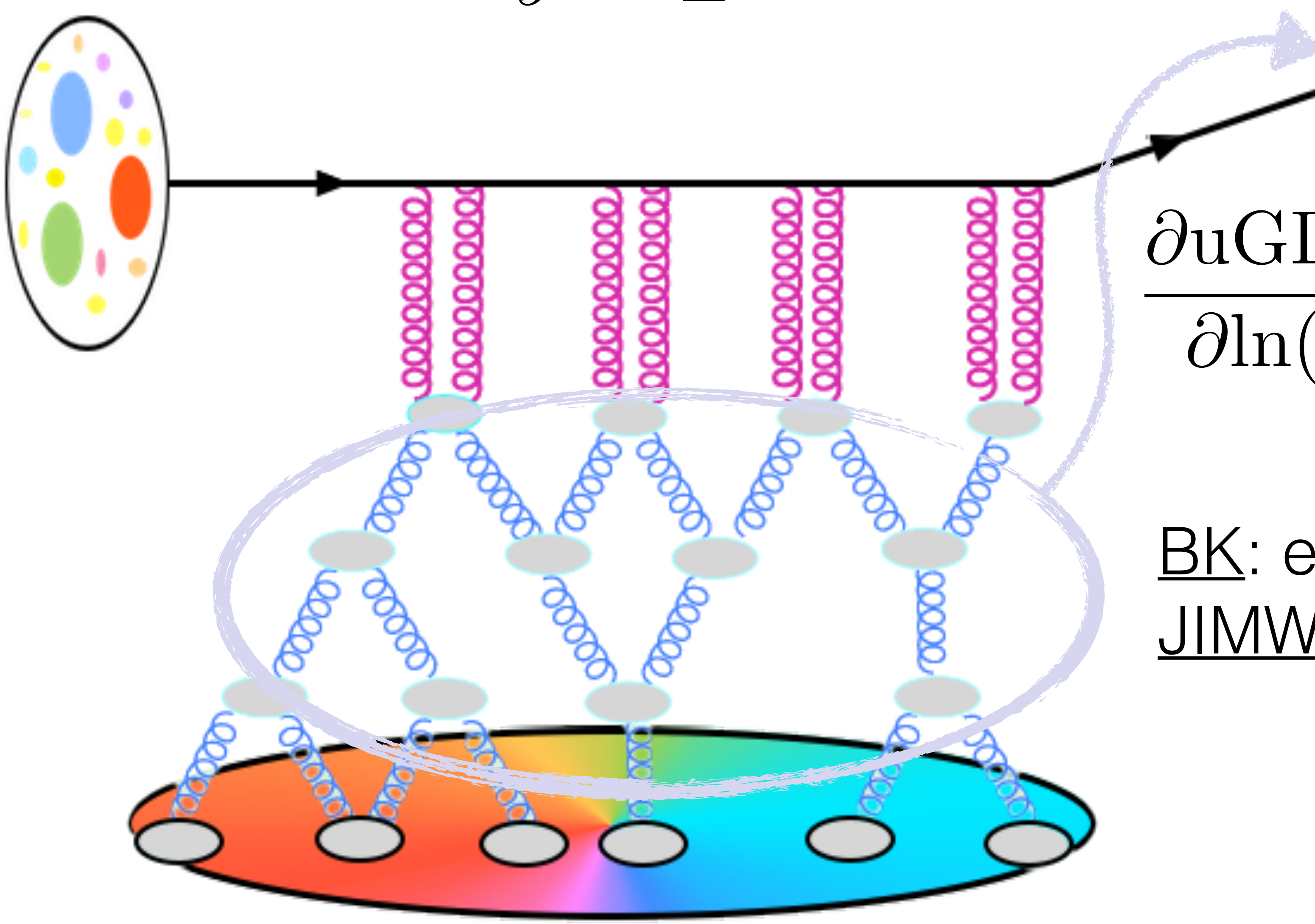

Non-linear small-x evolution:
BK-JIMWLK equations:

$$\frac{\partial \mathrm{uGD}(x, k_t)}{\partial \ln(x_0/x)} \sim \underbrace{\mathcal{K} \otimes \mathrm{uGD}}_{\text{Radiation}} - \underbrace{\mathrm{uGD}^2}_{\text{Recombination}}$$

BK: evolution of 2-point function
JIMWLK: (coupled) evolution of all n-point functions

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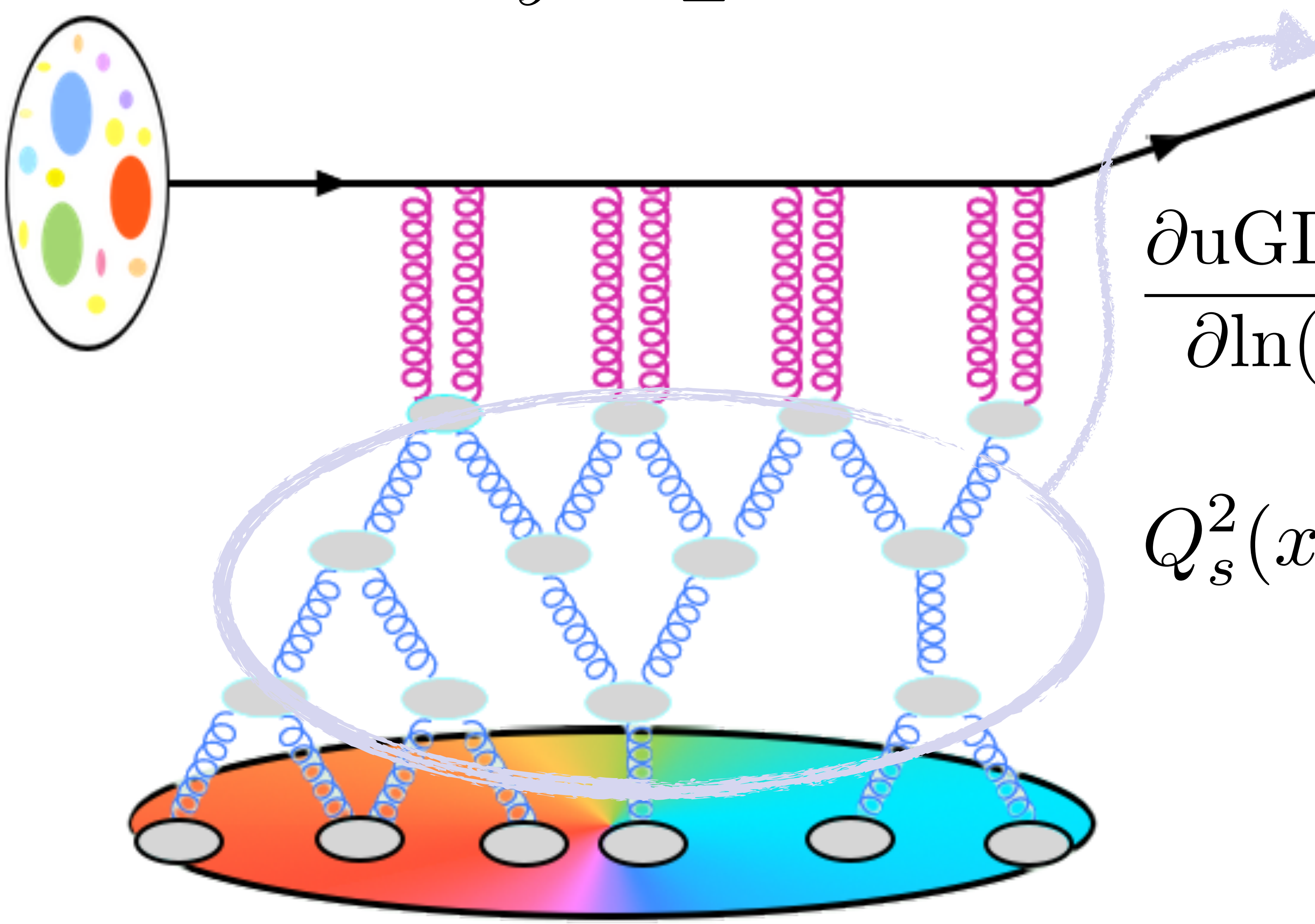

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$Q_s^2(x)$: **Signals when radiation and recombination terms become parametrically of the same order**

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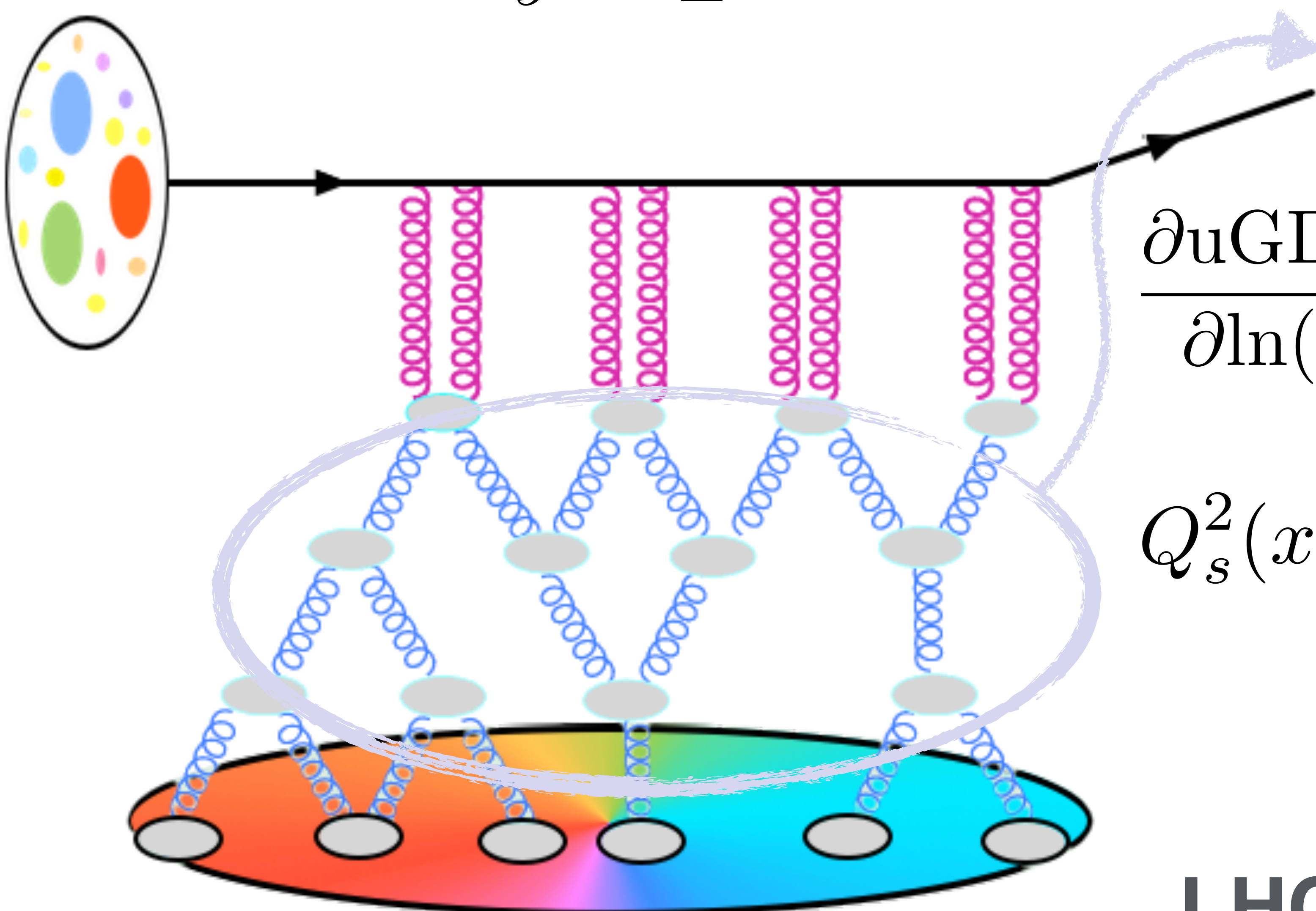

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$Q_s^2(x)$: **Signals when radiation and recombination terms become parametrically of the same order**

LHCf: (p-p) $\boxed{Q_s \gtrsim 1 \text{ GeV}}$

2. The Monte-Carlo event generator

Perturbative parton production: implementation of DHJ formula

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$$\frac{d\sigma^{h_1 h_2 \to (q/g) X}}{dy d^2 k_t} = \frac{K}{(2\pi)^2} \frac{\sigma_0}{2} x_p f_{(q/g)/h_1}(x_p, \mu^2) N_{(F/A),h_2}(x_t, k_t^2)$$

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  - Proton PDF: CTEQ6 LO set (J. Pumplin et. al., JHEP 07 (2002) 012)
  - Default factorization scale:

LHCf: $\mu = \max\{k_t, Q_s\}$

RHIC (forward): $Q_s < 1\,\mathrm{GeV} \longrightarrow \mu = 1\,\mathrm{GeV}$

(LHCf data description insensitive to cutoff)

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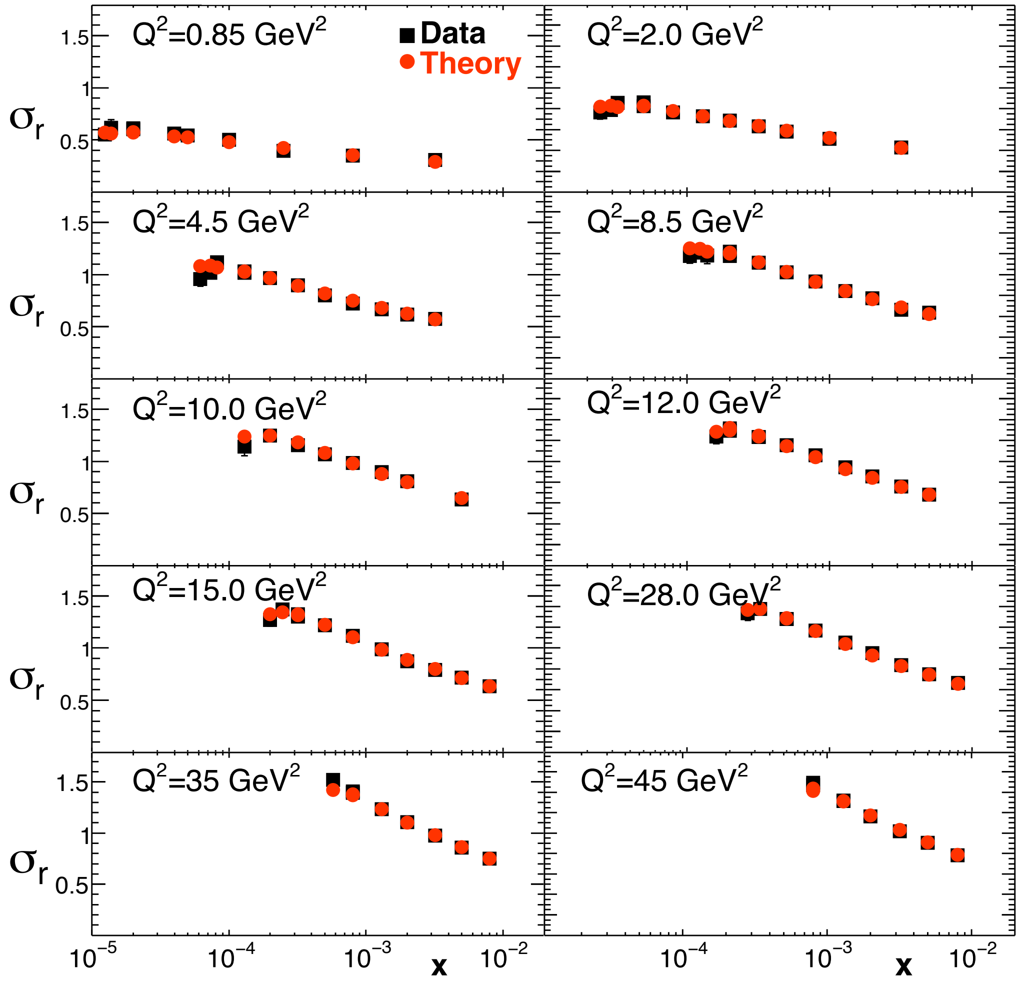
- uGD's: Fourier transforms of dipole scattering amplitudes.

$$N_{F(A)}(x, k_t) = \int d^2\mathbf{r}\, e^{-i\mathbf{k_t}\cdot\mathbf{r}} \left[1 - \mathcal{N}_{F(A)}(x, r)\right].$$

- Small-x evolution: We take parametrization of $\mathcal{N}_{F(A)}(x, r)$ from the *AAMQS* fits to data on the structure functions measured in e+p scattering at HERA:

rc-BK evolution

Fit with only light quarks



J. L. Albacete, N. Armesto, J. G. Milhano and C. A. Salgado, Phys. Rev. D80 (2009) 034031.
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rc-BK evolution

- Initial conditions for evolution:

$$\mathcal{N}_F(x_0, r) = 1 - \exp\left[-\frac{\left(r^2 Q_{s0}^2\right)^{\gamma}}{4}\log\left(\frac{1}{\Lambda r} + e\right)\right]$$

$$x_0 = 10^{-2} \quad \gamma = 1.101 \quad Q_{s0}^2 = 0.157\,\mathrm{GeV}^2$$

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- uGD's for nuclear target:

$$Q_{s0,nucleus}^2 = \underset{\substack{\uparrow \\ \textit{Oomph}\text{ factor}}}{A^{1/3}} Q_{s0,proton}^2$$

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  - Implicit integration in impact parameter $\vec{b}$: $\sigma_0/2$

