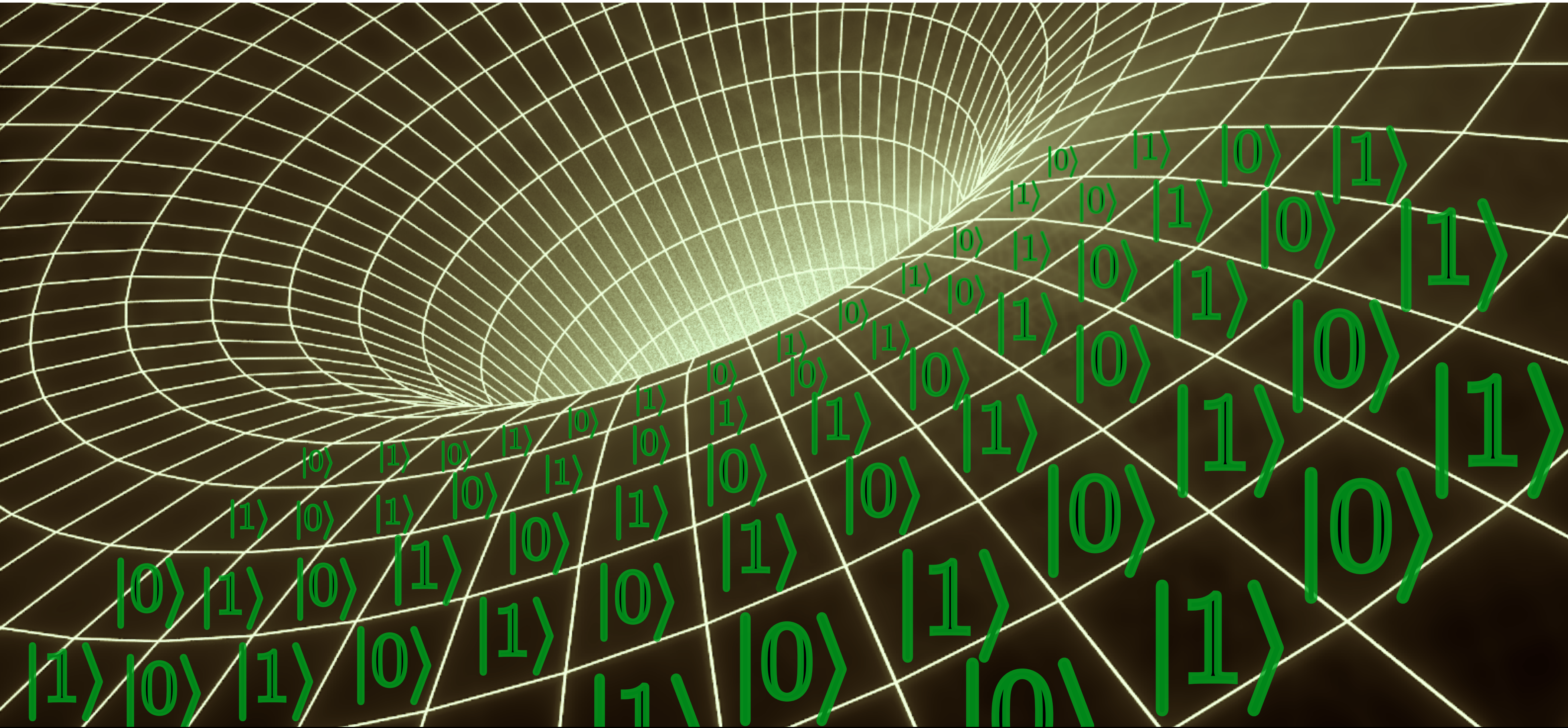
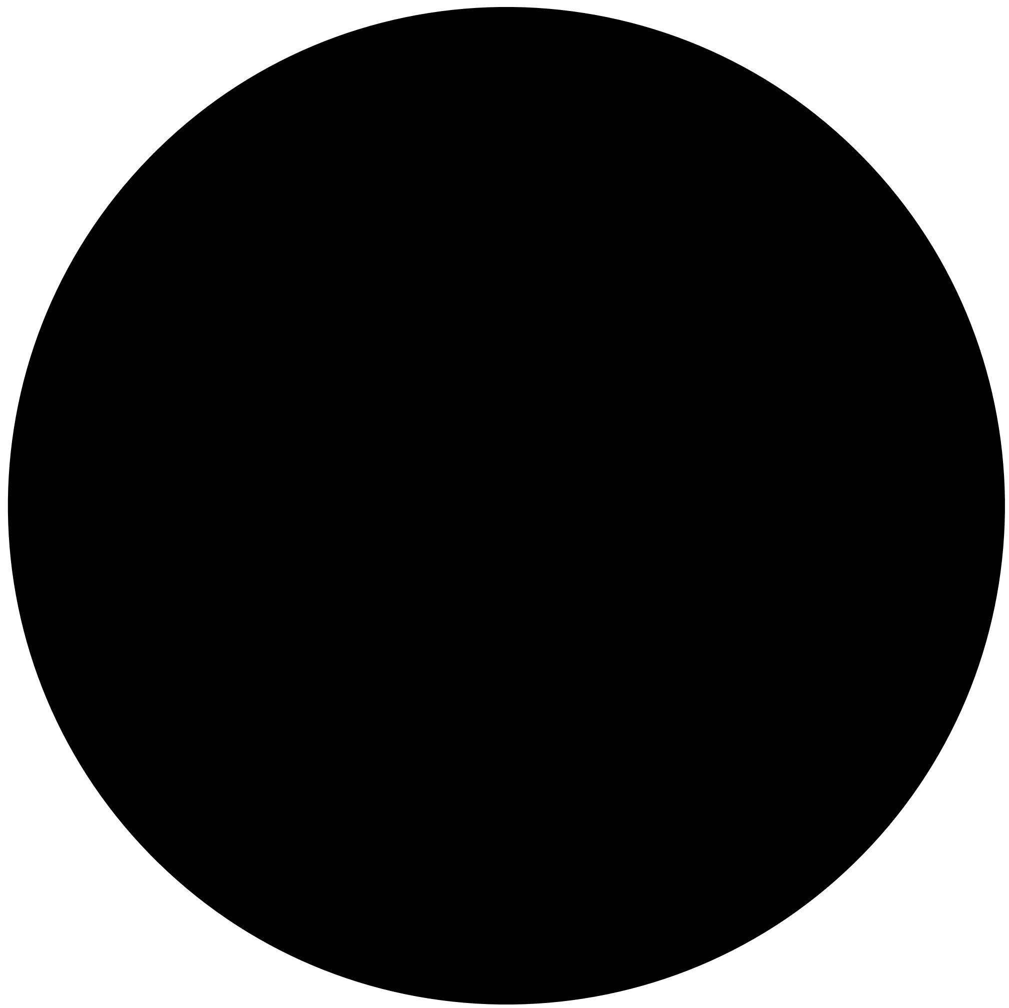


Spacetime wormholes in quantum gravity

Tom Hartman
Cornell University

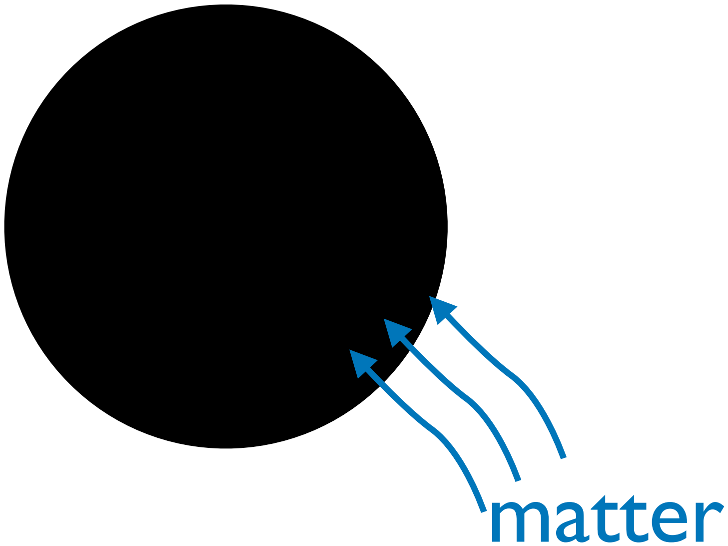
IFT Christmas Workshop ✦ Madrid ✦ December 16, 2022

Classical black holes



are *featureless* objects — pure spacetime curvature

However, they behave like thermodynamic systems with a very large number of microscopic degrees of freedom:

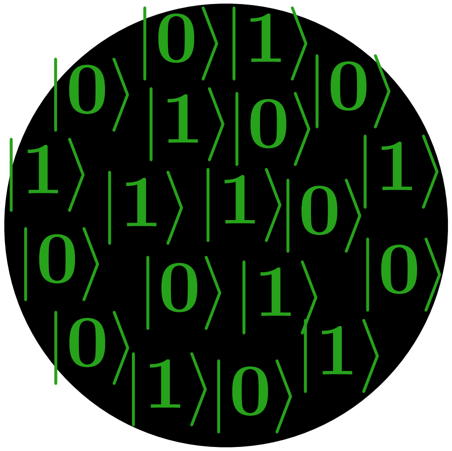


Hawking temperature $T = \frac{\hbar}{4\pi R}$

entropy $S = \frac{\text{Area}}{4\hbar G}$

Sagitarrius A^* $S = 2^{88}$

In quantum gravity,



$$\#\ \text{states} \approx e^S$$

How is this quantum information encoded?

How does it escape when a black hole evaporates?

What else does this tell us about the UV completion?

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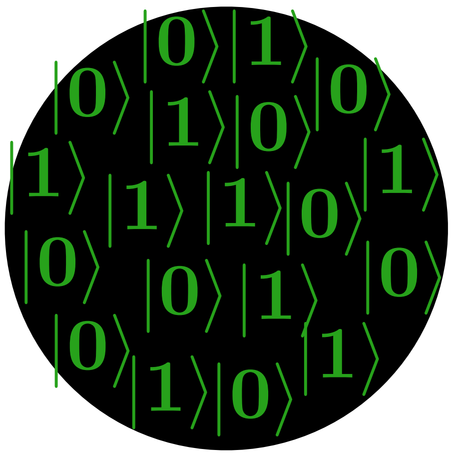

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What else does this tell us about the UV completion?

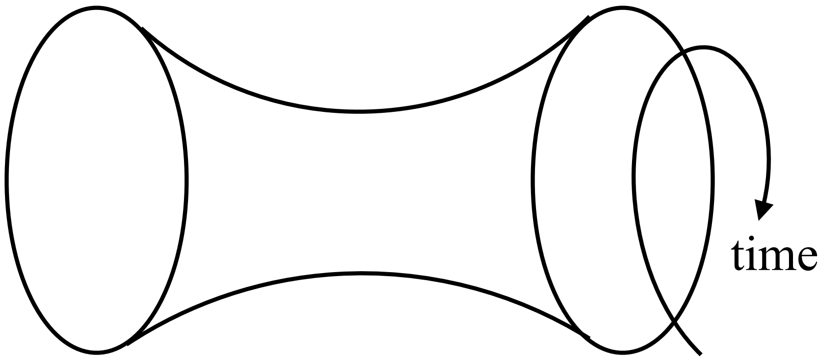
Recent progress

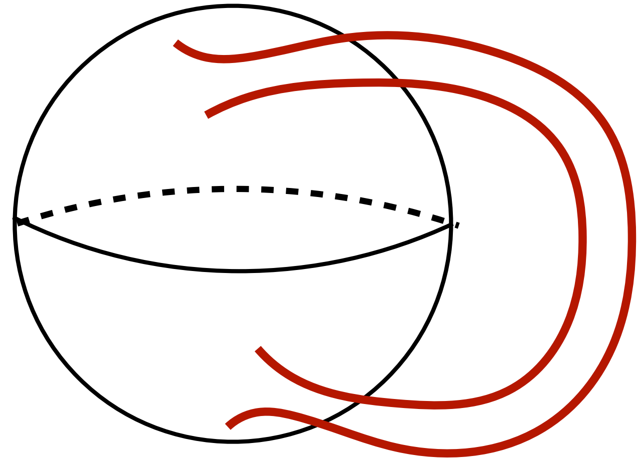
- **Modern tools of quantum information science**

$$S = -\mathrm{tr}\,\rho \ln \rho\ , \qquad I(A:B)\ , \qquad \mathcal{N}_{A\to B}$$

