Tensionless Strings and the Swampland

1808.05958 w/ Seung-Joo Lee and Wolfgang Lerche
+ work in progress

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Motivation

Quantum Gravity conjectures testable in string theory

- Check of swampland conjectures and sharper formulation
- Study manifestations of swampland conjectures in string geometry: (Why) Does mathematics know about QG?

This talk: **Swampland/QG Conjectures in 'open string theories'**

1. No Global Symmetries: [Banks,Dixon'88]
   Gauge symmetries cannot become global in presence of gravity.
   What goes wrong if they (seemingly) do?
2. Completeness Conjecture: [Polchinski'03]
   The full charge lattice should be populated.
   Open string charges seem limited - where do the states come from?
3. Weak Gravity Conjecture: [Arkani-Hamed,Motl,Nicolis,Vafa'06]
   (Sub)Lattice of charged particles with $q^2 g_{\rm YM}^2 \geq \# M^2$
   Can the numerical bound be tested?

Main Result

Consider most general F-theory compactification to 6d with 8 supercharges.

Whenever there exists a geometric limit where $g_{\rm YM} \to 0$ while $M_{\rm Pl}$ fixed, we

- prove the Sublattice Weak Gravity Conjecture
- including the effect of scalar fields
- and determine the index of the relevant sublattice of **non-BPS states**.

✓ Implies by duality similar result for 6d heterotic string (N=(1,0))

✓ Related to BPS state counting in 5d M-theory

F-theory in 6d

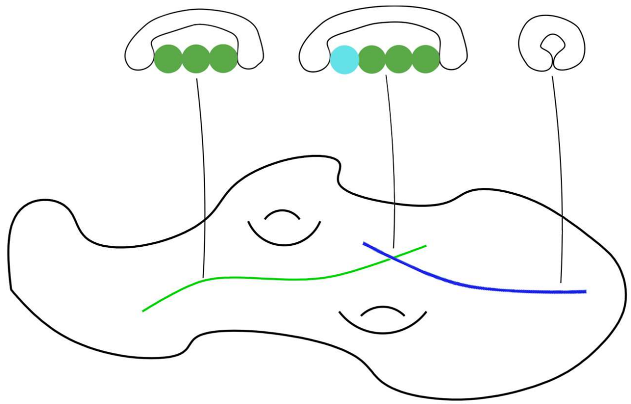
Context: F-theory compactified on elliptic CY_3 $Y_3 \to B_2$

Effective theory in $\mathbb{R}^{1,5}$:

$N = (1,0)$ supergravity (8 SUSYs)

base B_2: complex Kähler surface

7-branes on complex curve $C \subset B_2$



Couplings:

$\mathbf{M_{Pl}^4 = 4\pi vol_J(B_2)}$ $\quad\quad$ $\mathbf{\frac{1}{g_{YM}^2} = \frac{1}{2\pi} vol_J(C)}$

- non-abelian gauge algebra $\mathfrak{g}$:
  C contained in discriminant of fibration (wrapped by brane stack)
- abelian gauge algebra $\mathfrak{g} = \mathfrak{u}(1)_A$:
  $C = -\pi_*(\sigma(S_A) \cdot \sigma(S_A))$ (height pairing of rational section S_A)

Gravity and U(1)s

What happens if take $g_{\mathrm{YM}} \to 0$ at M_{Pl} finite?

Field theory intuition:

1. Weak Gravity Conjecture [Arkani-Hamed,Motl,Nicolis,Vafa'06], ...
   Gravity is weakest force.
2. In presence of gravity, no global symmetries. [Banks,Dixon'88], ...

General expectation: [Ooguri,Vafa'06], ...

