Conjeturas de la Ciénaga, Agujeros Negros y Estados BPS

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Based on work to appear w/ Murad Alim, Iñaki García Etxebarria, Ben Heidenreich, and Matt Reece

Outline

- Review of Swampland Conjectures
- A Tower/Sublattice WGC Counterexample?
- Black Holes and BPS States

Review of Swampland Conjectures

The Swampland Distance Conjecture

Ooguri, Vafa ‘06

At large distances in moduli space, a tower of resonances becomes light exponentially quickly with increasing distance:

$$m_n^2(\phi) \sim e^{-\alpha n \phi}$$

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa, '06

In any U(1) gauge theory that admits a UV completion with gravity, there must exist a "superextremal" state of charge q, mass m:

$$\frac{q}{m} \geq \frac{Q}{M}\Big|_{\text{ext}}$$

Strong Forms of the WGC

- Superextremal particles at:
  - Infinitely many sites in the charge lattice (Tower WGC) Andriolo, Junghans, Noemi, Shiu '18
  - Every charge site in a sublattice (sLWGC)
  - ~~Every site in the charge lattice (LWGC)~~

Heidenreich, Reece, T.R. '16

$$\mathrm{WGC}_d \overset{\sim}{\Rightarrow} \mathrm{T/sLWGC}_{d+1}$$

BPS States

- In theories with extended SUSY, massive states must satisfy BPS bound:

$$m \geq |\zeta_{q_i}(a_i)|$$

- Central charge ζ depends on moduli a_i, charge q_i of state
- In 4d $\mathcal{N} = 2$ theories from IIA on CY_3, D2-brane BPS states counted by Gopakumar-Vafa invariants

Gopakumar, Vafa, '98

A T/sLWGC Counterexample?

A T/sLWGC Counterexample?

- At conifold singularity in IIA, BPS bound becomes trivial:

$$m_{q=1} \to 0, \quad m_{q\neq 1} \not\to 0.$$

- No tower of BPS states!

- GMSV Conifold:

  Greene, Morrison, Strominger '95

  Greene, Morrison, Vafa '96

Empty "wedge" w/o BPS states

q_2 q_1	0	1	2	3	4
0	–	640	10032	288384	10979984
1	16	2144	231888	23953120	2388434784
2	0	120	356368	144785584	36512550816
3	0	−32	14608	144051072	115675981232
4	0	3	−4920	5273880	85456640608
5	0	0	1680	−1505472	3018009984
6	0	0	−480	512136	−748922304
7	0	0	80	−209856	218062416
8	0	0	−6	75300	−90910176
9	0	0	0	−21600	37721680
10	0	0	0	4312	−15086208
11	0	0	0	−512	5300736

4d Conifolds

(see also Grimm, Patti, Valenzuela '18)

- For IIA at conifold singularity, logarithmic running of gauge coupling:

$$g_4 \sim \log m \to 0$$

- Violation of T/sLWGC in 4d
- Need to renormalize WGC bound,

$$q(m)/m \geq 1/M_{\rm Pl}$$

- Recall: $\mathrm{WGC}_d \overset{\sim}{\Rightarrow} \mathrm{T/sLWGC}_{d+1}$
- WGC_3 not necessarily valid (see however Montero, Shiu, Soler '16)

5d Conifolds

- For 5d conifold, $g_5 \not\to 0$
- To be consistent with T/sLWGC, must have infinite tower of states becoming light whenever 5d gauge coupling vanishes
- Recall SDC: Infinite tower of light states $\Rightarrow$ expect divergence of field space metric
- Indeed, can prove zero coupling points are at infinite distance for M-theory compactifications on a CY_3 (up to certain regularity assumptions)

5d Conifolds (cont.)

- Claim: zero coupling points are at infinite distance for M-theory compactifications on CY_3
- Proof (sketch):
  - Write $\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K = 1$, with $C_{IJK}, Y^I \geq 0$.
  - Assume Laurent expansion for $Y^I(t)$ for path ending at t=0.
  - Show that metric behaves as $1/t^2$ whenever gauge kinetic term $a_{IJ}(t \to 0)$ blows up

