DERIVING THE WGC FROM ENTANGLEMENT ENTROPY THEOREMS

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  - The **WGC:** Three proofs in one workshop (see Shiu & Remmen's talks)!

- I will present a "physics proof" of the WGC, to appear (very?) soon.
- Like any other proof, it has some **assumptions:**
    - **Holography:** Statements only in AdS, use CFT dual
    - Validity of **large N expansion/semiclassical bulk EFT**
- **No use** of usual crutches: SUSY or even (directly) String Theory
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    Exactly stable extremal black branes lead to contradiction with rigorous quantum info theorems.

- Argument by contradiction: Take a WGC-violating EFT in AdS, and derive an inconsistency.

- For concreteness, focus on Einstein-Maxwell-AdS (+possibly charged matter).

$$\int d^{d+1}x \left(\frac{R}{2\kappa_{d+1}^2} + \frac{d(d-1)}{\ell^2} \right) - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}$$

- This theory has black brane solutions

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + \frac{r^2}{\ell^2} \left(\sum_{i=1}^{d-1} dx_i^2 \right)$$

$$U(r) \equiv \frac{r^2}{\ell^2} - \frac{m}{r^{d-2}} + \frac{q^2}{r^{2d-4}} \qquad A = \mu - \frac{1}{c} \frac{\sqrt{2}g}{\kappa_{d+1}} \frac{q}{r^{d-2}} dt$$

- There is a horizon at a finite radius

$$\frac{d}{d-2} r_+^{2d-2} \geq q^2 \ell^2$$

- We will play a game with **two players**:
  - **Thermodynamics**
  - Holographic **entanglement entropy**
- We will look at these two things on states described by the **extremal black brane** mentioned above.
- Always work on CFT on R^d / AdS_{d+1} in the Poincaré patch.

- Holography 101: In the large N limit, the bulk EFT tells us about the thermodynamics of the dual CFT!

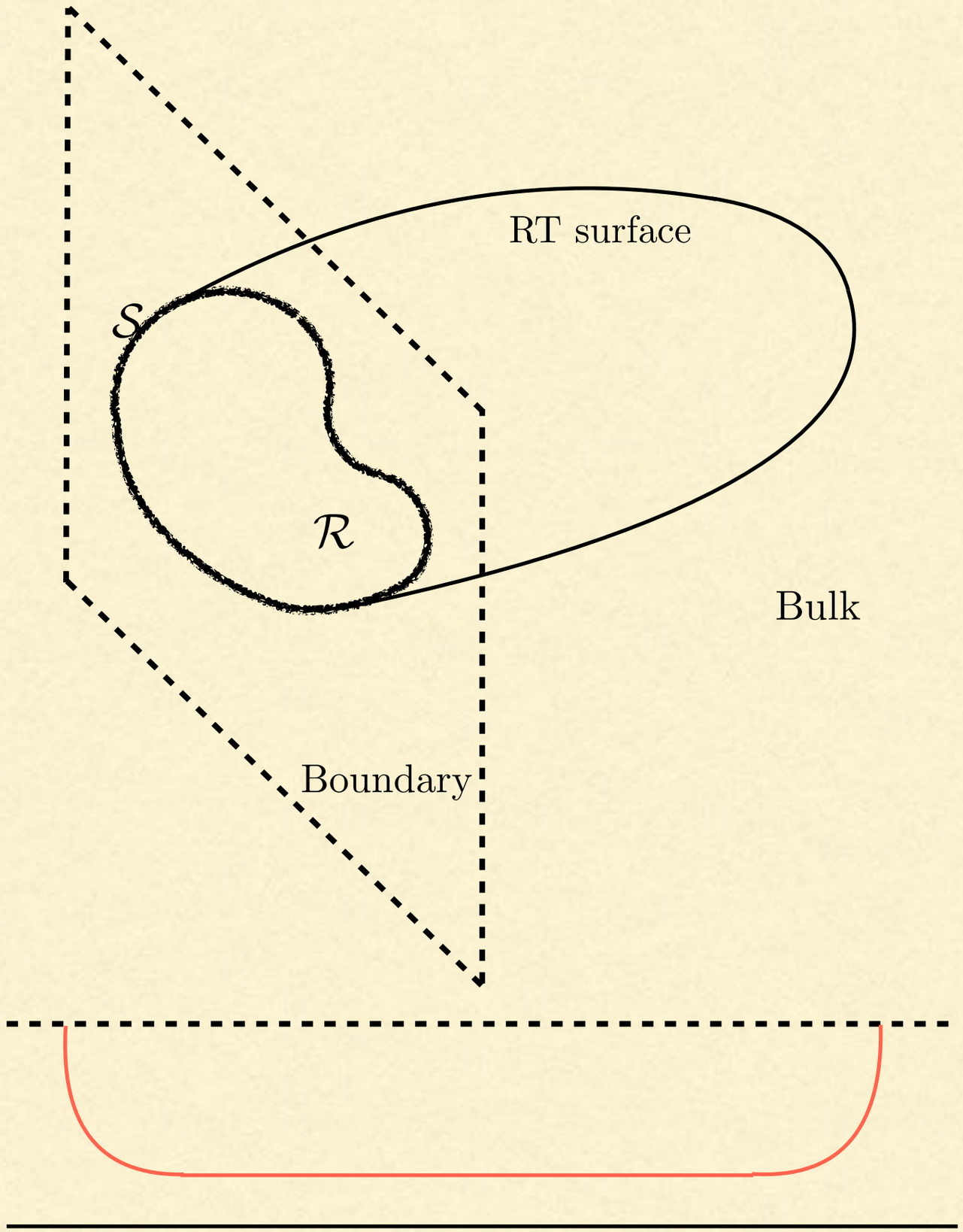
$$Z_{CFT} = \sum_{\text{Saddles}} e^{-S}$$