Free fit parameter of *AAMQS* fits:

$$\boxed{\frac{\sigma_0}{2} = 16.5 \text{ mb}}$$

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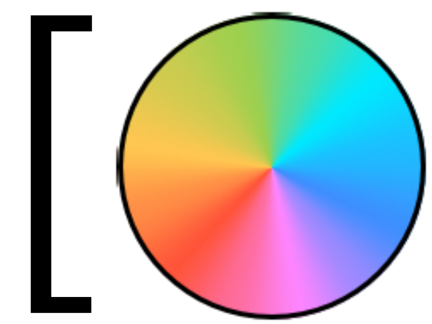


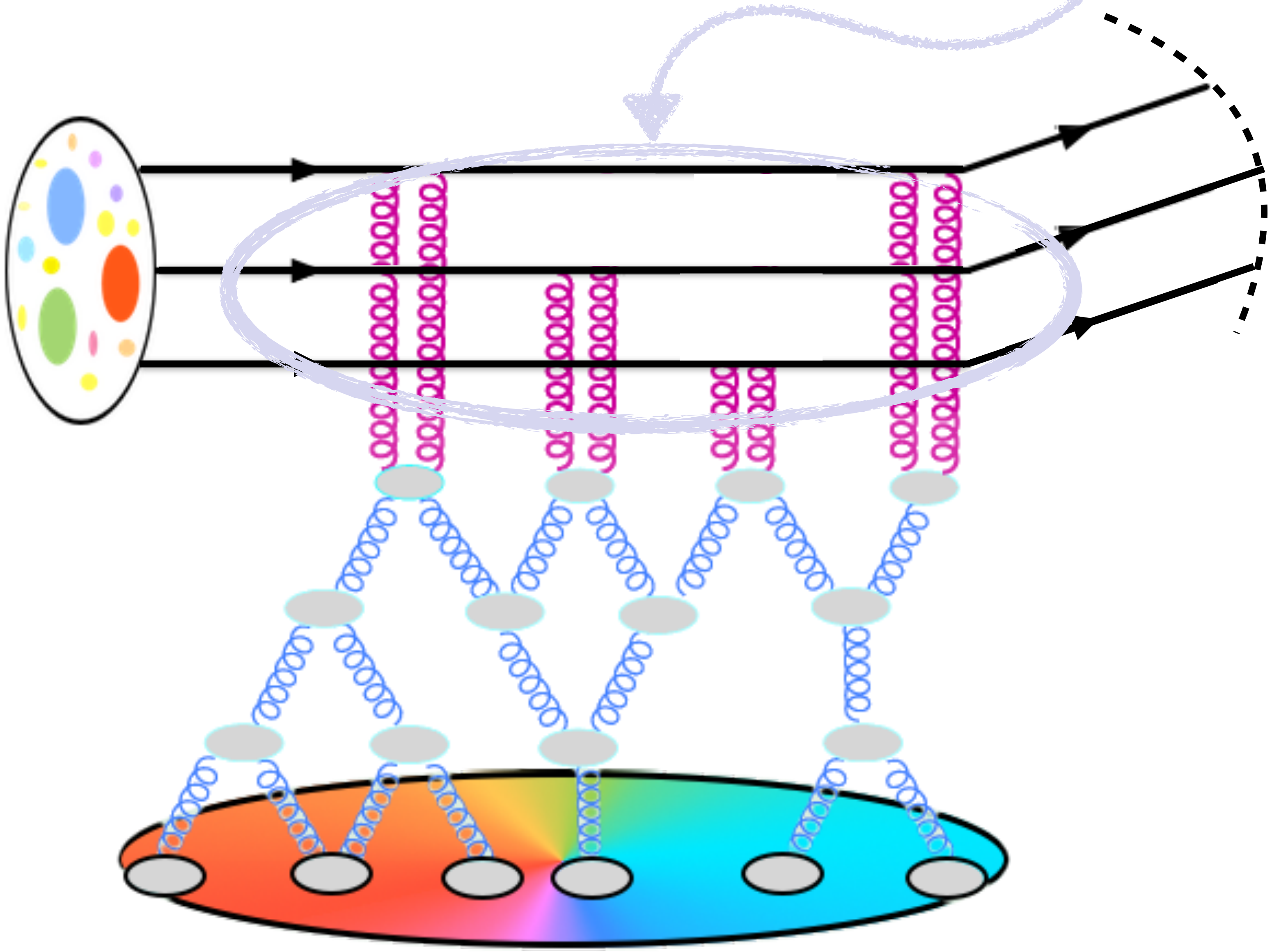
- K-factor: not the result of any calculation. May account for:
  - Higher order corrections
  - Non-perturbative effects
  - (…)

Multiple scattering: eikonal model

- Our approach:

  Monte-Carlo implementation of

$$\text{Hybrid formalism} + \underline{\text{Multiple parton scattering}}$$



Multiple scattering: eikonal model

- Number of **independent** hard scatterings according to Poisson probability distribution of mean n, where:

$$n(b, s) = T_{\mathrm{pp}}(b)\sigma_{\mathrm{DHJ}}(s)$$

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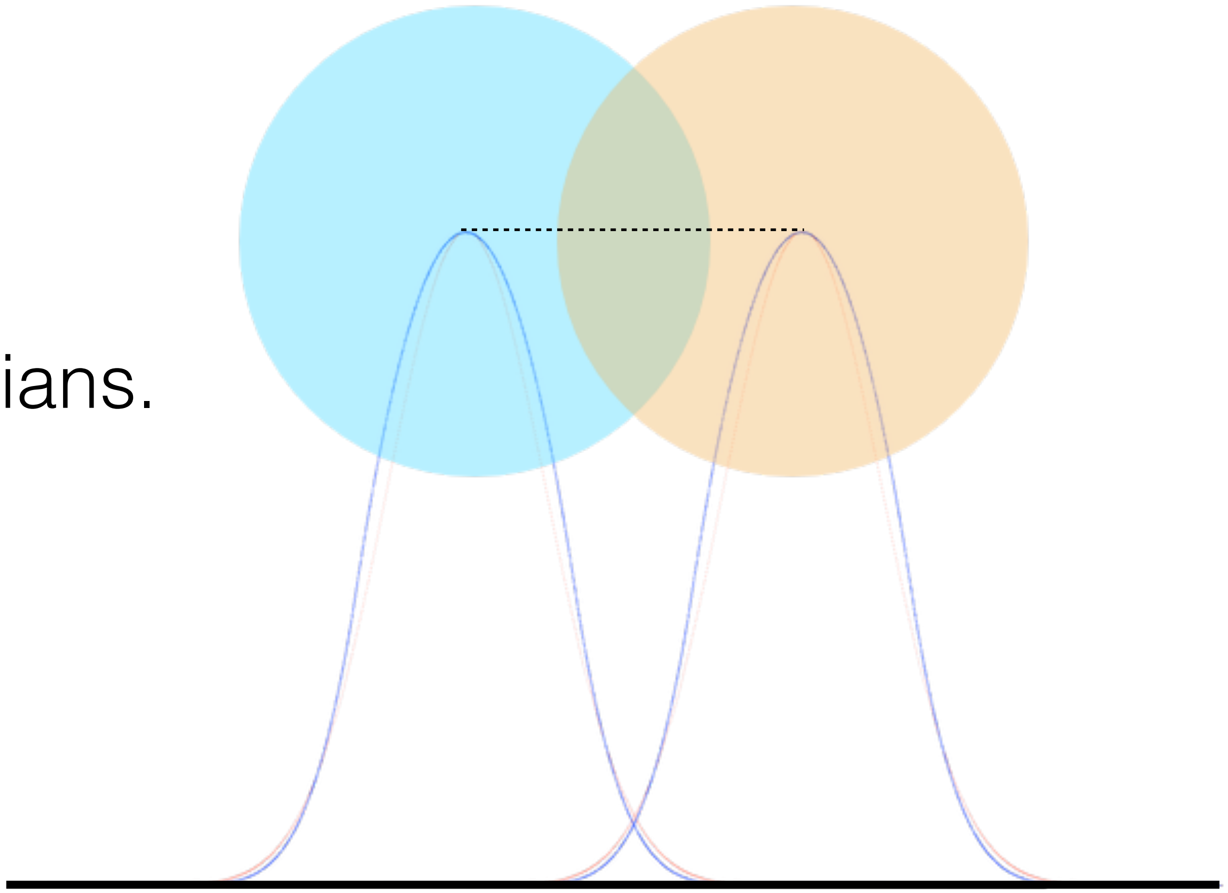
$$n(b, s) = T_{\rm pp}(b)\sigma_{\rm DHJ}(s)$$

- b randomly generated between 0 and b_{max}:

$$b_{max} = \sqrt{\frac{\sigma_{nd}}{\pi}}$$

- Spatial overlap: convolution of two Gaussians.

$$T_{\rm pp}(b) = \frac{1}{4\pi B}\exp\left(-\frac{b^2}{4B}\right)$$



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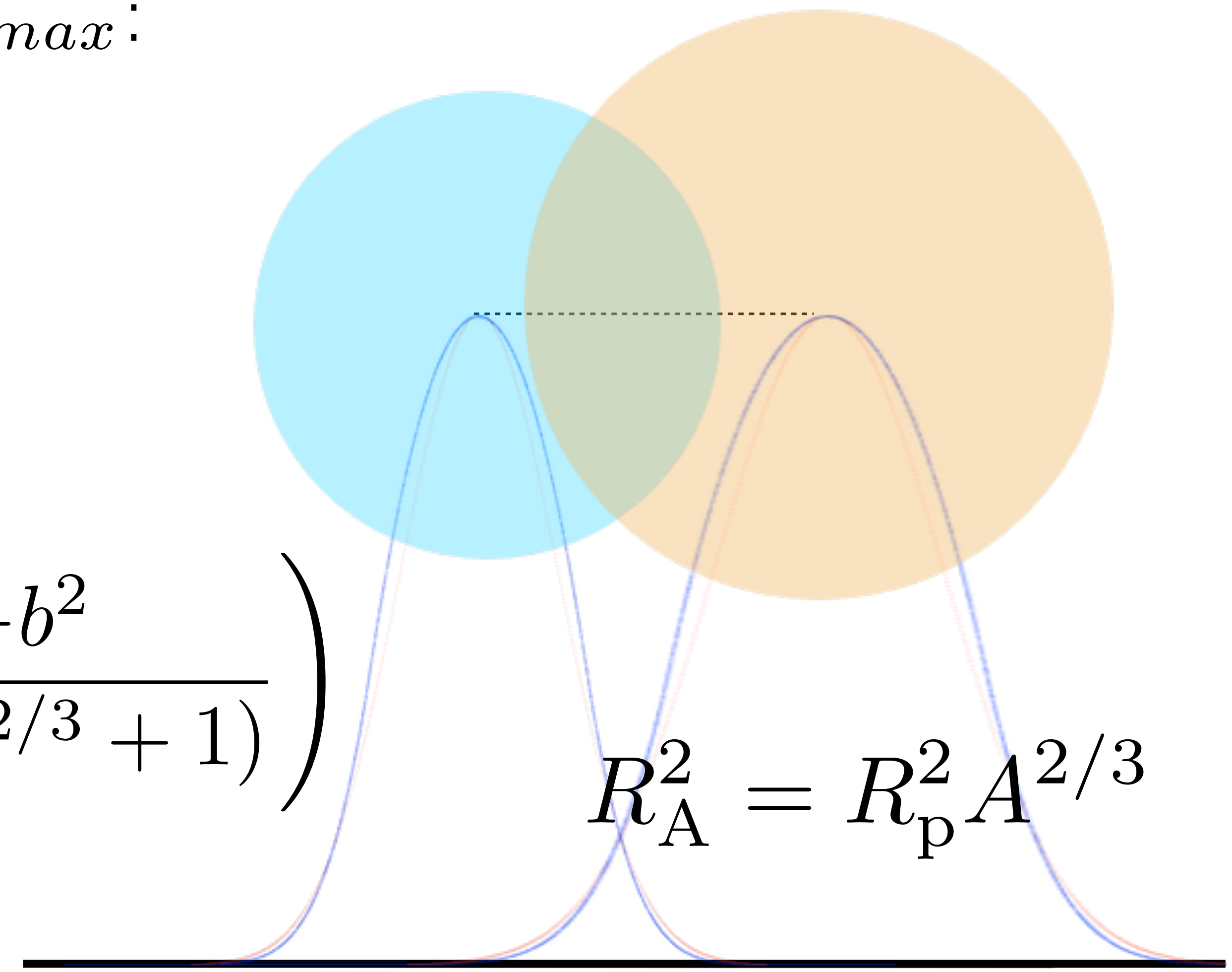
- For a nuclear target of mass number A:

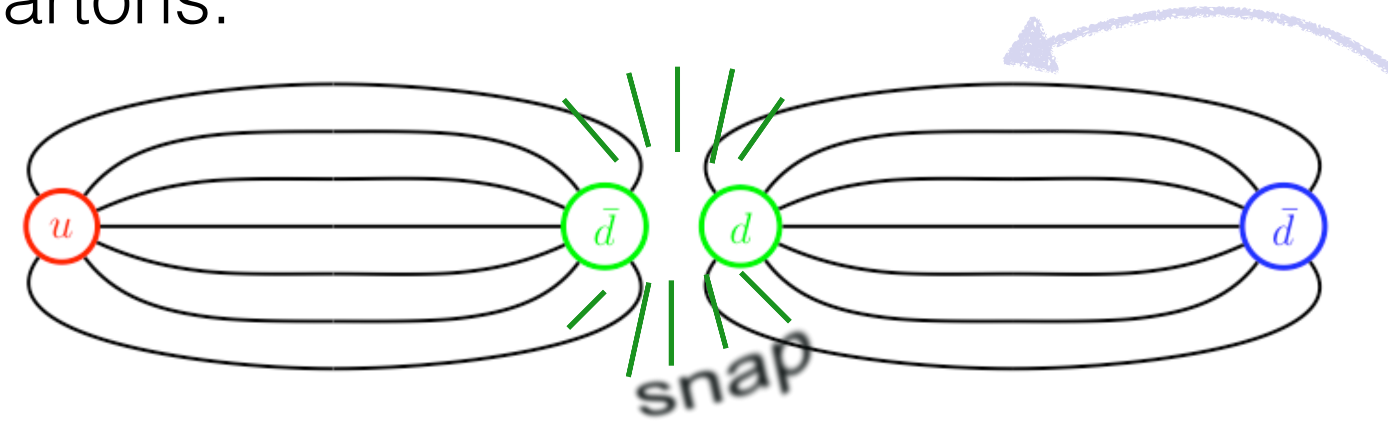
$$T_{\mathrm{pA}}(b) = \frac{1}{\pi R_{\mathrm{p}}^2(A^{2/3}+1)} \exp\left(\frac{-b^2}{R_{\mathrm{p}}^2(A^{2/3}+1)}\right)$$



$$R_{\mathrm{A}}^2 = R_{\mathrm{p}}^2 A^{2/3}$$

Hadronization: Lund fragmentation model

- Simple but powerful picture of hadron production based on the breaking of strings between partons:



String tension: κ

- Probability of string breaking by quark pair with $m_\perp^2 = m_q^2 + p_{\perp q}^2$:

$$\mathrm{Prob}(m_q^2, p_{\perp q}^2) \;\propto\; \exp\left(\frac{-\pi m_q^2}{\kappa}\right) \exp\left(\frac{-\pi p_{\perp q}^2}{\kappa}\right)$$

As implemented in: **PYTHIA 8**

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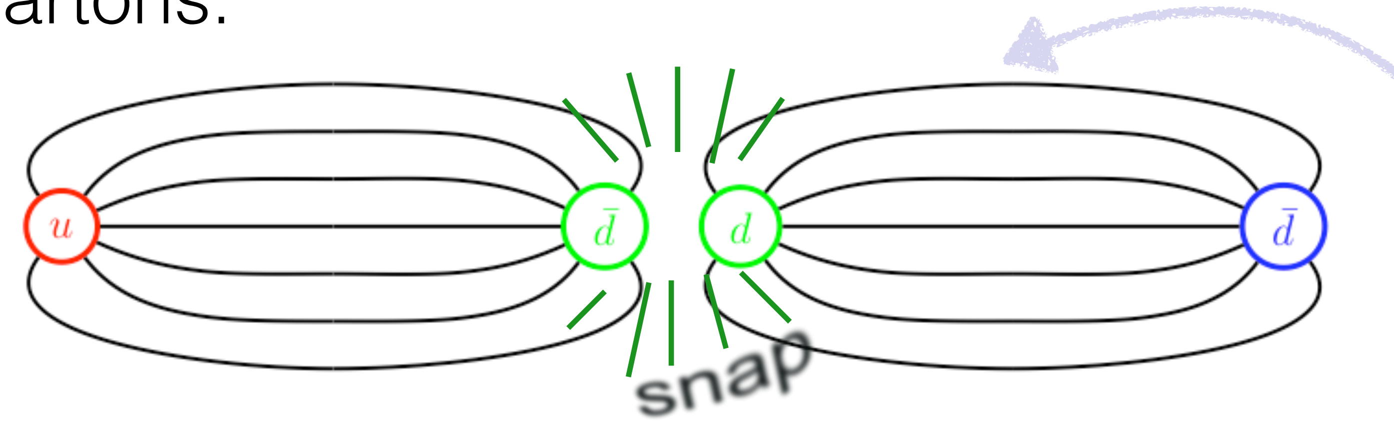