✓ Offensive limit should be at infinite distance (beyond reach)

✓ Effective theory must break down (quantum gravity censorship)

[Kläwer,Palti'16] [Palti'17] [Grimm,Palti,Valenzuela'18] [Heidenreich,Reece,Rudelius'16/'18] [Montero,Shiu,Soler'16] [Andriolo,Junghans,Noumi,Shiu'18] [Blumenhagen et al.'18] [Hebecker et al.'15]

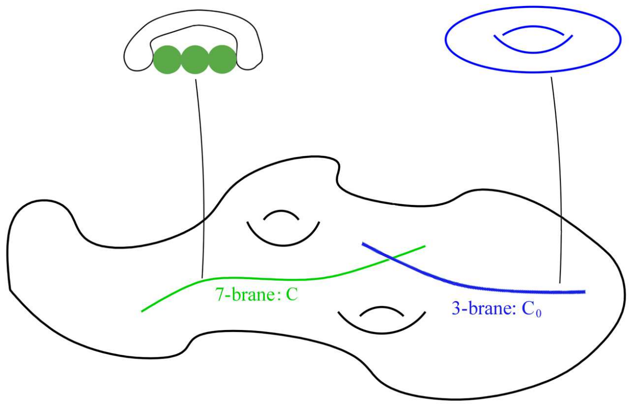
talks by Valenzuela, Palti, Shiu, Reece, Rudelius, Soler, Montero, ...

New: Quantum Gravity Conjectures and $U(1)$/gauge symmetries in 'open string sector'

Summary of results

Main results: [Lee,Lerche,TW'18]

1. For fixed $M_{\rm Pl}$, limit $g_{\rm YM} \to 0$ lies at infinite distance in Kähler moduli space of base B_2.
2. As $g_{\rm YM} \to 0$, necessarily charged tensionless weakly coupled strings arise in the 6d compactification.
   Math: **Mori's cone theorem**
   - **7-brane on C**
     $\mathrm{vol}_J(C) \sim t \sim \frac{1}{g^2_{\rm YM}} \to \infty$
   - **curve C_0 with $C_0 \cdot C \neq 0$**
     $\mathrm{vol}_J(C_0) \sim \frac{1}{t} \to 0$



3. The charged tensionless strings imply a breakdown of the effective theory.

Summary of results

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1. For fixed $M_{\rm Pl}$, limit $g_{\rm YM} \to 0$ lies at infinite distance in Kähler moduli space of base B_2.
2. As $g_{\rm YM} \to 0$, necessarily charged tensionless strings arise in the 6d compactification.
3. The charged tensionless strings imply a breakdown of the effective theory.
4. The tensionless string is the critical 6d heterotic string and is weakly coupled in the tensionless limit.

   Its tower of particle excitations can be analyzed quantitatively:

   - Charged string excitations satisfy the Completeness Hypothesis.
   - A sublattice of charged states satisfies the Sublattice Weak Gravity Conjecture bound - including the effect of scalar fields!

Global limit in Kähler geometry

[Lee,Lerche,TW'18]

Aim: $$\frac{1}{g^2_{\rm YM}} \sim {\rm vol}_J(C) \to \infty \quad \text{while} \quad M_{\rm Pl} \sim {\rm vol}_J(B_2) \equiv 1 \quad (*)$$

Result: There must exist another curve C_0 with

$$C_0 \cdot C \neq 0 \qquad \text{and} \qquad {\rm vol}_J(C_0) \to 0$$

General intuition

"On finite volume surface, if one direction gets big, normal direction must get very small".

Step 1) Limit $(*)$ requires asymptotically - (in full generality!)

$$J = tJ_0 + N \qquad \text{with} \qquad t \to \infty, \qquad N \text{ finite}$$

- ${\rm vol}_J(C) \stackrel{!}{\to} \infty \Rightarrow \int_C J_0 \geq 1 \Rightarrow {\rm vol}_J(C) \to \infty$ as $t \to \infty$
- ${\rm vol}(B_2) = \frac{1}{2}\int_{B_2} J^2 \stackrel{!}{=} 1 \Rightarrow \int_{B_2} J_0 \cdot J_0 = 0, \quad \int_{B_2} J_0 \cdot N \to \frac{1}{t}$

Global limit in Kähler geometry

Step 1) $J \sim tJ_0 + N$ with $\int_{B_2} J_0^2 = 0,\quad \int_{B_2} J_0 \cdot N \to \frac{1}{t}$

