Heavy quarks and effective field theories in and for lattice QCD

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Motivation 1

B-physics still is a possible window for new physics

- much non-perturbative theory (= lattice QCD) is needed
- is it always understood to the level that is claimed?
- reason to doubt

$$\underbrace{m_{\rm b}}_{5\,\rm GeV} > \underbrace{\Lambda_{\rm cut}^{\rm UV} = \frac{1}{a}}_{1.5\,\rm GeV\ldots 4 GeV} \quad \text{when} \quad \Lambda_{\rm cut}^{\rm IR} = \frac{1}{L} \ll m_\pi$$

$$\left(\frac{L}{a}\right)^4 = \#\ \text{points} \quad \text{limited}$$

Motivation 2

EFT can provide unexpected help to “solve” fundamental theory

- α_s from decoupling

Lattice for EFTs, EFTs for the lattice, and EFT on the lattice

RS: Les Houches Lect.Notes 108 (2020)

Table 1.2 Examples for the interplay of EFT and lattice QCD. Special considerations beyond $E \ll a^{-1}$ are marked in the last double-column.

		applicability range	
rôle	EFT	of EFT	of lattice QCD
1. Lattice for EFT			
determine LEC's	Chiral PT	low energy QCD	
2. EFT for Lattice			
discretisation effects	Symanzik EFT (Sect. 1.5.2)	$E \ll a^{-1}$	
finite volume effects	Chiral PT [3, 4]	$L^{-1} \ll m_\pi, \Lambda_{\mathrm{QCD}}$	
quark mass effects	Chiral PT [5]	$m_\mathrm{u}, m_\mathrm{d} \ll \Lambda_{\mathrm{QCD}}$	
	Heavy Meson Chiral PT [6–8]	$m_\mathrm{b} \gg \Lambda_{\mathrm{QCD}}$ $m_\mathrm{u}, m_\mathrm{d} \ll \Lambda_{\mathrm{QCD}}$	$m_\mathrm{b} \ll a^{-1}$
combined effects	HMrsChPT [9]	$m_\mathrm{u}, m_\mathrm{d} \ll \Lambda_{\mathrm{QCD}}$ $m_\mathrm{b} \gg \Lambda_{\mathrm{QCD}}$,	$m_\mathrm{b} \ll a^{-1}$
3. EFT on the Lattice			
NP EFT	$\mathrm{QCD}^{(3)}$	$E \ll m_\mathrm{c}, m_\mathrm{b}, m_\mathrm{t}$	
NP EFT	HQET	$E, \Lambda_{\mathrm{QCD}} \ll m_\mathrm{b}$	
NP EFT	NRQCD	$E, \Lambda_{\mathrm{QCD}} \ll a^{-1} \ll m_\mathrm{c}, m_\mathrm{b}$	
NP EFT	Nuclear EFT	see the literature	

+ more e.g. boundary effects

EFTs beyond perturbation theory

a bit of formalism

$$\Phi^{\mathrm{LO}} = \langle O \rangle_{\mathrm{LO}} = \frac{1}{Z_{\mathrm{LO}}} \int_{\mathrm{fields}} \mathrm{e}^{-S^{\mathrm{LO}}} O \,, \quad S^{\mathrm{LO}} = \int \mathrm{d}^4 x \, \mathscr{L}_{\mathrm{LO}}(x) \,,$$

$$\mathscr{L}_{\mathrm{LO}}(x) = \sum_i \omega_i^{\mathrm{LO}} \, \mathscr{O}_i^{\mathrm{LO}}(x) \,, \quad [\mathscr{O}_i^{\mathrm{LO}}] \leq 4 \quad [\omega_i^{\mathrm{LO}}] \geq 0 \,,$$

$$O = \varphi(x)\varphi(y)$$

as in any fundamental QFT

$$\mathscr{L}_{\mathrm{NLO}} = \sum_j \omega_j \mathscr{O}_j \,, \quad \omega_j = \frac{1}{m_{\mathrm{h}}} \hat{\omega}_j \,, \quad [\mathscr{O}_j] = 5 \,, \quad [\hat{\omega}_j] = 0 \,,$$

$$O_{\mathrm{eff}} = O_{\mathrm{LO}} + O_{\mathrm{NLO}} + \ldots \,.$$

EFTs beyond perturbation theory

At NLO in 1/m

$$\mathscr{L}_{\mathrm{NLO}} = \sum_j \omega_j \mathscr{O}_j \,, \quad \omega_j = \frac{1}{m_{\mathrm{h}}} \hat{\omega}_j \,, \quad [\mathscr{O}_j] = 5 \,, \quad [\hat{\omega}_j] = 0 \,,$$

$$O_{\mathrm{eff}} = O_{\mathrm{LO}} + O_{\mathrm{NLO}} + \ldots \,.$$

part of the definition of the EFT is

$$\Phi \equiv \langle O \rangle = \frac{\int_{\mathrm{fields}} \mathrm{e}^{-S} O}{\int_{\mathrm{fields}} \mathrm{e}^{-S}} \qquad \mathrm{e}^{-S} \to \mathrm{e}^{-S^{\mathrm{LO}}} \{ 1 - S^{\mathrm{NLO}} + \ldots \}$$

which yields

$$\Phi_{\mathrm{eff}}^{\mathrm{LO}} = \langle O^{\mathrm{LO}} \rangle_{\mathrm{LO}}$$

$$\Phi_{\mathrm{eff}}^{\mathrm{NLO}} = \langle O^{\mathrm{NLO}} \rangle_{\mathrm{LO}} - \Big(\langle O^{\mathrm{LO}} S^{\mathrm{NLO}} \rangle_{\mathrm{LO}} - \langle O^{\mathrm{LO}} \rangle_{\mathrm{LO}} \langle S^{\mathrm{NLO}} \rangle_{\mathrm{LO}} \Big)$$

higher order corrections as insertions into correlators

EFTs beyond perturbation theory

$$\Phi_{\mathrm{eff}}^{\mathrm{LO}} = \langle O^{\mathrm{LO}} \rangle_{\mathrm{LO}}$$

$$\Phi_{\mathrm{eff}}^{\mathrm{NLO}} = \langle O^{\mathrm{NLO}} \rangle_{\mathrm{LO}} - \left(\langle O^{\mathrm{LO}} S^{\mathrm{NLO}} \rangle_{\mathrm{LO}} - \langle O^{\mathrm{LO}} \rangle_{\mathrm{LO}} \langle S^{\mathrm{NLO}} \rangle_{\mathrm{LO}} \right)$$

higher order corrections as insertions into correlators

=> renormalizable

also non-perturbatively (in coupling expansion)

why?

OPE + renormalization of local fields + renormalizability of LO theory

(different when the LO theory is not renormalizable, e.g. NRQCD)

note that for the lattice theory:

renormalizability <—> existence of continuum limit

Renormalization and matching at LO in 1/m

Example: HQET: b-valence quark —> h(eavy)

$$\mathscr{L}^{\rm LO} = \mathscr{L}^{\rm stat} = \bar\psi_h(D_0 + m)\psi_h \quad \text{static}$$

h-light axial current

$$A_0^{\rm stat} = \bar\psi_u\gamma_0\gamma_5\psi_h\,, \qquad \mathscr{M}^{\rm stat} \equiv \langle 0\,|\,A_0(0)\,|\,B\rangle$$

no chiral symmetry for h —> renormalization, AD for $A_0^{\rm stat}$

renormalize + match to QCD

$$\mathscr{M}^{\rm QCD}(m_b) = C_{\rm match}(m_b, \mu) \times \mathscr{M}^{\rm stat}(\mu)$$

renormalization group improvement, pass to renormalization group invariant

$$\mathscr{M}(\mu) = \frac{\mathscr{M}^{\rm RGI}}{\varphi(\bar g(\mu))}\,, \qquad \varphi(\bar g) = \left[2b_0\bar g^2\right]^{-\gamma_0/2b_0} \underbrace{\exp\left\{-\int_0^{\bar g} x\left[\frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{b_0 x}\right]\right\}}_{1+\mathrm{O}(\bar g^2)}$$

Renormalization and matching at LO in 1/m

renormalize + match to QCD

$$\mathcal{M}^{\mathrm{QCD}}(m_b) = C_{\mathrm{match}}(m_b, \mu) \times \mathcal{M}^{\mathrm{stat}}(\mu)$$

renormalization group improvement, pass to renormalization group invariant

$$\mathcal{M}(\mu) = \frac{\mathcal{M}^{\mathrm{RGI}}}{\varphi(\bar{g}(\mu))}\,, \qquad \varphi(\bar{g}) = \left[2b_0\bar{g}^2\right]^{-\gamma_0/2b_0} \underbrace{\exp\left\{-\int_0^{\bar{g}} x \left[\frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{b_0 x}\right]\right\}}_{1+\mathrm{O}(\bar{g}^2)}$$

in perturbation theory

$$\mathcal{M}^{\mathrm{QCD}}(m_b) = [\bar{g}^2(m_b)]^{\hat{\gamma}}\left\{1 + \mathrm{O}(\bar{g}^2(m_b))\right\} \times \mathcal{M}^{\mathrm{RGI}}$$

Beyond LO in 1/m

renormalize + match to QCD in perturbation theory at LO in EFT

$$\mathcal{M}^{\rm QCD}(m_b) = [\bar{g}^2(m_b)]^{\hat{\gamma}}\left\{1 + \mathrm{O}(\bar{g}^2(m_b))\right\} \times \mathcal{M}^{\rm RGI}$$

perturbative uncertainty

$$\mathrm{O}([\bar{g}^2(m_b)]^{\#\,\rm loops})$$

This is not good enough if NLO accuracy is desired :

pert. errors = $[\bar{g}^2(m_b)]^{\#\,\rm loops} \gg \dfrac{\Lambda_{\rm QCD}}{m_b}$ for $m_b \gg \Lambda_{\rm QCD}$

instead need to perform renormalization and matching non-perturbatively (on the lattice)

in fact: even more severe reasons:

power-law divergences $\sim \left(\dfrac{g^2}{a^k}\right)$, for N^{k}LO EFT

Beyond LO in 1/m

more precisely the requirement is:

$$\Phi = \underbrace{\Phi^{\rm LO}}_{\text{NP}} + \underbrace{\Phi^{\rm NLO}}_{\text{perturbative renormalization + matching}}$$

perturbative renormalization+matching is okay (with perturbative errors) for the highest EFT-order correction

