Large Field Ranges, UV Cutoffs, and Photon Masses

Matthew Reece

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Based on: Heidenreich, MR, Rudelius 1606.08347, 1712.01868, 1802.08698; MR 1808.09966

Introduction

Swampland c. 2006:

Ooguri/Vafa: large $d(\phi) \Rightarrow$ breakdown of EFT.

Large field-space distance $d(\phi)$: tower of light modes

$$m \propto \mathrm{e}^{-c d(\phi)/M_{\mathrm{Pl}}}$$

Weak Gravity Conjecture: gauge coupling $g \to 0$ forbidden by QG, (magnetic) cutoff $\Lambda \lesssim g M_{\mathrm{Pl}}$.

More recently:

These are one and the same phenomenon, via *sublattice/tower* forms of WGC.

Heidenreich, MR, Rudelius 1509.06734, 1606.08437; Montero, Shiu, Soler, 1606.08438; Andriolo, Junghans, Noumi, Shiu 1802.02487

Introduction

Sublattice/Tower WGC

Beginning at the magnetic cutoff $\Lambda \lesssim g M_{\mathrm{Pl}}$, find an infinite tower of particles in different representations (different charges, for U(1)), *each* obeying WGC.

Many particles imply **low UV cutoffs**, as all of these particles can run in loops.

Recent progress: a new assumption, of a ***universal strong coupling scale***, together with towers of particles, can unify some of the Swampland ideas.

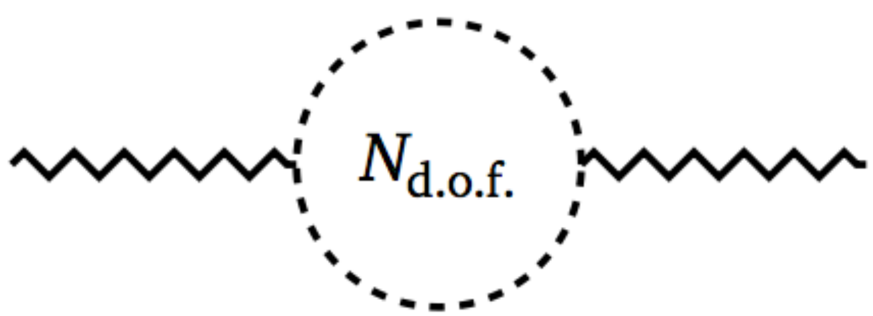
Furthermore, **quantitative predictions of UV cutoffs** help make clearer connections to phenomenology.

The Species Bound: Low Cutoffs from Many Weakly-Coupled Particles

In a theory with many light, weakly-coupled degrees of freedom, the UV cutoff at which gravity becomes strong is:

$$\Lambda_{\mathrm{QG}} \lesssim \frac{M_{\mathrm{Pl}}}{\left(N_{\mathrm{d.o.f.}}(\Lambda_{\mathrm{QG}})\right)^{1/(D-2)}}$$

The simplest argument for this is perturbative. Loops



renormalize the graviton kinetic term:

$$\delta M_{\mathrm{Pl}}^{D-2} \sim N_{\mathrm{d.o.f}} \Lambda_{\mathrm{QG}}^{D-2}$$

e.g. G. Dvali, 0706.2050 and G. Dvali & M. Redi, 0710.4344

Towers and Cutoffs

In theories with a field-dependent tower of states,

$$N(\phi) \sim N_0 e^{\alpha d(\phi)/M_{\mathrm{Pl}}}$$

the UV cutoff (in Einstein frame) decreases exponentially with field distance. Another perspective on Dine-Seiberg:

$$V(\phi) < \Lambda_{\mathrm{QG}}^2 M_{\mathrm{Pl}}^2 \lesssim e^{-\alpha d(\phi)/M_{\mathrm{Pl}}} M_{\mathrm{Pl}}^4$$

In natural theories with broken SUSY, we expect that

$$V(\phi) \sim m_{\mathrm{SUSY}}^2 \Lambda_{\mathrm{QG}}^2 \sim e^{-\beta d(\phi)/M_{\mathrm{Pl}}}$$

Don't need the quasi-dS assumption invoked by Vafa and Palti yesterday or any discussion of entropy, but do need some assumption about scaling with cutoff. (Still, examples work even with tuning, e.g. compactify Standard Model.)

Tower of States and Cutoffs

(in 4d for concreteness)

Single U(1):

QeM_{Pl} —— $q = Q$

$\vdots$

$3eM_{\mathrm{Pl}}$ —— $q = 3$

$2eM_{\mathrm{Pl}}$ —— $q = 2$

eM_{Pl} —— $q = 1$

Lattice versions of the WGC imply the existence of a tower of charged particles.

$$N_{\mathrm{d.o.f}}(\Lambda) \gtrsim \frac{\Lambda}{eM_{\mathrm{Pl}}}$$

(A sparse sublattice could change this, but we know no examples.)

$$\Lambda_{QG}^2 \lesssim \frac{1}{N_{\mathrm{d.o.f}}(\Lambda_{\mathrm{QG}})} M_{\mathrm{Pl}}^2$$

$$\Rightarrow \Lambda_{\mathrm{QG}} \lesssim e^{1/3} M_{\mathrm{Pl}}$$

Heidenreich, MR, Rudelius ’17

Phenomenological Consequences?

We don't know any very weakly coupled U(1) gauge theory in nature. But *B*-*L* could be gauged. Current bounds (Wagner et al. 1207.2442, Heeck 1408.6845) tell us that a massless *B*-*L* force, ***if it exists***, must have

$$e_{B-L} \lesssim 10^{-24}$$

so the lattice WGC species bound argument tells us that ***if*** we ever discover such a force, we would conclude the fundamental cutoff scale of gravity in our universe is

$$\Lambda_{\rm QG} \lesssim 10^{-8} M_{\rm Pl} \sim 10^{10}\ \mathrm{GeV}$$

this in turn would strongly constrain inflation, neutrino mass generation, SUSY breaking, …..