Step 2) J_0 is the class of a holomorphic curve C_0, i.e.

$$J_0 \sim [C_0] \qquad [C_0] \text{ is } \mathbb{P}^1$$

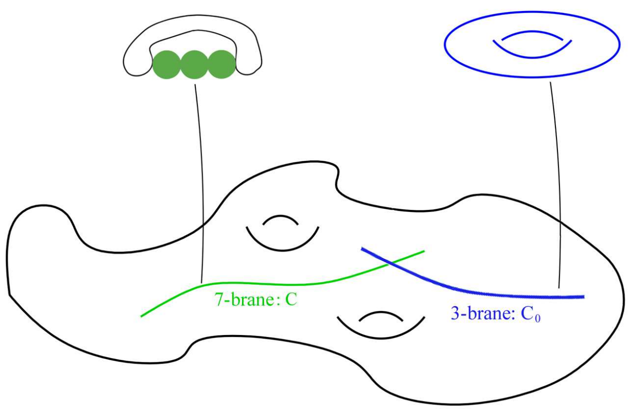
✓ $\mathrm{vol}_J(C_0) = \int_{C_0}(\cancel{tJ_0} + N) \to \frac{1}{t} \to 0$ as $t \to \infty$

✓ $C_0 \cdot C \neq 0$ because $\int_C J_0 \neq 0$

✓ $C_0 \cdot C_0 = 0$ because $\int_{C_0} J_0 = 0$

- **7-brane on C**
  $\mathrm{vol}_J(C) \sim t \sim \frac{1}{g_{\mathrm{YM}}^2} \to \infty$

- **curve C_0 with $C_0 \cdot C \neq 0$**
  $\mathrm{vol}_J(C_0) \sim \frac{1}{t} \to 0$



Quantum Gravity Conjectures

1) No global symmetries.

$\Rightarrow$ *The limit must be at infinite distance in moduli space.*

Indeed this is the case here.

Result: Limit $t \to \infty$ at distance $\Delta \sim \log(t) \to \infty$

2) Swampland Distance Conjecture:

[Ooguri,Vafa'06] [Kläwer,Palti'16] [Palti'17] [Heidenreich,Reece,Rudelius'17,'18]

[Grimm,Palti,Valenzuela'18] [Andriolo,Junghans,Noumi,Shiu'18]

[Blumenhagen,Kläwer,Schlechter,Wolf'18]

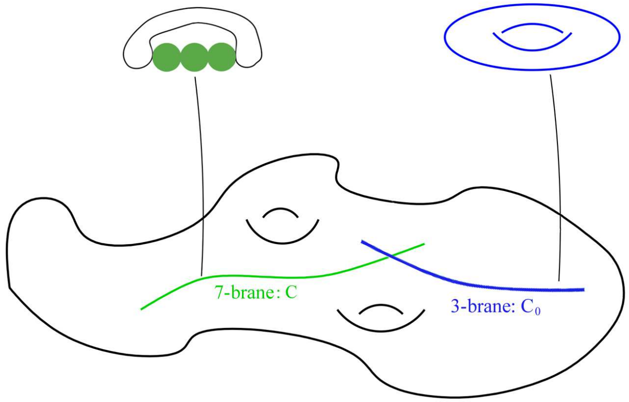
Infinitely many (charged!) states should become massless at exponential rate.

Present case: **Tensionless strings**

$$\Delta \sim \log(t), \qquad \textbf{tension} \sim T \sim \text{vol}_J(C_0) \sim \frac{1}{t} \sim e^{-\Delta}$$

Tensionless strings

- 7-brane on C
- D3-brane on $\mathbb{P}^1$ C_0 with $C_0 \cdot C \neq 0$ and $C_0^2 = 0$
  $\Rightarrow$ effective string in 6d



Twisted reduction of $N = 4$ SYM with varying gauge coupling along C_0

[Martucci'14][Haghighat,Murthy,Vafa,Vandoren'15][Lawrie,Schafer-Nameki,TW'16]

- 2d $N = (0, 4)$ effective theory describes worldsheet theory
- At intersection $C_0 \cap C$:
  isolated 3-7 string modes **charged** under 7-brane gauge group

Analysis of worldsheet theory on $C_0 = \mathbb{P}^1$ with $C_0^2 = 0$:

The effective string is the critical heterotic string propagating in 6d.