Black Holes and BPS States

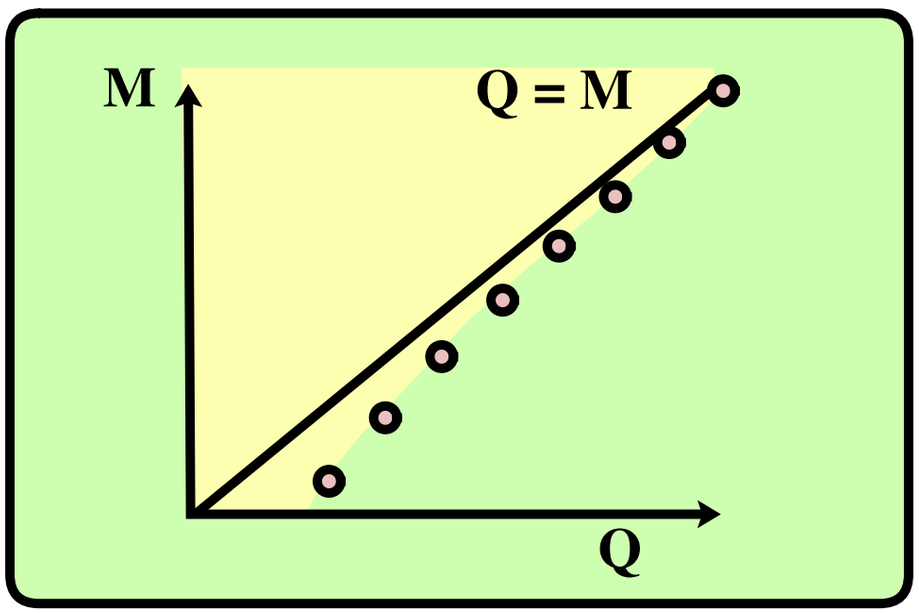
An Important Reminder

- In general,

$$\text{BPS} \neq \text{BHE}$$

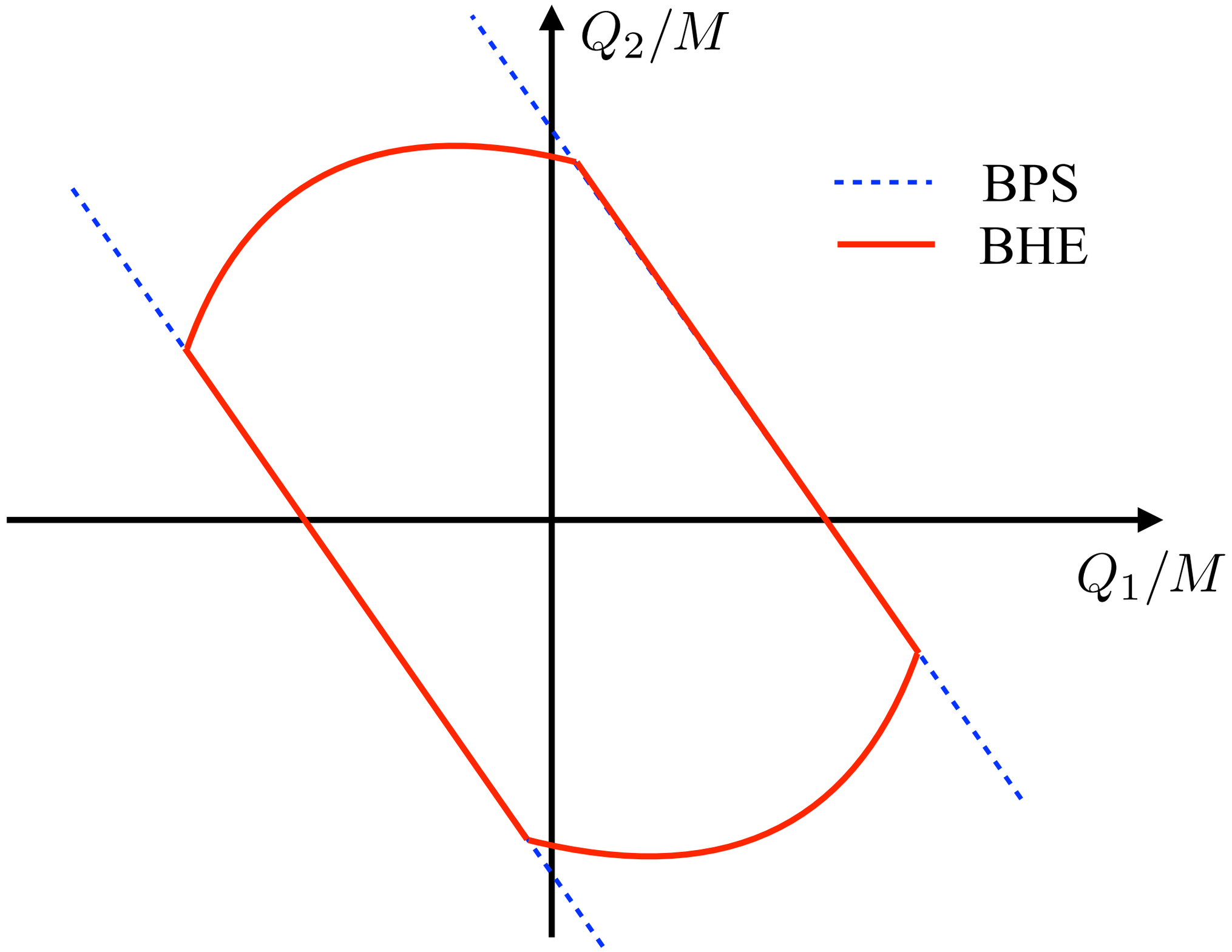
(even in theories with BPS states)