Emergent Gauge Fields?

(in 4d for concreteness)

Single U(1):

$$N_{\mathrm{d.o.f}}(\Lambda) \gtrsim \frac{\Lambda}{eM_{\mathrm{Pl}}} \Rightarrow \Lambda_{\mathrm{QG}} \lesssim e^{1/3} M_{\mathrm{Pl}}$$

QeM_{Pl} — $q = Q$

$\vdots$

$3eM_{\mathrm{Pl}}$ — $q = 3$

$2eM_{\mathrm{Pl}}$ — $q = 2$

eM_{Pl} — $q = 1$

This suggests the tower hits strong coupling at level

$$Q \sim e^{-2/3}$$

Integrating out charged degrees of freedom:

$$\frac{1}{e^2} = \frac{1}{e_{\mathrm{UV}}^2} + \sum_{q=1}^{Q} \frac{q^2}{12\pi^2} \log \frac{\Lambda}{eqM_{\mathrm{Pl}}}$$

Ignoring logs and constants, the sum is:

$$Q^3 \sim \frac{1}{e^2}$$

Heidenreich, MR, Rudelius '17

Emergent Gauge Fields?

Single U(1):

$$
\begin{array}{ll}
QeM_{\rm Pl} & \underline{\qquad\qquad} \\
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 & q = 2 \\
eM_{\rm Pl} & \underline{\qquad\qquad} \\
 & q = 1
\end{array}
$$

Put differently: a tower of states of different charges leads to a Landau pole for the U(1) coupling in the UV.

It also renormalizes the Planck scale, leading to a low gravitational cutoff.

A Lattice WGC tower is one for which these are (at least up to constant factors) the *same* scale!

Extends to general gauge groups!

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Nonabelian tower and UV cutoffs

SU(2): states charged under Cartan U(1)

$$QgM_{\rm Pl} \qquad q = Q, Q-1, \ldots, -(Q-1), -Q$$

$$\vdots$$

$$3gM_{\rm Pl} \qquad q = 3, 2, 1, 0, -1, -2, -3$$

$$2gM_{\rm Pl} \qquad q = 2, 1, 0, -1, -2$$

$$gM_{\rm Pl} \qquad q = 1, 0, -1$$

States come in multiplets: linear tower leads to quadratic growth in density of states

$$N_{\rm d.o.f.}(\Lambda) \gtrsim \left(\frac{\Lambda}{gM_{\rm Pl}}\right)^2$$

$$\Rightarrow \Lambda_{\rm QG} \lesssim g^{1/2} M_{\rm Pl}$$

Heidenreich, MR, Rudelius ’17

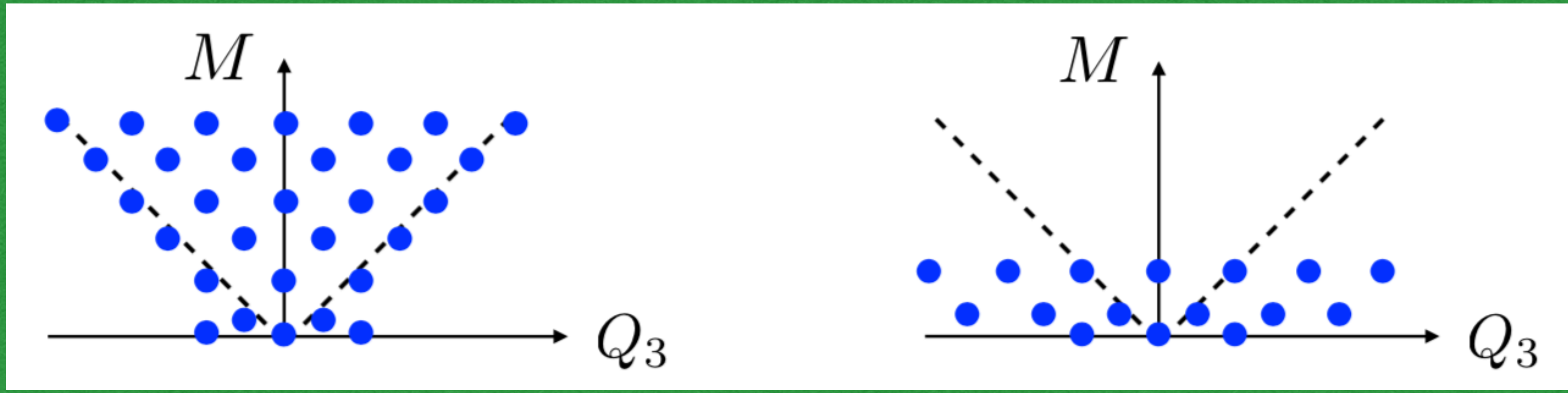
Nonabelian tower and UV cutoffs

Need to be careful about correct form of nonabelian WGC. *Not* just the abelian WGC applied to the Cartan, in which case we can "skip rungs" in the ladder by putting bigger reps at lower mass. Rather, it's about *representations* of the nonabelian group.