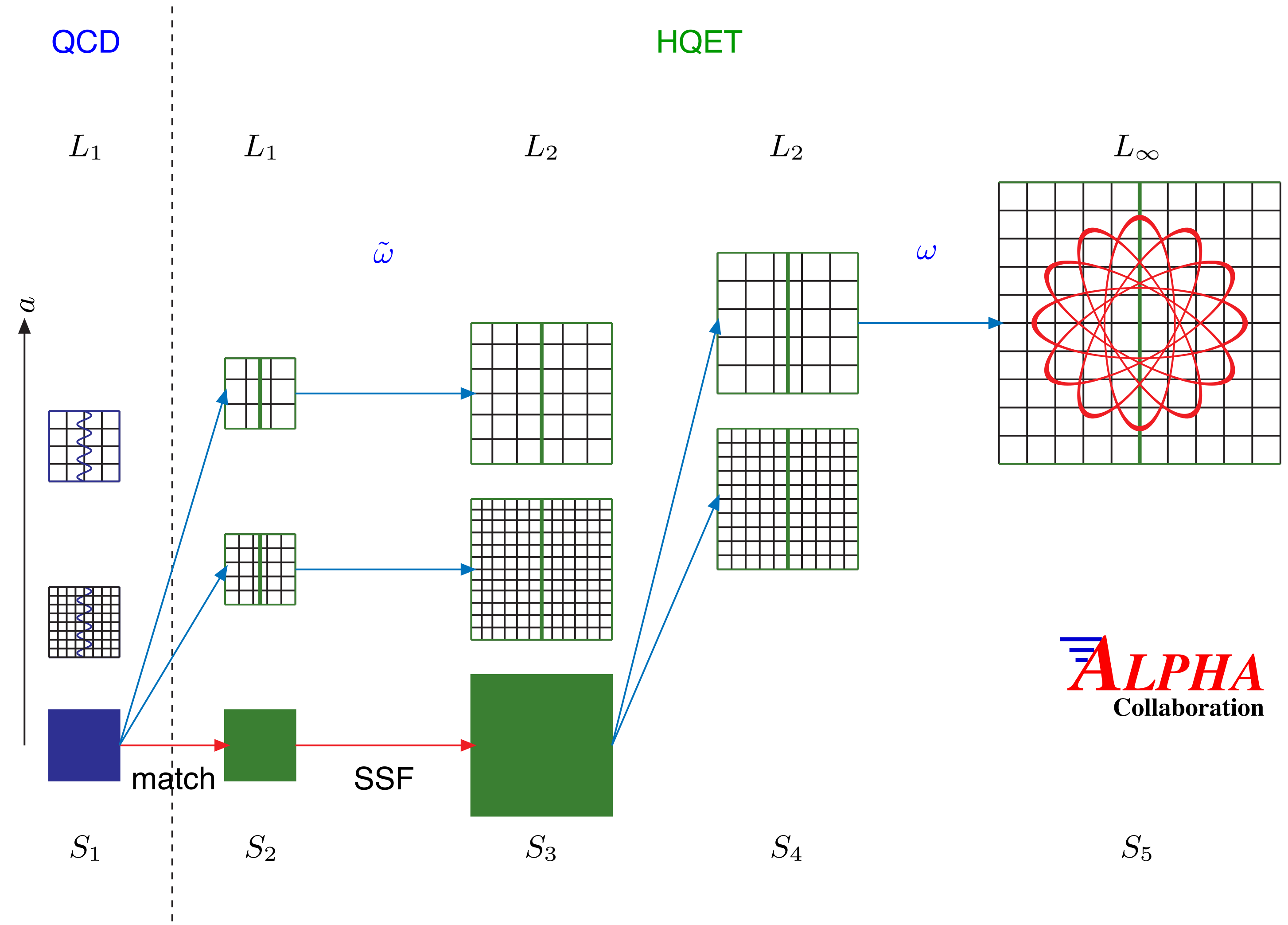
everything else needs to be done non-perturbatively (NP)

PT is useful for the highest EFT-order, in particular, to estimate the size of a small higher order EFT term

Non-perturbative HQET

Beautiful strategy
[Heitger, S., Heitger:2003nj]

Unfortunately very few phenomenological applications

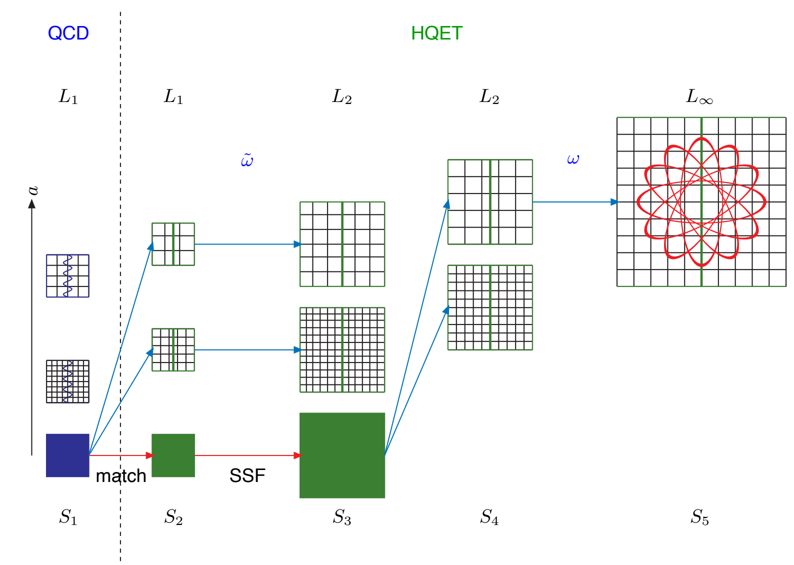


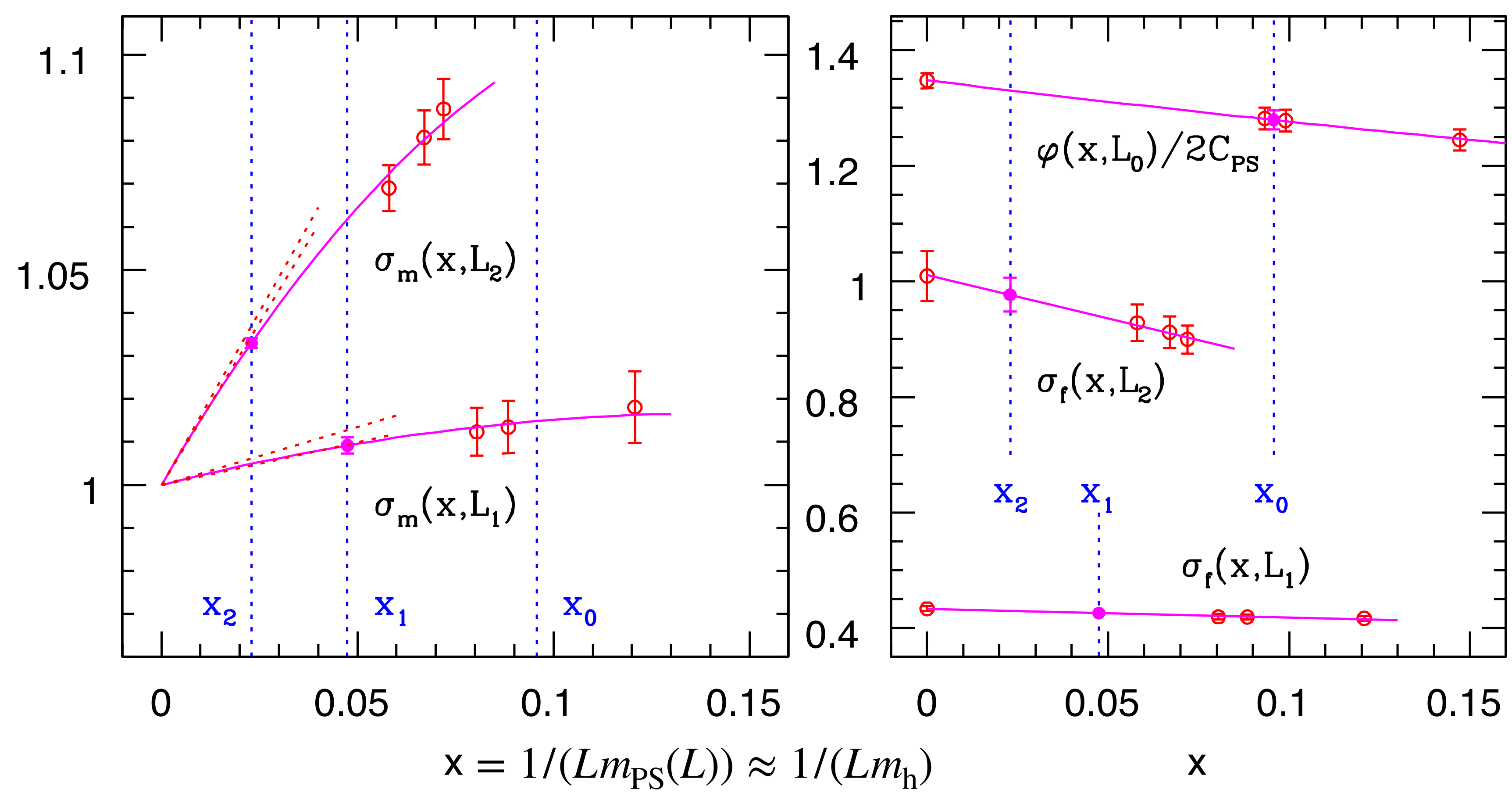
But there is a twist to it:

combine HQET step scaling functions with relativistic ones
to obtain a stabilized mass-dependence in the full range

Combine HQET
step scaling functions
with relativistic ones
to obtain a stabilized mass-dependence

[Guazzini, S., Tantalo; Guazzini:2007ja]