Tensionless 6d strings

Case 1)

F-theory base B_2 is Hirzebruch

$$p: \quad \mathbb{P}^1_f = C_0 \;\rightarrow\; \begin{array}{c}\mathbb{F}_a \\ \downarrow \\ \mathbb{P}^1_b\end{array}$$

Exists perturbative heterotic dual on K3 $\mathcal{K}$

$$r: \quad T^2 \;\rightarrow\; \begin{array}{c}\mathcal{K} \\ \downarrow \\ \mathbb{P}^1_b\end{array}$$

$$(g_s^h)^2 = \frac{\mathrm{vol}_J(\mathbb{P}^1_f)}{\mathrm{vol}_J(\mathbb{P}^1_b)} \rightarrow 0 \quad \text{in tensionless limit}$$

Case 2) General base B_2

- Dual heterotic string on $\mathcal{K}$ is in general not perturbative due to presence of extra NS 5-branes on heterotic side
- Working assumption: Away from these defects heterotic string can be treated 'quasi-perturbatively' (justified in examples)

The elliptic genus

Elliptic genus of 6d het. string $\leftrightarrow$ Subsector of charged non-BPS states

[Schellekens,Warner'87] [Witten'87], ...

$$Z_{\mathcal{K}}(\tau, z) \;\equiv\; \mathrm{Tr}_R \left[(-1)^F F^2 q^{H_L} \bar{q}^{H_R} \xi^J\right]$$

$q = e^{2\pi i \tau}$: τ complex structure of T^2

$\xi^J = e^{2\pi i z J}$: fugacity w.r.t. flavour symmetry $U(1)$

- Contains information only about trace of left-moving excitations of N=(0,4) worldsheet CFT
- Level-matched (!) physical states by pairing with right-movers
  $\Rightarrow$ Subsector of particle excitations of the 6d string

A chain of arguments identifies [Lee,Lerche,TW'18]

$$Z_{\mathcal{K}}(\tau, z) = -q^{-1}\mathcal{F}^{(0)}_{C_0}(\tau, z) = -q^{-1}\sum N^{(0)}_{C_0}(n, r)\, q^n\, \xi^r\,.$$

$\mathcal{F}^{(0)}_{C_0}(\tau, z)$: Genus zero prepotential of topological string on elliptic Y_3

The elliptic genus

6d F-theory on S^1

string wrapped on S^1
wrapping number w and
KK momentum k

5d M-theory

BPS particle in 5d
M2 brane on
$wC_0 + k\mathbb{E}_\tau$

Relation: $w\,k = n$ [Klemm,Mayr,Vafa'96]

Index 6d string non-BPS excitations

Gopakumar-Vafa invariants of 5d BPS states

[Haghighat,Iqbal,Kozaz,Lockhart,Vafa'13] [Haghighat,Klemm,Lockhart,Vafa'14], + many works!

$$Z_{\mathcal{K}}(\tau, z) = -q^{-1}\mathcal{F}^{(0)}_{C_0}(\tau, z) = -q^{-1}\sum N^{(0)}_{C_0}(n, r)\, q^n\, \xi^r$$

- (quasi-) modular form of U(1) fugacity index $m = \frac{1}{2}C \cdot C_0$

  [Lee,Lerche,TW'18]

  $\leftrightarrow$ Theory of weak Jacobi forms

- Computable via mirror symmetry for elliptic Y_3

  [Klemm,Mayr,Vafa'96][Klemm,Manschot,Wotschke'12][Huang,Katz,Klemm'15], ...