J(1)

States come in multiplets: linear tower leads to quadratic growth in density of states

$gM_{\rm Pl}$ $\overline{}$ $\overline{q = 1}, 0,$



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SU(2) and Emergence

As in the U(1) case, we can integrate out the tower to find corrections to the low-energy gauge coupling:

$$\frac{1}{g^2} = \frac{1}{g_{\mathrm{UV}}^2} + \sum_{j=1/2}^{J} \frac{I(j)}{16\pi^2} \log \frac{\Lambda}{j g M_{\mathrm{Pl}}}$$

where $I(j) = \frac{1}{3} j(j+1)(2j+1) \sim j^3$ is the Dynkin index of the spin-*j* representation. The sum scales like:

$$\frac{1}{g^2} \sim \sum_j j^3 \sim J^4 \sim \left(\frac{\Lambda}{g M_{\mathrm{Pl}}}\right)^4$$

So again with $\Lambda \sim g^{1/2} M_{\mathrm{Pl}}$ we have the gauge Landau pole and the gravitational cutoff at the ***same parametric scale***.

Generalizes to ***arbitrary gauge groups***.

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The Converse

Some interesting converse statements are true. Requiring that a gauge theory becomes strongly coupled *at or below* the quantum gravity scale implies at least one parametrically WGC-obeying charged particle exists. Even stronger:

$$\Lambda_{\rm gauge}^2 \lesssim e^2 \langle q^2 \rangle_{\Lambda_{\rm gauge}} M_{\rm Pl}^{D-2}$$

That is, particles below the UV cutoff will, *on average*, obey the WGC bound (parametrically).

This suggests a tight link between the WGC and UV cutoffs. A weak point arises for theories where the charged states are extended objects, and our one-loop estimates are not accurate.

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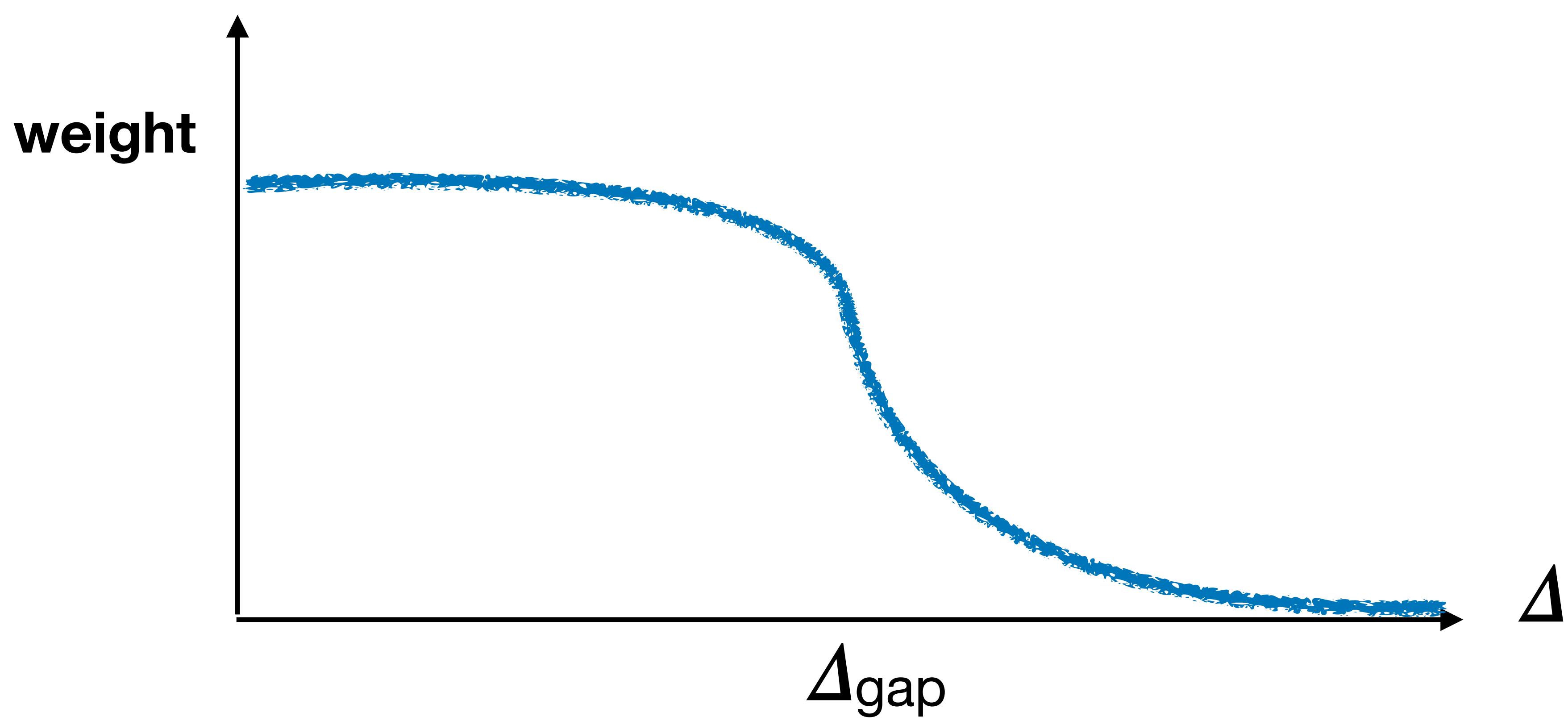
Wild Speculation / Wishful Thinking