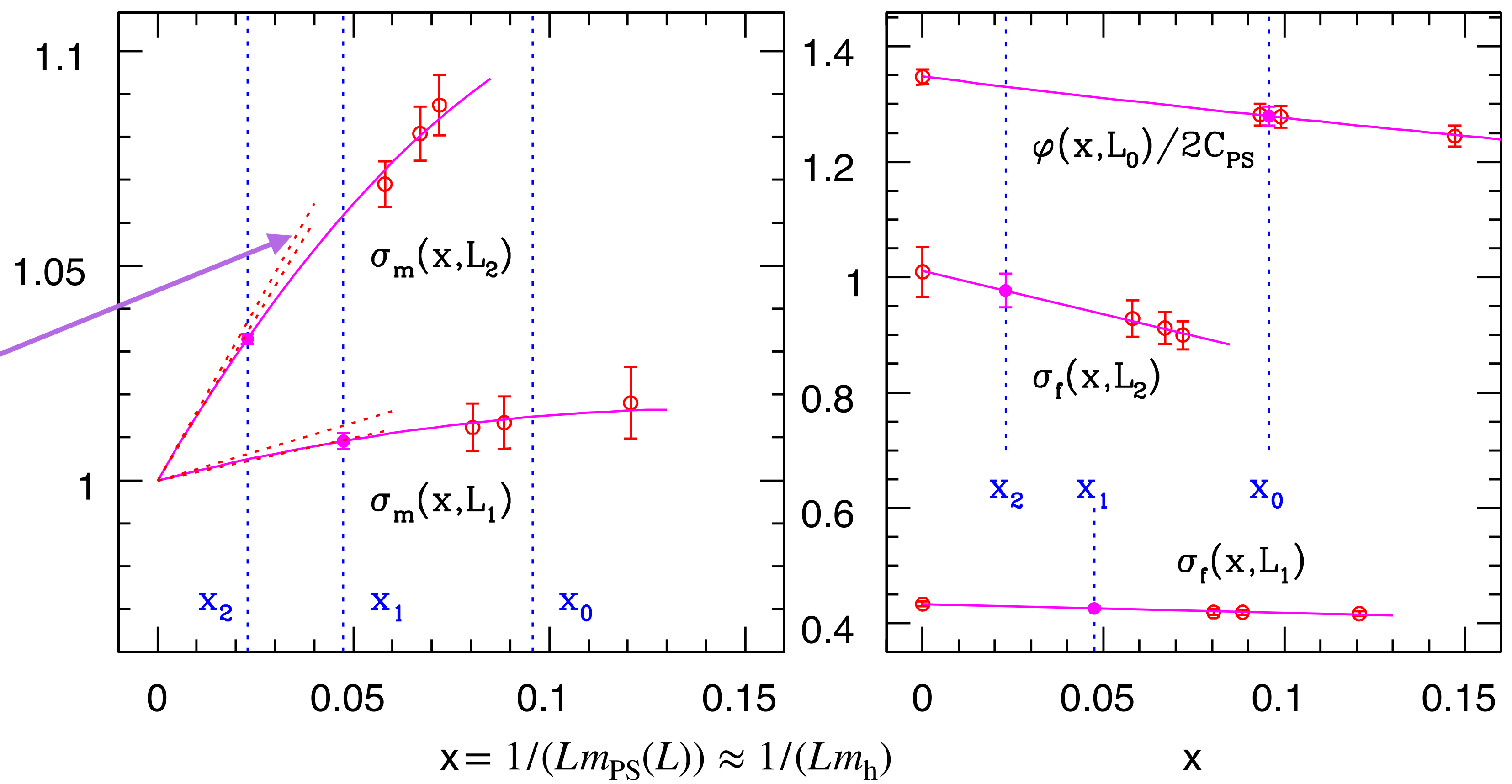




Non-perturbative HQET + QCD

$$\sigma_\mathrm{m} = \frac{m_\mathrm{PS}(2L)}{m_\mathrm{PS}(L)}$$

static approx



Presently revival with

- three dynamical quarks
- a twist in the quark-mass term
- a generalization to allow for form-factors such as B $\rightarrow \pi \ell \nu$

stay tuned

[A. Conigli, J. Frison, P. Fritsch, J. Heitger, G. Herdoiza, S. Kuberski, C. Pena, H. Simma, R.S.]

A radical switch …

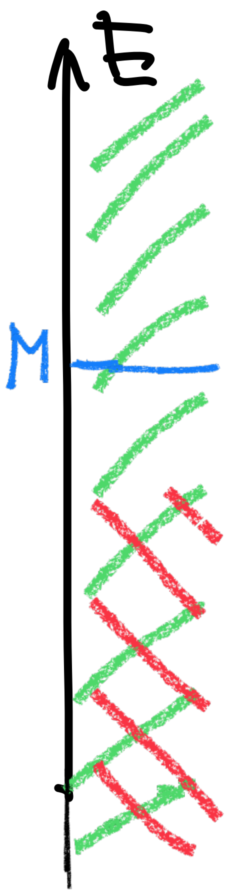
to a project with a phenomenologically relevant result

$$\alpha_s(m_Z) = 0.11823(69)(42)_{b_\mathrm{g}}(20)_{\Gamma_\mathrm{m}}(6)_{3\to5,\mathrm{PT}}(7)_{3\to5,\mathrm{NP}}$$

[M.DallaBrida,R.Hoellwieser,F.~Knechtli,T.Korzec,A.Ramos,S.Sint,R.S., 2022]

- try to explain some of the EFT (fun) aspects

Decoupling of heavy quarks



QCD$_{N_\mathrm{f}}$ with N_f quarks (6 in Nature)

N_ℓ light (neglect mass (uninteresting)),

$N_\ell - N_\mathrm{f}$ heavy, RGI mass M

For energy scales far below M: EFT = QCD$_{N_\ell}$ + $\underbrace{\mathrm{O}\left(E^2/M^2\right)}_{\mathrm{NLO}}$

renormalization + matching at LO: $\bar{g}^2_\mathrm{f} \leftrightarrow \bar{g}^2_\ell$, or $\Lambda_\mathrm{f} \leftrightarrow \Lambda_\ell$

reminder:

$$\Lambda_s = \mu\, \varphi_s(\bar{g}_s(\mu))\,, \qquad \mathrm{d}/\mathrm{d}\mu\, \Lambda_s = 0$$

$$\varphi_s(\bar{g}_s) = (b_0\bar{g}_s^2)^{-b_1/(2b_0^2)}\, \mathrm{e}^{-1/(2b_0\bar{g}_s^2)} \times \exp\left\{ -\int_0^{\bar{g}_s} \mathrm{d}x \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x} \right] \right\},$$

How large are $1/M^2$ corrections?

Avoid $\Lambda_\mathrm{f} \leftrightarrow \Lambda_\ell$ relation for this question

Any physical (μ -independent) mass-scale $\mathcal{S}_i$, in particular a particle mass satisfies

$\mathcal{S}_{\ell,i} = k_i \Lambda_\ell$ mass-less EFT, LO ($[k_i] = 0$)

and

$\mathcal{S}_{\mathrm{f},i} = h_i(M/\Lambda_\mathrm{f})\,\Lambda_\mathrm{f}$ full theory ($[h_i] = 0$)

For ratios the Λ - dependence drops out

$$\frac{\mathcal{S}_{\mathrm{f},i}}{\mathcal{S}_{\mathrm{f},j}} = \frac{h_i(M/\Lambda_\mathrm{f})}{h_j(M/\Lambda_\mathrm{f})} = \frac{k_i}{k_j} + \mathrm{O}(\Lambda_\mathrm{f}^2/M^2)$$

we tested this for $N_\mathrm{f} = 2 \rightarrow N_\ell = 0$ (QCD-like toy model)

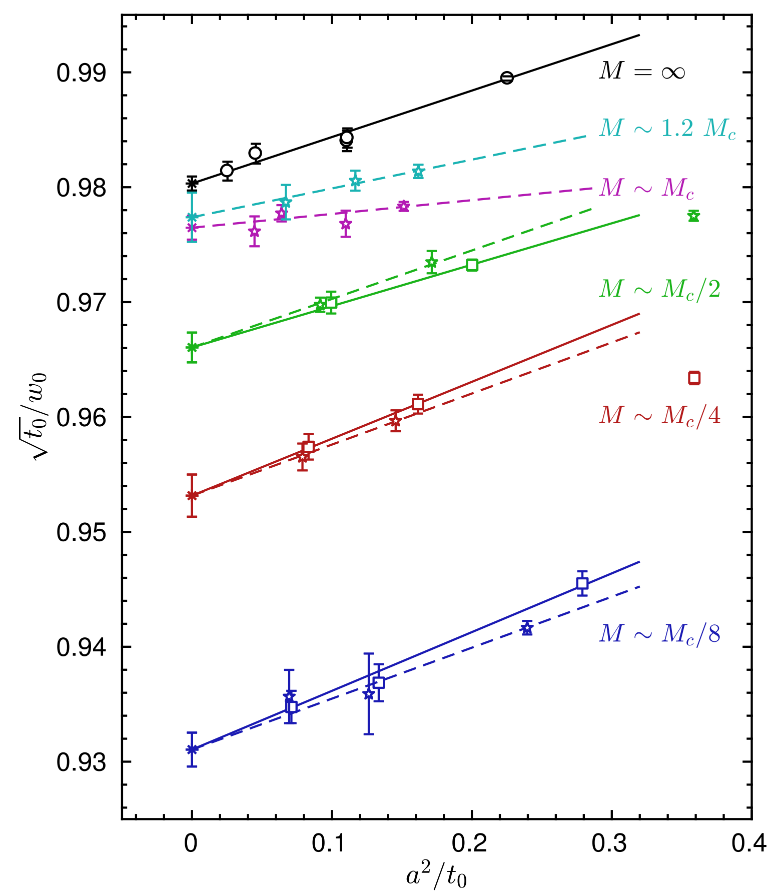
Decoupling: How large are $1/M^2$ corrections?

$$\frac{\mathcal{S}_{\mathrm{f},i}}{\mathcal{S}_{\mathrm{f},j}} = \frac{h_i(M)}{h_j(M)} = \frac{k_i}{k_j} + \mathrm{O}(\Lambda_\mathrm{f}^2/M^2)\,,$$

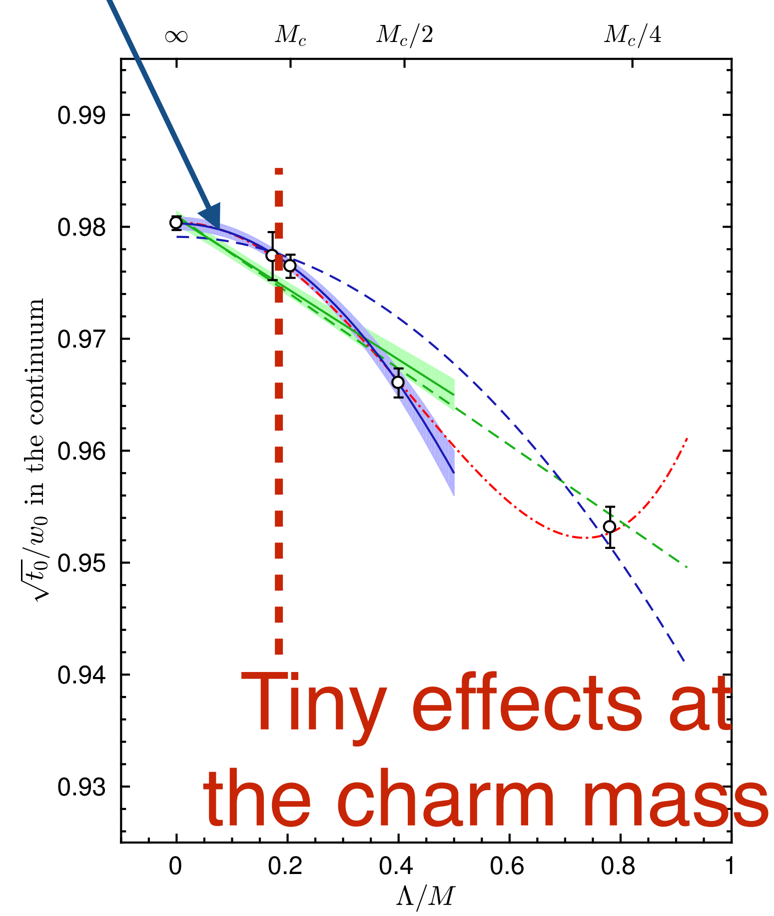
use $\mathcal{S}_i$ = gradient flow scales because of excellent precision

for $N_\mathrm{f} = 2 \to N_\ell = 0$

continuum limits



mass behavior



[Bruno, Finkenrath, Knechtli, Leder, S., Bruno:2014ufa]

