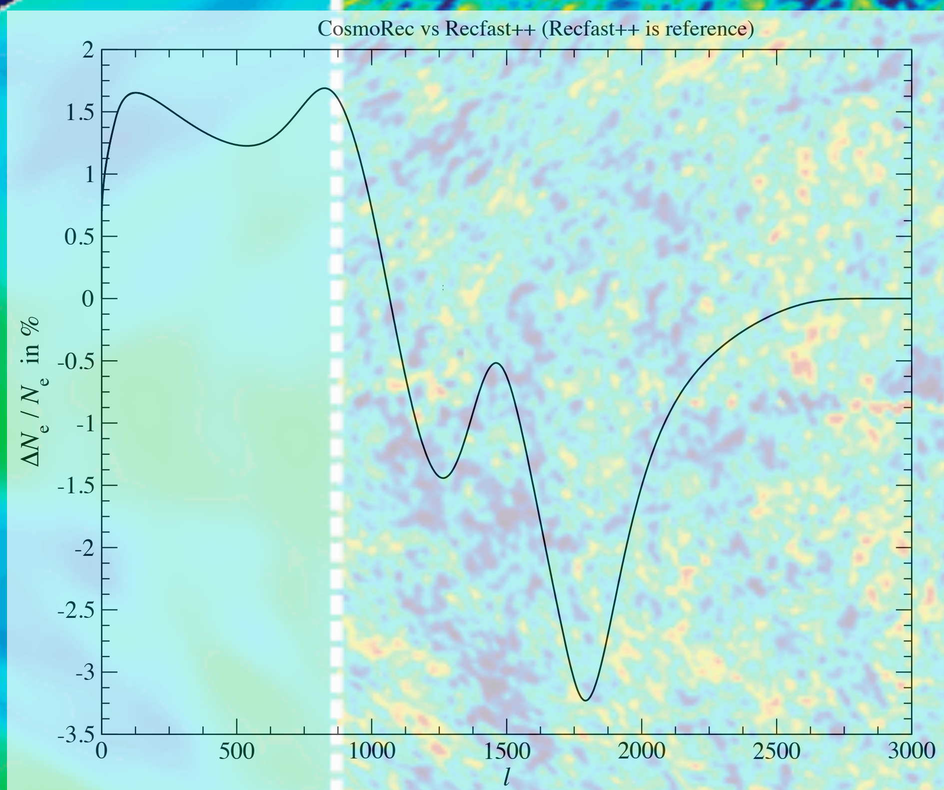
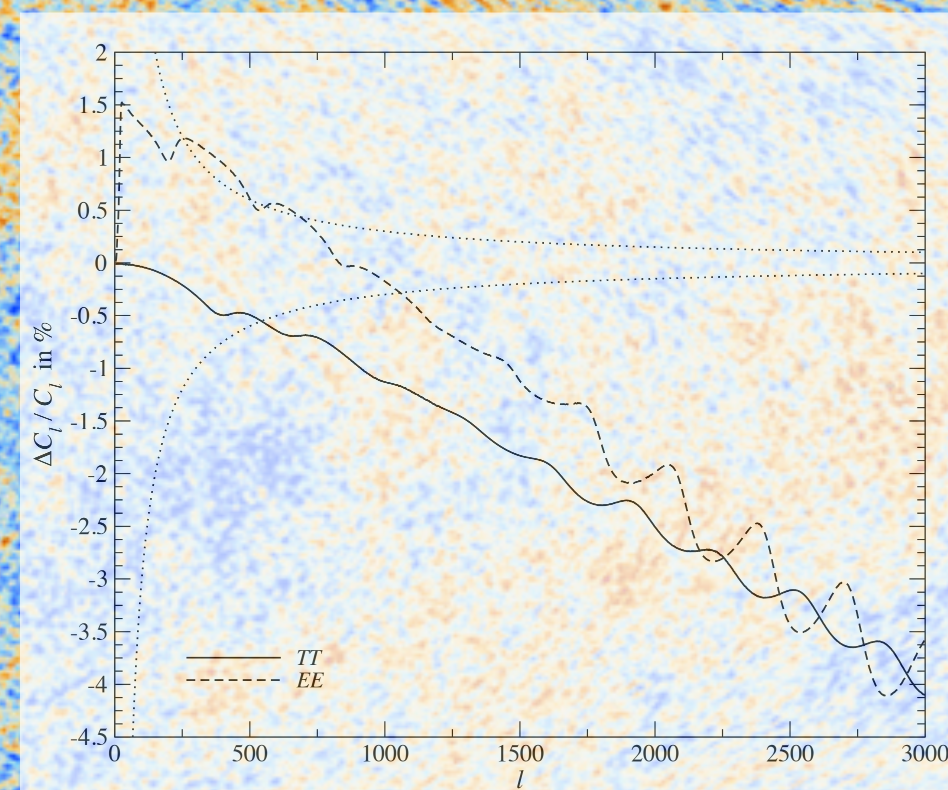
Overview of different recombination codes





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Madrid, Spain, Nov 12th - 15th, 2013

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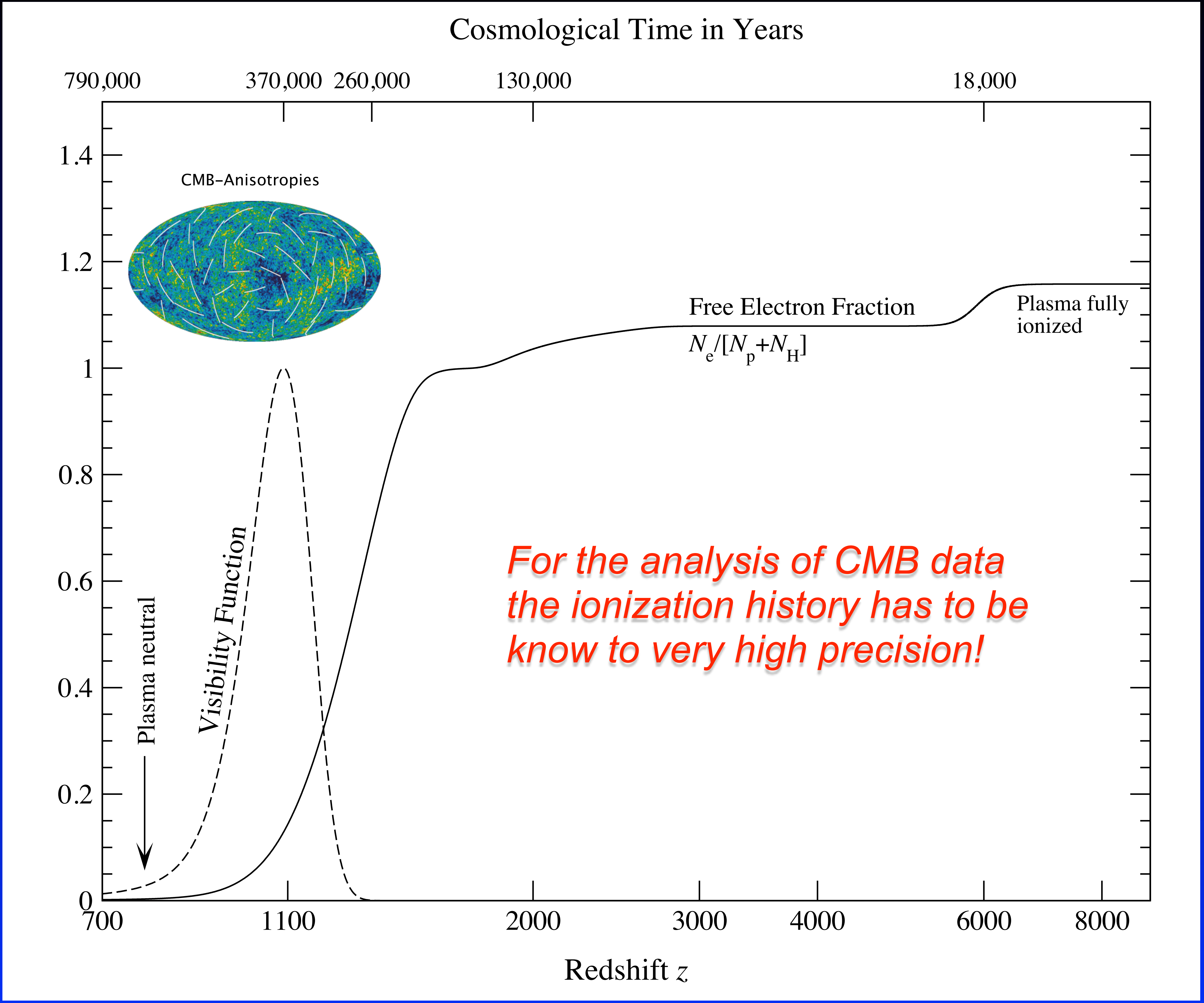
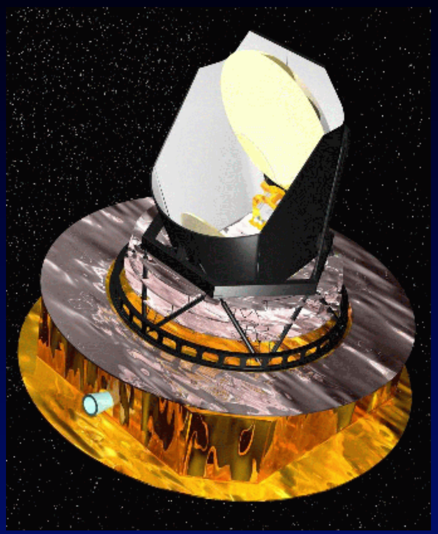
Canadian Institute for Theoretical Astrophysics
L'institut canadien d'astrophysique theorique



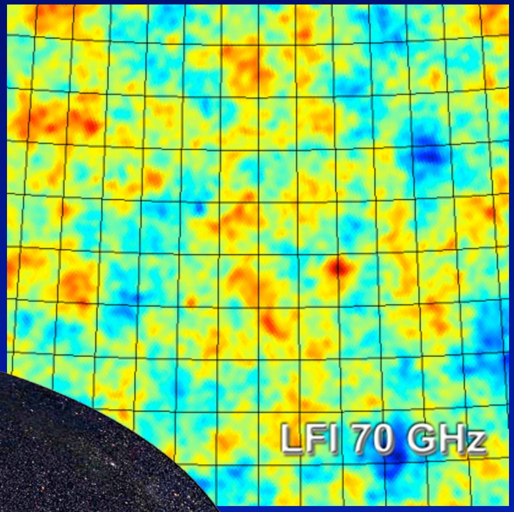
For the analysis of CMB data the ionization history has to be know to very high precision!

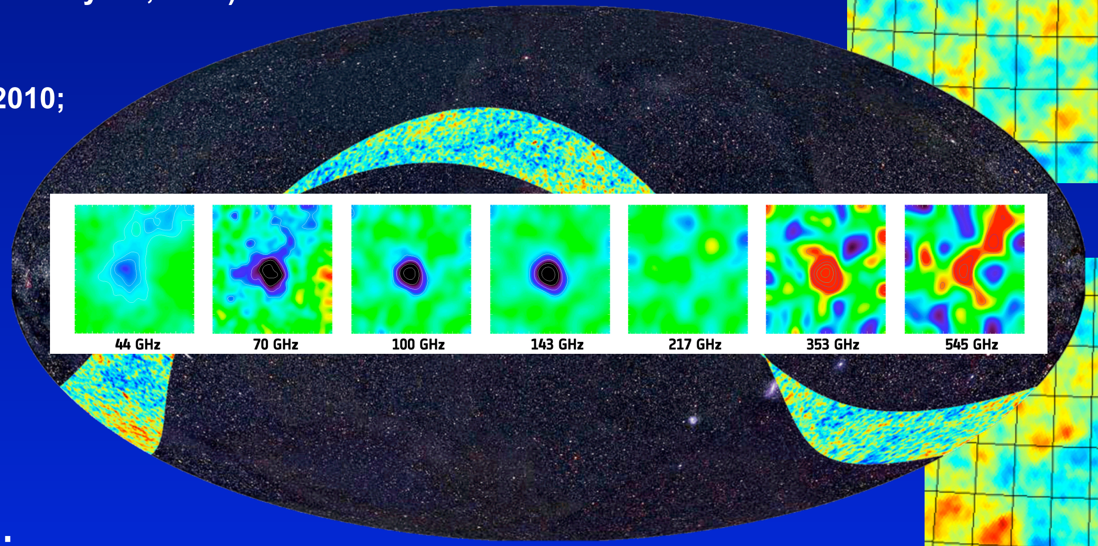
Getting the job done for Planck

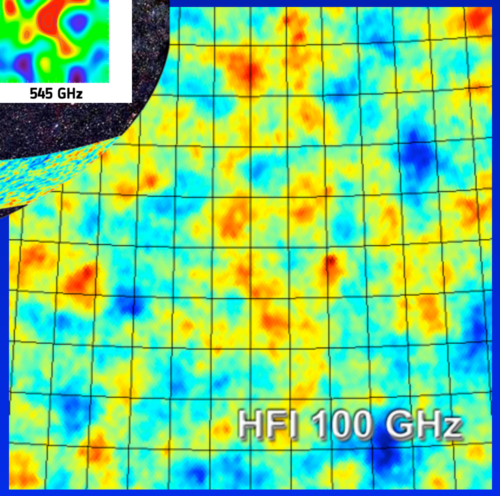


Hydrogen recombination

- Two-photon decays from higher levels
  (Dubrovich & Grachev, 2005, Astr. Lett., 31, 359; Wong & Scott, 2007; JC & Sunyaev, 2007; Hirata, 2008; JC & Sunyaev 2009)
- Induced 2s two-photon decay for hydrogen
  (JC & Sunyaev, 2006, A&A, 446, 39; Hirata 2008)
- Feedback of the Lyman-α distortion on the 1s-2s two-photon absorption rate
  (Kholupenko & Ivanchik, 2006, Astr. Lett.; Fendt et al. 2008; Hirata 2008)
- Non-equilibrium effects in the angular momentum sub-states
  (Rubiño-Martín, JC & Sunyaev, 2006, MNRAS; JC, Rubiño-Martín & Sunyaev, 2007, MNRAS; Grin & Hirata, 2009; JC, Vasil & Dursi, 2010)
- Feedback of Lyman-series photons (Ly[n] → Ly[n-1])
  (JC & Sunyaev, 2007, A&A; Kholupenko et al. 2010; Haimoud, Grin & Hirata, 2010)
- Lyman-α escape problem (*atomic recoil, time-dependence, partial redistribution*)
  (Dubrovich & Grachev, 2008; JC & Sunyaev, 2008; Forbes & Hirata, 2009; JC & Sunyaev, 2009)
- Collisions and Quadrupole lines
  (JC, Rubiño-Martín & Sunyaev, 2007; Grin & Hirata, 2009; JC, Vasil & Dursi, 2010; JC, Fung & Switzer, 2011)
- Raman scattering
  (Hirata 2008; JC & Thomas , 2010; Haimoud & Hirata, 2010)







Helium recombination

- Similar list of processes as for hydrogen
  (Switzer & Hirata, 2007a&b; Hirata & Switzer, 2007)
- Spin forbidden 2p-1s triplet-singlet transitions
  (Dubrovich & Grachev, 2005, Astr. Lett.; Wong & Scott, 2007; Switzer & Hirata, 2007; Kholupenko, Ivanchik&Varshalovich, 2007)
- Hydrogen continuum opacity during He I recombination
  (Switzer & Hirata, 2007; Kholupenko, Ivanchik & Varshalovich, 2007; Rubiño-Martín, JC & Sunyaev, 2007; JC, Fung & Switzer, 2011)
- Detailed feedback of helium photons
  (Switzer & Hirata, 2007a; JC & Sunyaev, 2009, MNRAS; JC, Fung & Switzer, 2011)

$\Delta N_e \; / \; N_e \sim 0.1 \; \%$

Recombination code overview

Code	Recfast	Recfast++	CosmoRec
Language	Fortran 77/90 & C	C++	C++
Requirements	-	-	GNU Scientific Lib (GSL)
Solves for	X_p, X_{HeI}, T_e	X_p, X_{HeI}, T_e	X_{1s}, X_{ns}, X_{np}, X_{nd}, T_e
Solves for	X_p, X_{HeI}, T_e	X_p, X_{HeI}, T_e	X_{1s}, X_{ns}, X_{np}, X_{nd}, T_e
ODE-Solver	explicit	implicit (Gears method)	implicit (Gears method)
PDE-Solver	-	-	semi-implicit (Crank-Nicolson)
Approach	derivative fudge	correction function	physics
Simplicity	simple	simpler	pretty big code
Flexibility	limited	better but limited	very flexible
Validity	close to standard cosmology	close to standard cosmology	wide range of cosmologies
Tools	-	ODE Solver	*HI & He Atom, Solvers, Quadrature routines*
Extras	-	DM annihilation	*DM annihilation, high-ν distortion*
Runtime	0.01 sec	0.08 sec	1.5 - 2 sec

Recfast Equations

$$\frac{dx_p}{dz} = \frac{[x_e x_p n_{\rm H}\alpha_{\rm H} - \beta_{\rm H}(1-x_p)e^{-h\nu_{\rm H\,2s}/kT_M}][1+K_{\rm H}\Lambda_{\rm H}n_{\rm H}(1-x_p)]}{H(z)(1+z)[1+K_{\rm H}(\Lambda_{\rm H}+\beta_{\rm H})n_{\rm H}(1-x_p)]}, \quad (1)$$

$$\frac{dx_{\rm He\,II}}{dz} = \{[x_{\rm He\,II}x_e n_{\rm H}\alpha_{\rm He\,I} - \beta_{\rm He\,I}(f_{\rm He}-x_{\rm He\,II})e^{-h\nu_{\rm He\,I\,2^1s}/kT_M}] \times [1+K_{\rm He\,I}\Lambda_{\rm He}n_{\rm H}(f_{\rm He}-x_{\rm He\,II})e^{-h\nu_{\rm He\,I\,2^1p2^1s}/kT_M}]\} \Big/ \{H(z)(1+z)[1+K_{\rm He\,I}(\Lambda_{\rm He}+\beta_{\rm He\,I})n_{\rm H} \times (f_{\rm He}-x_{\rm He\,II})e^{-h\nu_{\rm He\,I\,2^1p2^1s}/kT_M}]\}, \quad (2)$$

$$\frac{dT_M}{dz} = \frac{8\sigma_{\rm T}a_R T_R^4}{3H(z)(1+z)m_e c}\frac{x_e}{1+f_{\rm He}+x_e}(T_M-T_R)+\frac{2T_M}{(1+z)}$$

Seager et al, 1999

- Old expressions from Peebles 1969
- second shell quasi-stationary
- recombination rates and escape probabilities fudged
- spin-forbidden transition added to helium equation (Wong, Moss & Scott, 2009)

Recfast

recfast.readme

```
The input interface was designed to look familiar to users of Seljak &
Zaldarriaga's code CMBFAST.  A convenient way to run the program is by using
a file recfast.run of the form:

output.file
Omega_B, Omega_DM, Omega_vac
H_0, T_0, Y_p
Hswitch
Heswitch

For example:
junk.out
0.04 0.20 0.76
70 2.725 0.25
1
6
```

meaning of parameters

write into **`recfast.ini`**

Execute code like `./recfast < recfast.ini`

recfast.for

```
c       Modification for H correction (Hswitch):
        write(*,*) 'Modification for H recombination:'
        write(*,*)'0) no change from old Recfast'
    write(*,*)'1) include correction'
        write(*,*)'Enter the choice of modification for H (0-1):'
    read(*,*)Hswitch

C   Fudge factor to approximate the low z out of equilibrium effect
    if (Hswitch .eq. 0) then
      fu=1.14d0
    else
      fu=1.125d0
    end if

C   Modification for HeI recombination (Heswitch):
    write(*,*)'Modification for HeI recombination:'
    write(*,*)'0) no change from old Recfast'
    write(*,*)'1) full expression for escape probability for singlet'
    write(*,*)'   1P-1S transition'
    write(*,*)'2) also including effect of contiuum opacity of H on HeI'
    write(*,*)'   singlet (based in fitting formula suggested by'
    write(*,*)'   Kholupenko, Ivanchik & Varshalovich, 2007)'
    write(*,*)'3) only including recombination through the triplets'
    write(*,*)'4) including 3 and the effect of the contiuum '
    write(*,*)'   (although this is probably negligible)'
    write(*,*)'5) including only 1, 2 and 3'
    write(*,*)'6) including all of 1 to 4'
    write(*,*)'Enter the choice of modification for HeI (0-6):'
    read(*,*)Heswitch
```

Recfast++

Initialization for Recfast++ uses same file as CosmoRec

```
//=========================================================================================
// the above parameters are (default values are given as examples)
//=========================================================================================

2000      == number of redshift points (for the range z= 50-3000 nz=500 is in principle sufficient)
3000      == starting redshift; above z=3400 the Recfast++ Solution should be used.
             This is automatically done in batch mode.
0         == ending redshift; below z=50 the Recfast++ system is solved with rescale dXe/dt

0.24      == Yp
2.725     == T0
0.2678    == Omega_m
0.0444    == Omega_b
0.7322    == Omega_L  (if <=0 it will be computed from the other variables)
0.0       == Omega_k
0.71      == h100
3.04      == N_nu
1.14      == Recfast++ fudge factor (usually leave unchanged)

3         == number of hydrogen shells for ODE problem (currently: 3, 4, 5 or 10; lite only 3)
500       == nS for effective HI rates (nS=10, 20, 50, 100, 128, 200, 300, 400 and 500; lite only 500)
1.0e-24   == dark matter annihilation efficiency in eV/sec (see Chluba 2009).
             Values <= 10^-23 eV/sec are recommended. For larger values the CosmoRec
             calculation breaks down. In Recfast-mode also larger values are possible.

3         == number of helium shells (currently: 2, 3, 5, or 10; lite only 3)
0         == HI absorption during HeI-recombination (0: off; 1: on; 2: on with Diffusion fudge)
0         == spin forbidden transitions for HeI-recombination (0: off; 1: on)
0         == Feedback in Helium levels (positive: no HI abs between the lines
                    negative: with HI abs between the lines)

1     == run PDE part (1) or not (0). In the latter case only ODE system will be solved.
         If this flag is set to 0 only the initial calculation without transfer corrections
         will be performed
2     == correction to 2s-1s channel; 0: no corr; 1: stim. 2s-1s; 2: full correction;
3     == nS for corrections because of two-photon decays.
         If set to <3 then only the diffusion correction is included.
2     == nS for corrections because of Raman-scattering
         If set to <2 then the 1+1 Raman rates are not corrected.

./outputs/ == path for output
.dat       == addition to name of files at the very end

//=========================================================================================
//=========================================================================================
```

./runfiles/parameters.dat

parameters for both Recfast++ & CosmoRec

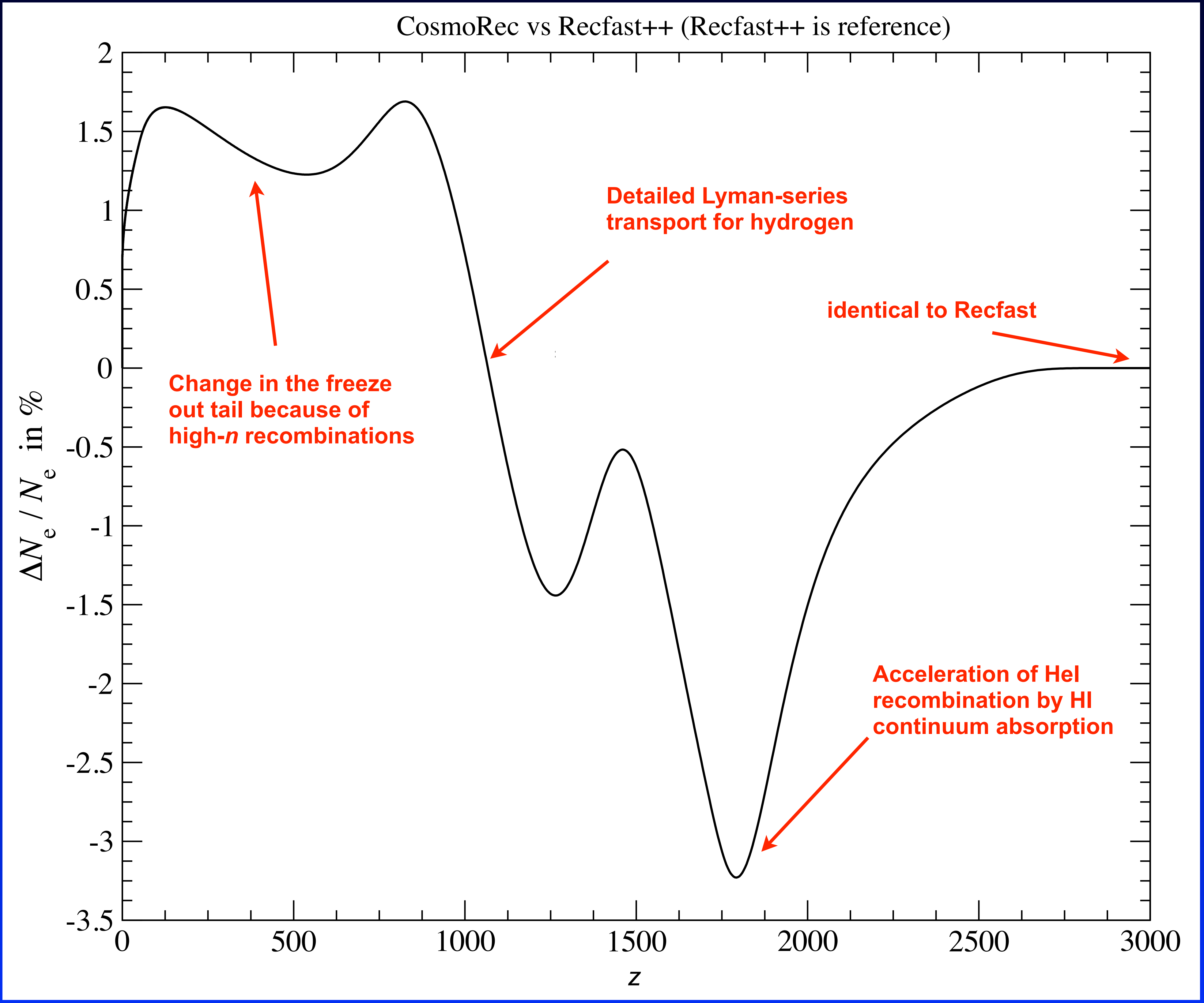
main CosmoRec parameters

Execute Recfast++ like

`./CosmoRec REC runfiles/parameters.dat` **(equivalent to old recfast)**

`./CosmoRec RECcf runfiles/parameters.dat` **(recfast + correction function)**

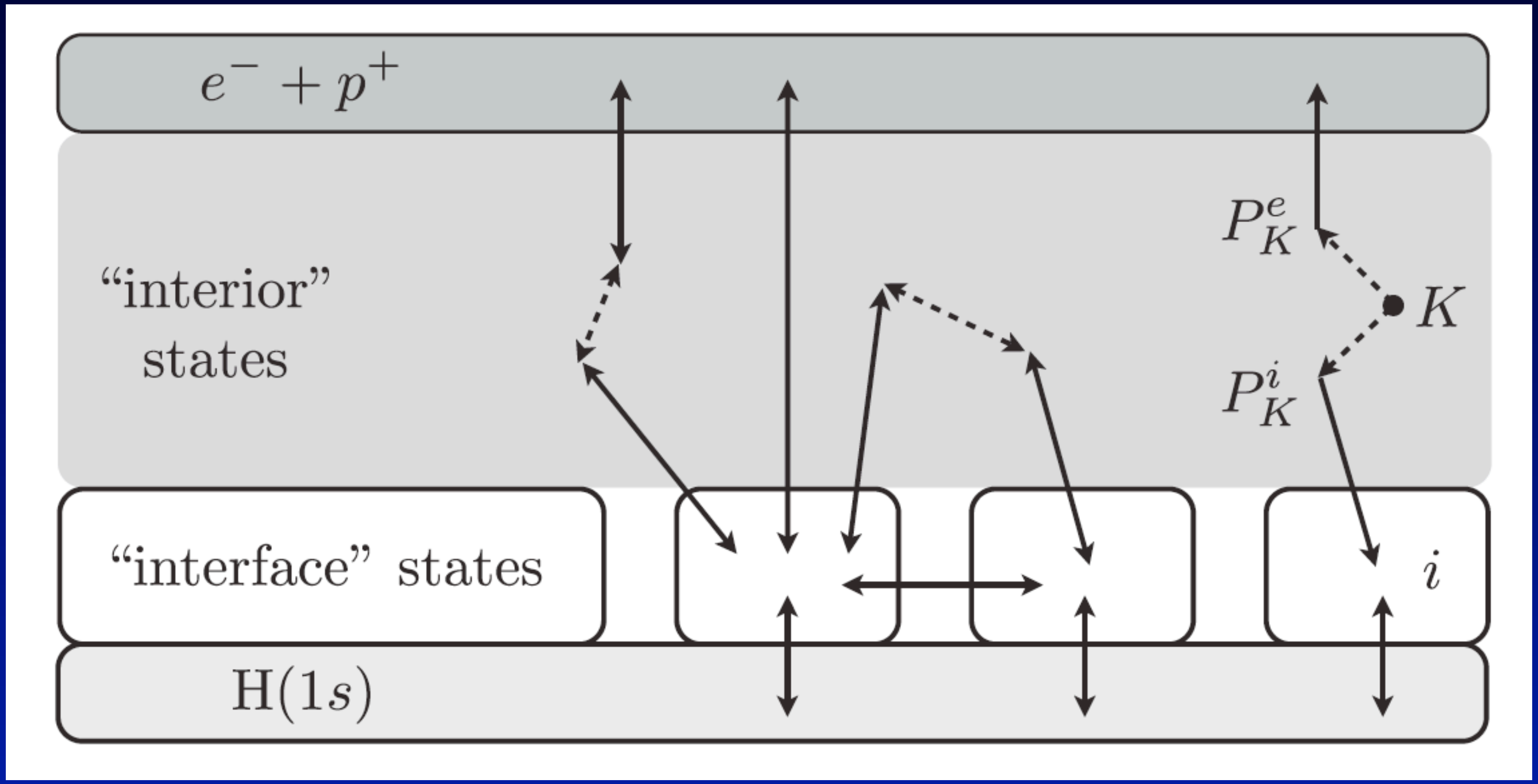
Correction function approach just uses full correction!



Introduced in Rubino-Martin et al, 2009

CosmoRec

Extended Effective Multi-level Atom



CosmoRec & HyRec

- need to treat angular momentum sub-levels separately
- Complexity of problem scales like ~ n^2_{max}
- Full problem pretty demanding (500 shells ≃ 130000 equations!)

$\Longrightarrow$ *effective multi-level approach* (Ali-Haimoud & Hirata, 2010)

- This allowed fast computation of the recombination problem!

CosmoRec parameters

./runfiles/parameters.dat

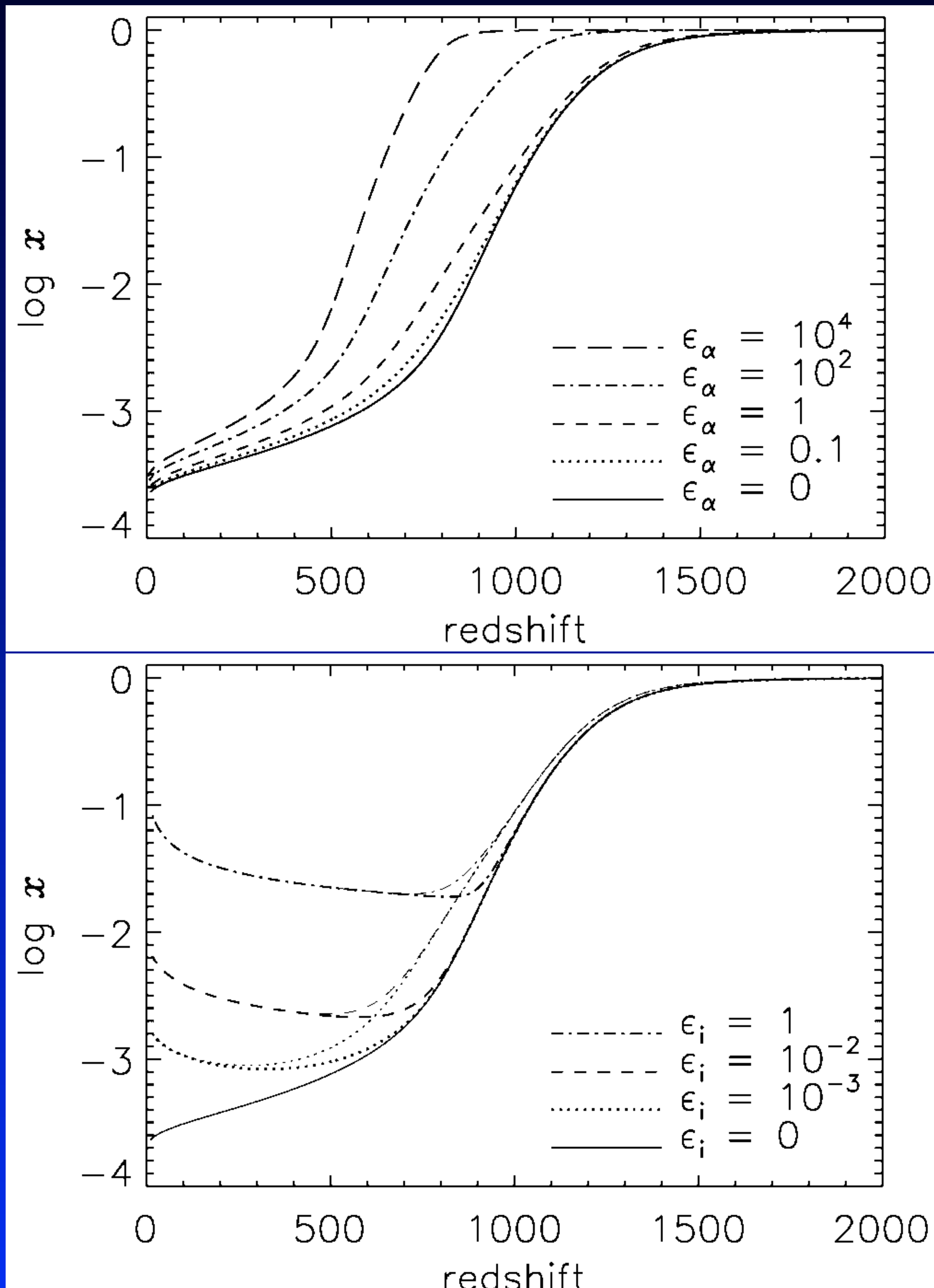
```
3         == number of hydrogen shells for ODE problem (currently: 3, 4, 5 or 10; lite only 3)
500       == nS for effective HI rates (nS=10, 20, 50, 100, 128, 200, 300, 400 and 500; lite only 500)
1.0e-24   == dark matter annihilation efficiency in eV/sec (see Chluba 2009).
             Values <= 10^-23 eV/sec are recommended. For larger values the CosmoRec
             calculation breaks down. In Recfast-mode also larger values are possible.

3         == number of helium shells (currently: 2, 3, 5, or 10; lite only 3)
0         == HI absorption during HeI-recombination (0: off; 1: on; 2: on with Diffusion fudge)
0         == spin forbidden transitions for HeI-recombination (0: off; 1: on)
0         == Feedback in Helium levels (positive: no HI abs between the lines
                    negative: with HI abs between the lines)

1     == run PDE part (1) or not (0). In the latter case only ODE system will be solved.
         If this flag is set to 0 only the initial calculation without transfer corrections
         will be performed
2     == correction to 2s-1s channel; 0: no corr; 1: stim. 2s-1s; 2: full correction;
3     == nS for corrections because of two-photon decays.
         If set to <3 then only the diffusion correction is included.
2     == nS for corrections because of Raman-scattering
         If set to <2 then the 1+1 Raman rates are not corrected.

./outputs/ == path for output
.dat       == addition to name of files at the very end
```

Execute CosmoRec like

./CosmoRec runfiles/parameters.dat

Annihilation and extra energy release

Extra Sources of Ionizations or Excitations

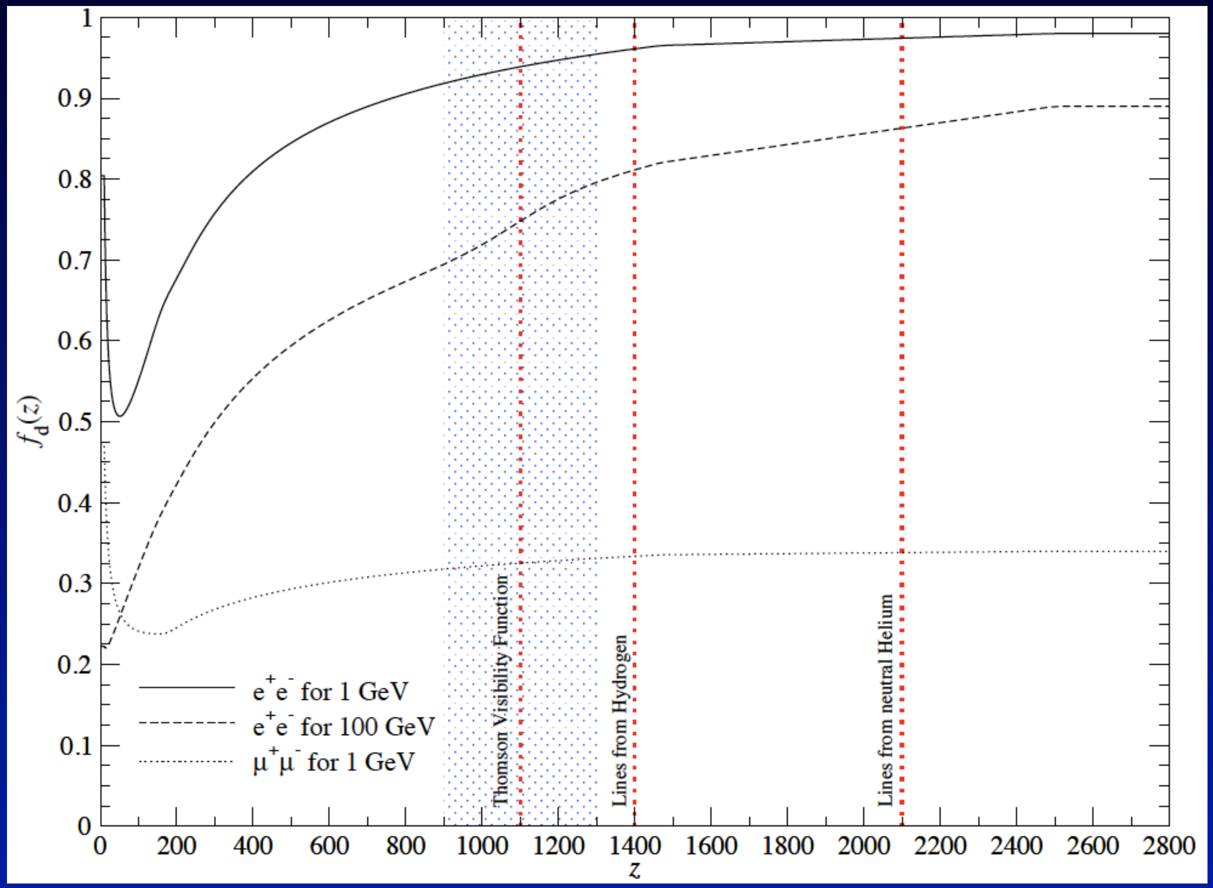


- ‚Hypothetical' source of extra photons parametrized by ε_α & ε_i
- Extra excitations ⇒ delay of Recombination
- Extra ionizations ⇒ affect 'freeze out' tail
- This affects the Thomson visibility function
- From WMAP ⇒ $\varepsilon_\alpha < 0.39$ & $\varepsilon_i < 0.058$ at 95% confidence level (Galli et al. 2008)

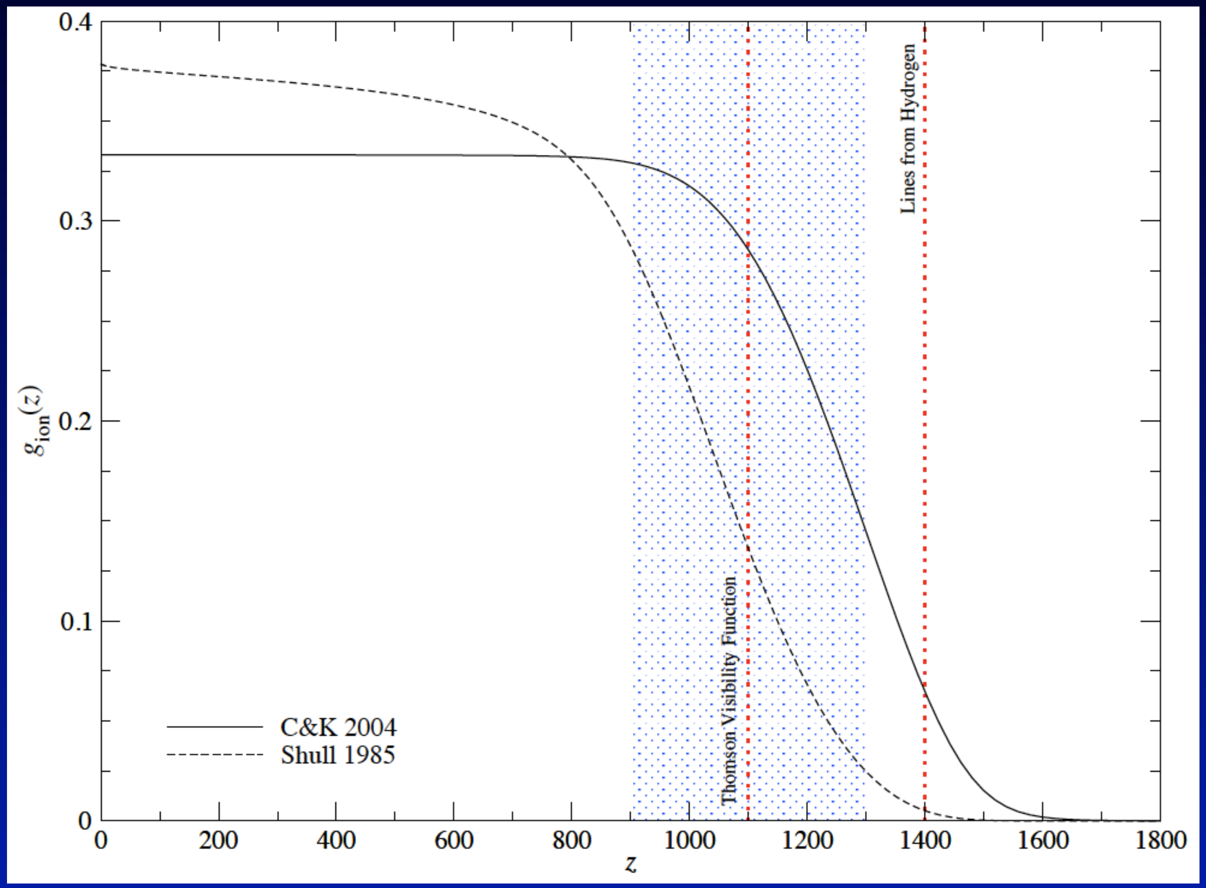
- Extra ionizations & excitations should also lead to additional photons in the recombination radiation!!!
- This in principle should allow us to check for such sources at z~1000

Peebles, Seager & Hu, ApJ, 2000

Dark Matter Annihilation: Energy Branching Ratios



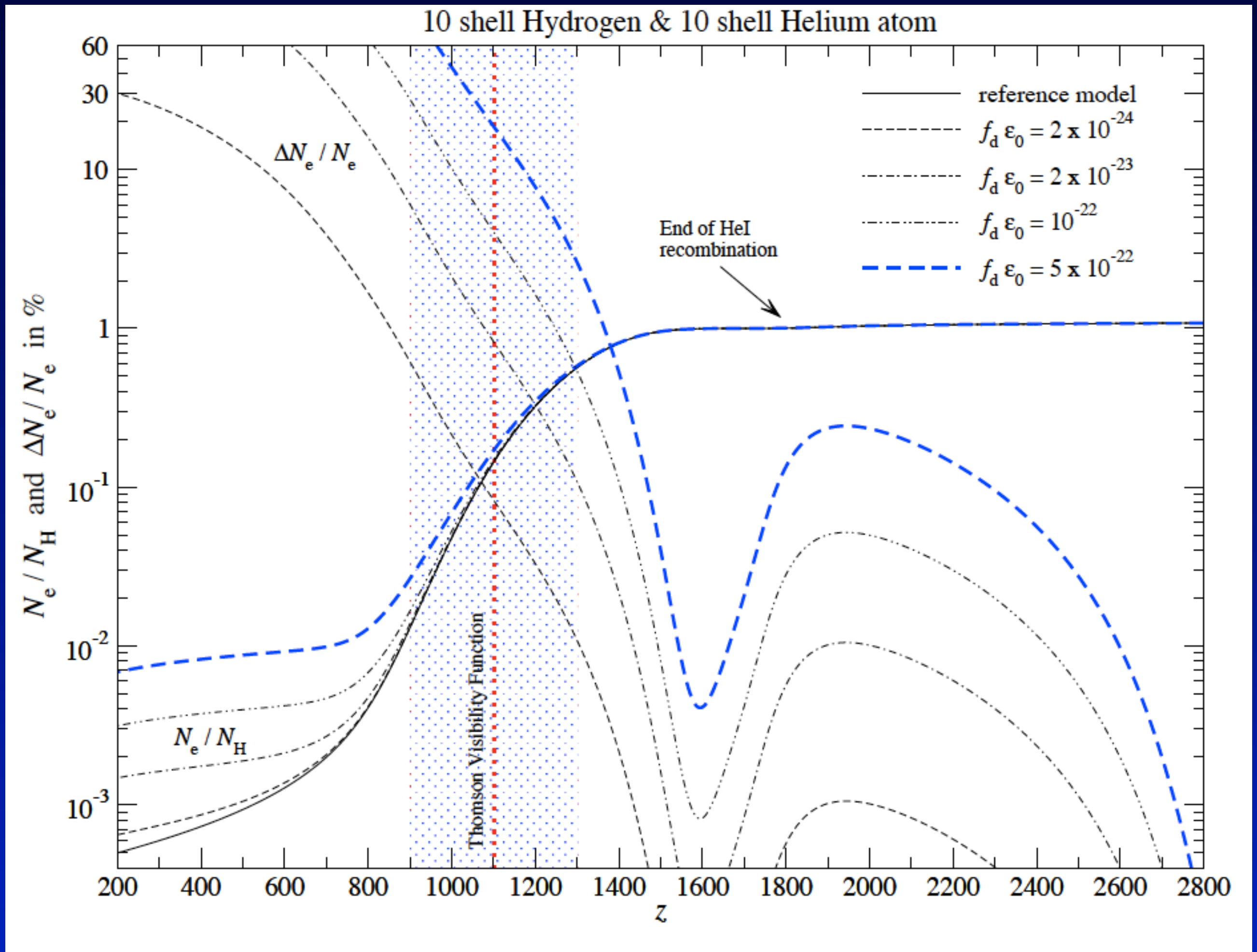
curves from Slatyer et al. 2009

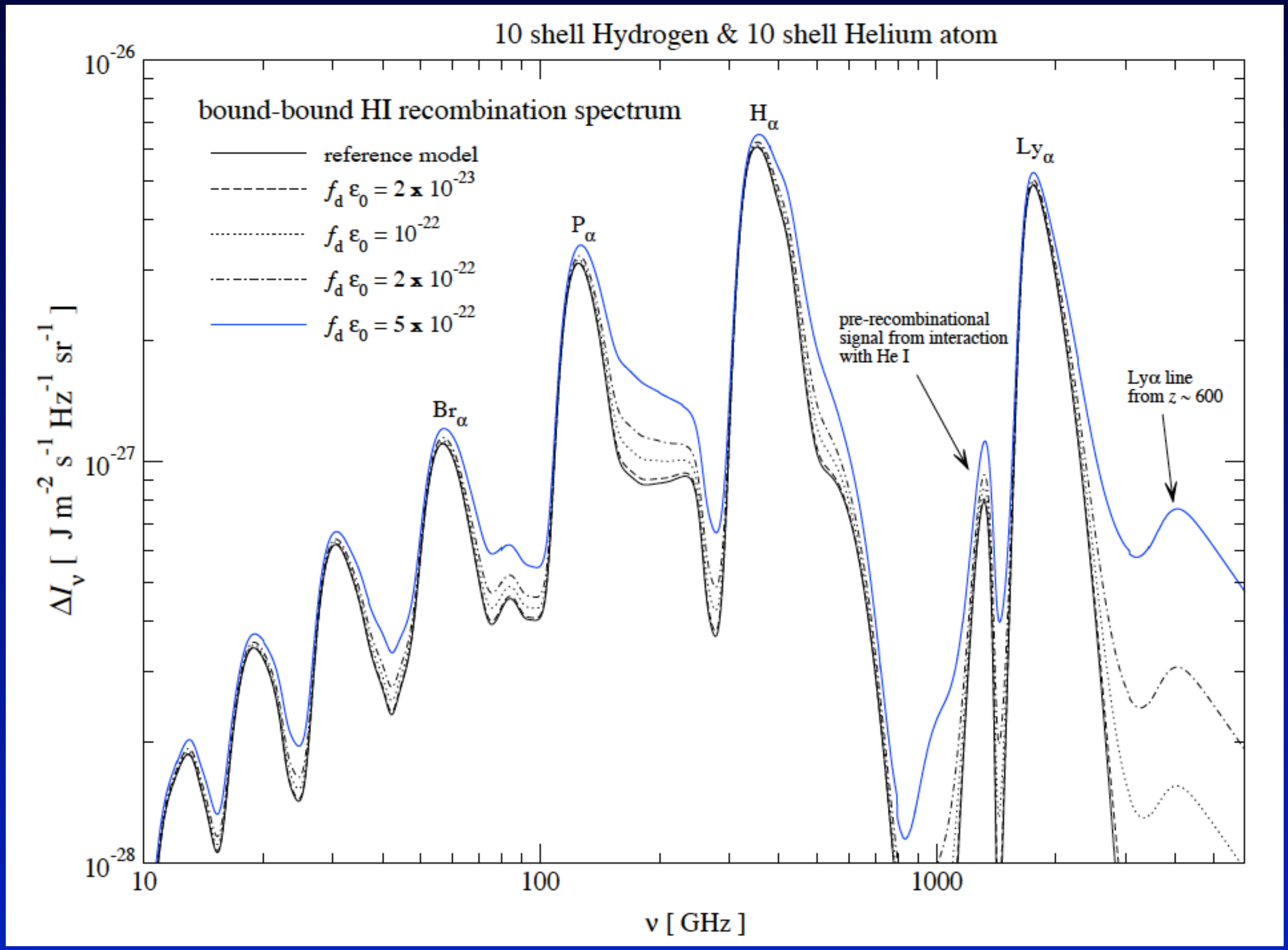


Efficiencies according to Chen & Kamionkowski, 2004 & Shull & van Steenberg 1985

- N^2 - dependence ⇒ $dE/dt \propto (1+z)^6$ and $dE/dz \propto (1+z)^{3...3.5}$
- only part of the energy is really deposited (f_d ~ 0.1)
- Branching into *heating* (100% at high z), *ionizations* and *excitations* (mainly during recombination)
- Branching depends on considered DM model