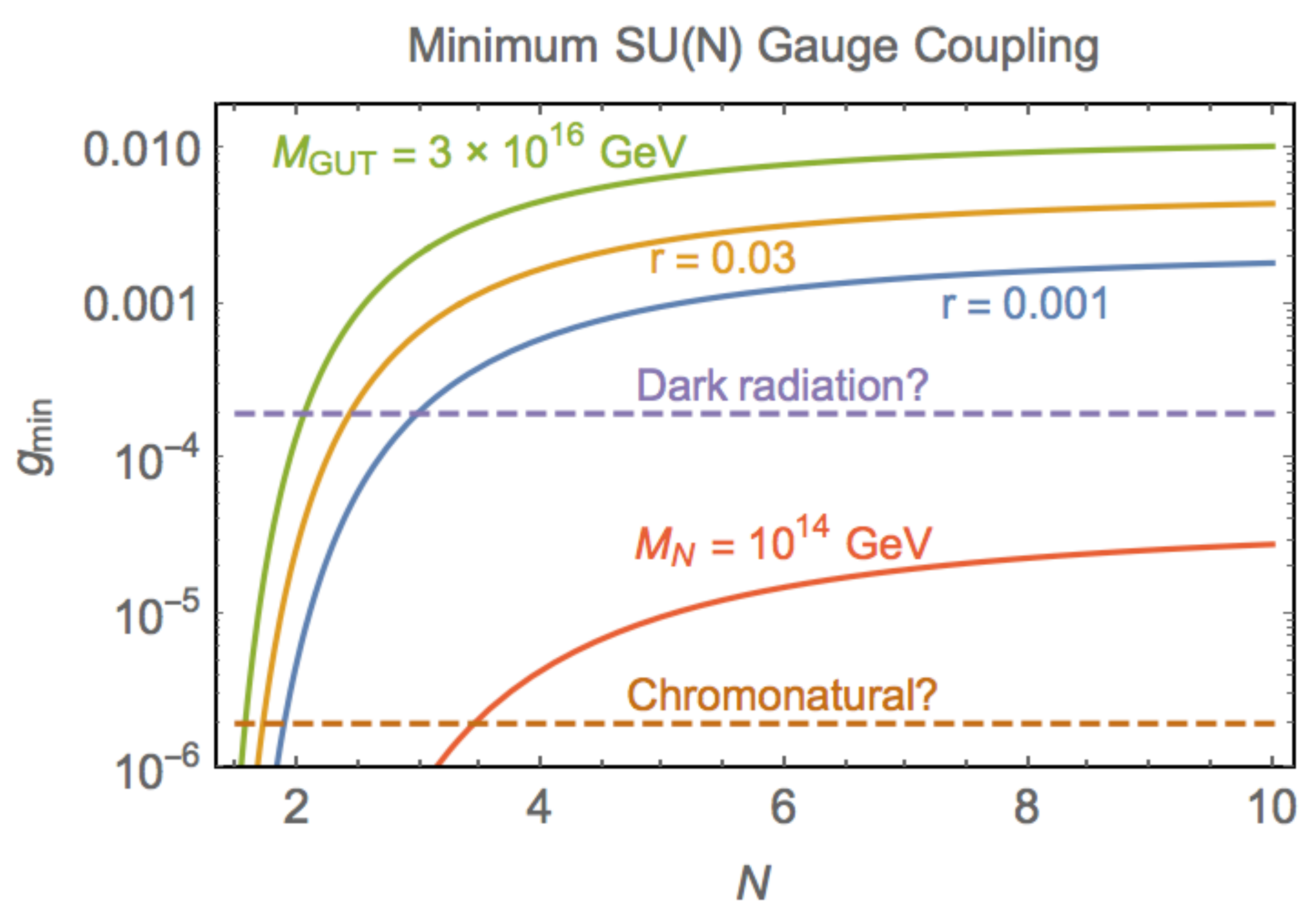
Suppose we had a ***sum rule*** in CFTs that told us the central charge counts all single-trace operators below Δ_{gap}, the analogue of the string scale or Λ_{QG}, but counts heavier states in some exponentially penalized way. And suppose that the coefficient *b* in the current-current 2-point function similarly counted charged operators proportional to q^2. This could make our ~ arguments rigorous, for AdS QG.



Applications

Dark matter coupled to **dark radiation** has a detectable imprint on large-scale structure: Buen-Abad, Marques Tavares, Schmaltz 1505.03542; Lesgourgues, … 1507.04351

$g \sim 2 \times 10^{-4}$ improves fit by 3σ



We expect small *r* ? Not *quite* constraining for SU(2).

Chromo-natural inflation? (Adshead, Wyman 1202.2366, Martinec, …) Benchmark $g \sim 2 \times 10^{-6}$ Marginal with WGC. It has other significant problems.