[Knechtli, Korzec, Leder, Moir, Knechtli:2017xgy]

Decoupling: relation of Λ parameters

Decoupling relation

$$\Lambda_\ell = P_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f})\,\Lambda_\mathrm{f} \qquad \text{(x)}$$

more precisely need to first specify the units (scale)

$$\frac{\Lambda_\ell}{\mathcal{S}_\ell} = P^{\mathcal{S}}_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f}) \times \frac{\Lambda_\mathrm{f}}{\mathcal{S}_\mathrm{f}(M)}$$

scales $\mathcal{S}$ can be dropped because $\dfrac{\mathcal{S}_{\mathrm{f},i}}{\mathcal{S}_{\mathrm{f},j}} = \dfrac{h_i(M/\Lambda)}{h_j(M/\Lambda)} = \dfrac{k_i}{k_j} + \mathrm{O}(\Lambda^2/M^2)$

so

$$P^{\mathcal{S}}_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f}) = P^{\mathcal{S}'}_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f}) \times (1 + \mathrm{O}(\Lambda_\mathrm{f}^2/M^2))$$

observations

- (x) is valid at LO in the EFT
- Λ_ℓ inherits a mass-dependence from (x)

  large (external) mass beyond the reach of QCD_ℓ

Decoupling: relation of Λ parameters

$$\Lambda_\ell = P_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f})\,\Lambda_\mathrm{f}\;, \qquad \Lambda_s = \mu\,\varphi_s(\bar{g}_s(\mu))$$

$$\varphi_s(\bar{g}_s) = (b_0\bar{g}_s^2)^{-b_1/(2b_0^2)}\,\mathrm{e}^{-1/(2b_0\bar{g}_s^2)} \times \exp\left\{-\int_0^{\bar{g}_s}\mathrm{d}x\,\left[\frac{1}{\beta_s(x)}+\frac{1}{b_0x^3}-\frac{b_1}{b_0^2x}\right]\right\}\,,$$

yields

$$P_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f}) = \frac{\varphi_\ell(\bar{g}_\ell(\mu))}{\varphi_\mathrm{f}(\bar{g}_\mathrm{f}(\mu))}$$

insert perturbative relation of couplings

$$\bar{g}_\ell^2(\mu) = C(\bar{g}_\mathrm{f}(m_\star))\;\bar{g}_\mathrm{f}^2(m_\star), \qquad\qquad m_\star = \overline{m}_{\overline{\mathrm{MS}}}(m_\star)$$

in $\overline{\mathrm{MS}}$ scheme: $\quad C(x) = 1 + c_2x^4 + c_3x^6 + c_4x^8 + \ldots\;,$

[Bernreuther:1981sg,Grozin:2011nk,Chetyrkin:2005ia,Schroder:2005hy,Kniehl:2006bg,Gerlach:2018hen]

Decoupling: relation of Λ parameters

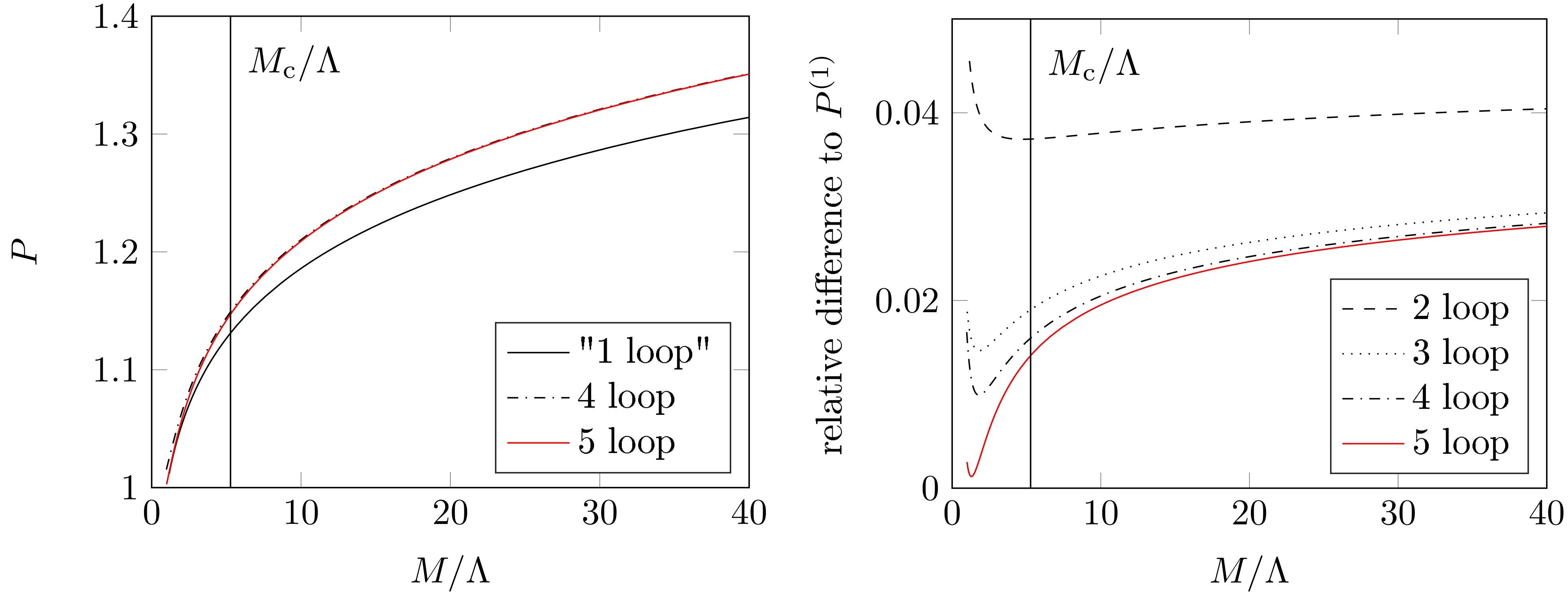
$$P_{\ell,\mathrm{f}}(M/\Lambda_\mathrm{f}) = \frac{\varphi_\ell(C(\bar{g}_\ell(\mu)))}{\varphi_\mathrm{f}(\bar{g}_\mathrm{f}(\mu))}$$

fantastic asymptotic convergence in $\overline{\mathrm{MS}}$ scheme: use C from [Bernreuther:1981sg,Grozin:2011nk,Chetyrkin:2005ia,Schroder:2005hy,Kniehl:2006bg,Gerlach:2018hen]

and 5-loop beta-function

example:

decoupling of charm



- NP "corrections" have been studied and are small as well

[A.Athenodorou,J.Finkenrath,F.~Knechtli,T.Korzec,B.Leder,M.Marinkovic,R.S., 2018]

- charm-mass dependence of proton mass can be computed in PT. It is very weak. $m_\mathrm{proton} = P(M/\Lambda_\mathrm{f}) \times \mathrm{const}$

Decoupling as a tool: determination of QCD Λ parameter(s)

- pure gauge theory $\Lambda_\ell = \Lambda^{(0)}$ known in terms of GF scale

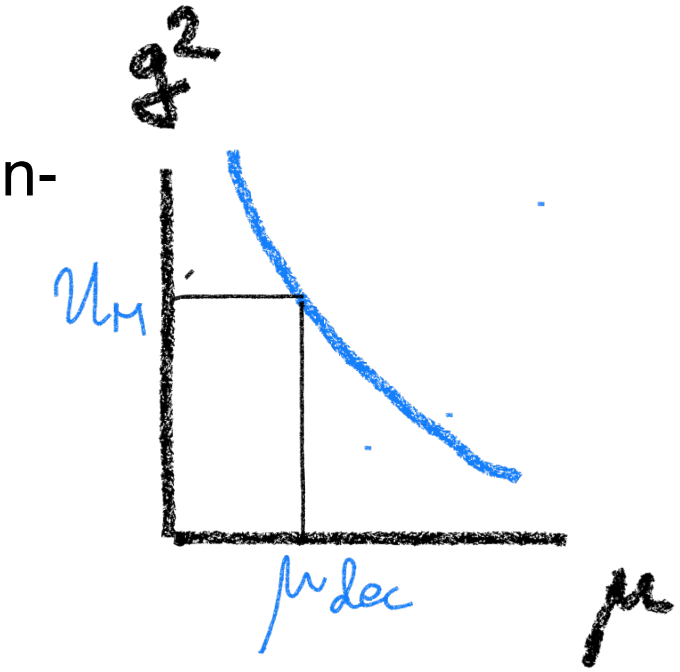
  [M. Dalla Brida & A. Ramos: DallaBrida:2019wur]

- decoupling relation relates it to $\Lambda_\mathrm{f} = \Lambda^{(3)}$ with 3 quarks in terms
  of the same GF scale in theory with
  3 artificially heavy quarks

Decoupling as a tool

decoupling relation

$$P_{\ell,\mathrm{f}}(M/\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}})\,\frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\mathrm{f}}(M)} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\ell}}$$

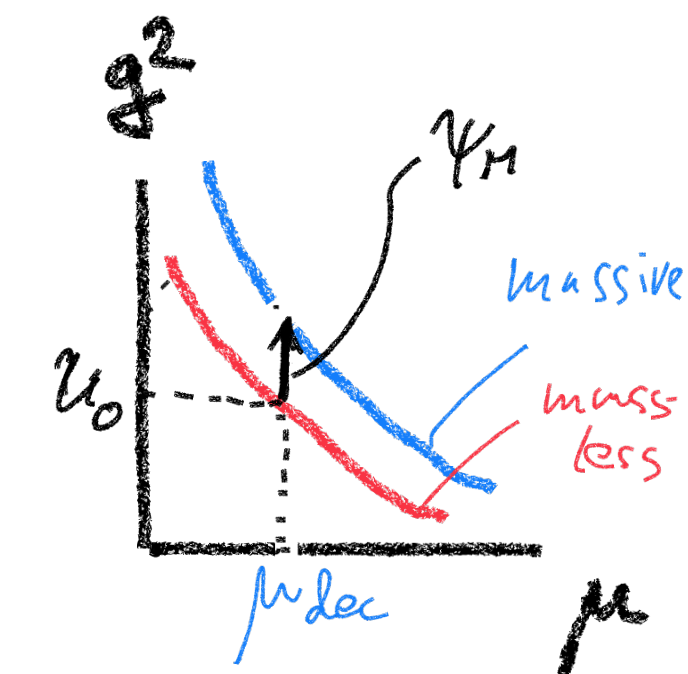


it is practical to define the scale through a specific value of a non-perturbatively defined running coupling

$$\mathcal{S} \equiv \mu_{\mathrm{dec}}\,, \quad \text{with } [\bar{g}^{\mathrm{f}}_{\mathrm{GF}}(\mu_{\mathrm{dec}}, M)]^2 = u_{\mathrm{M}}$$

rewrite

$$\frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mathcal{S}} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\Lambda^{\ell}_{\mathrm{GF}}}\,\frac{\Lambda^{\ell}_{\mathrm{GF}}}{\mu_{\mathrm{dec}}} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\Lambda^{\ell}_{\mathrm{GF}}}\,\varphi^{\ell}_{\mathrm{g,GF}}(\sqrt{u_{\mathrm{M}}})$$



introduce the function which relates the coupling in the full theory with the massive quarks and the one with all massless ones:

$$u_{\mathrm{M}} = \Psi_{\mathrm{M}}(u_0, z)\,, \quad \text{with } u_0 = [\bar{g}^{\mathrm{f}}_{\mathrm{GF}}(\mu_{\mathrm{dec}}, 0)]^2\,, \quad z = M/\mu_{\mathrm{dec}}$$