- Black brane has temp. β and voltage μ. It controls thermo of CFT in R^d with temp. β and chemical pot μ [Chamblin-Emparan-Johnson-Myers '99]

$$H \to H - \mu Q, \quad Q = \int J^0 d^{d-1}x$$

- At $\beta \to \infty$, ground state of CFT becomes dual to the extremal black brane. This has **finite** entropy density and linear heat capacity at low T.

$$S = \frac{A}{4G} = \frac{2\pi r_+^{d-1}}{\kappa_{d+1}} \qquad \frac{d\langle H - \mu Q\rangle_\beta}{d\beta} \propto T$$



- Ryu-Takayanagi '06 told us what the large N limit of EE (on semiclassical states) is:

$$S_{EE} = \frac{A_{\text{Extremal}}}{4G}$$

- We like to study scaling of EE with size of S. Several possibilities:
  - Volume law
  - Area law (typical of ground states)

- The black brane linear heat capacity leads to an upper **bound** on the entanglement entropy of ground states via the following Theorem: [Brandao-Cramer '14]

Consider a translationally invariant local Hamiltonian on a (d-1)—dimensional lattice. Let S be a cubic region of edge length L, and let |ψ> be a ground state. If the heat capacity is proportional to T^γ for low T, then the entanglement entropy of S satisfies a sub-volume law

$$S_{|\Psi\rangle}(L) \leq C_0 L^{d-2+\frac{1}{\gamma+1}}$$

for L sufficiently large and some constant C_0 independent of L

- In our case, this means that the ent. entropy of translationally inv. ground states scales at most as **subvolume**