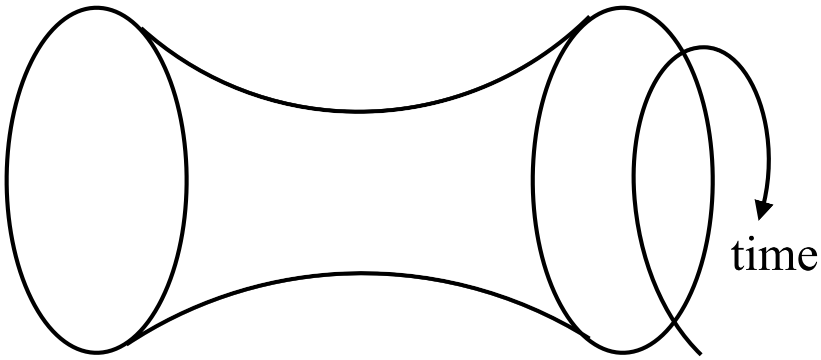
- **Spacetime wormholes**

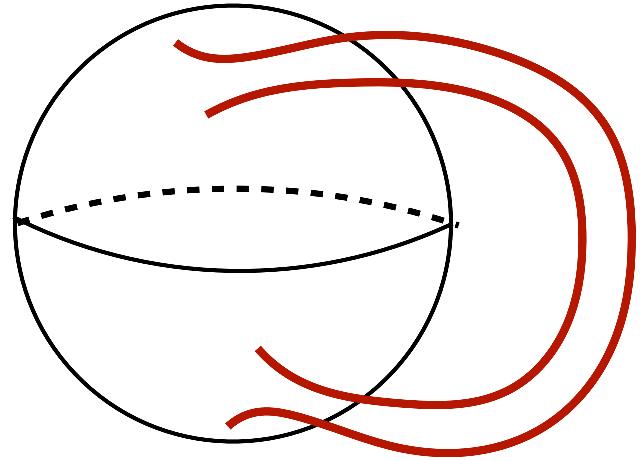
Higher topology spacetime manifolds that contribute to the gravitational path integral





These are not the wormholes of science fiction — those are *spatial* wormholes (and don’t exist in theories with positive energy!)





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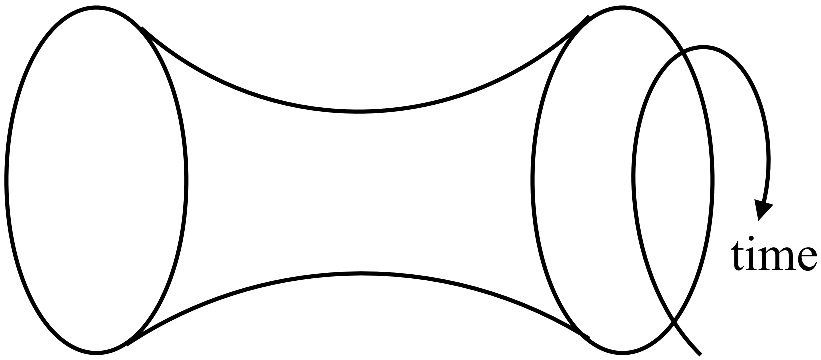
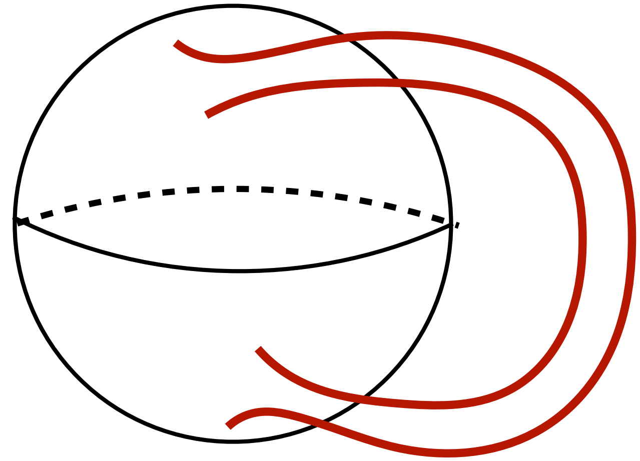
Spacetime wormholes are 4-dimensional solutions of General Relativity that exist whenever there are black holes.





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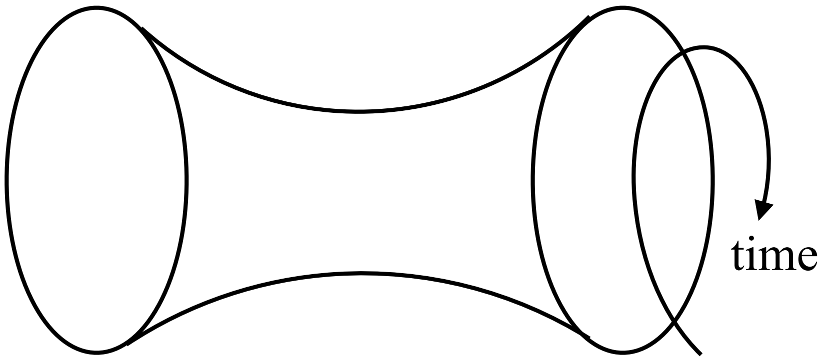
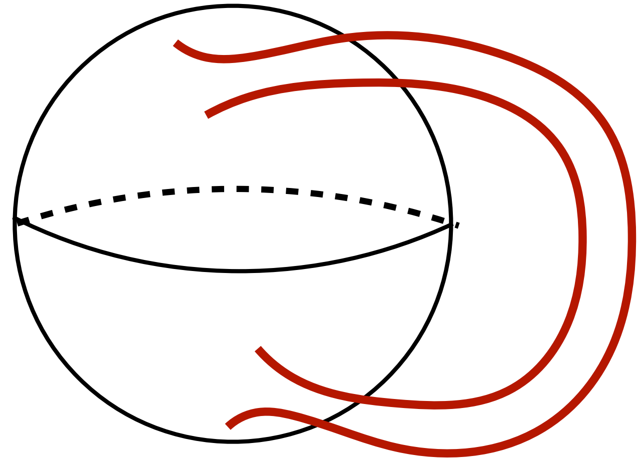
Generic prediction of GR — any Λ, any matter content, no strings required





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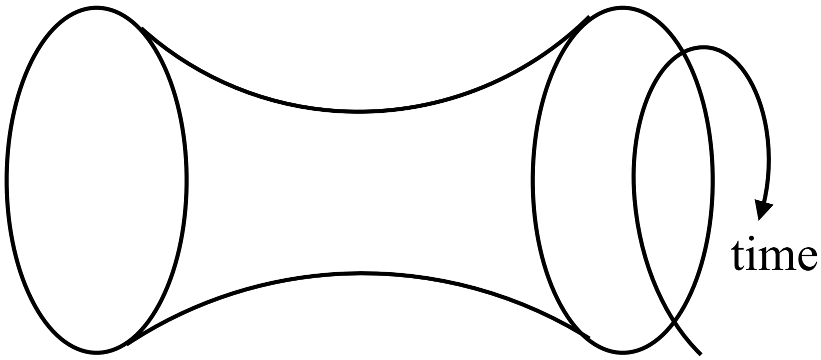

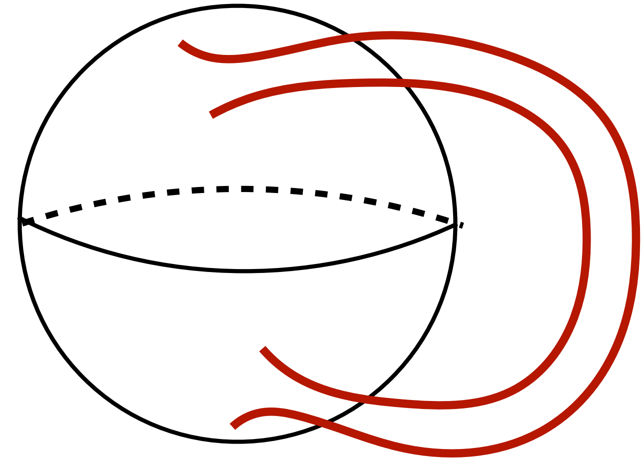

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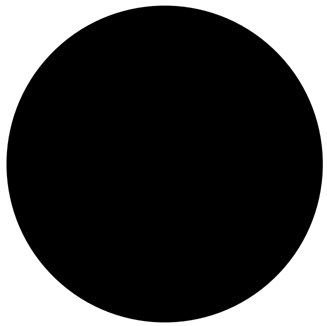
Complex-valued metric.

Gravitational instantons.

Black hole entropy tells us the average density of states in the UV theory,



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(Averaged over a small energy window)

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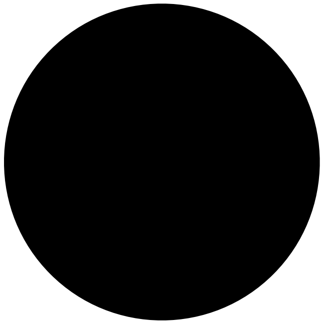

$$\Rightarrow \quad \overline{\rho(E)}$$

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Spacetime wormholes tell us more detailed statistical information about the UV.

I will discuss two examples:

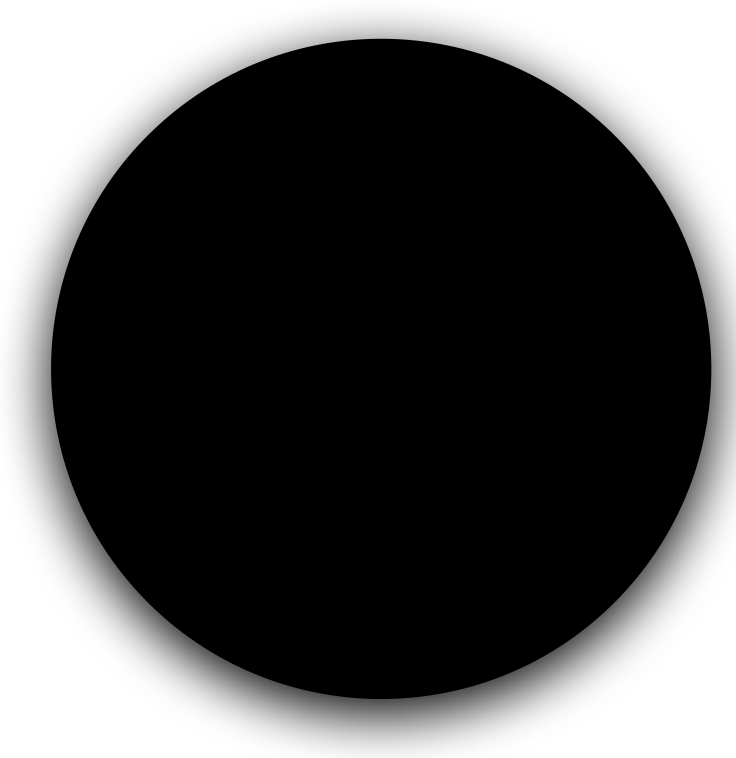
I. Replica wormholes $\Rightarrow$ entropy of Hawking radiation

II. Thin-shell wormholes $\Rightarrow$ operator statistics and global symmetry violation

I. Replica Wormholes

Based on:
[Almheiri, TH, Maldacena, Shaghoulian, Tajdini ’19]
[Penington, Shenker, Stanford, Yang ’19]