Dark Matter Annihilation: Effect on CMB Anisotropies and the Recombination Spectrum





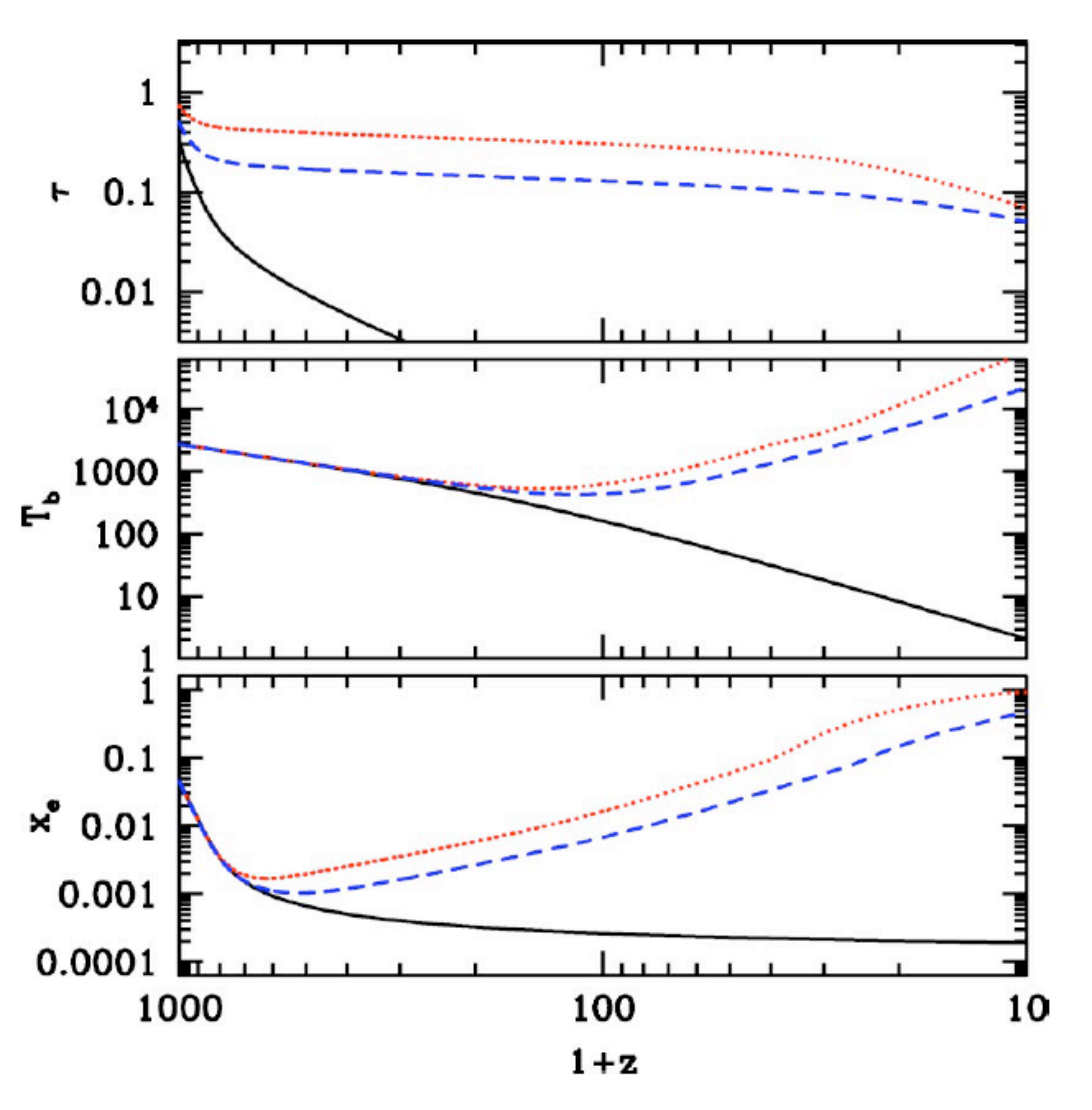
- ‘Delay of recombination’
- Affects Thomson visibility function
- Possibility of Sommerfeld-enhancement
- Clumpiness of matter at z<100

- Additional photons at all frequencies
- Broadening of spectral features
- Shifts in the positions

compare also Padmanabhan & Finkbeiner, Phys. Rev. D, 2005

JC, 2009, arXiv:0910.3663

Decaying particle during & after recombination

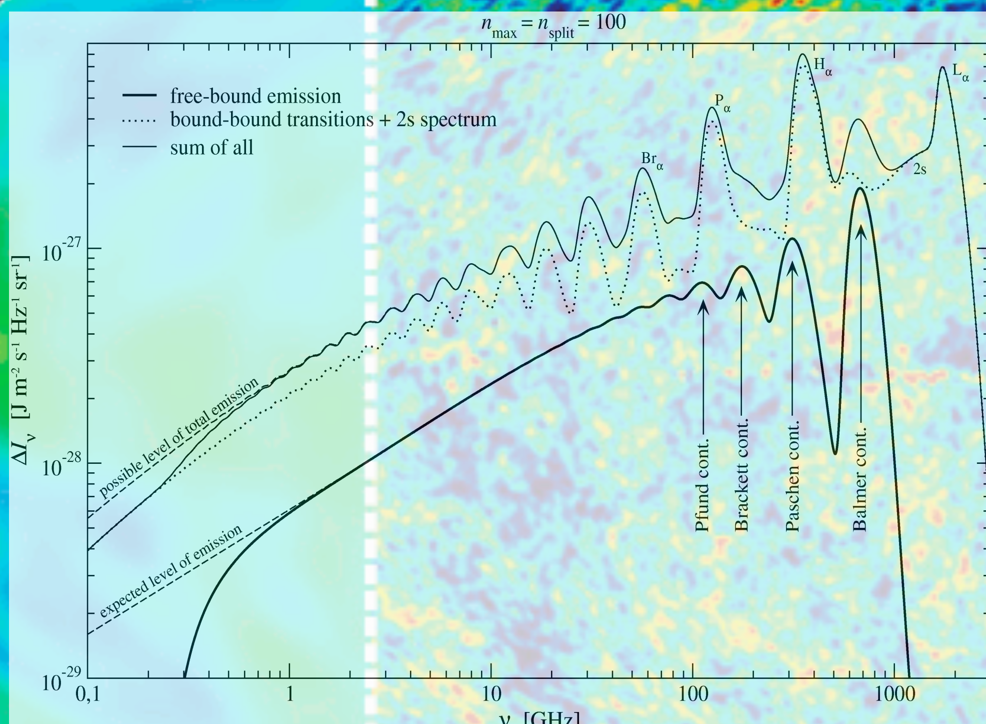


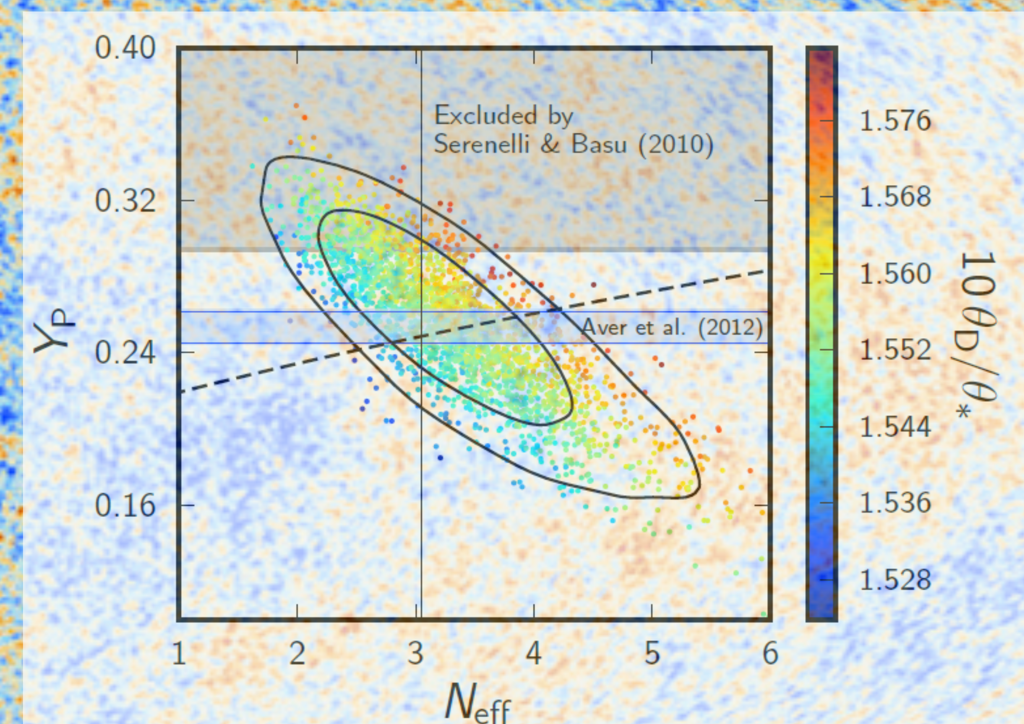
Chen & Kamionkowski, 2004

- Modify recombination history
- this changes Thomson visibility function and thus the CMB temperature and polarization power spectra
- ⇒ CMB anisotropies allow

  probing particles with lifetimes $\gtrsim 10^{12}$ sec
- CMB spectral distortions provide complementary probe! (more tomorrow)

Recombination Physics and What this has to do with Cosmology and Particle Physics





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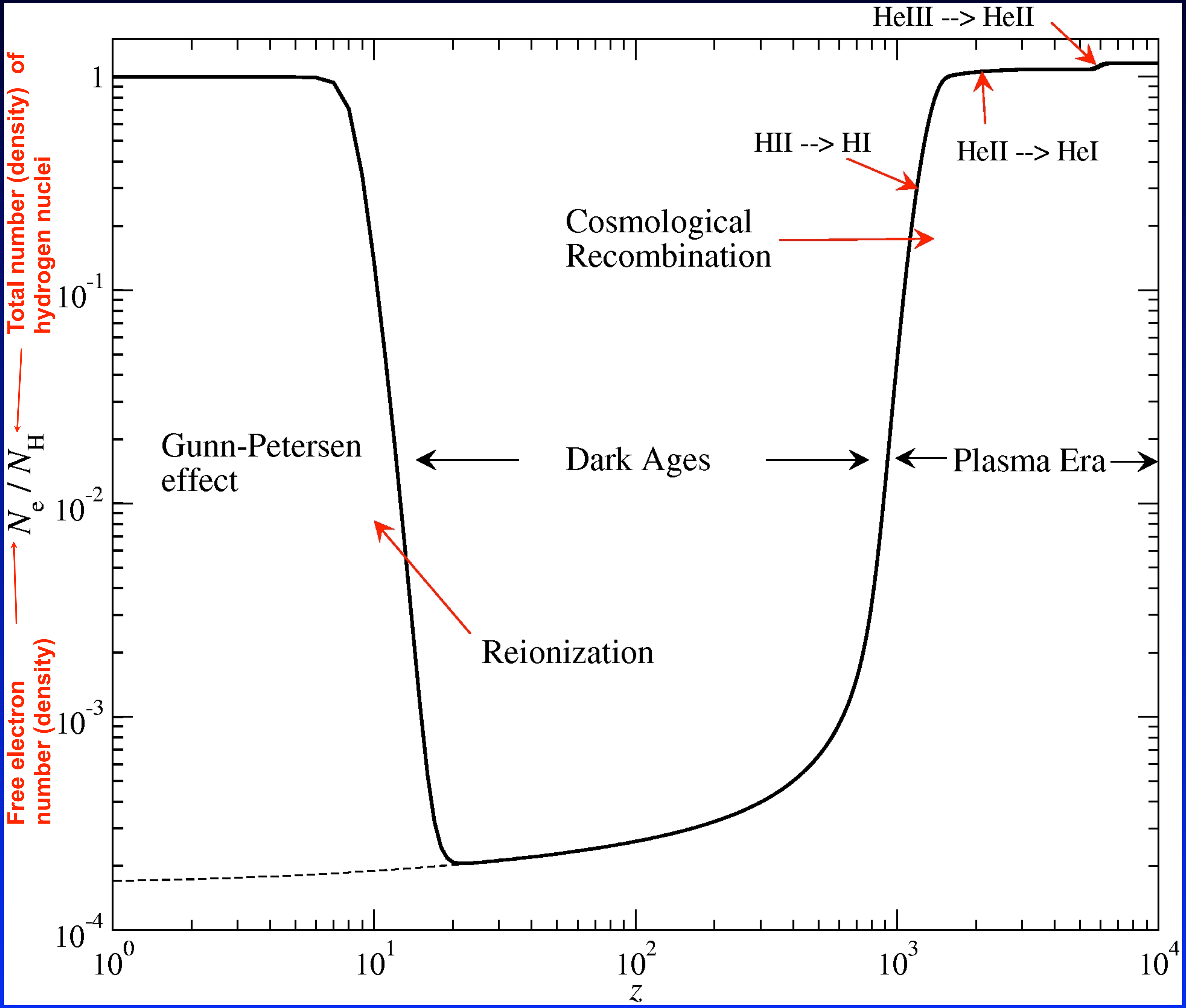
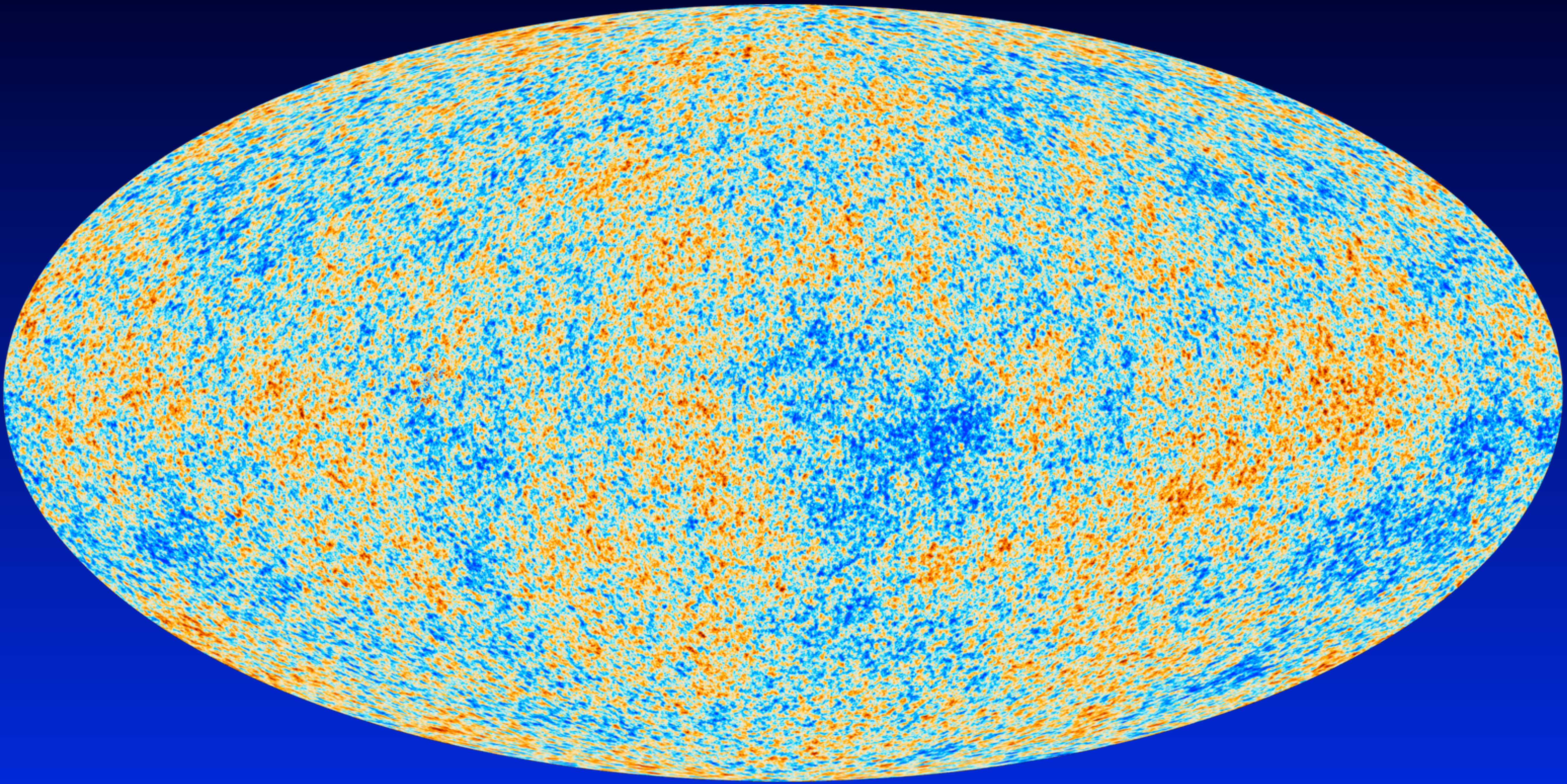
Sketch of the Cosmic Ionization History



- **at redshifts higher than ~10^4 Universe → *fully ionized***
- **z ≥ 10^4 → *free electron fraction* N_e/N_H ~ 1.16** (Helium has 2 electrons and abundance ~ 8%)
- **HeIII → HeII recombination at z~6000**
- **HeII → HeI recombination at z~2000**
- **HII → HI recombination at z~1000**

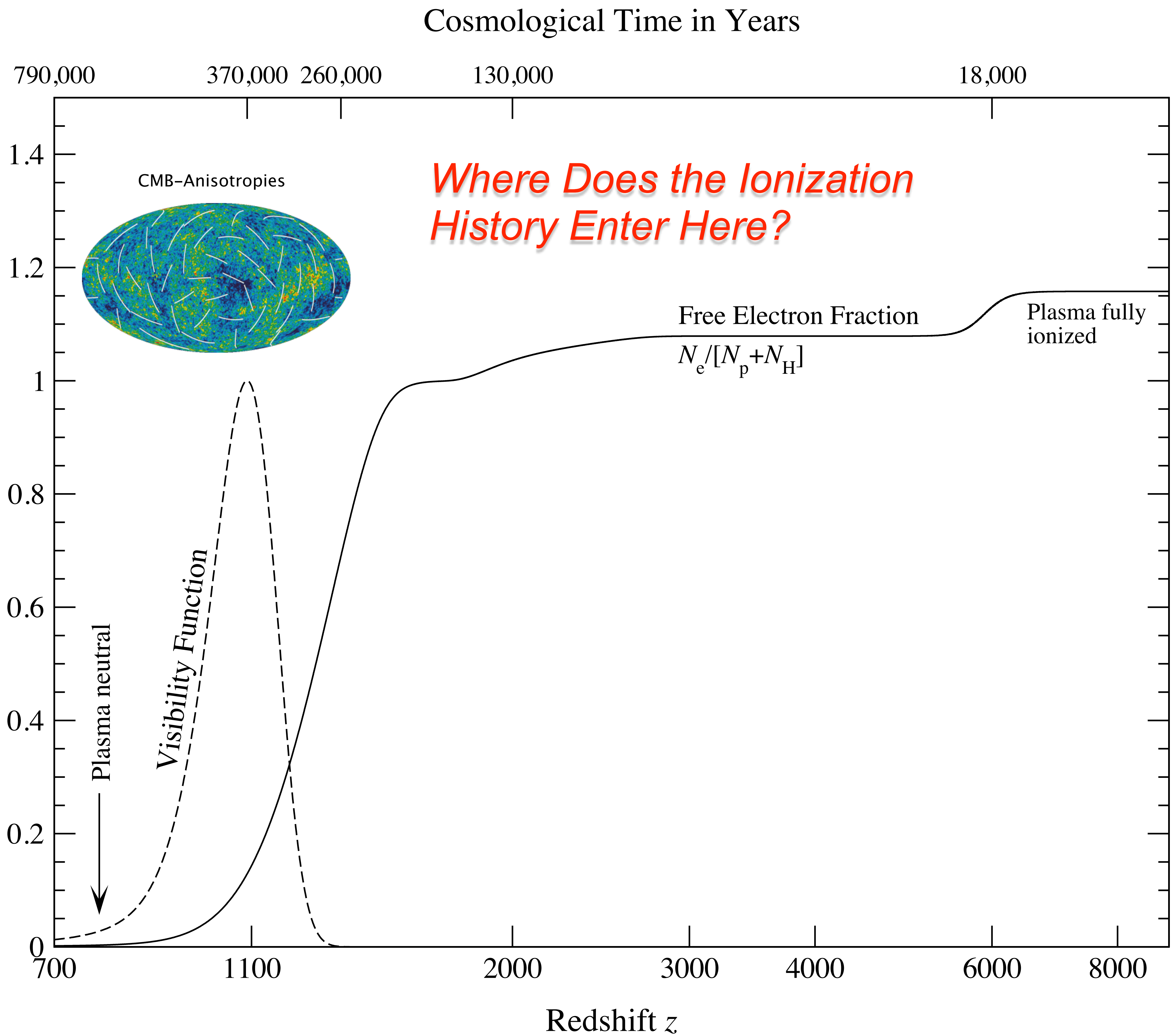
Cosmic Microwave Background Anisotropies

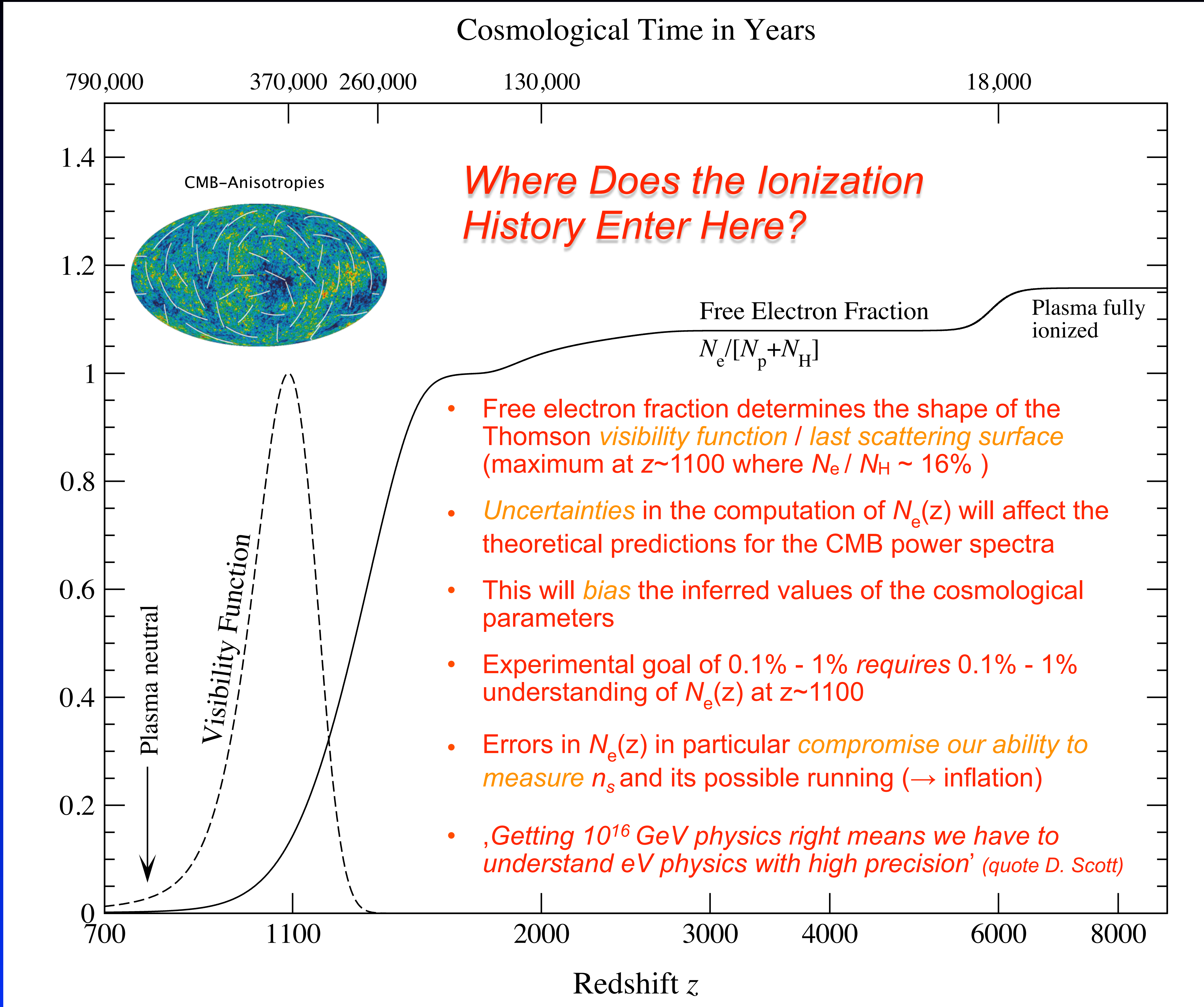


Planck all sky map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

Where Does the Ionization History Enter Here?

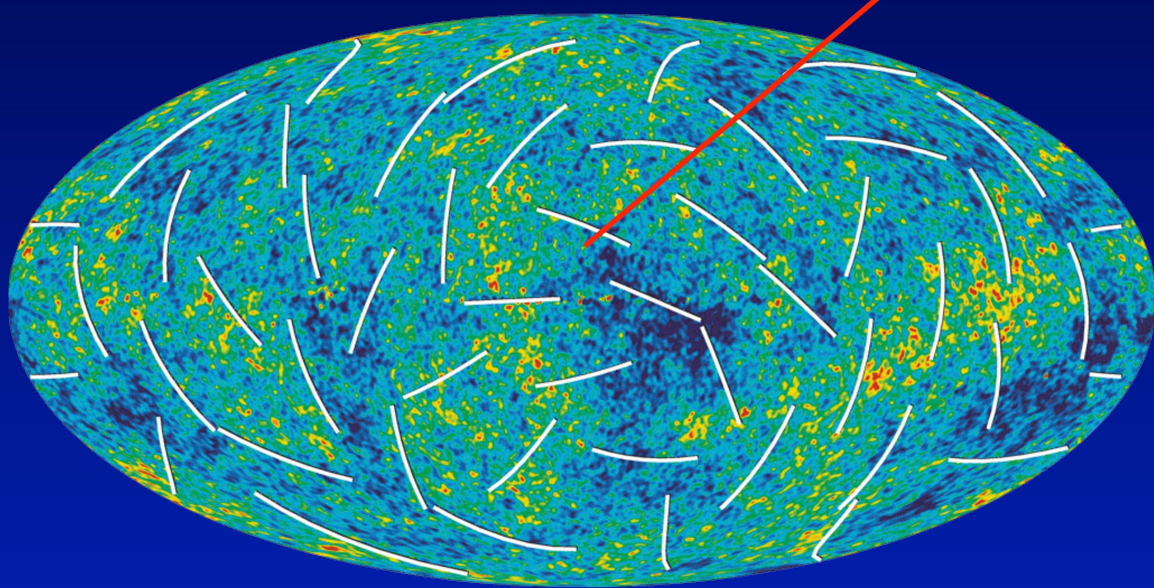




Cosmological Time in Years
790,000
370,000
260,000
130,000
18,000
Where Does the Ionization History Enter Here?
CMB-Anisotropies
Free Electron Fraction
$N_e/[N_p+N_H]$
Plasma fully ionized
Visibility Function
Plasma neutral
• Free electron fraction determines the shape of the Thomson visibility function / last scattering surface (maximum at z~1100 where N_e / N_H ~ 16%)
• Uncertainties in the computation of $N_e(z)$ will affect the theoretical predictions for the CMB power spectra
• This will bias the inferred values of the cosmological parameters
• Experimental goal of 0.1% - 1% requires 0.1% - 1% understanding of $N_e(z)$ at z~1100
• Errors in $N_e(z)$ in particular compromise our ability to measure n_s and its possible running (→ inflation)
• ‚Getting 10^{16} GeV physics right means we have to understand eV physics with high precision' (quote D. Scott)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
700
1100
2000
3000
4000
6000
8000
Redshift z

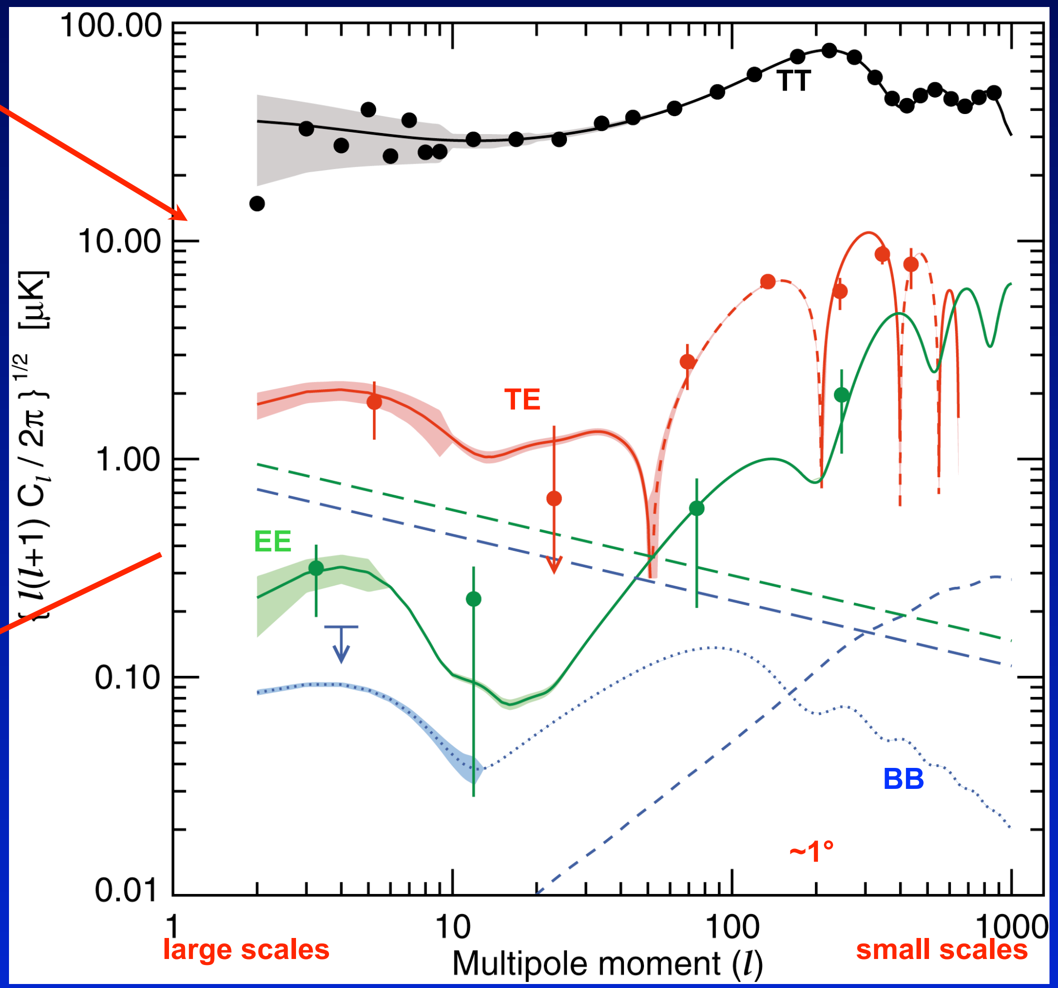
CMB Sky → Cosmology

WMAP CMB Sky



a_{lm}

Power spectra



(Joint) analysis

Cosmological Parameters

Ω_{tot}, Ω_m, Ω_b, Ω_Λ, h, τ, n_s,...

Other cosmological Dataset:

small-scale CMB, Supernovae, large-scale structure/ BAO, Lyman-α forest, lensing, ...

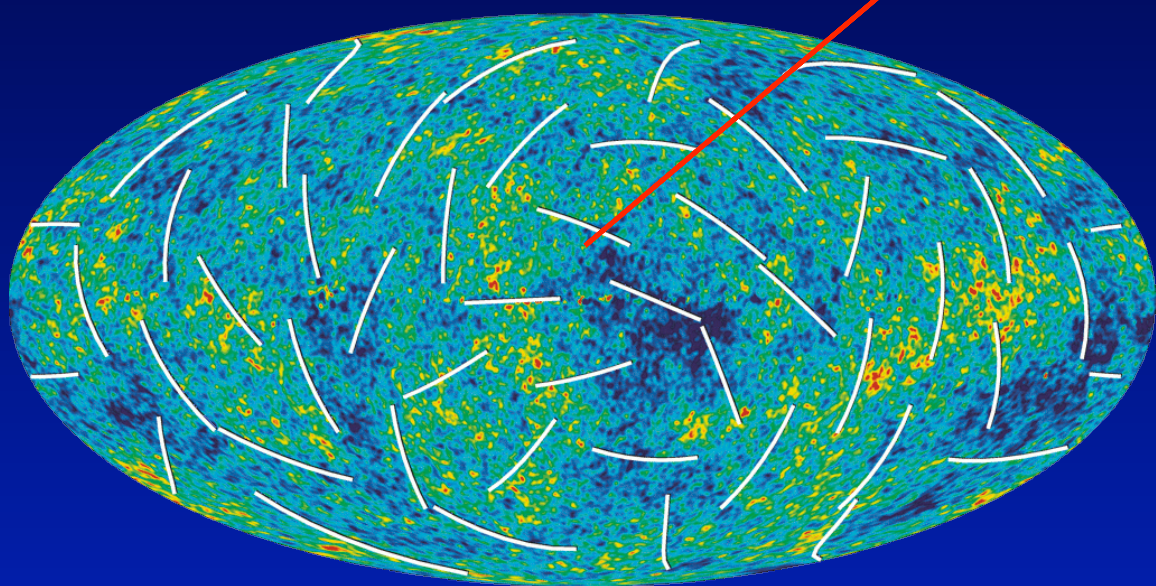
CMB Sky → Cosmology

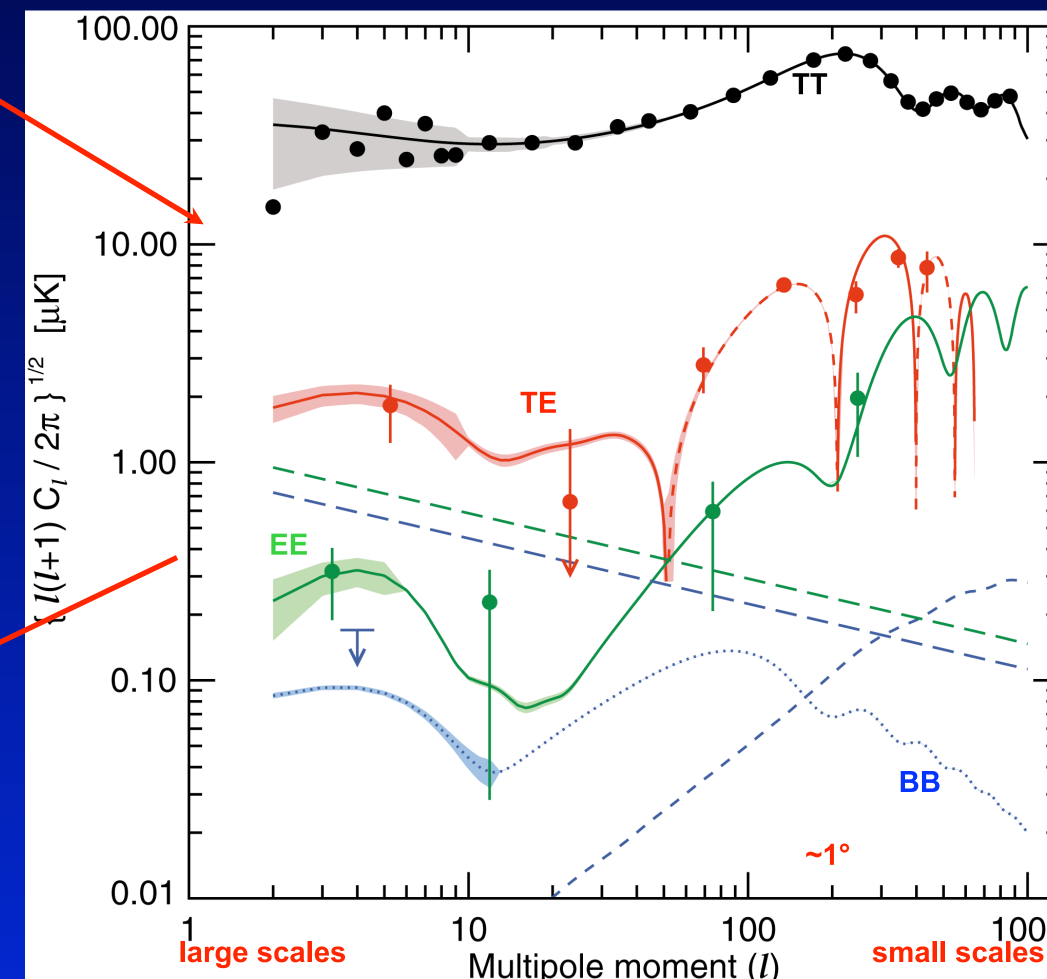
$N_e(z)$ is a *crucial* input

a_{lm}

Power spectra

WMAP CMB Sky





(Joint) analysis

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h, τ, n_s,...

Other cosmological Dataset:

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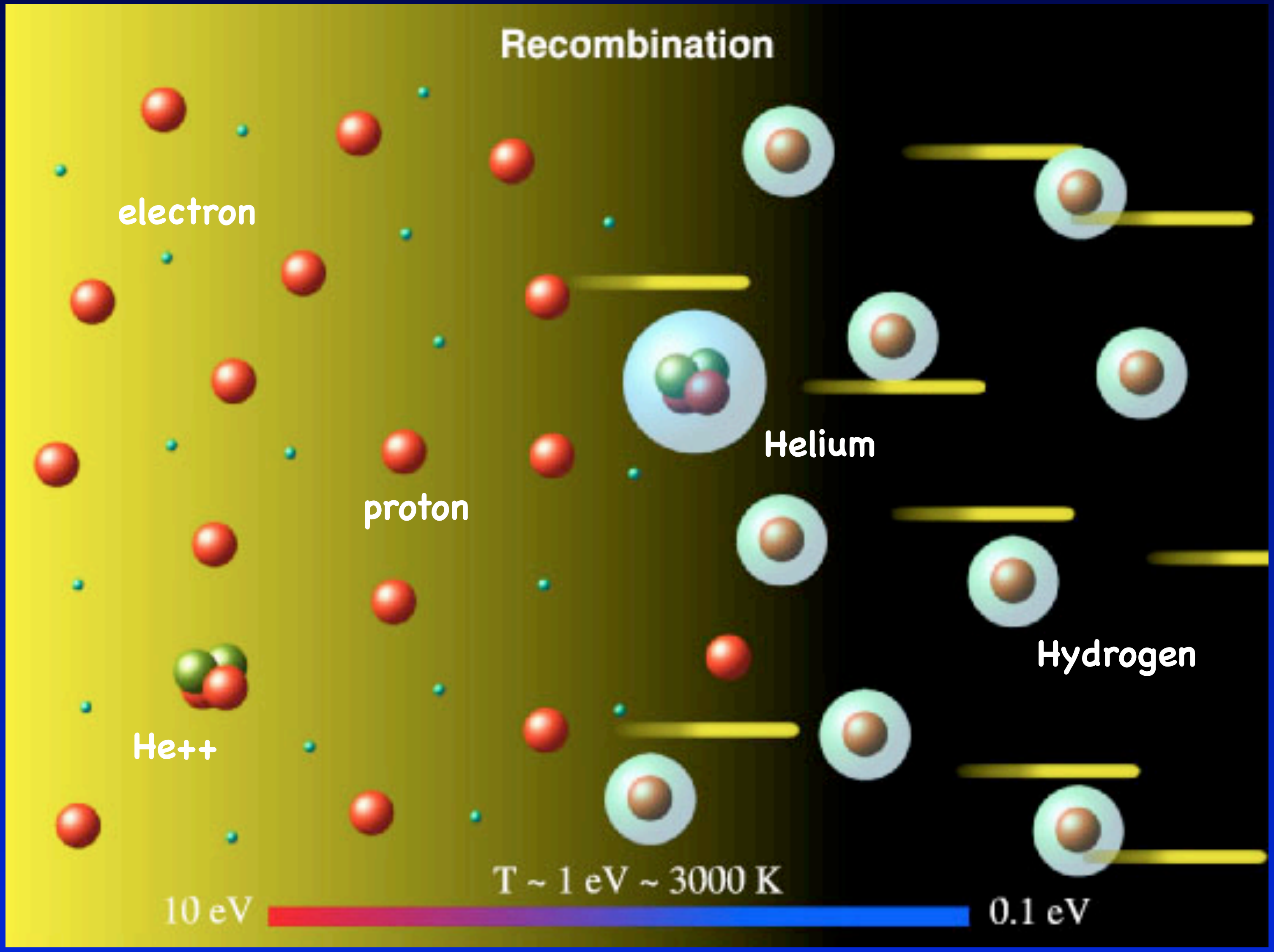
BAO, Lyman-α forest, lensing, ...

How does cosmological recombination work?

What is the recombination problem about?

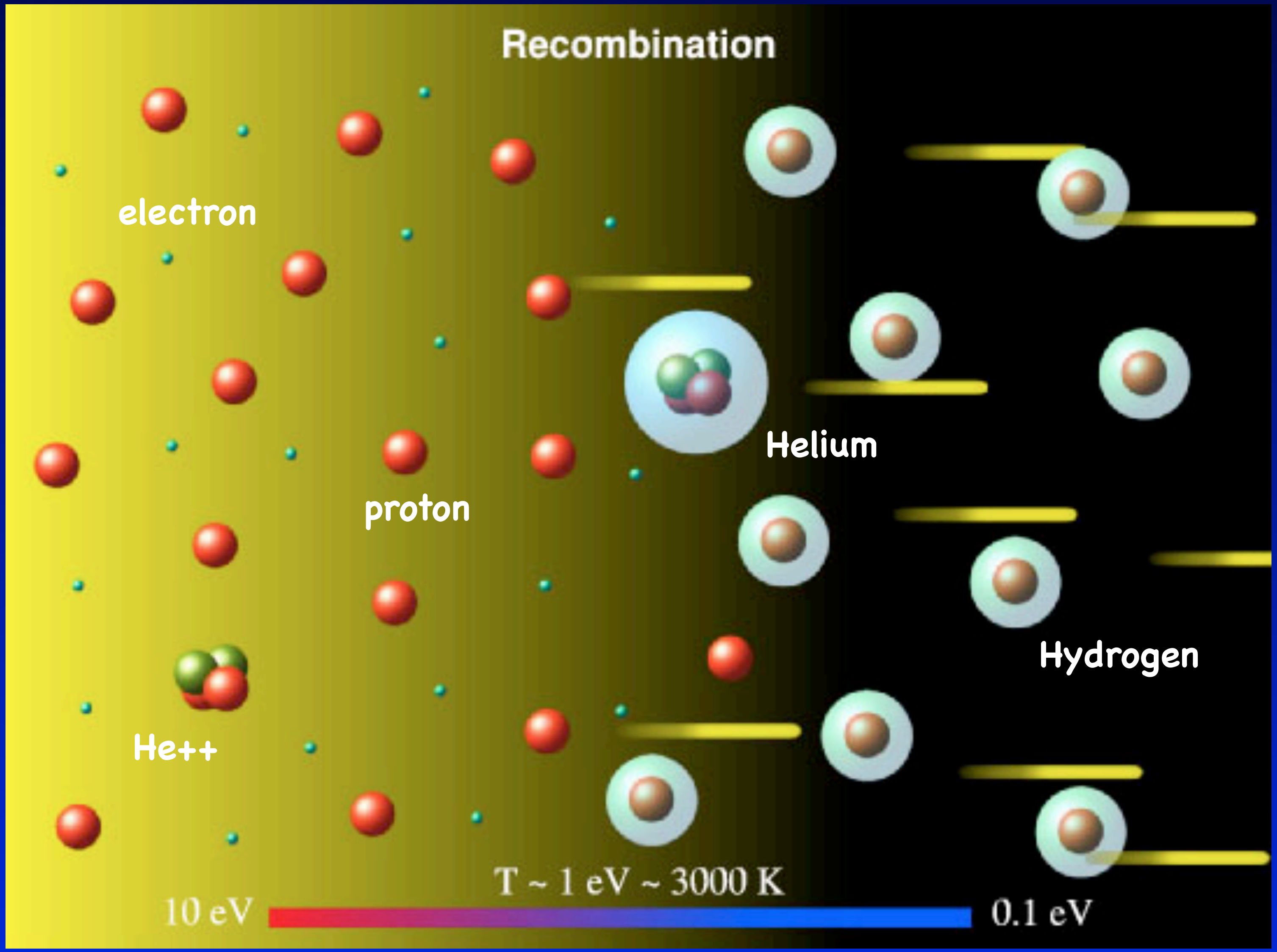


- coupled system describing the interaction of *matter* with the ambient CMB *photon* field
- atoms can be in different excitation states

  θ *lots of levels to worry about*
- recombination process changes Wien tail of CMB and this affects the recombination dynamics

  $\Longrightarrow$ *radiative transfer problem*

What is the recombination problem about?



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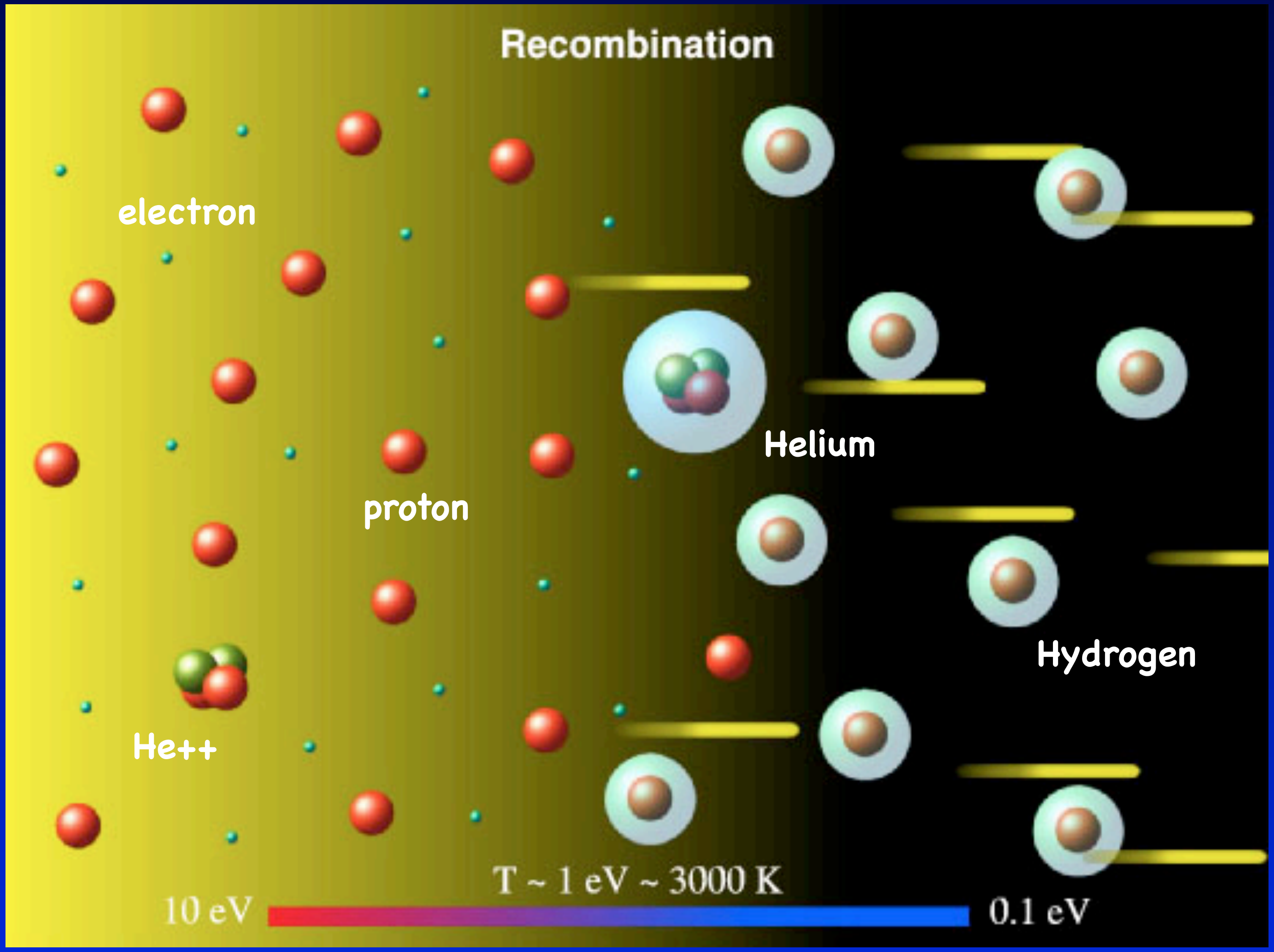
  $\Longrightarrow$ *radiative transfer problem*

Have to follow evolution of: $N_{\rm e}, T_{\rm e}, N_{\rm p}, N_i$ and ΔI_ν

What is the recombination problem about?