Decoupling as a tool

decoupling relation now is

$$\underbrace{\rho \quad \times P_{\ell,\mathrm{f}}(z/\rho)}_{\text{High order PT}} = \underbrace{\frac{\Lambda_{\overline{\mathrm{MS}}}^{\ell}}{\Lambda_{\mathrm{GF}}^{\ell}}}_{\text{1-lp exact}} \underbrace{\varphi_{\mathrm{GF}}^{\ell}}_{\text{YM}}\left(\underbrace{\sqrt{\Psi_{\mathrm{M}}(u_0, z)}}_{\text{full}}\right), \qquad \rho = \frac{\Lambda_{\overline{\mathrm{MS}}}^{\mathrm{f}}}{\mu_{\mathrm{dec}}}$$

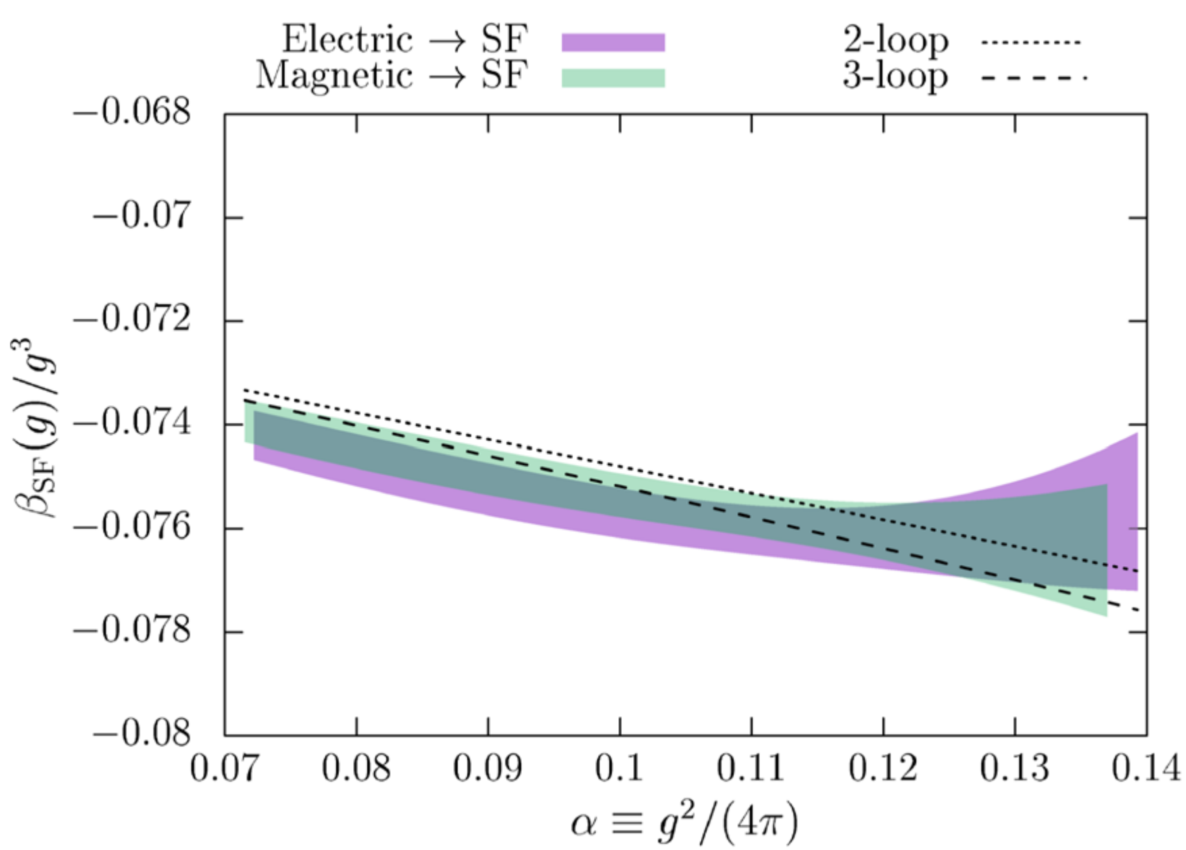
$$z = M/\mu_{\mathrm{dec}}$$

Decoupling as a tool

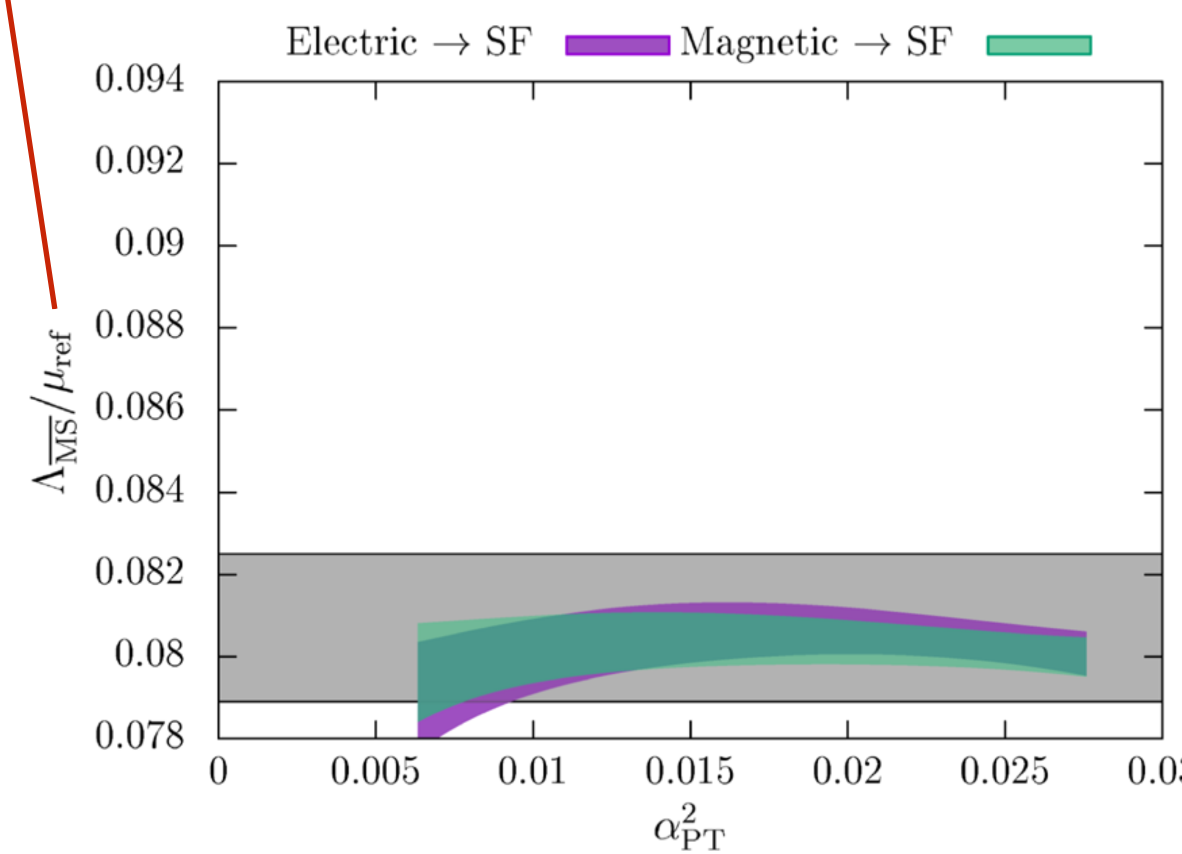
decoupling relation

$$\underbrace{\rho \times P_{\ell,\mathrm{f}}(z/\rho)}_{\text{High order PT}} = \underbrace{\frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\Lambda^{\ell}_{\mathrm{GF}}}}_{\text{1-lp exact}} \underbrace{\varphi^{\ell}_{\mathrm{GF}}}_{\text{YM}}\Big(\underbrace{\sqrt{\Psi_{\mathrm{M}}(u_0, z)}}_{\text{full}}\Big)\,, \qquad \rho = \frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}}$$

$$z = M/\mu_{\mathrm{dec}}$$



finite volume step scaling + …



[M. Dalla Brida & A. Ramos, DallaBrida:2019wur]