Information loss in black hole evaporation



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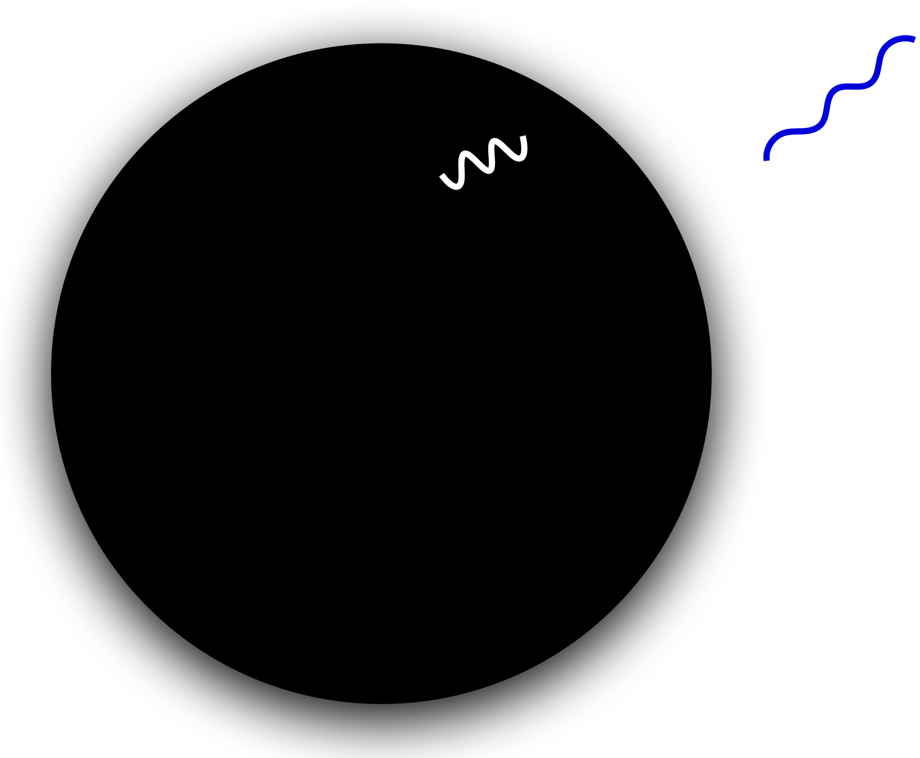

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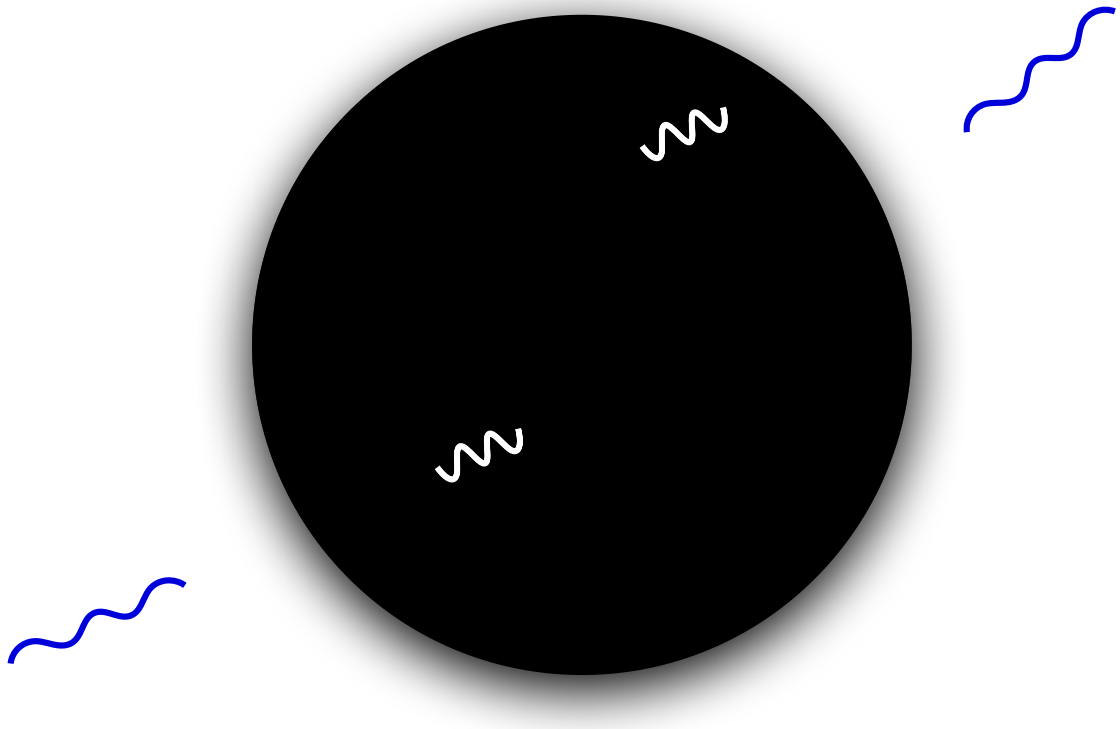

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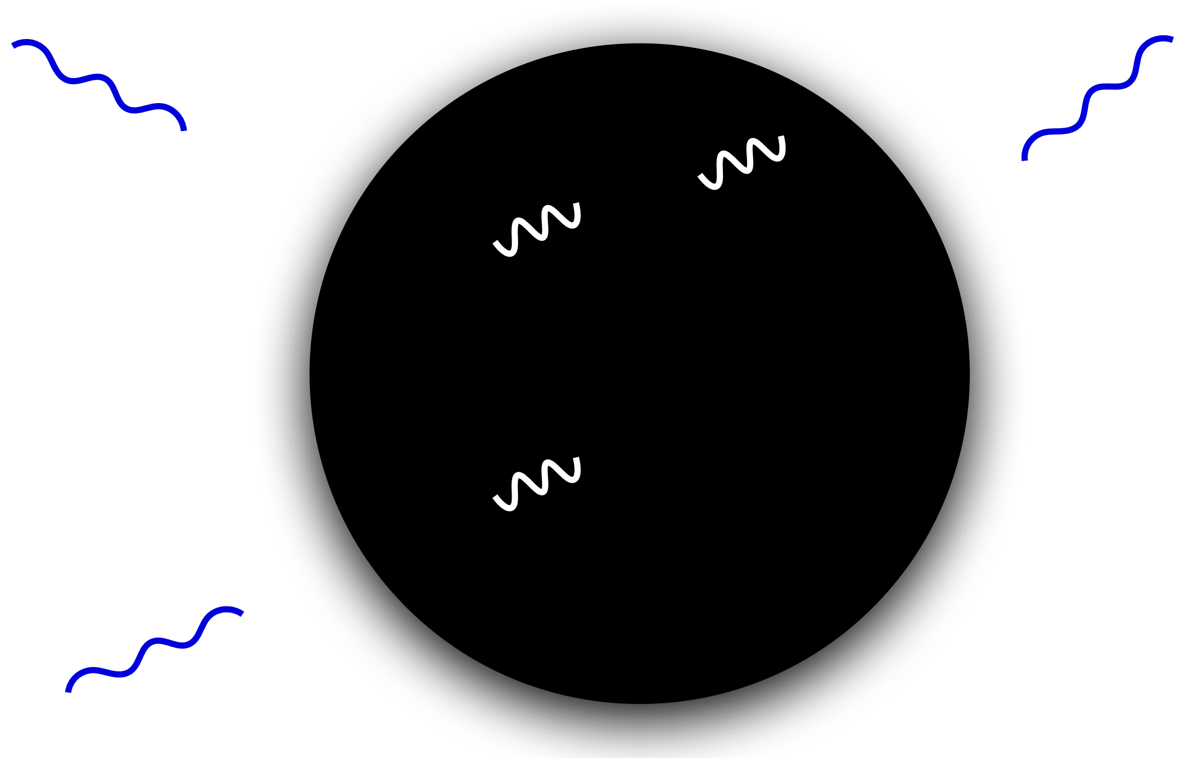

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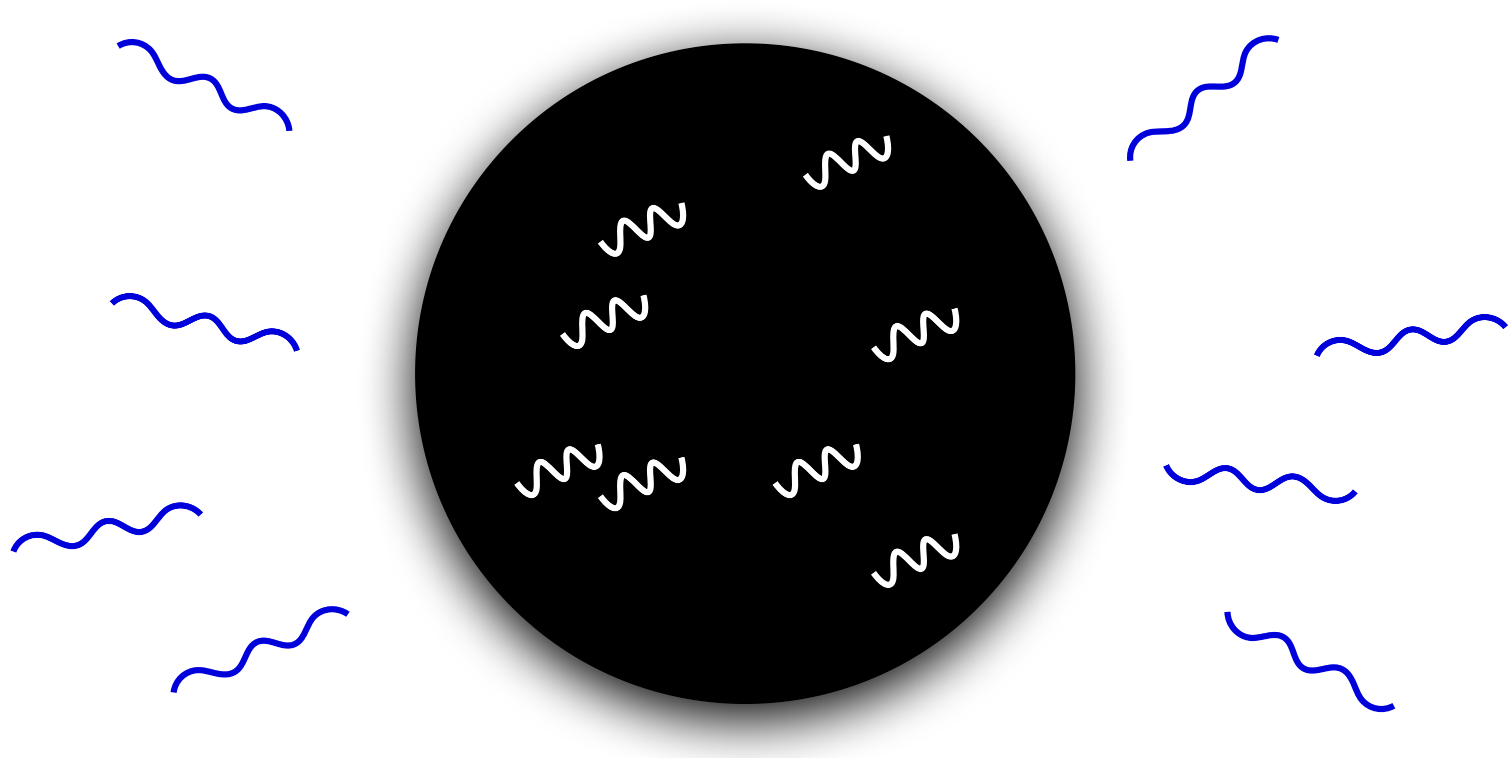