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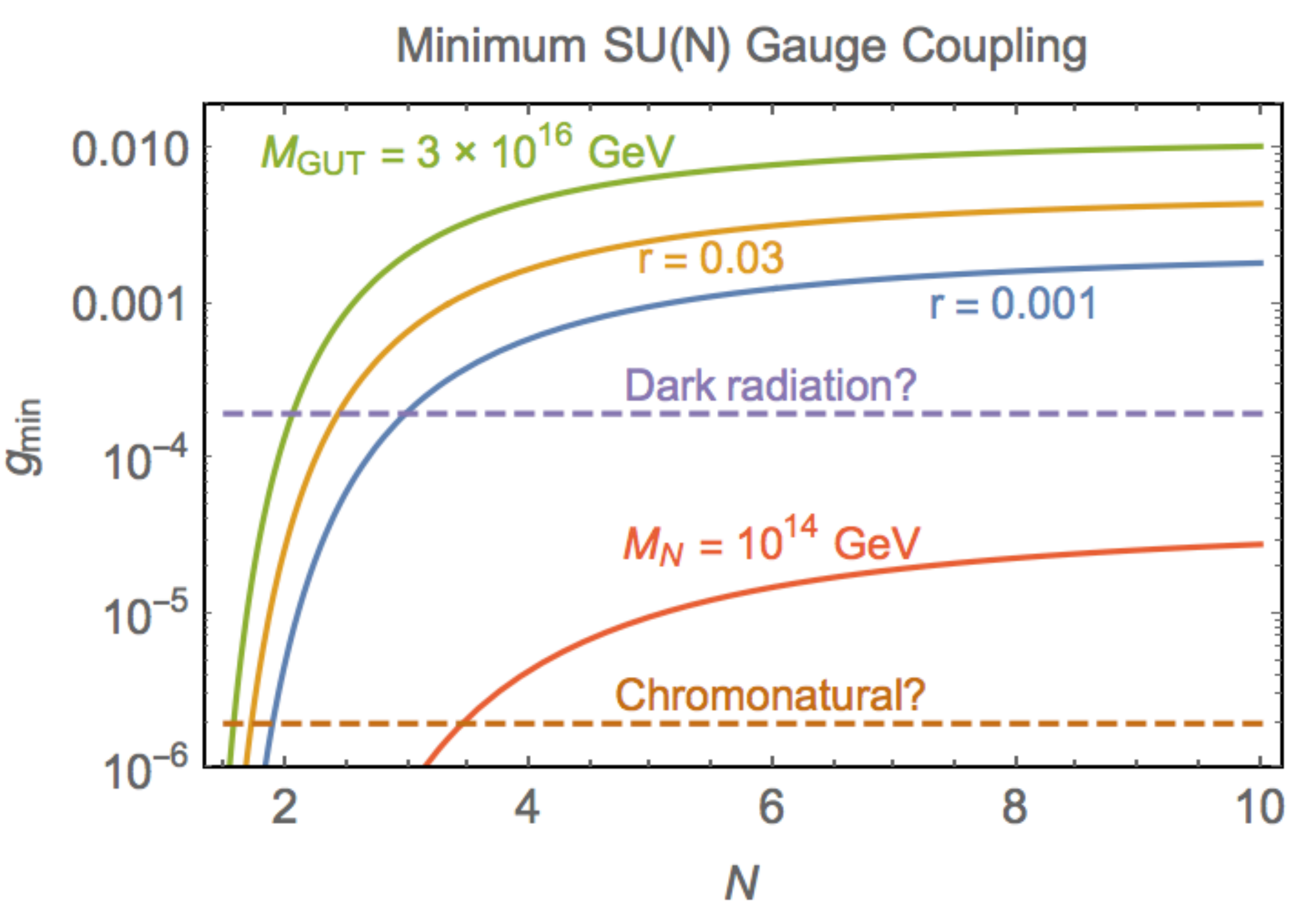


Suggests our WGC-inspired UV cutoff conjectures could be falsifiable with data.

Other applications?

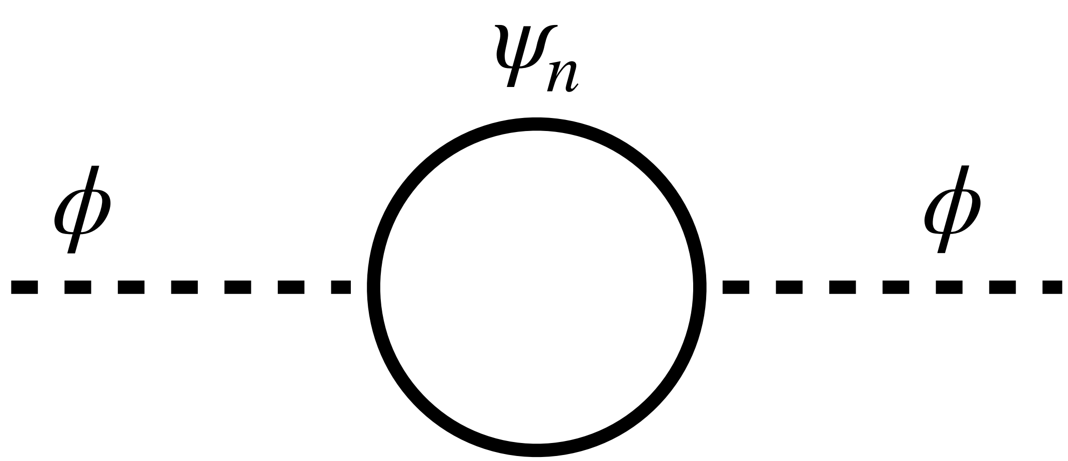
Chromo-natural inflation?
(Adshead, Wyman 1202.2366, Martinec, …)
Benchmark $g \sim 2 \times 10^{-6}$
Marginal with WGC. It has other significant problems.

Moduli and the Quantum Gravity Scale

Assume fields becoming light at a special point $\phi = 0$.

$$\mathscr{L} = \frac{1}{2}K(\phi)(\partial\phi)^2 + \sum_n \bar{\psi}_n(\mathrm{i}\not{\partial} - m_n(\phi))\psi_n$$

Loops:



$$\sim \frac{1}{K(\phi_0)}\left(\frac{\partial m_n}{\partial \phi}\right)^2$$

Strong coupling at same scale as species bound:

$$K(\phi_0) \sim \sum_{m_n<\Lambda_{\mathrm{QG}}}\left(\frac{\partial m_n}{\partial \phi}\right)^2 \sim \frac{1}{\phi_0^2}\sum_{m_n<\Lambda_{\mathrm{QG}}} m_n^2 \sim \frac{1}{\phi_0^2}N\Lambda_{\mathrm{QG}}^2 \sim \frac{M_{\mathrm{Pl}}^2}{\phi_0^2}$$

Heidenreich, MR, Rudelius ’18; also Grimm, Palti, Valenzuela ’18 (see Irene’s talk)

Moduli and the Quantum Gravity Scale

Ooguri/Vafa 2006 *conjectured* towers become light at a rate exponential in field space distance.

Here we see it is an *output* of assuming a universal strong-coupling scale, implying a kinetic term:

$$\mathscr{L} \sim \frac{M_{\mathrm{Pl}}^2}{\phi^2} \partial_\mu \phi \partial^\mu \phi$$

Applying a similar argument to *axion* fields:

$$\left\langle (\Delta m)^2 \right\rangle \sim \Lambda_{\mathrm{QG}}^2 \frac{d(\phi)^2}{M_{\mathrm{Pl}}^2}$$

Super-Planckian field traversals require O(1) fraction of modes to pass through QG cutoff!