Decoupling as a tool

decoupling relation

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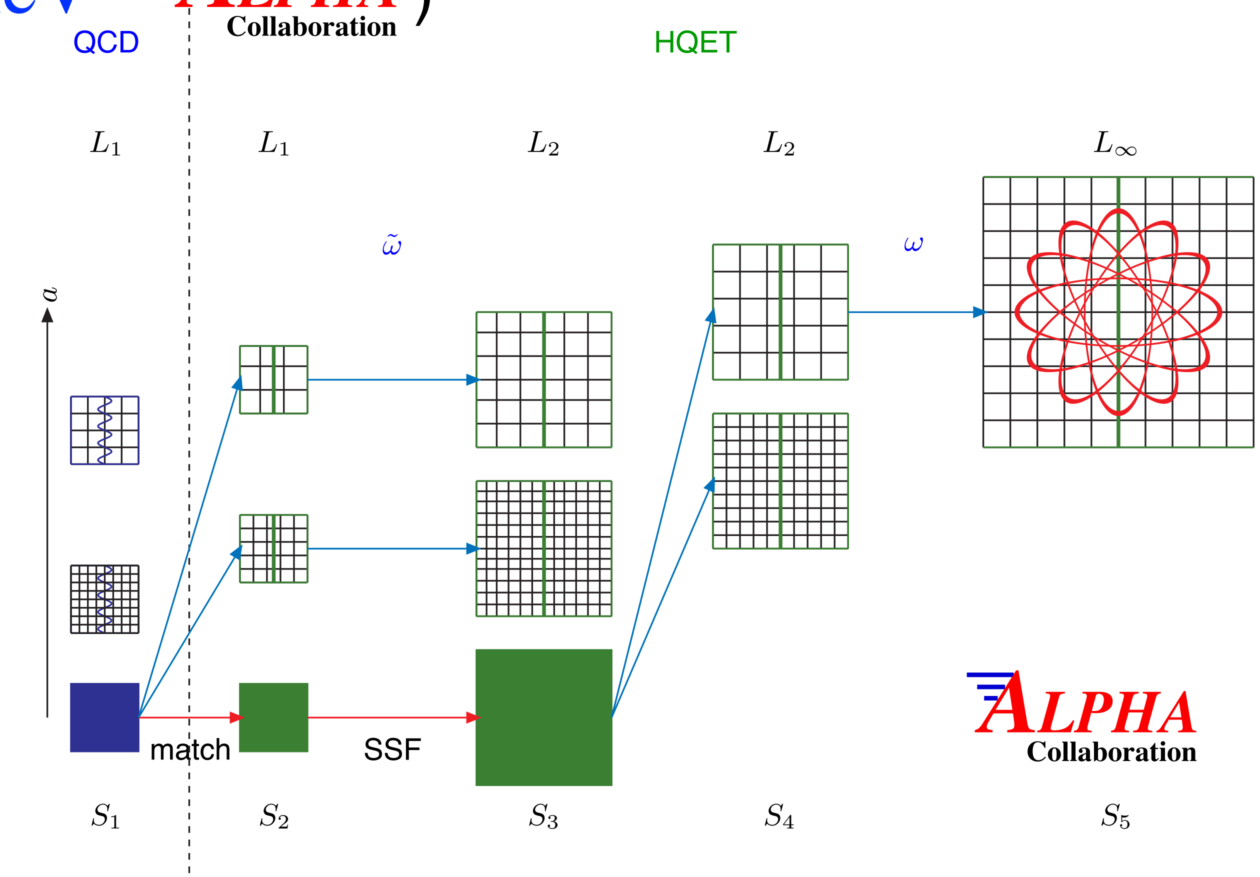
$$z = M/\mu_{\mathrm{dec}}$$

choice of GF-scheme + scale, $\bar{g}^2_{\mathrm{GF}}(\mu_{\mathrm{dec}} = 2/L_1, M = 0) = u_0 = 3.949$,

as in HQET project $(\mu_{\mathrm{dec}} = 789(15)\,\mathrm{MeV}$ ALPHA Collaboration)

L/a	β	κ	$\bar{g}^2_{\mathrm{GF}}$	β_{LCP}
12	4.3030	0.1359947(18)	3.9461(41)	4.3019(16)
16	4.4662	0.1355985(9)	3.9475(61)	4.4656(23)
20	4.6017	0.1352848(2)	3.9493(63)	4.6018(24)
24	4.7165	0.1350181(20)	3.9492(64)	4.7166(25)
32	4.9000	0.1345991(8)	3.949(11)	4.9000(42)
40	–	–	–	5.0497(41)
48	–	–	–	5.1741(54)

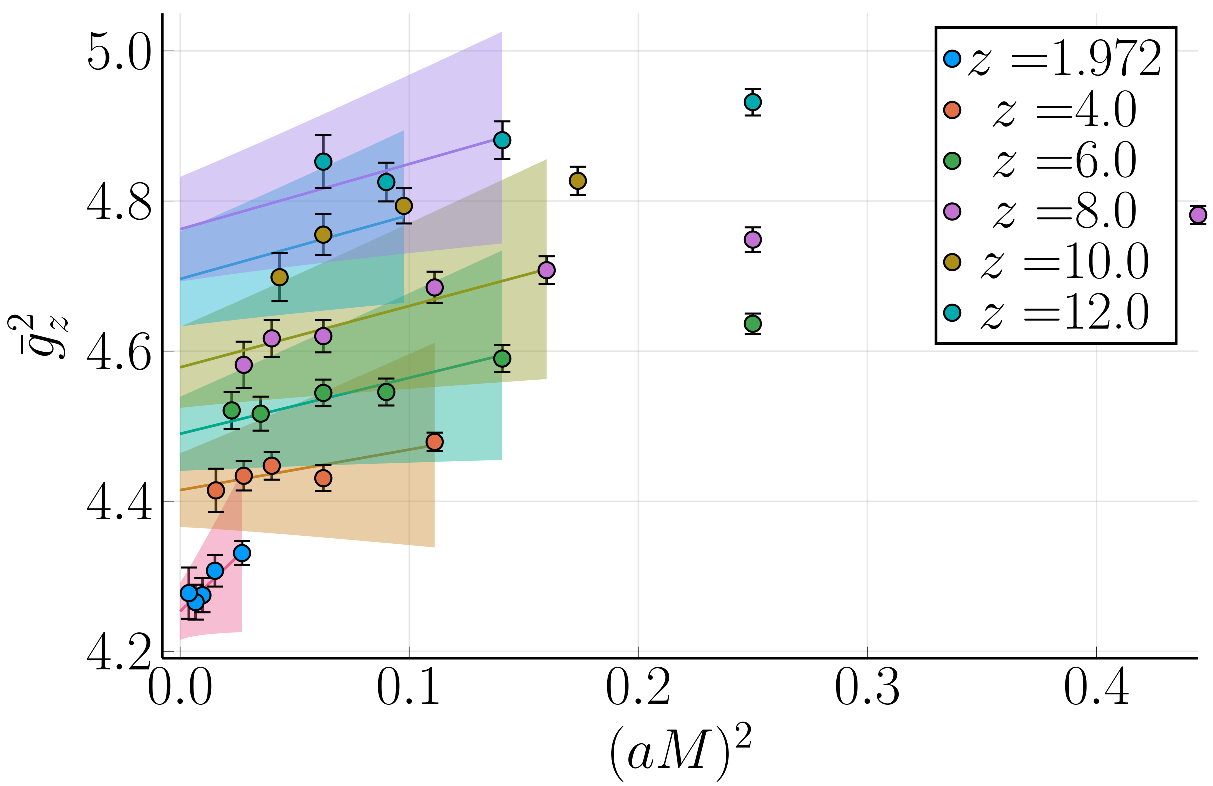


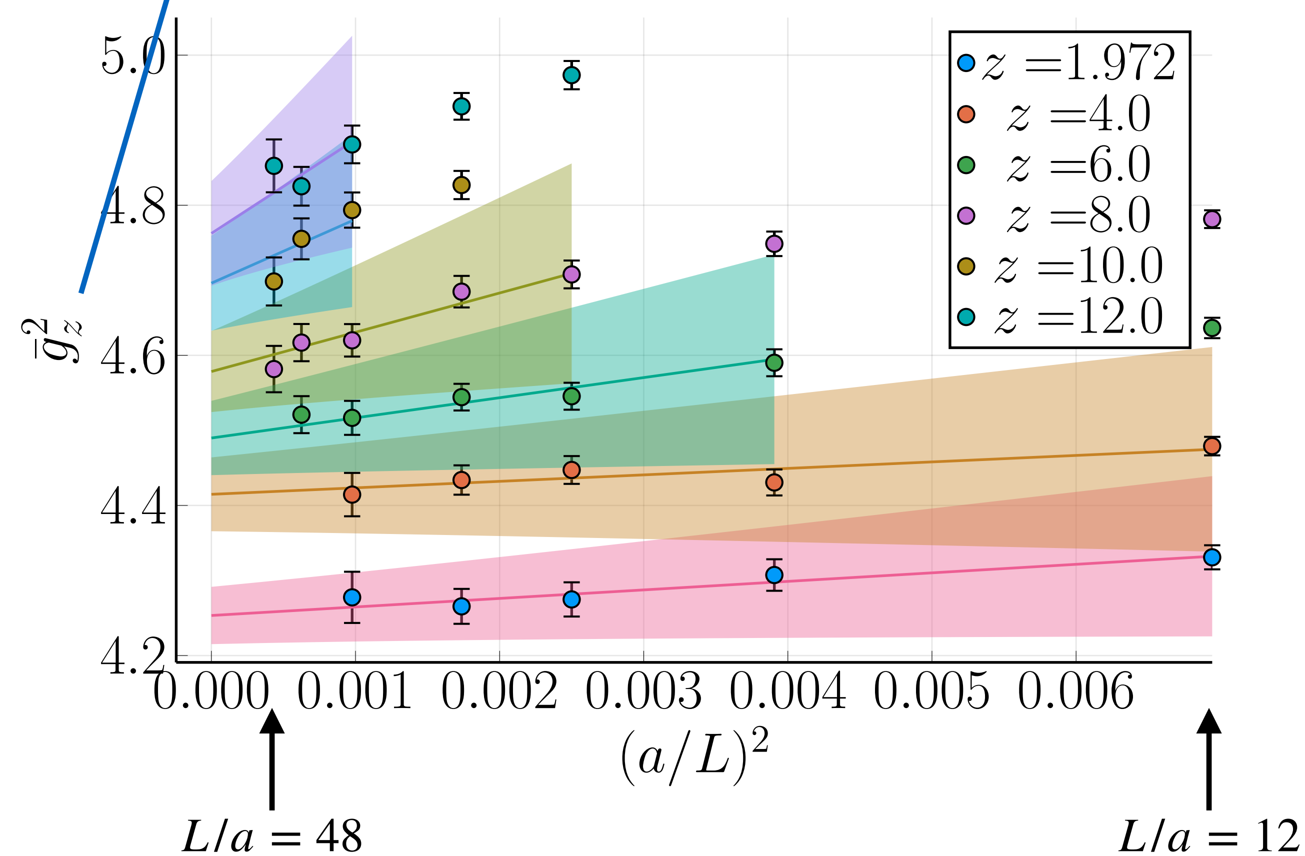
Decoupling as a tool

decoupling relation now is

$$\underbrace{\rho \times P_{\ell,\mathrm{f}}(z/\rho)}_{\text{High order PT}} = \underbrace{\frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\Lambda^{\ell}_{\mathrm{GF}}}}_{\text{1-lp exact}} \underbrace{\varphi^{\ell}_{\mathrm{GF}}}_{\text{YM}}\left(\underbrace{\sqrt{\Psi_{\mathrm{M}}(u_0, z)}}_{\text{full}}\right), \qquad \rho = \frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}}$$