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  θ *lots of levels to worry about*
- recombination process changes Wien tail of CMB and this affects the recombination dynamics

  $\Longrightarrow$ *radiative transfer problem*

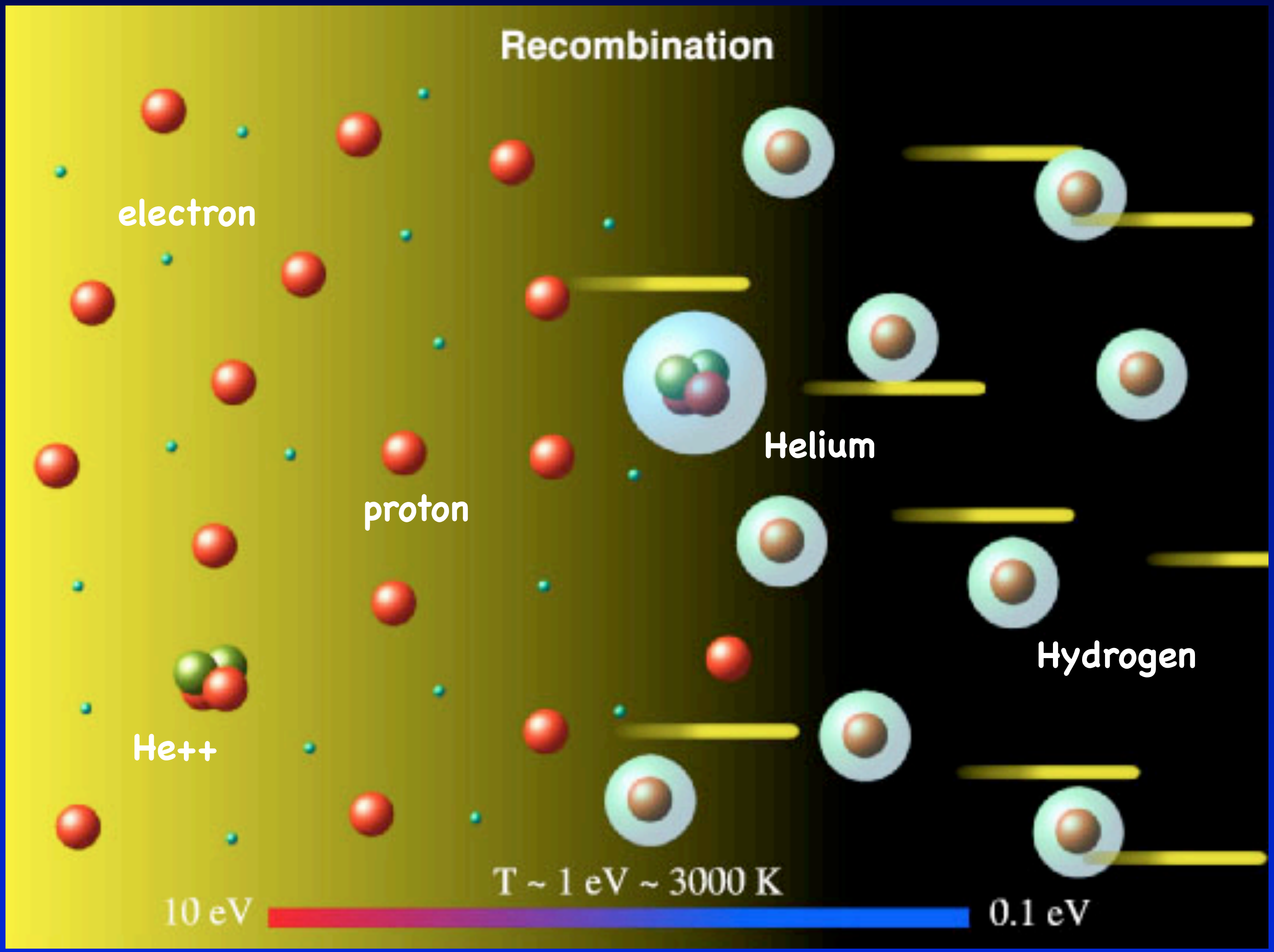
electron temperature

Have to follow evolution of: $N_{\rm e}, T_{\rm e}, N_{\rm p}, N_i$ and ΔI_ν

number densities

non-thermal photons

What is the recombination problem about?



- coupled system describing the interaction of *matter* with the ambient CMB *photon* field
- atoms can be in different excitation states

  θ *lots of levels to worry about*
- recombination process changes Wien tail of CMB and this affects the recombination dynamics

  $\Longrightarrow$ *radiative transfer problem*

Only problem in time!

electron temperature

Have to follow evolution of: N_e, T_e, N_p, N_i and ΔI_ν

number densities

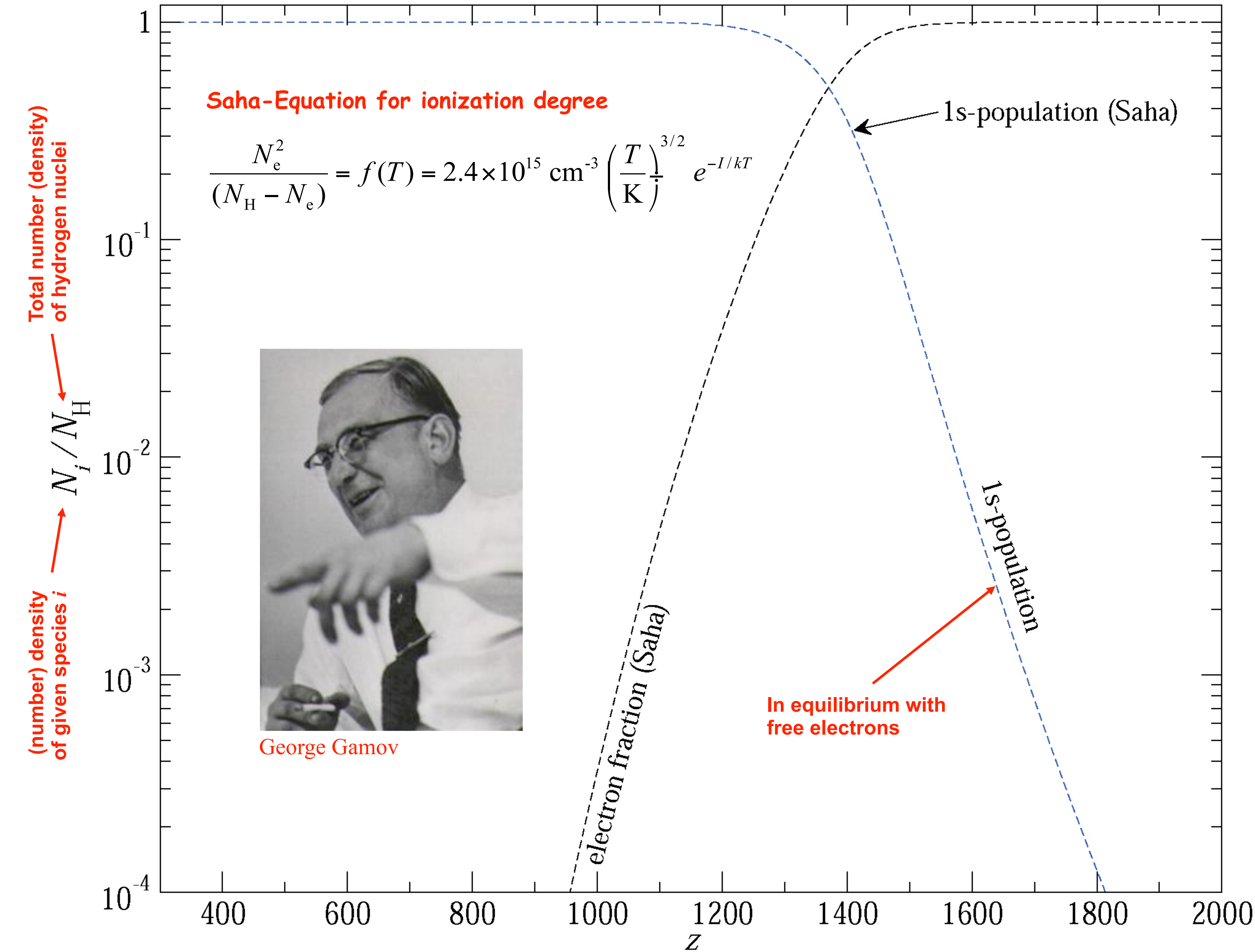
non-thermal photons

Physical Conditions during Recombination

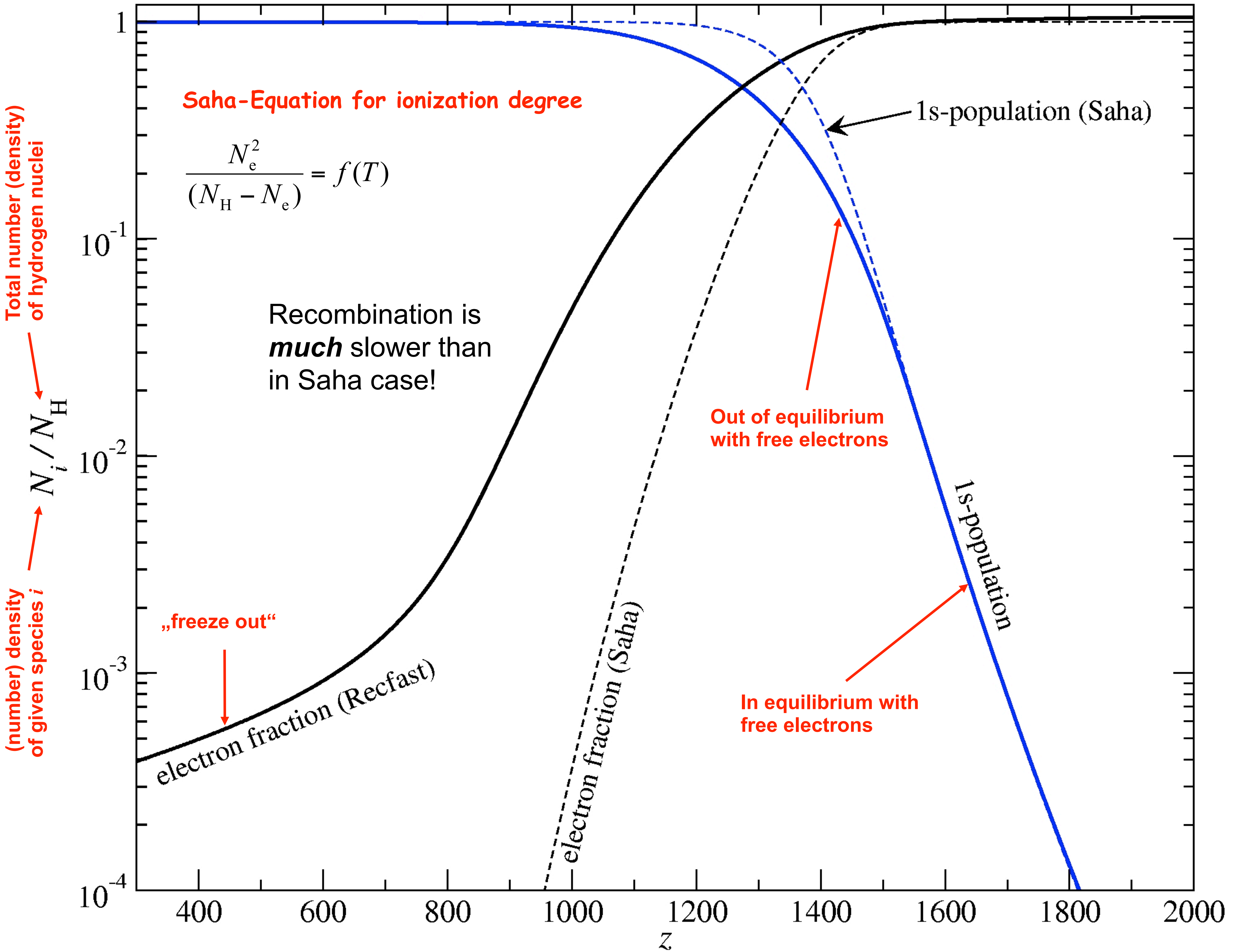
- Temperature $T_\gamma \sim 2.725\,(1+z)$ K ~ 3000 K
- Baryon number density $N_b \sim 2.5\text{x}10^{-7}\text{cm}^{-3}\,(1+z)^3 \sim 330\ \text{cm}^{-3}$
- Photon number density $N_\gamma \sim 410\ \text{cm}^{-3}\,(1+z)^3 \sim 2\times10^9\,N_b$

  ⇒ *photons in very distant Wien tail of blackbody spectrum can keep hydrogen ionized until* $h\nu_\alpha \sim 40\,kT_\gamma \iff T_\gamma \sim 0.26$ eV
- Collisional processes negligible (*completely different in stars!!!*)
- Rates dominated by radiative processes

  (*e.g. stimulated emission & stimulated recombination*)
- Compton interaction couples electrons very tightly to photons until $z \sim 200 \Rightarrow T_\gamma \sim T_e \sim T_m$

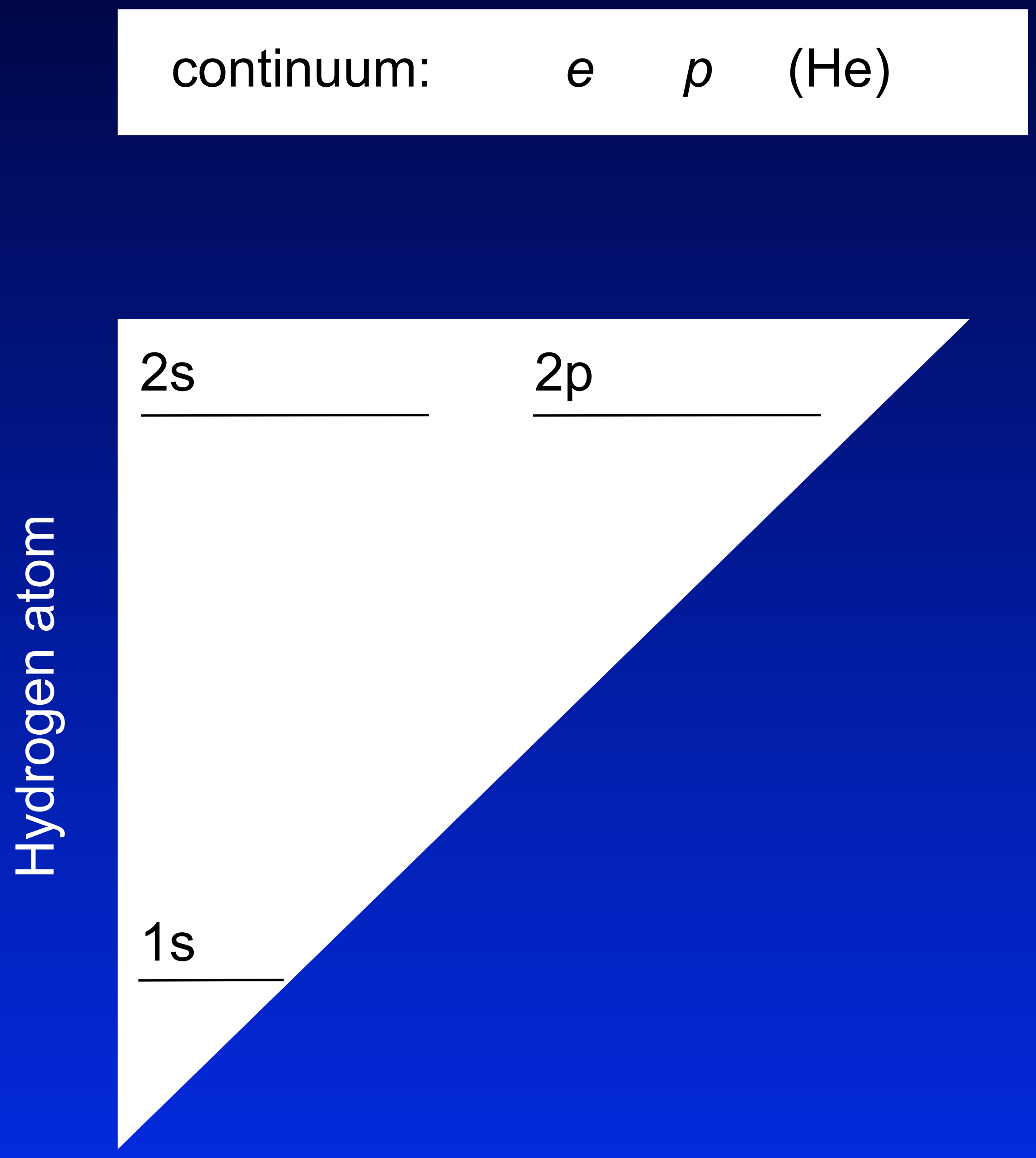


George Gamov



Saha-Equation for ionization degree
$$\frac{N_e^2}{(N_H - N_e)} = f(T)$$
Recombination is *much* slower than in Saha case!
1s-population (Saha)
Out of equilibrium with free electrons
In equilibrium with free electrons
1s-population
„freeze out“
electron fraction (Recfast)
electron fraction (Saha)
Total number (density) of hydrogen nuclei
(number) density of given species i
N_i / N_H
z
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
400
600
800
1000
1200
1400
1600
1800
2000

3-level Hydrogen Atom and Continuum



Routes to the ground state ?

Zeldovich, Kurt & Sunyaev, 1968, ZhETF, 55, 278
Peebles, 1968, ApJ, 153, 1

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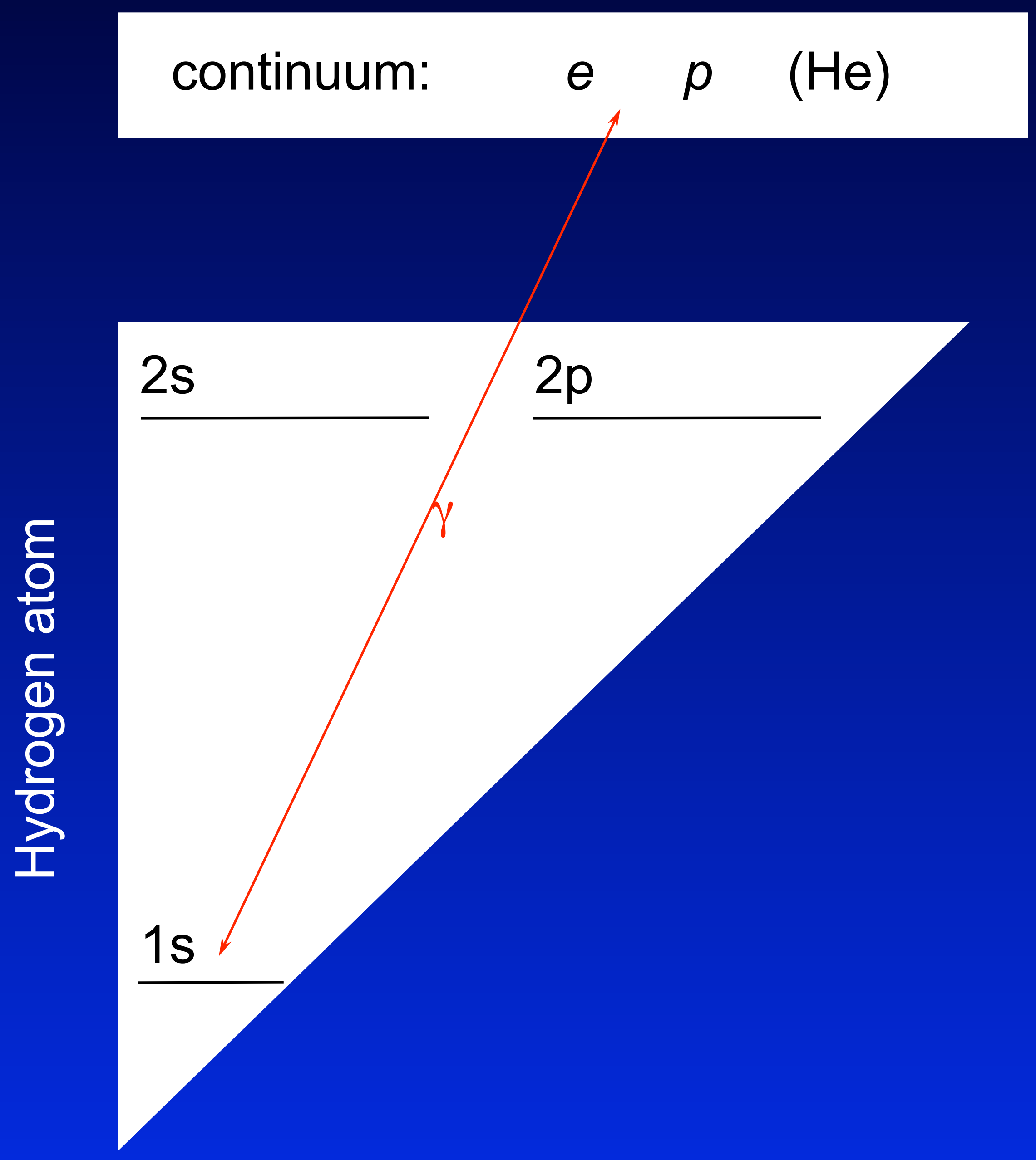

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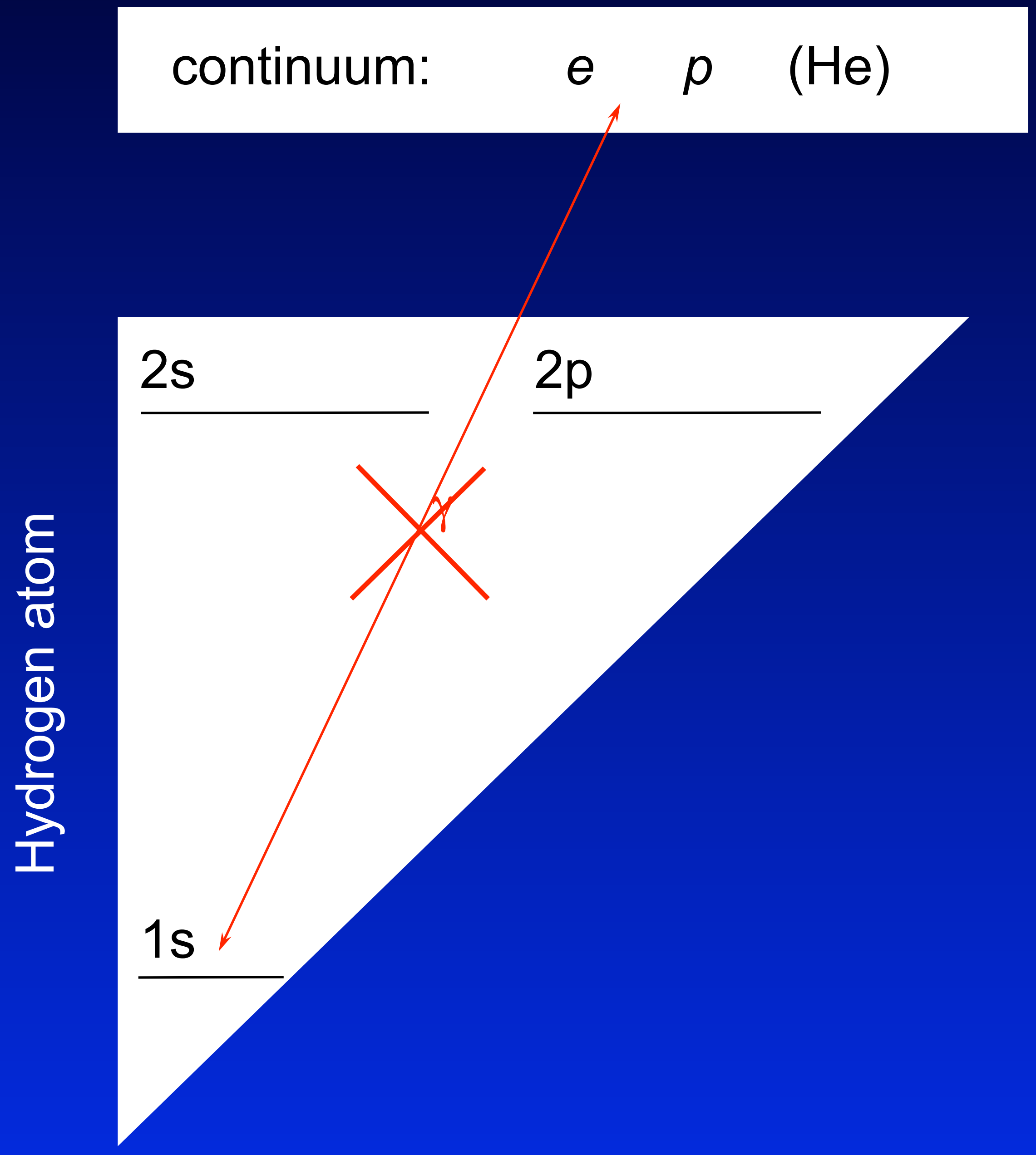

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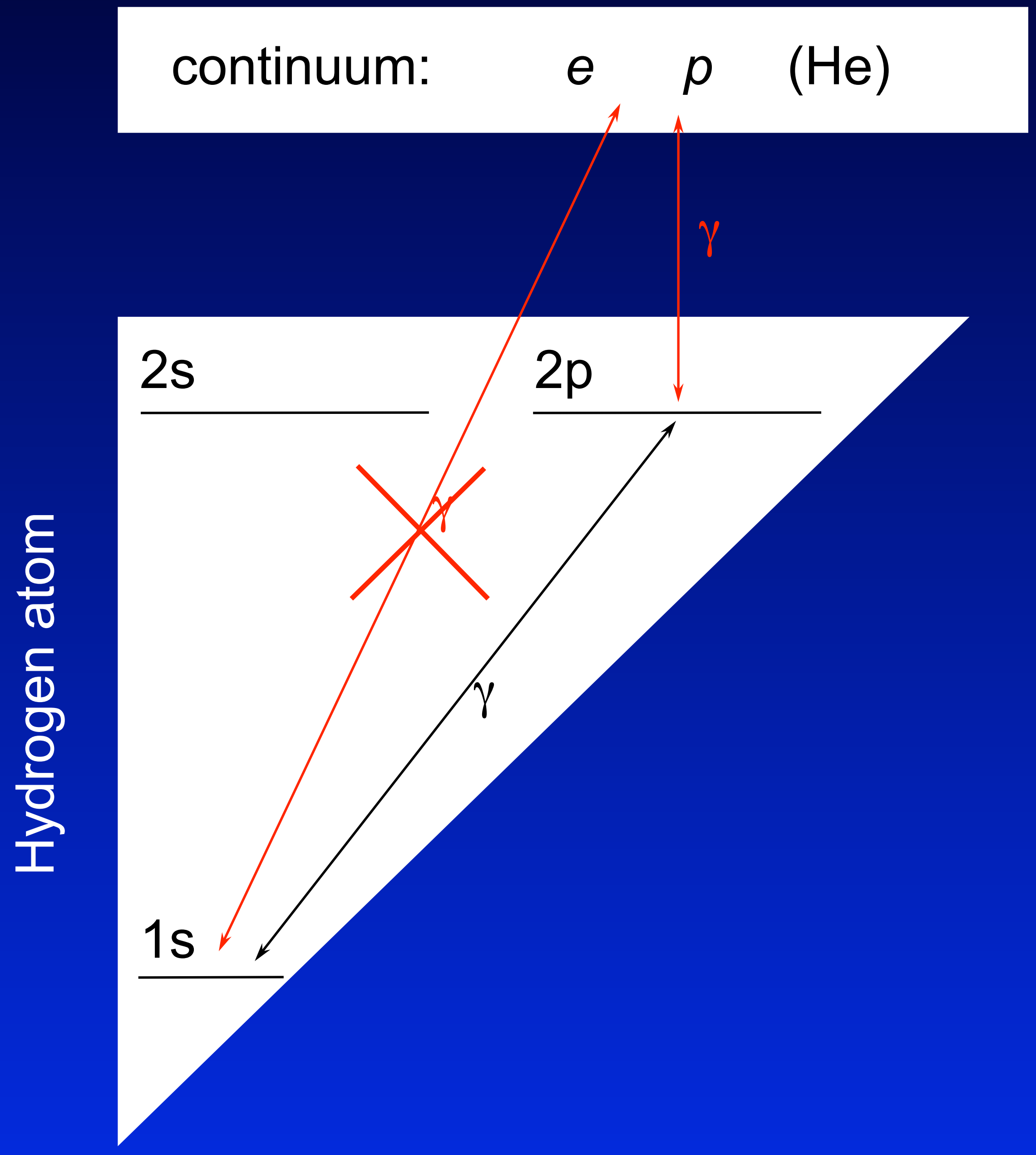

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  - **medium optically thick to Ly-α phot.**
  - **many resonant scatterings**
  - **escape very hard (*p* ~10^{-9} @ *z* ~1100)**

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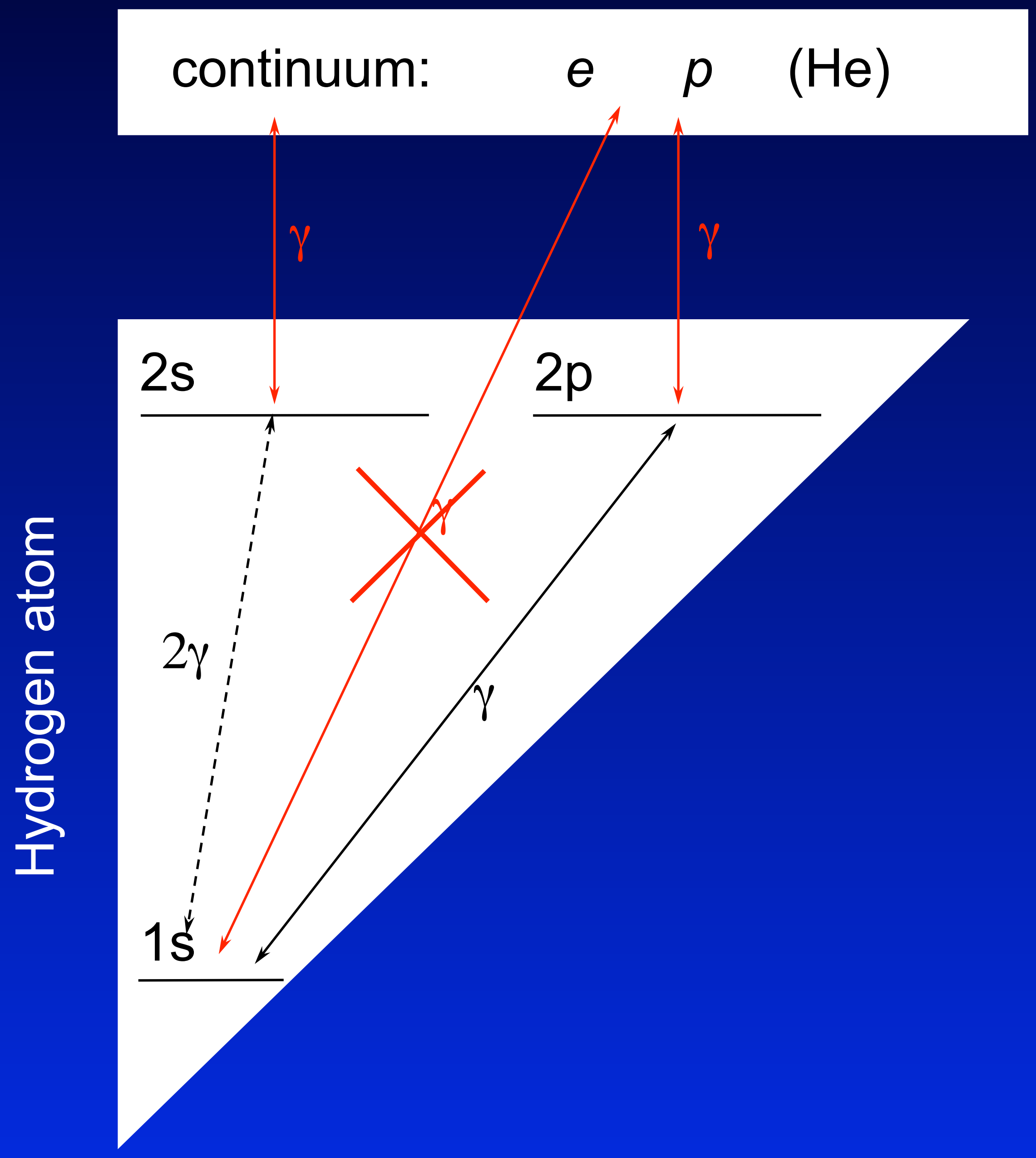

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  - **2s → 1s ~10^8 times slower than Ly-α**
  - **2s two-photon decay profile → maximum at $\nu \sim 1/2\, \nu_\alpha$**
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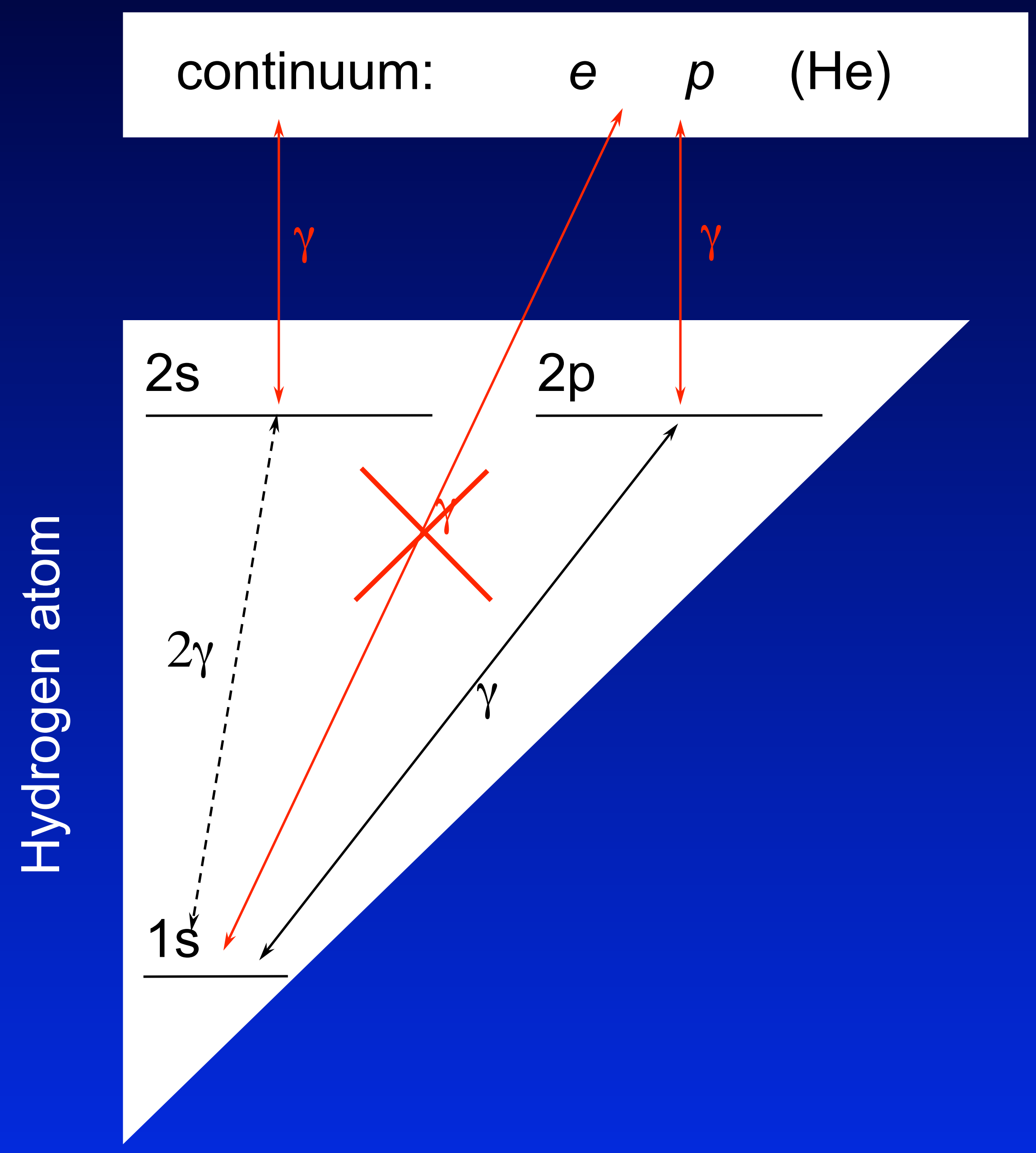

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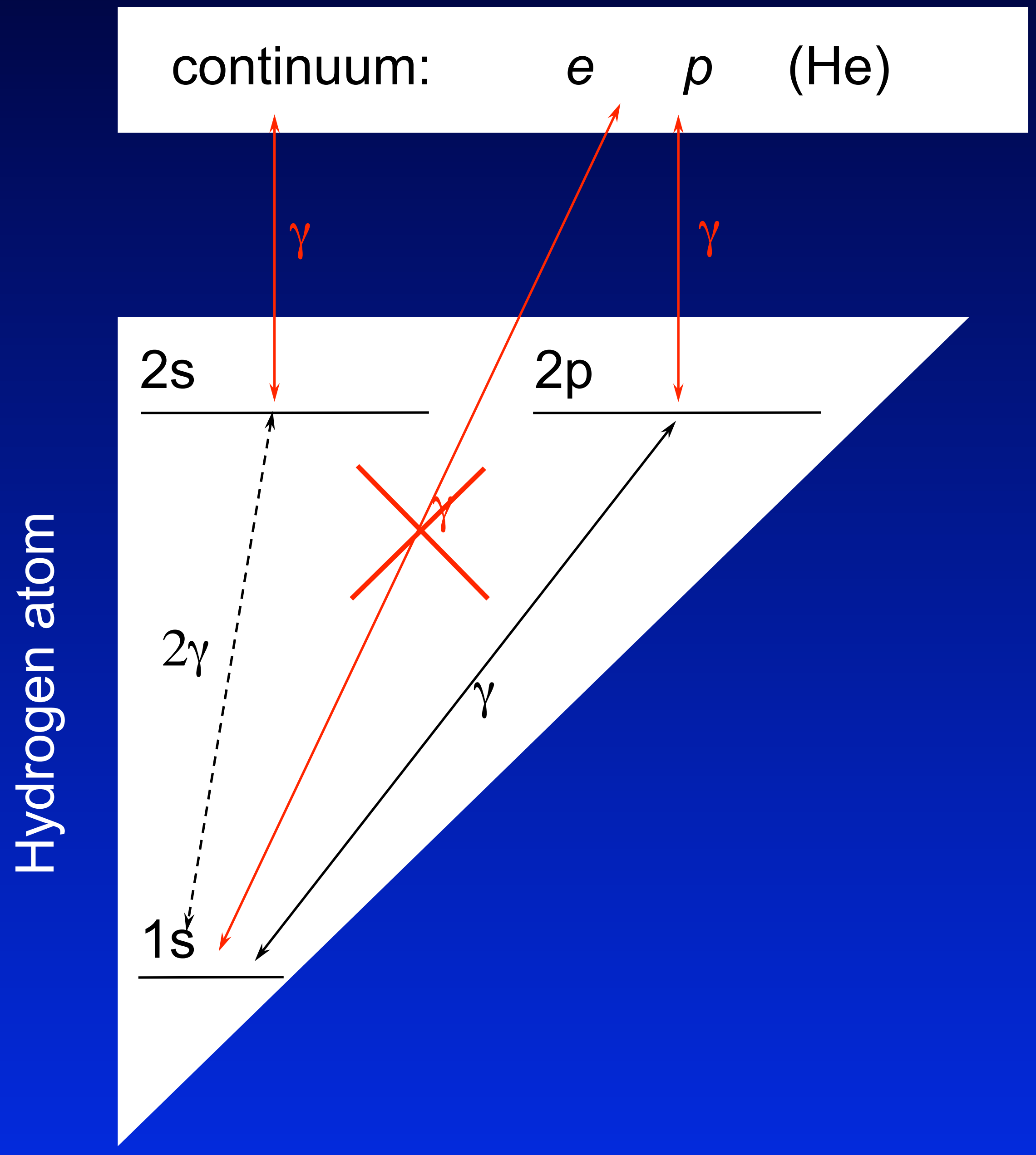

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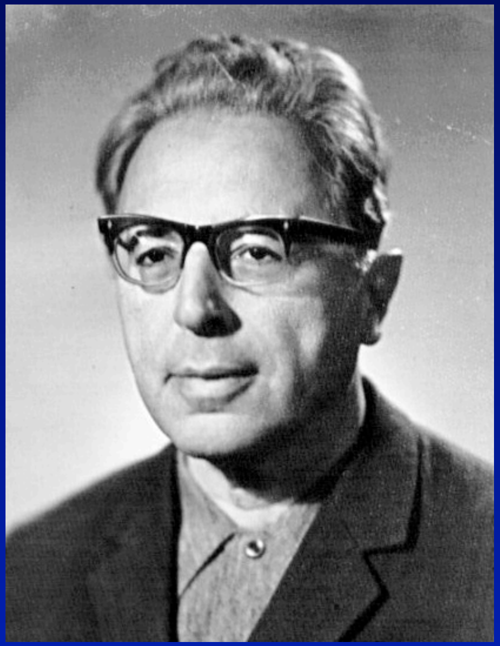
$\Delta N_e\ /\ N_e \sim 10\% - 20\%$

These first computations were completed in 1968!

