The Weak Gravity Conjecture and Cosmic Censorship

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In collaboration with G. T. Horowitz and J. E. Santos
1 Present numerical evidence that the Weak Cosmic Censorship Conjecture is violated for the Einstein-Maxwell equations on AdS$_4$. 
Outline

1. Present numerical evidence that the Weak Cosmic Censorship Conjecture is violated for the Einstein-Maxwell equations on AdS$_4$.

2. Argue that these counter-examples are avoided if we include a sufficiently charged scalar field in our action, as the Weak Gravity Conjecture tells us to do.
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2. Argue that these counter-examples are avoided if we include a sufficiently charged scalar field in our action, as the Weak Gravity Conjecture tells us to do.

3. Time permitting, discuss a possible violation of Cosmic Censorship for the vacuum Einstein equations on AdS$_4$. 
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- Rough statement of Cosmic Censorship: This always happens...
What do we mean by Cosmic Censorship?

Why is it important?

- **Blessing:** There should be no need to worry about Quantum Gravity in Astrophysics
- **Curse:** Difficult to do Quantum Gravity experiments if you want to be able to tell people the results
What do we mean by Cosmic Censorship?

Precise statement for our purposes

We say that Cosmic Censorship is violated if there exists an open set of smooth geodesically complete initial data such that for all solutions constructed from these initial data, some curvature invariant is unbounded in a region visible to observers at infinity.
Is Cosmic Censorship ever violated?

Yes! It is believed to be violated in higher dimensions through the Gregory-Lafflamme instability (R. Gregory and R. Laflamme 1993, L. Lehner and F. Pretorius 2010). But it is believed to be true in 3 + 1 dimensional asymptotically flat space-times.

What about 3 + 1 dimensional asymptotically Anti-de Sitter space-times? Super-radiance thought to violate Cosmic Censorship (B.E. Niehoff, J.E. Santos, B. Way 2015). We have numerical evidence for Cosmic Censorship violation when a gauge field is included without a corresponding charged scalar.
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Set-up

We take the action

\[
S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R - F^{ab} F_{ab} + \frac{6}{L^2} \right)
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- Initial conditions: Gauge field vanishes and the metric is pure AdS (we work in the Poincaré patch):

\[ ds^2 = \frac{L^2}{z^2} (-dt^2 + dz^2 + dR^2 + R^2 d\phi^2) \]
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- Boundary conditions: Flat boundary metric and for the gauge field we impose:

\[ F_{tR} \big|_{z=0}(t, R, \phi) = \frac{a(v) R}{(1 + R^2)^{3/2}} \]

where \( v = t - \sqrt{1 + R^2} \).
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Key points

- No stationary solutions with a connected horizon exist above a critical amplitude $a = a_{\text{max}}$. (G. T. Horowitz, N. Iqbal, J. E. Santos and B. Way 2014)
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- In the sub-critical stationary solutions, the curvature diverges on the horizon as \( a \to a_{\text{max}} \).

Plan: Increase \( a \) with time from an initial value of 0, up to a value \( a_0 > a_{\text{max}} \) and wait for the system to settle down. (G. T. Horowitz, J. E. Santos and B. Way 2016)
Results

$F^2$ is measured at the apparent horizon on the axis of symmetry. $a_{\text{max}} = 0.678$. 

$a_0 = 0.9899$

$a_0 = 0.8485$

$a_0 = 0.7071$

$a_0 = 0.5657$

$a_0 = 0.4243$
$F^2$ is measured at the apparent horizon on the axis of symmetry
Results

Summary

- If a distant observer waits for long enough, they should be able to see arbitrarily large curvatures.
- So Cosmic Censorship is violated for the Einstein-Maxwell equations in AdS$_4$. 
What is the Weak Gravity Conjecture?

The Weak Gravity Conjecture (in asymptotically flat space)

Any consistent quantum theory of gravity with a gauge field should contain a particle with charge $q$ and mass $m$ such that $q \geq m$. (N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa 2006)
What is the Weak Gravity Conjecture?

Generalization to AdS:

- We want extremal black holes to be unstable to Schwinger pair production.
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Generalization to AdS:

- We want extremal black holes to be unstable to Schwinger pair production.
- In AdS, this instability is present at the classical level. It is the charged superradiant instability.
- By requiring that arbitrarily small extremal black holes are unstable we obtain the bound:

\[ q \geq \frac{\Delta}{L} \]

where

\[ \Delta = \frac{3}{2} + \sqrt{\frac{9}{4} + L^2 m^2} \]
Question: If we take the Weak Gravity Conjecture seriously and include a scalar field with $qL \geq \Delta$ in our action, does our counter-example to Cosmic Censorship still work?
Cosmic Censorship meets Weak Gravity

Idea: Are the stationary solutions we had previously unstable to forming scalar hair?
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Idea: Are the stationary solutions we had previously unstable to forming scalar hair? Do these hairy solutions persist for arbitrarily large amplitudes?
Cosmic Censorship meets Weak Gravity

Answer: Yes! And the required charge appears to agree precisely with the Weak Gravity bound!
Numerical Method

- Take the sub-critical stationary solutions we had previously.
- Solve the scalar field equation on these fixed backgrounds.
- Look for zero modes to detect when scalar hair can form.
- Compute QNMs: if the solutions are unstable to forming scalar hair, Cosmic Censorship is likely preserved.
Results

Minimum charge required for a zero mode to exist ($\Delta = 4$)
Results

Remaining question: Could the full non-linear solutions with scalar hair also become *singular* above a certain amplitude?
Results

The Weak Gravity Conjecture and Cosmic Censorship

Saving Cosmic Censorship with the Weak Gravity Conjecture
Results

Caveat: Hovering Black Holes could form now that charged matter is present, and would be an alternative way of avoiding Cosmic Censorship Violation.
It is important to know whether distant observers have to worry about curvature singularities or not.

We have numerically constructed a time dependent solution to the Einstein-Maxwell equations in 3+1 dimensional Anti-de Sitter space, and found evidence that a “naked singularity” can form, although it takes an infinite time.

Stationary solutions including a charged scalar suggest that the Weak Gravity Conjecture is sufficient, and may be necessary, to avoid these counter-examples to Cosmic Censorship, suggesting an interesting connection between the two conjectures.
But...
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Similar counter examples to cosmic censorship may occur in vacuum.
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Similar counter examples to cosmic censorship may occur in vacuum.

Set-up

No gauge field, but impose a boundary metric:

$$ds^2 = -dt^2 + dR^2 + R^2 (d\phi + A_t(t, R)dt + A_R(t, R)dR)^2$$
Important differences:

- Finite temperature counter-examples appear to exist.
- Positivity of energy less clear.
- Boundary stress-tensor blows up.