$$z = M/\mu_{\mathrm{dec}}$$





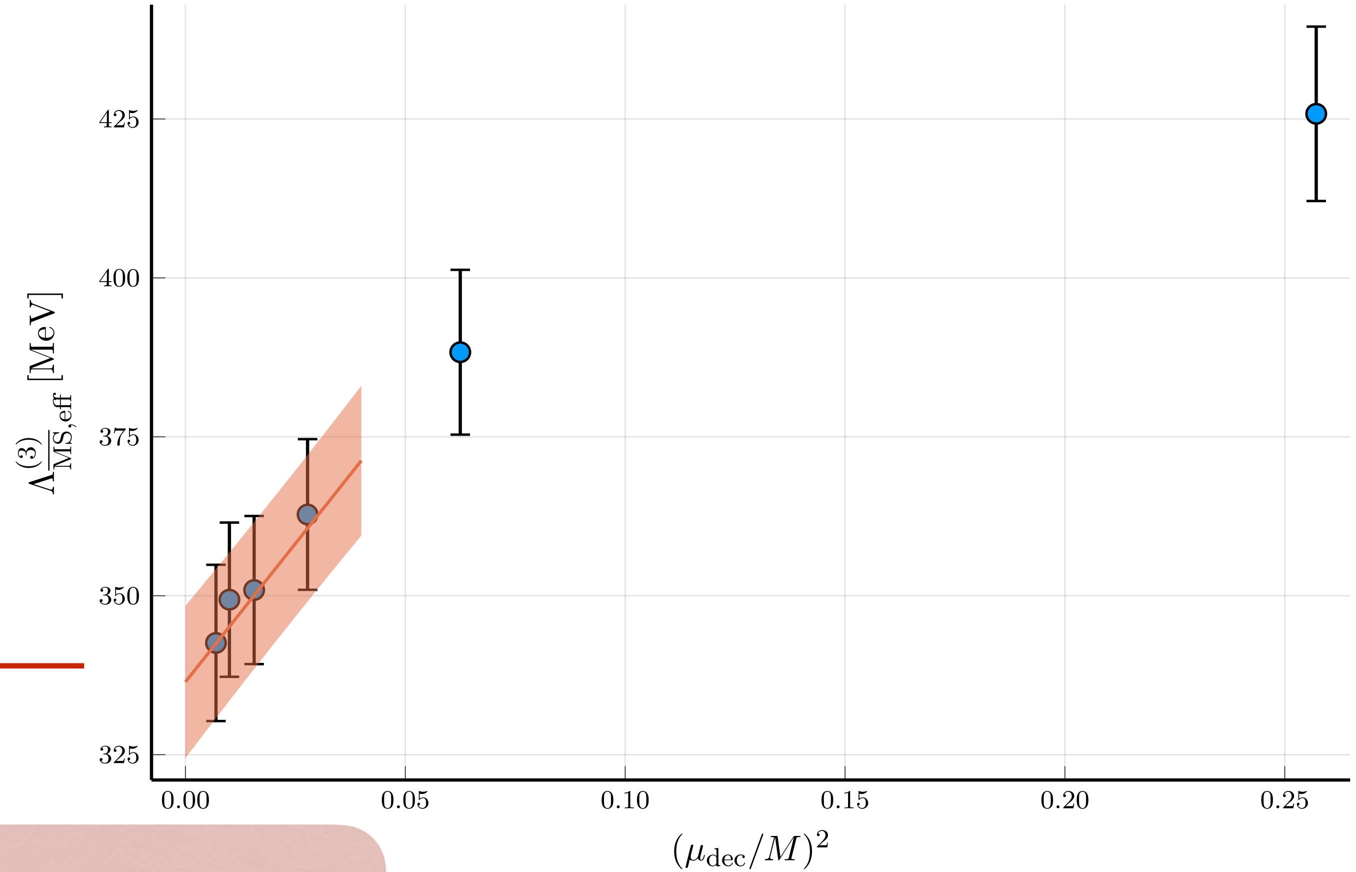
Determination of α_s

decoupling relation

$$\underbrace{\rho \times P_{\ell,\mathrm{f}}(z/\rho)}_{\text{High order PT}} = \underbrace{\frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\Lambda^{\ell}_{\mathrm{GF}}}}_{\text{1-lp exact}} \underbrace{\varphi^{\ell}_{\mathrm{GF}}}_{\text{YM}}\left(\underbrace{\sqrt{\Psi_{\mathrm{M}}(u_0, z)}}_{\text{full}}\right), \qquad \rho = \frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}}$$

solve for ρ at each z

extrapolate to $z = M/\mu_{\mathrm{dec}} \to \infty$



$$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 336(10)(6)_{b_{\mathrm{g}}}(3)_{\Gamma_m}\ \mathrm{MeV}$$

...

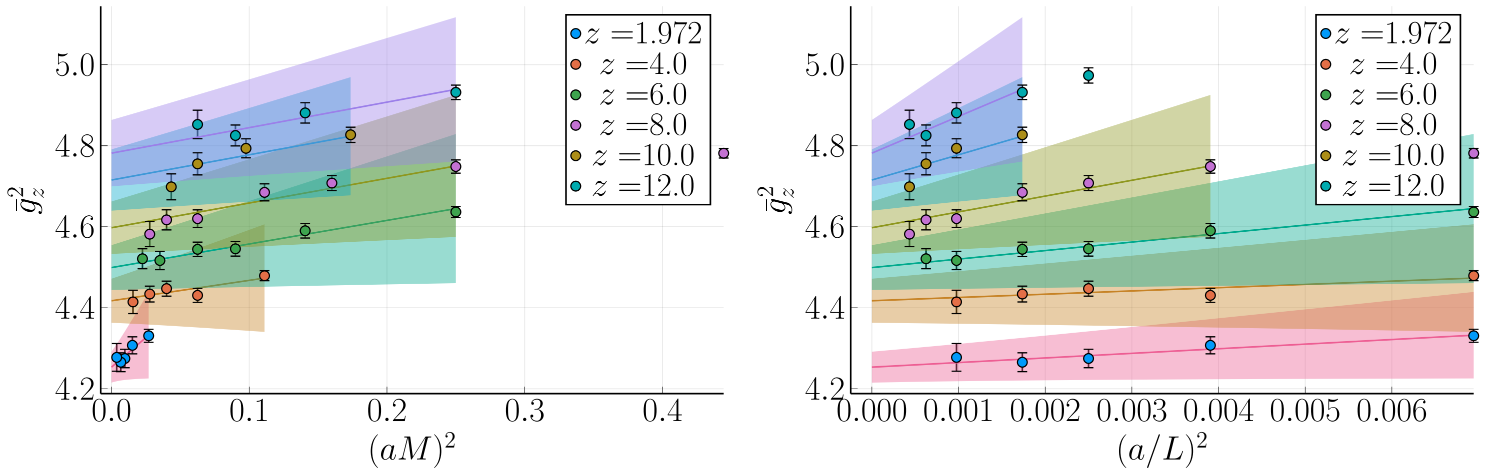
$$\alpha_s(m_Z) = 0.11823(69)(42)_{b_{\mathrm{g}}}(20)_{\Gamma_{\mathrm{m}}}(6)_{3\to5,\mathrm{PT}}(7)_{3\to5,\mathrm{NP}}$$

Sources of errors

$$\alpha_s(m_Z) = 0.11823(69)(42)_{b_\mathrm{g}}(20)_{\Gamma_\mathrm{m}}(6)_{3\to5,\mathrm{PT}}(7)_{3\to5,\mathrm{NP}}$$

- uncertainty is very small
- many small uncertainties play a role and need to be understood
- EFTs help

Discretization errors / the continuum limit



SymEFT: expansion in $a \sim 1/m_{\mathrm{h}}$

symmetries: lattice symmetries

LO: a^0: $\mathscr{L}_0 = \mathscr{L}_{\mathrm{YM}}$

NNLO: a^2: $$\mathscr{L}_2 = \omega_1 \underbrace{\sum_{\mu\nu} \mathrm{tr}\, D_\mu F_{\mu\nu} D_\mu F_{\mu\nu}}_{O(4)\text{ violating}} + \ldots + \omega_{m,1}\, m^2 \sum_{\mu\nu} \mathrm{tr}\, F_{\mu\nu} F_{\mu\nu} + \ldots$$

Discretization errors / the continuum limit

E

a^{-1} lattice QCD

M SymEFT $\mathscr{L} = \mathscr{L}_{\text{QCD}} + a^2 \mathscr{L}_2^{\text{Sym}} + \ldots$

$$\mathscr{L}_2^{\text{Sym}} = \omega_1 \sum_{\mu\nu} \operatorname{tr} D_\mu F_{\mu\nu} D_\mu F_{\mu\nu} + \ldots + \omega_{\text{m},1} M^2 \sum_{\mu\nu} \operatorname{tr} F_{\mu\nu} F_{\mu\nu} + \ldots$$
$$+ \omega_{\text{m},2} M^2 \, \bar{\psi} M \psi + \ldots$$

expand SymEFT (continuum EFT) in $1/M$

μ_{dec} decSymEFT $\mathscr{L} = \mathscr{L}_{\text{QCD}} + \frac{1}{M^2} \mathscr{L}_2^{\text{dec}} + a^2 M^2 \mathscr{D}_0 + a^2 \mathscr{L}_2^{\text{decSym}} + \ldots$

$$\mathscr{L}_2^{\text{dec}} = \sigma_1 \mathscr{D}_1 + \sigma_2 \mathscr{D}_2 \,, \quad \mathscr{L}_2^{\text{decSym}} = \omega_1 \mathscr{D}_1 + \omega_2 \mathscr{D}_2 + \omega_3 \mathscr{D}_3$$

$$\mathscr{D}_1 = \sum_{\mu\nu\rho} \operatorname{tr} D_\rho F_{\mu\nu} D_\rho F_{\mu\nu} \,, \quad \underbrace{\mathscr{D}_2 = \sum_{\mu\nu\rho} \operatorname{tr} D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}}_{\text{GF modifies e.o.m.}} \,, \quad \underbrace{\mathscr{D}_3 = \sum_{\mu\nu} \operatorname{tr} D_\mu F_{\mu\nu} D_\mu F_{\mu\nu}}_{O(4)\text{ violating}}$$