- Ex: heterotic string on torus



$$M^2 = Q^2 - 1$$

BPS vs. BHE



BPS vs. BHE

- Since $g_5 \not\to 0, \zeta \to 0$, must have BPS $\neq$ BHE for conifold
- Q: When does BPS = BHE?
- A: BPS = BHE $\Leftrightarrow \zeta_{q_i}(a_i) > 0 \, \forall a_i$

$$\Leftrightarrow \; q_i \in \bigcap_{\text{All phases}} \text{Mori Cone}$$

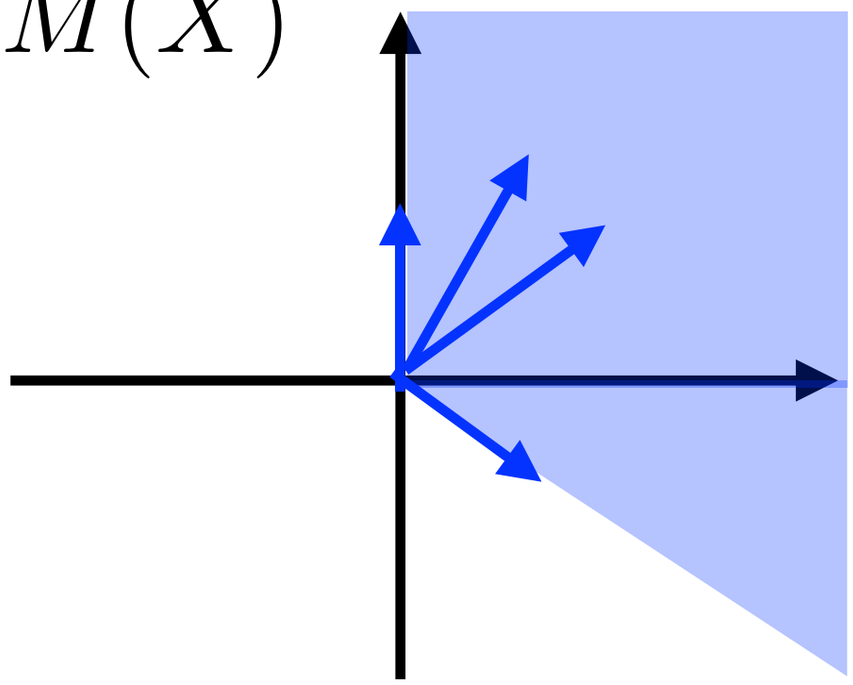
BPS vs. BHE (cont.)

- Claim: BPS = BHE $\Leftrightarrow \zeta_{q_i}(a_i) > 0 \, \forall a_i$
- Proof (sketch):
  - Critical points of ζ_{q_i} are all local minima (maxima) with $\zeta_{q_i} > 0$ ($\zeta_{q_i} < 0$).
  - By Morse theory, such local minima are therefore global minima
  - So, either:
    - $\zeta_{q_i} > 0$ everywhere (BPS=BHE)
    - $\zeta_{q_i} = 0$ somewhere, no critical points (no BPS BH solutions exist)