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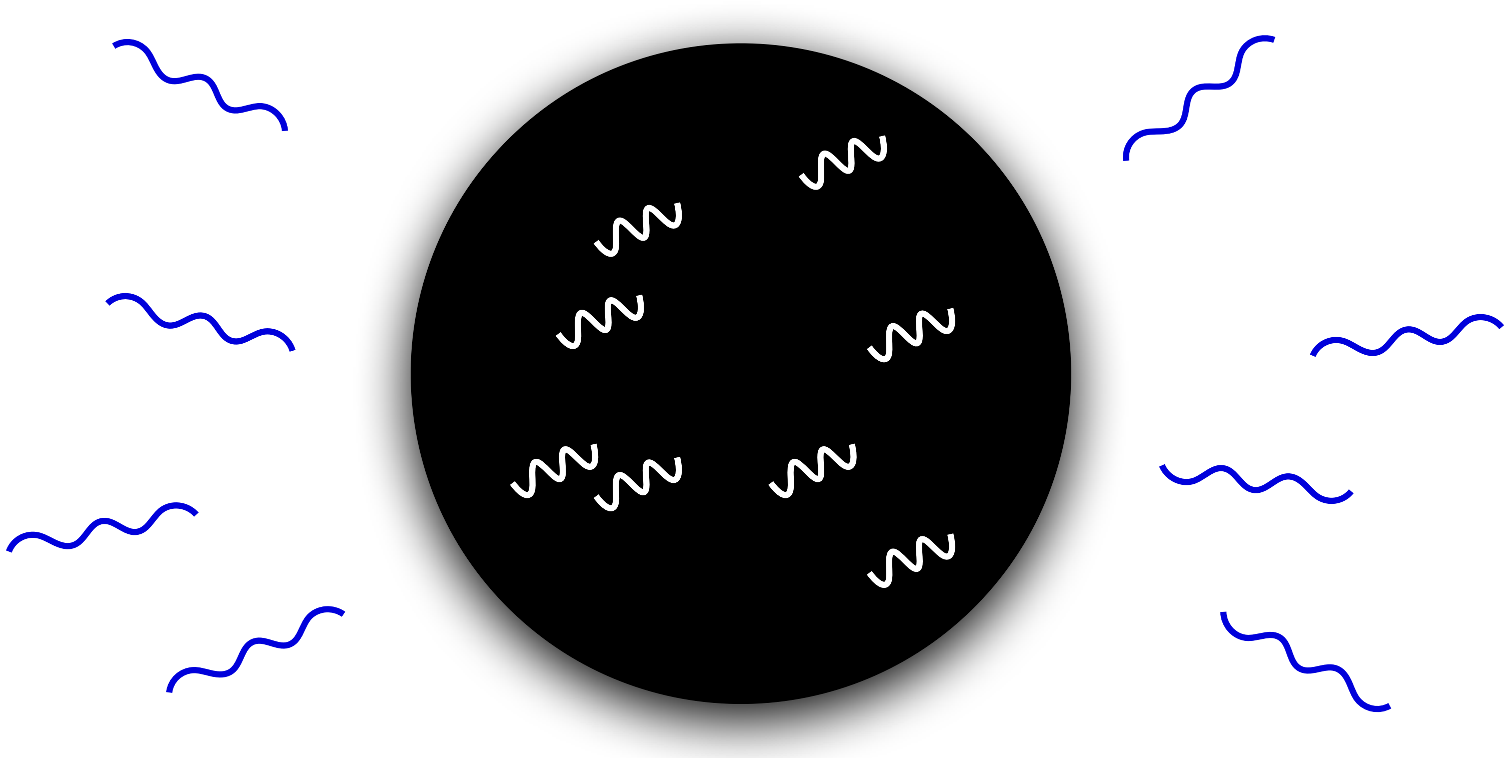
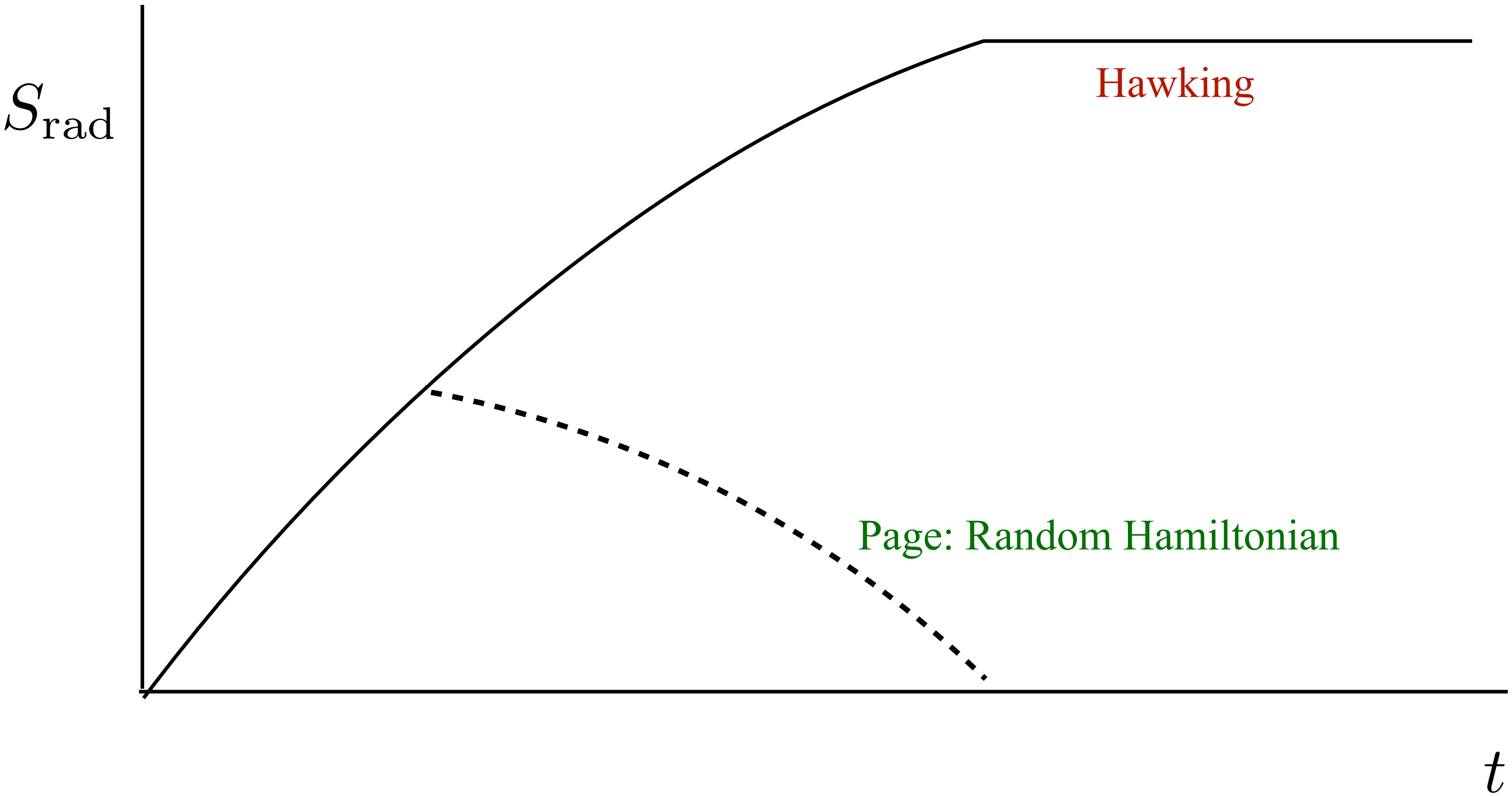


Hawking radiation is a process of *entanglement production* between the black hole interior and the radiation.

The “paradox” is that at the end, there is nothing for the radiation to be entangled with — QIS.

Entropy of Hawking radiation



The Island Effect

[Penington, '19]
[Almheiri, Engelhardt, Marolf, Maxfield '19]

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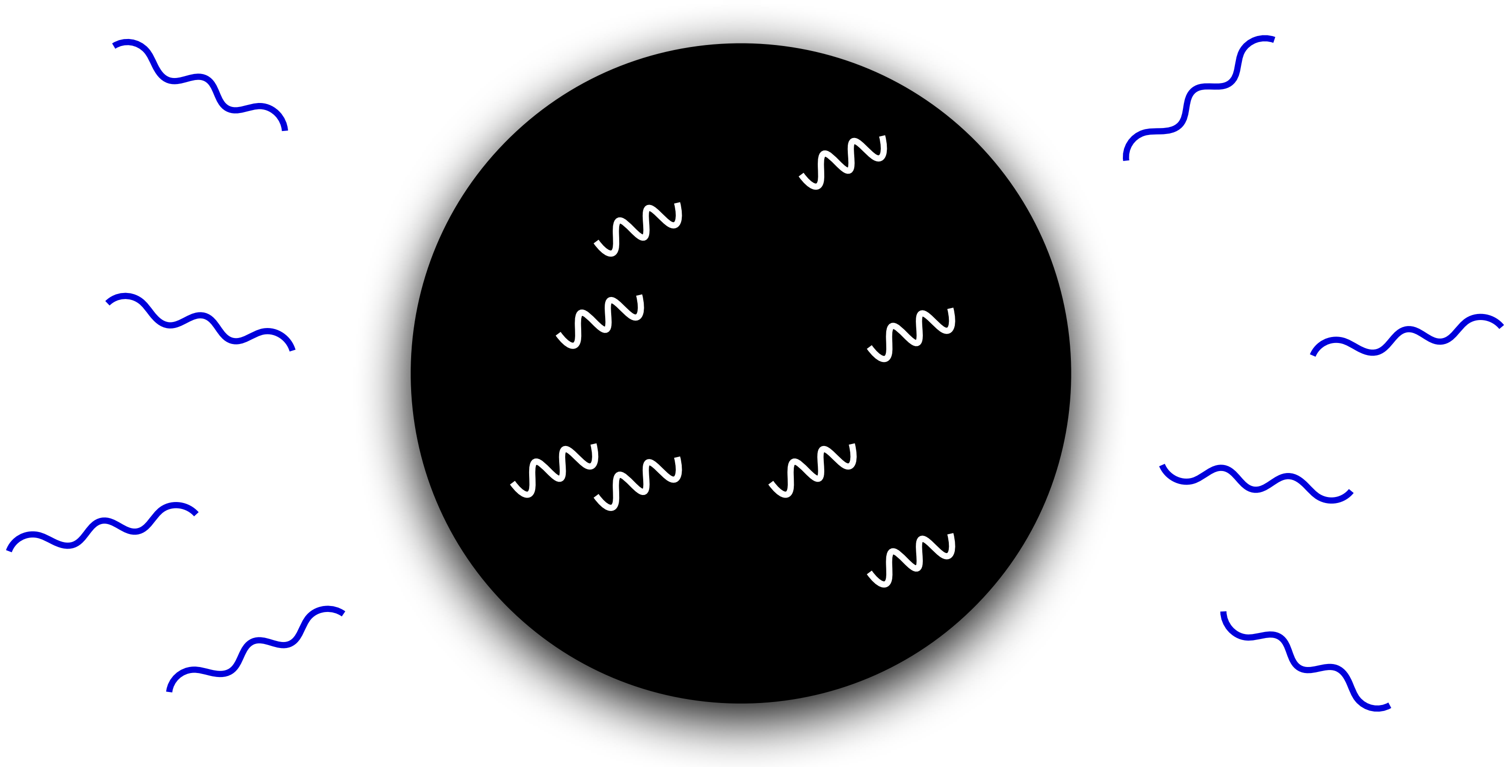