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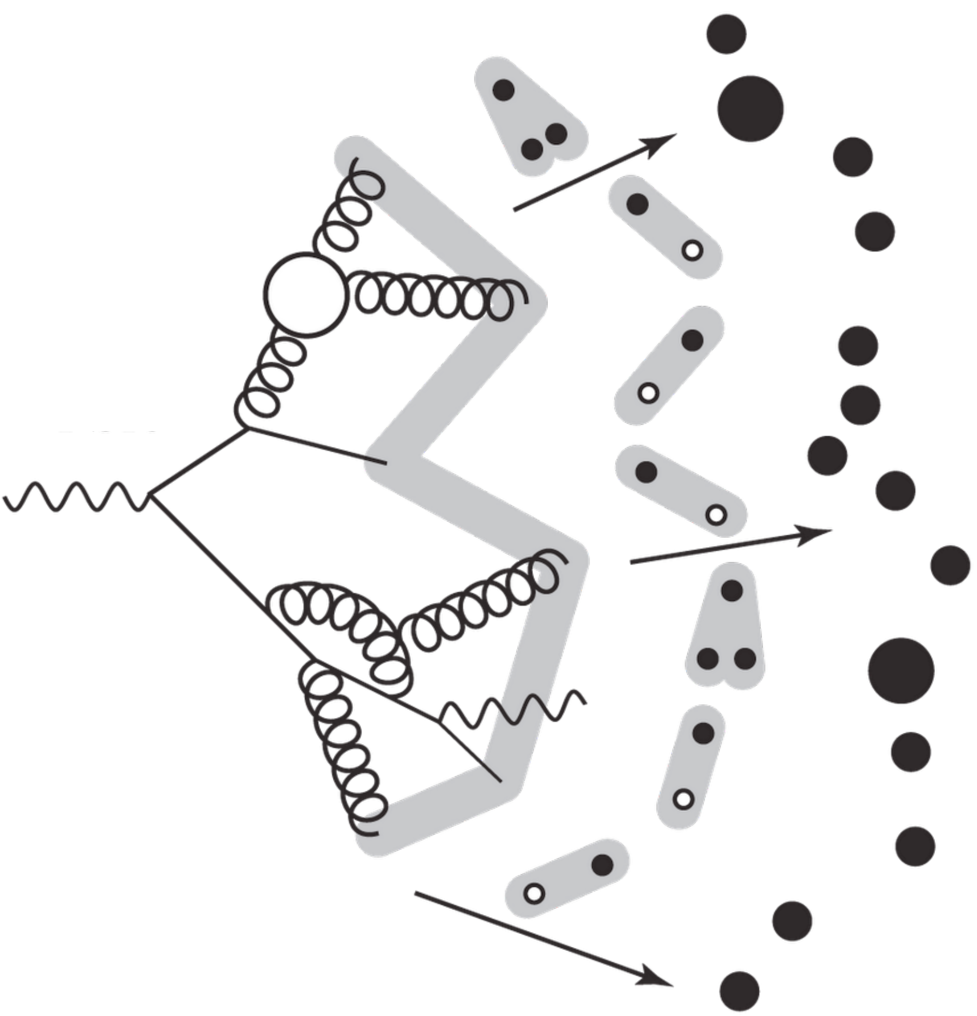


$$\mathrm{Prob}(m_q^2, p_{\perp q}^2) \;\propto\; \exp\left(\frac{-\pi m_q^2}{\kappa}\right) \exp\left(\frac{-\pi p_{\perp q}^2}{\kappa}\right)$$

- Lund fragmentation function:

$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{b\,(m_h^2 + p_{\perp h}^2)}{z}\right)$$

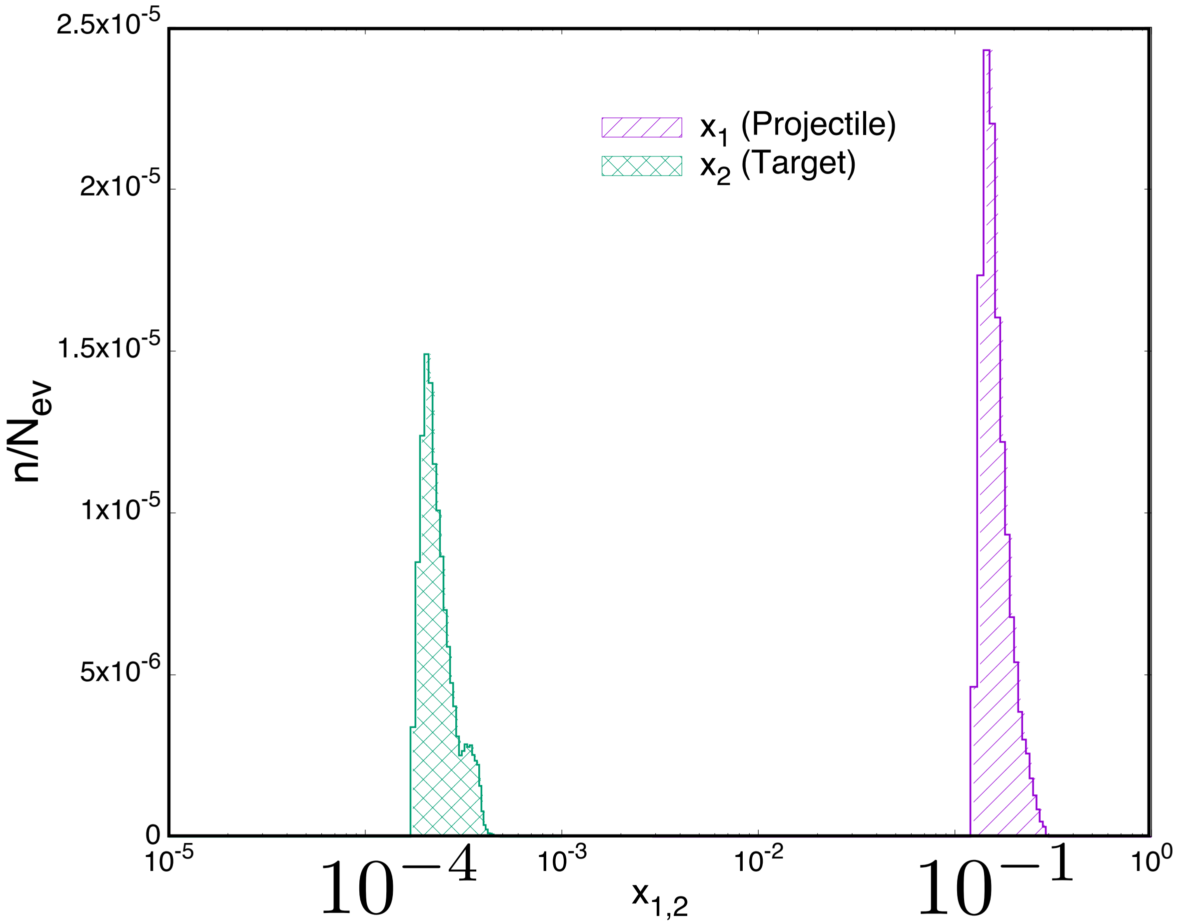
As implemented in: **PYTHIA 8**

3. Results

RHIC: d-Au @ 200 GeV

- Forward spectra observed at RHIC allows for a description in terms of CGC:

RHIC:

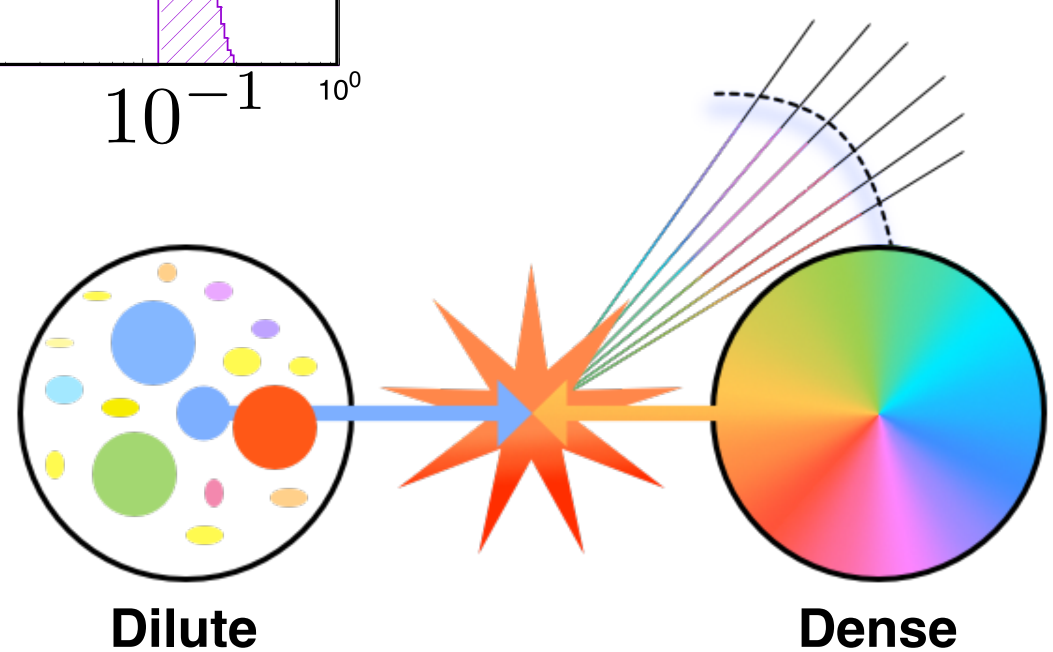


$$\sqrt{s} = 200\ \text{GeV}$$

$$1 < p_t < 2\ \text{GeV}$$

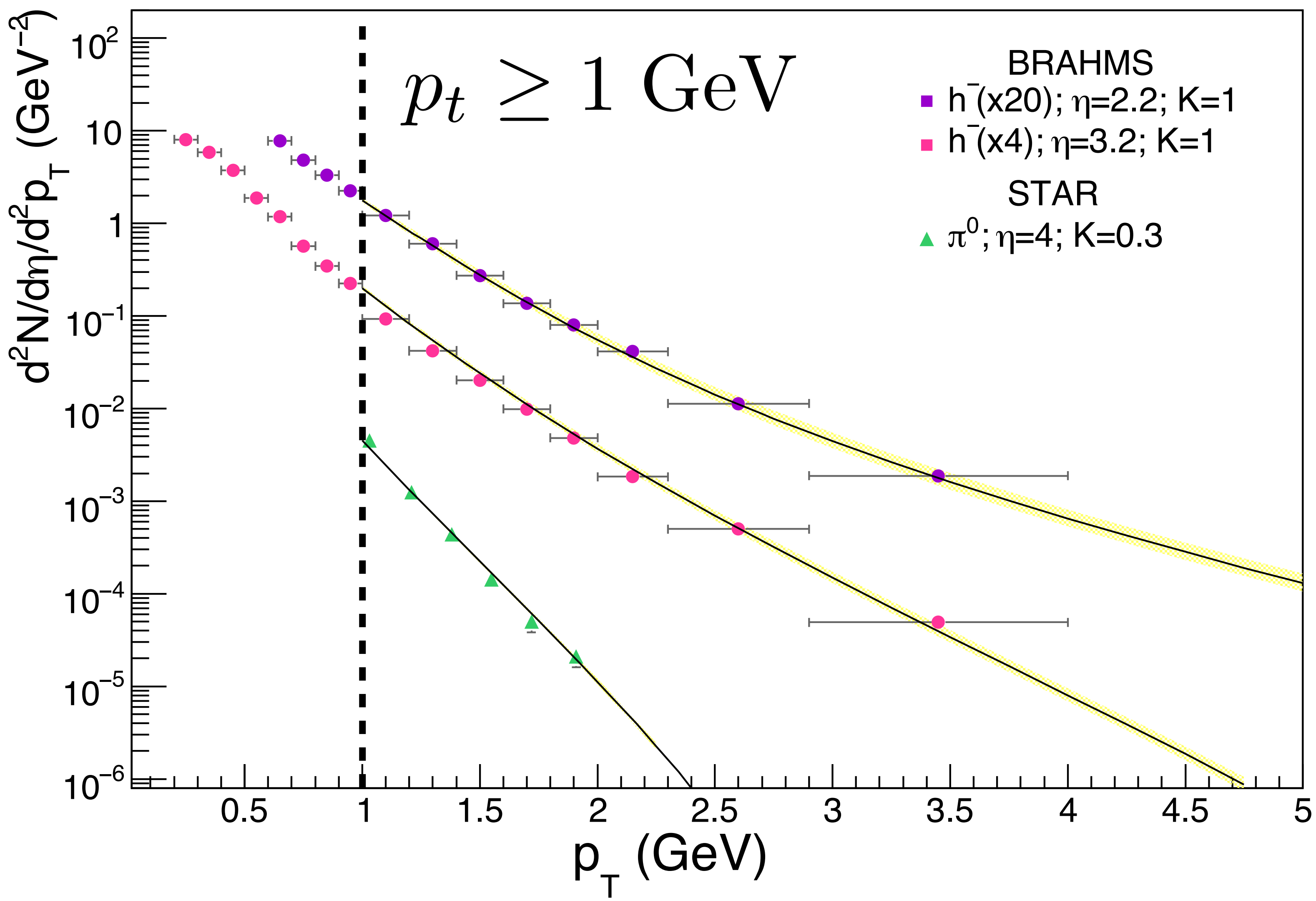
$$3.2 < y < 3.4$$

$$\boxed{\begin{array}{l} x_p \sim 10^{-1} \\ x_t \sim 10^{-4} \end{array}}$$



- Previous approaches:

$$\frac{d\sigma^{hadrons}}{d^2k_\perp dy} = \frac{d\sigma^{partons}_{\mathrm{DHJ}}}{d^2k_\perp dy} \otimes D_{h/p}$$



Albacete, Javier L. et al. Phys.Lett. B687 (2010) 174-179 arXiv:1001.1378 [hep-ph]