=> $[\alpha(1/a)]^{\hat{\Gamma}_i} [\alpha(M)]^{\hat{\Sigma}_j} \mathscr{M}(\mu_{\text{dec}})$

partial knowledge on $\hat{\Gamma}_i \,, \hat{\Sigma}_j$

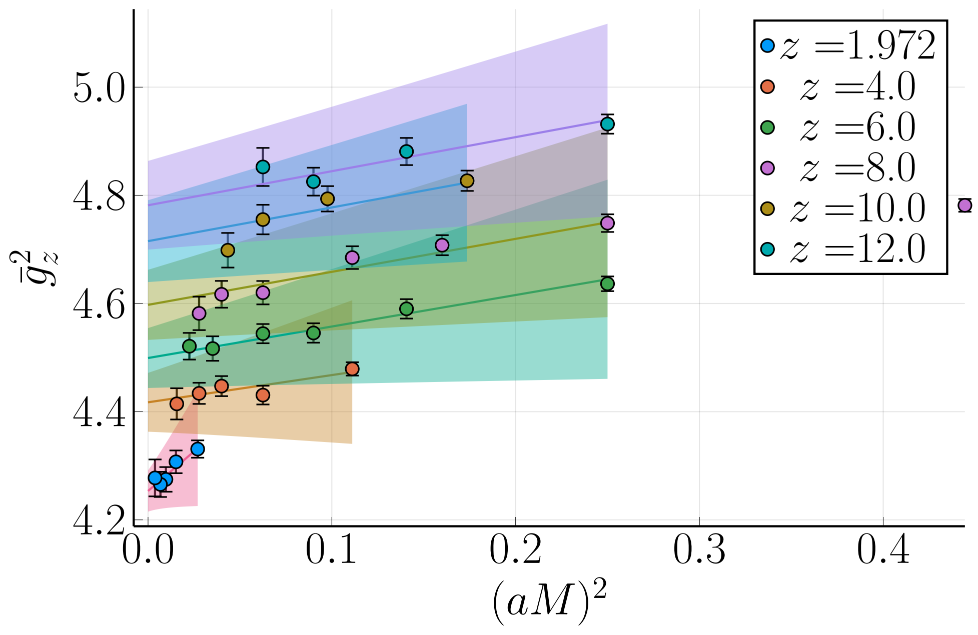
Discretization errors / the continuum limit

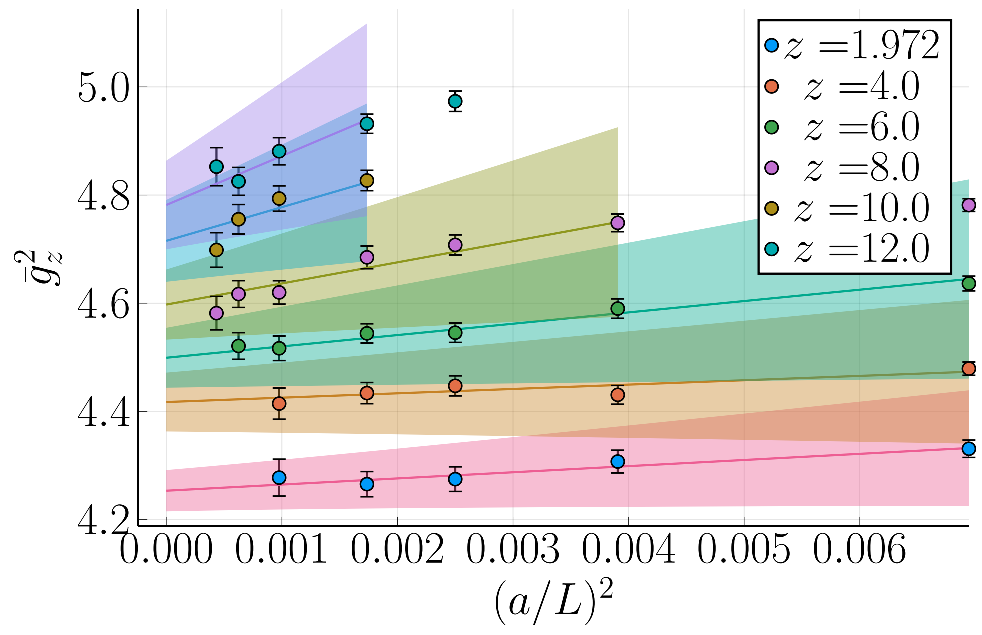
crucial conclusion:

global continuum limit for several, large M

$$\bar{g}^2(z_i, a) = C_i + p_1[\alpha_{\overline{\mathrm{MS}}}(a^{-1})]^{\hat{\Gamma}}(a\mu_{\mathrm{dec}})^2 + p_2[\alpha_{\overline{\mathrm{MS}}}(a^{-1})]^{\hat{\Gamma}'}(aM_i)^2$$

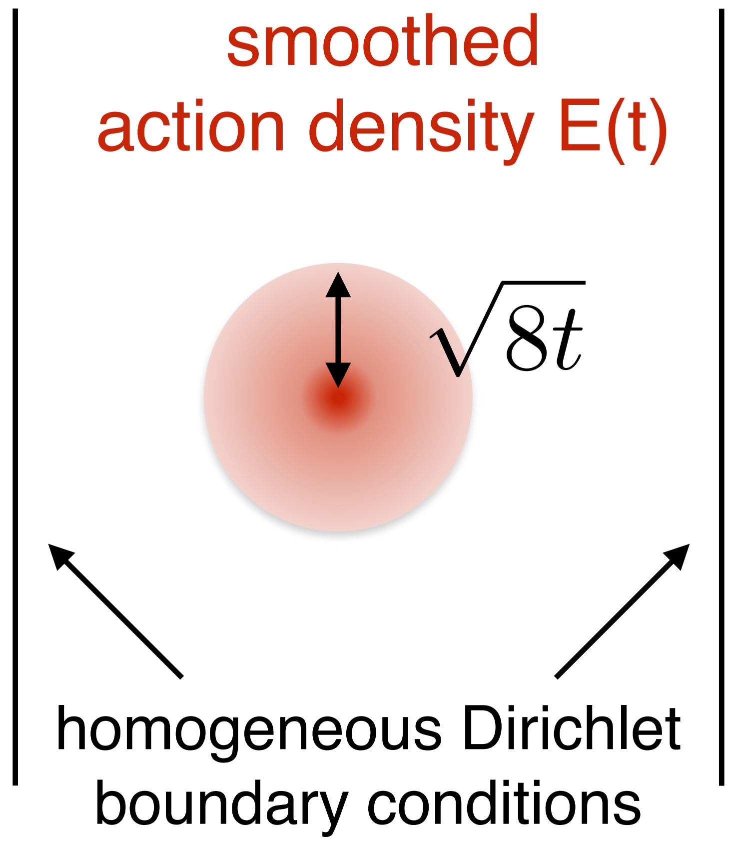
p_1, p_2 common for all z_i





Boundary effects

For various reasons we use the Coupling definition



$$\bar{g}^2_{\mathrm{GF}}(\mu = 1/L) = \# \times \langle E(t) \rangle$$

$$\sqrt{8t} = 0.3\,L$$

smoothing by GF
=
heat equation
for gauge fields

Boundaries:

effects kept small by keeping $E(t)$ away from the boundary

but they introduce $1/M$ effects in the decoupling

a single term $\mathscr{L}_1^{\mathrm{dec}} = \omega_{\mathrm{b}} \left\{ \mathrm{tr}\, F_{0k} F_{0k} \left[\delta(x_0) + \delta(x_0 - T) \right] \right\}$

Boundary effects

Boundary $1/M$ effects due to

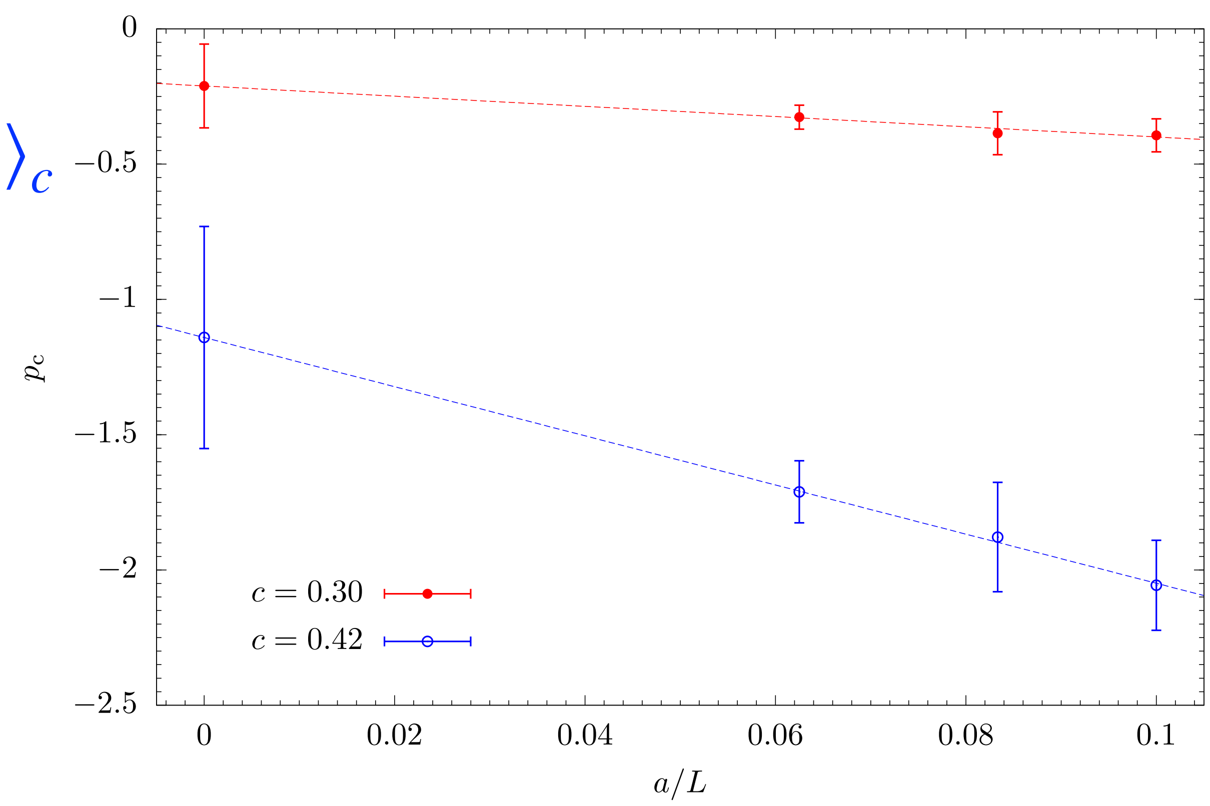
$$\mathscr{L}_1^{\mathrm{dec}} = \hat{\omega}_{\mathrm{b}} \frac{1}{M} \left\{ \mathrm{tr}\, F_{0k} F_{0k} \left[\delta(x_0) + \delta(x_0 - T) \right] \right\}$$

evaluated $\hat{\omega}_{\mathrm{b}}$ in leading order (1-loop) PT

and

$$\mathscr{M} = \langle \mathrm{tr}\, F_{0k} F_{0k} \left[\delta(x_0) + \delta(x_0 - T) \right] E(t) \rangle_c$$

non-perturbatively by simulation in YM



=> the total effect is negligible compared

to our statistical errors

Summary

EFTs are important for lattice QCD

- SymEFT crucial for understanding the continuum limit
  log-corrections recently [N. Husung et al]

- HQET solves heavy-quark-on-the-lattice problem (valence quark)
  but has not reached its potential

- decoupling of heavy sea quarks:

  — already charm can usually be neglected
  — can be turned into a tool:
  world-highest precision already for $\alpha_s(M_Z)$
  higher when a technical problem will be solved (b_g uncertainty)

Thanks for your patience

have a nice holiday season