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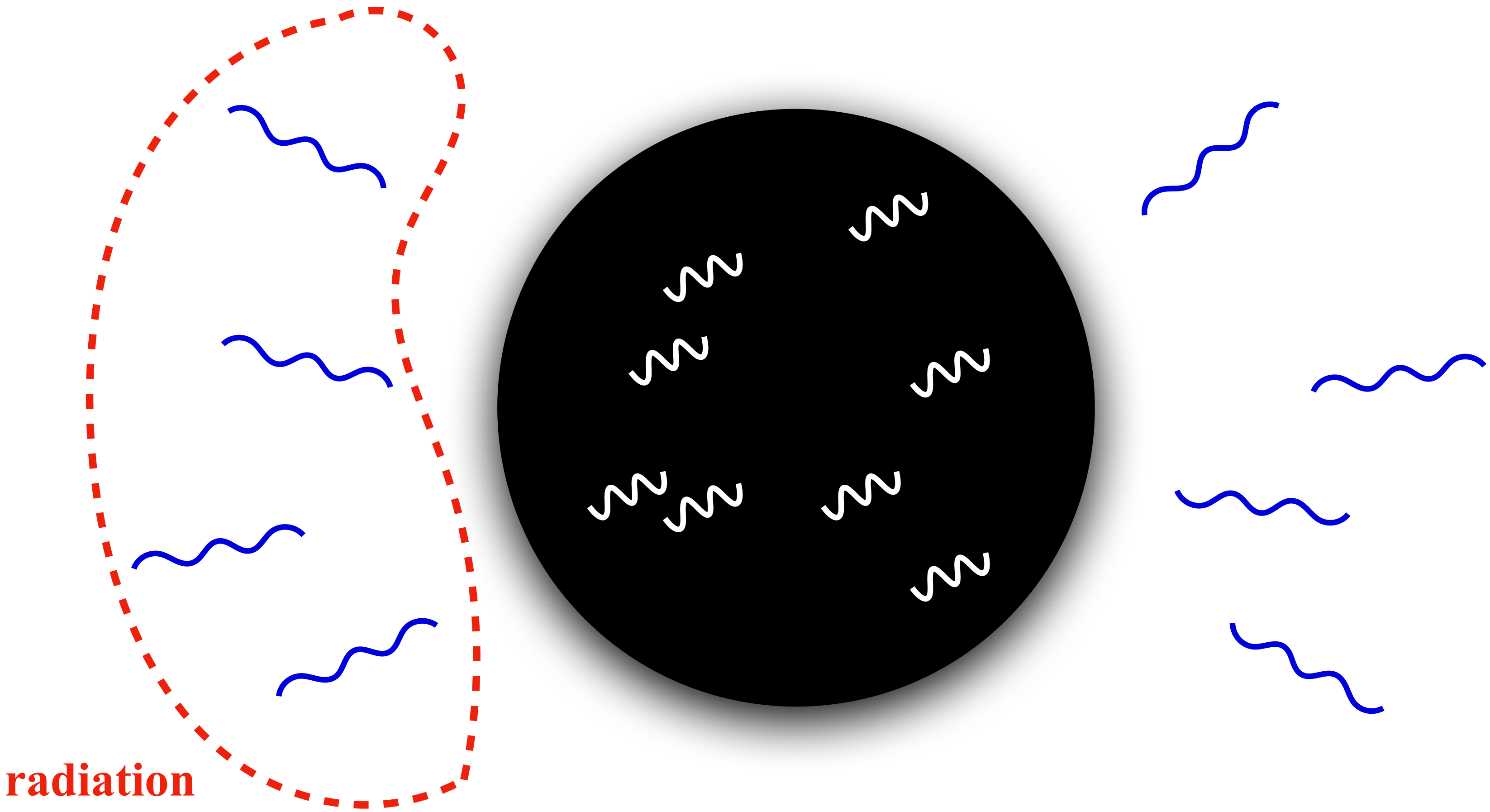

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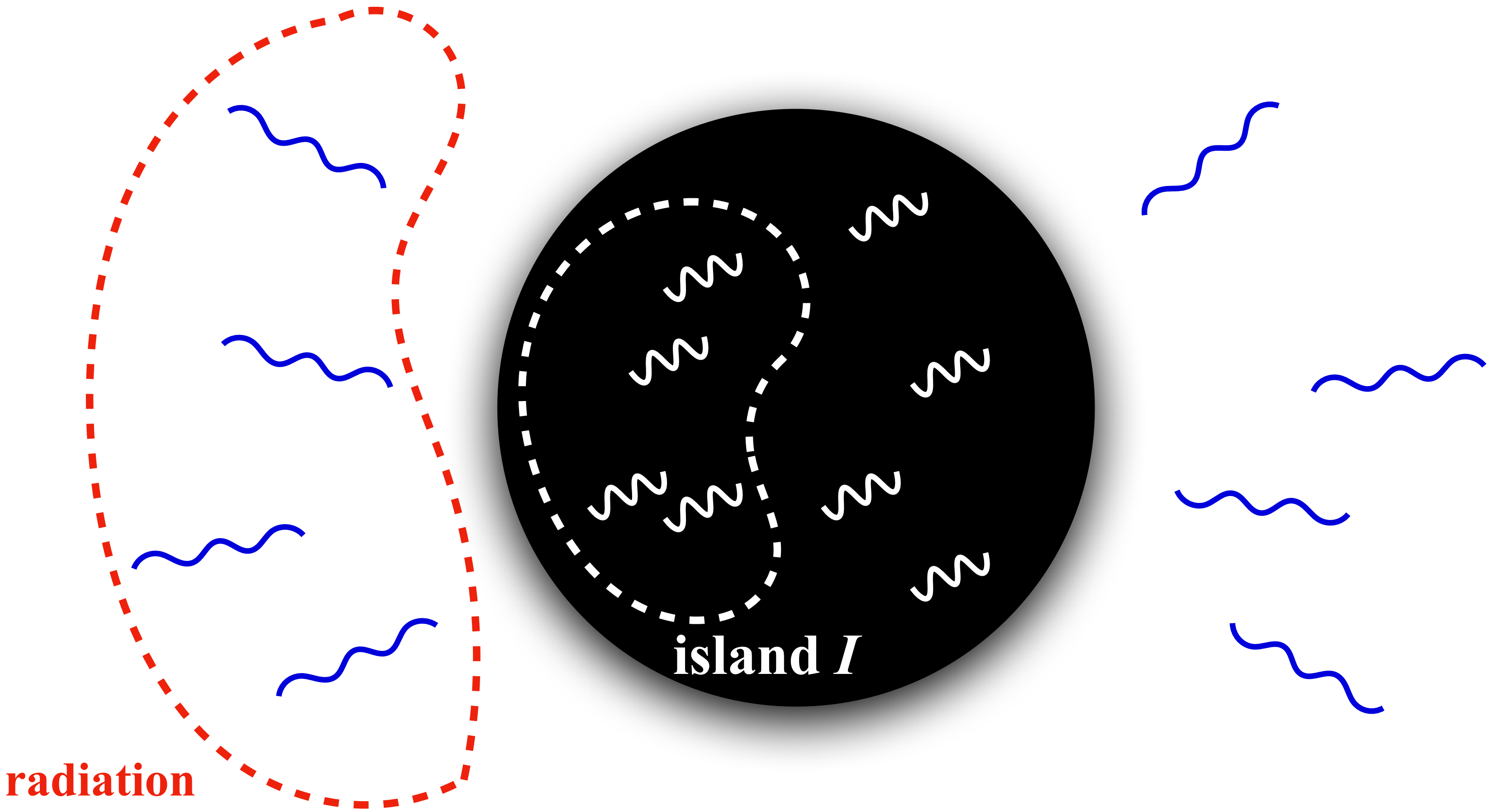

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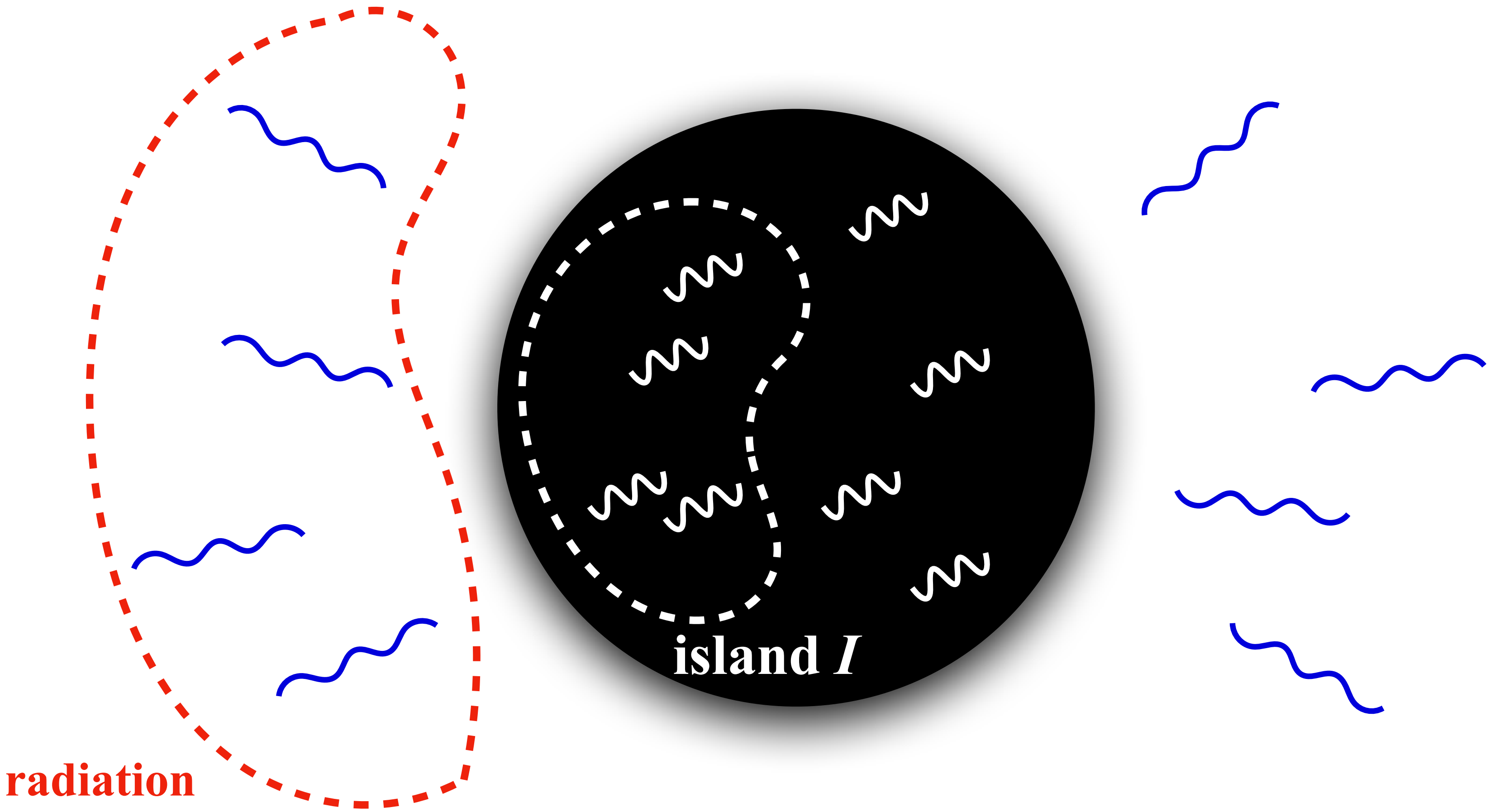

Entropy formula: $S(\rho_{\rm rad}) = \min_I \text{ext}_I \left[\frac{\text{Area}(\partial I)}{4} + S_{\rm QFT}(I \cup \text{rad}) \right]$

[Penington, '19]
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Replica wormholes

A “replica wormhole” is a gravitational instanton supported by matter entanglement.