$$S(L) \propto L^{d-3/2} \quad (\text{Area} \propto L^{d-2})$$

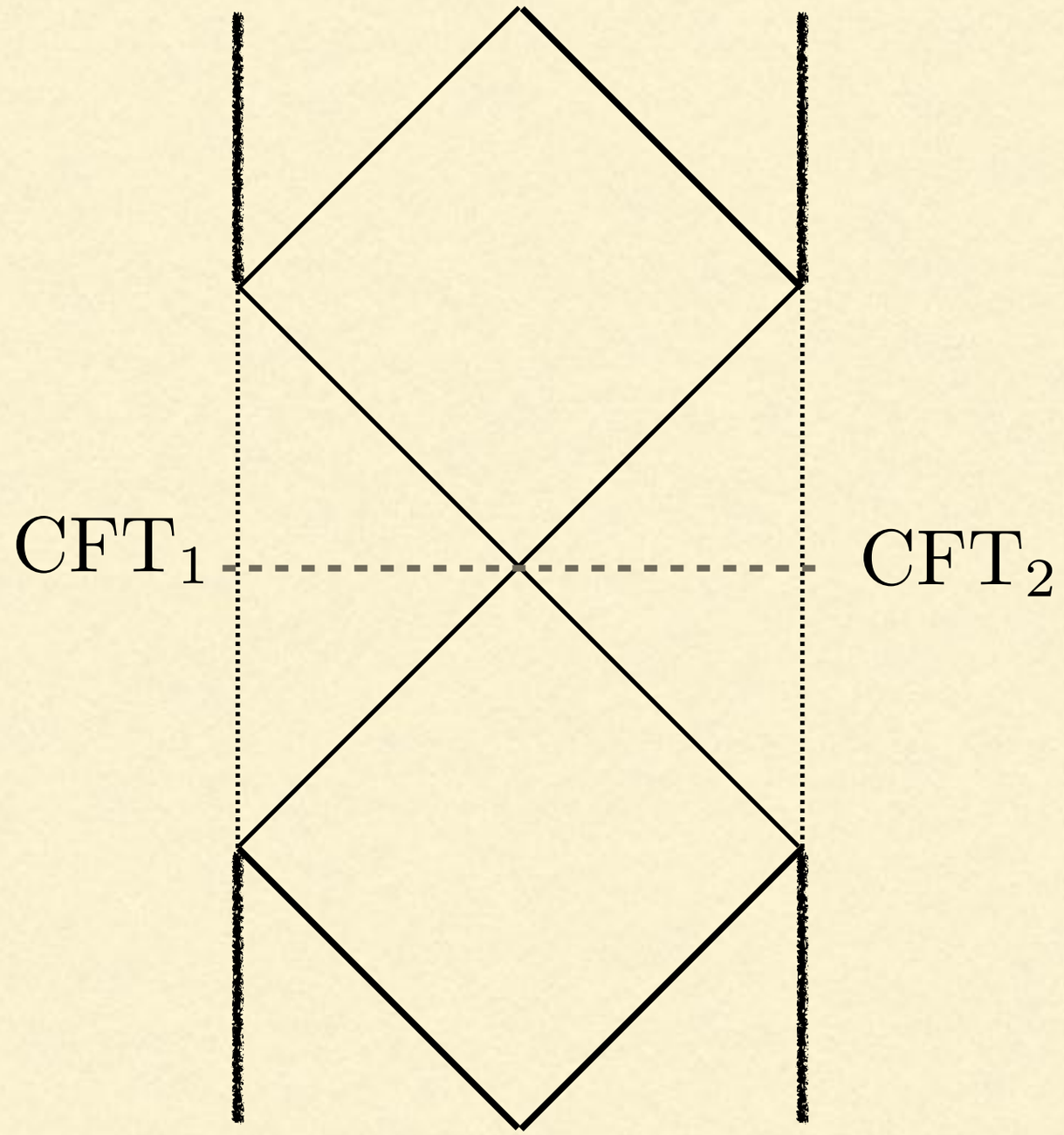
$$(\text{Volume} \propto L^{d-1})$$

Let's check the hypotheses:

- Lattice? We regularize the CFT to compute EE.
- Local Hamiltonian (nearest-neighbour)? Ensured by locality
- Linear heat capacity, $\gamma = 1$.

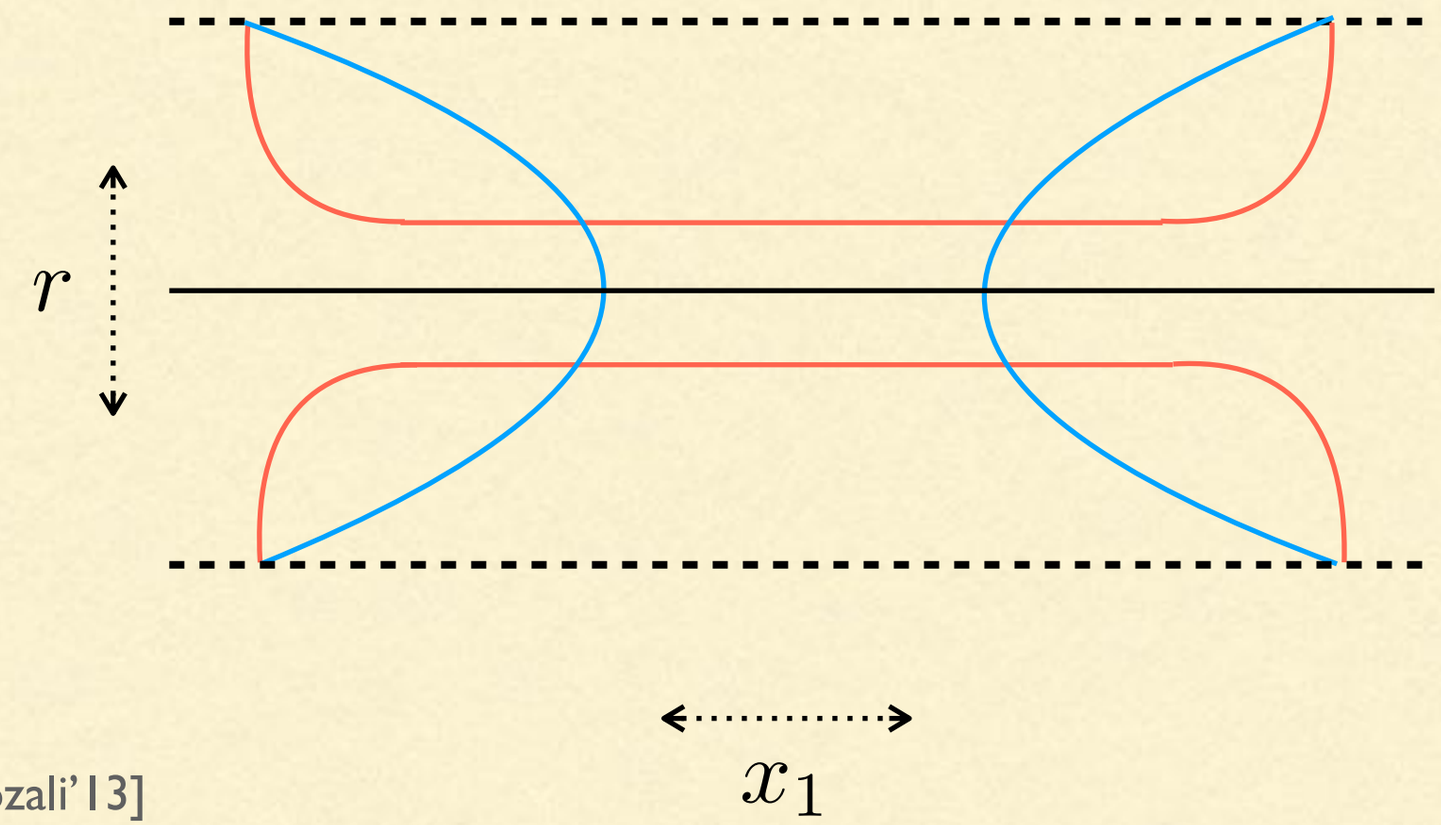
- We still need a **translationally invariant ground state.**
- We take the **thermofield double state** at zero temp.