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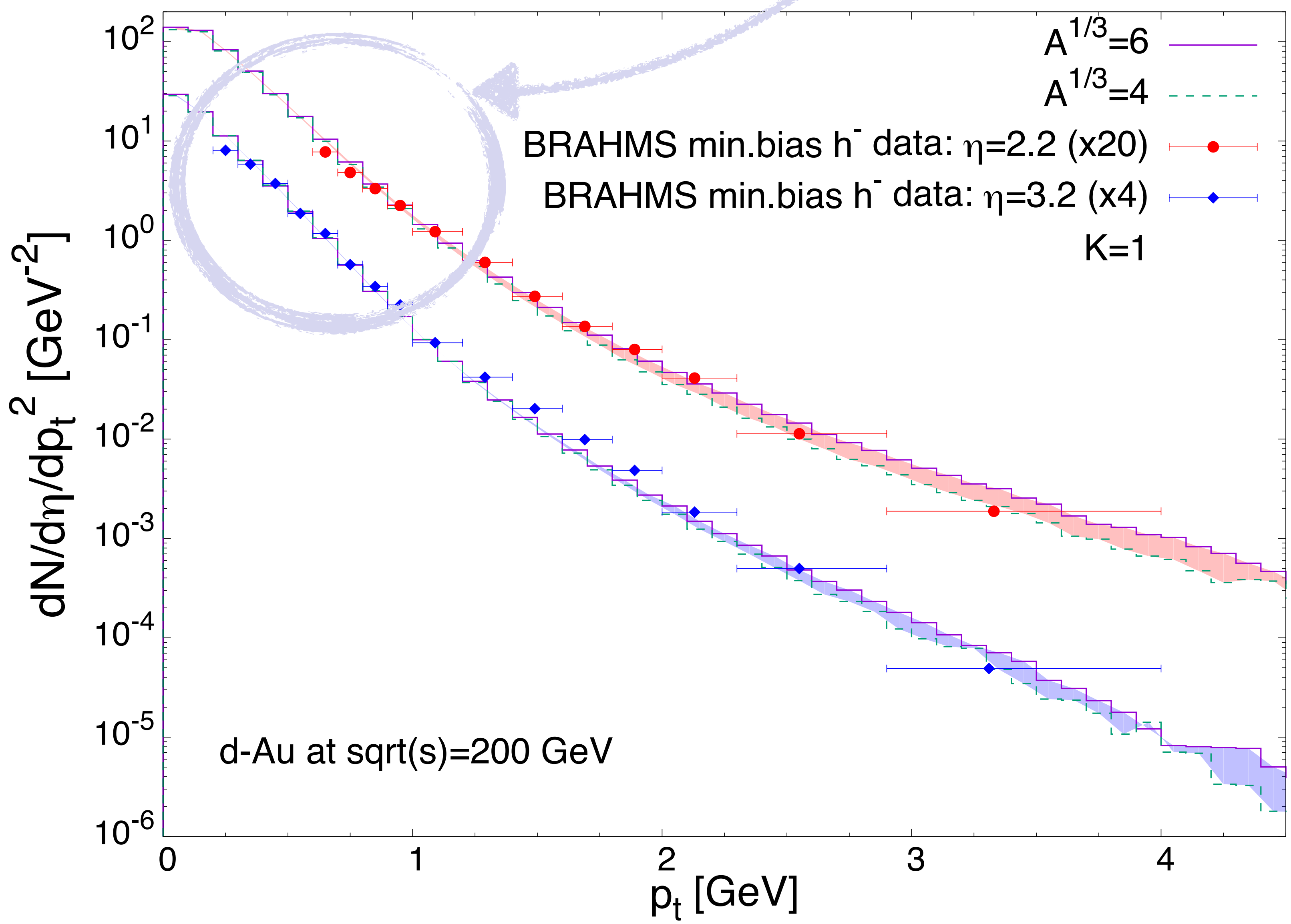
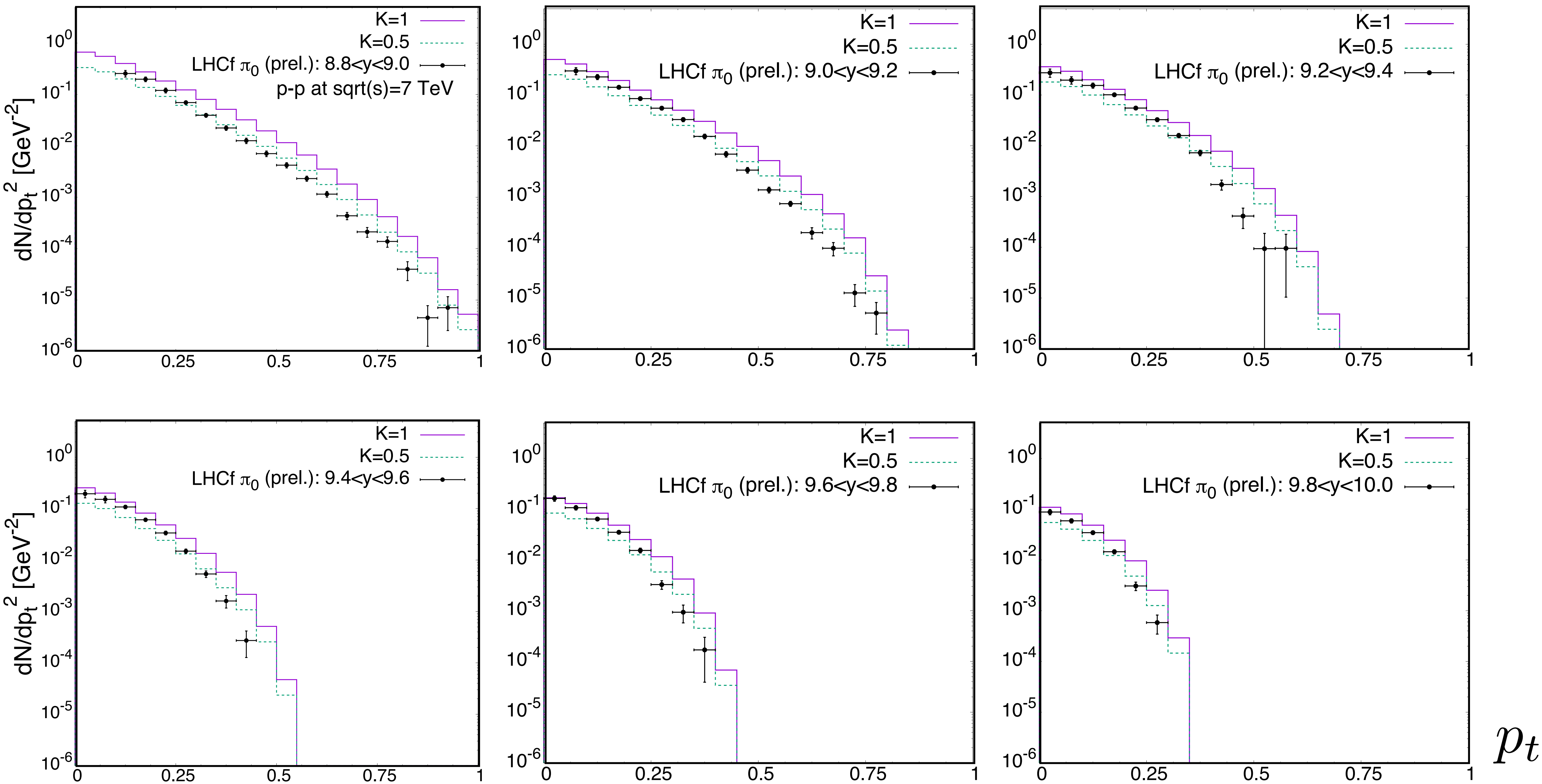
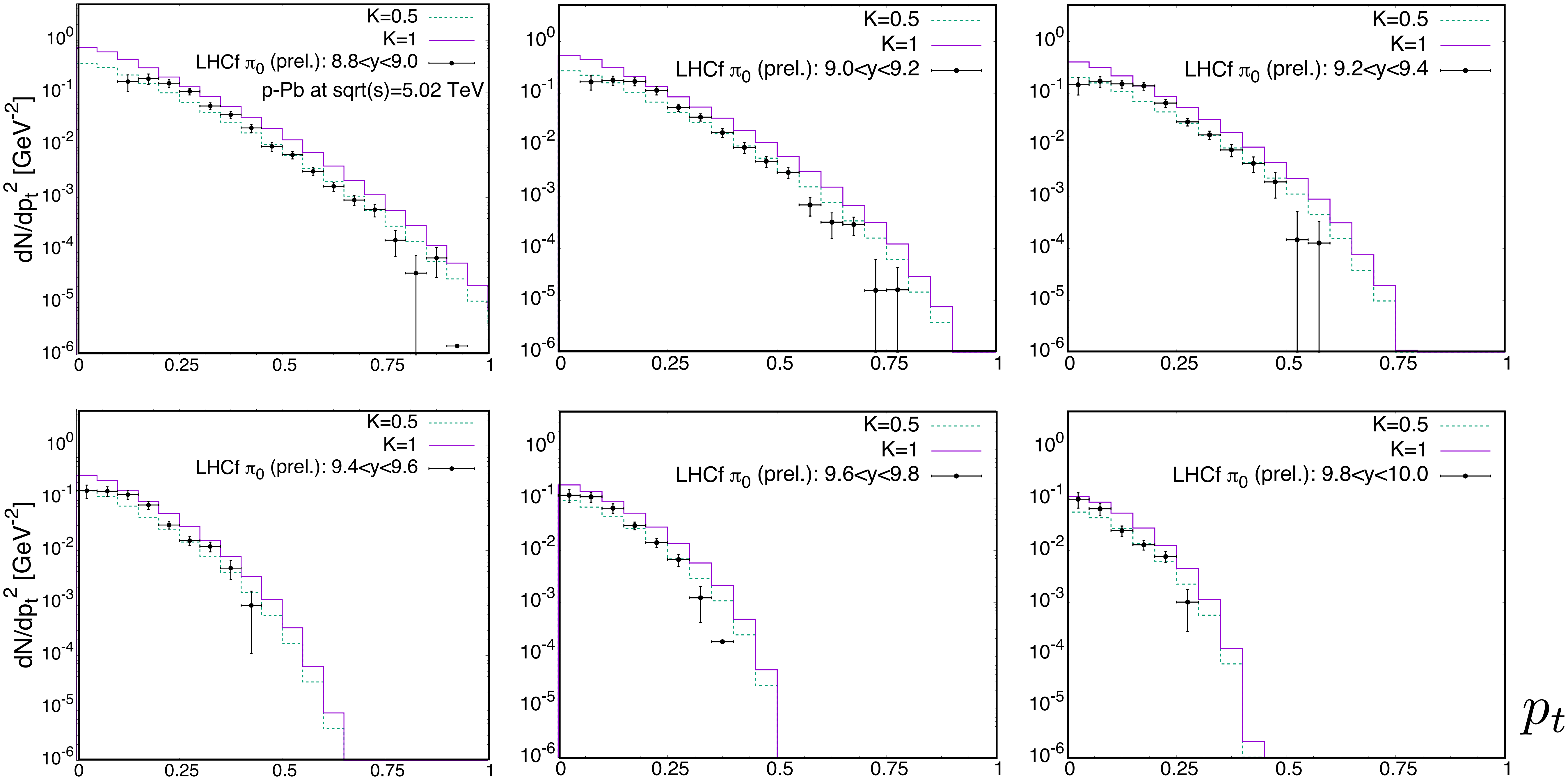


As implemented in: **PYTHIA 8**

LHCf: p-p @ 7 TeV

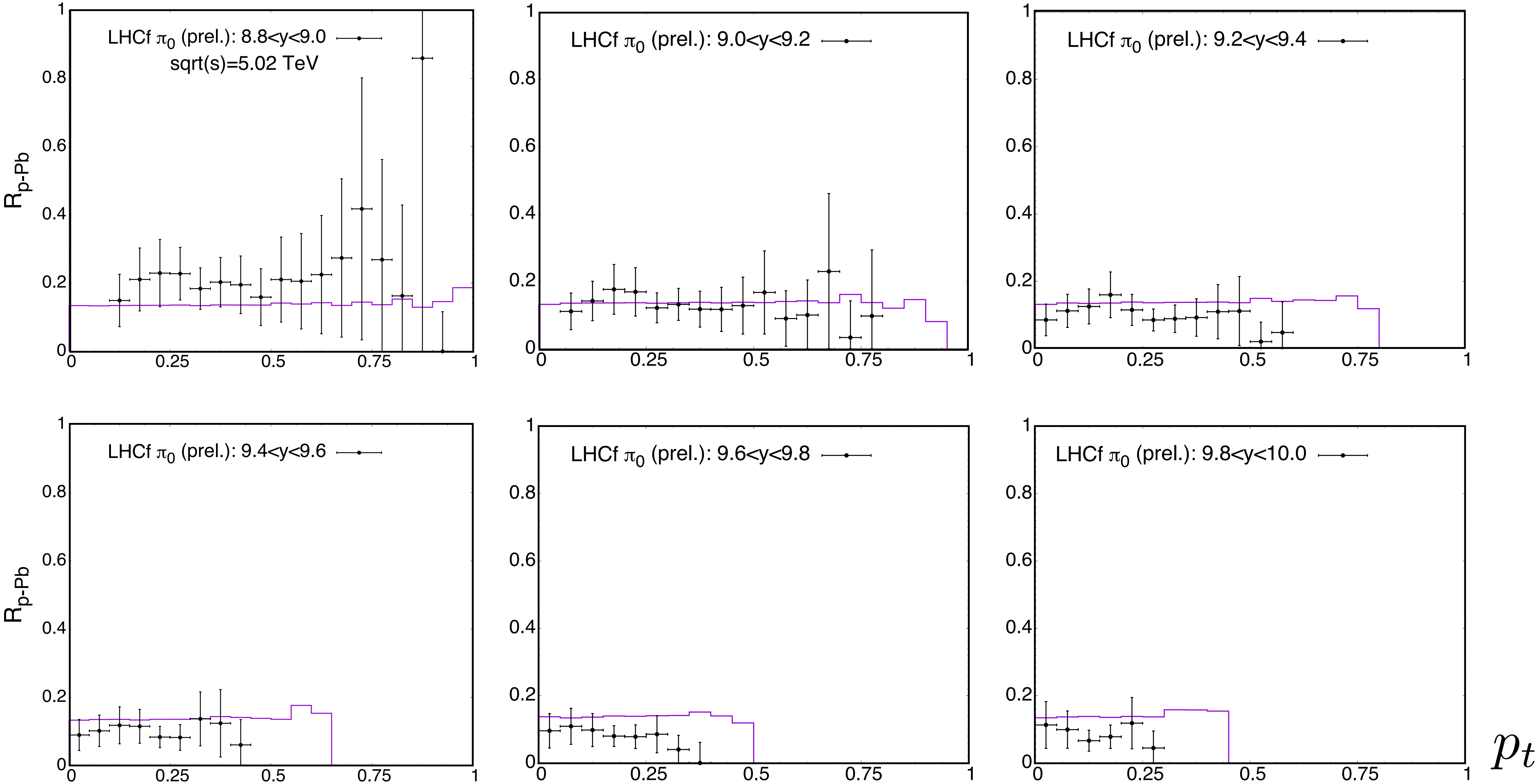


- Increment of evolution rapidity with respect to RHIC: $\boxed{\Delta Y \sim \ln\left(\frac{x_0}{x}\right) \sim 14}$
- Only difference with respect to RHIC set: **dynamical evolution of uGD's according to rcBK equation**.



- Similar situation that in the proton-proton case.
- Plenty of room for improvement in the proton-nucleus implementation.
- Low momentum region well described.

LHCf: nuclear modification factor $R_{\text{p-Pb}}$ @ 5.02 TeV



$$R^{\pi^0}_{\text{p-Pb}} \equiv \frac{1}{\langle N_{coll}\rangle} \frac{dN^{\text{pPb}\to\pi^0 X}/dyd^2p_t}{dN^{\text{pp}\to\pi^0 X}/dyd^2p_t}$$

- Approximate constant flat suppression of: $0.15 \approx 1/\langle N_{coll}\rangle$

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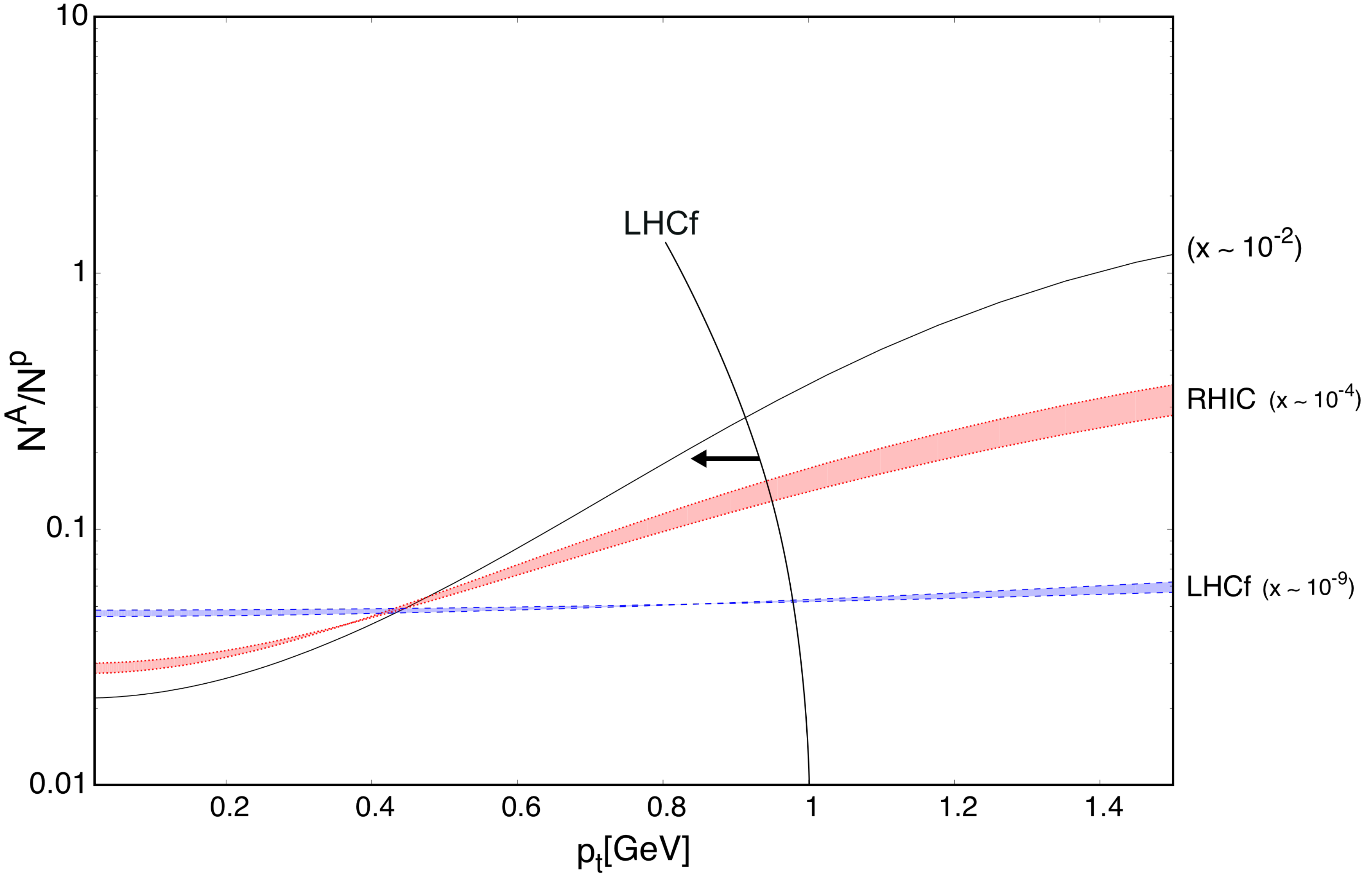
- Approximate constant flat suppression of: $0.15 \approx 1/\langle N_{coll}\rangle$

- This behavior can be understood as a direct consequence of the behavior of the ratios of the uGD's:



4. Conclusions, future prospects

Conclusions, future prospects

- We achieve a good description of single inclusive spectra of charged particles and neutral pions at RHIC and the LHC respectively, and nuclear modification factors for proton-lead collisions at the LHC.

  $\hookrightarrow$ This adds evidence to the idea that the **main properties of forward data are dominated by the saturation effects** encoded in the unintegrated gluon distribution of the target

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- There is still a **lot of room for improvement!** (NLO corrections, proper Monte-carlo implementation of proton-nucleus, etc.)