$\Longrightarrow$ island effect

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Consider the "**purity**"

$$Z_2 := \mathrm{tr}\,(\rho^2)$$

pure: $Z_2 = 1$

mixed: $Z_2 < 1$

Calculate the purity of the Hawking radiation by a gravitational path integral:

$$\mathrm{tr}(\rho^2) \approx$$

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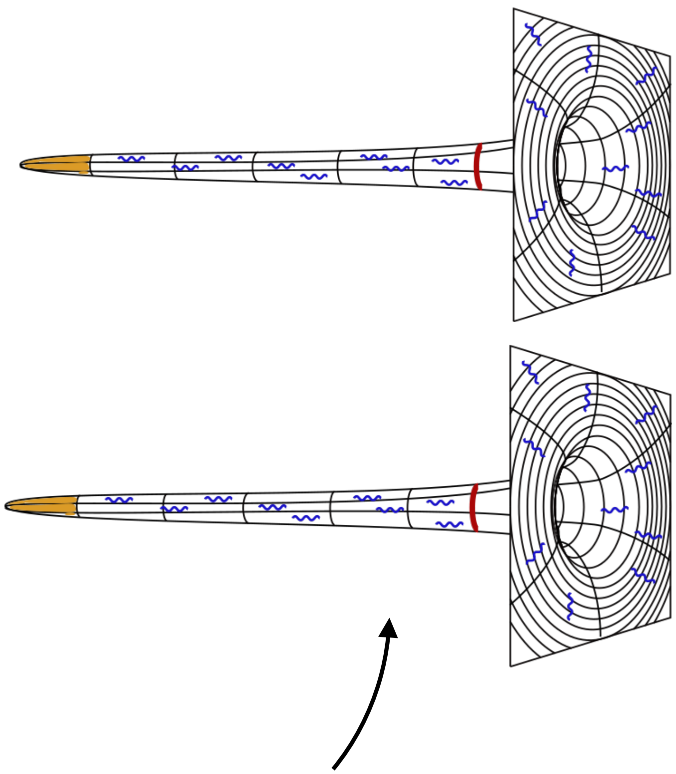

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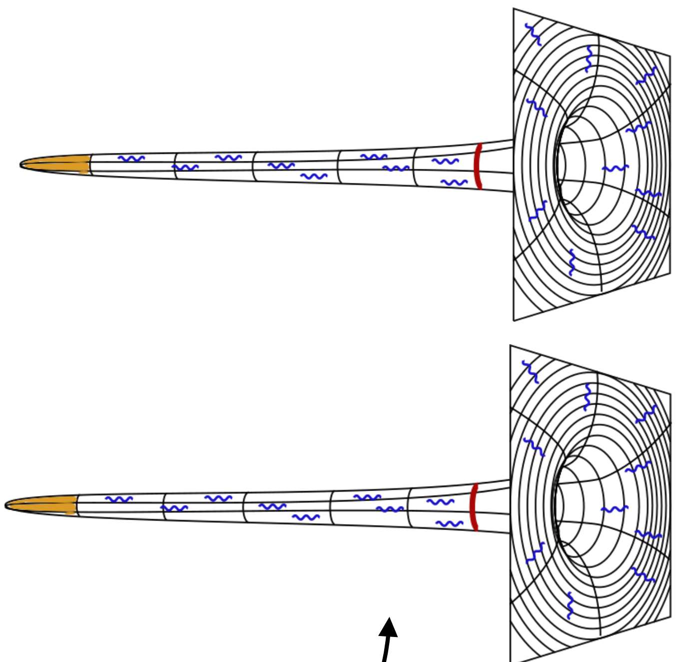
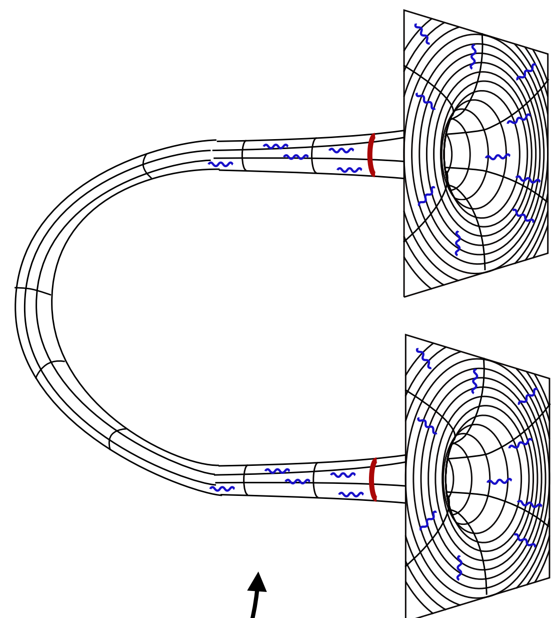


$+$



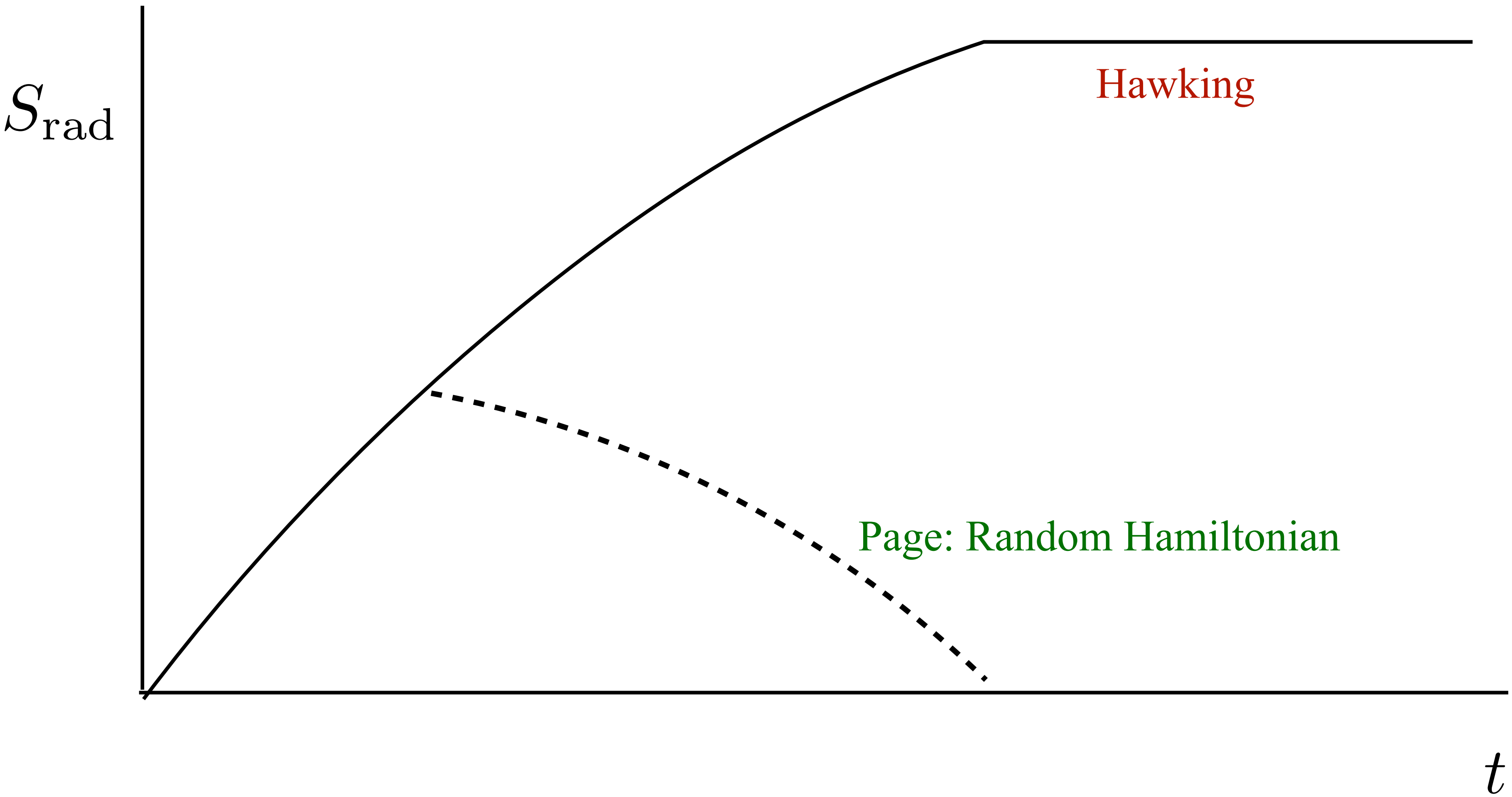
$+ \cdots$

Hawking radiation
is highly mixed

$$G_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle$$

Hawking radiation re-purifies

Intriguingly: The resulting entropy agrees precisely with the “Random Hamiltonian” prediction of Page.



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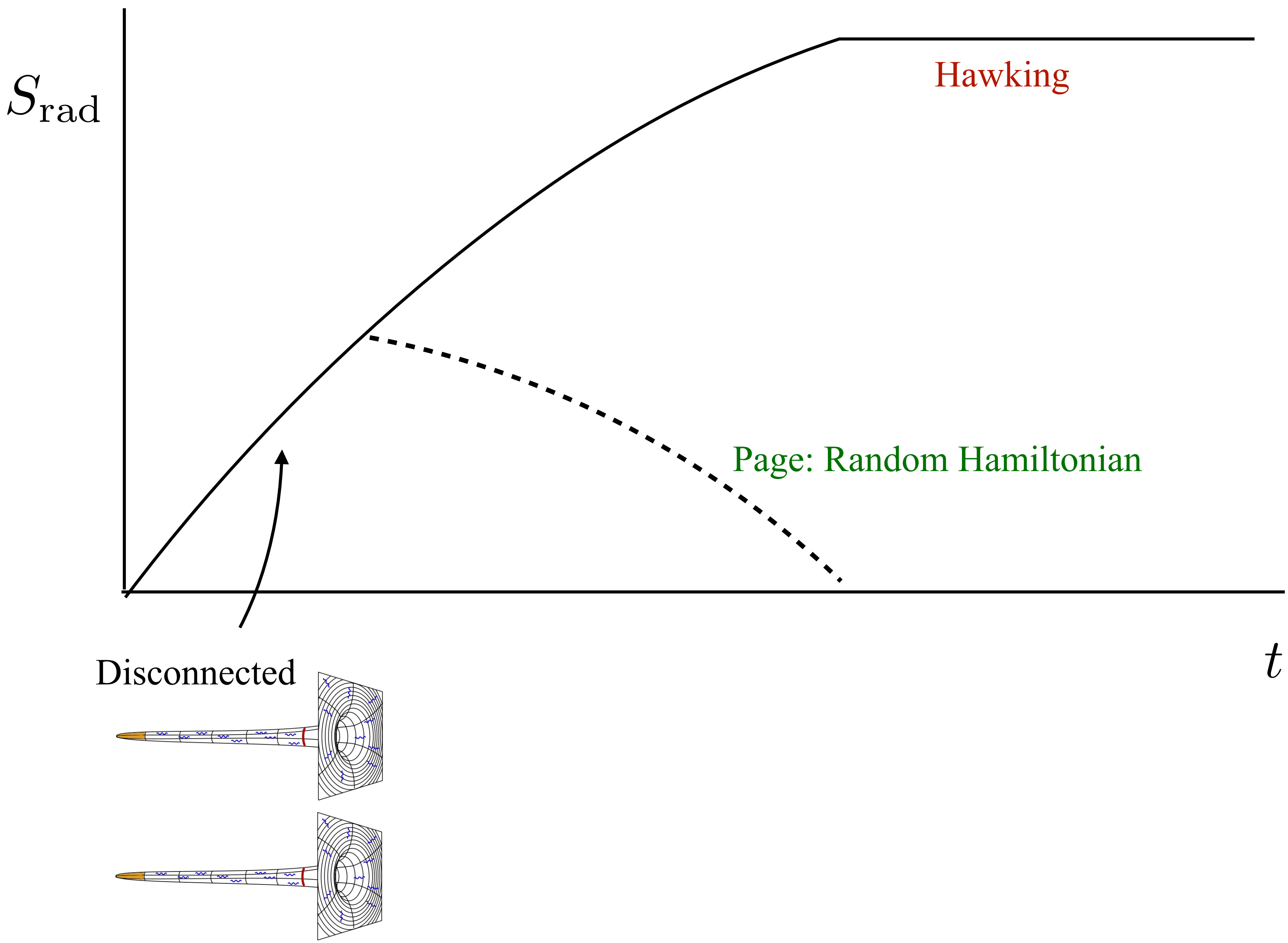