The elliptic genus

General properties of weak Jacobi forms suffice to show: [Lee,Lerche,TW'18]

Exists charge sublattice of finite index (at most) $2m = C \cdot C_0$ such that to each charge $\mathfrak{q}_k = 2m\,k\,,\,k \in \mathbb{Z}$ there exists state at excitation level n with

$$\mathfrak{q}_k^2 = 4m\,n(k)$$

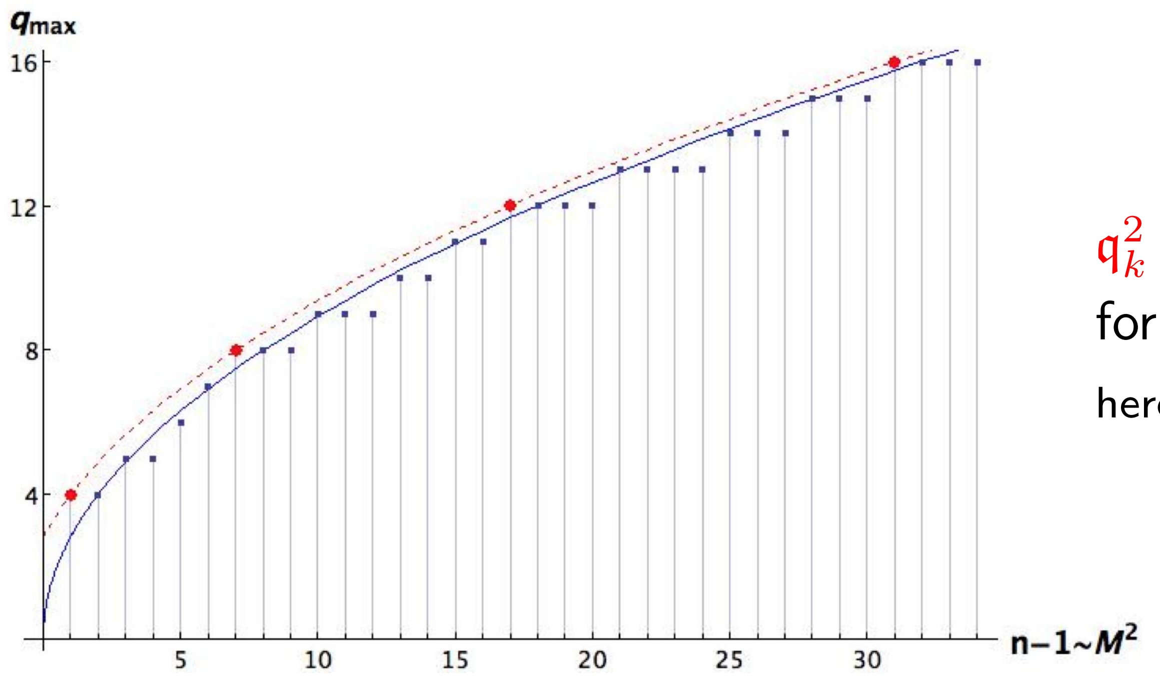
Heterotic on T^4: [Heidenreich,Reece,Rudelius]

Example: $B_2 = dP_2$

topological type of $U(1)$ model with $m_A = 2$

$$\begin{aligned}
\mathcal{F}^{(0)}_{C_0}(\tau,\xi) &= \frac{q}{1728\eta^{24}}\Big(-23E_4^2E_6\varphi_{-2,1}^2 + 27E_4^3\varphi_{-2,1}\varphi_{0,1} + 19E_6^2\varphi_{-2,1}\varphi_{0,1} - 23E_4E_6\varphi_{0,1}^2 \\
&\quad +E_2(-E_6^2\varphi_{-2,1}^2 + 2E_4E_6\varphi_{-2,1}\varphi_{0,1} - E_4^2\varphi_{0,1}^2)\Big) \\
&= -2 + \left(252 + 84\xi^{\pm 1}\right) q \\
&\quad + \left(116580 + 65164\xi^{\pm 1} + 9448\xi^{\pm 2} + 84\xi^{\pm 3} - 2\xi^{\pm 4}\right) q^2 \\
&\quad + \left(6238536 + 3986964\xi^{\pm 1} + 965232\xi^{\pm 2} + 65164\xi^{\pm 3} + 252\xi^{\pm 4}\right) q^3 \\
&\quad + \mathcal{O}(q^4)\,.
\end{aligned}$$

The elliptic genus



$\mathfrak{q}_k^2 = 4m\, n(k)$

for $\mathfrak{q}_k = 2m\, k$

here: $m = 2$, on dP_2

1) Completeness Hypothesis satisfied:

- Each charge q is populated by some state
- Note: Not the case for purely open [p,q] string sector!