* Back-up

Perturbative parton production: implementation of DHJ formula

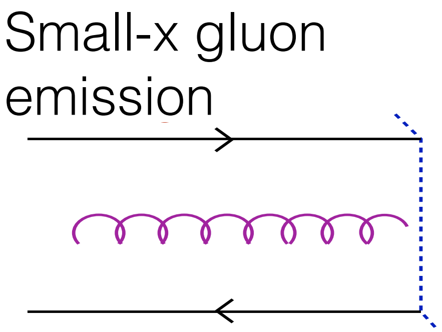
- Degree of accuracy of our approach:
  - ✦ DHJ formula ⟶ **leading logarithmic (LL)**
  - ✦ Scale dependence of PDF's ⟶ **LO DGLAP evolution**
  - ✦ Scale dependence of UGD's ⟶ **rc-BK evolution**

- *State-of-the-art* degree of accuracy:
  - ✦ DHJ formula ⟶ **NLO**[1, 2]
  - ✦ Scale dependence of PDF's ⟶ **DGLAP NNLO**[3]
  - ✦ Scale dependence of UGD's ⟶ **BK NLO**[4, 5]

1 T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and M. Lublinsky, Phys. Rev. D91 (2015)no. 9 094016
2 G. A. Chirilli, B.-W. Xiao and F. Yuan, Phys.Rev. D86 (2012) 054005
3 Gao, Jun et al. Phys.Rev. D89 (2014) no.3, 033009
4 I. Balitsky and G. A. Chirilli, Phys. Rev. D77 (2008) 014019
5 I. Balitsky and G. A. Chirilli, Phys. Rev. D88 (2013) 111501

BACK-UP: BK equation with running coupling

- LO BK equation resumming $\alpha_s \ln(1/x)$ contributions to all orders:



$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \int d\mathbf{r_1}\, K^{\mathrm{LO}}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) \times \left[\mathcal{N}(r_1,Y) + \mathcal{N}(r_2,Y) - \mathcal{N}(r,Y) - \mathcal{N}(r_1,Y)\,\mathcal{N}(r_2,Y)\right]$$

LO Evolution Kernel:

$$\boxed{K^{\mathrm{LO}}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) = \frac{N_c\,\alpha_s}{2\pi^2}\,\frac{r^2}{r_1^2\,r_2^2}}$$

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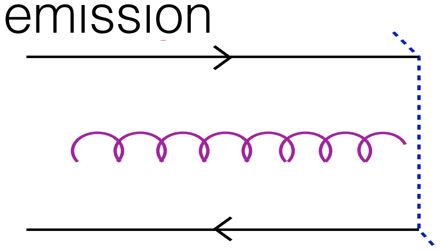

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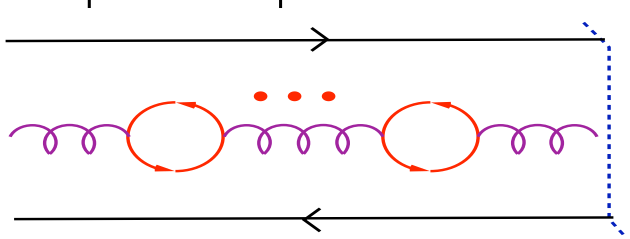
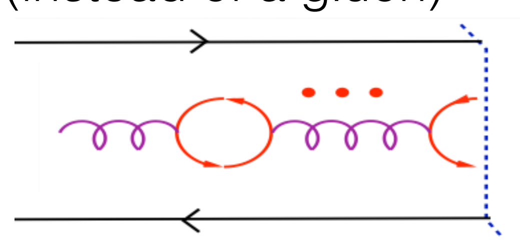
- Considering $\alpha_s N_f$ corrections:

Running coupling: chains of quark loops



$+$

Emission of $q\bar{q}$ pair (instead of a gluon)



(According to some separation scheme)

$$\frac{\partial \mathcal{N}(r,Y)}{\partial Y} = \overbrace{\underbrace{\mathcal{R}[\mathcal{N}]} - \mathcal{S}[\mathcal{N}]}$$

Running coupling term: gathers all the $\alpha_s N_f$ factors that complete the β-function

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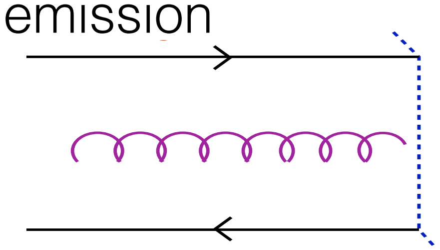

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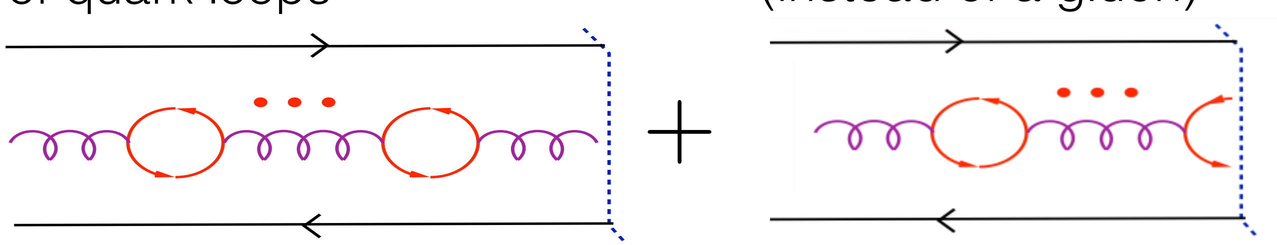

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- Numerical evaluation of substraction term $\mathcal{S}[\mathcal{N}]$ demands very large computing time.

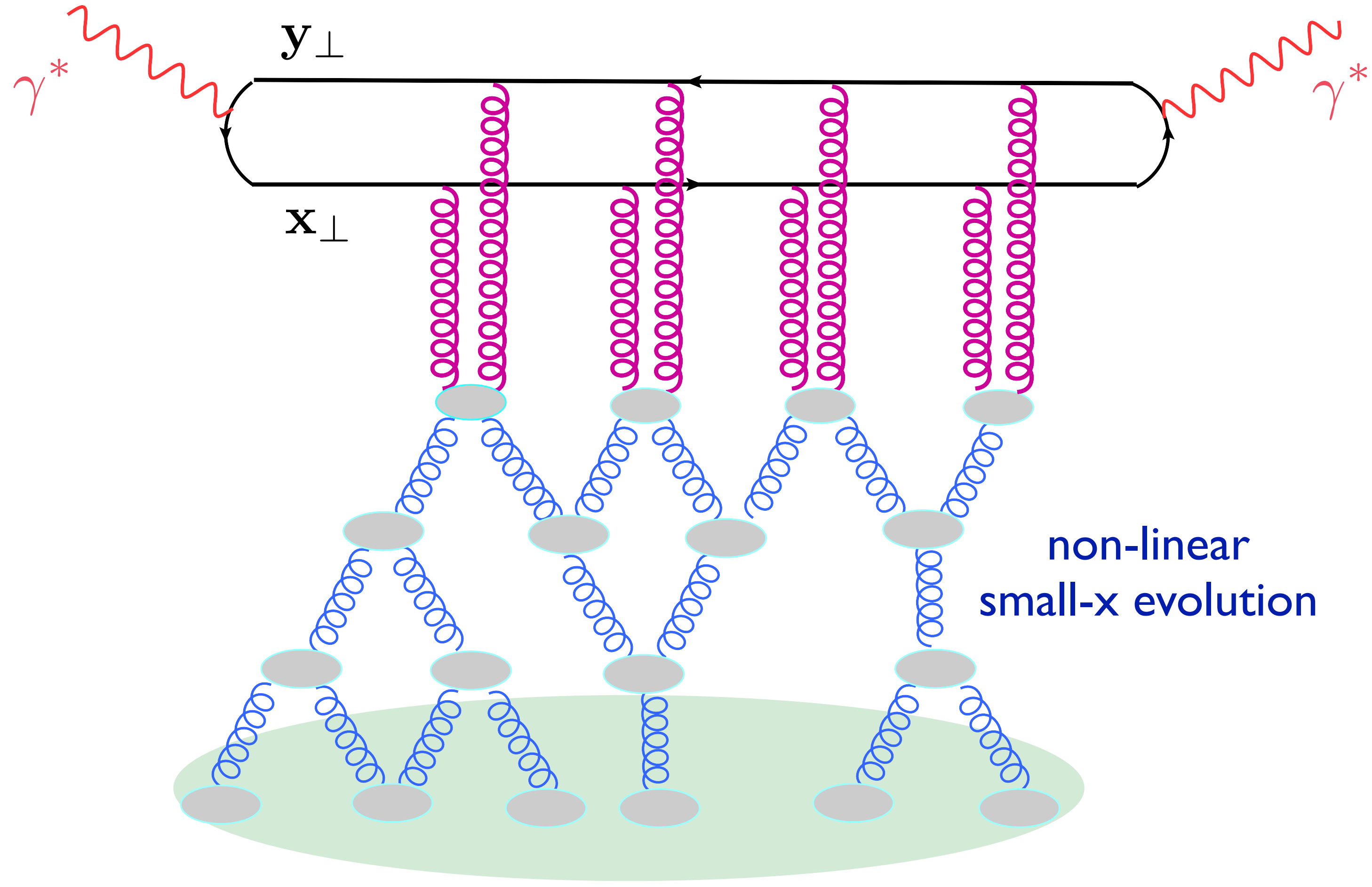
  → We only consider the running term $\mathcal{R}[\mathcal{N}]$ (prescription proposed by Balitsky[1])

Running coupling Kernel: $\boxed{K^{\mathrm{Bal}}(\mathbf{r},\mathbf{r_1},\mathbf{r_2}) = \frac{N_c\,\alpha_s(r^2)}{2\pi^2}\left[\frac{r^2}{r_1^2\,r_2^2}+\frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)}-1\right)+\frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)}-1\right)\right]}$

1 **I. I. Balitsky, *Quark Contribution to the Small-x Evolution of Color Dipole*, Phys. Rev. D 75 (2007) 014001**

BACK-UP: Dipole models, Wilson lines

- Dipole models are simple formulations for the description of Deep Inelastic Scattering processes (such as those observed in e-p collisions at HERA).
- We describe the effect of the small-x gluon field over the projectile as the multiple gluon exchange with a virtual quark-antiquark dipole.



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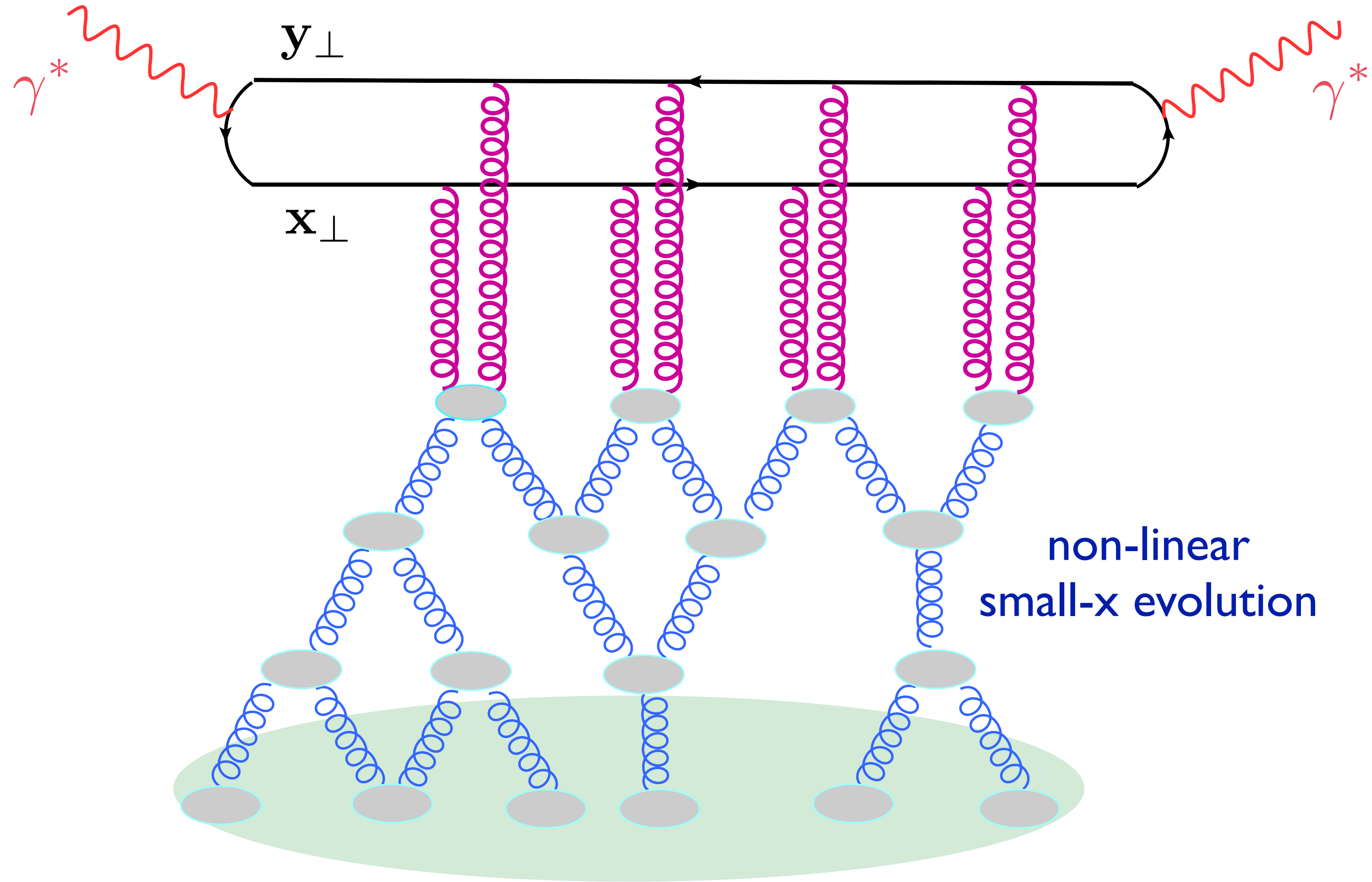
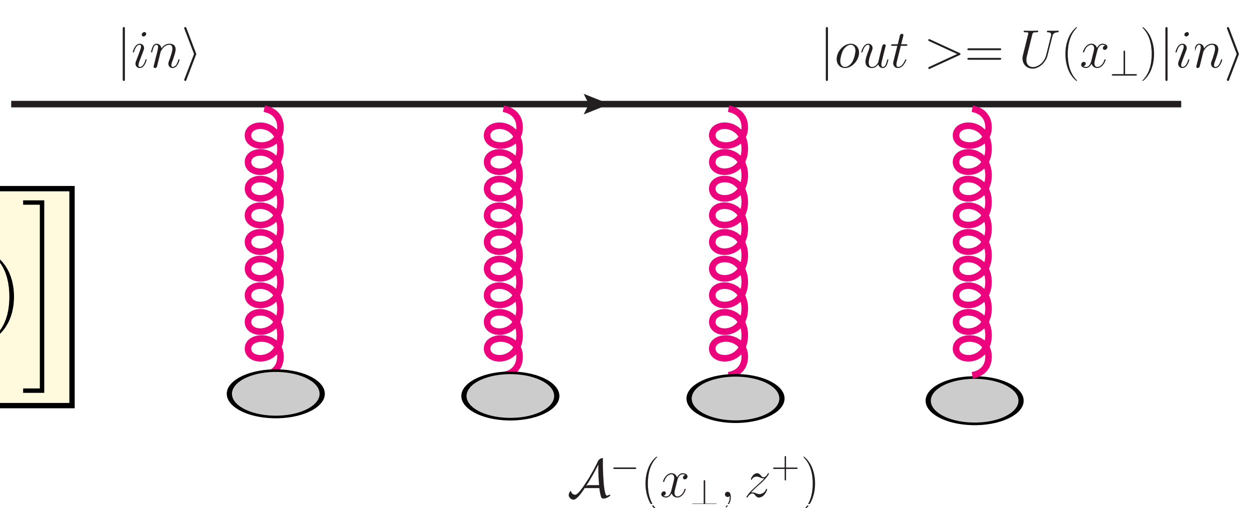


- Multiple gluon scattering in the eikonal approximation: definition of **WILSON LINES**:

$$\boxed{U(x_\perp) = \mathcal{P}\exp\left[ig\int dx^- A^+(x^-, x_\perp)\right]}$$



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$$\boxed{U(x_\perp) = \mathcal{P} \exp\left[ig \int dx^- A^+(x^-, x_\perp)\right]}$$

- Dipole scattering amplitudes: two-point correlators of Wilson Lines:

$$\mathcal{N}(\mathbf{r}, \mathbf{b}, x) = 1 - \frac{1}{N_c} \langle \mathrm{tr}\{U(x_{1\perp}) U^\dagger(x_{2\perp})\}\rangle_x$$

- Unintegrated gluon distributions (uGD's) defined as the Fourier transform of dipole scattering amplitude. We take the uGD's as universal objects that represent the effect of gluon-saturated target over hadronic projectiles.