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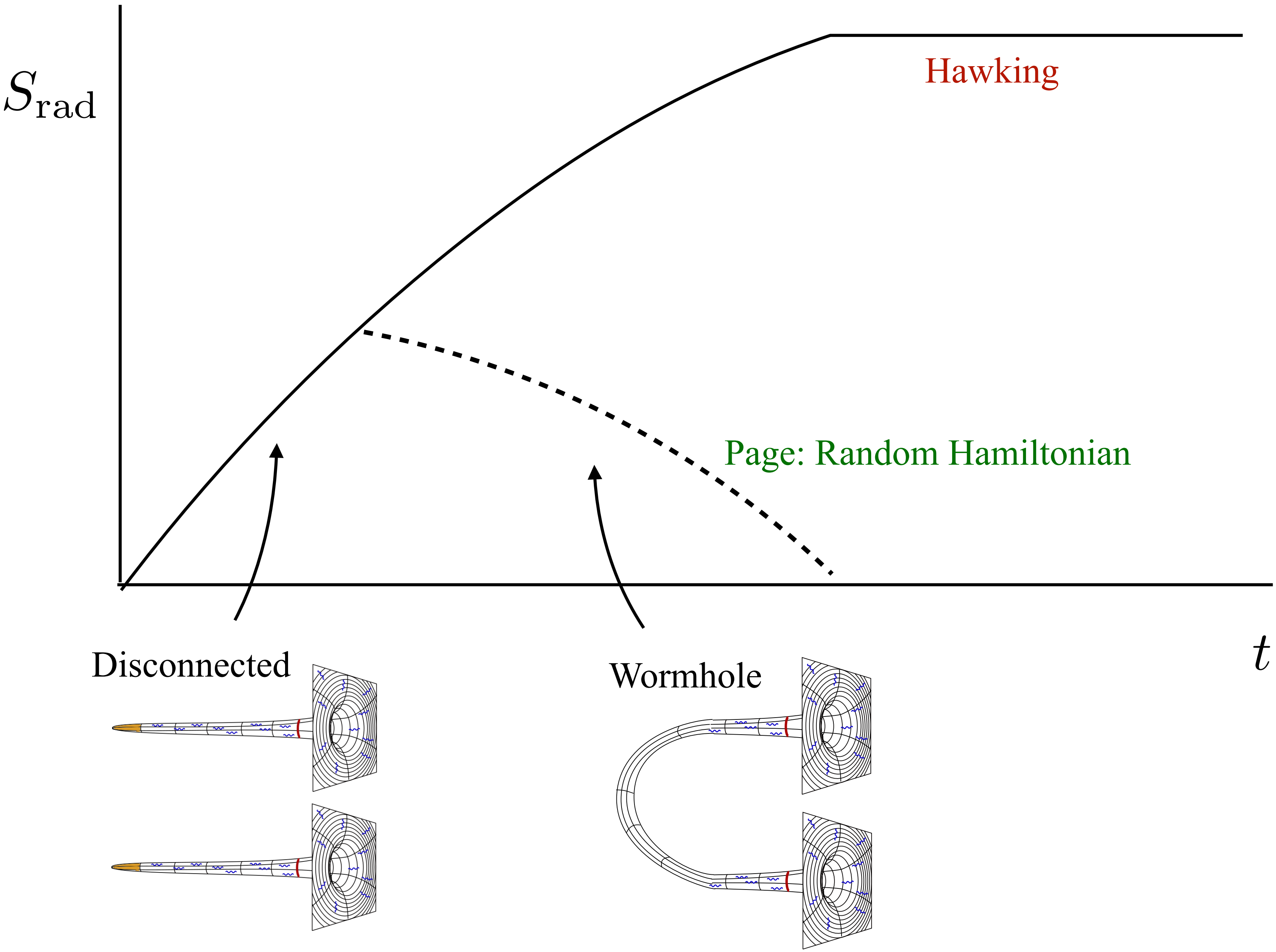
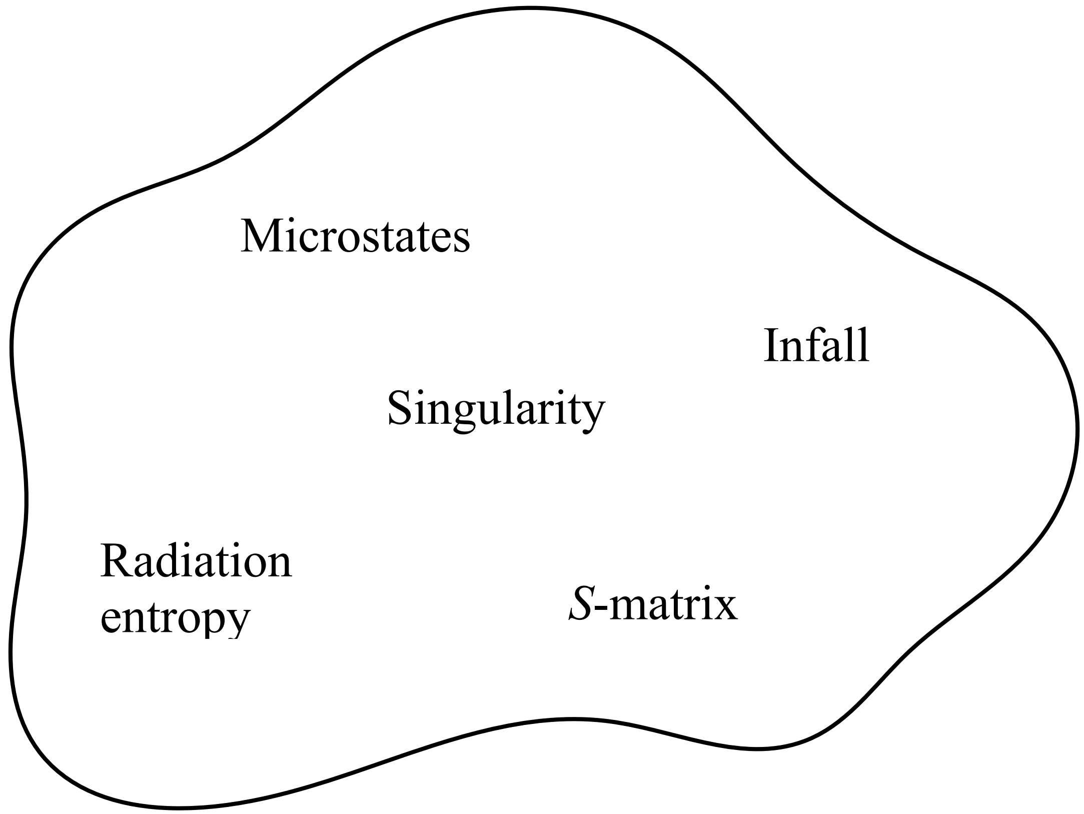


Status of the information problem



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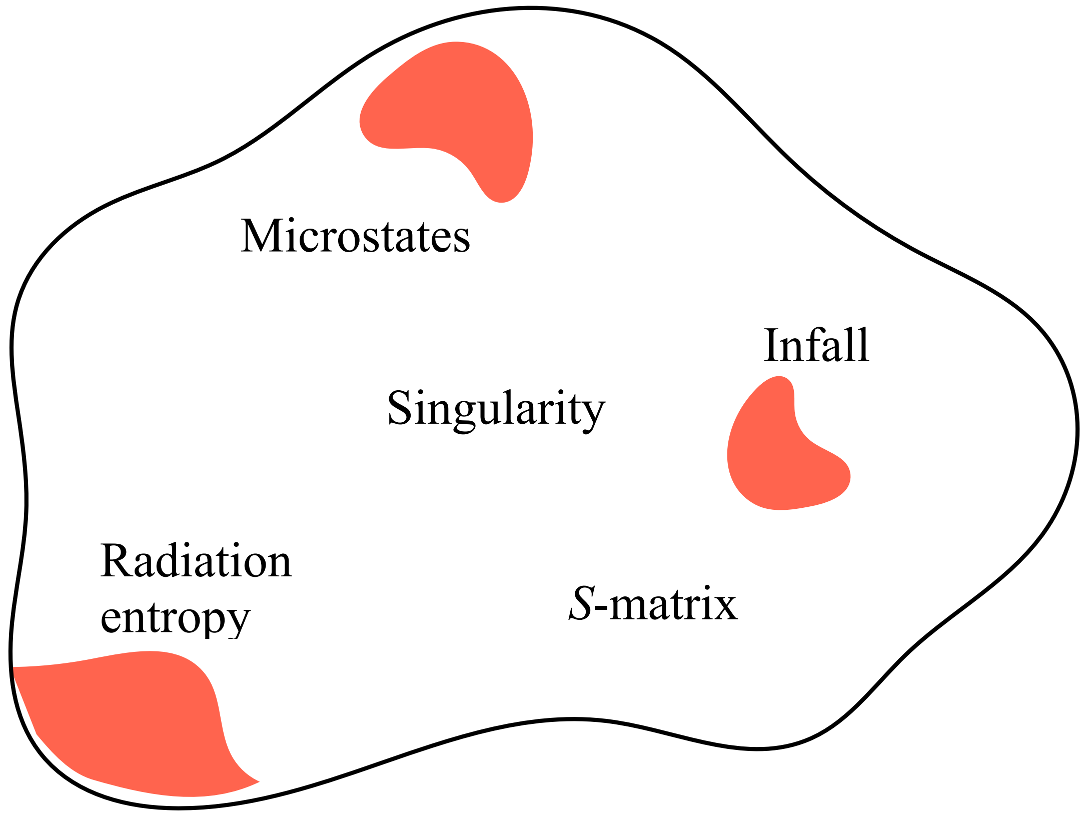

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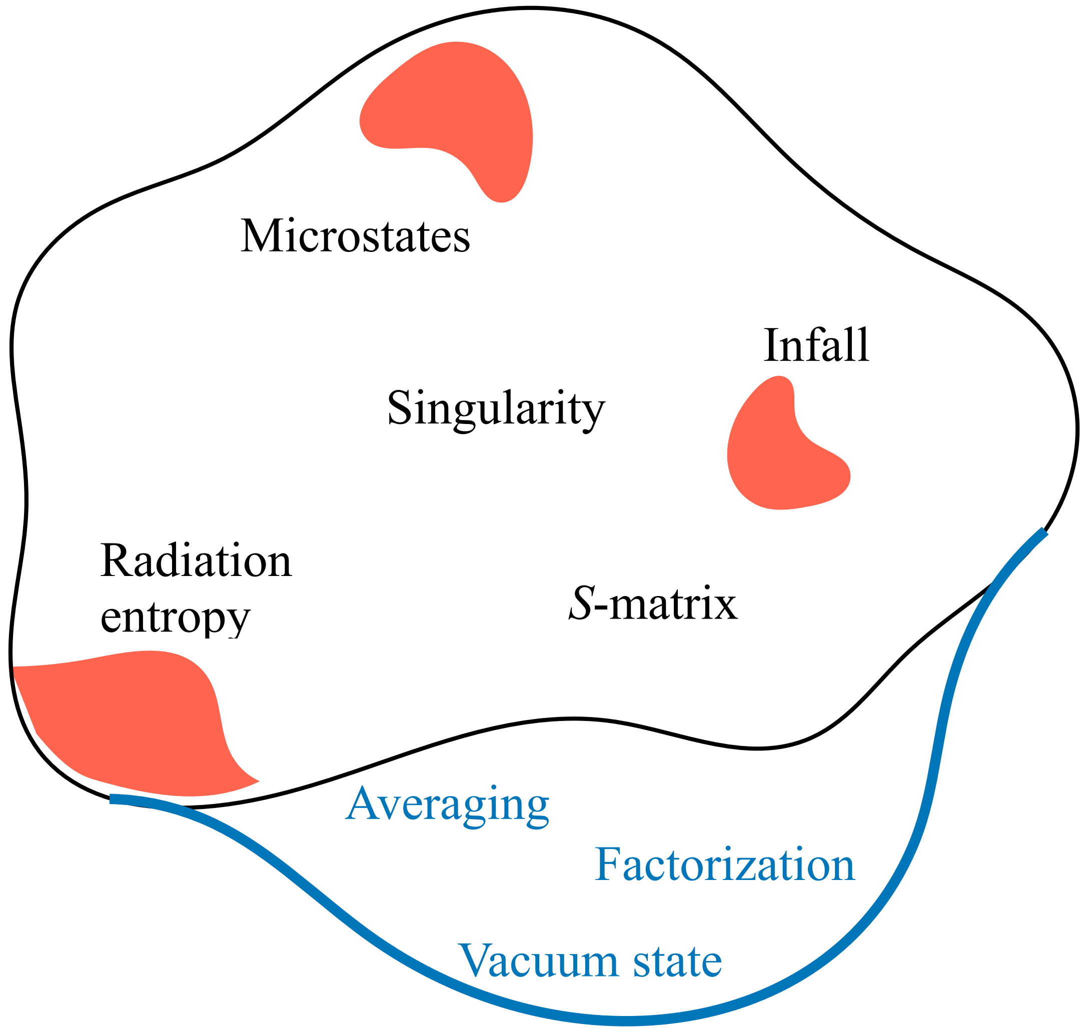


Some corners have been solved; but the scope of the problem is now **BIGGER**!