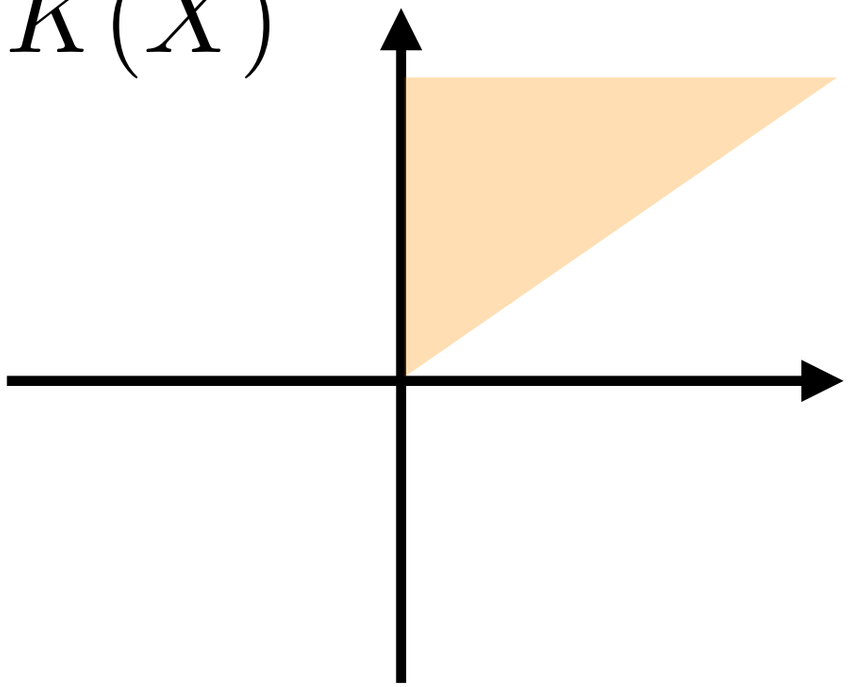
Primer on Algebraic Geometry

- Calabi-Yau X, irreducible curves $\{C_i\}$
- Mori Cone $M(X) := \{\sum a_i [C_i], 0 \leq a_i\}$
- Kähler Form $J \in H^{1,1}(X)$
- Kähler Cone $K(X) := \{J, \mathrm{Vol}(C_i) = \int_{C_i} J > 0\}$