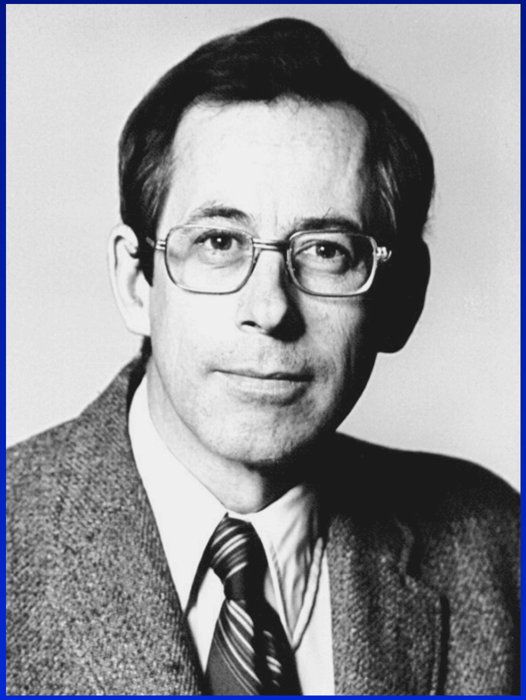
Moscow

Yakov Zeldovich



Iosif Shklovskii

Princeton

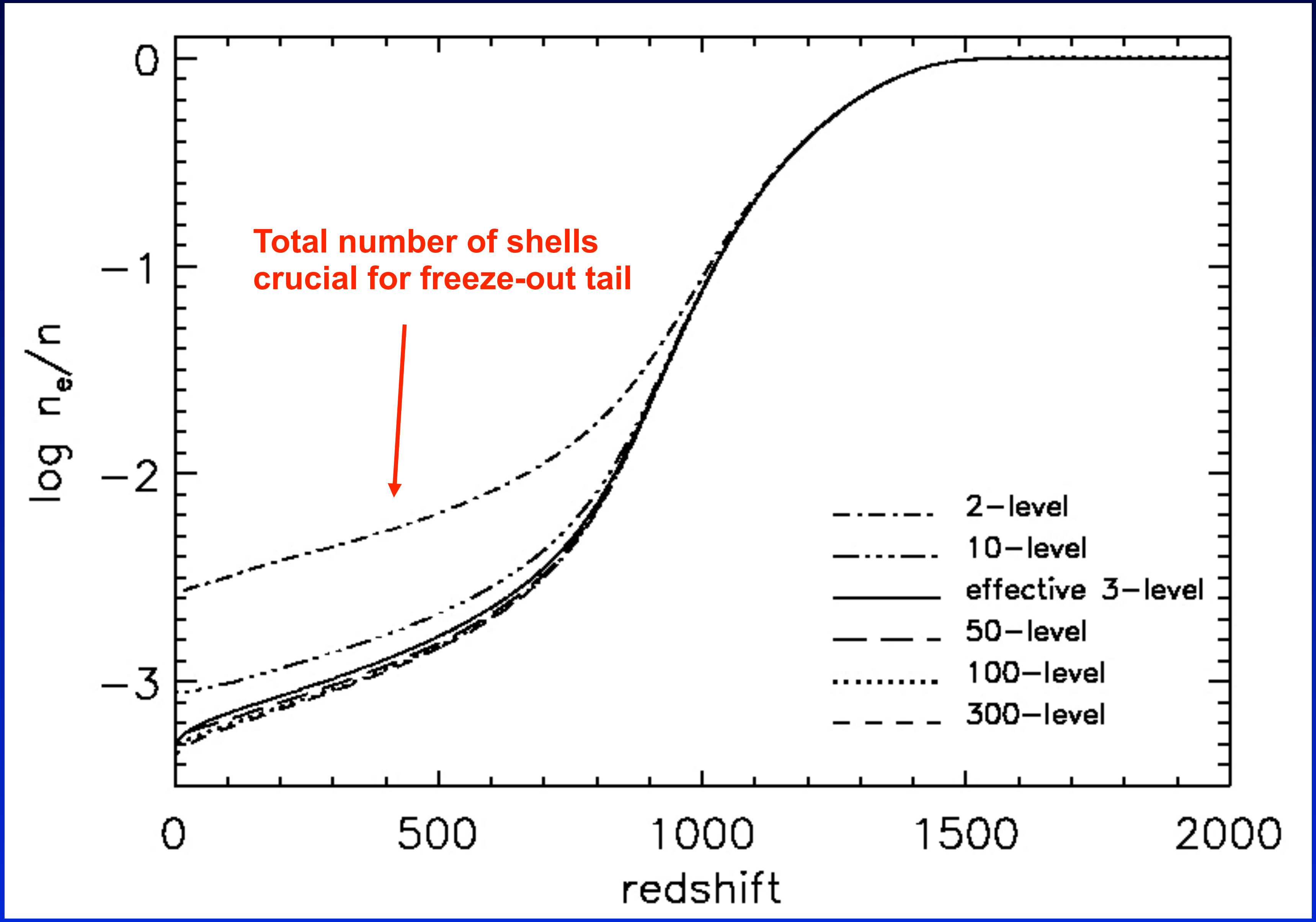


Jim Peebles

Vladimir Kurt
(UV astronomer)

Rashid Sunyaev

Multi-level Atom ⟺ Recfast-Code



Output of N_e/N_H

Hydrogen:

- **up to 300 levels (shells)**
- **$n \geq 2$ → full SE for l-sub-states**

Helium:

- **HeI 200-levels (z ~ 1400-1500)**
- **HeII 100-levels (z ~ 6000-6500)**
- **HeIII 1 equation**

Low Redshifts:

- **H chemistry (only at low z)**
- **cooling of matter (Bremsstrahlung, collisional cooling, line cooling)**

Seager, Sasselov & Scott, 1999, ApJL, 523, L1
Seager, Sasselov & Scott, 2000, ApJS, 128, 407

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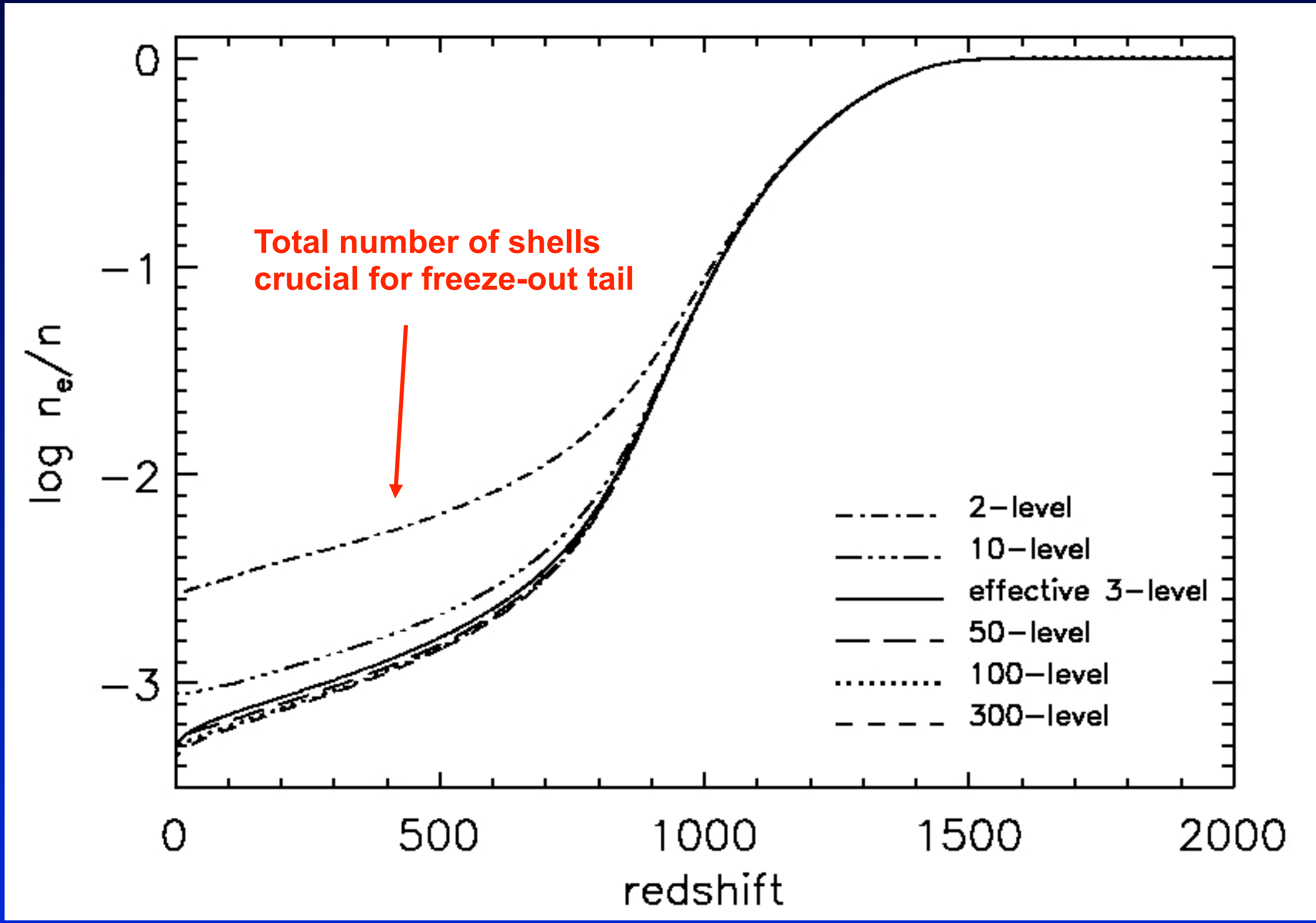

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- HeIII 1 equation

Low Redshifts:

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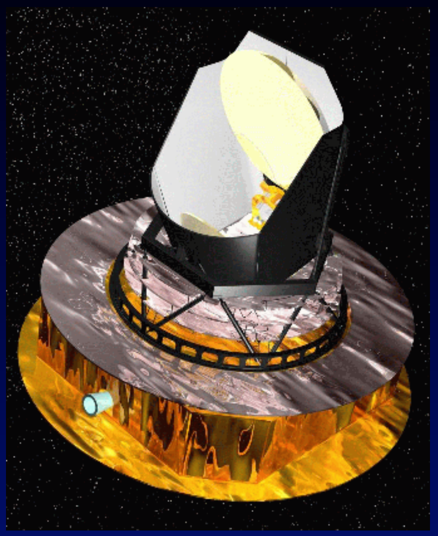
- cooling of matter (Bremsstrahlung, collisional cooling, line cooling)

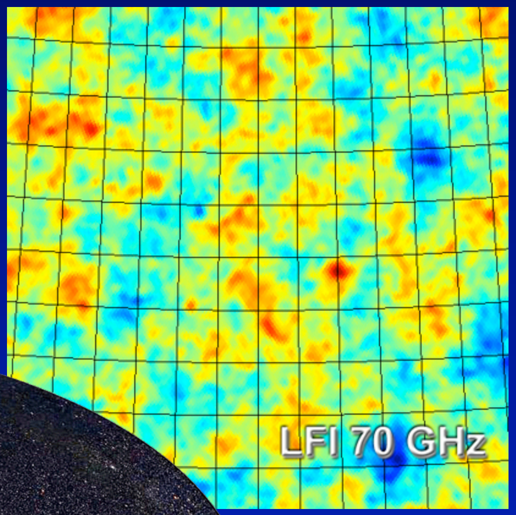
$\Delta N_e \,/\, N_e \sim 1\% - 3\%$

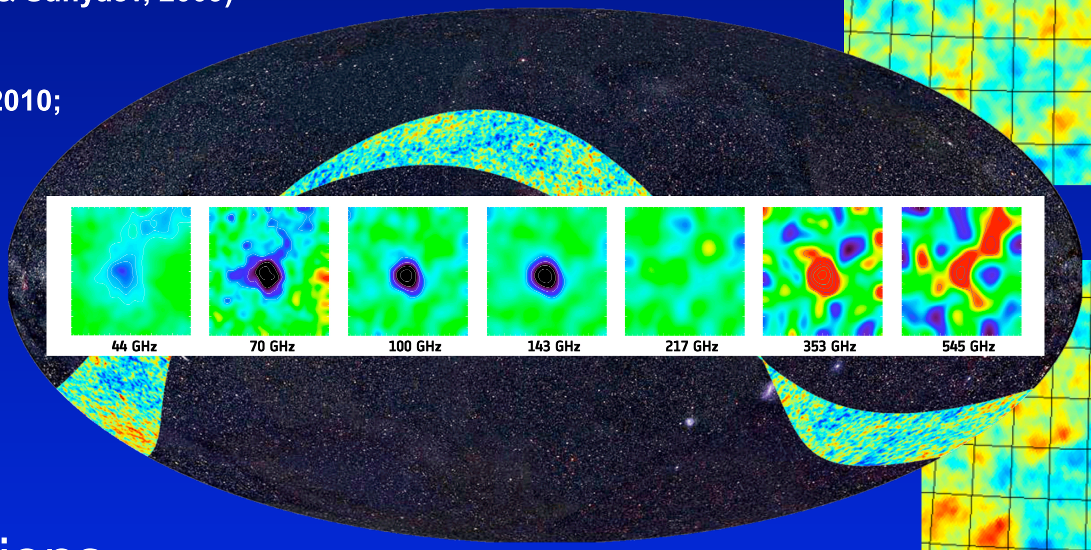
Getting the job done for Planck

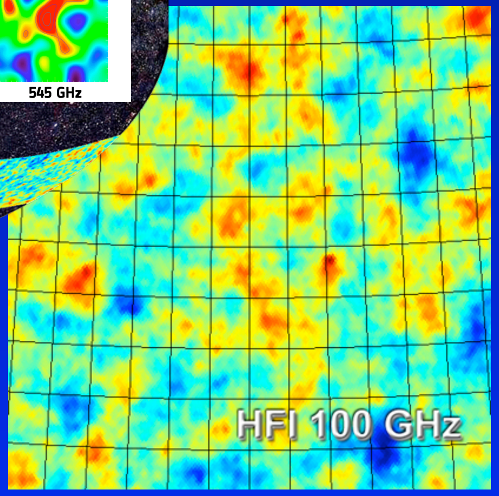


Hydrogen recombination

- Two-photon decays from higher levels
  (Dubrovich & Grachev, 2005, Astr. Lett., 31, 359; Wong & Scott, 2007; JC & Sunyaev, 2007; Hirata, 2008; JC & Sunyaev 2009)
- Induced 2s two-photon decay for hydrogen
  (JC & Sunyaev, 2006, A&A, 446, 39; Hirata 2008)
- Feedback of the Lyman-α distortion on the 1s-2s two-photon absorption rate
  (Kholupenko & Ivanchik, 2006, Astr. Lett.; Fendt et al. 2008; Hirata 2008)
- Non-equilibrium effects in the angular momentum sub-states
  (Rubiño-Martín, JC & Sunyaev, 2006, MNRAS; JC, Rubiño-Martín & Sunyaev, 2007, MNRAS; Grin & Hirata, 2009; JC, Vasil & Dursi, 2010)
- Feedback of Lyman-series photons (Ly[n] → Ly[n-1])
  (JC & Sunyaev, 2007, A&A; Kholupenko et al. 2010; Haimoud, Grin & Hirata, 2010)
- Lyman-α escape problem (*atomic recoil, time-dependence, partial redistribution*)
  (Dubrovich & Grachev, 2008; JC & Sunyaev, 2008; Forbes & Hirata, 2009; JC & Sunyaev, 2009)
- Collisions and Quadrupole lines
  (JC, Rubiño-Martín & Sunyaev, 2007; Grin & Hirata, 2009; JC, Vasil & Dursi, 2010; JC, Fung & Switzer, 2011)
- Raman scattering
  (Hirata 2008; JC & Thomas , 2010; Haimoud & Hirata, 2010)







Helium recombination

- Similar list of processes as for hydrogen
  (Switzer & Hirata, 2007a&b; Hirata & Switzer, 2007)
- Spin forbidden 2p-1s triplet-singlet transitions
  (Dubrovich & Grachev, 2005, Astr. Lett.; Wong & Scott, 2007; Switzer & Hirata, 2007; Kholupenko, Ivanchik&Varshalovich, 2007)
- Hydrogen continuum opacity during He I recombination
  (Switzer & Hirata, 2007; Kholupenko, Ivanchik & Varshalovich, 2007; Rubiño-Martín, JC & Sunyaev, 2007; JC, Fung & Switzer, 2011)
- Detailed feedback of helium photons
  (Switzer & Hirata, 2007a; JC & Sunyaev, 2009, MNRAS; JC, Fung & Switzer, 2011)

$\Delta N_e \,/\, N_e \sim 0.1\ \%$

Solving the problem for the Planck Collaboration was a common effort!

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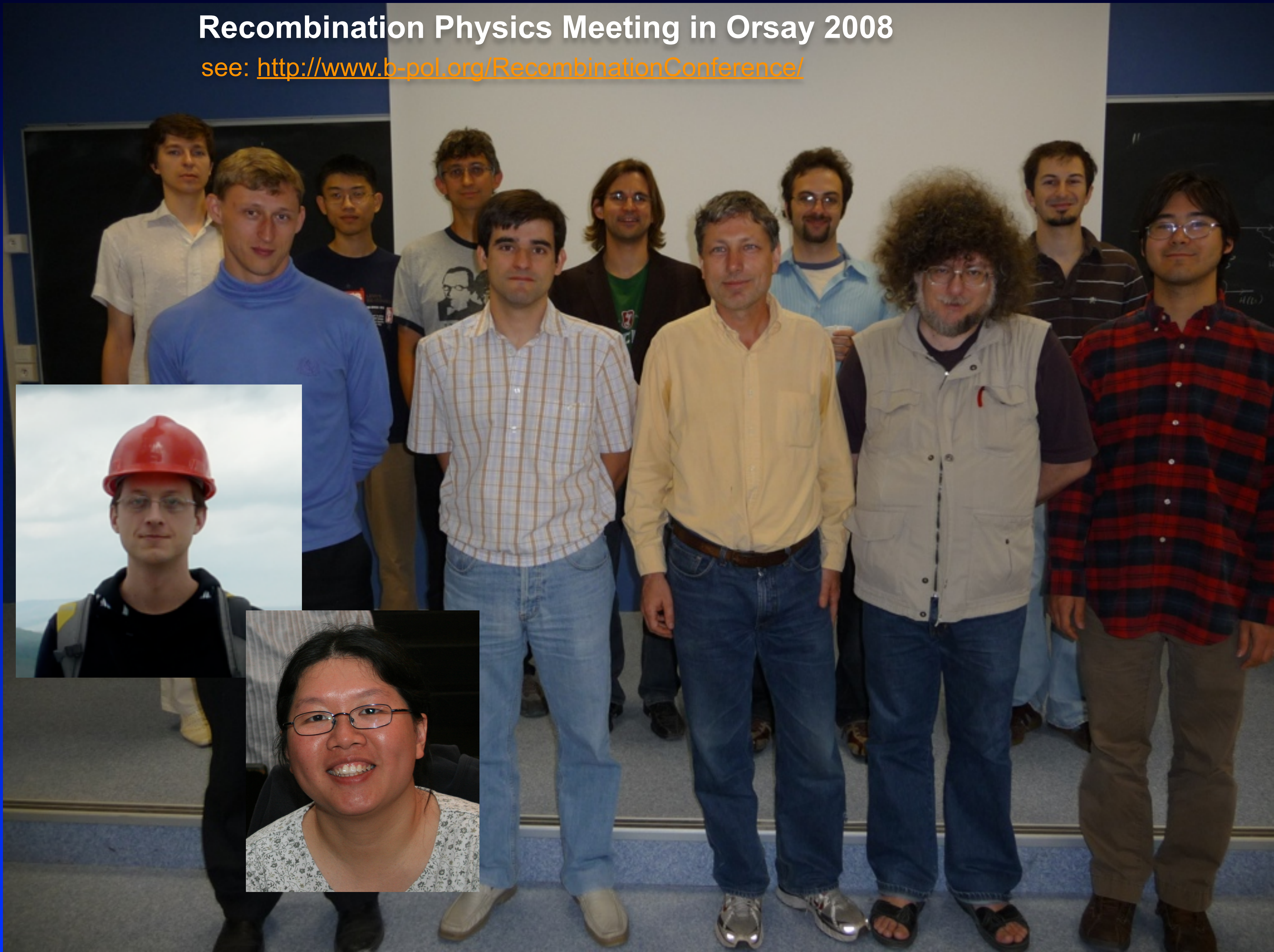

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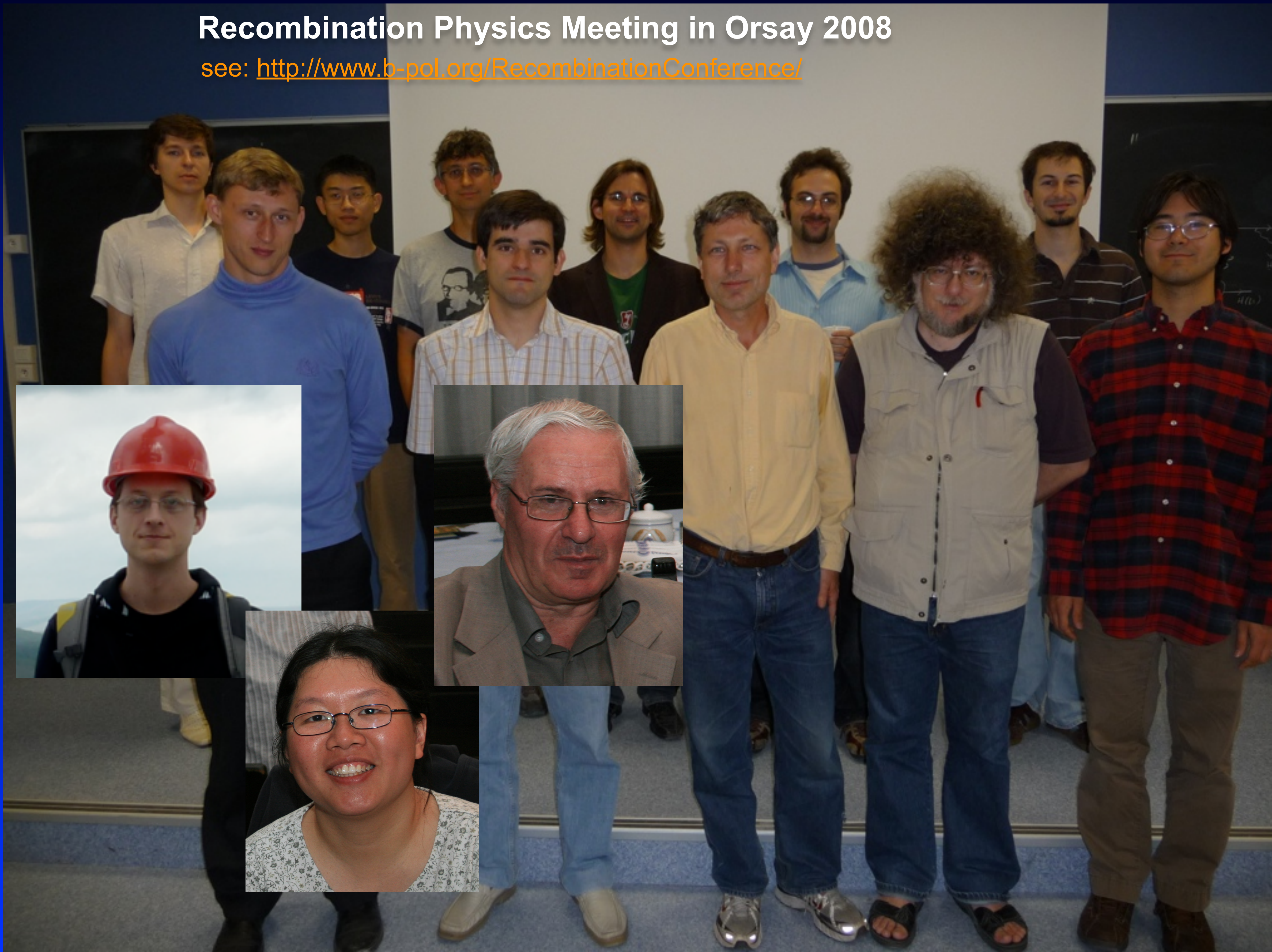

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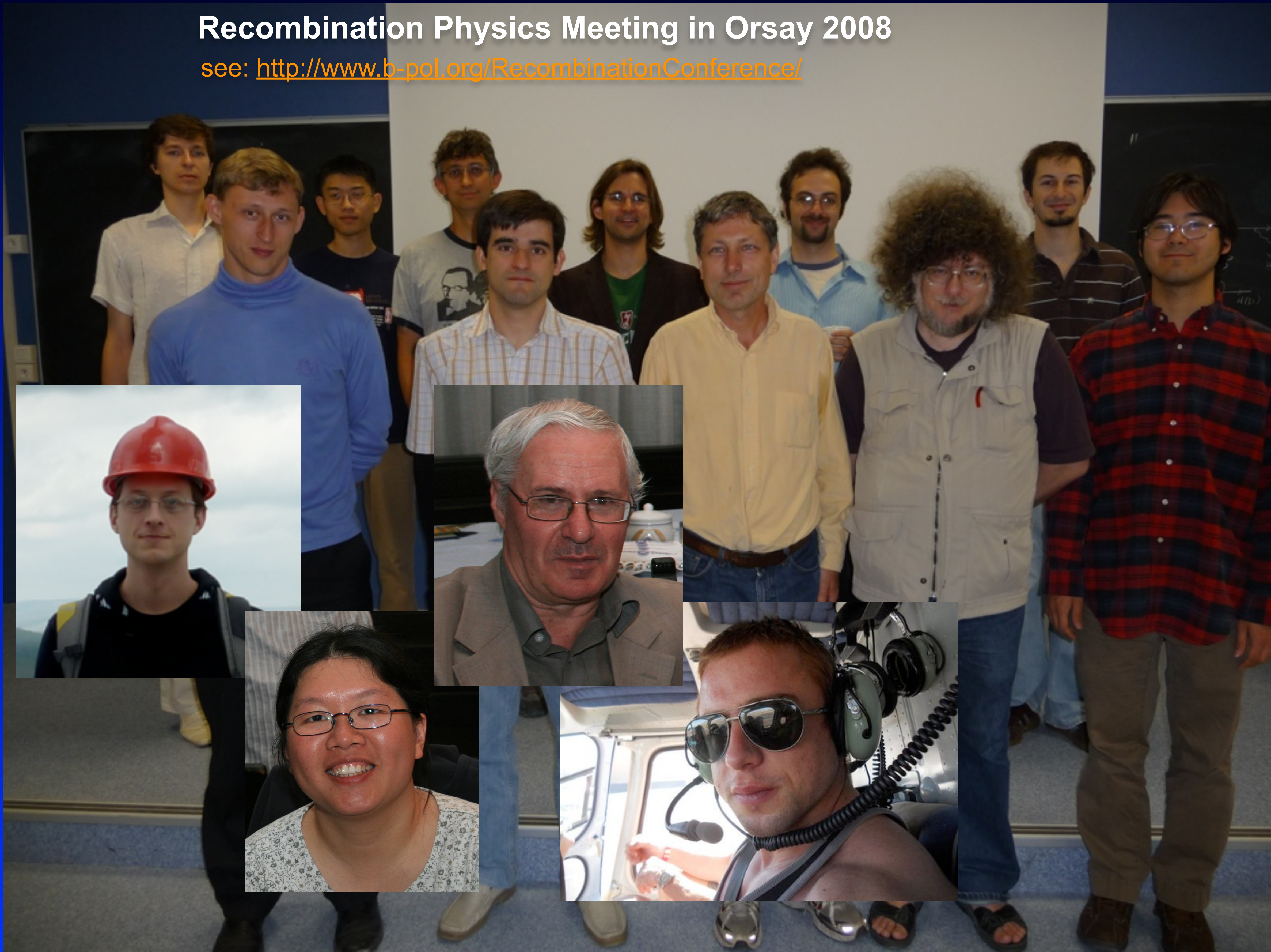

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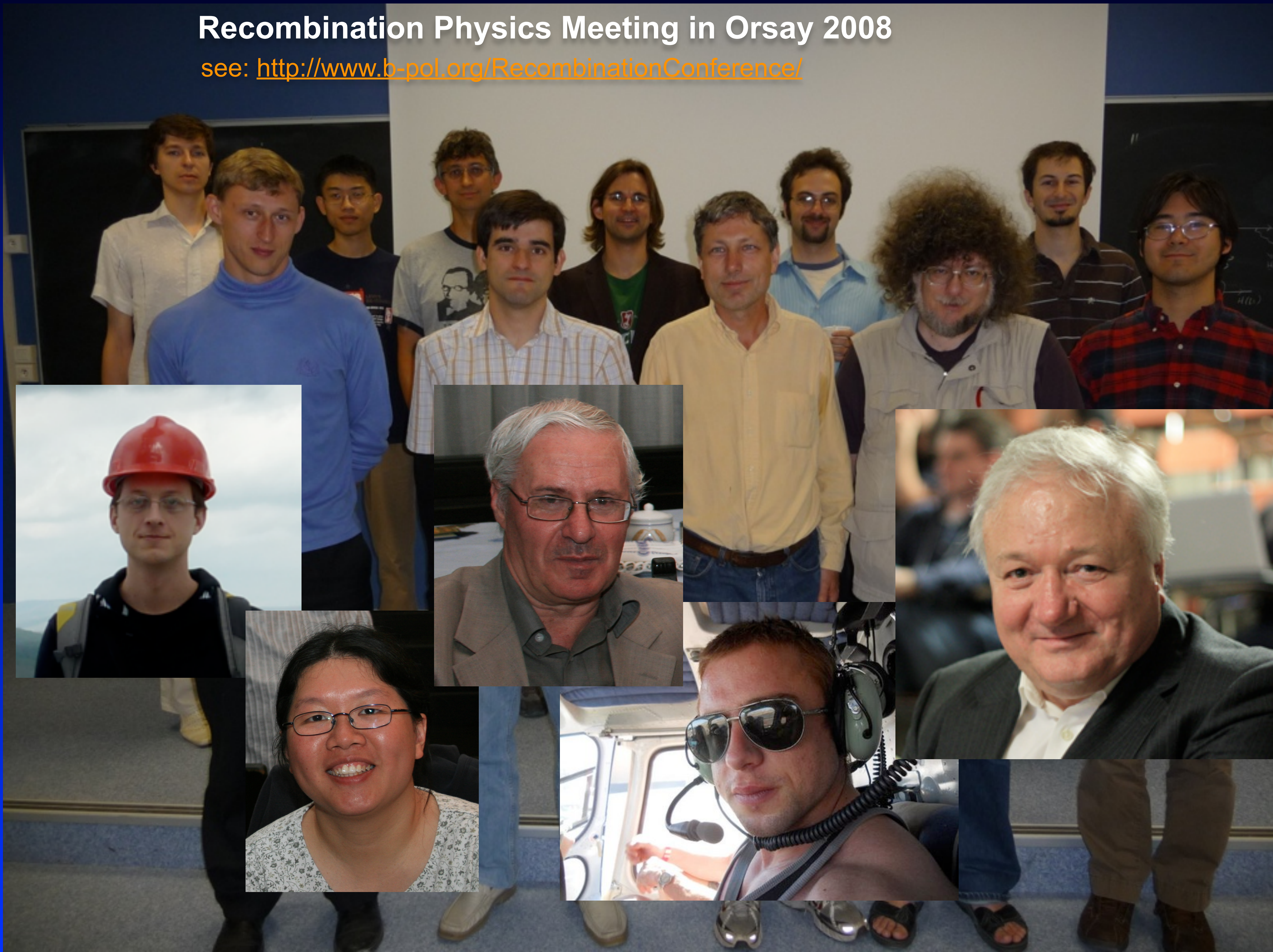
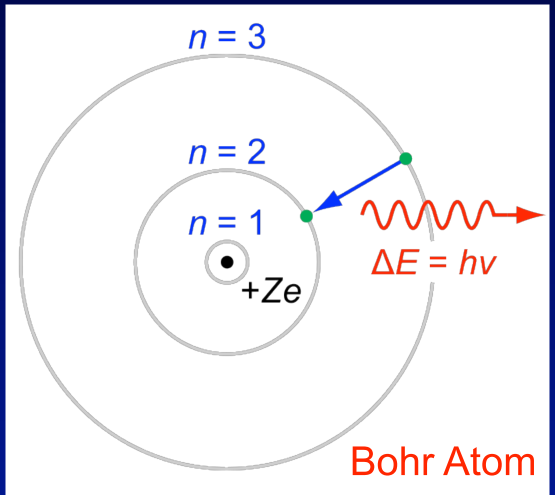


Atomic Physics Challenges

Hydrogen Atom & Hydrogenic Helium

- Rather simple and basically analytic (e.g., Karzas & Latter, 1961)
- Even 2γ rates can be computed precisely (e.g., Goeppert-Mayer, 1931)
- Collision rates less robust, but effect small (new rates became available!)
- Biggest computational challenge is the number of levels (~ n^2)



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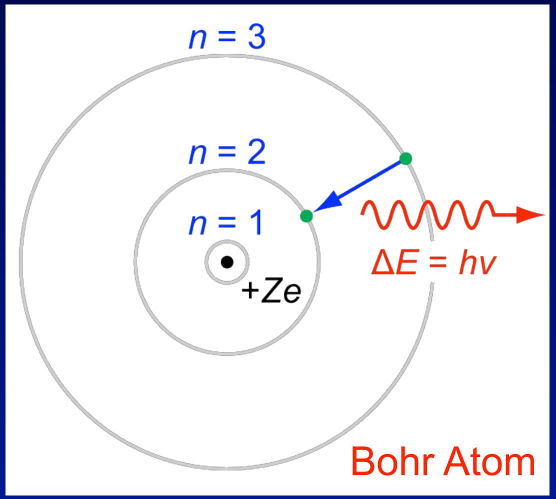
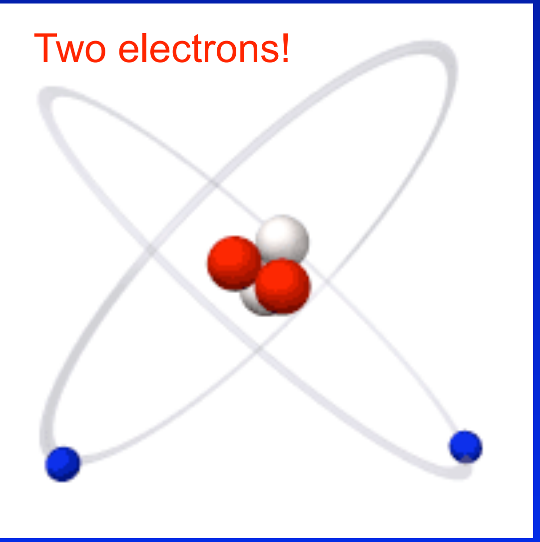
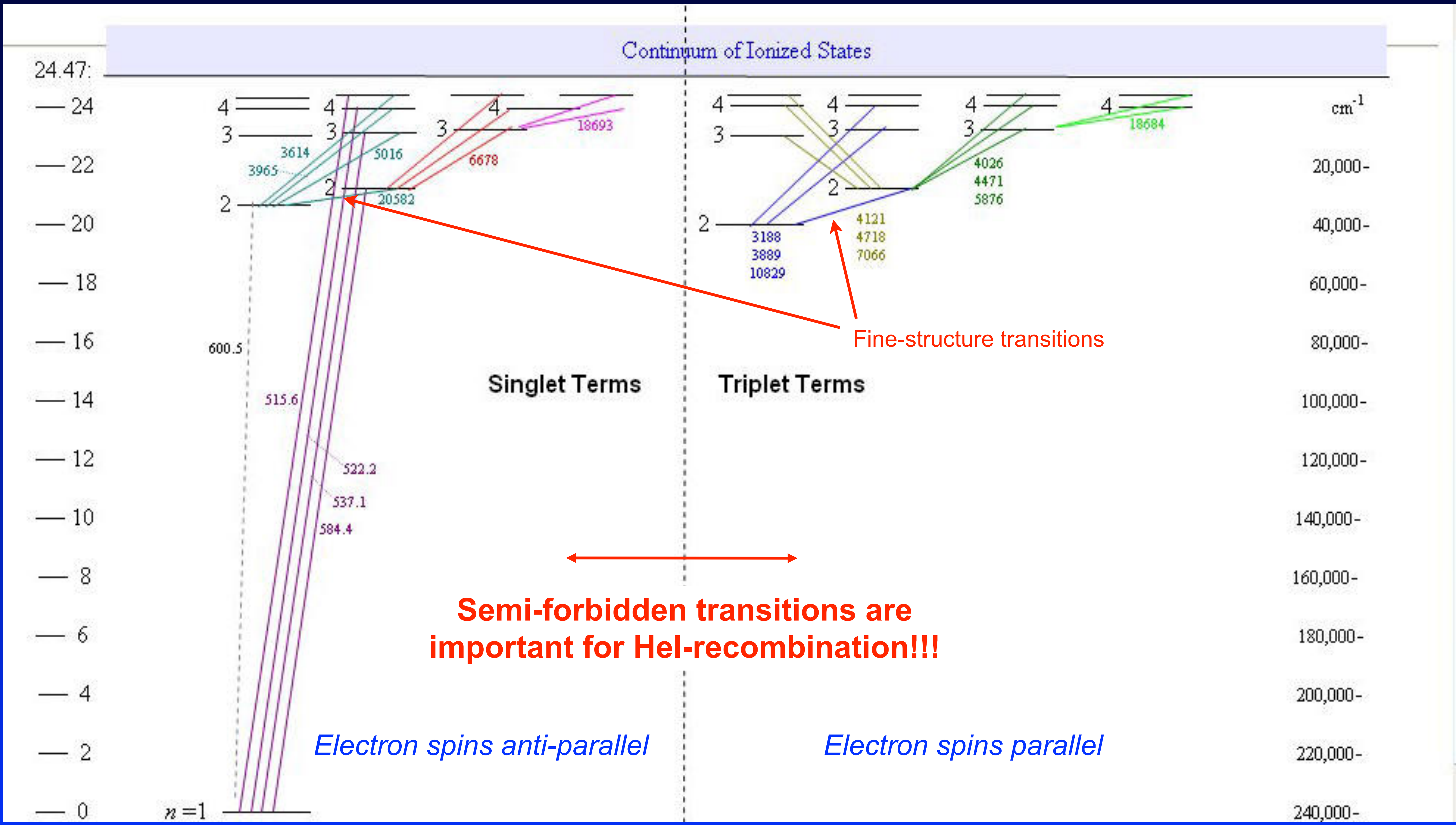


Neutral Helium

- Lower levels non-hydrogenic (perturbative approach needed)
- Spectrum complicated and data (was) rather sparse (e.g., Drake & Morton, 2007)



Grotrian diagram for neutral helium



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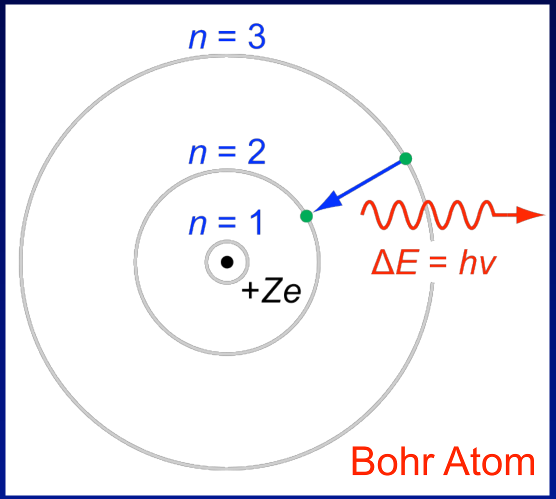

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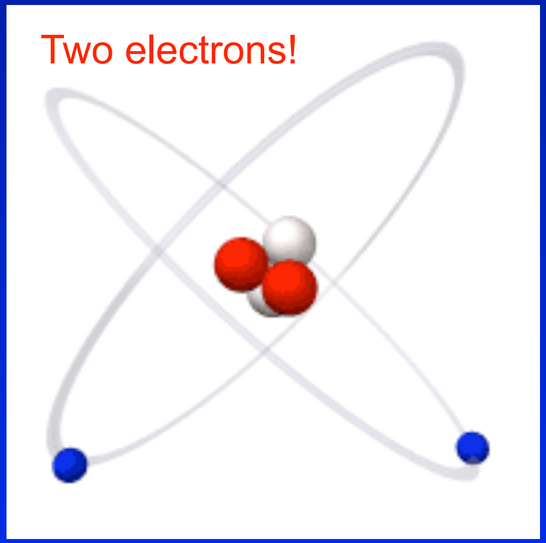
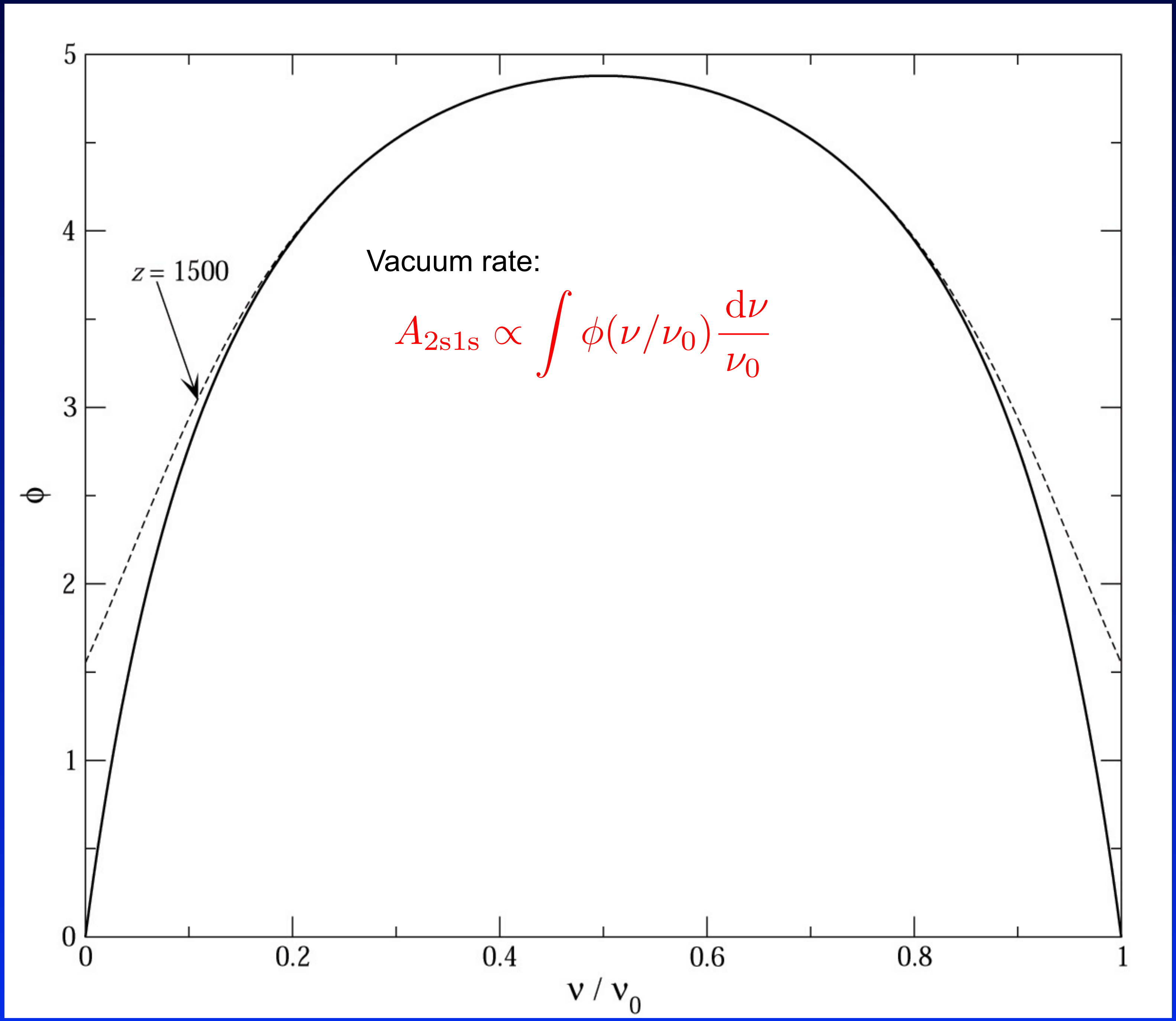
- Collision rates pretty rough (important for distortions...)
- Computational challenge because of levels not as severe



Stimulated 2s → 1s decay



2s-1s emission profile

Transition rate in vacuum

→ A_{2s1s} ~ 8.22 sec^{-1}

CMB ambient photons field

→ A_{2s1s} increased by ~1%-2%

→ HI - recombination faster by $\Delta N_e/N_e$ ~ 1.3%

JC & Sunyaev, 2006, A&A

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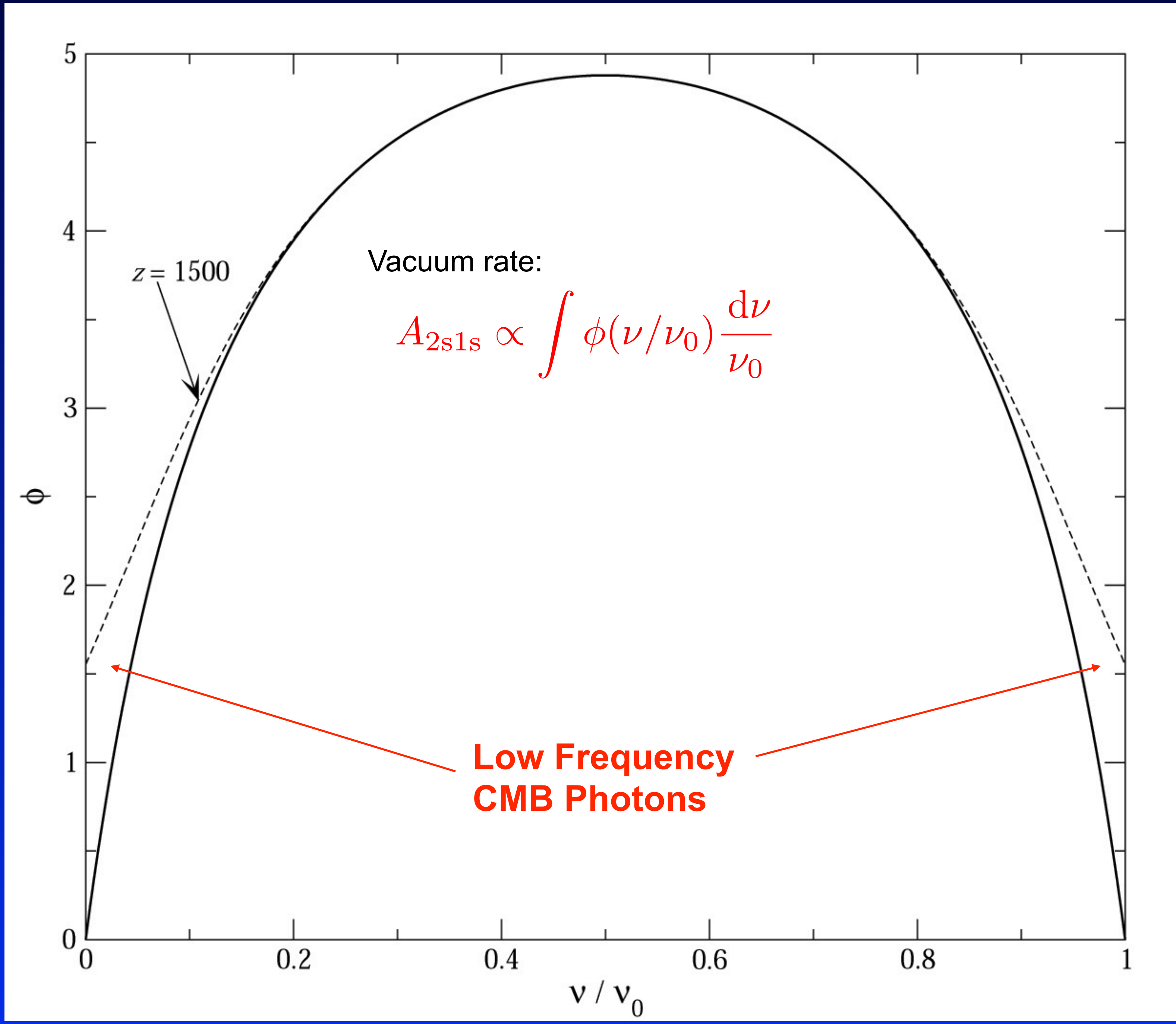

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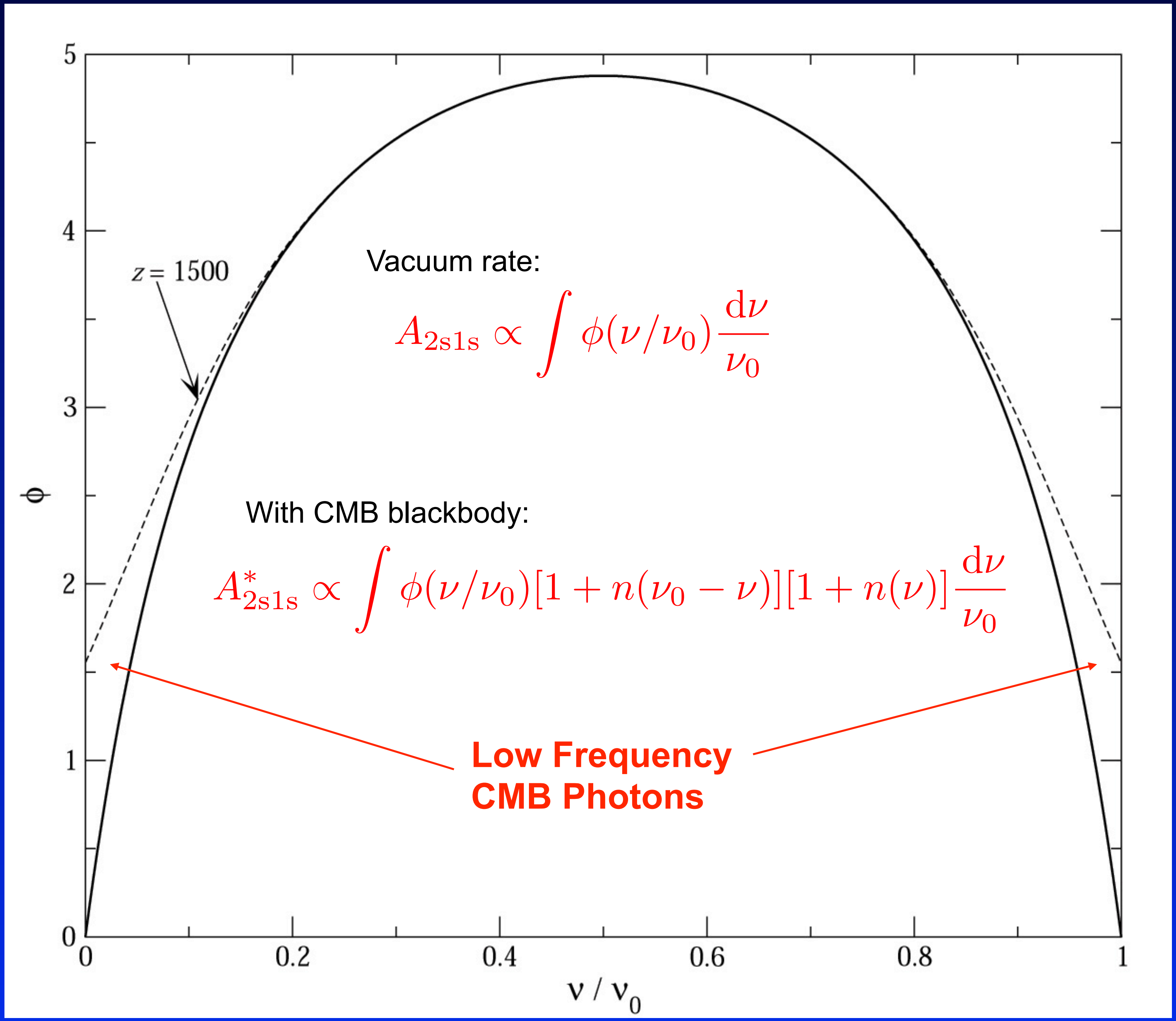

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Feedback of Ly-α on the 1s →2s transition

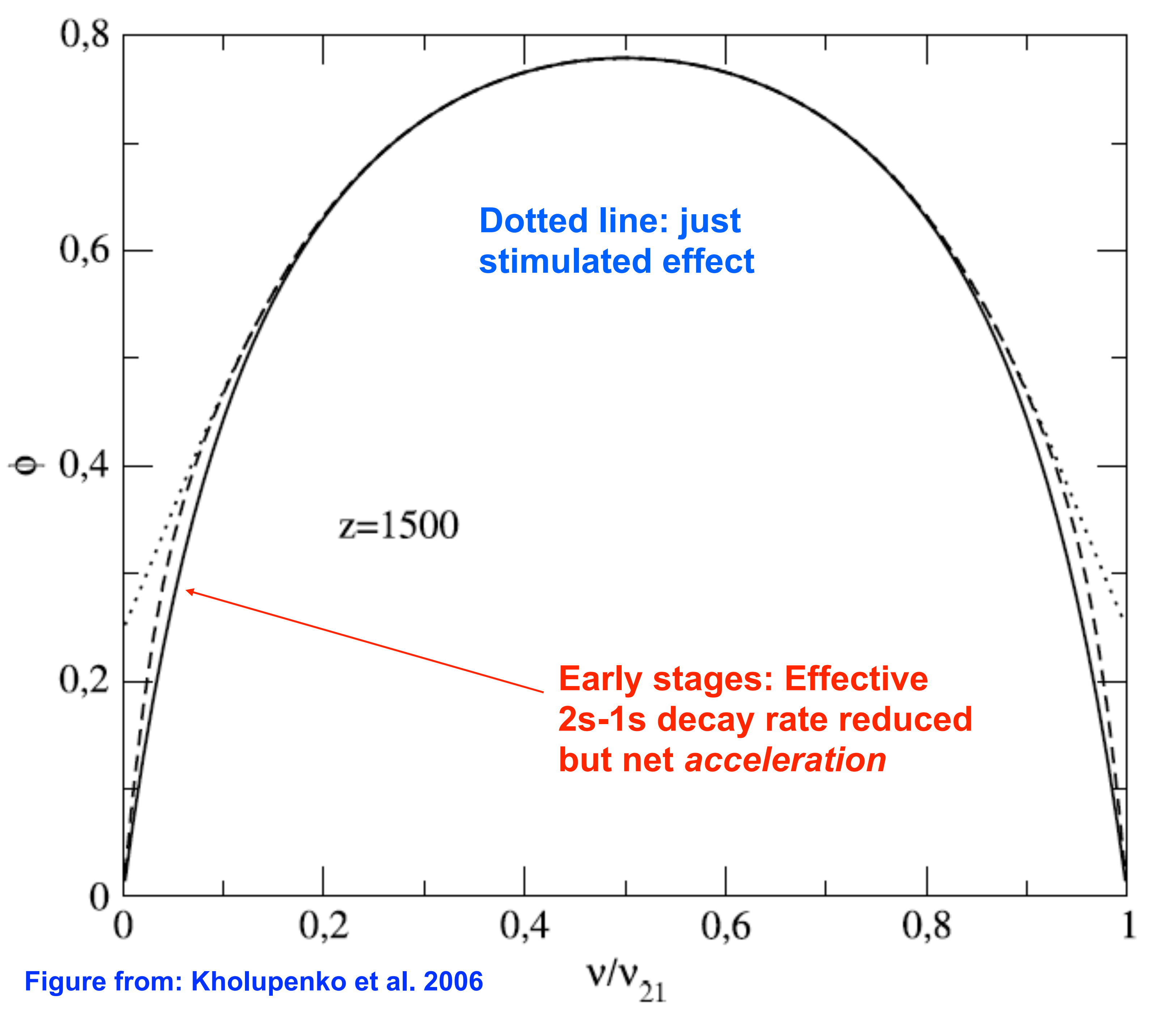

Figure from: Kholupenko et al. 2006

- Some Ly-α photon are re-absorbed in the 1s-2s channel
- delays recombination
- net effect on 2s-1s channel $\Delta N_e/N_e$ ~ 0.6% around z~1100
- 2s-1s *self-feedback* $\Delta N_e/N_e$ ~ -0.08% around z~1100 (JC & Thomas, 2010)