Scalar Weak Gravity Conjecture

2) Sublattice version of Scalar Weak Gravity Conjecture satisfied (in perturbative asymptotic limit):

- Mass shell condition for perturbative string states:

$$M_n^2 = 8\pi T(n-1)$$

Crucial: Valid in pert. limit of het. string = tensionless limit $t \to \infty$

- Sublattice of states satisfies: $\mathfrak{q}_k^2 = 4m\, n(k)$

$$\begin{aligned} M_{\rm Pl}^4 &= 4\pi \,\mathrm{vol}(B_2) \equiv 4\pi\,, \qquad \frac{1}{g_{\rm YM}^2} = \frac{1}{2\pi}\mathrm{vol}(C) \\ T &= 2\pi\,\mathrm{vol}(C_0)\,. \end{aligned}$$

with $\mathrm{vol}(C)\,\mathrm{vol}(C_0) \to 2m$ as $t \to \infty$

$$\mathfrak{q}_k^2\, g_{\rm YM}^2 = \frac{M_n^2}{M_{\rm Pl}^4} + 1 > \frac{M_n^2}{M_{\rm Pl}^4} \qquad \text{as} \quad t \to \infty$$

Scalar Weak Gravity Conjecture

Viewpoint 1: Decay of dilatonic RN BH [AMNV'06]

- $S = \int_{6\mathrm{d}} \frac{1}{2} dx \wedge *dx + \frac{1}{4g_{\mathrm{YM}}^2} e^{\alpha x} F_{\mu\nu} F^{\mu\nu} + \ldots$ with $\alpha = 1$
- WGC bound for decay of (d=6) dilatonic RN black hole:

$$q^2 g_{\mathrm{YM}}^2 \overset{!}{\geq} \frac{M^2}{M_{\mathrm{Pl}}^2} \left(\frac{d-3}{d-2} + \frac{\alpha^2}{4} \right)$$

[Heidenreich,Reece,Rudelius'15]

- Present case: Tensor scalar field enters g_{YM} as 'dilaton' with $\alpha = 1$!

Viewpoint 2: Scalar Weak Gravity Conjecture [Palti'17]

- Must exist WGC particle such that

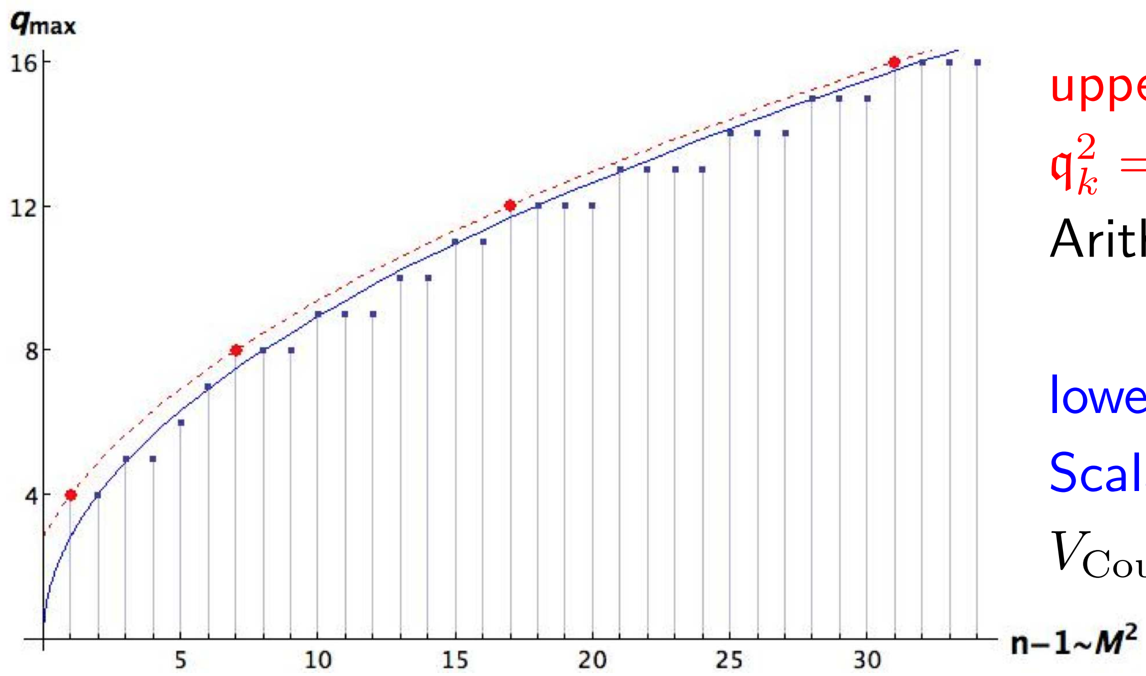
  Coulomb force $\overset{!}{\geq}$ Gravitational force in d=6 + Yukawa force