Heidenreich, MR, Rudelius ’18

Moduli and the Quantum Gravity Scale

This is not a sharp no-go theorem, but it does suggest that one should be very careful trusting the validity of EFTs over super-Planckian field ranges in quantum gravity.

One loophole: this refers to the *field-space distance* (geodesic distance), while the *potential* might steer fields along non-geodesic paths.

(See: Hebecker, Henkenjohann, Witkowski ’17; Landete, Shiu ’18)

Super-Planckian field traversals require O(1) fraction of modes to pass through QG cutoff!

Stückelberg in the Swampland

In effective field theory we can add masses to abelian gauge bosons and they're harmless. At small enough mass, the longitudinal mode is very weakly coupled.

We can view a photon mass as a *Stückelberg* mass, introducing a Goldstone boson that shifts:

$$\frac{1}{2}f^2(\partial_\mu\theta - e\hat{A}_\mu)^2$$

In string theory, such masses are ubiquitous. SUSY implies that a radial mode exists. Distinguishing feature is the kinetic term:

$$K(\Phi, \Phi^\dagger, V) = -M^2 \log(\Phi + \Phi^\dagger - cV)$$

Stückelberg in the Swampland

The point of zero photon mass lies at infinite distance,

$$\mathrm{Re}\,\Phi \to \infty, \quad m_V \sim \frac{M^2}{(\Phi + \Phi^\dagger)^2}$$

Dualize the eaten Goldstone boson to a 2-form gauge field *B*:

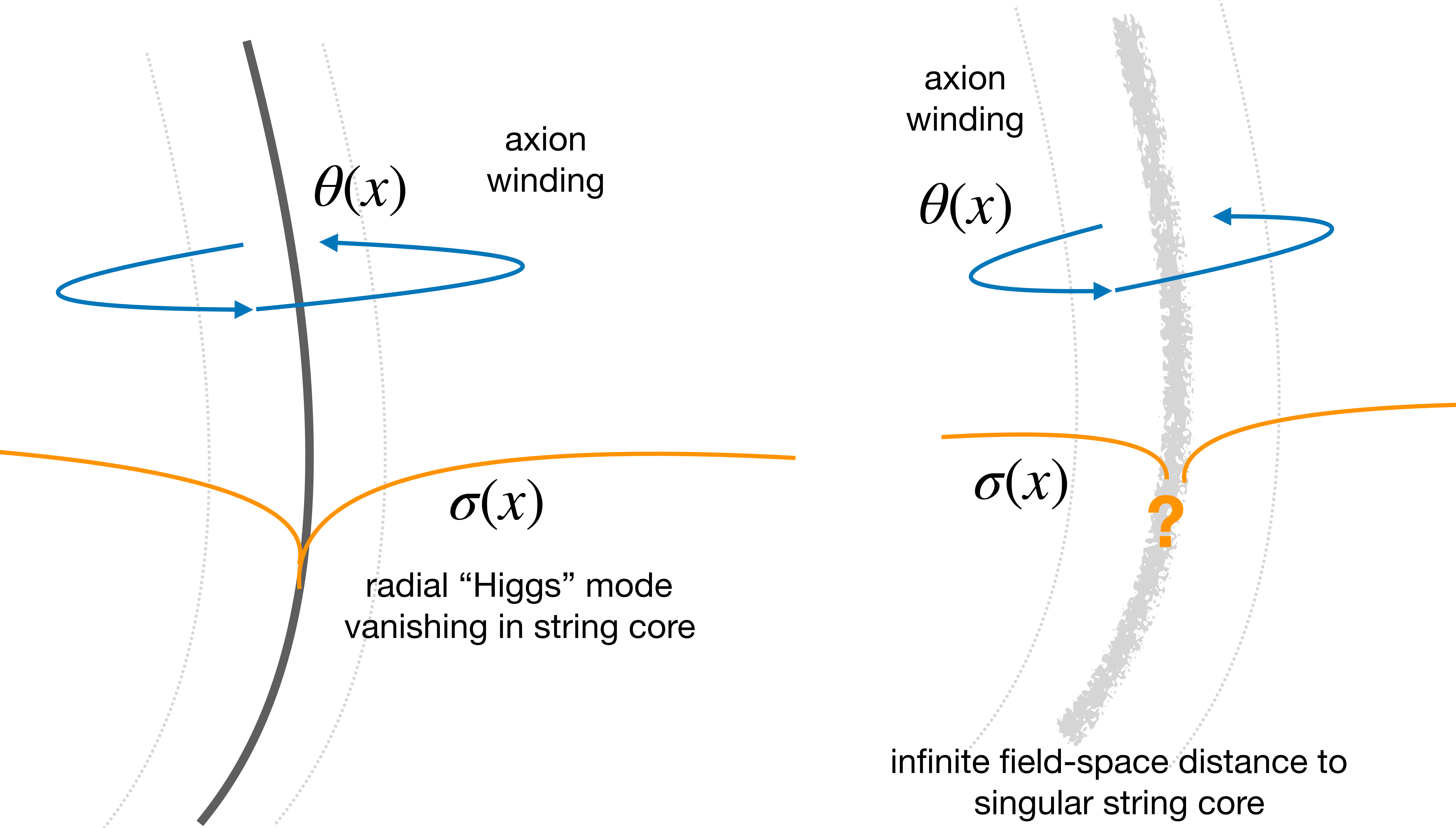
$$\epsilon^{\mu\nu\rho\lambda}\partial_{[\mu}B_{\nu\rho]} = f^2\partial^\lambda\theta$$

Now apply the **WGC** to the *B*-field: charged strings exist with tension $T \lesssim f\, M_{\mathrm{Pl}}$. (see Hebecker, Soler '17)

For Stückelberg masses—unlike the Higgs mechanism—these are *fundamental* strings.