Kholupenko et al. 2006
Fendt, JC, Rubino-Martin & Wandelt, 2009

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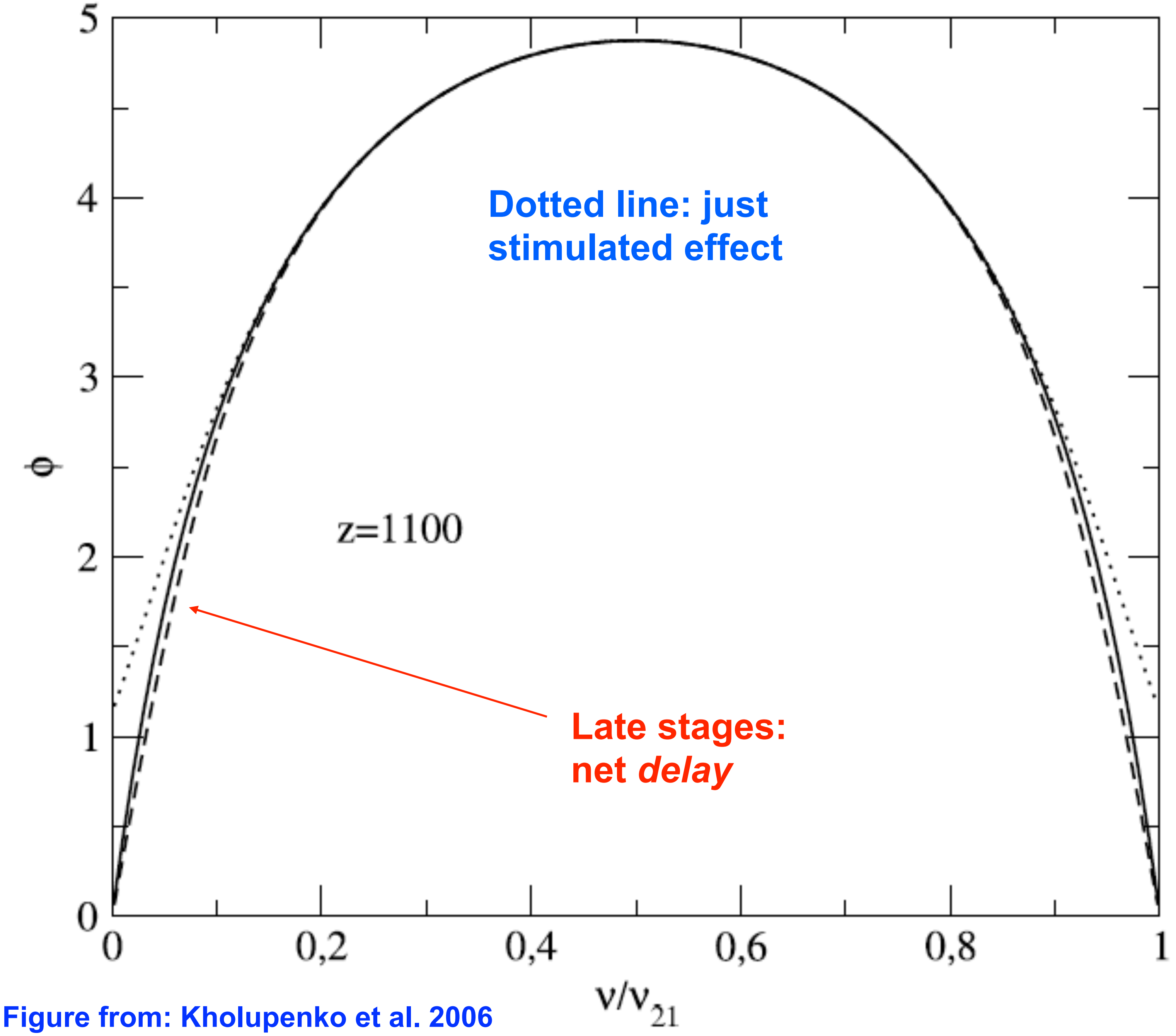

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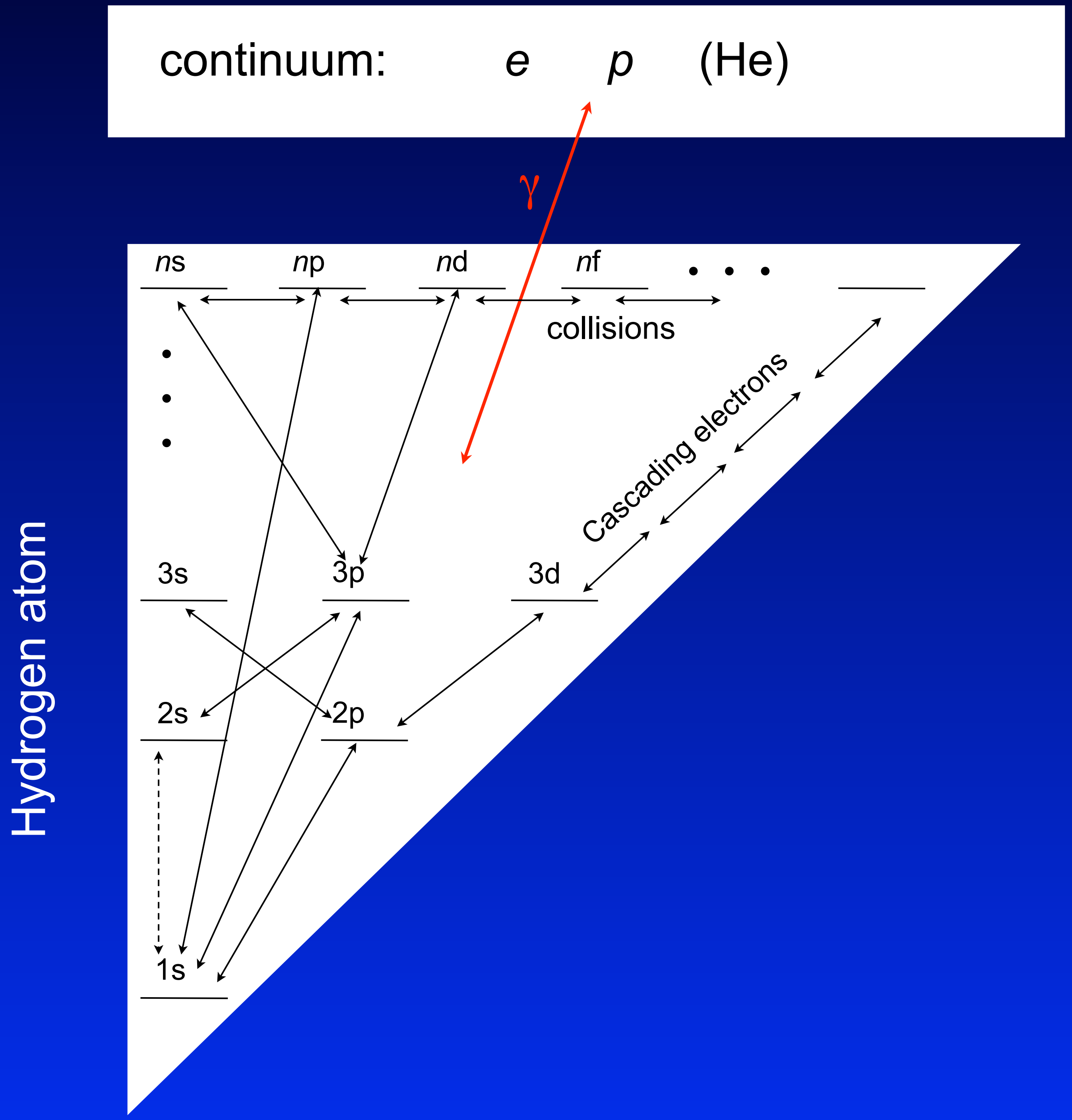
Deviations from Statistical Equilibrium in the upper levels

Basis for Recfast computation (Seager et al. 2000)

$$N_{nl} = \frac{2l+1}{n^2} N_{\mathrm{tot},n}$$

- l-dependence of populations neglected
- Levels in a given shell assumed to be in Statistical Equilibrium (SE)
- Complexity of problem scales like ~ $n_{\max}$

Processes for the upper levels



- **recombination & photoionization**
  - ***n*** **small → *l*-dependence not drastic**
  - **high shells → more likely to *l*<<*n***
  - **large *n* → *induced* recombination**
- **many radiative dipole transitions**
  - **Lyman-series optically thick**
  - **$\Delta l = \pm 1$ restriction (electron cascade)**
  - **large *n* & small Δn → *induced* emission**
- ***l*-changing collisions**
  - **help to establish full SE within the shell**
  - **only effective for *n* > 25-30**
- ***n*-changing collisions**
- **Collisional photoionization**
- **Three-body-recombination**

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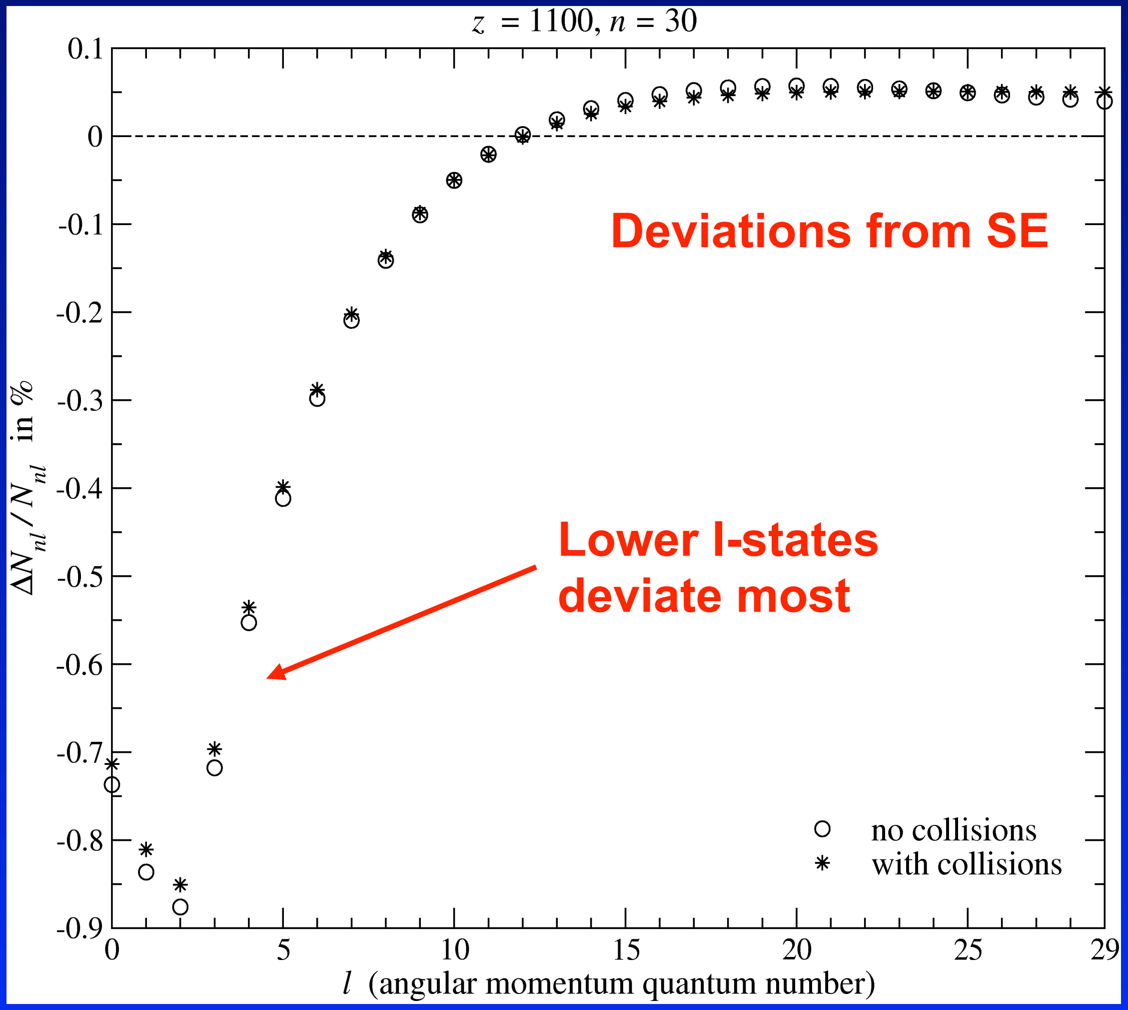

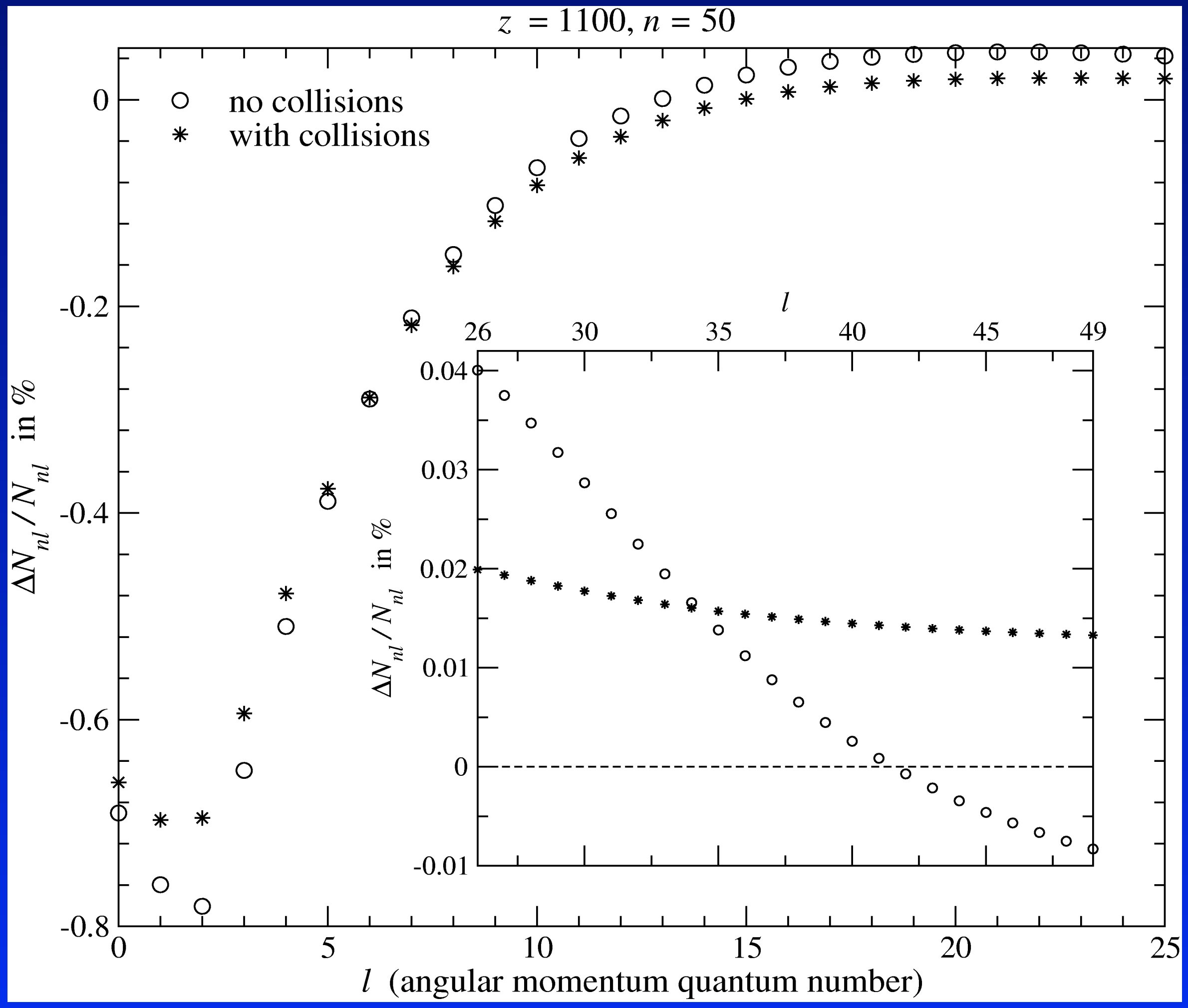

JC, Rubino-Martin & Sunyaev, MNRAS, 2007

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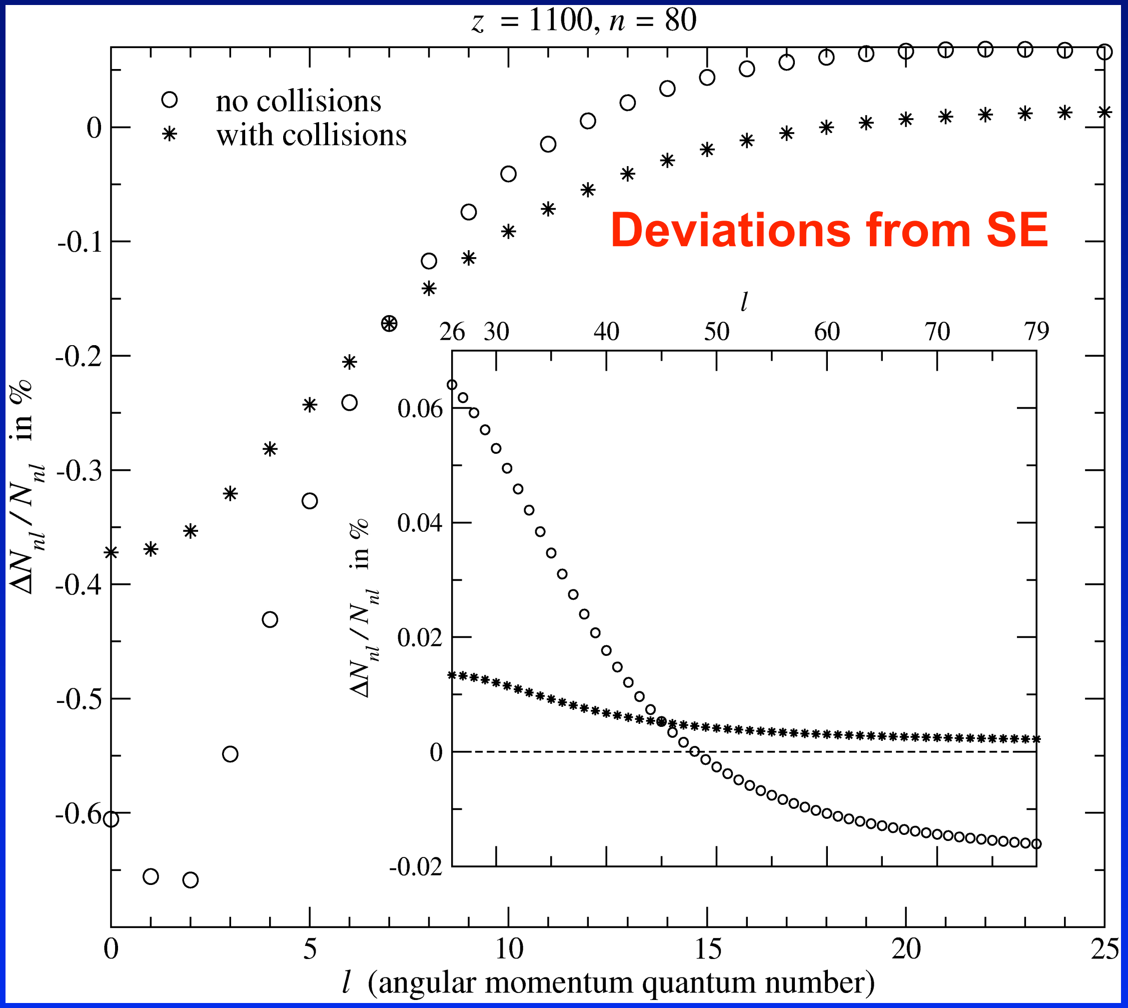
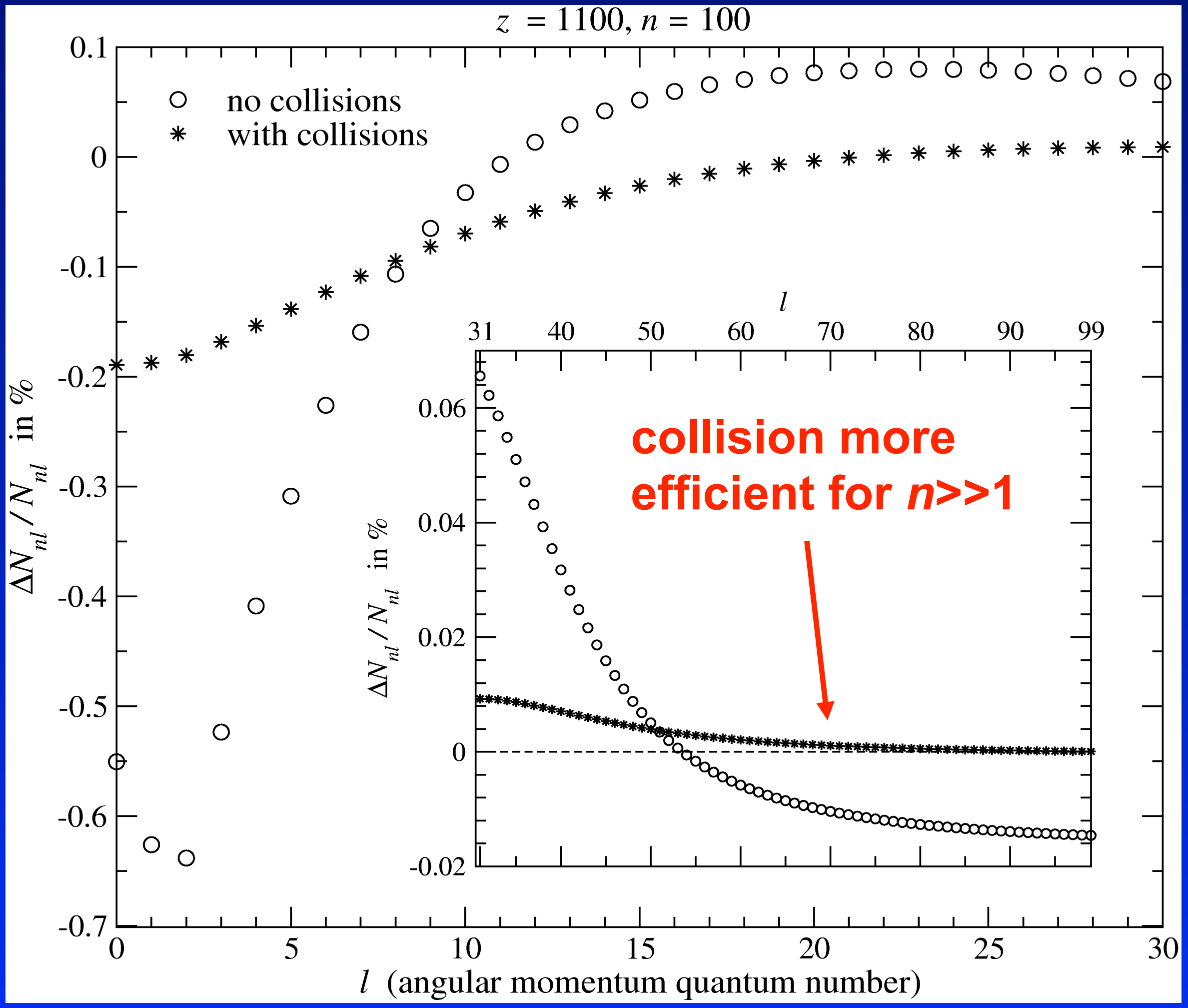
$$N_{nl} = \frac{2l+1}{n^2} N_{\mathrm{tot},n}$$

- l-dependence of populations neglected
- Levels in a given shell assumed to be in Statistical Equilibrium (SE)
- Complexity of problem scales like ~ $n_{\max}$





JC, Rubino-Martin & Sunyaev, MNRAS, 2007

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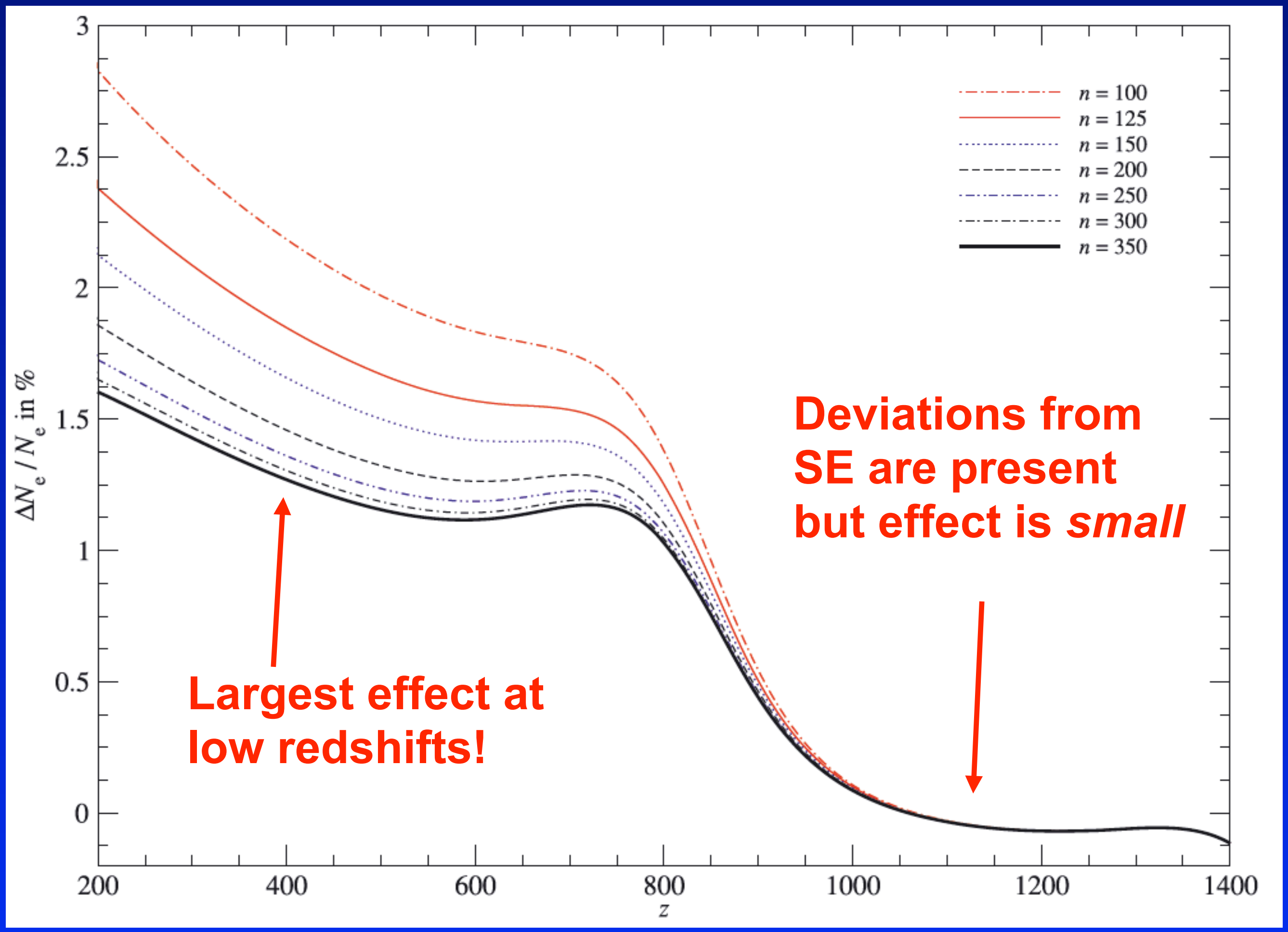
Refined computation
(JC, Rubino-Martin & Sunyaev, 2007)

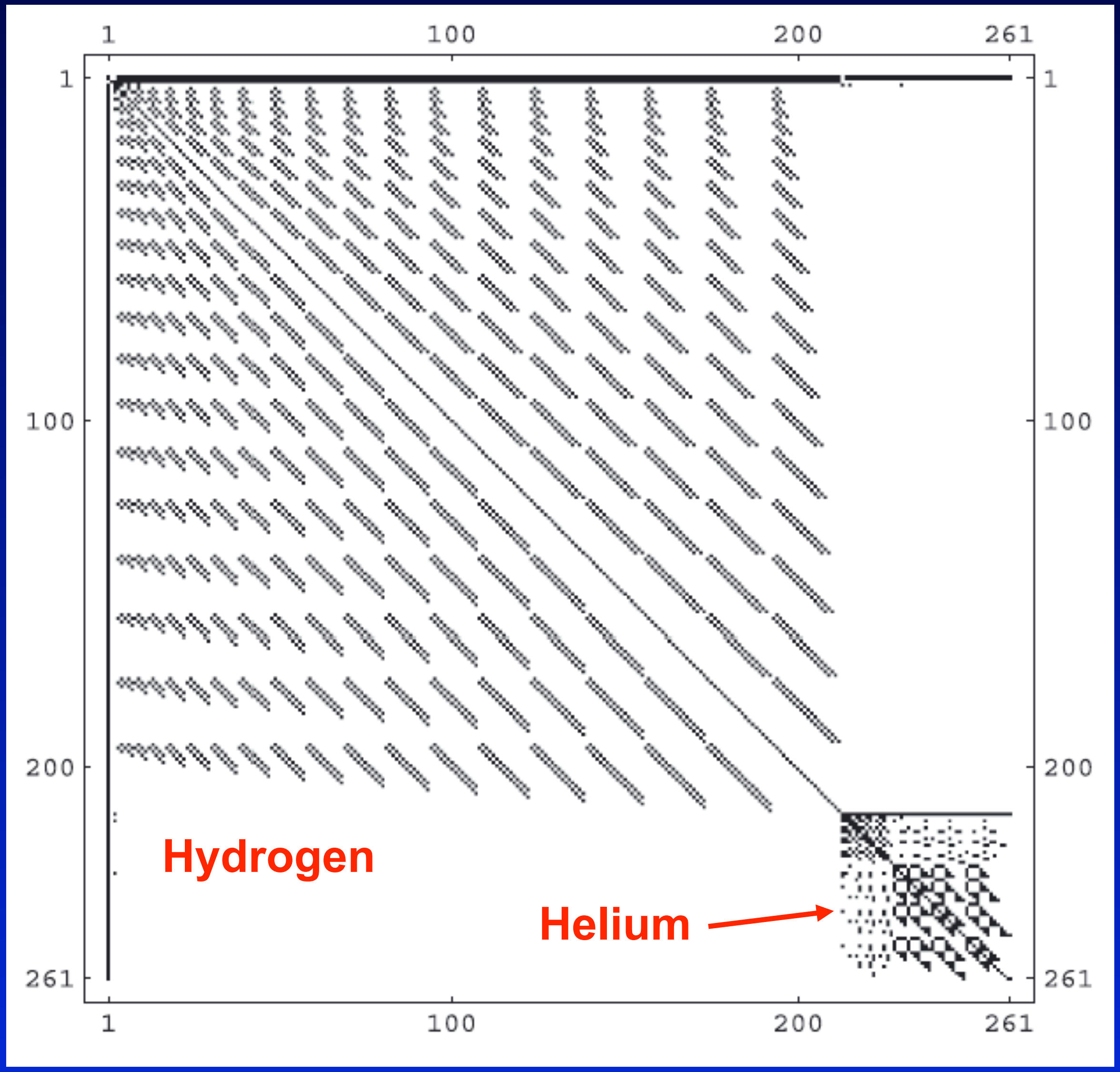
- need to treat angular momentum sub-levels separately!
- include collision to understand how close things are to SE
- Complexity of problem scales like ~ n^2_{max}
- But problem very *sparse* (Grin & Hirata, 2010; JC, Vasil & Dursi, 2010)

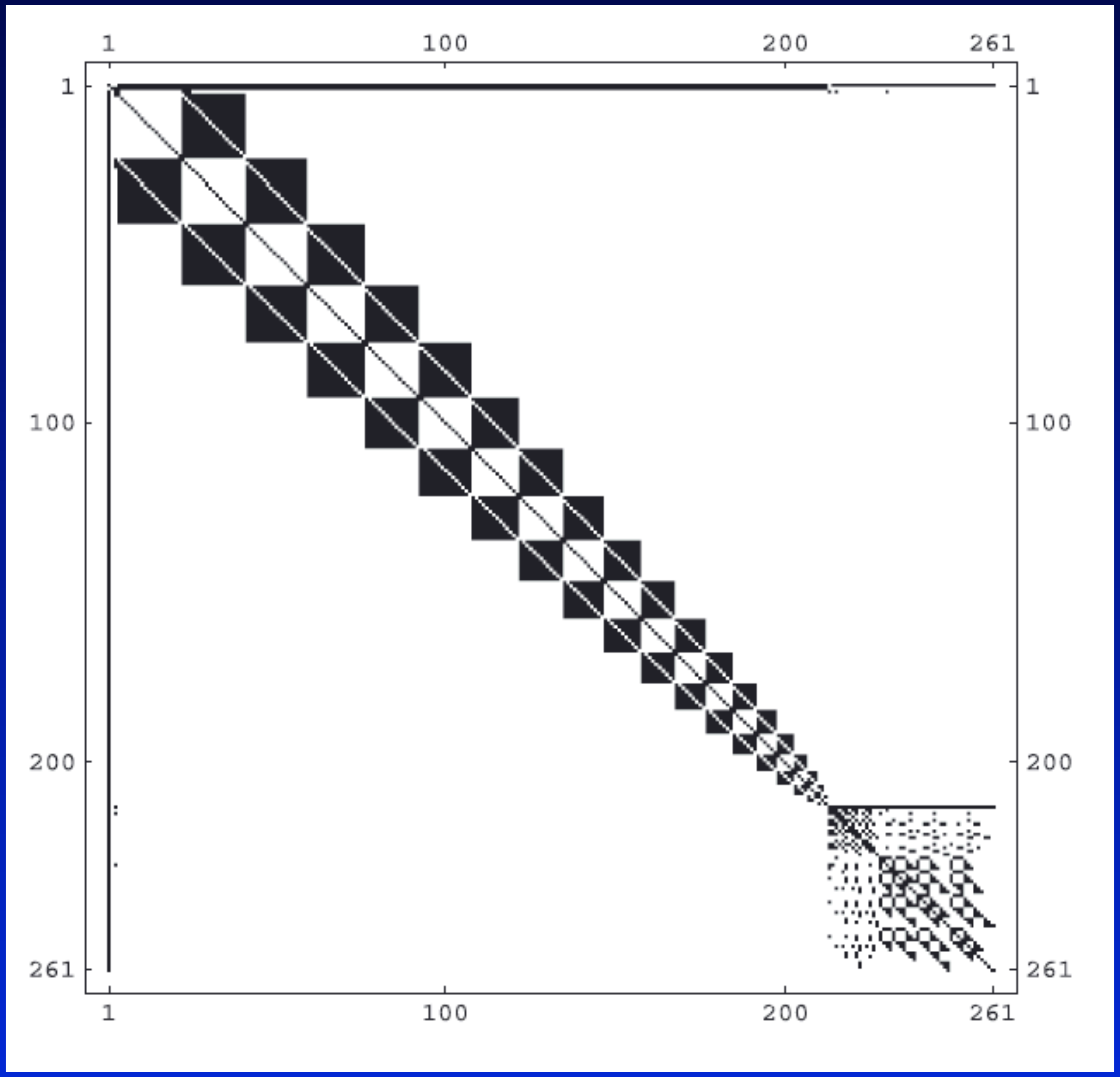


JC, Vasil & Dursi, MNRAS, 2010

Sparsity of the problem and effect of ordering

20 shell Hydrogen + 5 shell Helium model





Shell-by-Shell ordering

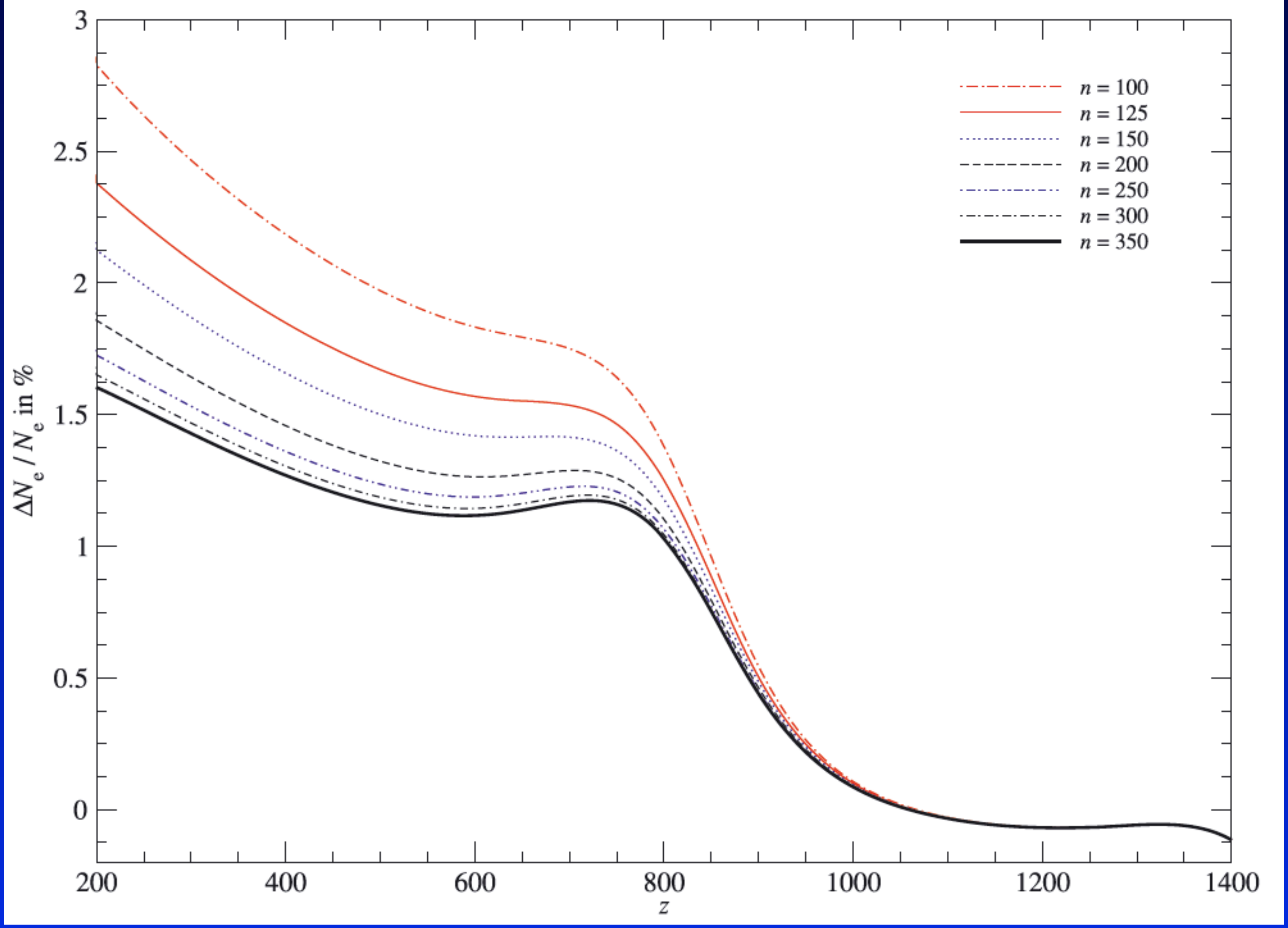
$$1s, 2s, 2p, 3s, 3p, 3d, ...$$

Angular momentum ordering

$$1s, 2s, 3s, \ldots, ns, 2s, 3p, \ldots, np, 3d, 4d, ...$$

Grin & Hirata, 2010
JC, Vasil & Dursi, MNRAS, 2010

Collisions during hydrogen recombination



- **effective recombination cross section of the atom matters most at low *z***

JC, Vasil & Dursi, MNRAS, 2010

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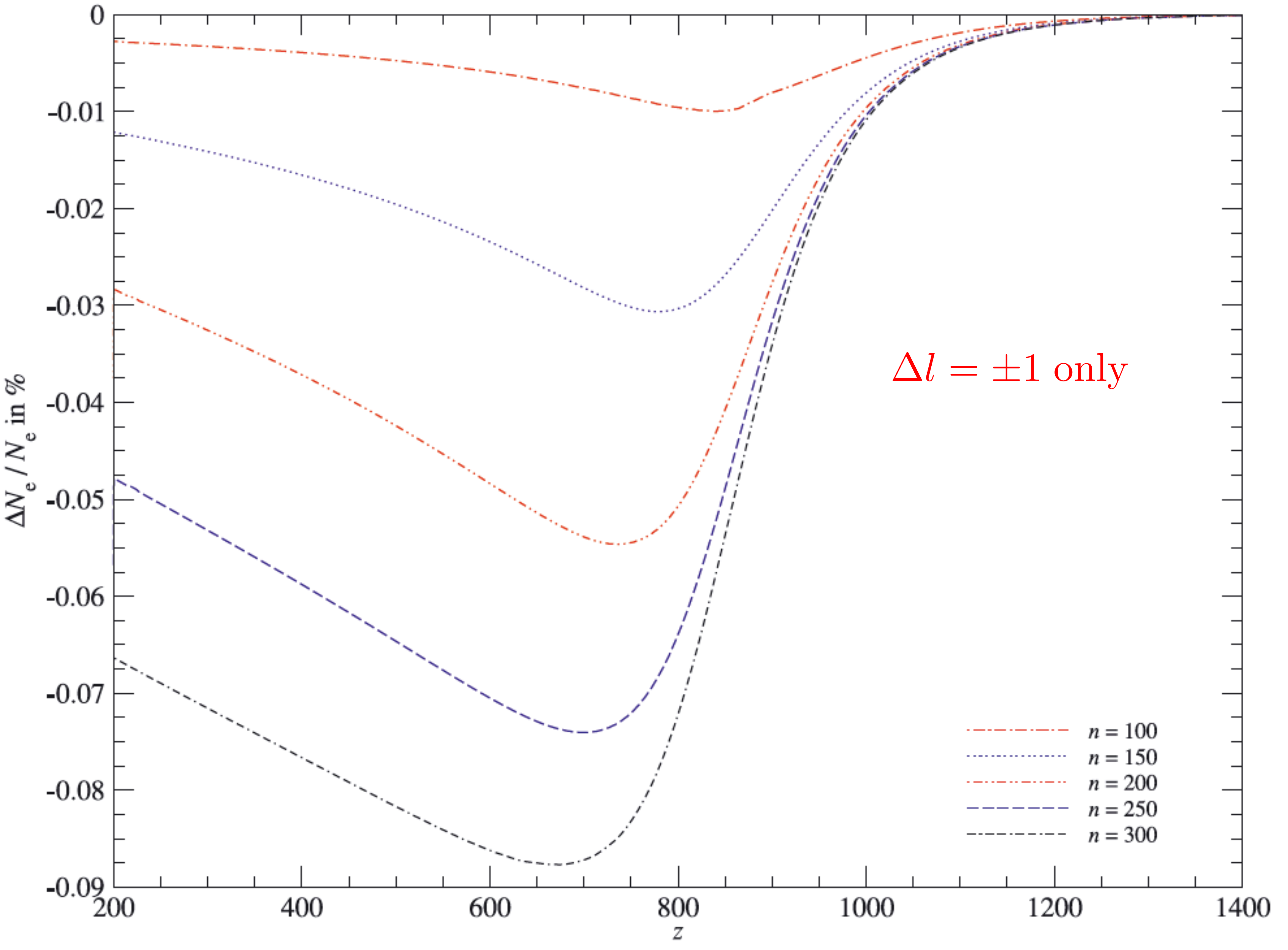

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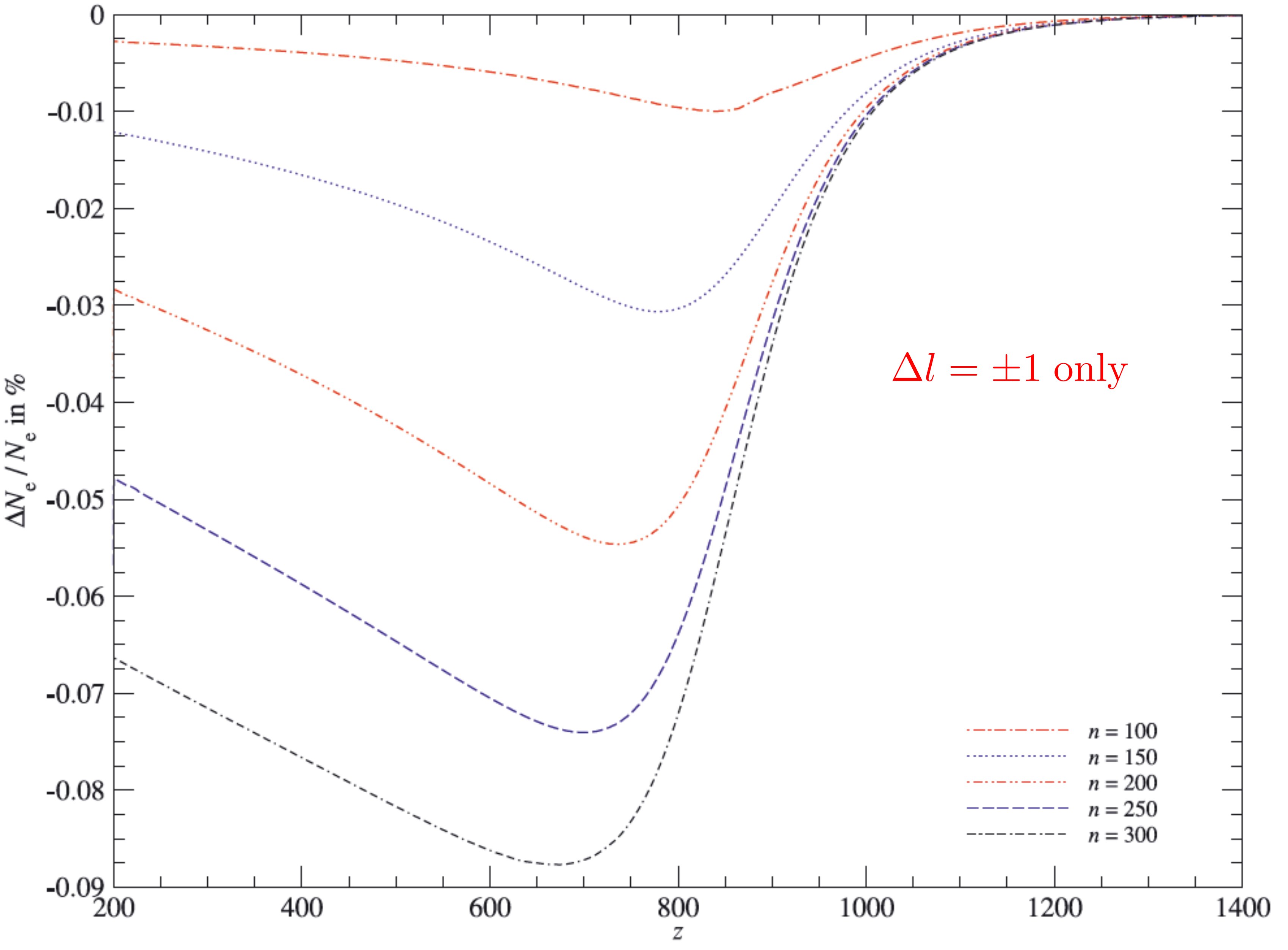

