The Weak Gravity Conjecture and Cosmic Censorship

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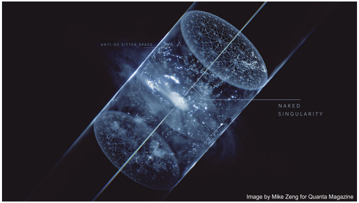
In collaboration with G. T. Horowitz and J. E. Santos



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2. Argue that these counter-examples are avoided if we include a sufficiently charged scalar field in our action, as the Weak Gravity Conjecture tells us to do.
3. Time permitting, discuss a possible violation of Cosmic Censorship for the *vacuum* Einstein equations on AdS_4.

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- In typical examples, these singularities are contained within Black Holes.
- Rough statement of Cosmic Censorship: This always happens

What do we mean by Cosmic Censorship?

Why is it important?

- Blessing: There should be no need to worry about Quantum Gravity in Astrophysics
- Curse: Difficult to do Quantum Gravity experiments if you want to be able to tell people the results

What do we mean by Cosmic Censorship?

Precise statement for our purposes

We say that Cosmic Censorship is violated if there exists an open set of smooth geodesically complete initial data such that for all solutions constructed from these initial data, some curvature invariant is unbounded in a region visible to observers at infinity.

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- What about $3+1$ dimensional asymptotically Anti-de Sitter space-times?
  - Super-radiance thought to violate Cosmic Censorship (B.E. Niehoff, J.E. Santos, B. Way 2015)
  - We have numerical evidence for Cosmic Censorship violation when a gauge field is included without a corresponding charged scalar.

Set-up

We take the action

$$S = \frac{1}{16\pi G} \int d^4x\sqrt{-g}\left(R - F^{ab}F_{ab} + \frac{6}{L^2}\right)$$

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- Boundary conditions: Flat boundary metric and for the gauge field we impose:

$$F_{tR}|_{z=0}(t, R, \phi) = \frac{a(v)R}{(1+R^2)^{3/2}}$$

where $v = t - \sqrt{1+R^2}$.

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Key points

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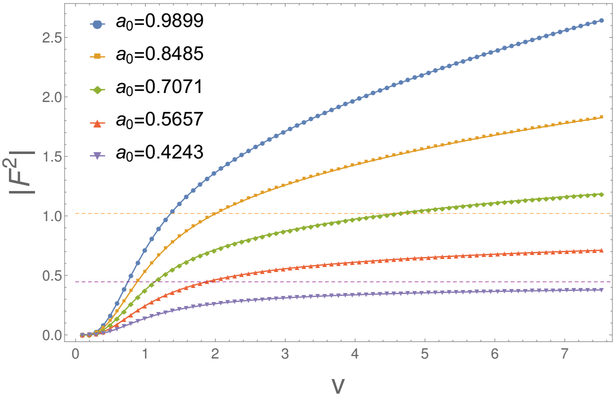
- In the sub-critical stationary solutions, the curvature diverges on the horizon as $a \to a_{max}$.

Plan: Increase a with time from an initial value of 0, up to a value $a_0 > a_{max}$ and wait for the system to settle down. (G. T. Horowitz, J. E. Santos and B. Way 2016)

Results



F^2 is measured at the apparent horizon on the axis of symmetry.
$a_{max} = 0.678$.

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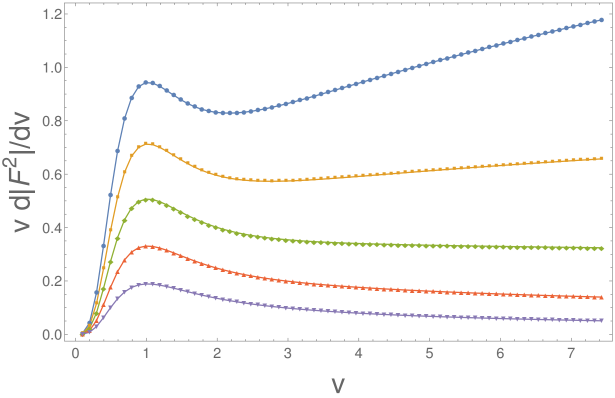

F^2 is measured at the apparent horizon on the axis of symmetry

Results

Summary

- If a distant observer waits for long enough, they should be able to see arbitrarily large curvatures.
- So Cosmic Censorship is violated for the Einstein-Maxwell equations in AdS_4.

What is the Weak Gravity Conjecture?