$$|TFD\rangle = \frac{1}{Z}\sum_n e^{-\frac{\beta}{2}(E_n - \mu Q_n)}|n\rangle_L |n\rangle_R$$



  - Ground state ✔
  - Translationally invariant ✔
- It has a **semiclassical bulk description:** The two-sided black brane [Maldacena '02…]

- There are two competing RT surfaces. We're supposed to take the one with lower A.
  - Blue: Area law
  - Red: Volume law



- [Hartman-Maldacena '13, Andrade-Fischetti-Marolf-Ross-Rozali'13]

- In the T=0 TFD, the blue surface has **divergent A**.

- We get a **contradiction**: Volume law when the theorem forbids it. One of our hypothesis must be **wrong**:
  - Validity of EFT/large N framework ✓
  - Don't like T=0? Can repeat analysis in the $T \to 0$ limit ✓
  - Einstein-Maxwell / Stable black brane ✗
- More generally:

  Electrically charged extremal black branes must* be unstable.

* Should have polynomial low-T heat capacity. Unstable means there's another solution w. lower free energy.

- Simplest modification of the EFT one can make to fix the above picture: Introduce **charged fields** with high-enough q/m

$$m^2 - \frac{g}{\kappa_{d+1}} q_\phi^2 \leq \frac{d(d-1)}{4\ell^2}$$

The (mild) **Weak Gravity Conjecture**.

- Comes with AdS correction; One of the AdS WGC variants advocated in [Nakayama-Nomura '15]

- BH is unstable; decays to horizonless "electron star".

- Only possibility I know of, in the EFT regime. **Higher derivative terms** suppressed by some UV scale should not be very important for huge black branes (with very high μ).

- Also, connection to another quantum information theorem [Brandao-Horodecki '13]

Exponential decay of **all** correlators → Area law for entanglement

- Can check exp. decay of correlators in the worldine approximation [Andrade-Fischetti-Marolf-Ross-Rozali'13]

- Connection to **ER=EPR**? The extremal thermofield double has a wormhole, but its length is infinite.

CONCLUSIONS

- Quantum Information Theorem = No stable electric black branes.
- EFT's that predict these are sick & in the Swampland, leading to the (mild) WGC in AdS.
- Take home message:

  Entanglement as a Swampland constraint machine

  - Check other conjectures: SDC[Ooguri-Vafa '06,…], Scalar-WGC [Palti '17], Convex-Hull [Cheung-Remmen '14]… or prove new ones!

Thank you!

¡Gracias!

FAQ

- **Q:** What about BPS black holes? They have an horizon & are stable.

  A: There are **no** examples of asymptotically AdS (only) electrically charged black branes which are BPS (that I know of). The extremality bound is **strictly above** the BPS bound.

- **Q:** Ok, so what about branes with magnetic charges?

  A: Turning on a magnetic charge = turning on a background magnetic field in the CFT. Breaks translational invariance of hamiltonian!

- **Q:** Bah, who cares. AdS is SUSY anyway. WGC=BPS bound.

  A: There can be U(1)'s which are not central charges (flavor U(1)'s). No BPS bound for these, yet the argument still applies! Also, saturating the BPS bound is not completely trivial…