- Present context:

  $M_n = \langle M_n \rangle e^{-x/2} \Rightarrow$ Yukawa interaction $\equiv$ dilatonic correction!

- Both criteria exactly equivalent in asymptotic limit $t \to \infty$ and just barely met by sublattice tower of states! [Lee,Lerche,TW to appear]

Scalar Weak Gravity Conjecture



upper line:

$\mathfrak{q}_k^2 = 4m\, n(k)$

Arithmetics of Jacobi forms

lower line:

Scalar WGC bound

$V_{\rm Coul} = V_{\rm Grav} + V_{\rm Yuk}$

Our derivation valid in limit $g_{\rm YM} \to 0$ = tensionless limit $t \to \infty$

- Validity of $M_n^2 = 8\pi T(n-1)$
- $\mathrm{vol}(C)\mathrm{vol}(C_0) \to 2m$

What about general points in moduli space?

Corrections? talks by Shiu, Remmen, Soler

Summary

Systematic study of Quantum Gravity constraints for 6d open string U(1)s

As gauge symmetry becomes global:
Tower of infinitely many charged from tensionless '6d critical heterotic string'
Different from behaviour of SCFT strings ($C^2 < 0$): Strongly coupled!

Beautiful interplay between geometry, arithmetics, CFT and Quantum Gravity constraints

No global symmetries	Kähler geometry of B_2
Swampland Distance Conjecture	Mori's cone theorem
Completeness Hypothesis	Weak Jacobi forms
Scalar Weak Gravity Conj.	and their modular properties

How do (Weak) Jacobi forms know about the Weak Gravity Conjectures?

String Pheno 2019

24-28 June 2019 @CERN

The elliptic genus

$Z_{\mathcal{K}}(\tau, z)$ is related to more general index for string on curve C_0 on B_2

$$Z_{C_0}(\tau, \lambda_s, z) = \mathrm{Tr}_R(-1)^F F^2 q^{H_L} \bar{q}^{H_R} \xi^J u^{2J_-}$$

$q = e^{2\pi i \tau}$: τ complex structure of T^2
$\xi = e^{2\pi i z}$: fugacity w.r.t. flavour symmetry Cartan $U(1)$
$u = e^{2\pi i \lambda_s}$: fugacity w.r.t. $SU(2)_- \supset SO(4)_T$ (6d spin)