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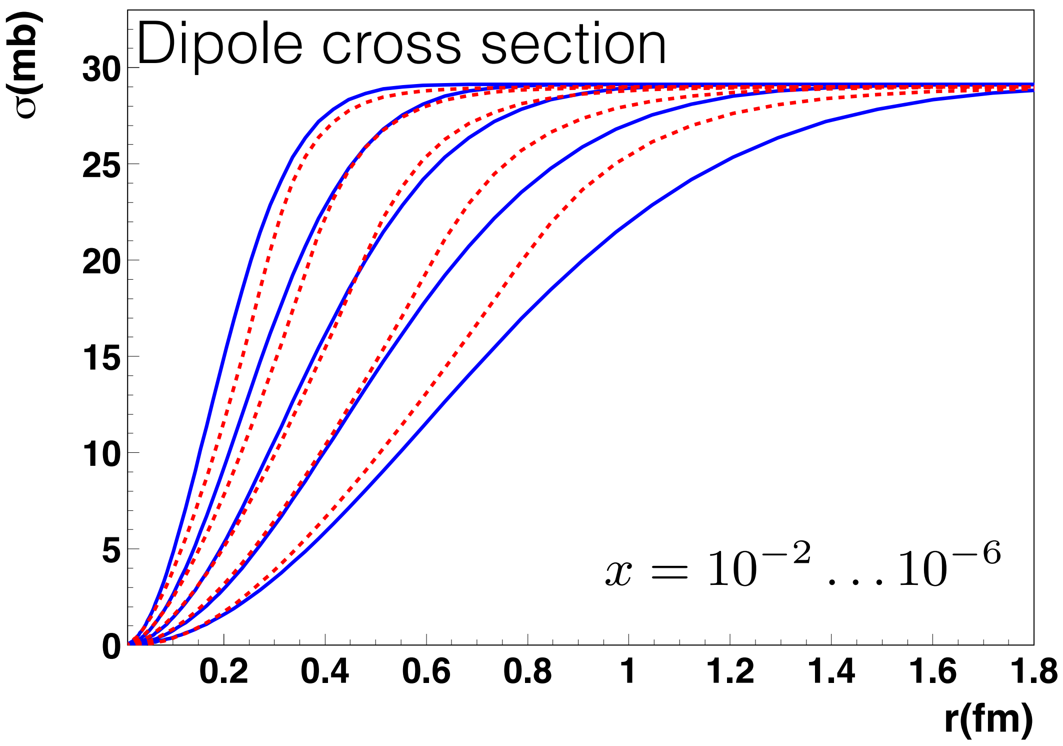
- Phenomenological models $\longrightarrow$ **modelization of dipole scattering amplitude**

For example: GBW model [1]

$$\mathcal{N}(\mathbf{r}, \mathbf{b}, x) = \theta(b_0 - b)(1 - \exp(-r^2 Q_s^2/4))$$



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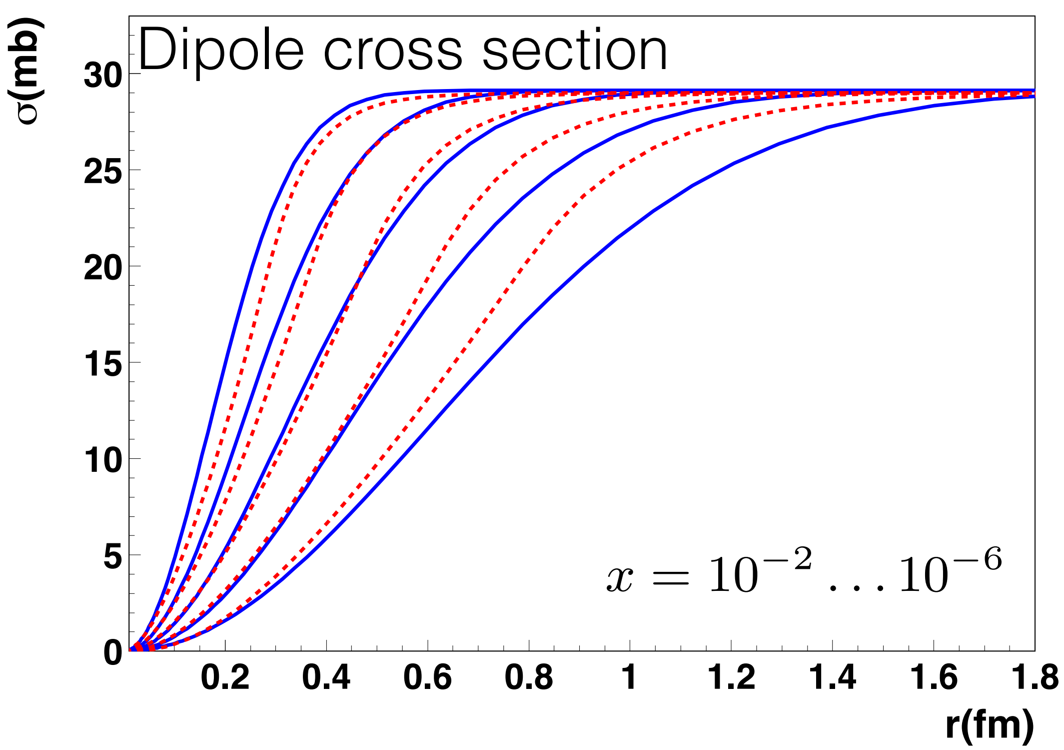
- Small-x evolution encoded in BK equation

$\downarrow$

Theoretically controlled tool for extrapolation!

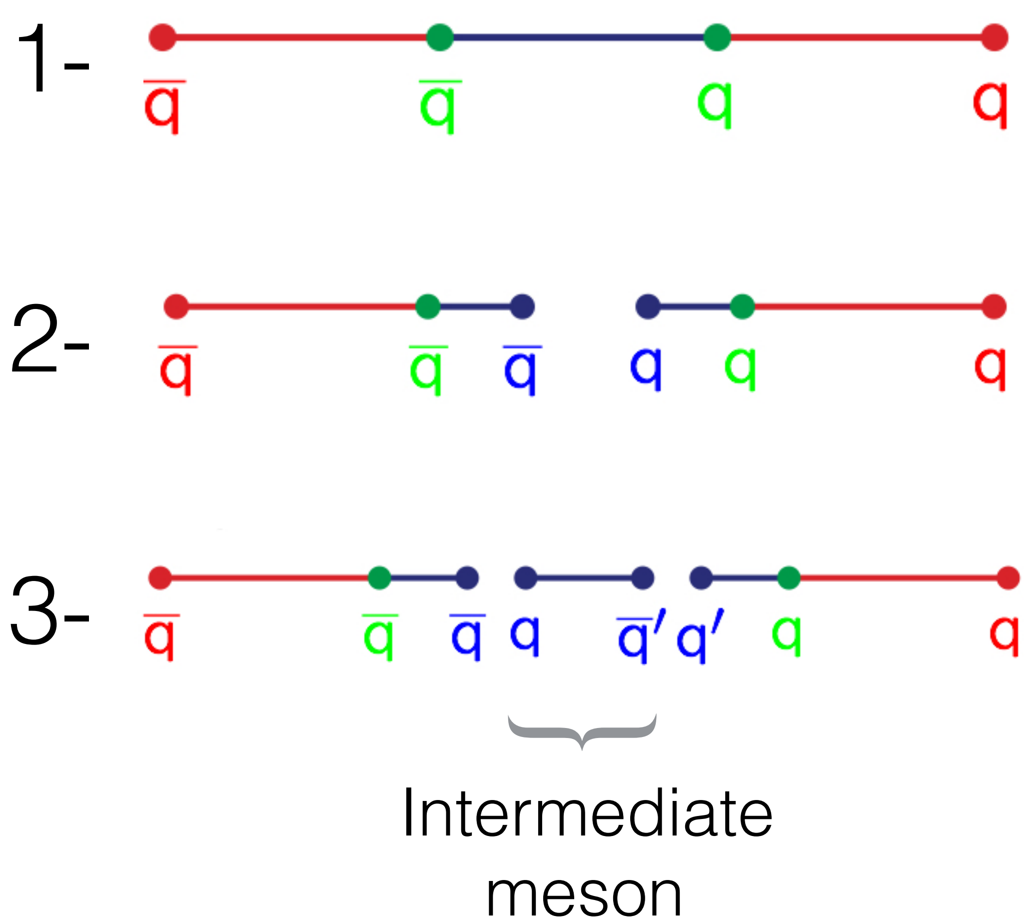


1 Golec-Biernat, K. et al. Phys.Rev. D79 (2009) 114010

BACK-UP: Model of baryon production in Lund formalism

- **Diquark model**: diquarks in color antitriplets are (effectively) fundamental objects of the theory $\longrightarrow$ diquark-antidiquarks fluctuations are an additional string breaking mechanism.

- **Popcorn model**: Quarks are the only fundamental objects. This model allows for the generation of intermediate mesons.



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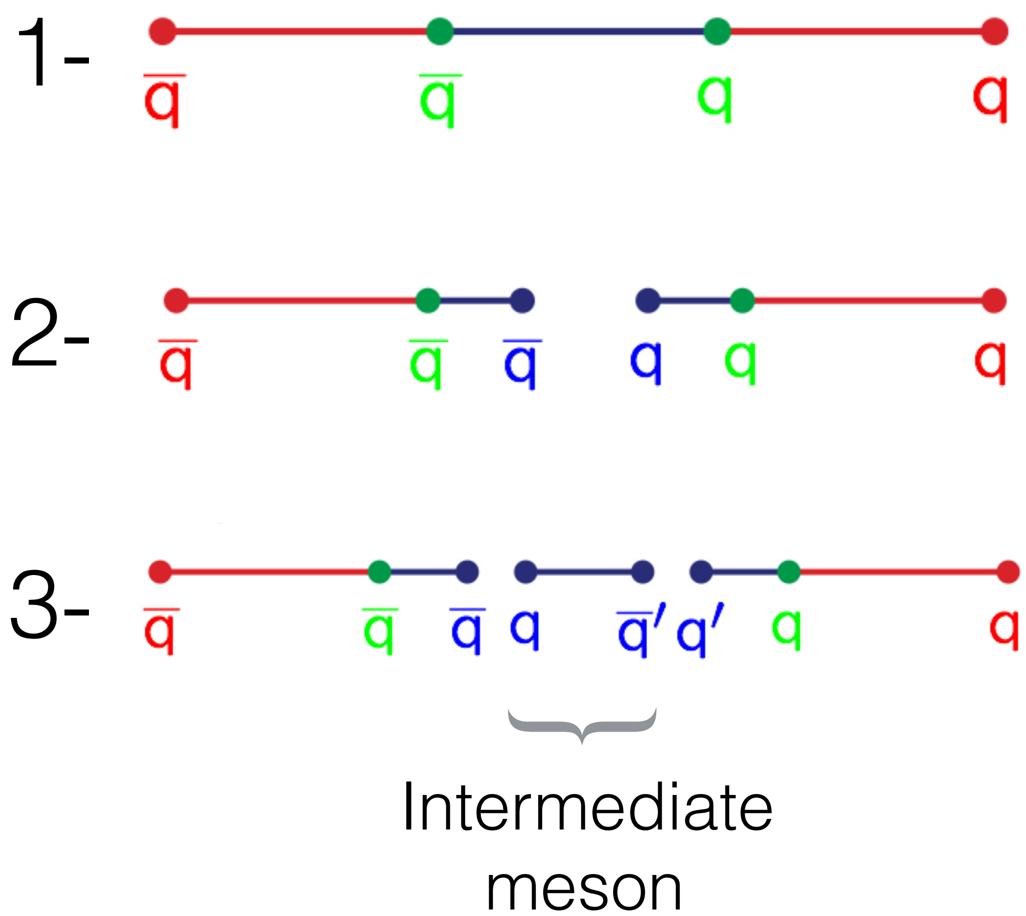
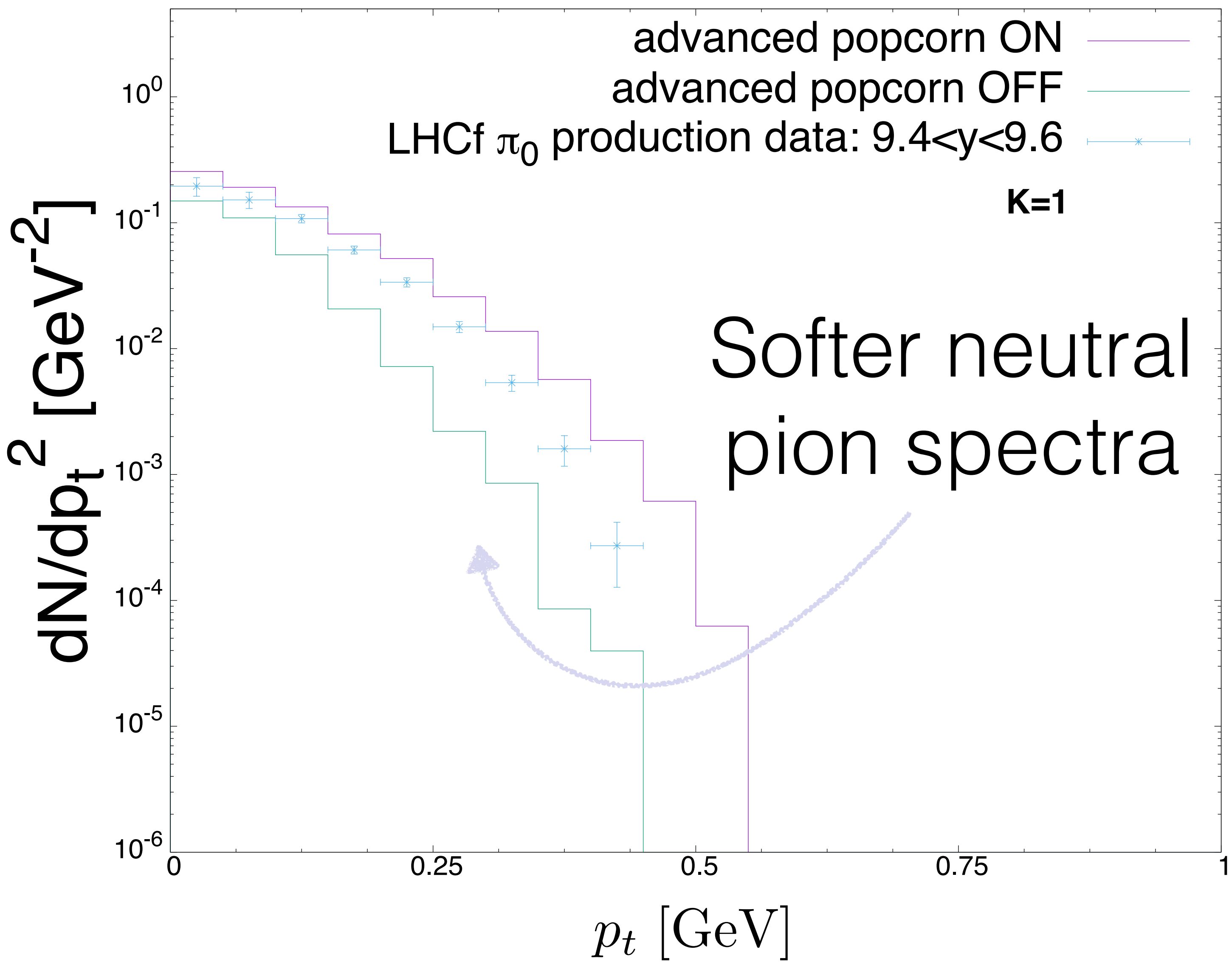
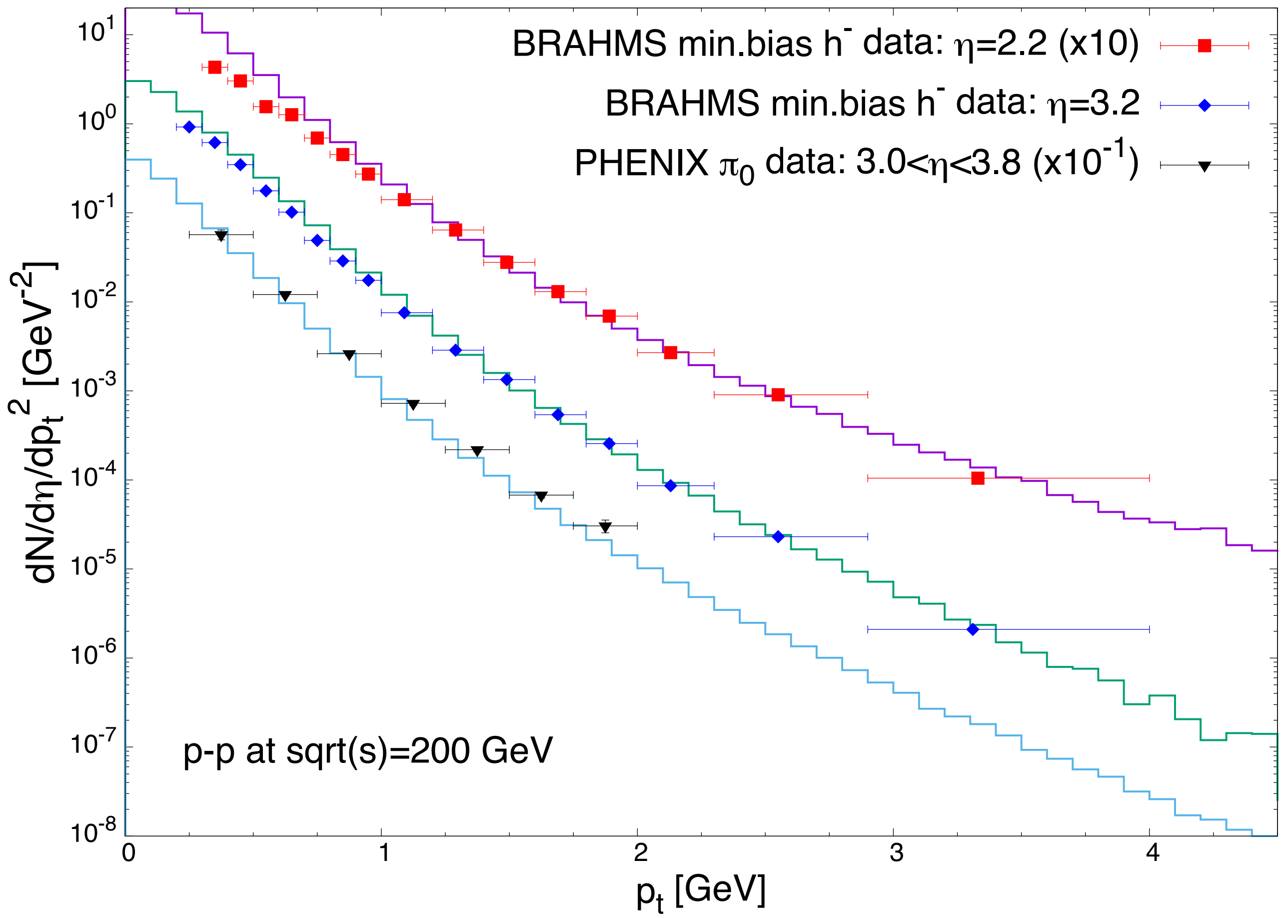
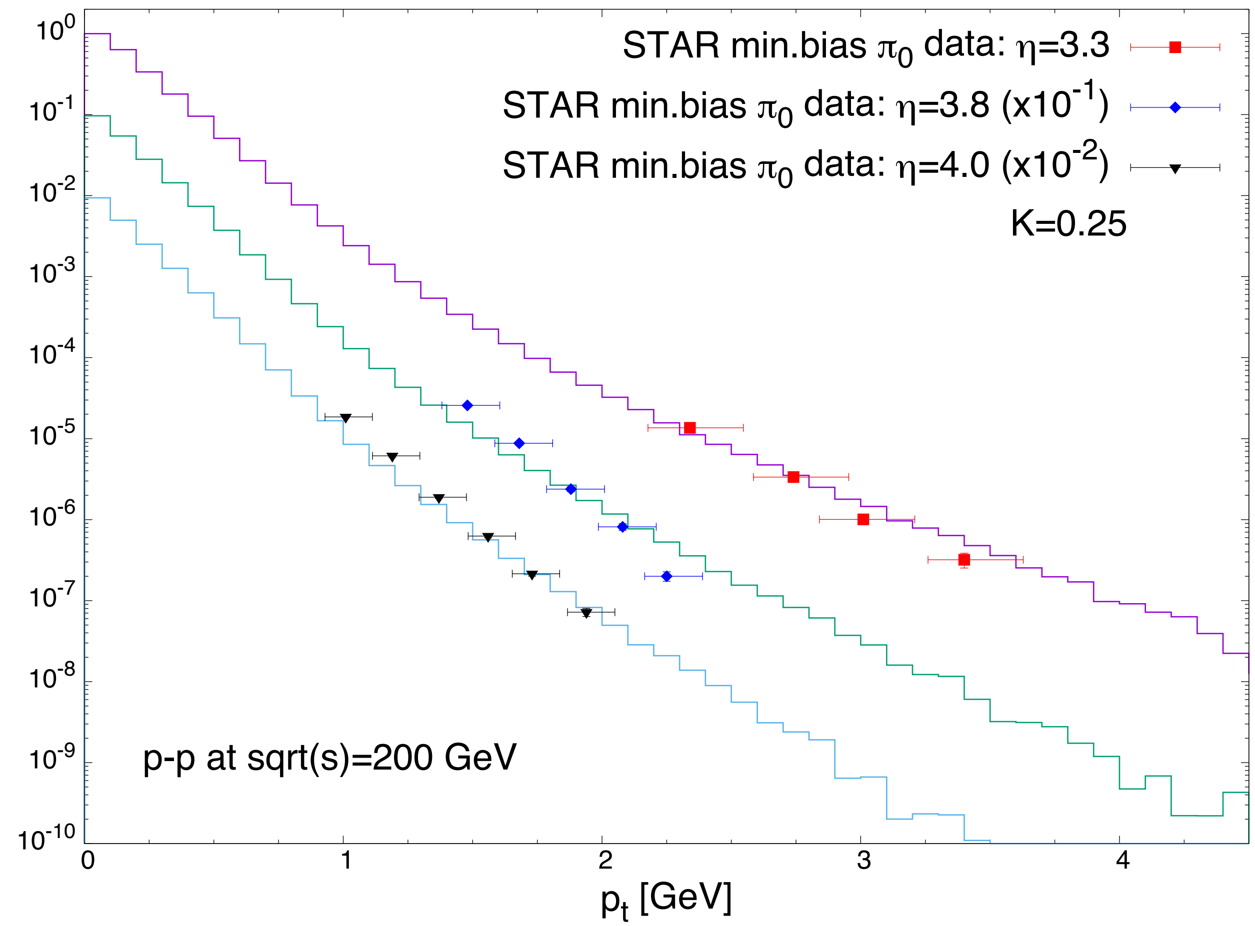