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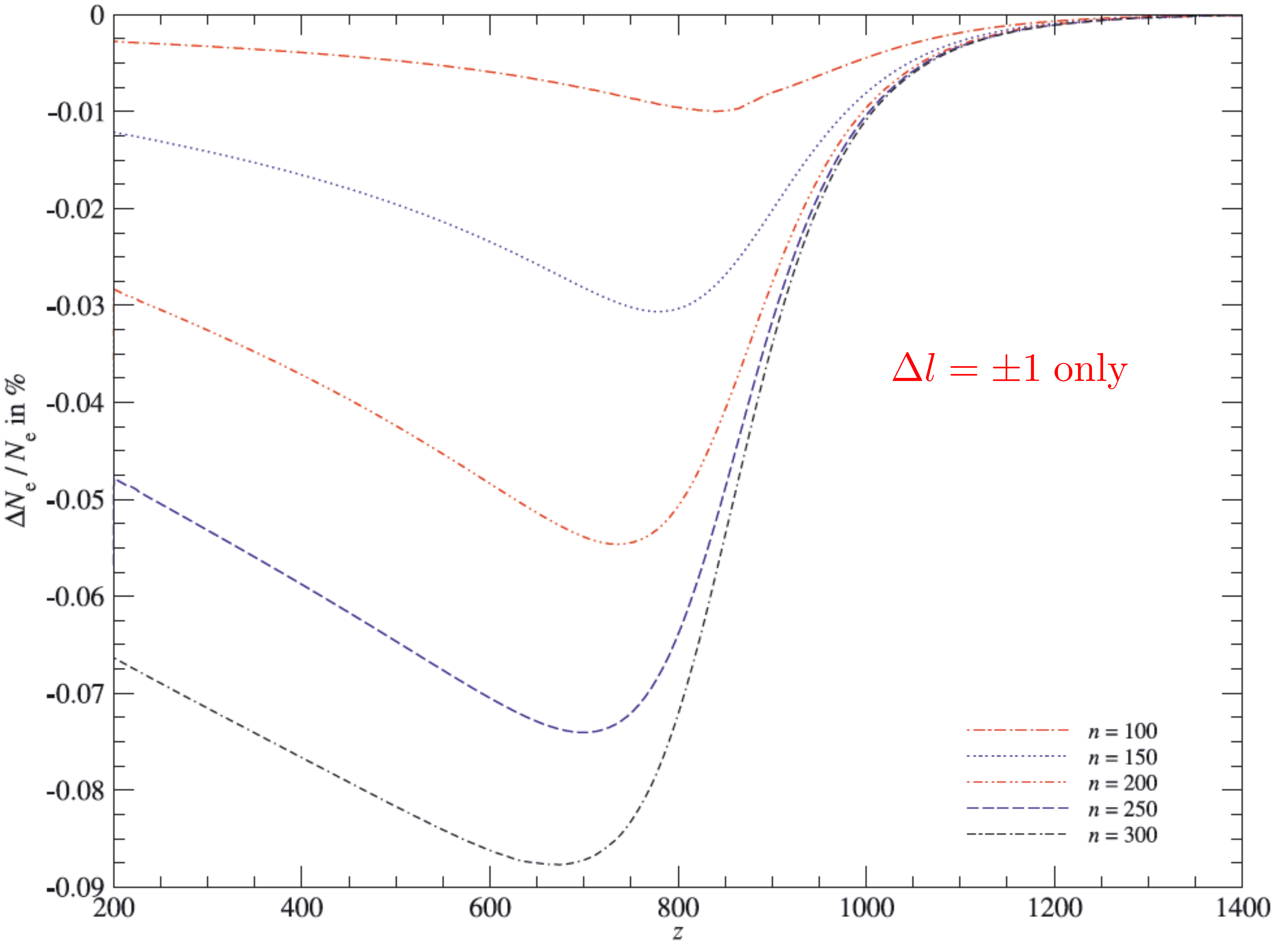

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CITA ICAT JC, Vasil & Dursi, MNRAS, 2010

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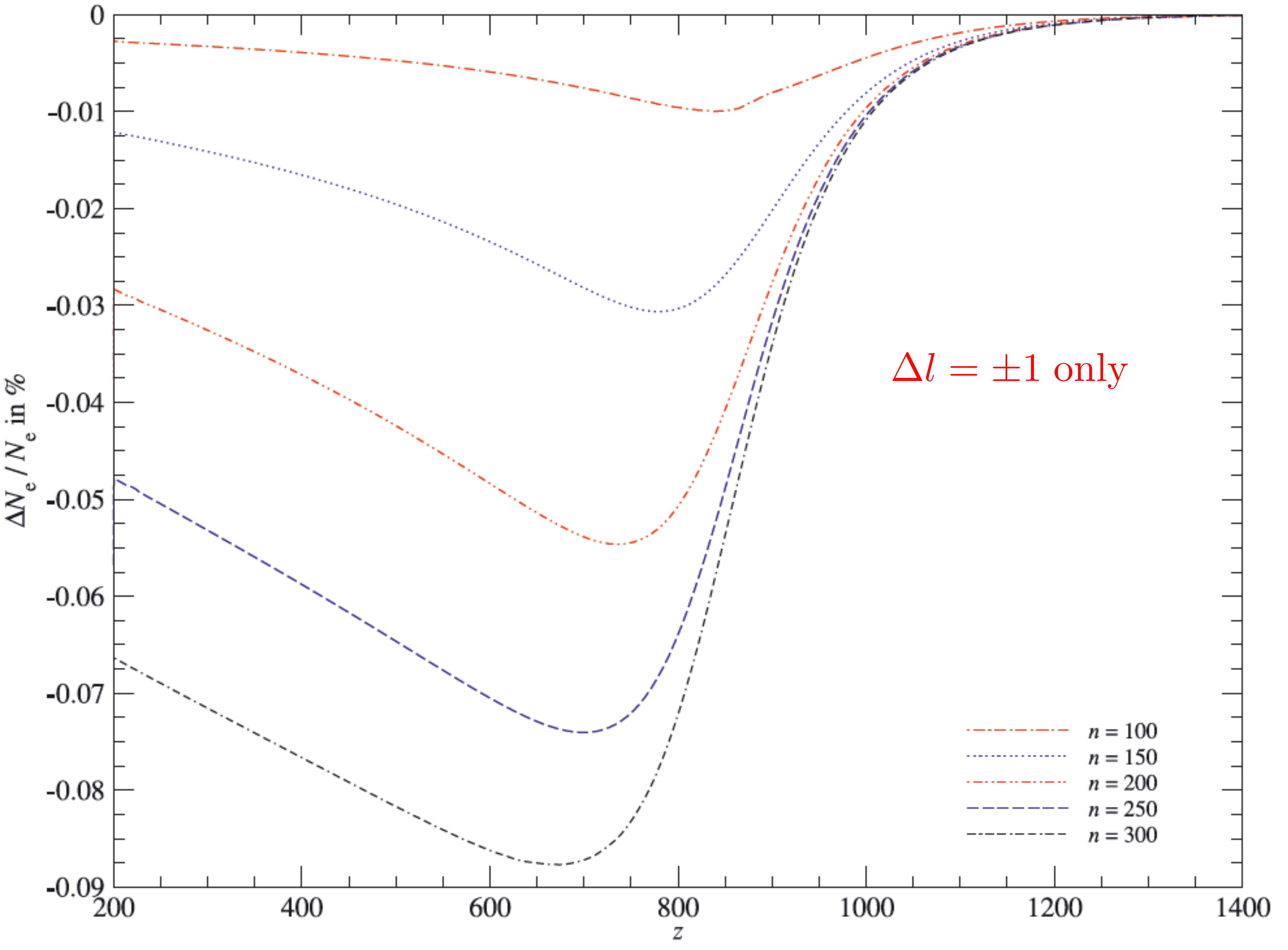

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CITA ICAT Canadian Institute for Theoretical Astrophysics L'institut canadien d'astrophysique théorique

JC, Vasil & Dursi, MNRAS, 2010

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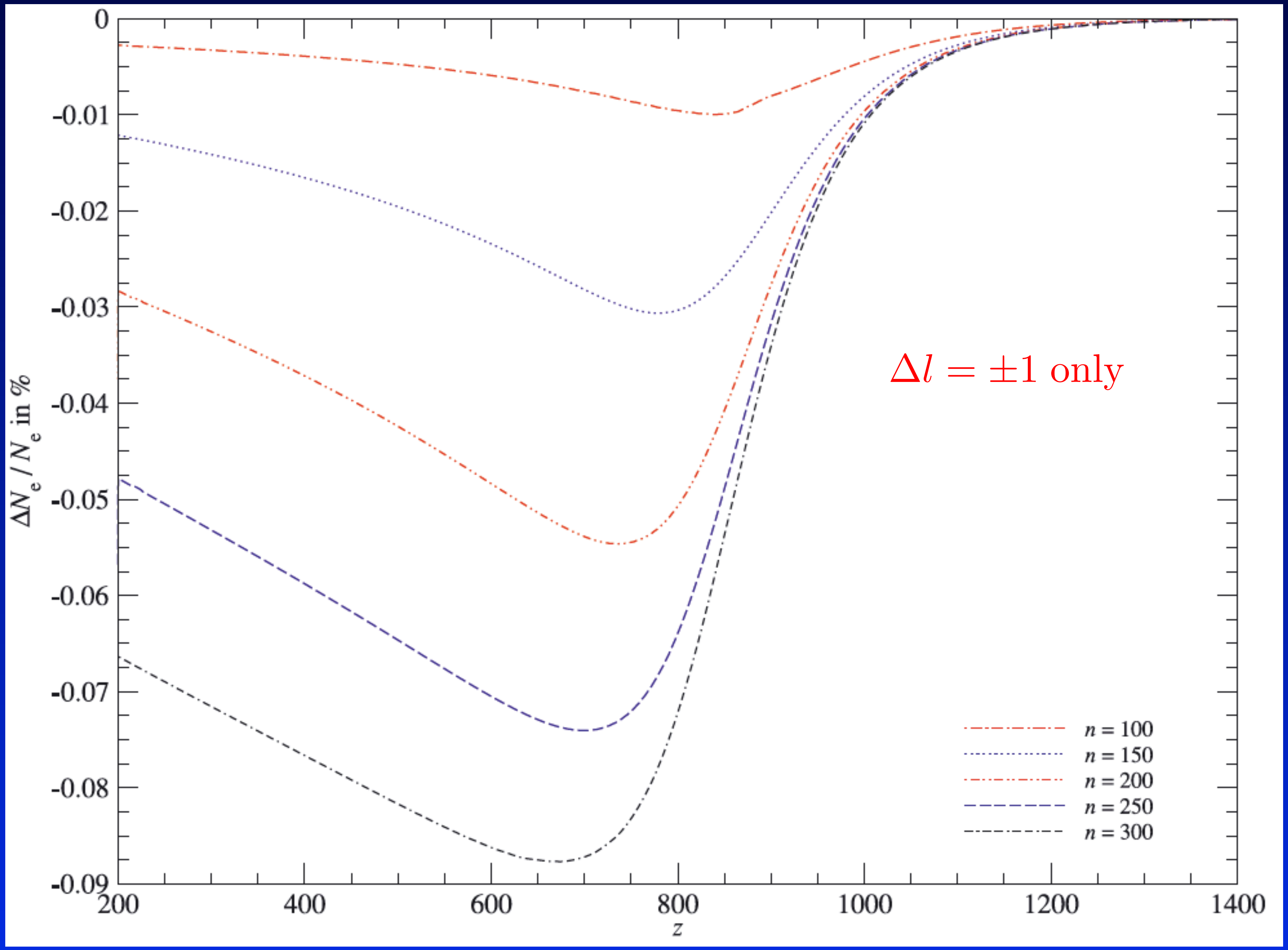

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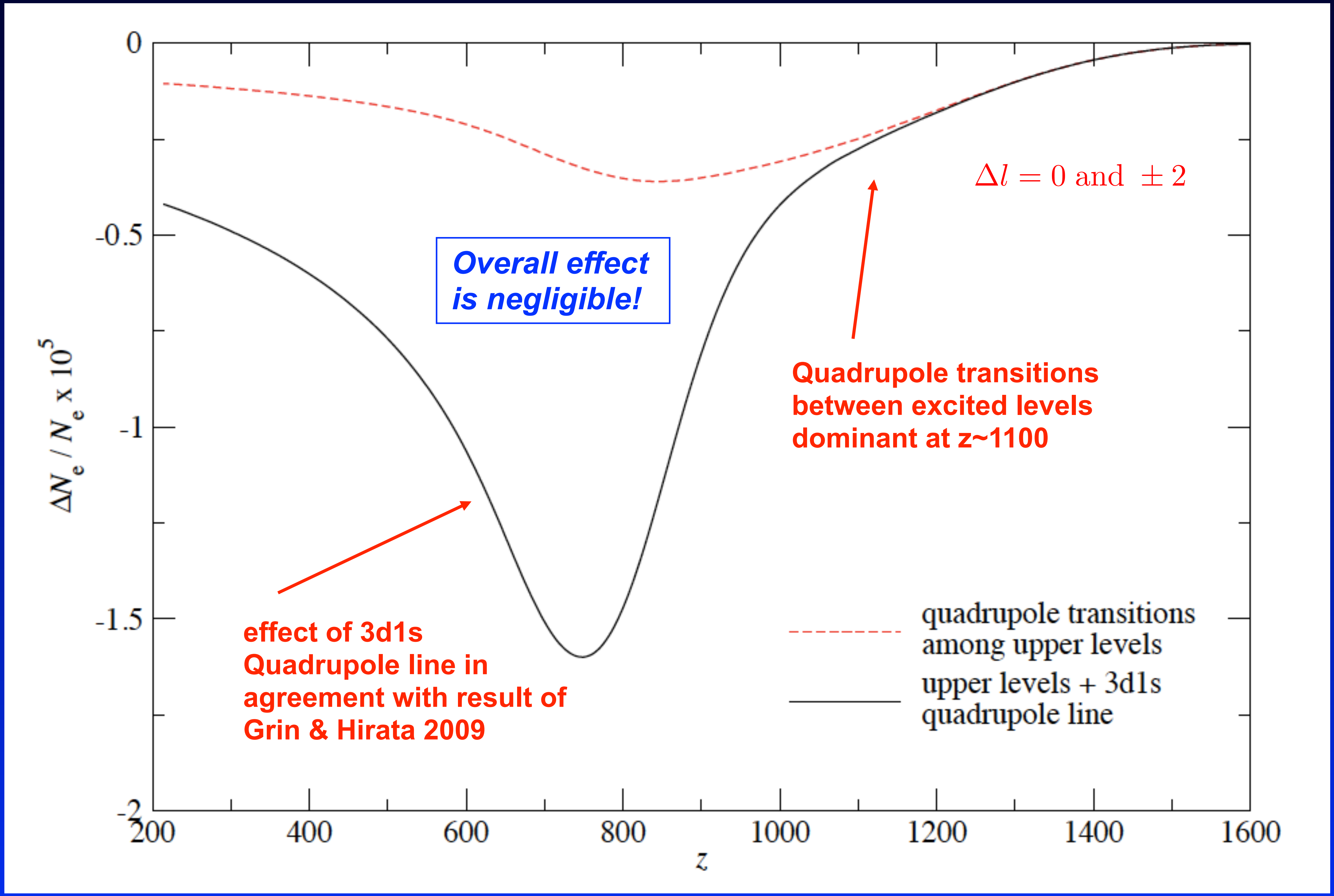
- **this should be checked, even if the final result may not dramatically change things**
- *updated rates (with large Δl) available!*

JC, Vasil & Dursi, MNRAS, 2010

Quadrupole lines during hydrogen recombination



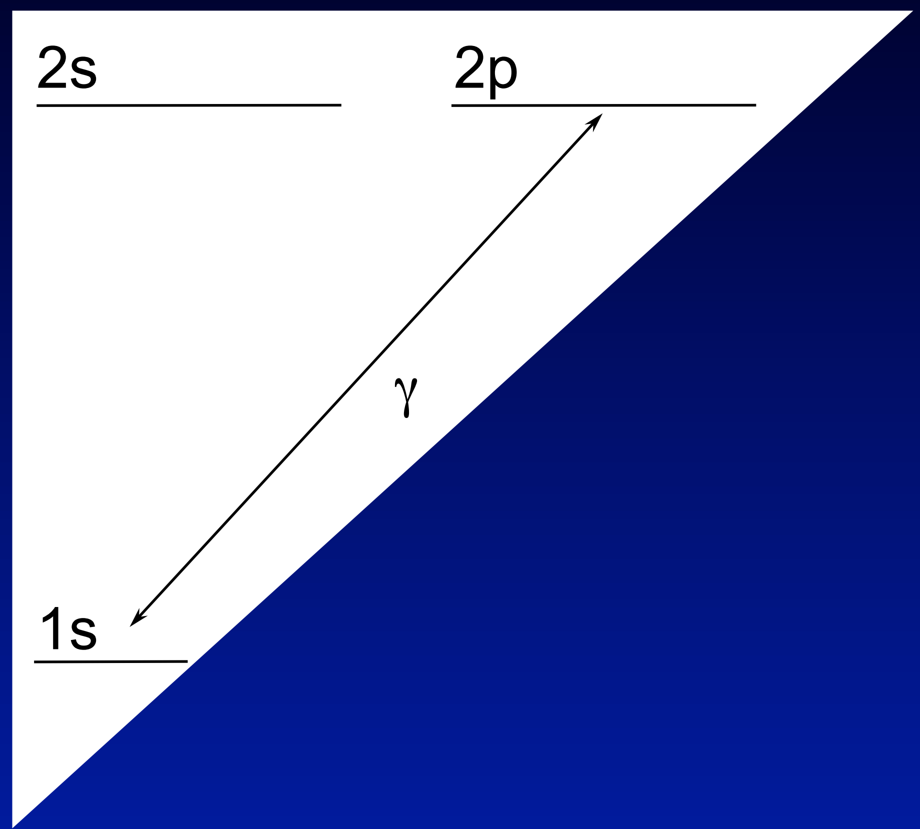
JC, Fung & Switzer, 2011

Two-photon transitions from the upper levels and the Lyman-α escape problem

Sobolev approximation

(developed in late 50‘s to model moving envelopes of stars)

Hydrogen atom



- To solve the coupled system of rate-equations

  → need to know mean intensity across the Ly-α (& Ly-n) resonance at different times

  → solution by introducing the *escape probability*

  → *Escape* == *photons stop interacting with Ly-α resonance*
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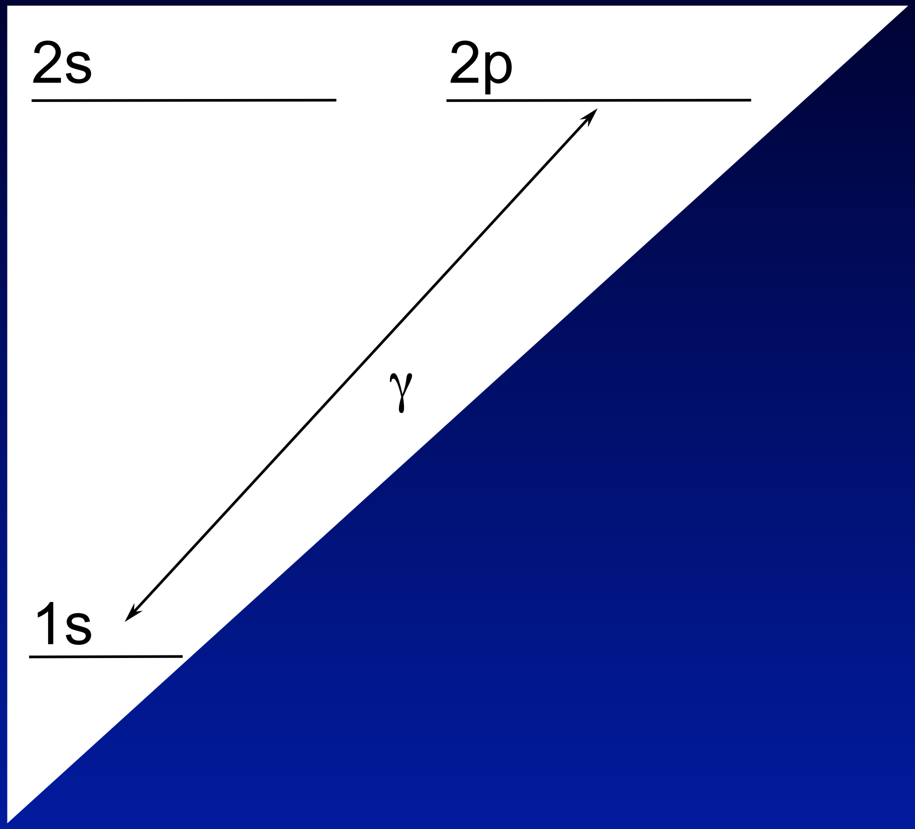

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  - populations of level + radiation field *quasi-stationary*
  - every 'scattering' leads to *complete redistribution*
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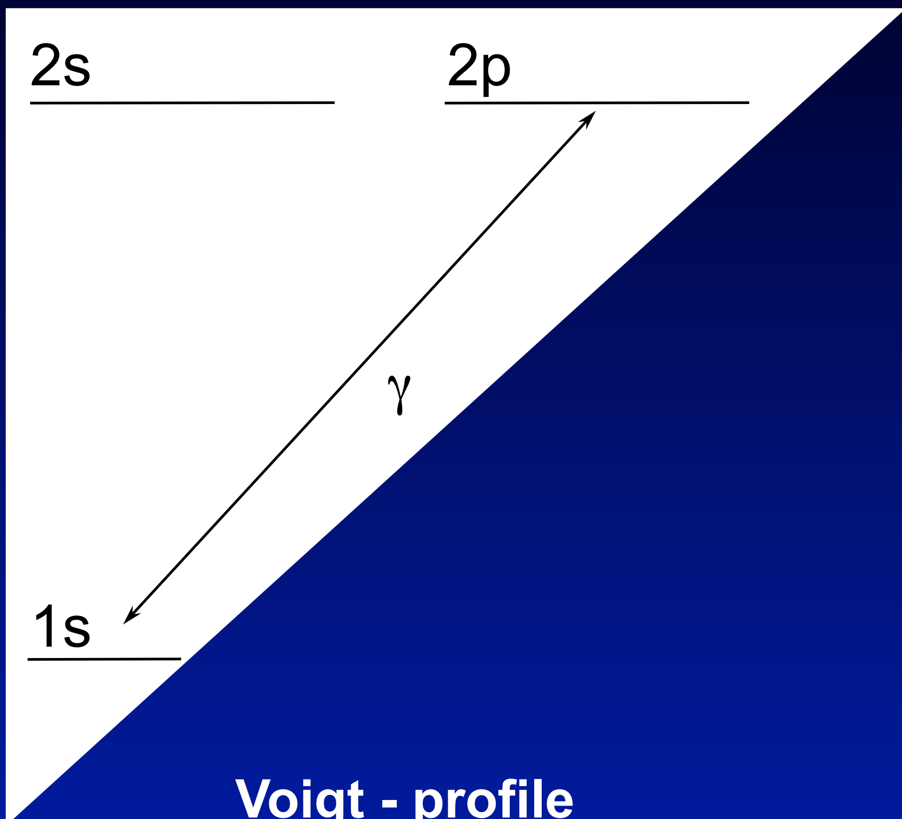
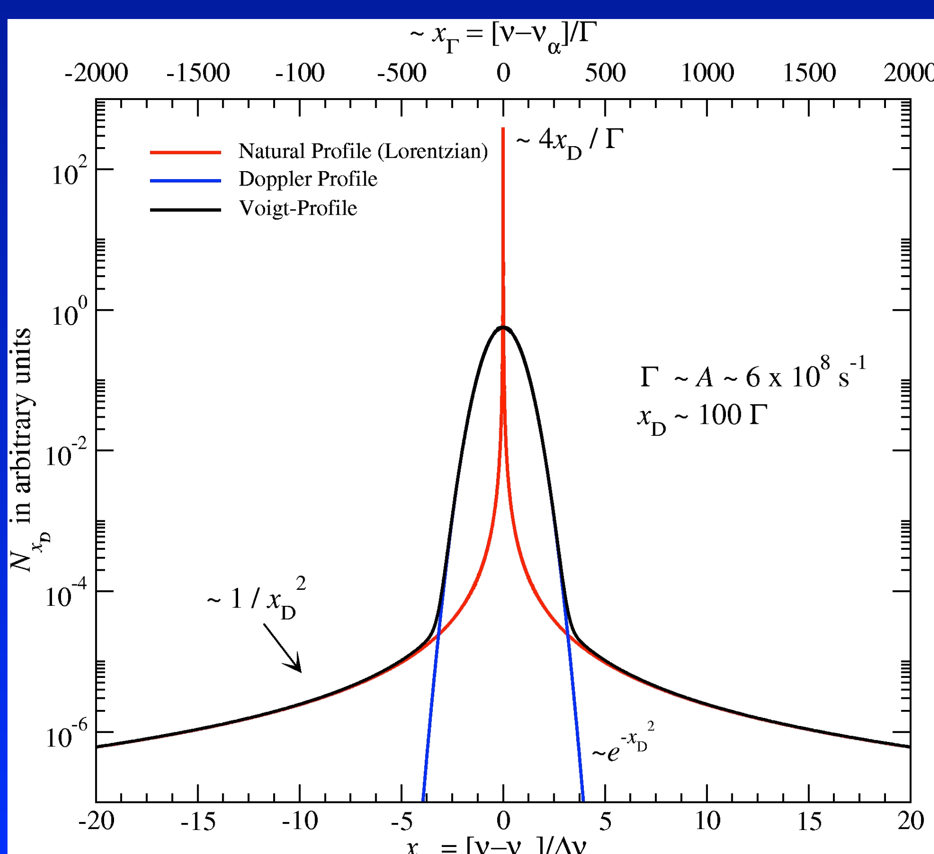


Voigt - profile



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Doppler width

$$\frac{\Delta\nu_D}{\nu} = \sqrt{\frac{2kT}{m_H c^2}} \simeq \text{few} \times 10^{-5}$$

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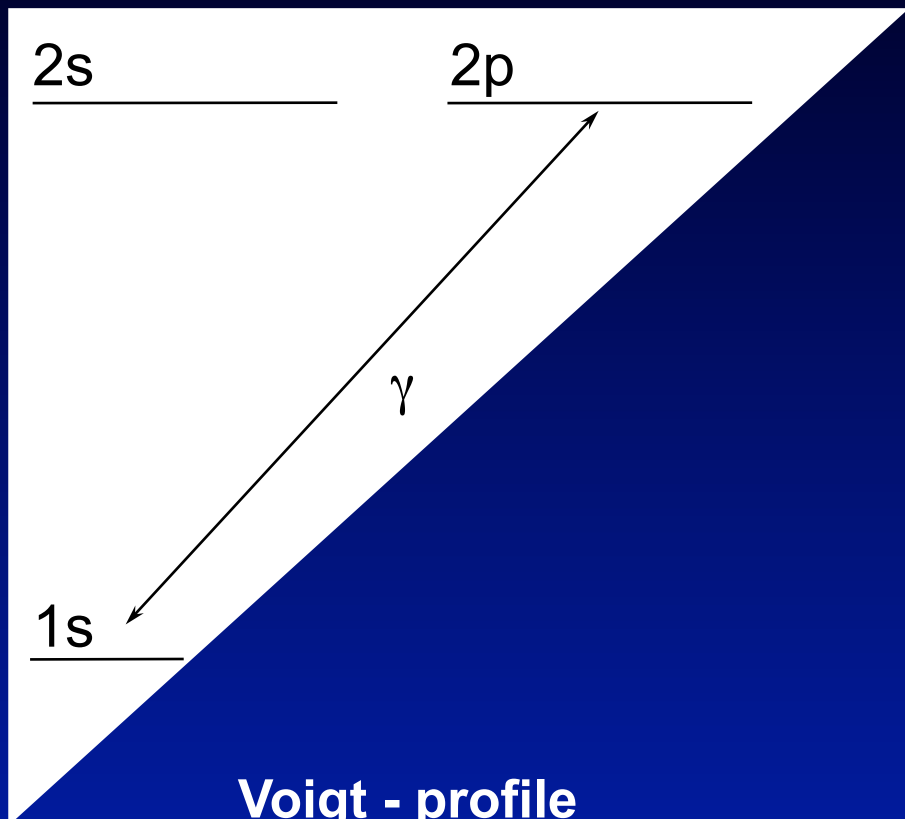

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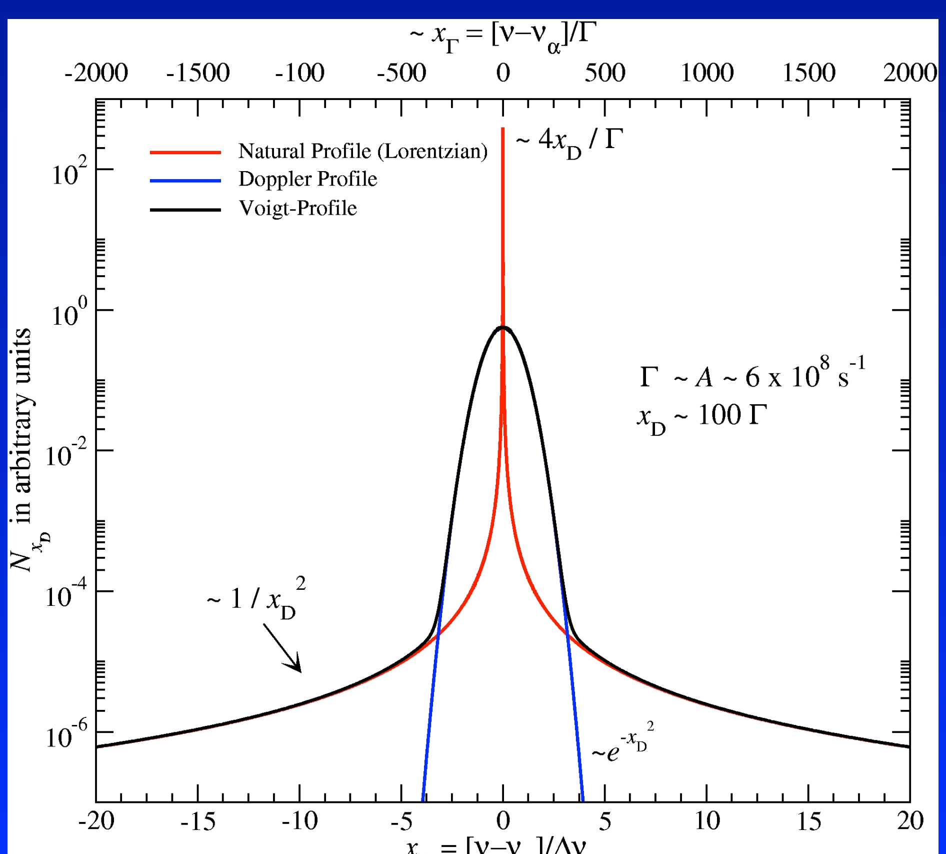
- Sobolev escape probability & optical depth

$$P_{\rm S} = \frac{1 - e^{-\tau_{\rm S}}}{\tau_{\rm S}} \simeq 10^{-8}$$

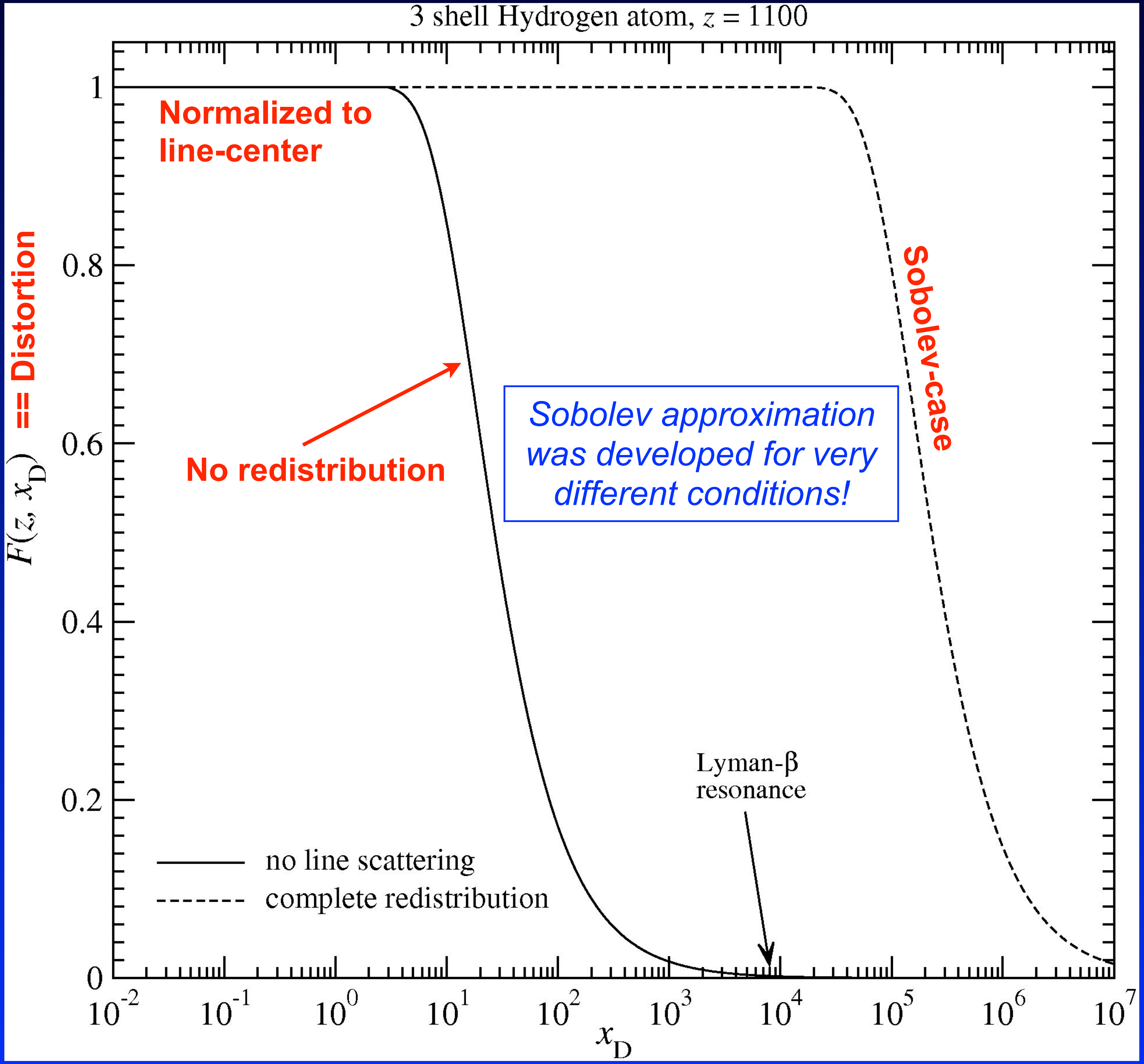
$$\tau_{\rm S} = \frac{c\,\sigma_{\rm r} N_{\rm 1s}}{H}\,\frac{\Delta\nu_{\rm D}}{\nu} = \frac{g_{\rm 2p}}{g_{\rm 1s}}\,\frac{A_{21}\lambda_{21}^3}{8\pi H}\,N_{\rm 1s}$$

Voigt - profile



Problems with Sobolev approximation:

Complete redistribution $\Longleftrightarrow$ partial redistribution



Sobolev-approximation:

- Important variation of the photon distribution at ~1.5 times the ionization energy!
- For 1% accuracy one has to integrate up to ~10^7 Doppler width!
- *Complete redistribution bad approximation and very unlikely* (p~10^{-4}-10^{-3})

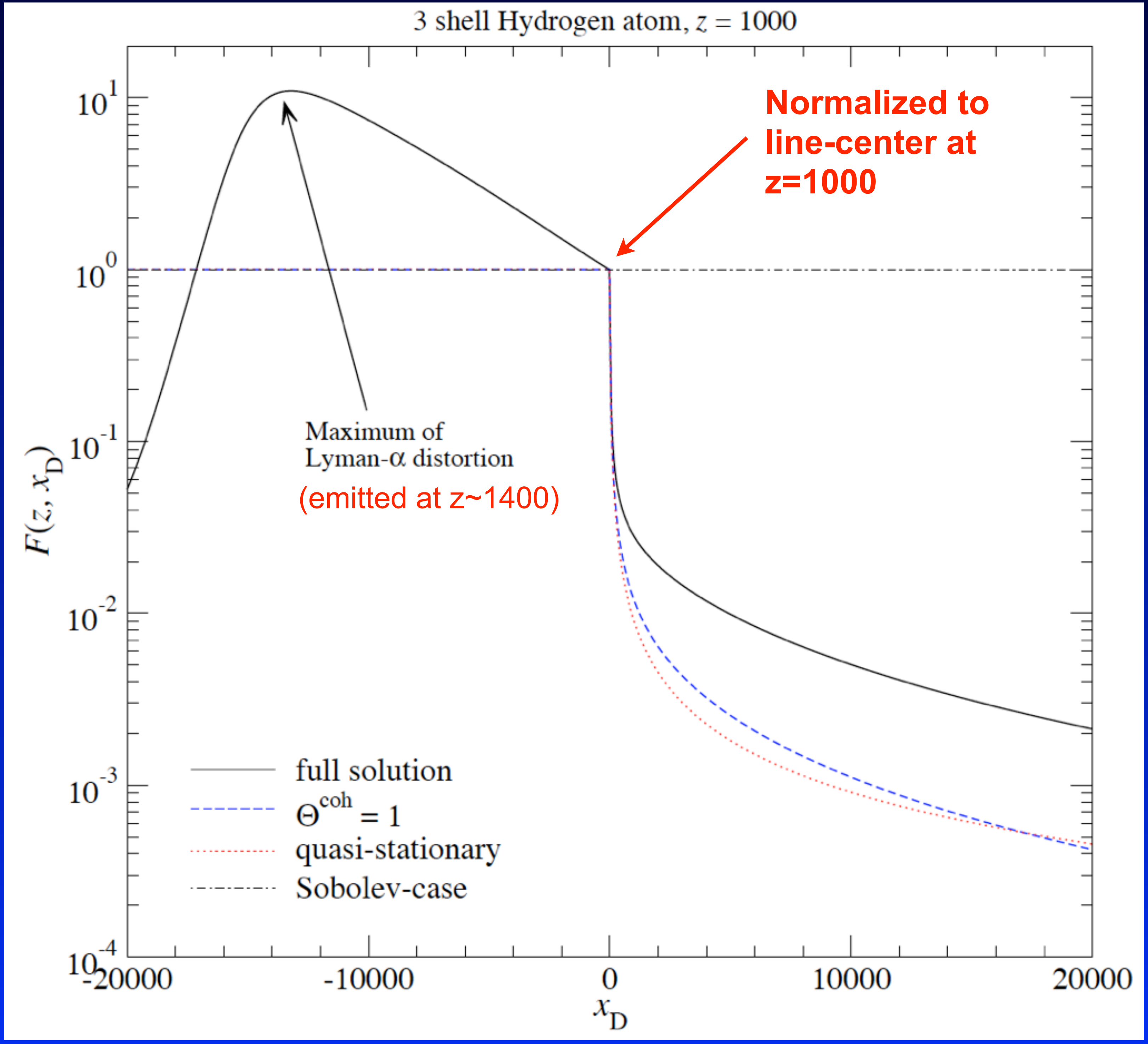
No redistribution case:

- Much closer to the correct solution (*partial redistribution*)
- Avoids some of the unphysical aspect

JC & Sunyaev, 2009

Problems with Sobolev approximation:

Time dependence of radiation field



- Evolution close to line center is indeed quasi-stationary
- non-stationarity important in the wings

  $\Longrightarrow$ *information* takes time to travel from line center to the wings
- For support of 2p level even spectrum up to $|x_D| \sim 10^4$ is important

  $\Longrightarrow$ *time dependence* has to be included

JC & Sunyaev, 2009

Problems with Sobolev approximation:

Difference between emission and absorption profile

- Standard textbook:

Normalized Ly-α profile $\int \phi(\nu)\,\mathrm{d}\nu\,\mathrm{d}\Omega = 1$

$$\frac{1}{c}\frac{\mathrm{d}N_\nu}{\mathrm{d}t}\bigg|_{\mathrm{Ly-}\alpha} = A_{21}\phi(\nu)\left[N_{\mathrm{2p}}(1+n_\nu) - \frac{g_{\mathrm{2p}}}{g_{\mathrm{1s}}}N_{\mathrm{1s}}n_\nu\right]$$

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Only fulfilled at line center!

Detailed balance not guaranteed in the line wings!

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Asymmetry of emission and absorption profile

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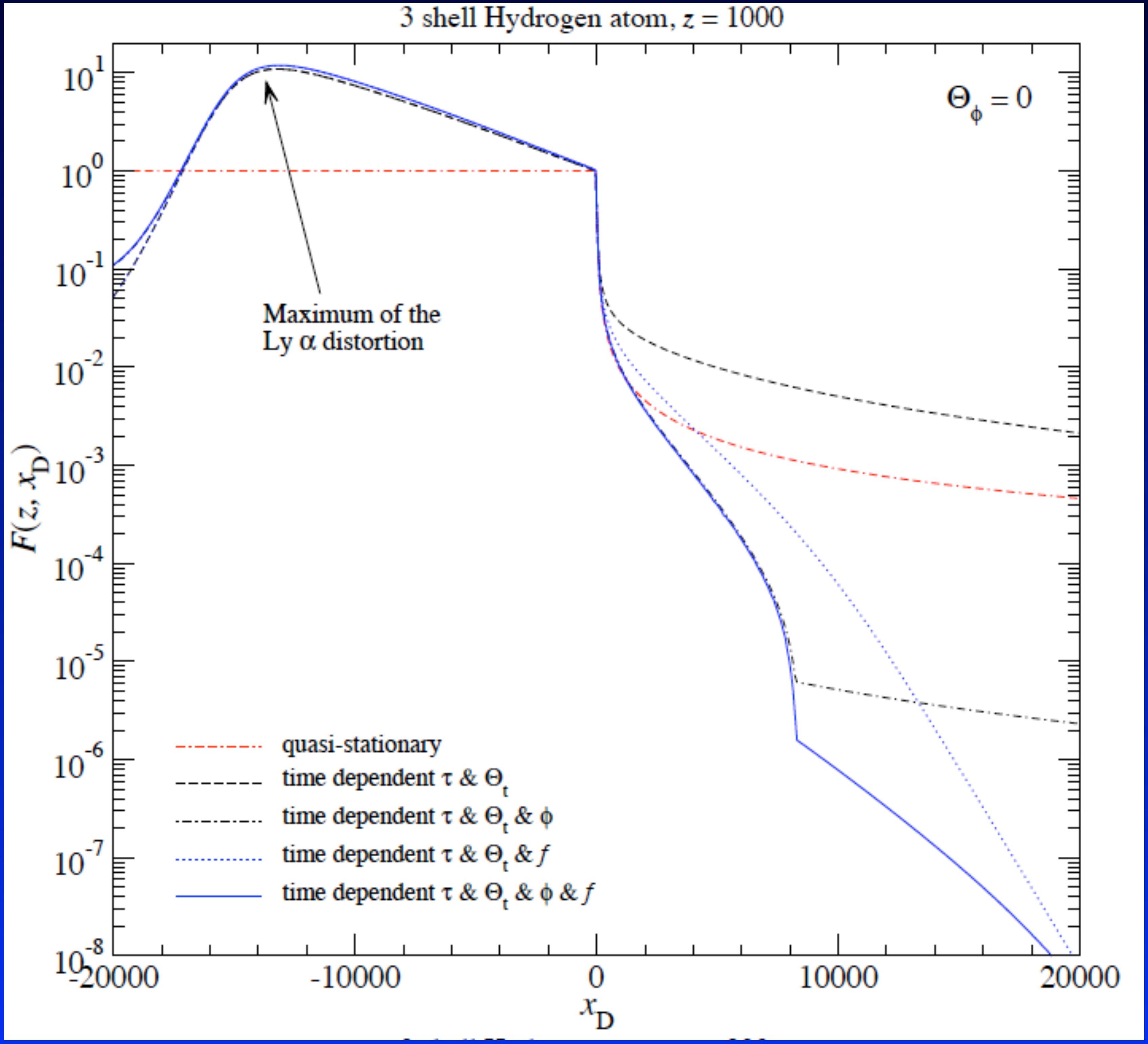
Asymmetry of emission and absorption profile

- Naturally comes out of 2γ treatment (JC & Sunyaev 2009)

Problems with Sobolev approximation:

Difference between emission and absorption profile



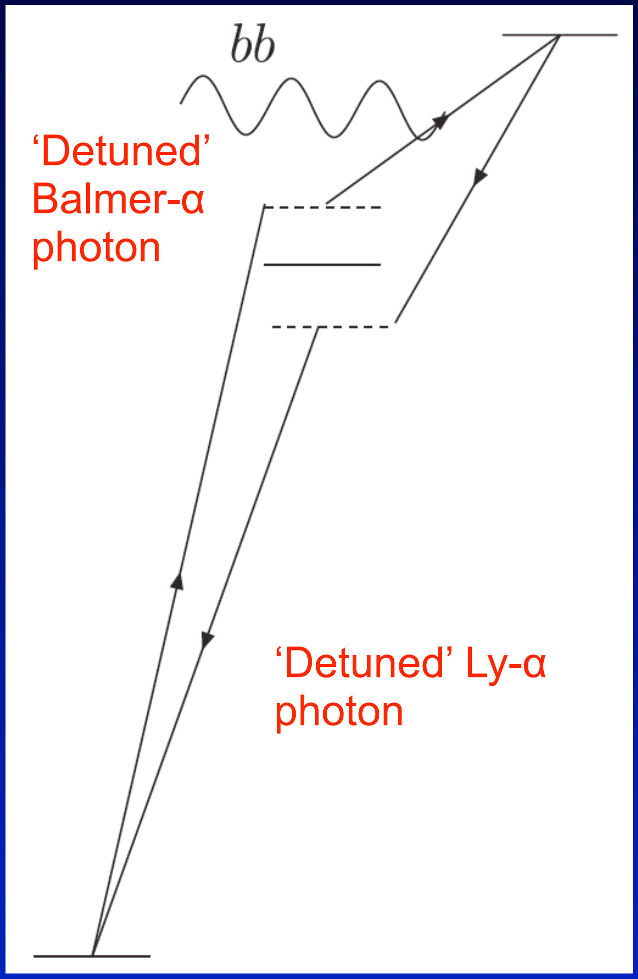


Illustration from Switzer & Hirata 2007 (meant for Helium)

- Real absorption & emission requires a second photon!