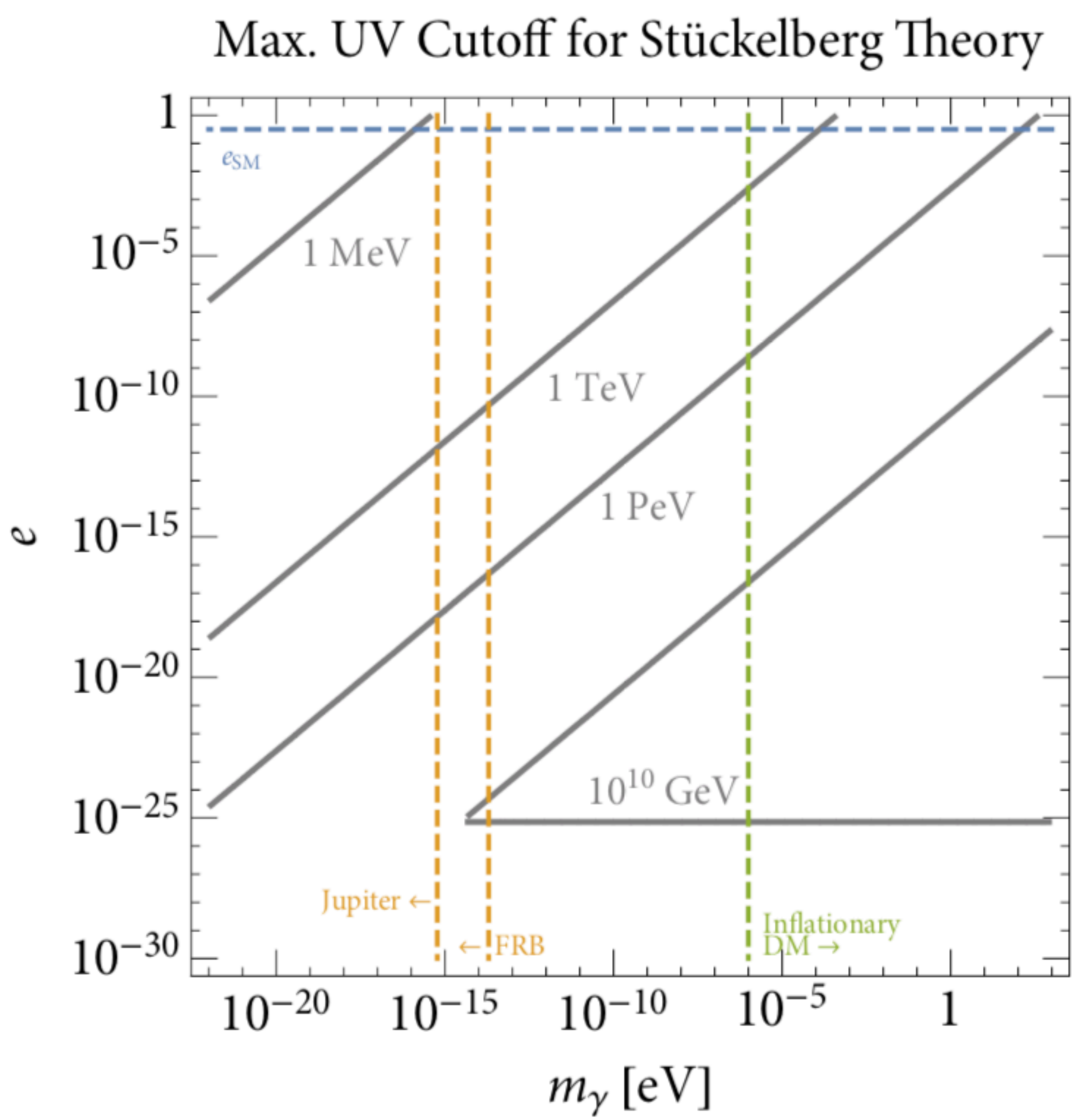
MR, '18

Abelian Higgs strings versus fundamental (Stückelberg) strings



MR, ’18

Ultraviolet cutoffs on Stückelberg photons



$$m_\gamma = ef$$

$e \to 0 : A_\mu$ weakly coupled

$f \to 0 : B_{\mu\nu}$ weakly coupled

$$\Lambda_{QG} \lesssim \min(e^{1/3} M_{Pl}, \sqrt{m_\gamma M_{Pl}/e})$$

the "inflationary DM" line is dark photon dark matter produced by inflationary fluctuations: Graham, Mardon, Rajendran 2015

MR, '18

Can the photon have a mass?

For the SM photon, very simple kinematic bounds (from fast radio bursts) tell us

$$m_\gamma \lesssim 10^{-14}\ \mathrm{eV}$$

A mass at this scale leads to local EFT breaking down at low energies:

$$\Lambda_{\mathrm{QG}} \lesssim \sqrt{m_\gamma M_{\mathrm{Pl}}/e} \lesssim 10\ \mathrm{MeV}$$

So the SM photon can't have a Stückelberg mass.
Loophole is the unit of charge: suppose the electron charge is N, i.e. what we know as e is really $e_0 N$ for $N >> 1$.

We can push the UV cutoff above a TeV if $N \sim 10^{14}$.
(Or Higgs mechanism: Higgs is millicharged, similarly huge N.)

Not very *plausible*, but not logically inconsistent?

MR, '18

Briefly: non-slow-roll inflation

One topic that hasn't been coming up much at this meeting is the possibility of non-slow-roll inflation models, if you suspect that quantum gravity is incompatible with slow roll.