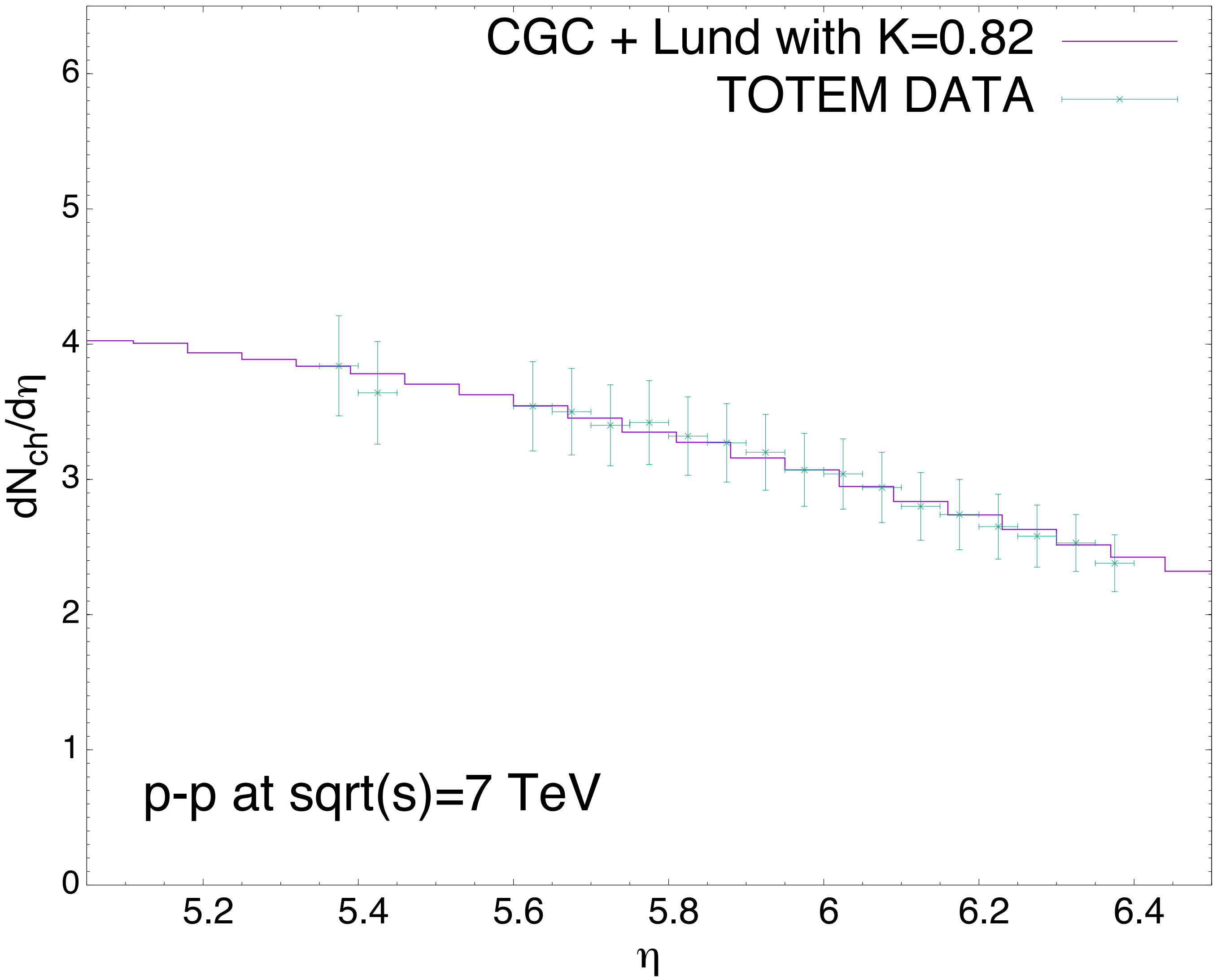
RHIC: p-p@ 200 GeV

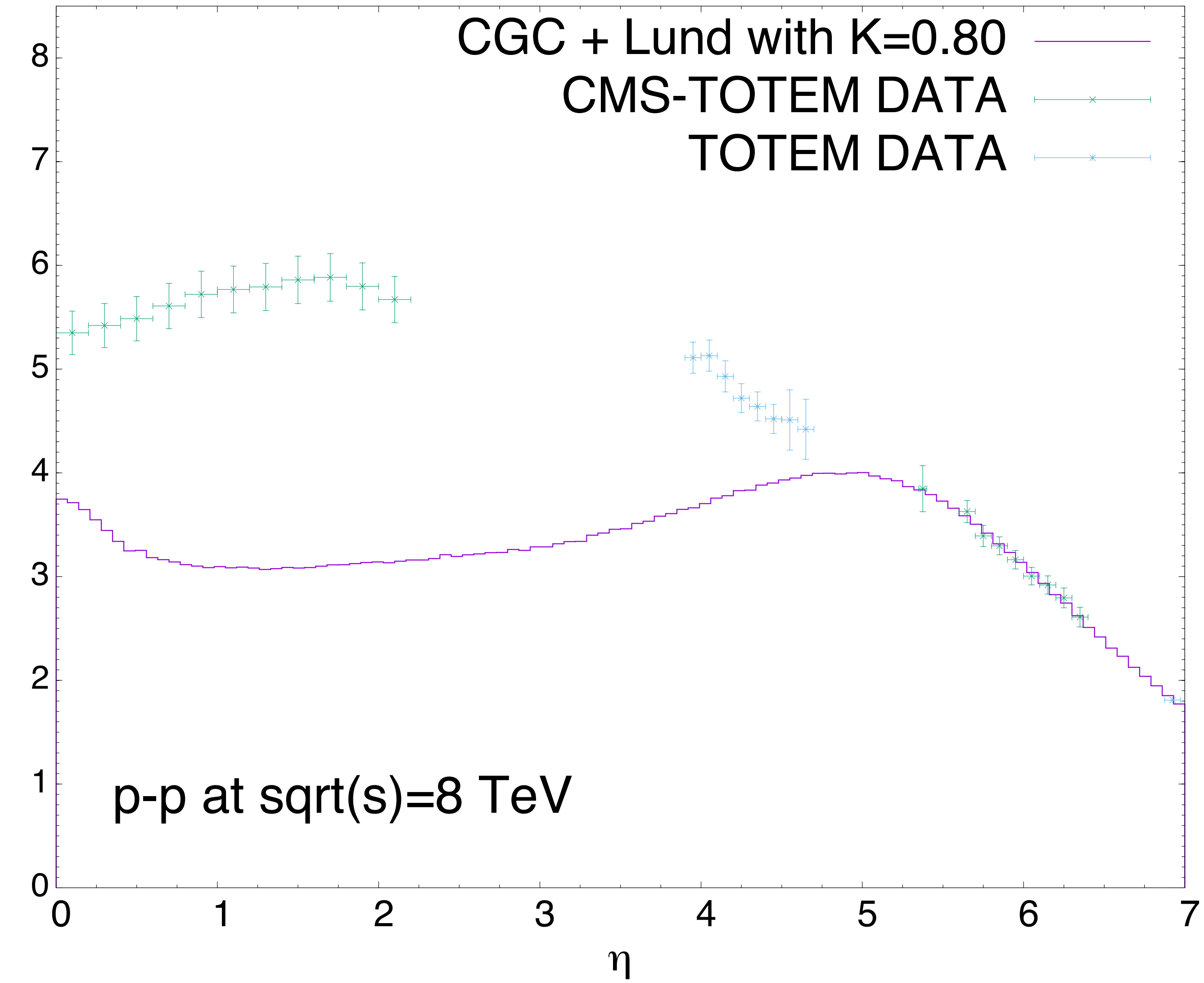




- Good agreement with data in the whole p_t range with $K = 1$ (except for data measured at STAR).
- CGC + Lund approach allows to reach p_t values as low as detected experimentally, $p_t \sim 0.2$ GeV

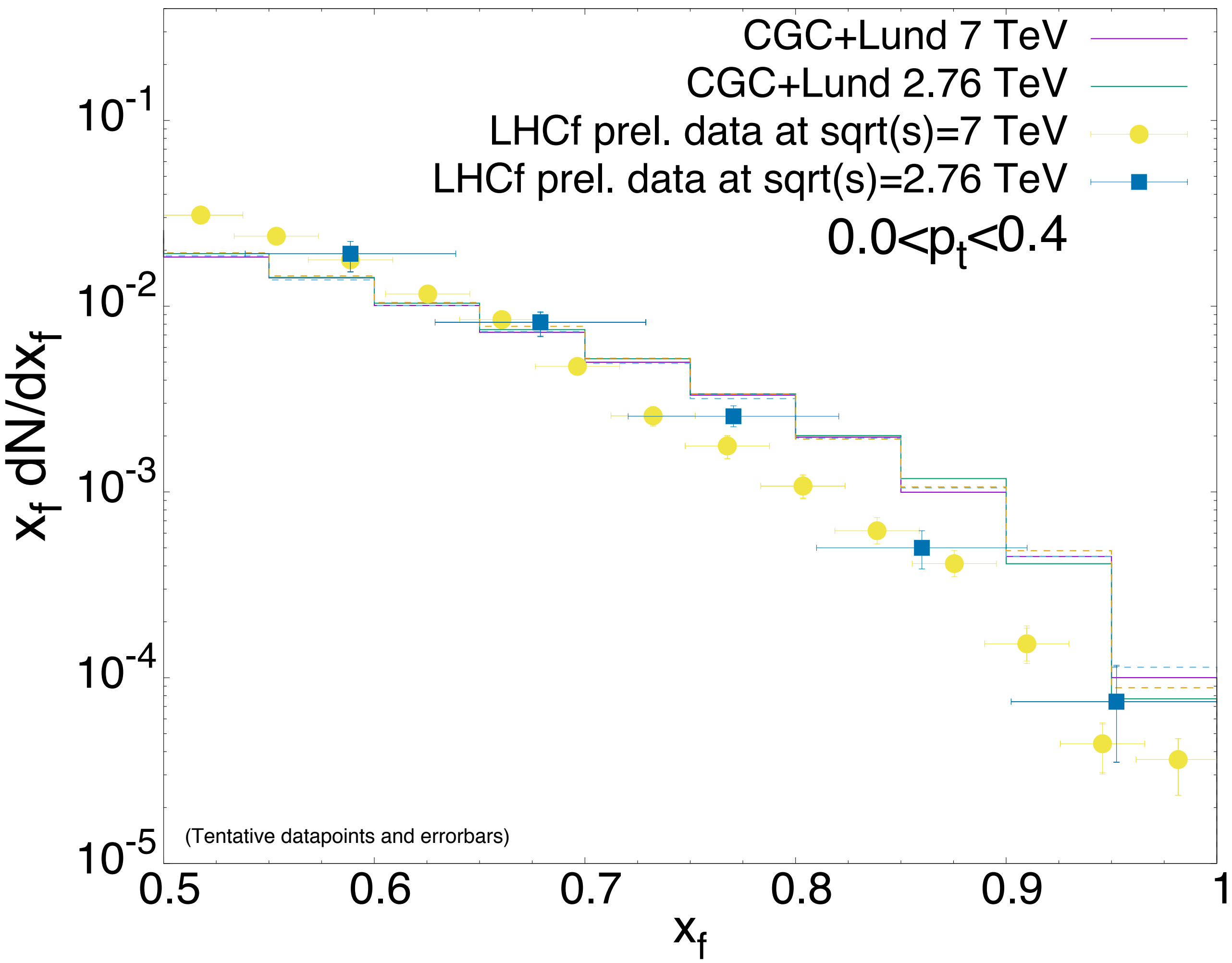
Multiplicity in p-p collisions: TOTEM data (sneak peek)