The Weak Gravity Conjecture (in asymptotically flat space)

Any consistent quantum theory of gravity with a gauge field should contain a particle with charge q and mass m such that $q \geq m$. (N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa 2006)

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Generalization to AdS:

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- We want extremal black holes to be unstable to Schwinger pair production.
- In AdS, this instability is present at the classical level. It is the charged superradiant instability.
- By requiring that arbitrarily small extremal black holes are unstable we obtain the bound:

$$q \geq \frac{\Delta}{L}$$

where

$$\Delta = \frac{3}{2} + \sqrt{\frac{9}{4} + L^2 m^2}$$

Cosmic Censorship meets Weak Gravity

Question: If we take the Weak Gravity Conjecture seriously and include a scalar field with $qL \geq \Delta$ in our action, does our counter-example to Cosmic Censorship still work?

Cosmic Censorship meets Weak Gravity

Idea: Are the stationary solutions we had previously unstable to forming scalar hair?

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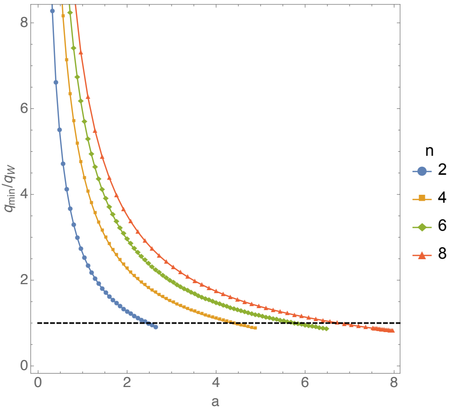
Cosmic Censorship meets Weak Gravity

Answer: Yes! And the required charge appears to agree precisely with the Weak Gravity bound!

Numerical Method

- Take the sub-critical stationary solutions we had previously.
- Solve the scalar field equation on these fixed backgrounds.
- Look for zero modes to detect when scalar hair can form.
- Compute QNMs: if the solutions are unstable to forming scalar hair, Cosmic Censorship is likely preserved.

Results

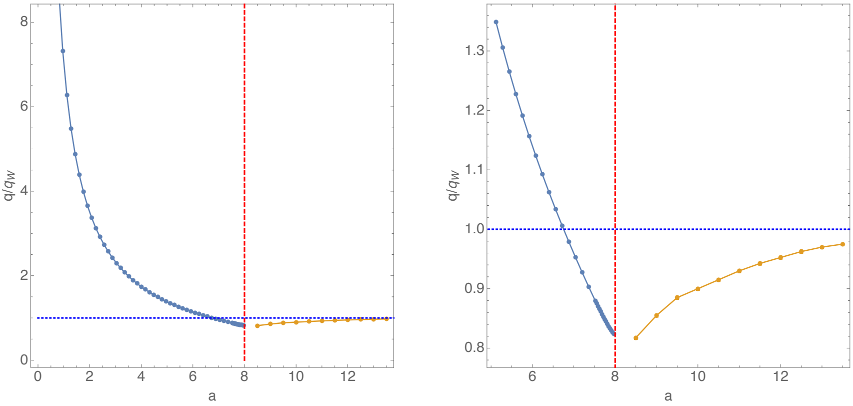


Minimum charge required for a zero mode to exist ($\Delta = 4$)

Results

Remaining question: Could the full non-linear solutions with scalar hair also become singular above a certain amplitude?

Results



Results

Caveat: Hovering Black Holes could form now that charged matter is present, and would be an alternative way of avoiding Cosmic Censorship Violation.

Summary

- It is important to know whether distant observers have to worry about curvature singularities or not.
- We have numerically constructed a time dependent solution to the Einstein-Maxwell equations in 3+1 dimensional Anti-de Sitter space, and found evidence that a “naked singularity” can form, although it takes an infinite time.
- Stationary solutions including a charged scalar suggest that the Weak Gravity Conjecture is sufficient, and may be necessary, to avoid these counter-examples to Cosmic Censorship, suggesting an interesting connection between the two conjectures.

But...

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Similar counter examples to cosmic censorship may occur *in vacuum*.

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Set-up

No gauge field, but impose a boundary metric:

$$ds^2 = -dt^2 + dR^2 + R^2(d\phi + A_t(t,R)dt + A_R(t,R)dR)^2$$

But...

Important differences:

- Finite temperature counter-examples appear to exist.
- Positivity of energy less clear.
- Boundary stress-tensor blows up.