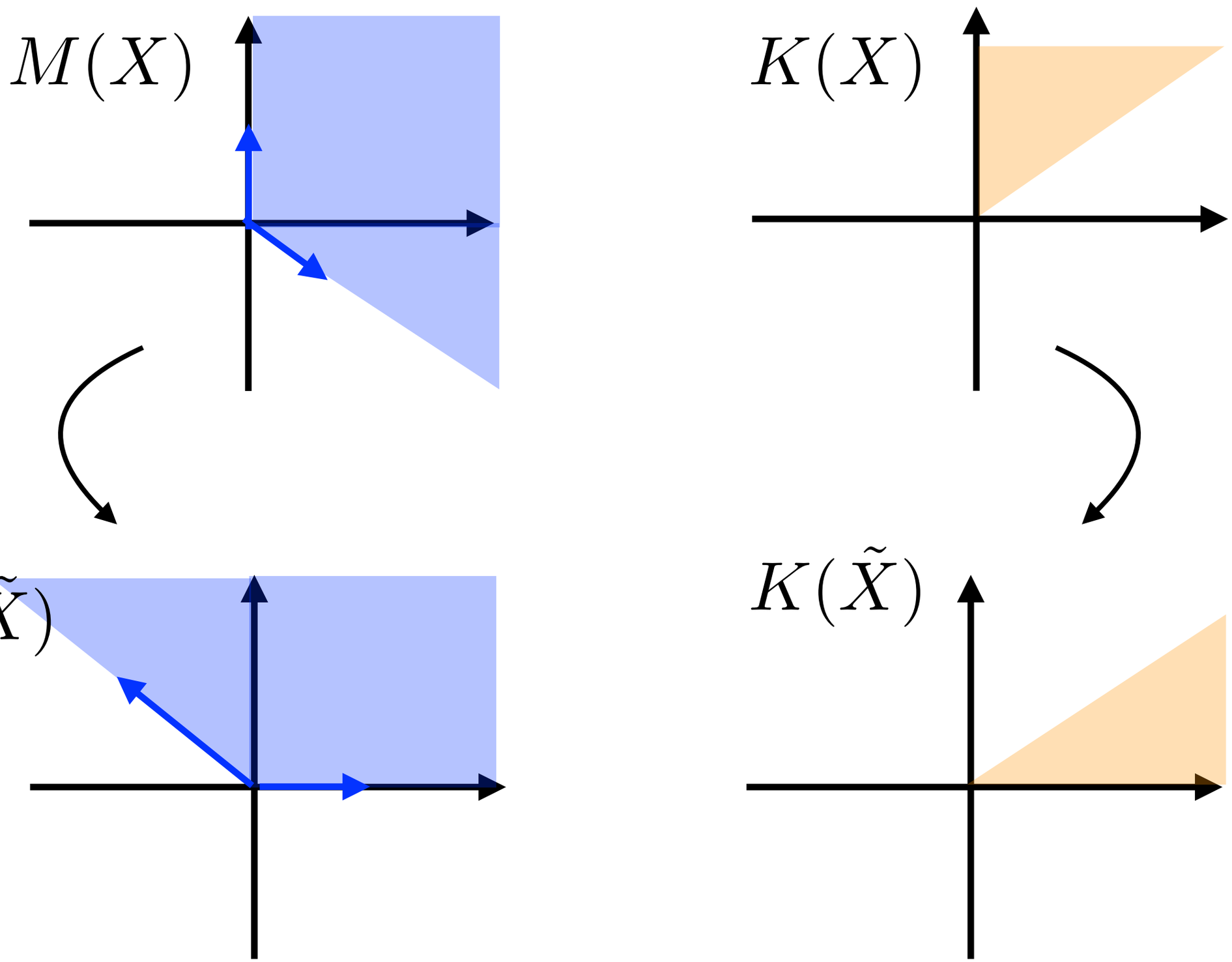
$M(X)$



$K(X)$



Flop $X \to \tilde{X}$

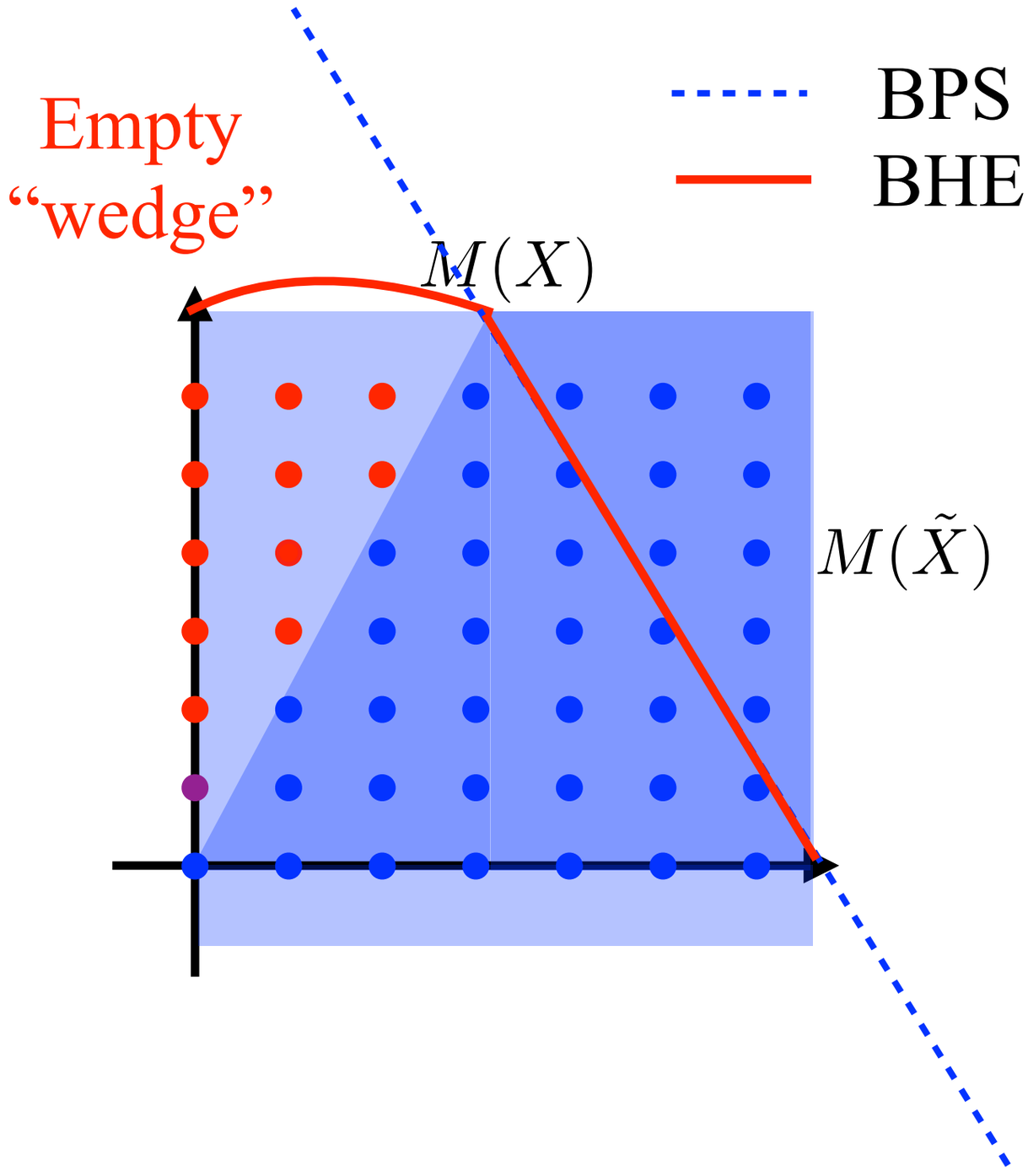


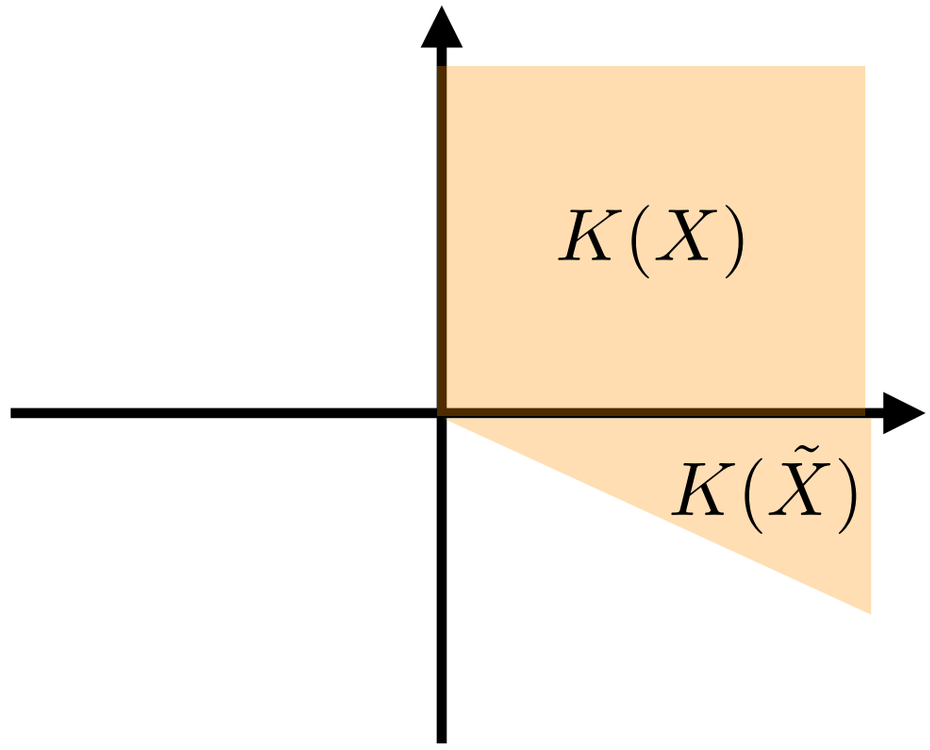
SUGRA and Geometry

- Curve $[C_i] \to$ charge $\vec{q} = \vec{q}([C_i])$
- $\zeta_{\vec{q}} > 0 \Rightarrow \mathrm{Vol}(C_i) > 0$
- So, BPS = BHE $\Leftrightarrow \zeta_{q_i}(a_i) > 0 \, \forall a_i$

$$\Leftrightarrow \; q_i \in \bigcap_{\text{All phases}} \text{Mori Cone}$$

GMSV Revisited





Summary

- Beautiful interplay between
  - BPS states and GV invariants
  - SUGRA central charges
  - Calabi-Yau geometry/intersection theory
  - Black hole extremality bounds

  leads to consistency with T/sLWGC and SDC
- Not to mention relationship between WGC, SDC and
  - Axion cosmology and inflation
  - Cosmic censorship (see Toby's talk)
  - Emergence (see Irene's and Matt's talks)
  - Gravitational instantons (see Pablo's talk)
  - AdS/CFT (see Miguel's talk)
  - Black hole entropy (see Grant's talk)
  - Scattering amplitudes (see Gary's talk)
  - F-theory and tensionless strings (see Timo's talk)
- Exciting time to be hiking through the Swampland!