JC & Sunyaev, 2009

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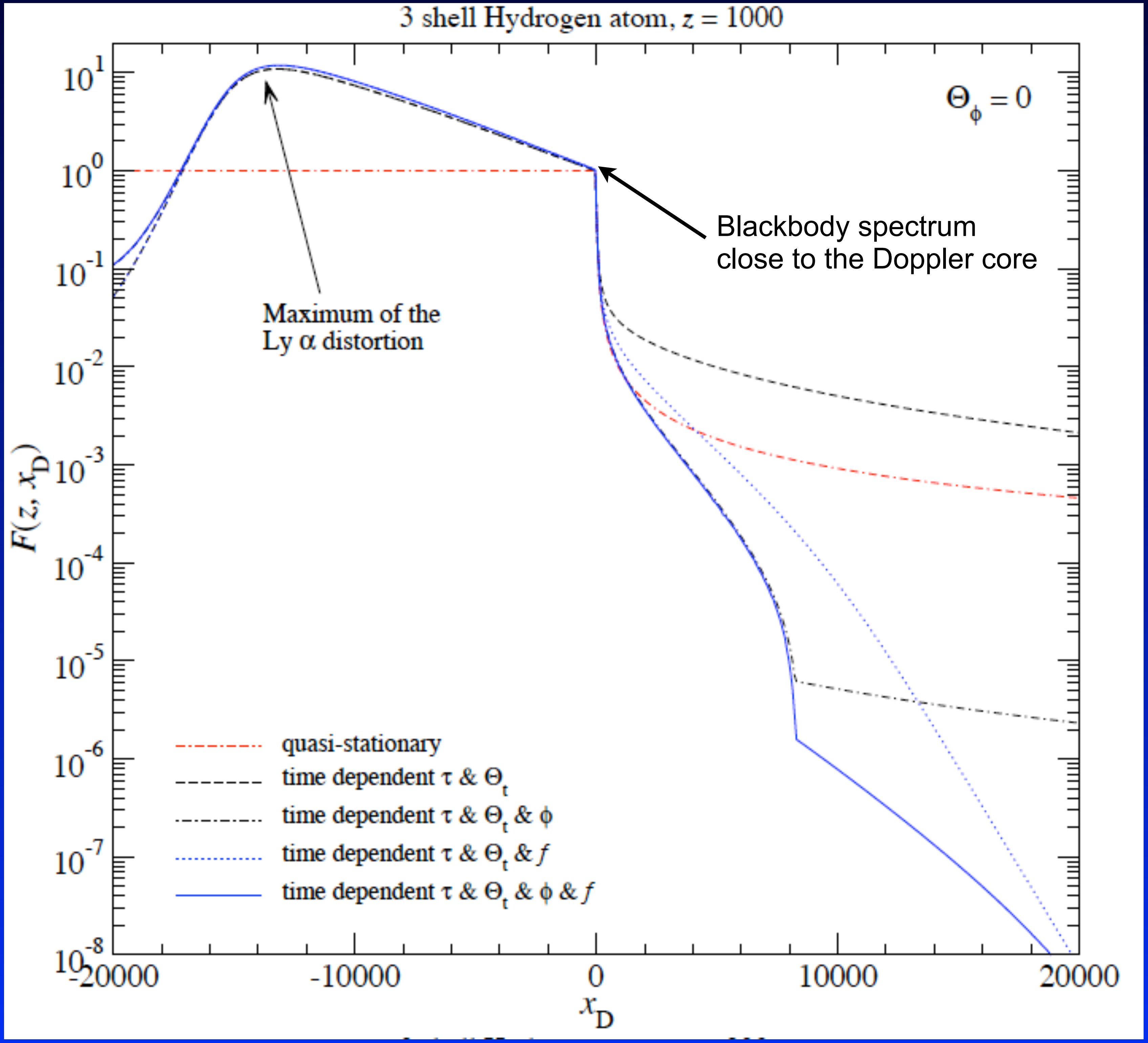

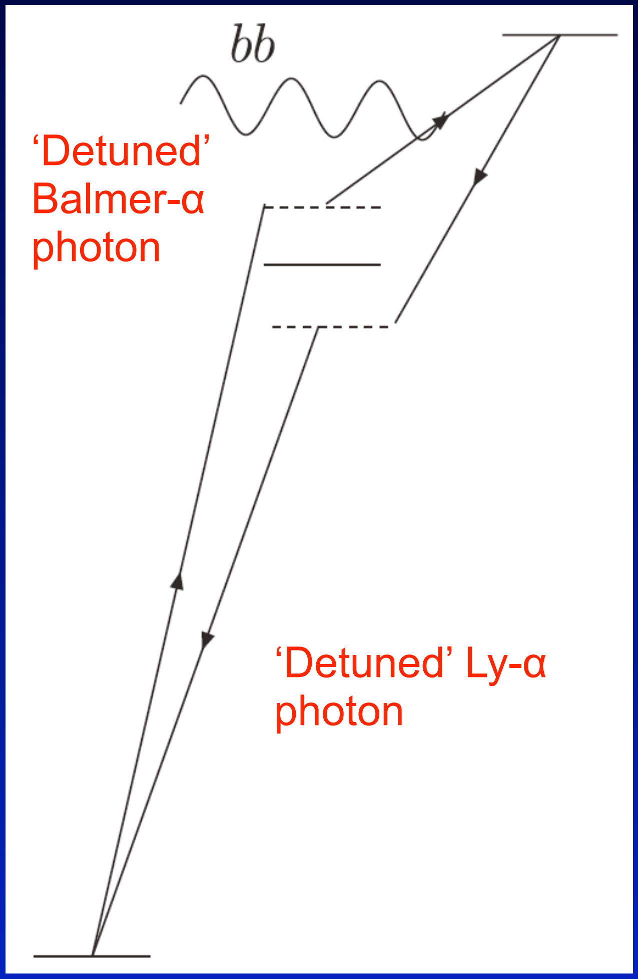

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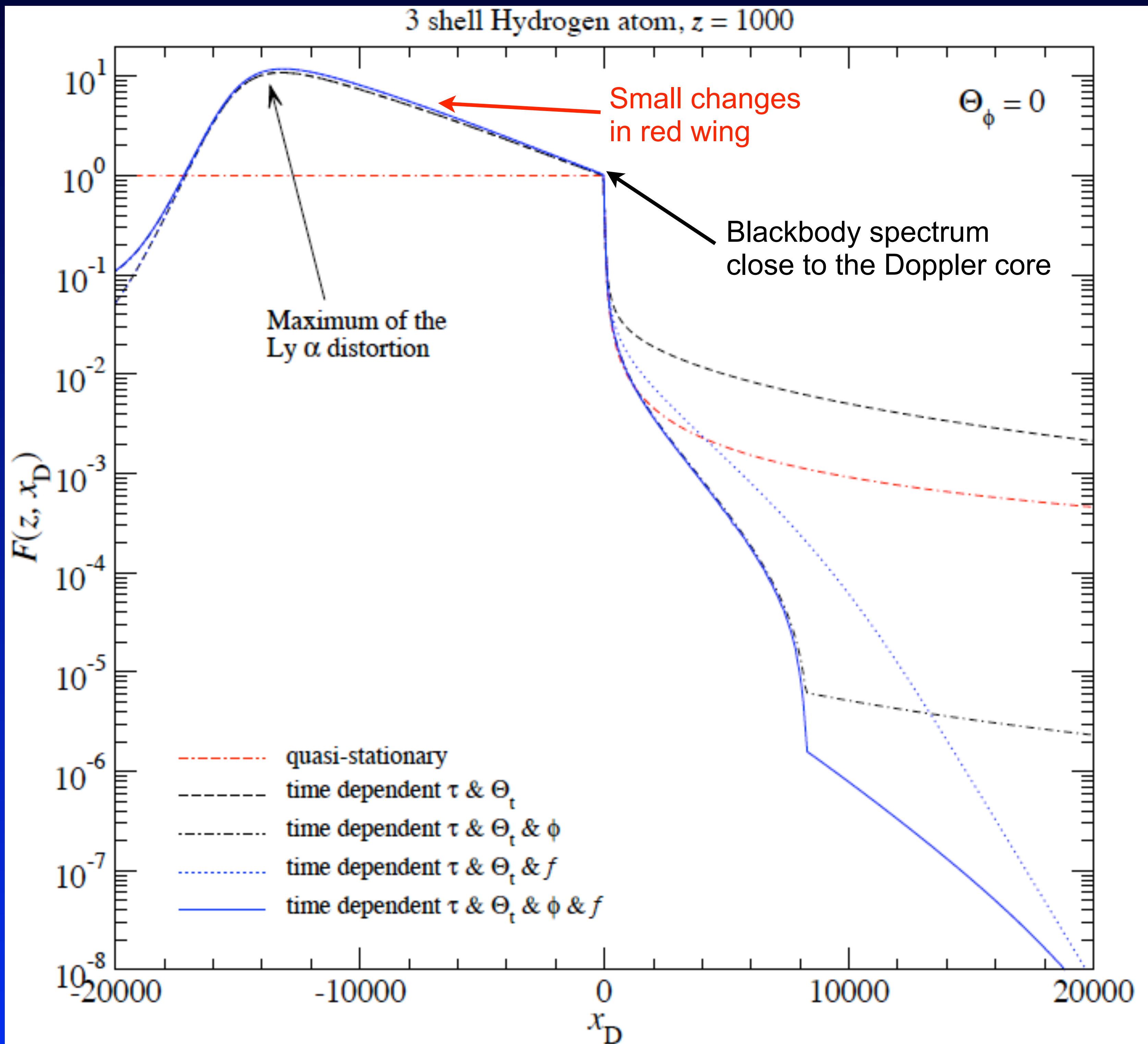

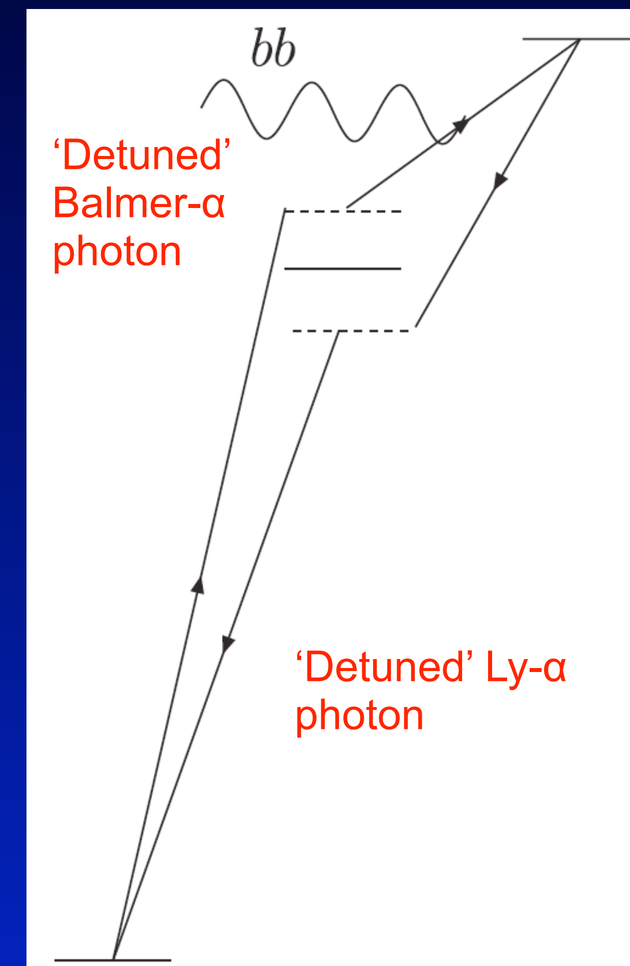

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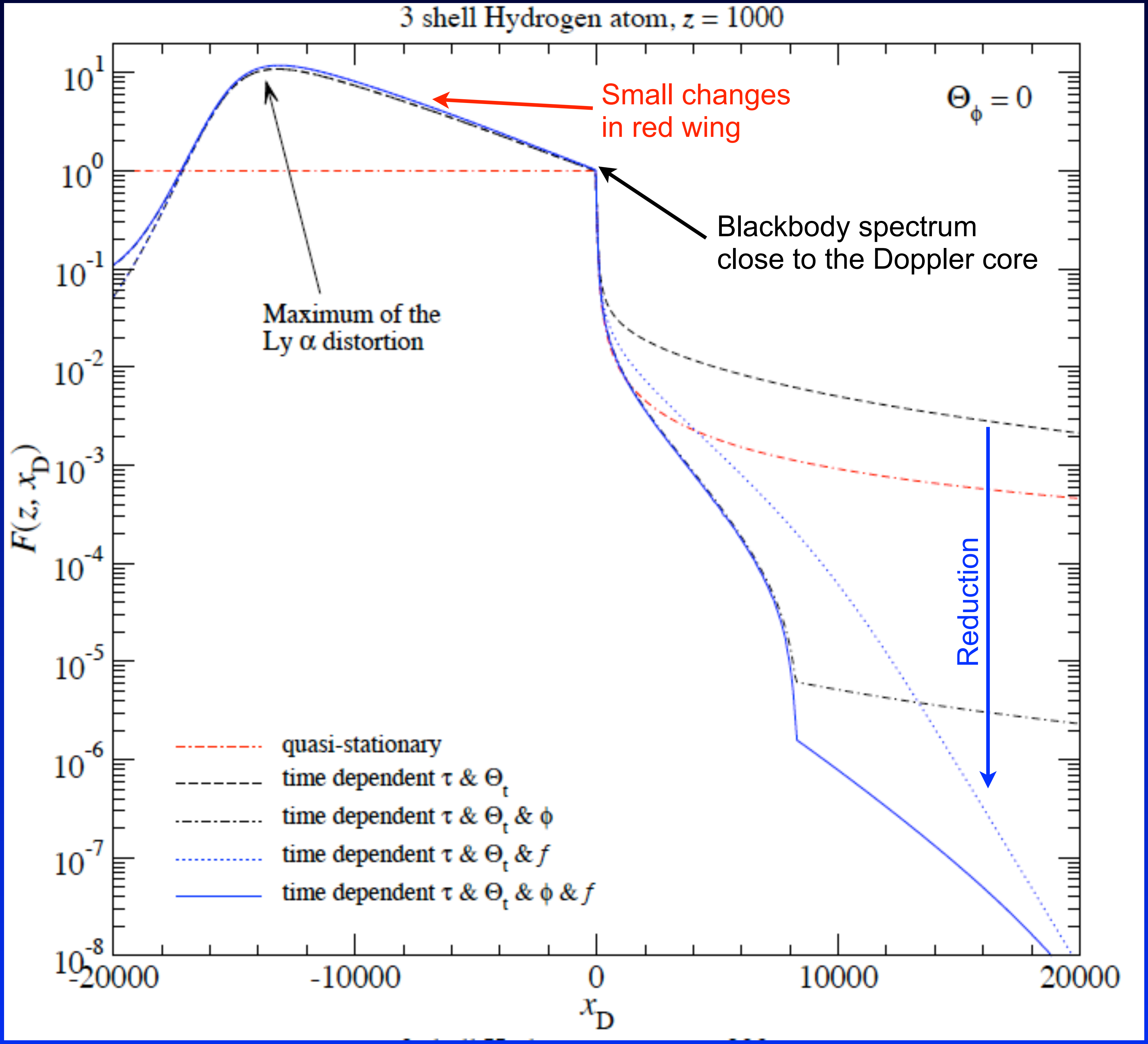

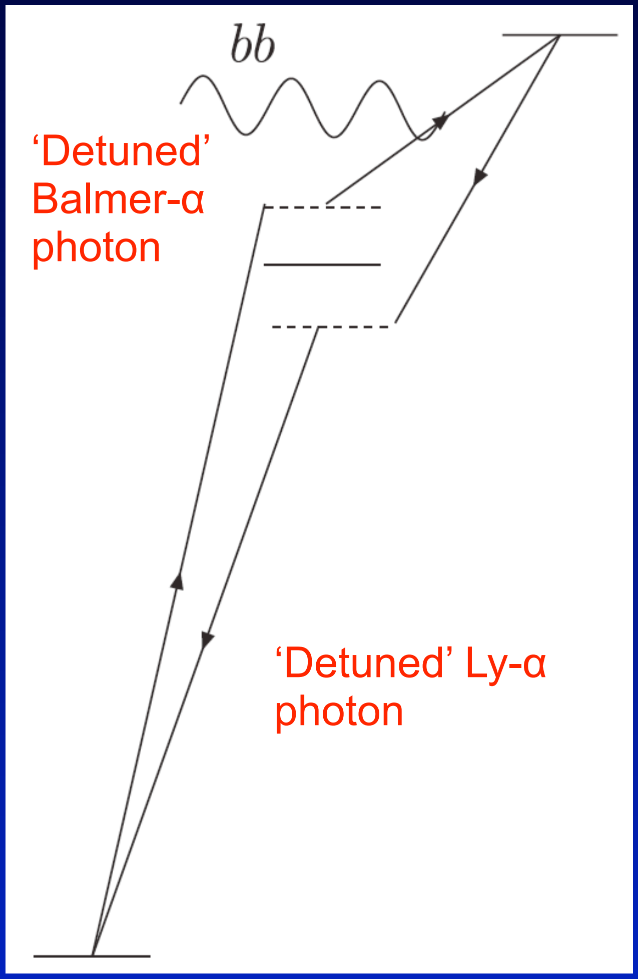

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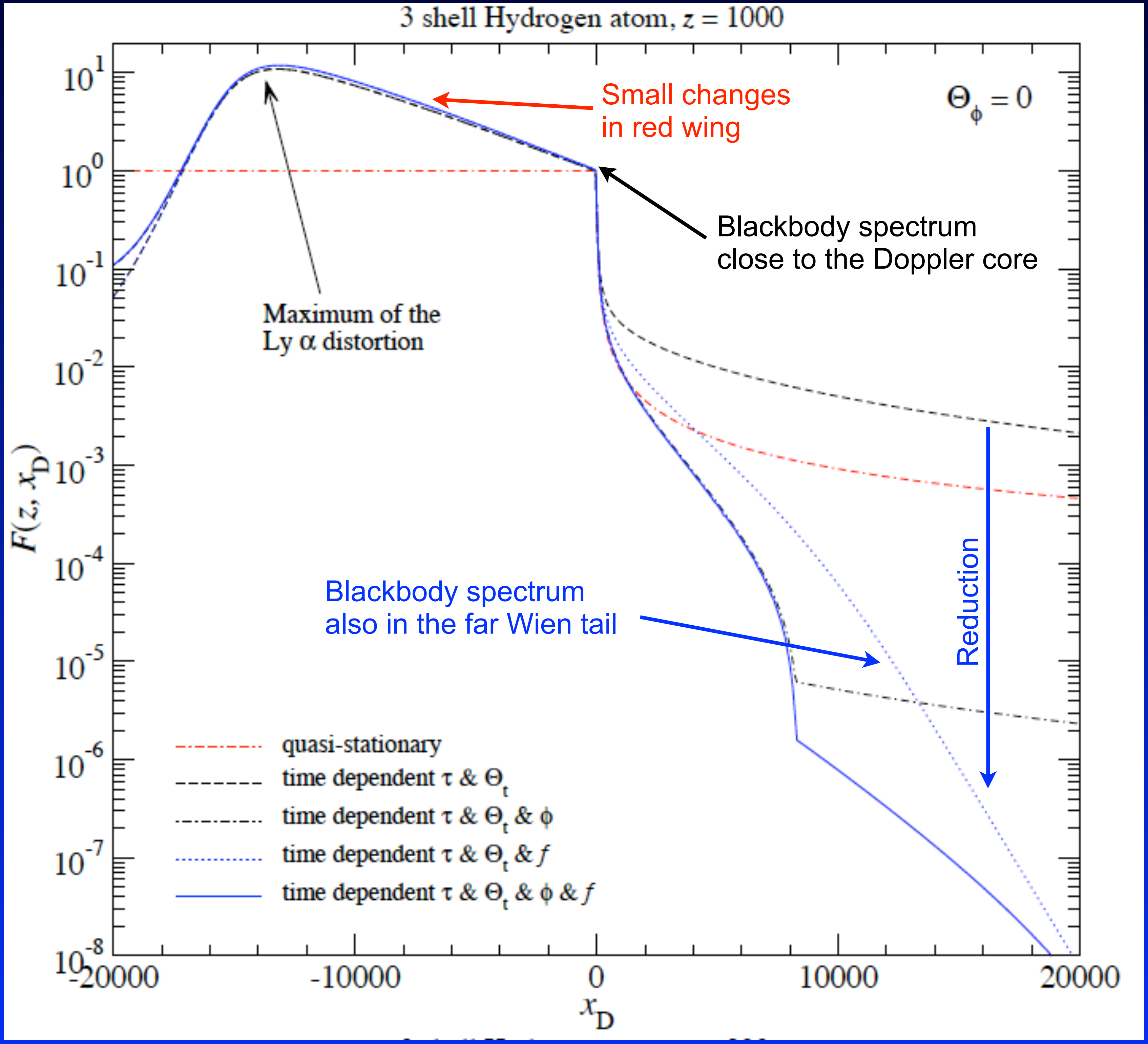


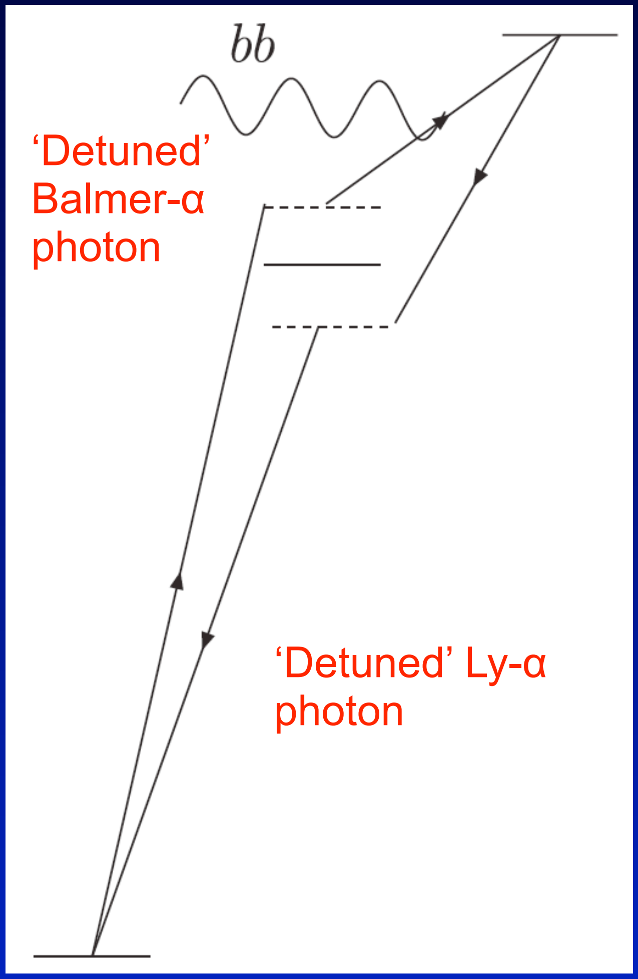


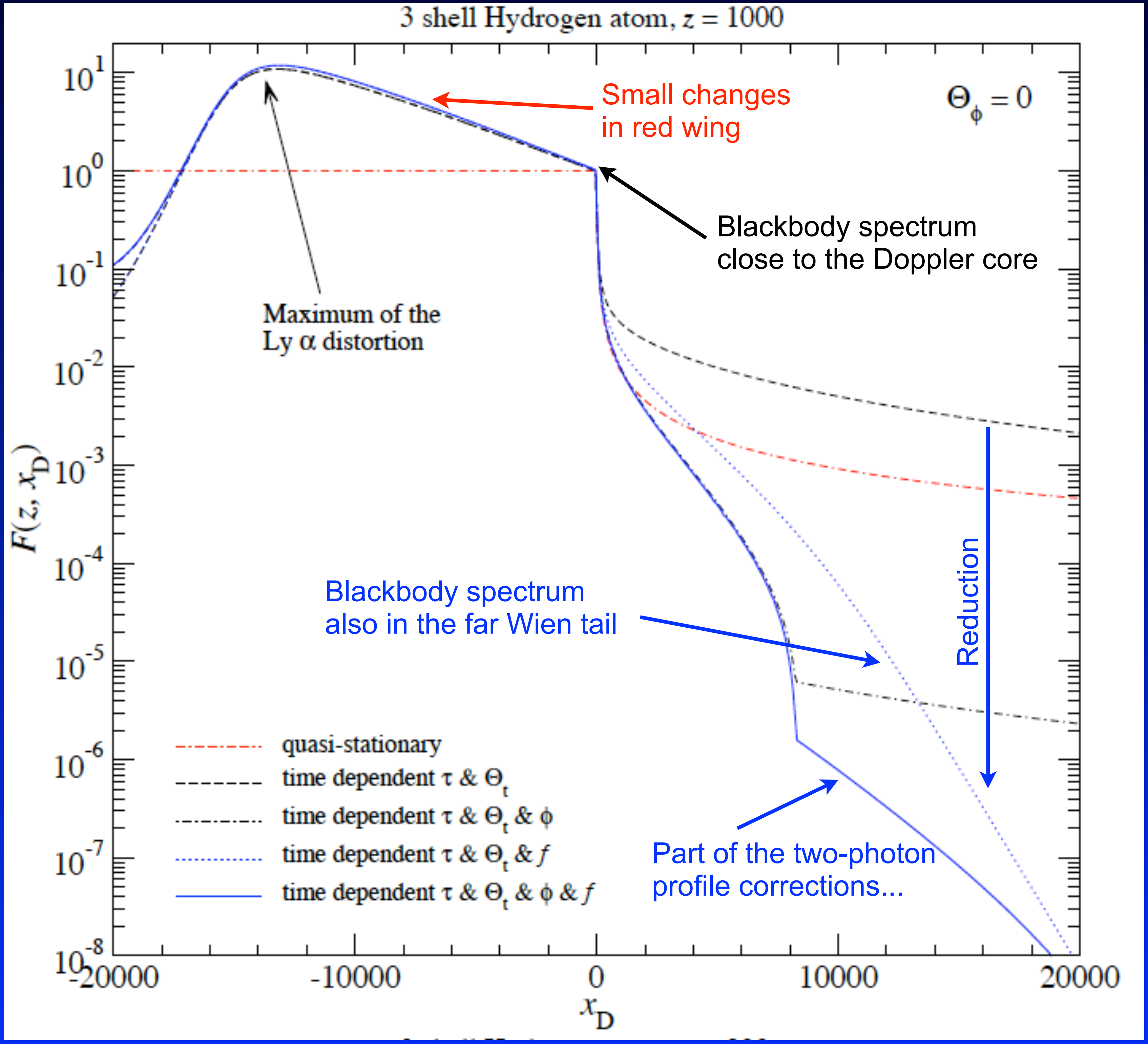
Illustration from Switzer & Hirata 2007 (meant for Helium)

- Real absorption & emission requires a second photon!

JC & Sunyaev, 2009

Problems with Sobolev approximation:

Difference between emission and absorption profile



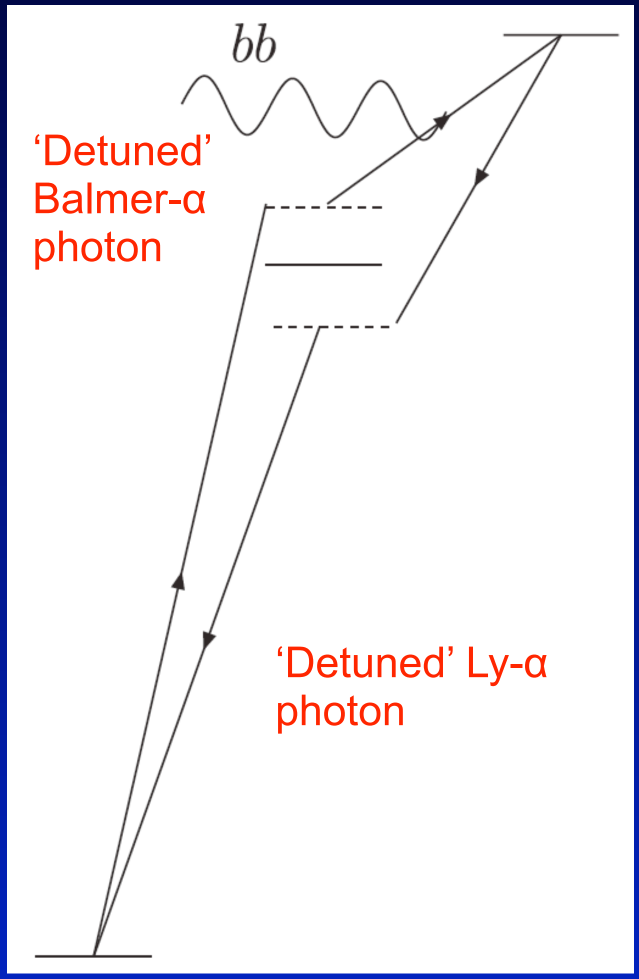


Illustration from Switzer & Hirata 2007 (meant for Helium)

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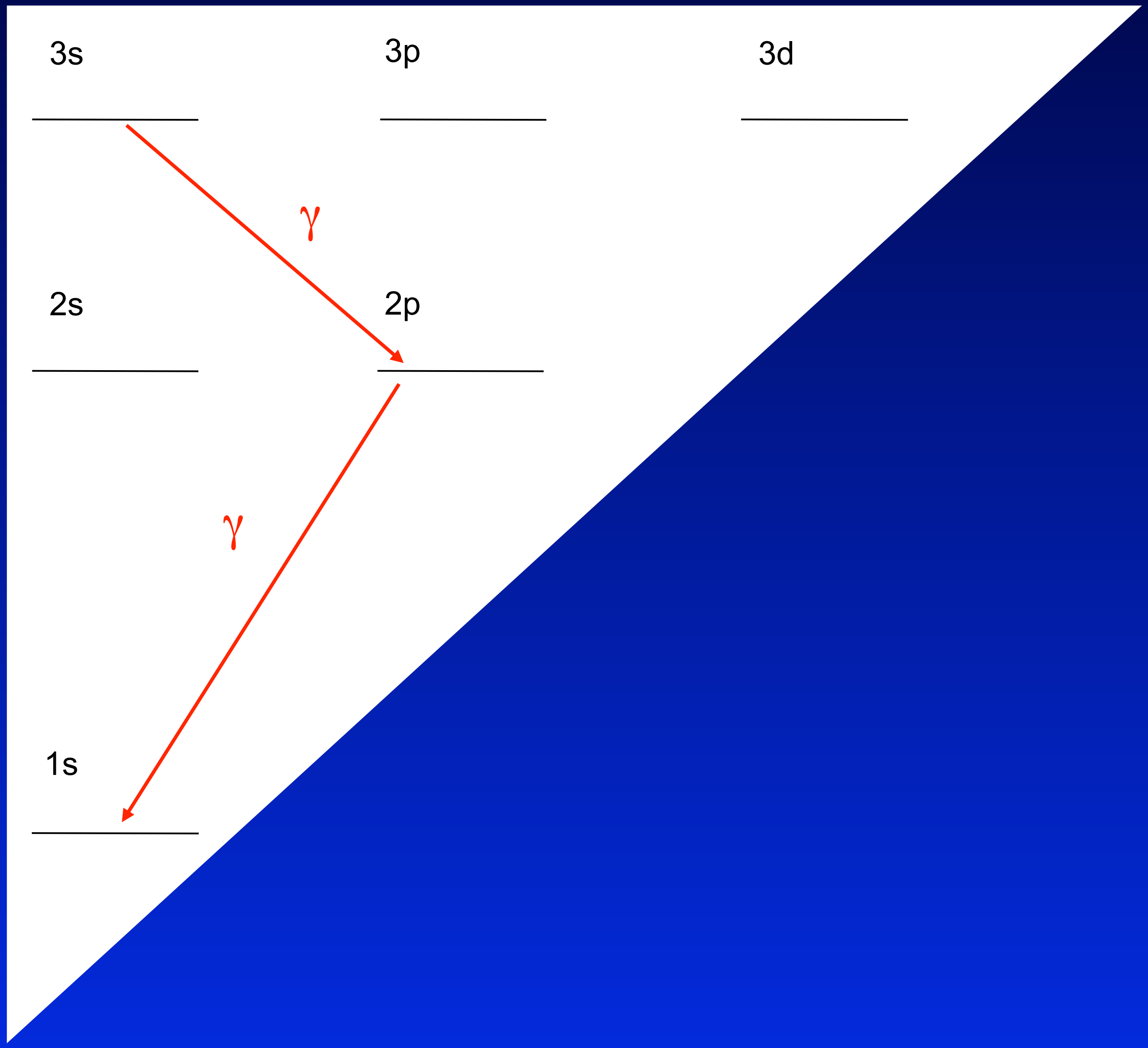
JC & Sunyaev, 2009

Two-photon emission profile



Seaton cascade (1+1 photon)

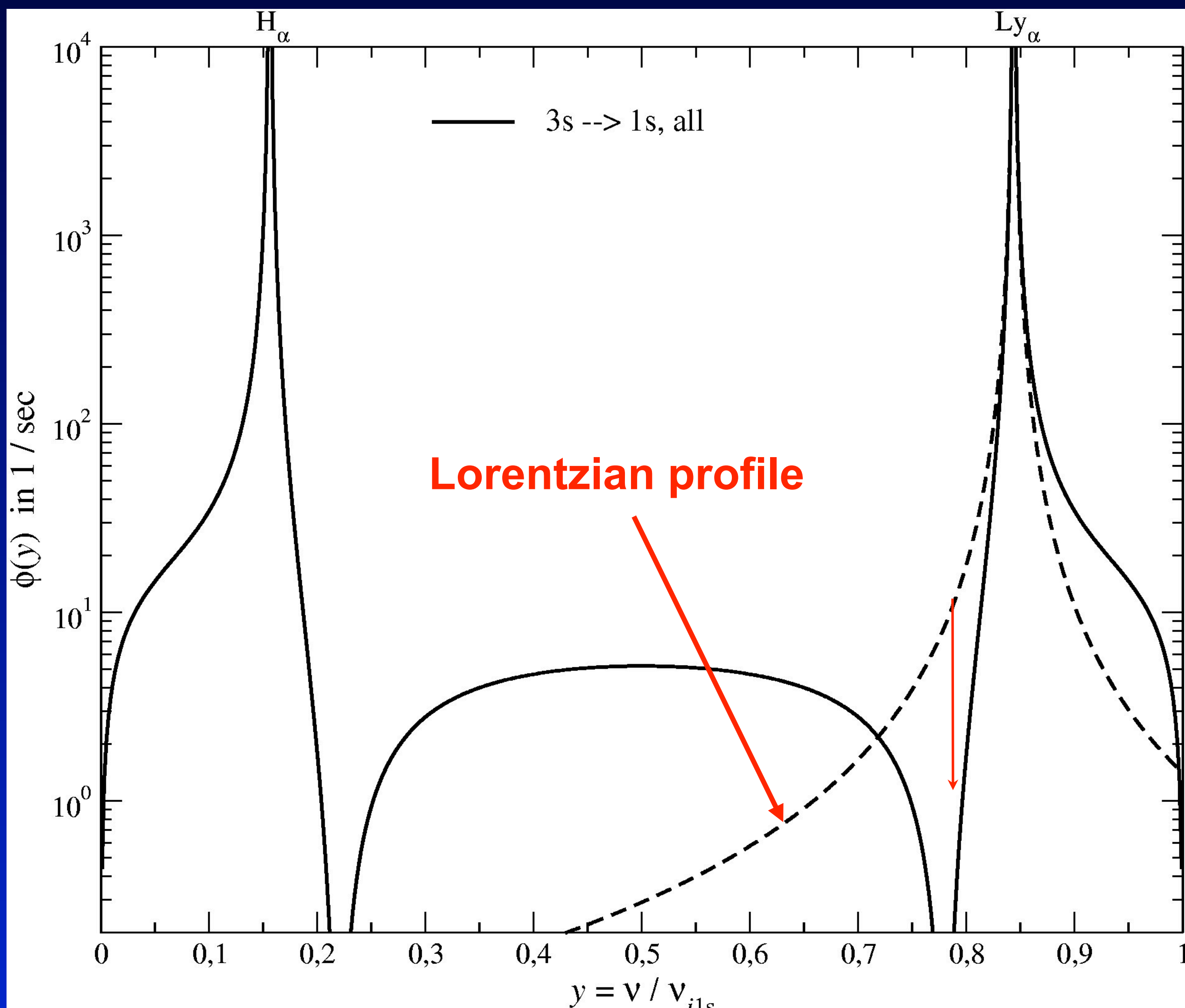
No collisions → two photons (mainly H-α and Ly-α) are emitted!

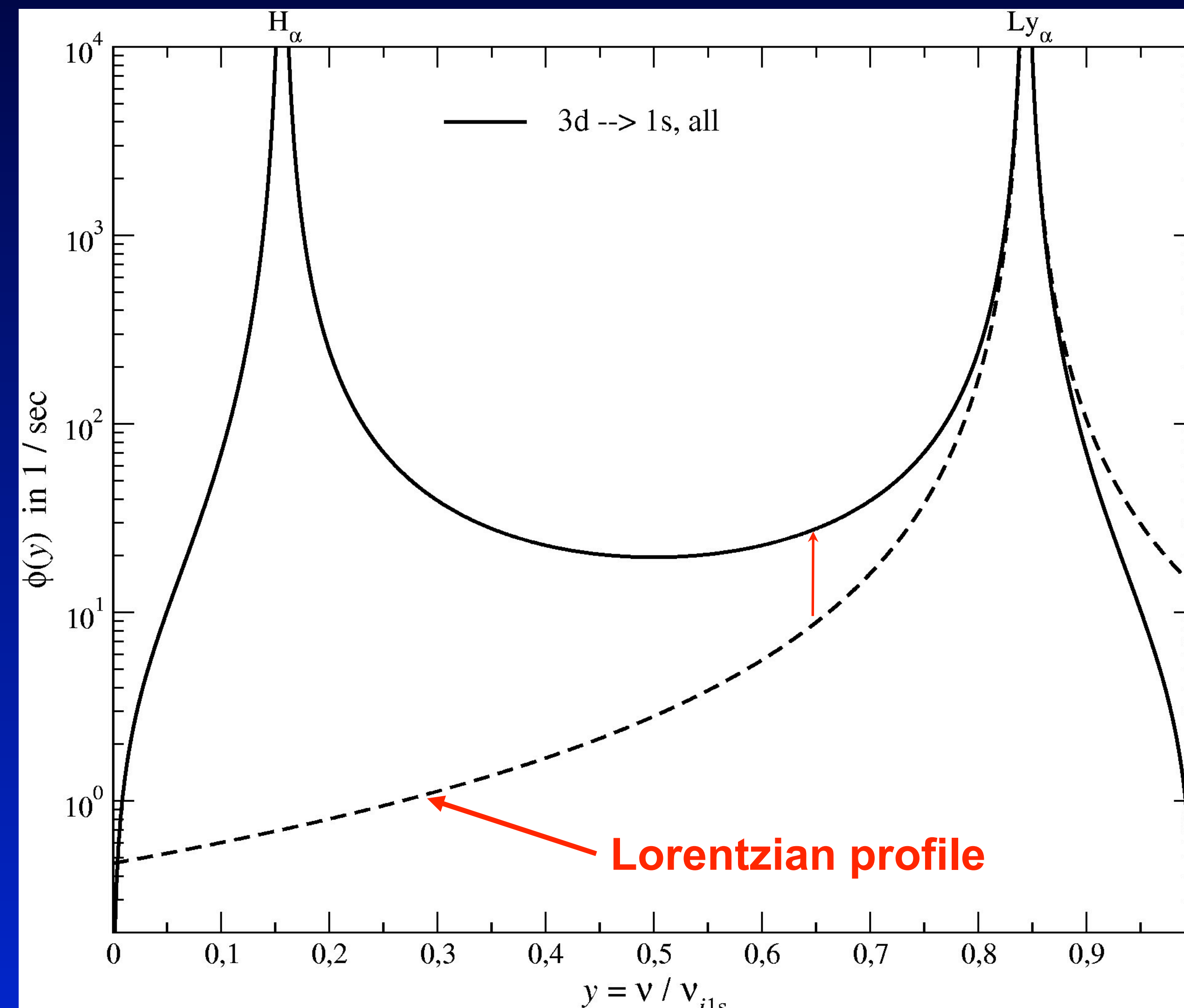
Maria-Göppert-Mayer (1931): description of two-photon emission as single process in Quantum Mechanics

→Deviations of the *two-photon line profile* from the Lorentzian in the damping wings

→Changes in the optically thin (below ~500-5000 Doppler width) parts of the line spectra

3s and 3d two-photon decay spectrum





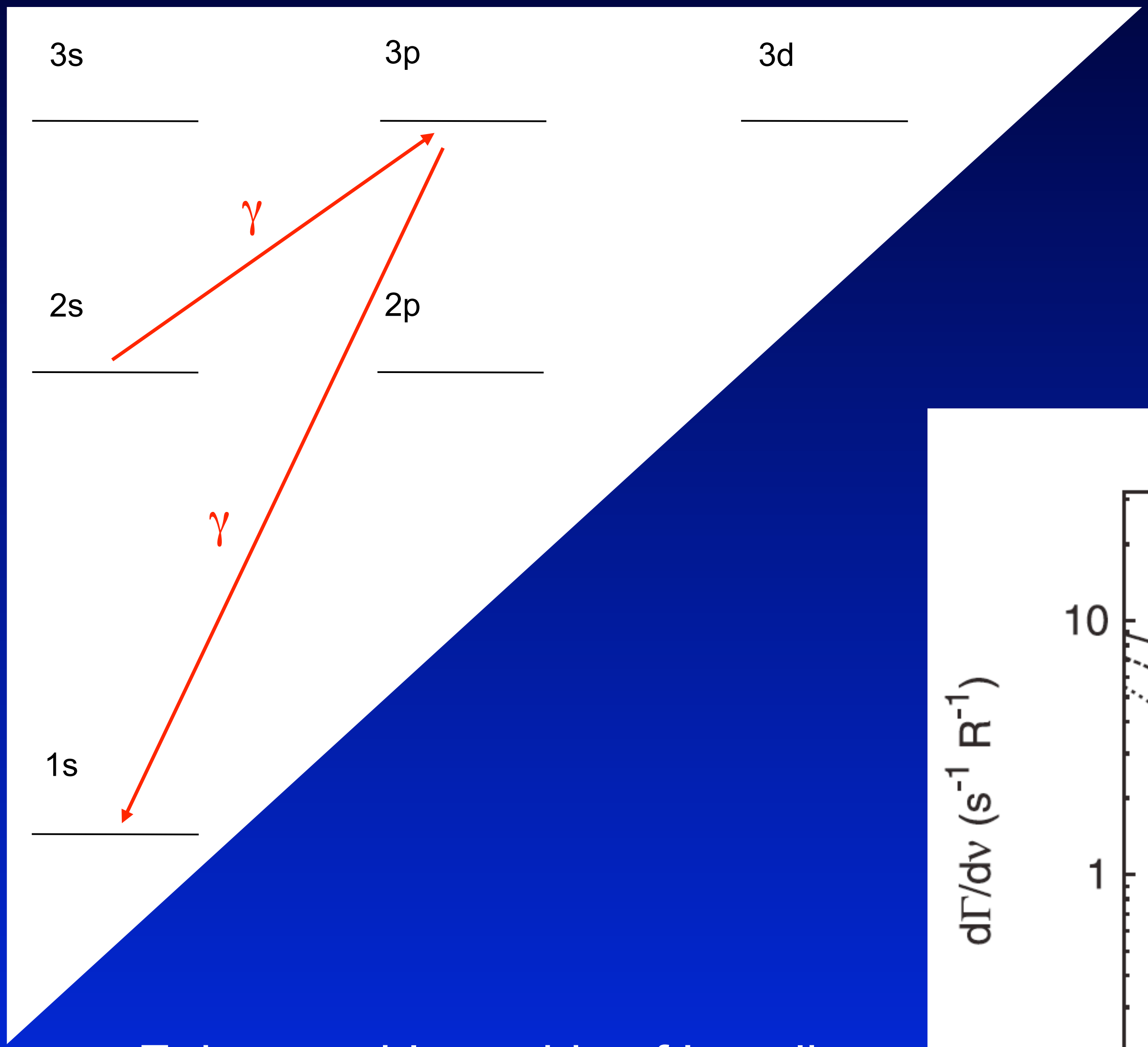
Direct Escape in optically thin regions:

→ HI -recombination is a bit *slower* due to 2γ-transitions from s-states

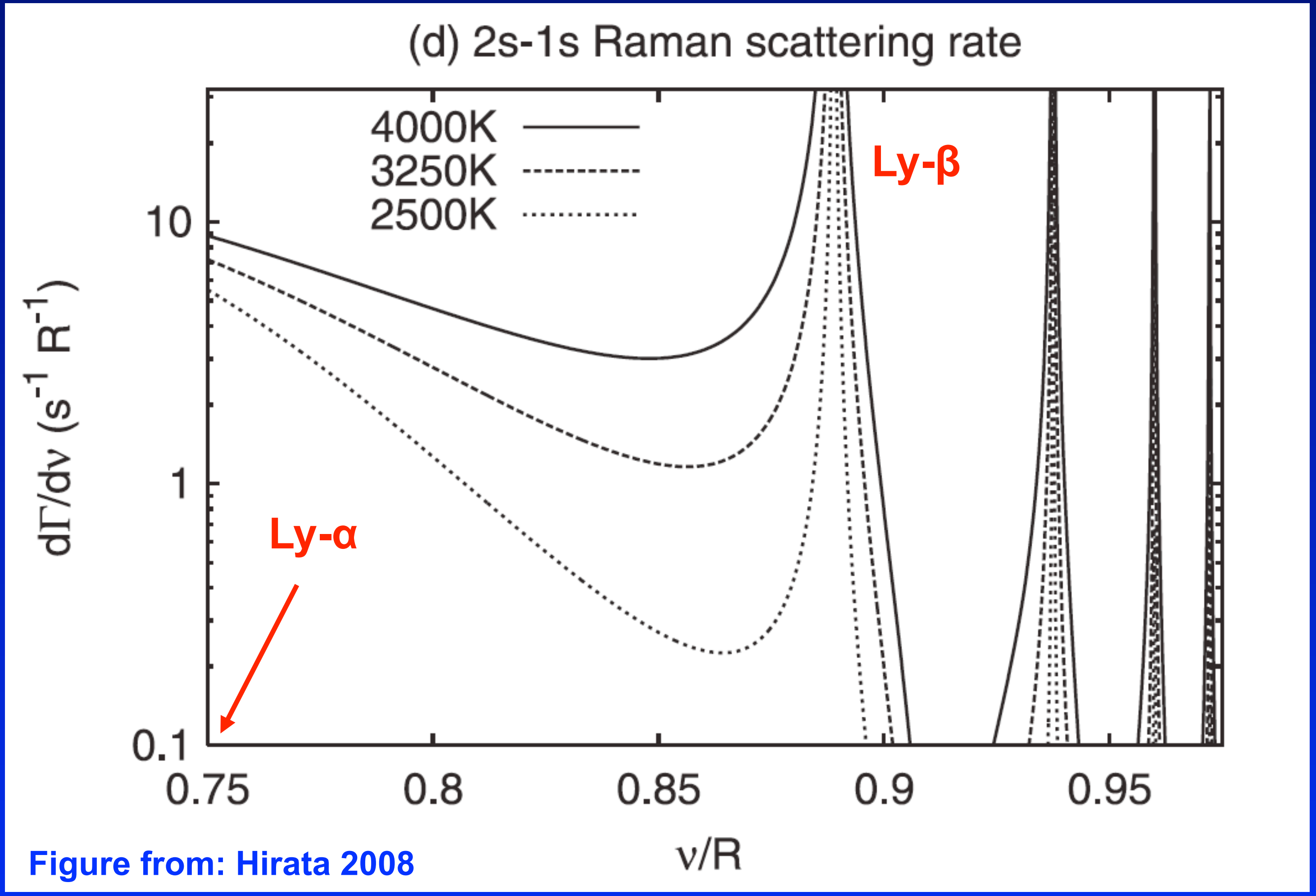
→ HI -recombination is a bit *faster* due to 2γ-transitions from d-states

JC & Sunyaev, A&A, 2008

2s-1s Raman scattering



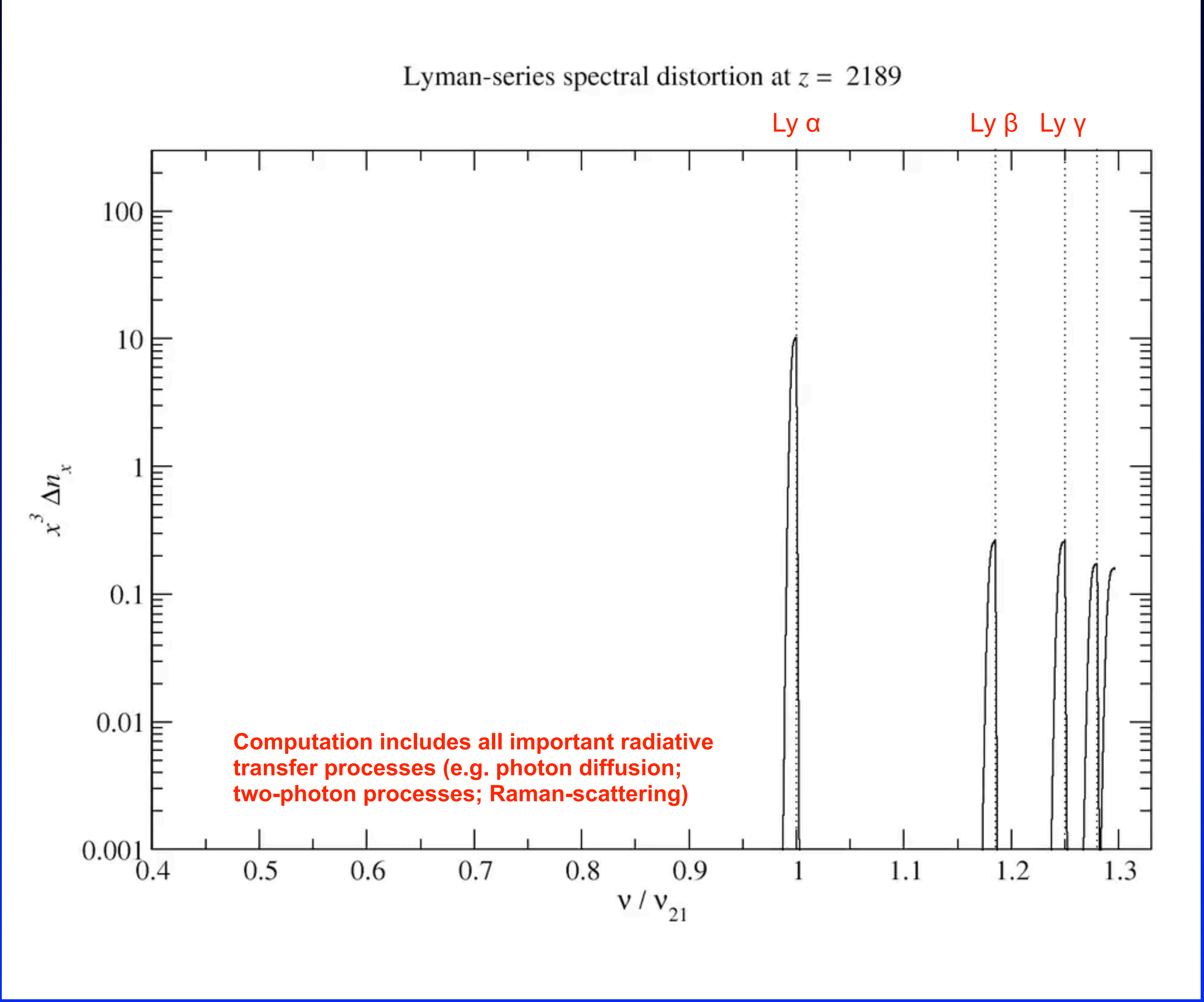
- Computation similar to two-photon decay profiles
- collisions weak $\Longrightarrow$ *process needs to be modeled as single quantum act*



- Enhances blues side of Ly-α line
- associated feedback delays recombination around z~900