II. Thin-shell wormholes

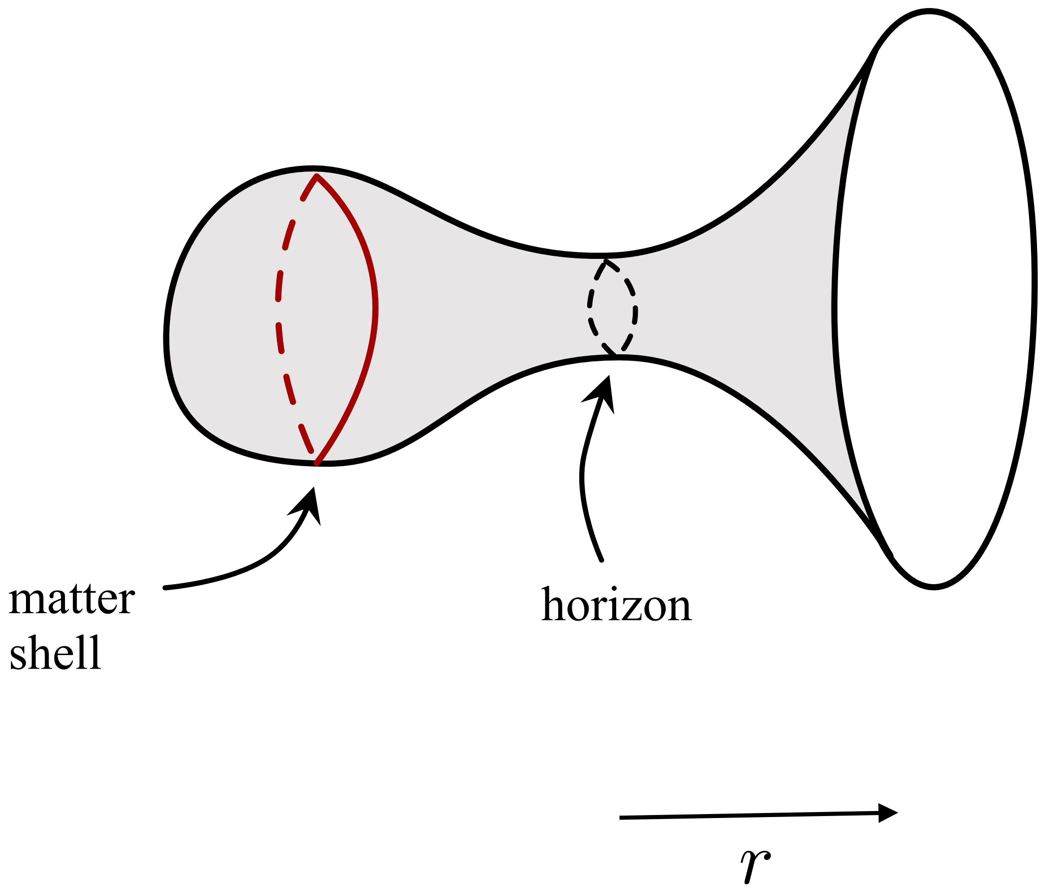
Based on:

[Chandra, Collier, TH, Maloney ’22]

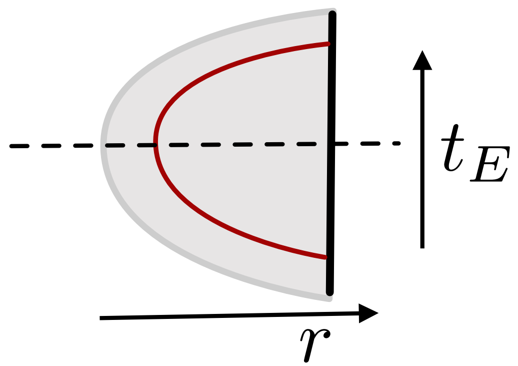
[Chandra, TH ’22]

Consider a black hole created by a spherically symmetric thin shell of massive particles in 4 spacetime dimensions.

Spatial geometry



In the Euclidean path integral, this black hole is a solution that looks like this:



Standard black hole thermodynamics from the path integral: [Gibbons, Hawking]

$$\langle \Psi | \Psi \rangle \approx e^{-I_{\rm cl}} \qquad \text{(WKB)}$$

$$|\Psi\rangle = \sum_n \psi_n |n\rangle$$

$$\langle \Psi | \Psi \rangle = \sum_n |\psi_n|^2$$

Therefore: General relativity "knows" the average value of $|\psi_n|^2$

Question: Does General Relativity know higher moments of black hole states?

$$\overline{\psi_{n_1}\psi_{n_2}\psi_{n_3}\psi_{n_4}\cdots}$$

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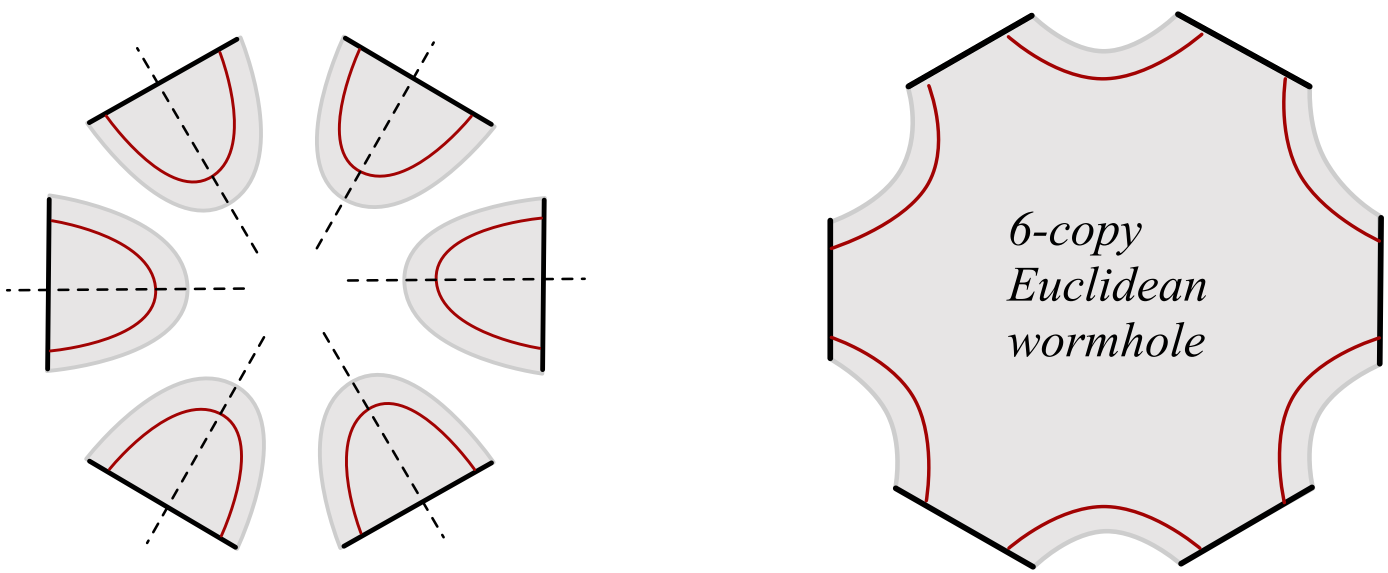
A partial answer:

Yes. At least some higher moments are encoded in spacetime wormholes connecting multiple copies of the original black hole.

$d = 2$ [Penington, Shenker, Stanford, Yang '19]
$d > 2$ [Chandra, TH '22]



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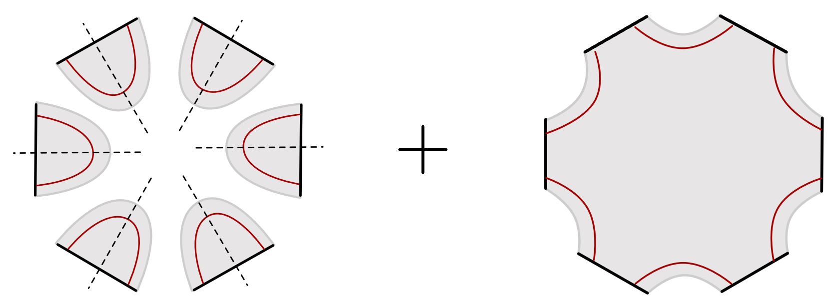
$=$  $+ \cdots$

In this case, and many similar examples,

General relativity	$\Leftrightarrow$	***Unitary quantum system that is "as random as possible"***

Subject to the constraints of gauge invariance and asymptotic locality.

Another example:

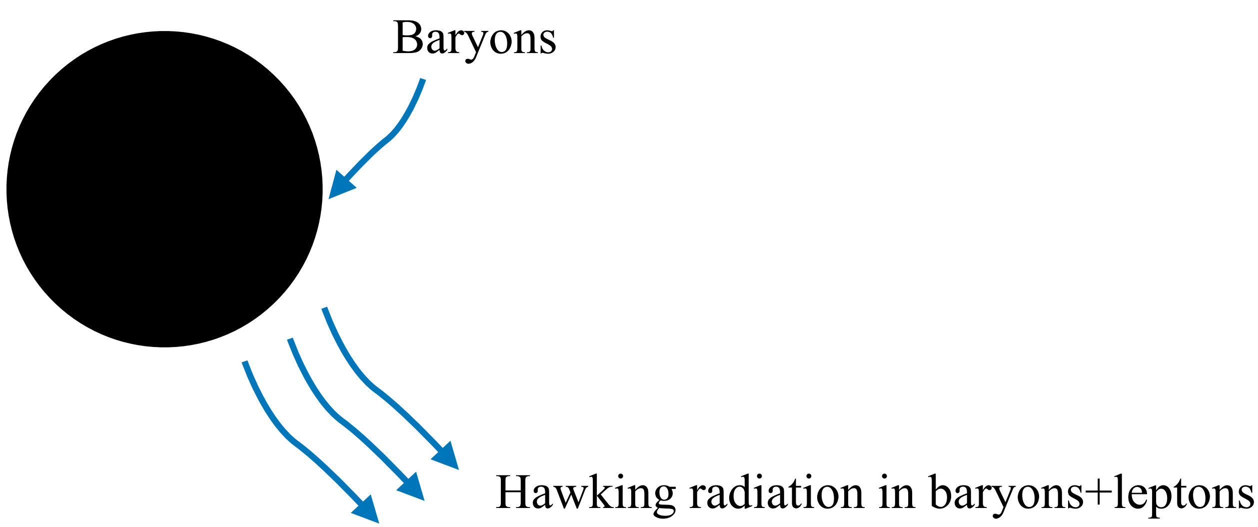
Einstein gravity in AdS_3 $=$ Average over $\mathrm{CFT}_2{}'\mathrm{s}$ subject to conformal bootstrap

[Chandra, Collier, TH, Maloney '22]

Violation of global symmetries

There is a general argument that quantum gravity violates all global symmetries.

e.g. [Misner, Wheeler '57]

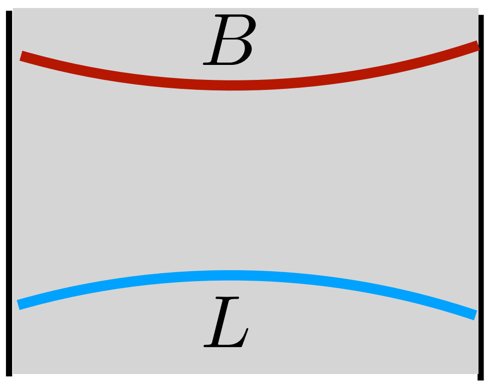


In the Standard Model:

$$U(1)_{B-L} \quad (qqq\ell)$$

Thin-shell wormholes can be used to calculate nonperturbative contributions to this effect.

Two-copy wormhole:



$$e^{-I_{\rm cl}} \approx \overline{|\langle B|L\rangle|^2}$$

Conclusion

The gravitational path integral encodes the statistics of the UV quantum theory.

Interplay between *quantum information* and *higher topologies*.

However we do not yet know the full story.

- Can all statistics be determined systematically?
- How does this compare to UV-complete examples in string theory?
- Is there a role for higher topology / quantum info in cosmology?

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