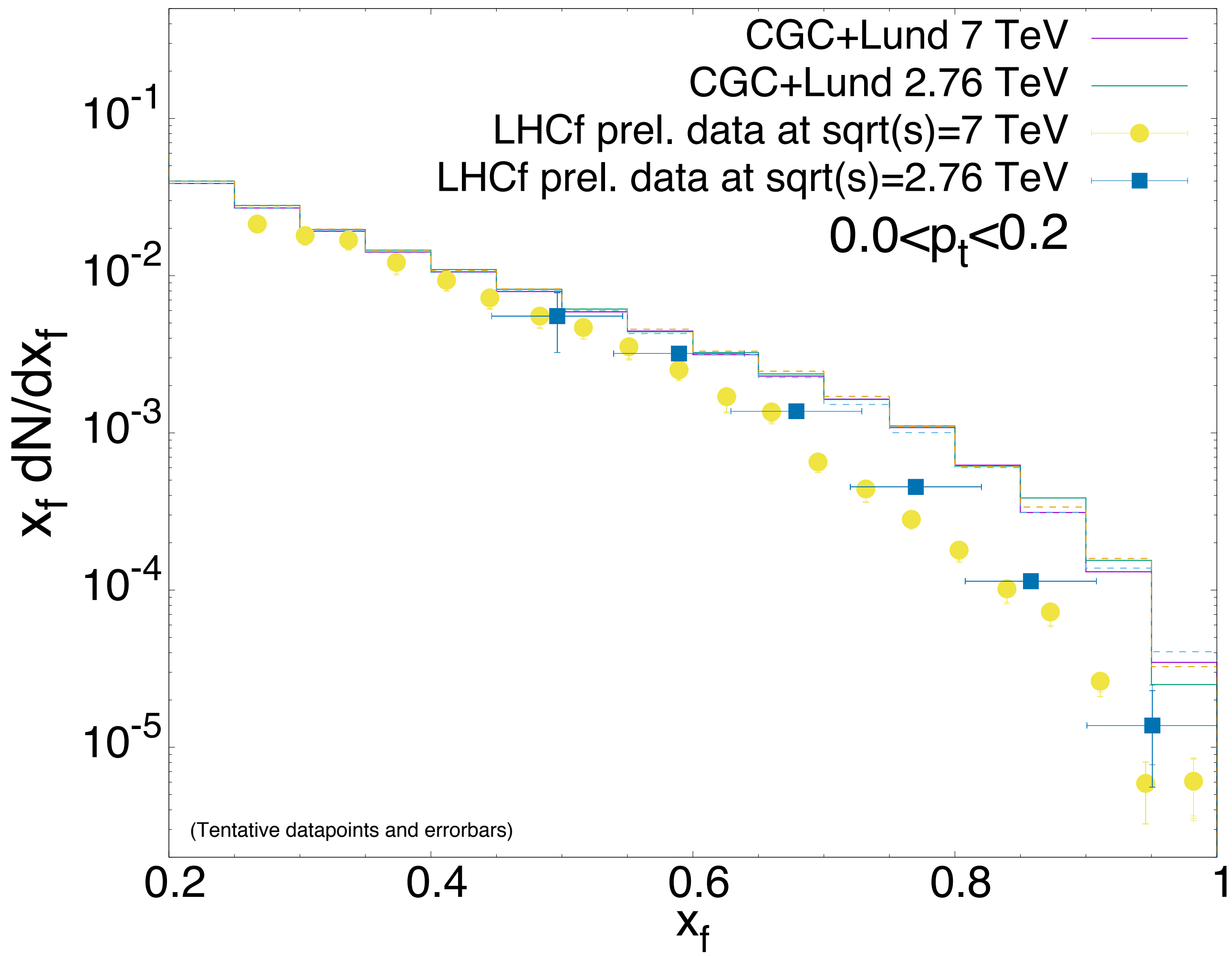




- Good reproduction of charged hadron multiplicity for high rapidities

Feynman scaling: LHCf (preliminary) data (sneak peek)





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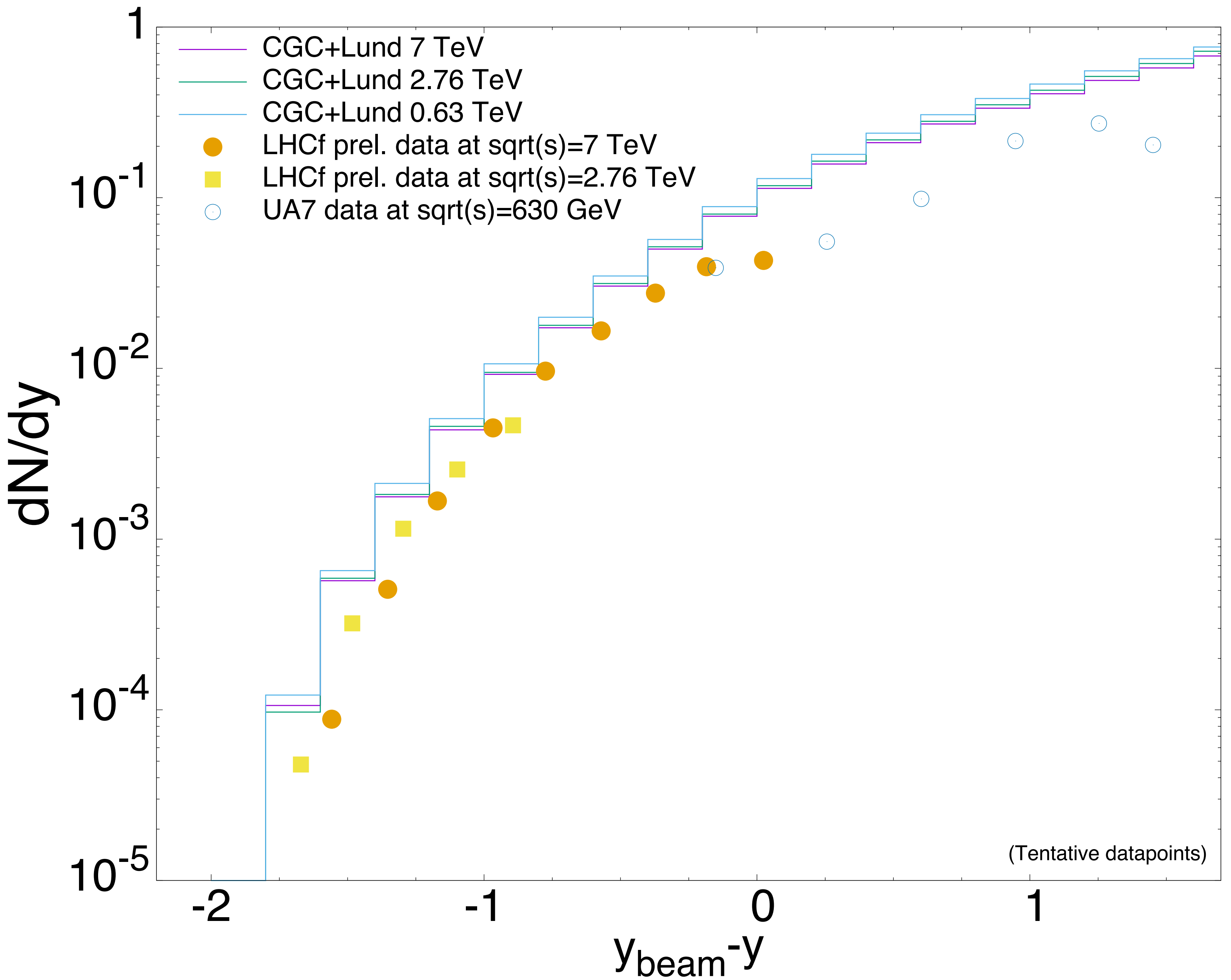

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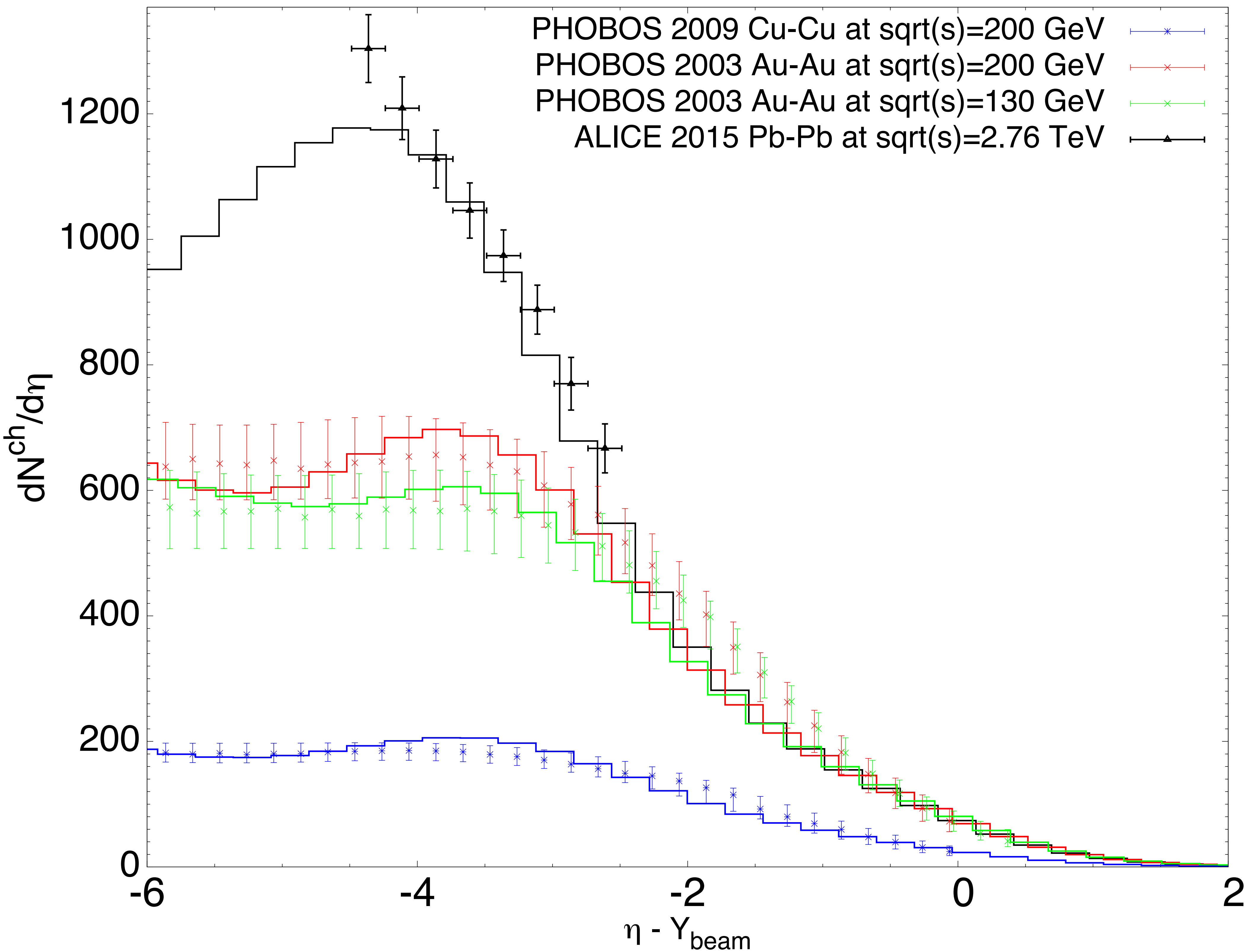
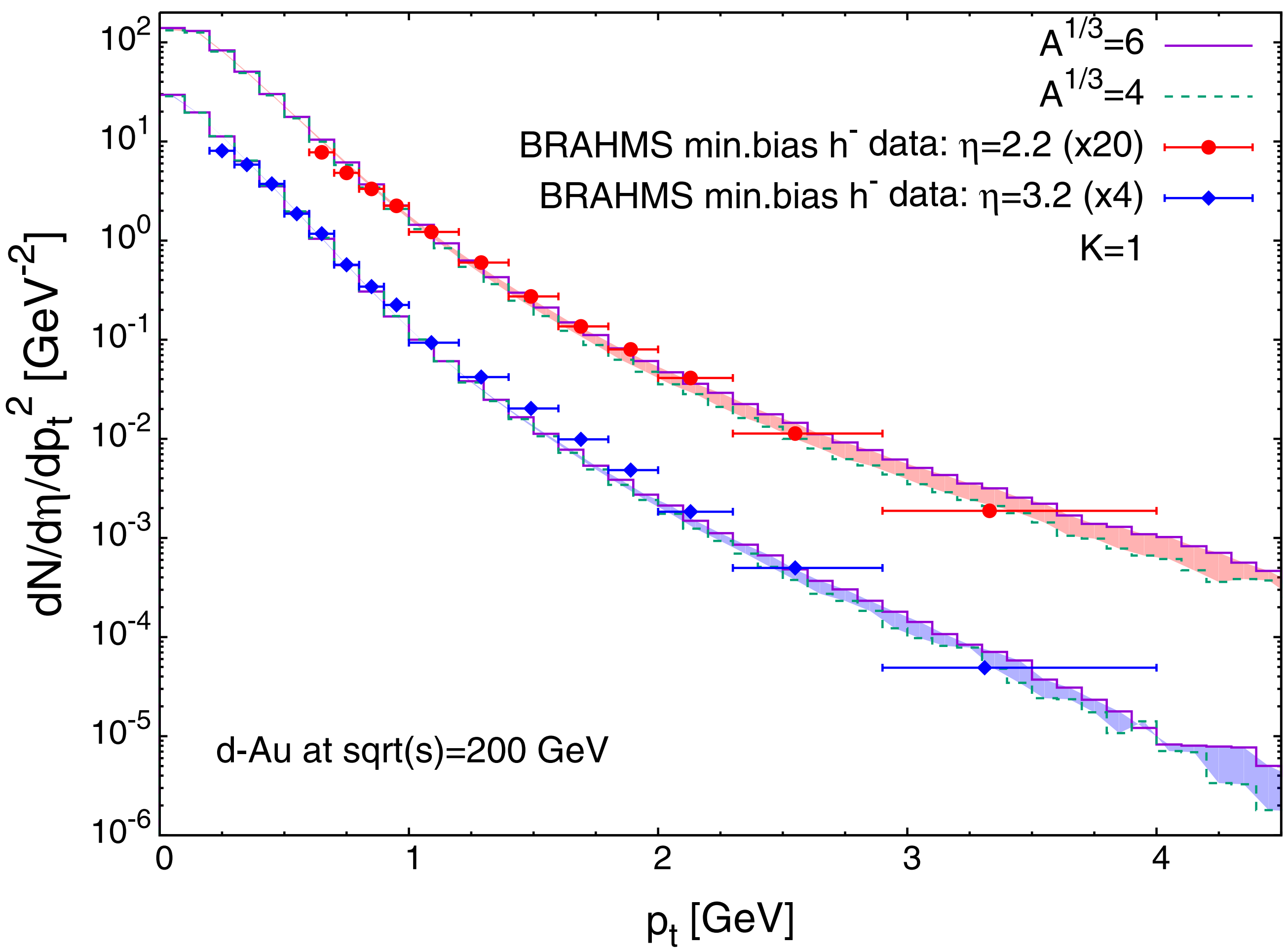
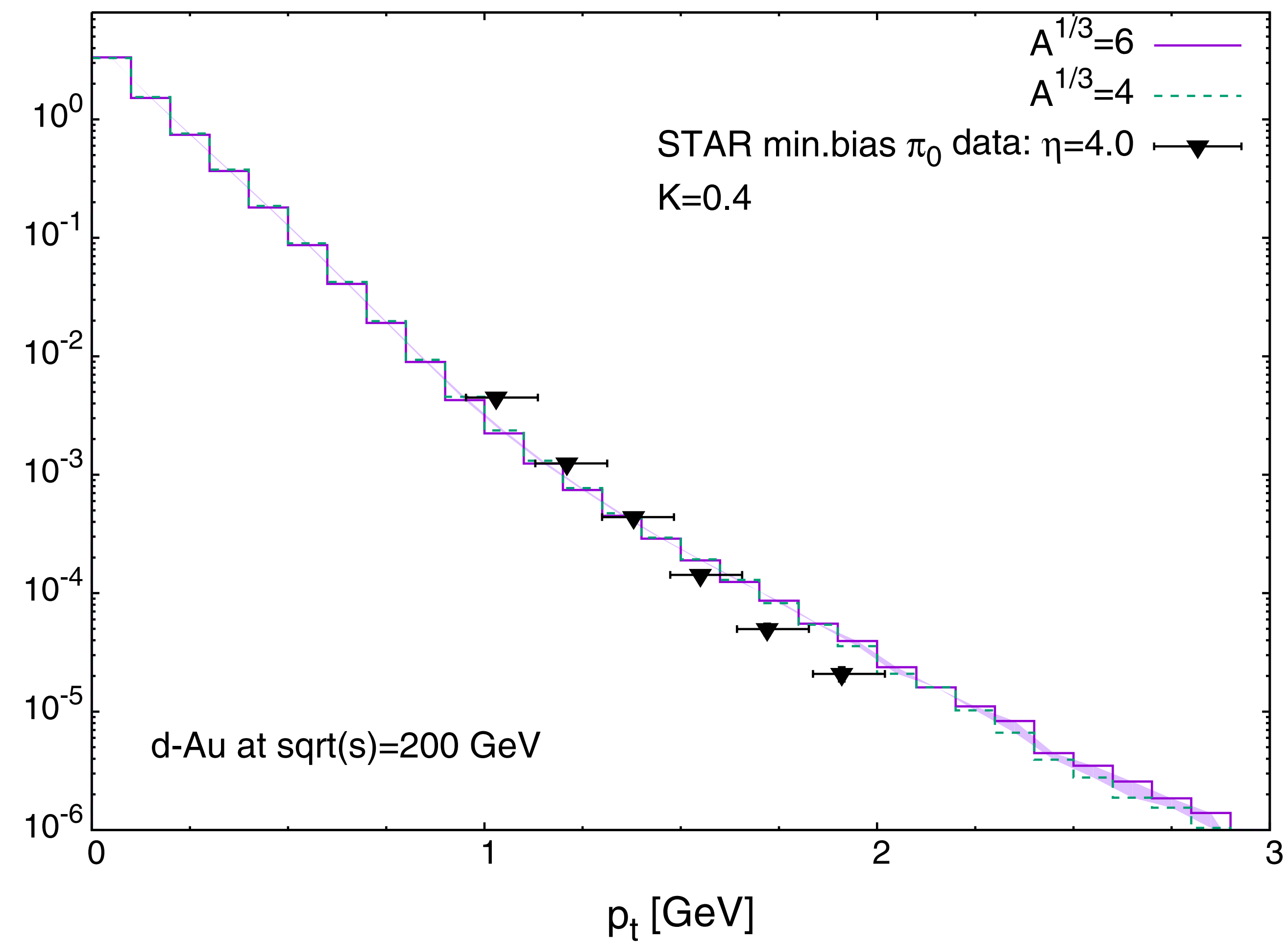
Nucleus-nucleus collisions: early results (sneak peek)



RHIC: d-Au @ 200 GeV





- CGC + Lund approach allows to reach p_t values as low as detected experimentally, $p_t \sim 0.2\ \mathrm{GeV}$.
- Little sensibility to number of participants,
- BRAHMS data well described with $K = 1$.
- STAR data well described with $K = 0.4$ (also observed in previous analysis of data).