Z_{C_0} is a Weyl invariant Jacobi form of weight $w = 0$
fugacity index m_u and m_ξ determined by the geometry

$$\begin{aligned} \varphi_{w,\mathbf{m}}\left(\frac{a\tau+b}{c\tau+d}, \frac{\xi}{c\tau+d}\right) &= (c\tau+d)^w e^{2\pi i \frac{\mathbf{m}\, c}{c\tau+d}\frac{(\xi,\xi)}{2}} \varphi_{w,\mathbf{m}}(\tau,\xi) \\ \varphi_{w,\mathbf{m}}(\tau, \xi+\lambda\tau+\mu) &= e^{-2\pi i \mathbf{m}\left(\frac{(\xi,\xi)}{2}\tau + 2\frac{(\lambda,\xi)}{2}\right)} \varphi_{w,\mathbf{m}}(\tau,\xi) \end{aligned}$$

[Klemm,Mayr,Vafa'96]

[Klemm,Manschot,Wotschke'12] [Haghighat,Lockhart,Vafa'14] [Haghighat,Klemm,Lockhart,Vafa'14] [Huang,Katz,Klemm'15] [Haghighat,Murthy,Vafa,Vandoren'15] . . .

The elliptic genus

$$Z_{C_0}(\tau, \lambda_s, z) = \mathrm{Tr}_R(-1)^F F^2 q^{H_L} \bar{q}^{H_R} u^{2J_-} \xi^J$$

Properties largely fixed by modular properties:

- Fix fugacity index w.r.t. $U(1)$ [Lee,Lerche,TW'18]

$$\begin{aligned} m_\xi &= \frac{1}{2} C_0 \cdot b \qquad b = -\pi_*(\sigma(S_A) \cdot \sigma(S_A)) \quad (U(1)\,\text{height pairing}) \\ &\equiv \frac{1}{2} C_0 \cdot C \end{aligned}$$

Anomaly arguments of [Schellekens,Warner]; [Benini,Eager,Hori,Tachikawa'13] [Haghighat,Lockhart,Vafa'14],. . . , [Xu,TW'17]

- Ansatz from analysis of pole structure:

[Haghighat,Murthy,Vafa,Vandoren'15], [Huang,Katz,Klemm'15], . . .

$$Z_{C_0}(\tau, \lambda_s, \mathbf{z}) = \left(\frac{1}{\eta^2(\tau)}\right)^{6C_0 \cdot \bar{K}} \frac{\Phi_{W,L,\mathbf{m}}(\tau, \lambda_s, \mathbf{z})}{\varphi_{-2,1}(\tau, \lambda_s)} .$$

$W = 6C_0 \cdot \bar{K} - 2 = 10, \qquad L = \frac{1}{2} C_0 \cdot (C_0 + K) + 1 = g(C_0) = 0$

The elliptic genus

- Encoded in free energy of topological string on same elliptic Y_3:

  $\mathcal{F}(\lambda_s, \tau, \mathbf{t}, \mathbf{z}) = \sum_{g \geq 0} \mathcal{F}^{(g)}(\tau, \mathbf{t}, \mathbf{z}) \lambda_s^{2g-2}$

  Topological string on elliptic Y_3 $\leftrightarrow$ Elliptic genus of strings

$$Z_{\text{top}} = \exp\left(\mathcal{F}(\lambda_s, \tau, \mathbf{t}, \mathbf{z})\right) \quad = \quad Z_0(\tau, \lambda_s) \left(1 + \sum_{C_\beta} Z_{C_\beta}(\tau, \lambda_s, \mathbf{z})\, e^{2\pi i t_\beta + \frac{1}{2}(C_\beta \cdot \bar{K})\tau}\right)$$

  [Haghighat,Iqbal,Kozaz,Lockhart,Vafa'13] [Haghighat,Klemm,Lockhart,Vafa'14]

- The non-spin refined object $Z_{\mathcal{K}}(\tau, z)$ identified from '1-string-irreducible' contribution to free energy at lowest order in λ_s:

$$Z_{\mathcal{K}}(\tau, z) = -q^{-1} \mathcal{F}^{(0)}_{C_0}(\tau, z) = -q^{-1} \sum N^{(0)}_{C_0}(n, r)\, q^n\, \xi^r$$

  - for Hirzebruch: modular form of same U(1) fugacity index - see [Schellekens,Warner'87]
  - general B_2: only quasi-modular, and consistent with tensor transition of dual het. string! [Lee,Lerche,TW'18]