A number of models rely on Lagrangians of the form:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m_{\gamma'}^2 A_\mu^2 - \frac{\beta}{4f_a}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)$$

As ϕ evolves, some Fourier modes of A become tachyonic:

$$\ddot{\mathbf{A}}_{\mathbf{k},\pm} + H\dot{\mathbf{A}}_{\mathbf{k},\pm} + \left(m_{\gamma'}^2 + \frac{k^2}{a^2} \mp \frac{k}{a}\frac{\beta\dot{\phi}}{f_a}\right)\mathbf{A}_{\mathbf{k},\pm} = 0$$

Then particle production can lead to friction slowing down the inflaton (Anber / Sorbo). In the nonabelian gauge theory case, interesting classical solutions (Adshead / Wyman "chromonatural").

Large photon-axion couplings?

For these applications the term $-\frac{\beta}{4f_a}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu}$ must have a large coefficient, typically

$$\beta \sim 100 \text{ to } 1000$$

But compactness of the axion and of U(1) leads us to expect

$$\beta = n\frac{g^2}{8\pi^2}, \quad n \in \mathbb{Z}$$

For chromonatural inflation, for instance, one needs $g \sim 10^{-6}$ and so $n \sim 10^{15}$. Enormous number! Obstacle to UV completions.

P. Agrawal, J. Fan, MR 1806.09621: $\Lambda_{\text{axion}} \lesssim f_a/N_{\text{clock}}$

In some models including Anber/Sorbo can explain large integer as product of small integers (KNP/clockwork). Chromonatural: cannot. Next option to explore: kinetic mixing with a lighter axion.

Dark Photon Dark Matter

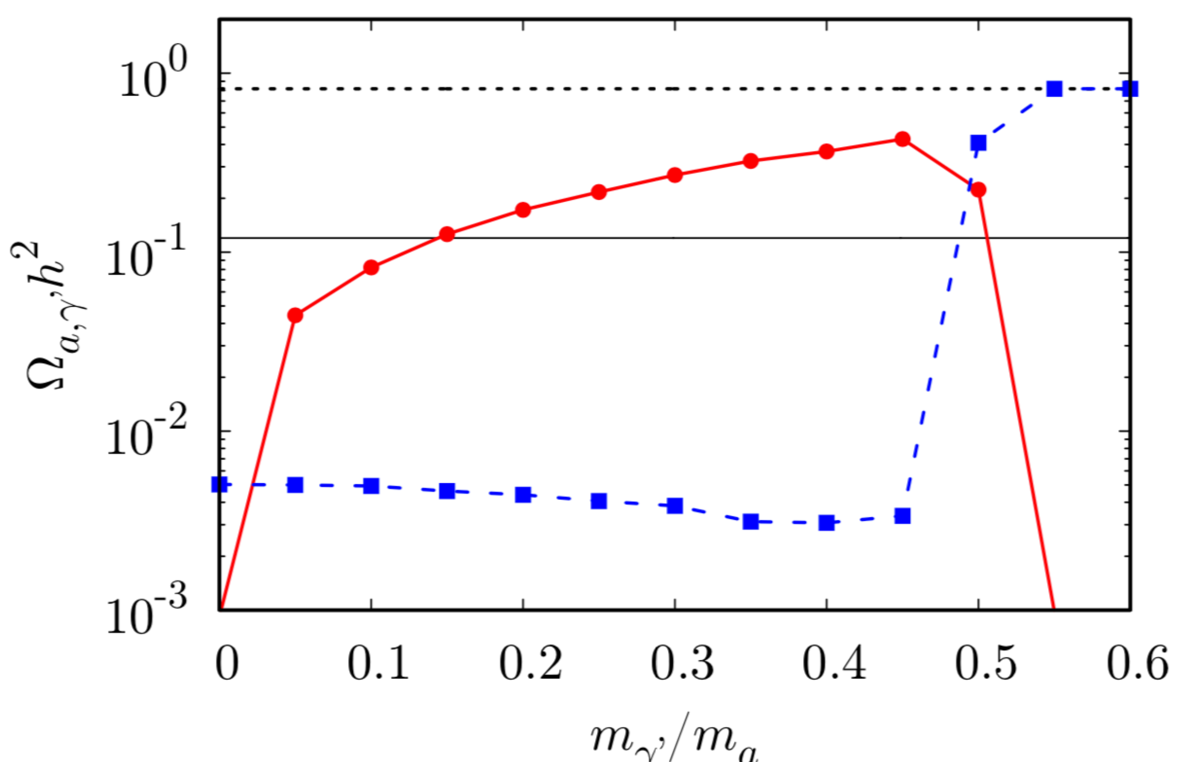
Very light dark photons are an interesting dark matter candidate, with many proposed experiments searching for them, but ***until now*** no model to generate their abundance.

$$-\frac{\beta}{4f_a}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

Can do it with tachyonic particle production from an oscillating axion.

Can be compatible with swampland photon mass constraints, but again need enormous enhancement of β.

Final dark photon DM relic abundance compared to the would-be axion relic abundance with no axion/dark photon coupling.



P. Agrawal, N. Kitajima, MR, T. Sekiguchi, F. Takahashi, to appear soon

Conclusions

The stronger *lattice/tower* forms of the WGC are tightly linked to the Swampland Distance Conjecture.

Crucially for many phenomenological applications, they imply bounds on the UV cutoff at which gravity is strongly coupled.

Sharpening this statement is a worthwhile goal, as it may be possible to *derive* the tower/lattice WGC from statements about a universal strong coupling scale.

Stückelberg masses in string theory are examples of infinite-distance points, and as such come with UV cutoffs. Strong bounds on photon (and dark photon) Stückelberg masses are of great real-world interest.