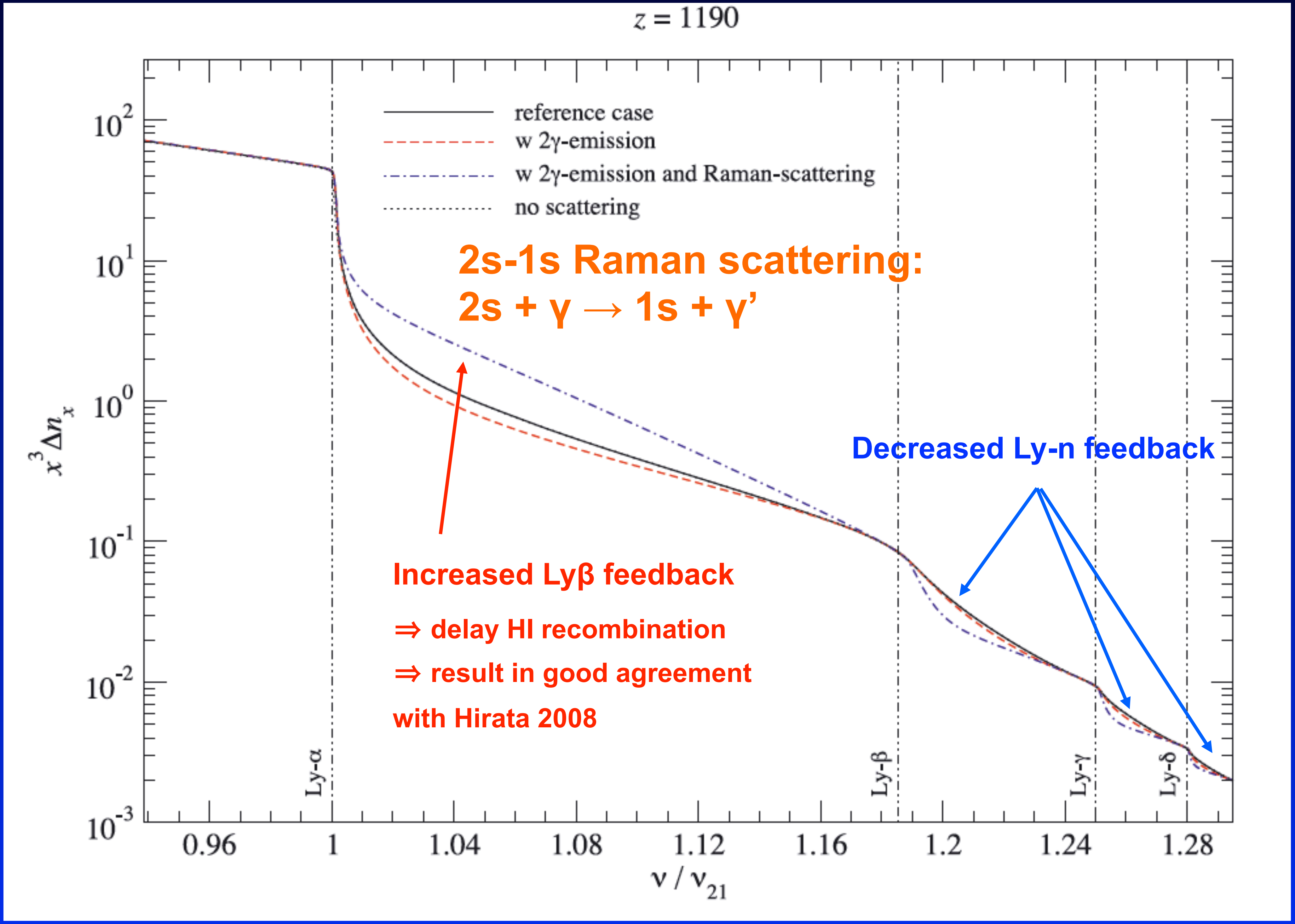
Hirata 2008
JC & Thomas, 2010

Evolution of the HI Lyman-series distortion



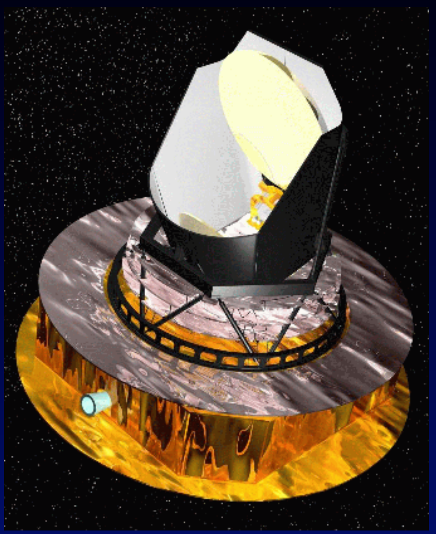
JC & Thomas, MNRAS, 2010

Effect of Raman scattering and 2γ decays



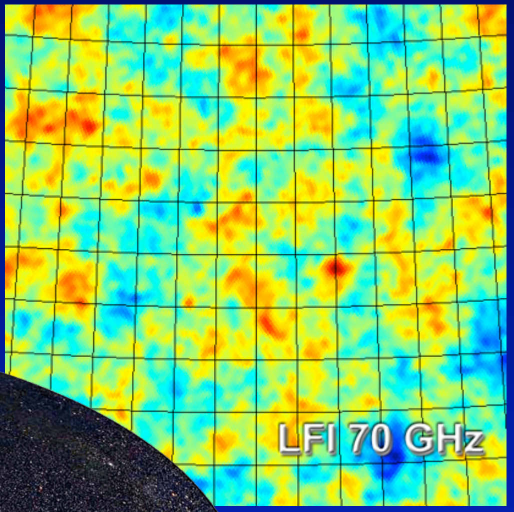
JC & Thomas, MNRAS, 2010

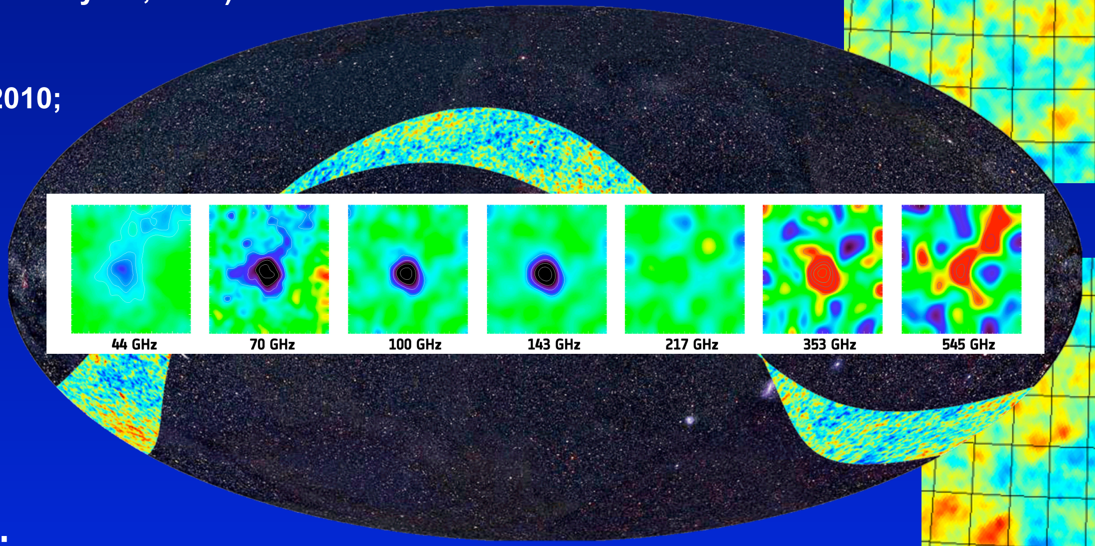
Getting Ready for Planck

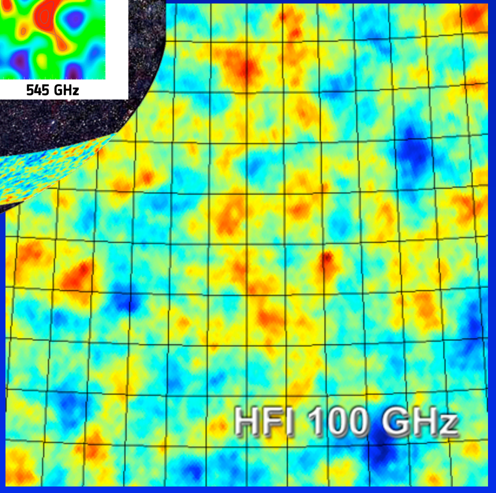


Hydrogen recombination

- Two-photon decays from higher levels
  (Dubrovich & Grachev, 2005, Astr. Lett., 31, 359; Wong & Scott, 2007; JC & Sunyaev, 2007; Hirata, 2008; JC & Sunyaev 2009)
- Induced 2s two-photon decay for hydrogen
  (JC & Sunyaev, 2006, A&A, 446, 39; Hirata 2008)
- Feedback of the Lyman-α distortion on the 1s-2s two-photon absorption rate
  (Kholupenko & Ivanchik, 2006, Astr. Lett.; Fendt et al. 2008; Hirata 2008)
- Non-equilibrium effects in the angular momentum sub-states
  (Rubiño-Martín, JC & Sunyaev, 2006, MNRAS; JC, Rubiño-Martín & Sunyaev, 2007, MNRAS; Grin & Hirata, 2009; JC, Vasil & Dursi, 2010)
- Feedback of Lyman-series photons (Ly[n] → Ly[n-1])
  (JC & Sunyaev, 2007, A&A; Kholupenko et al. 2010; Haimoud, Grin & Hirata, 2010)
- Lyman-α escape problem (*atomic recoil, time-dependence, partial redistribution*)
  (Dubrovich & Grachev, 2008; JC & Sunyaev, 2008; Forbes & Hirata, 2009; JC & Sunyaev, 2009)
- Collisions and Quadrupole lines
  (JC, Rubiño-Martín & Sunyaev, 2007; Grin & Hirata, 2009; JC, Vasil & Dursi, 2010; JC, Fung & Switzer, 2011)
- Raman scattering
  (Hirata 2008; JC & Thomas , 2010; Haimoud & Hirata, 2010)





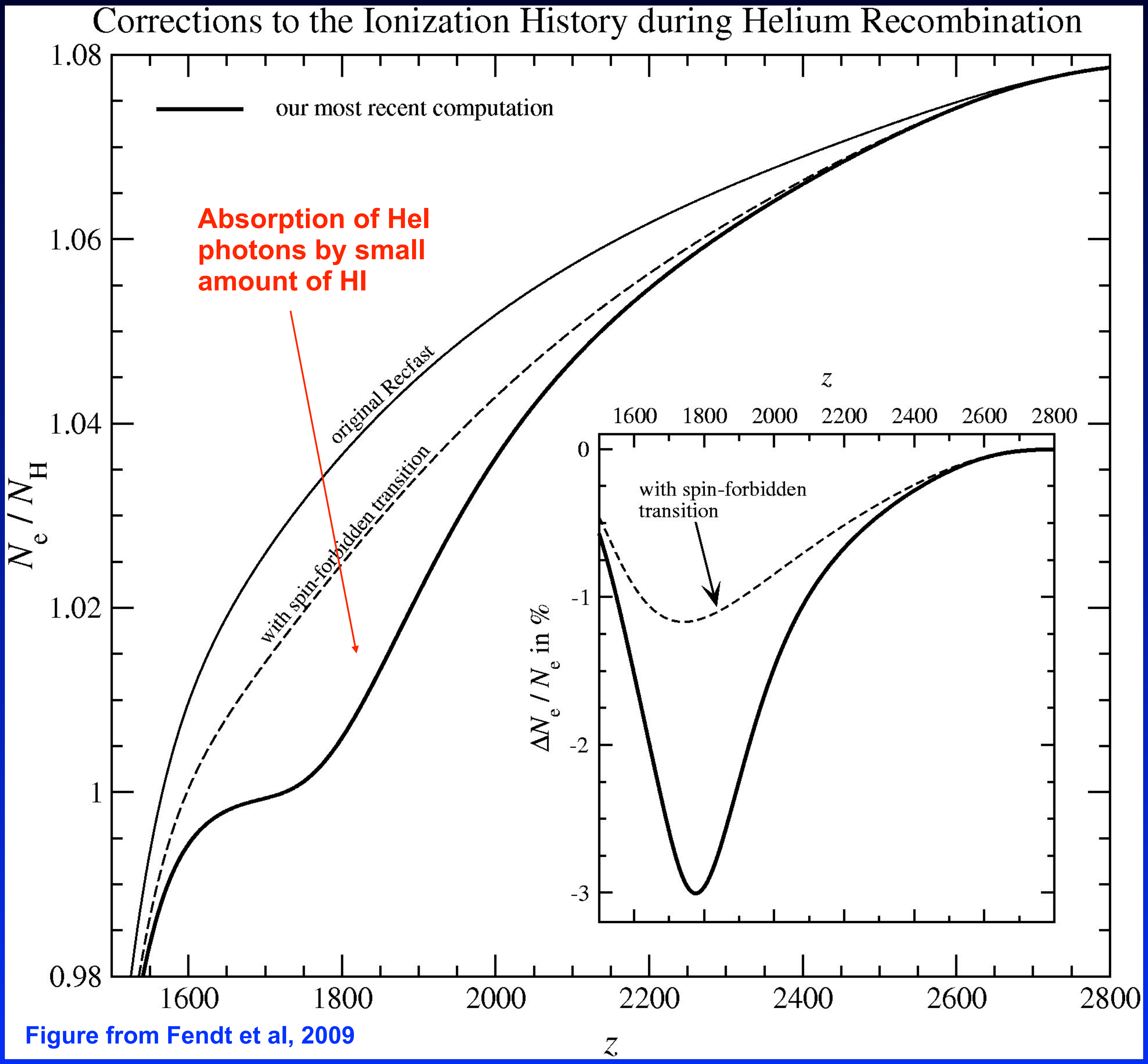


Helium recombination

- Similar list of processes as for hydrogen
  (Switzer & Hirata, 2007a&b; Hirata & Switzer, 2007)
- Spin forbidden 2p-1s triplet-singlet transitions
  (Dubrovich & Grachev, 2005, Astr. Lett.; Wong & Scott, 2007; Switzer & Hirata, 2007; Kholupenko, Ivanchik&Varshalovich, 2007)
- Hydrogen continuum opacity during He I recombination
  (Switzer & Hirata, 2007; Kholupenko, Ivanchik & Varshalovich, 2007; Rubiño-Martín, JC & Sunyaev, 2007; JC, Fung & Switzer, 2011)
- Detailed feedback of helium photons
  (Switzer & Hirata, 2007a; JC & Sunyaev, 2009, MNRAS; JC, Fung & Switzer, 2011)

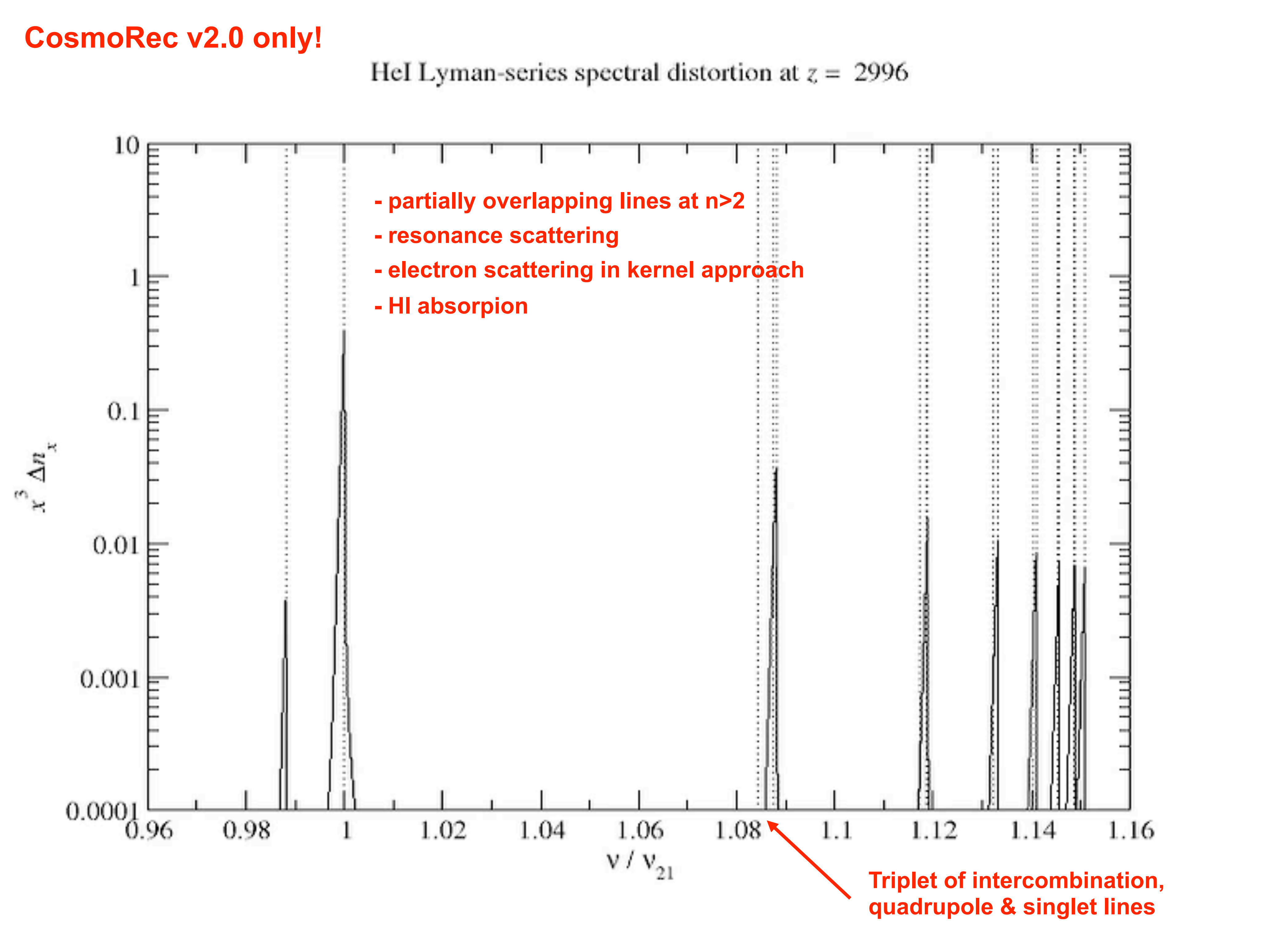
$\Delta N_e \,/\, N_e \sim 0.1\ \%$

Main corrections during HeI Recombination



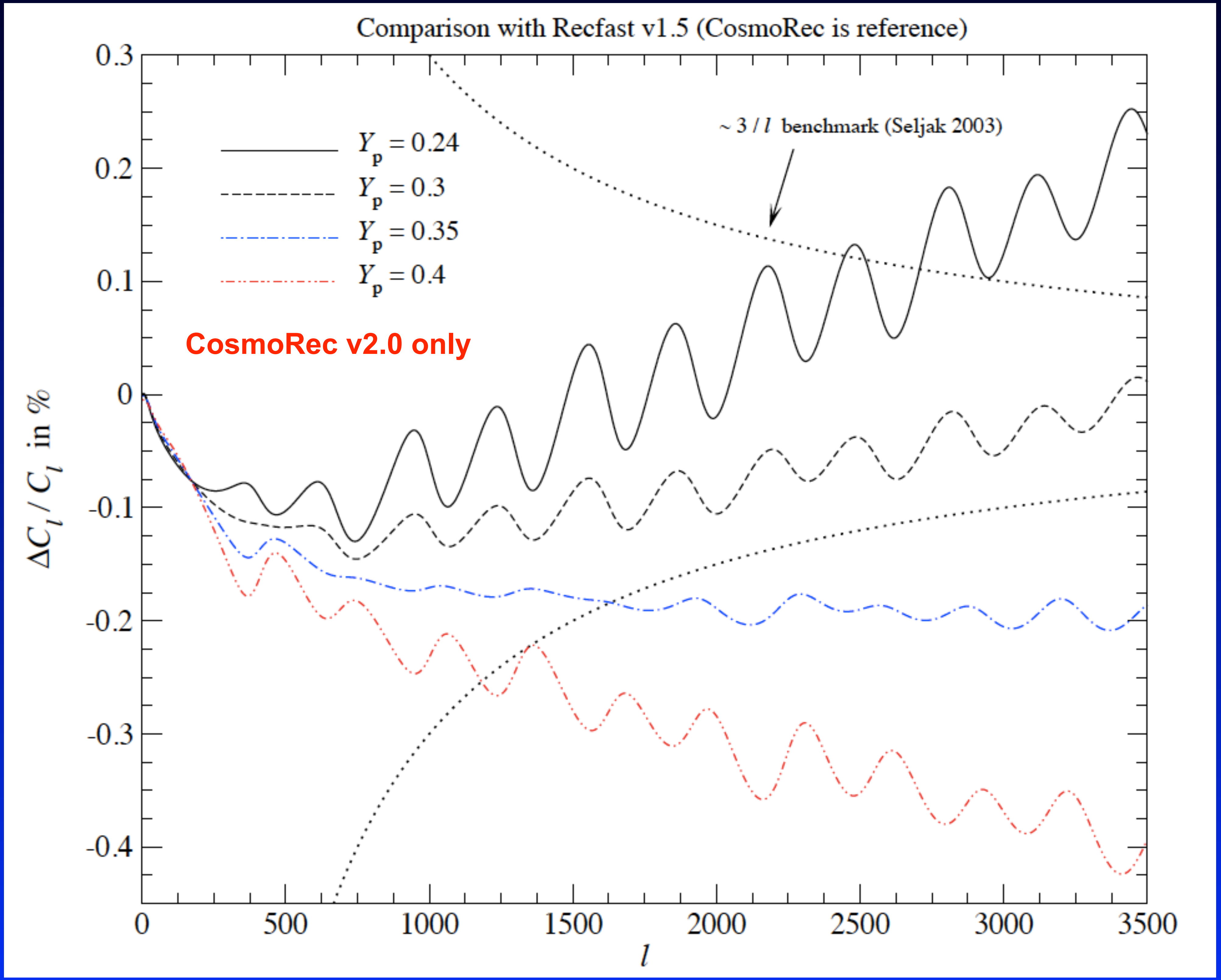
Kholupenko et al, 2007
Switzer & Hirata, 2007

Evolution of the HeI high frequency distortion



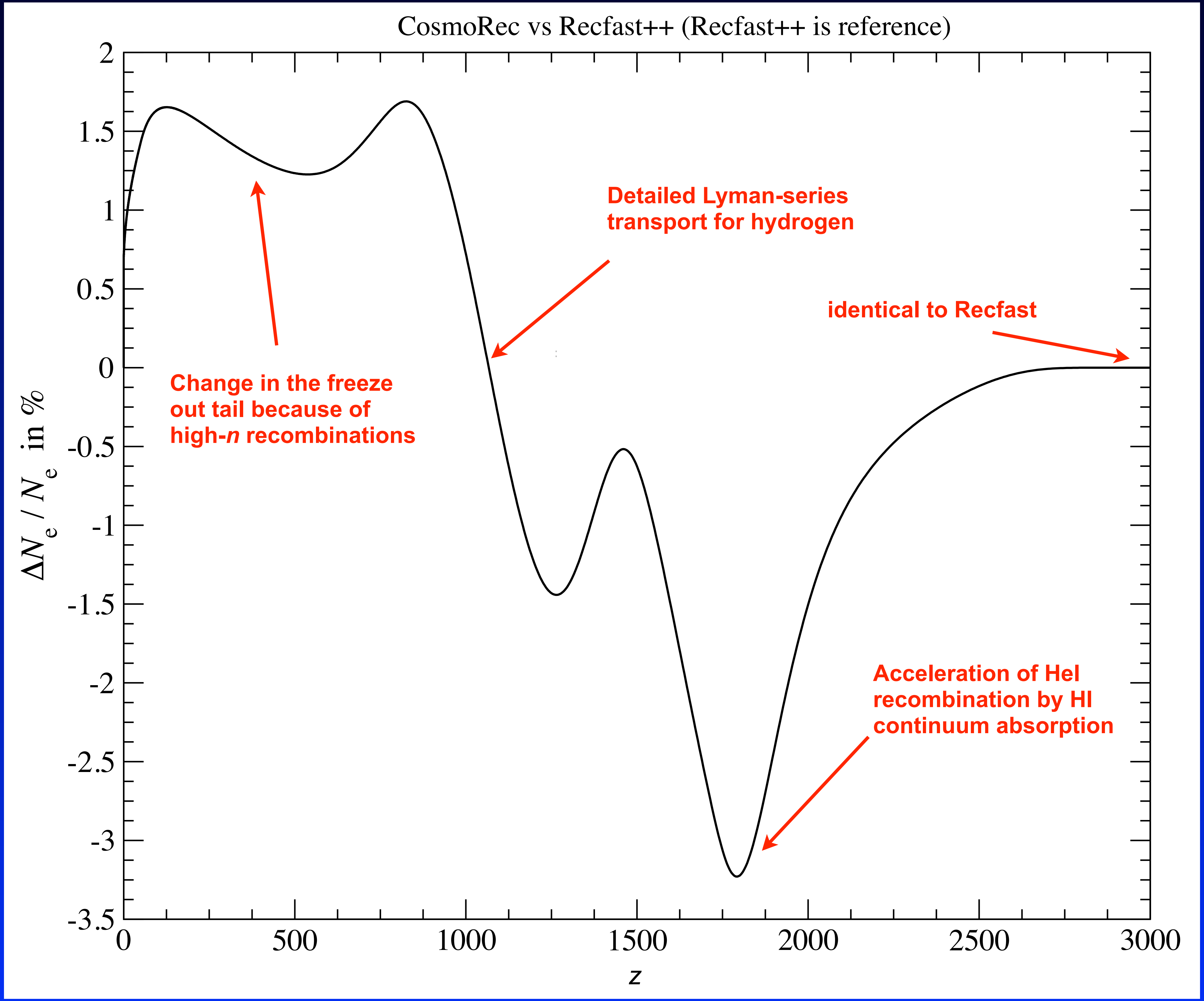
JC, Fung & Switzer, 2011

Overall effect of detailed HeI radiative transfer



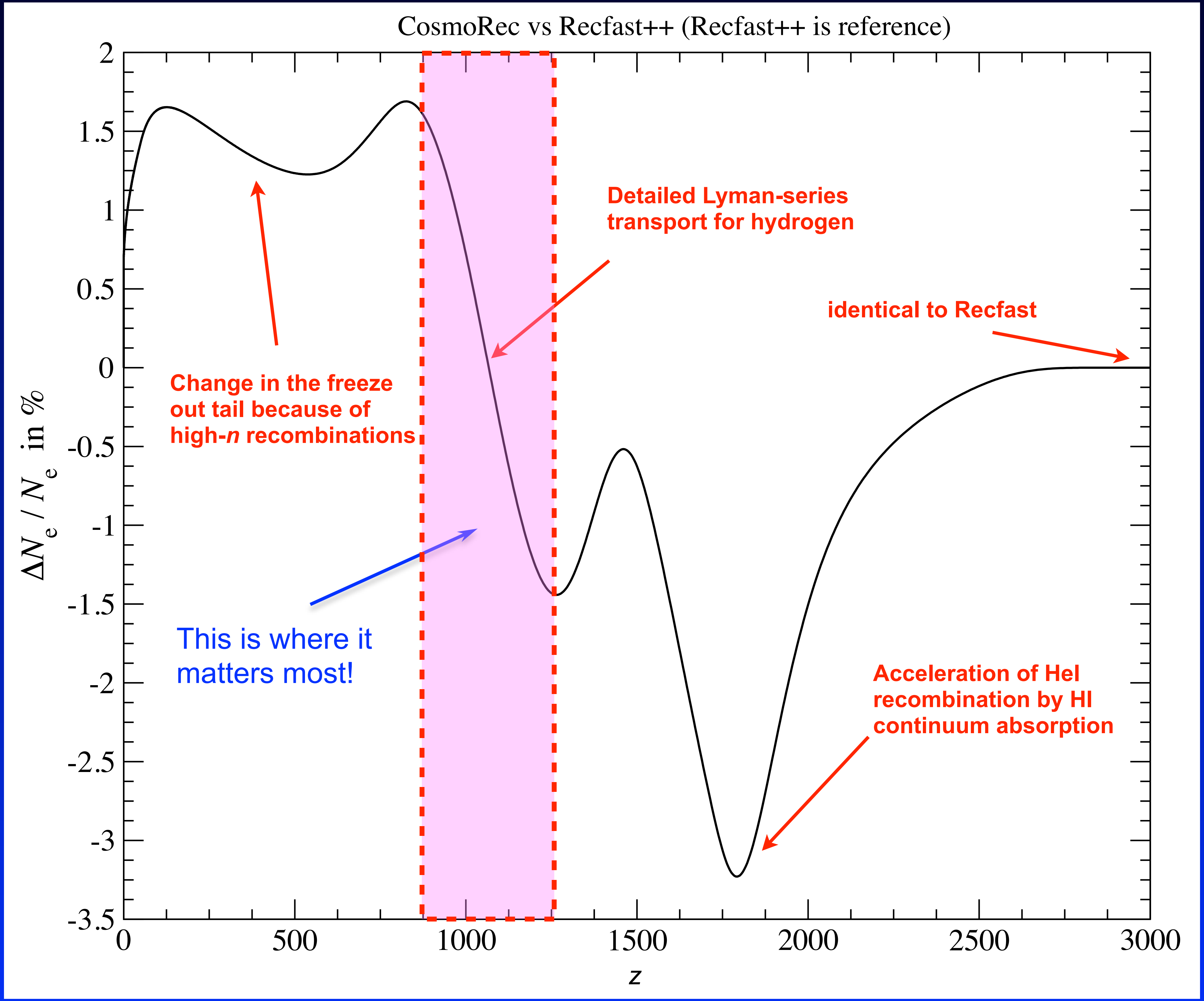
JC, Fung & Switzer, 2011

Cumulative Changes to the Ionization History



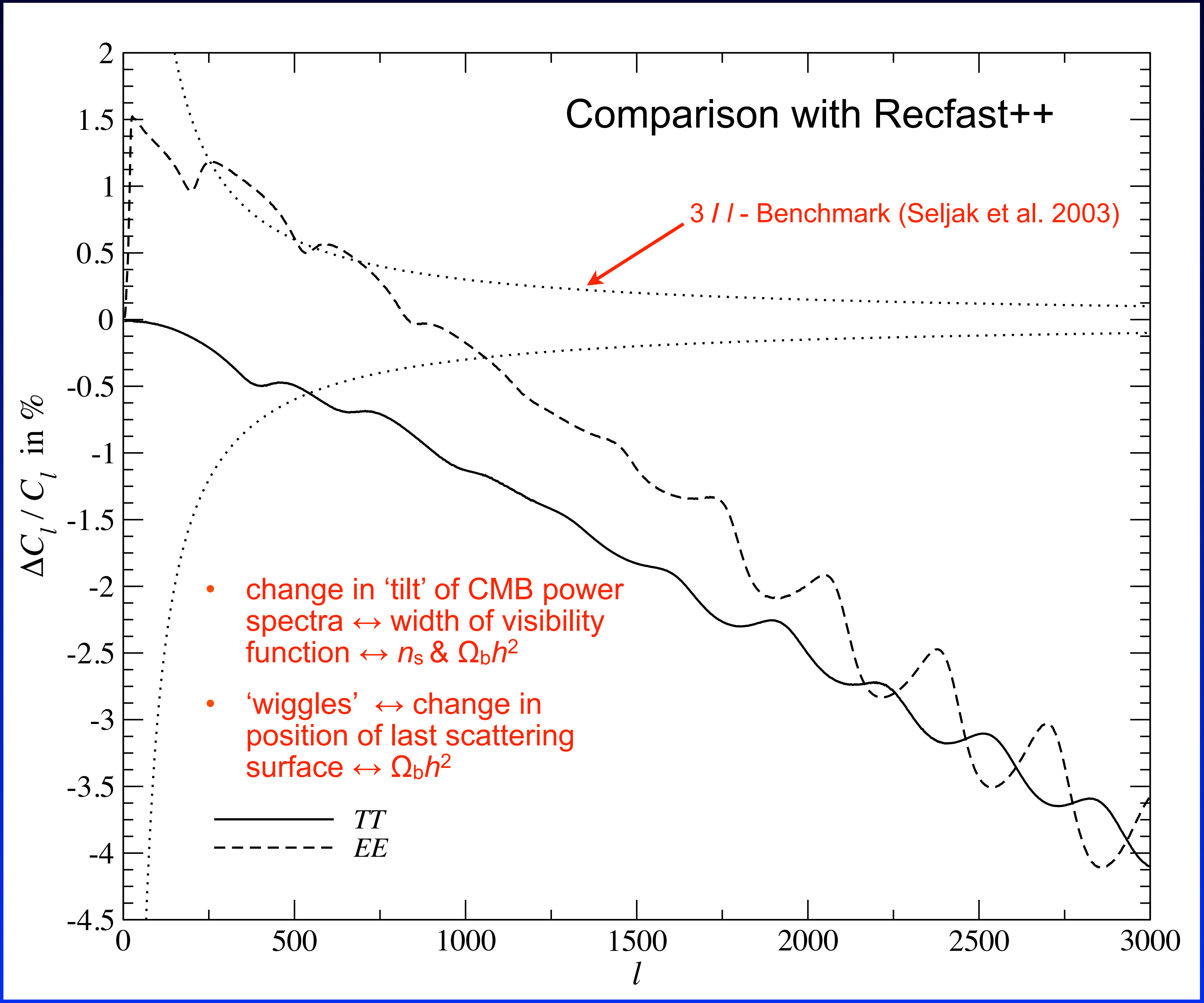
CITA ICAT JC & Thomas, MNRAS, 2010; Shaw & JC, MNRAS, 2011

Cumulative Changes to the Ionization History



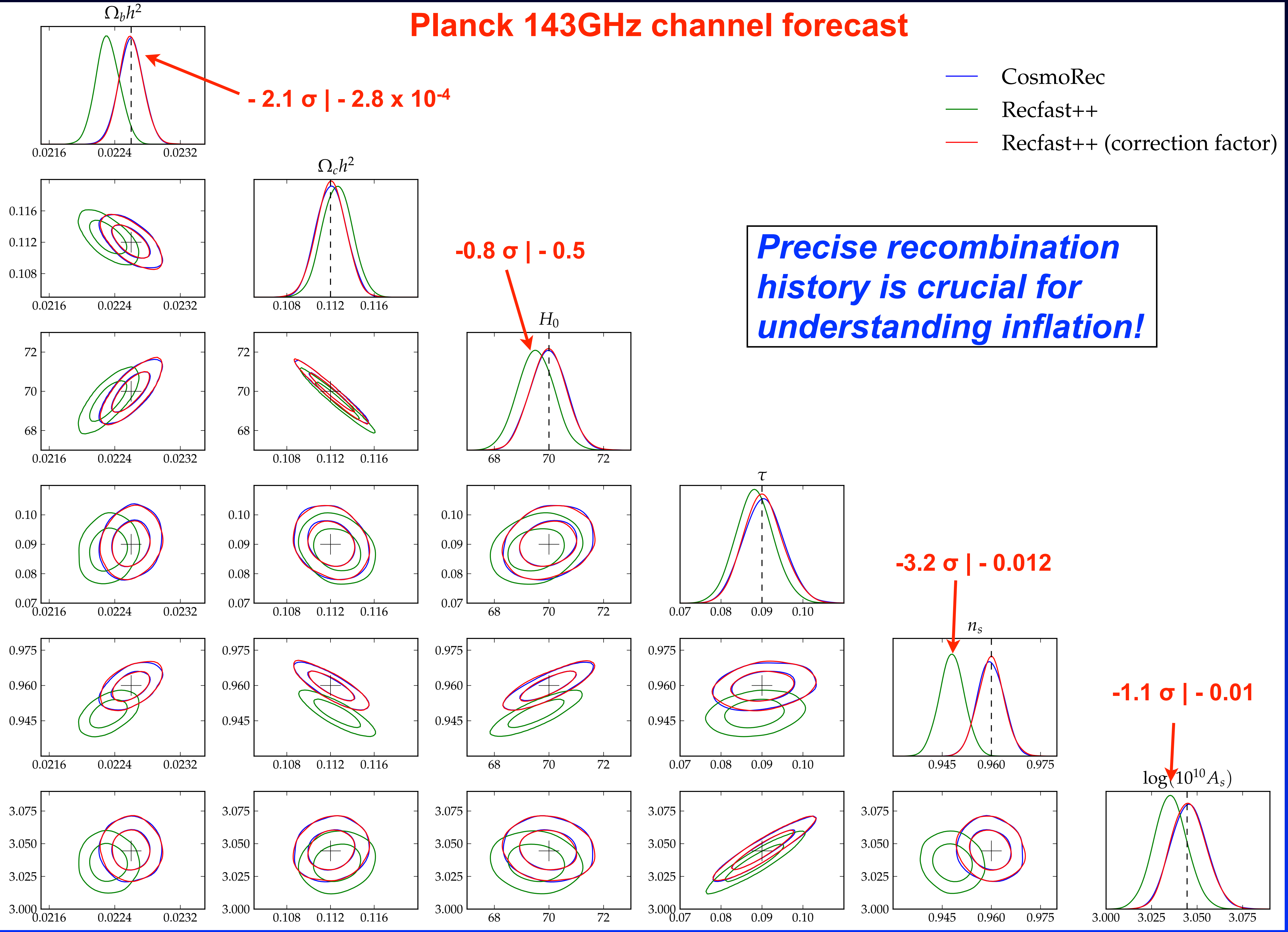
CITA ICAT JC & Thomas, MNRAS, 2010; Shaw & JC, MNRAS, 2011

Cumulative Change in the CMB Power Spectra



Shaw & JC, MNRAS, 2011

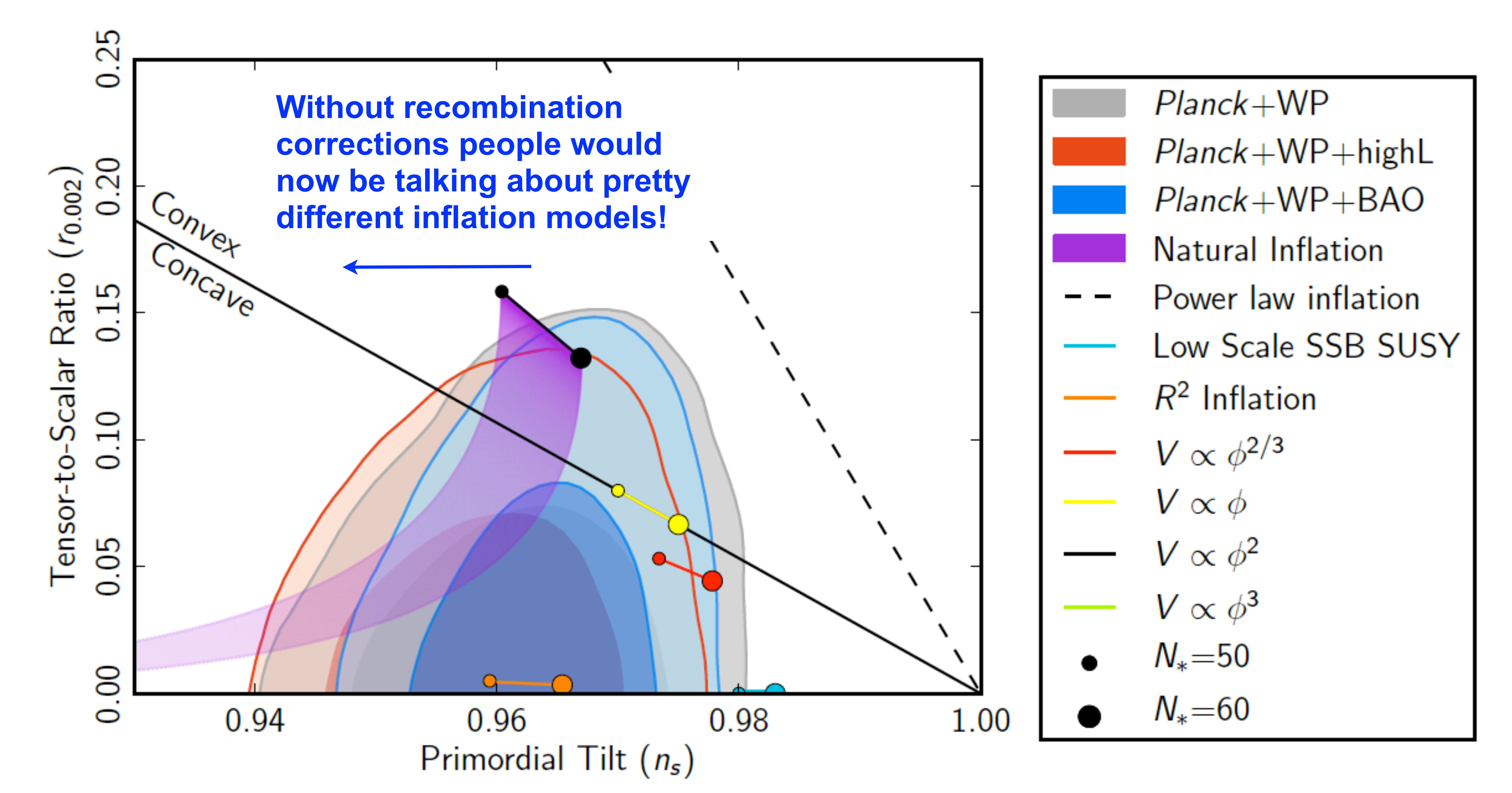
Importance of recombination for inflation



Precise recombination history is crucial for understanding inflation!

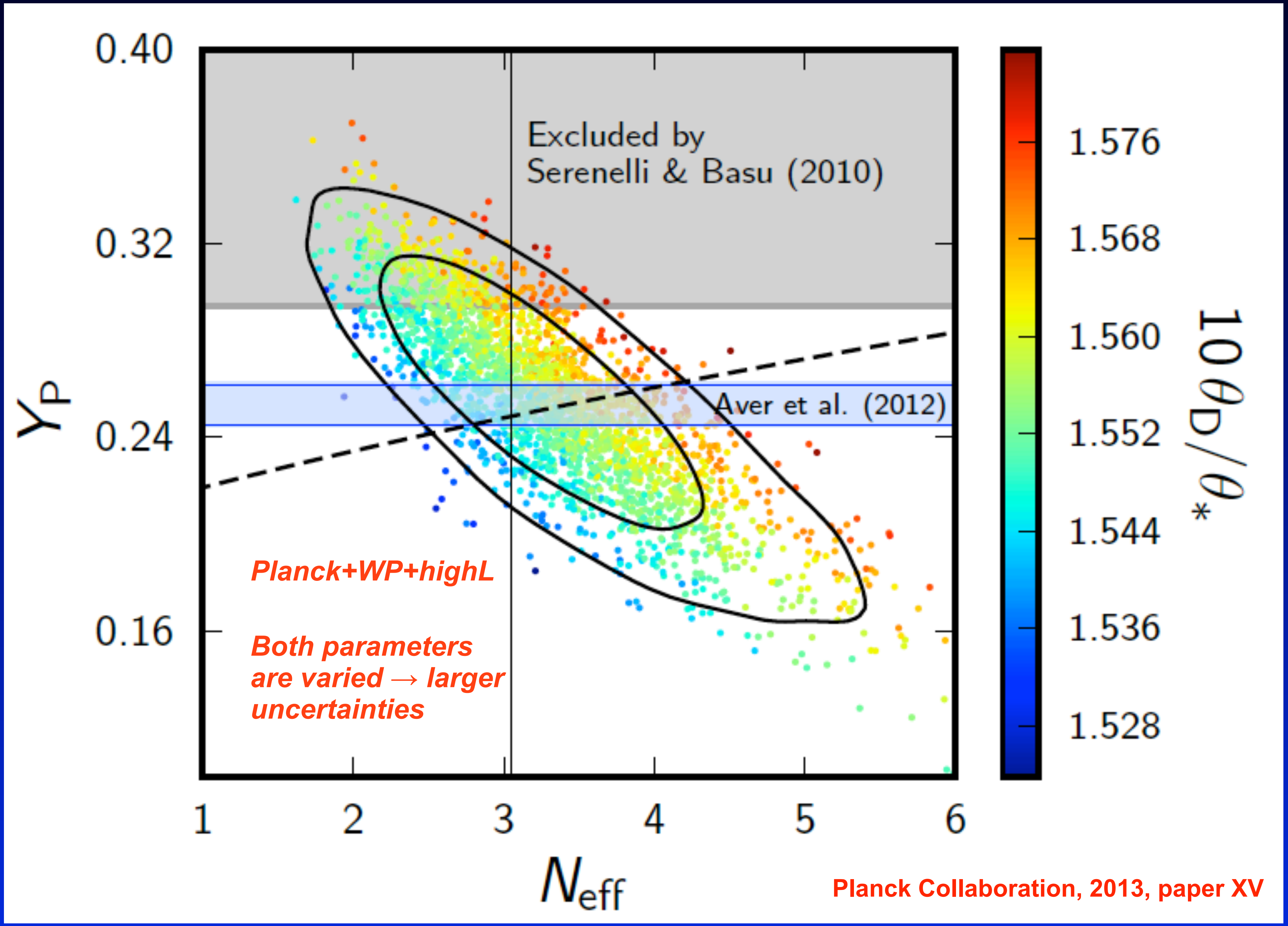
CITA ICAT Shaw & JC, 2011, and references therein

Importance of recombination for inflation



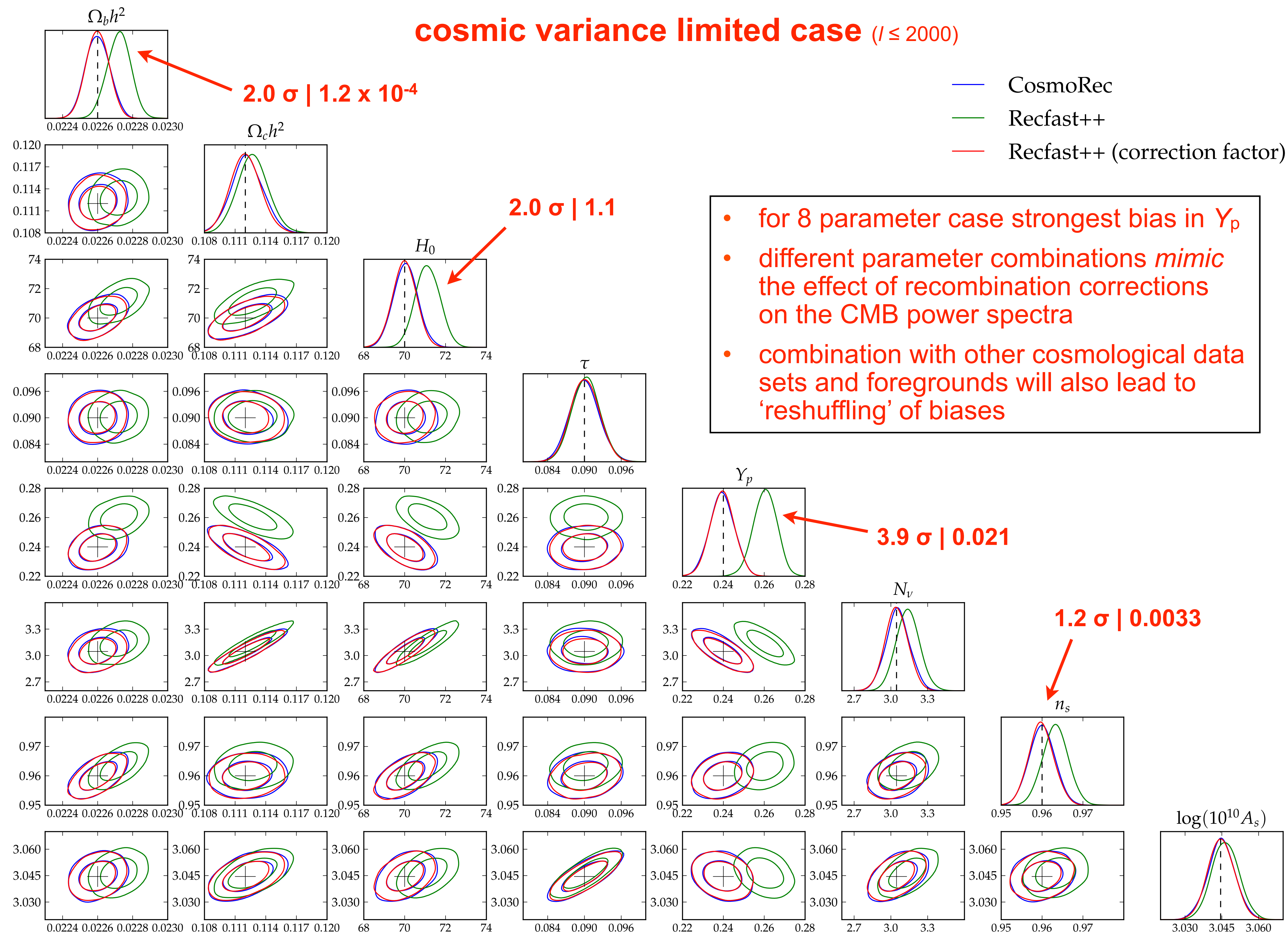
Planck Collaboration, 2013, paper XXII

CMB constraints on N_{eff} and Y_p



- *Consistent with SBBN and standard value for N_{eff}*
- *Future CMB constraints (SPTPol & ACTPol) on Y_p will reach 1% level*

Importance of recombination for measuring helium



- for 8 parameter case strongest bias in Y_p
- different parameter combinations *mimic* the effect of recombination corrections on the CMB power spectra
- combination with other cosmological data sets and foregrounds will also lead to 'reshuffling' of biases

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Shaw & JC, 2011, and references therein

What if something unexpected happened?

- E.g., something *standard* was missed, or something *non-standard* happened !?
- A *non-parametric estimation* of possible *corrections* to the recombination history would be very useful → *Principle component analysis* (PCA)

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Farhang, Bond & JC, 2012

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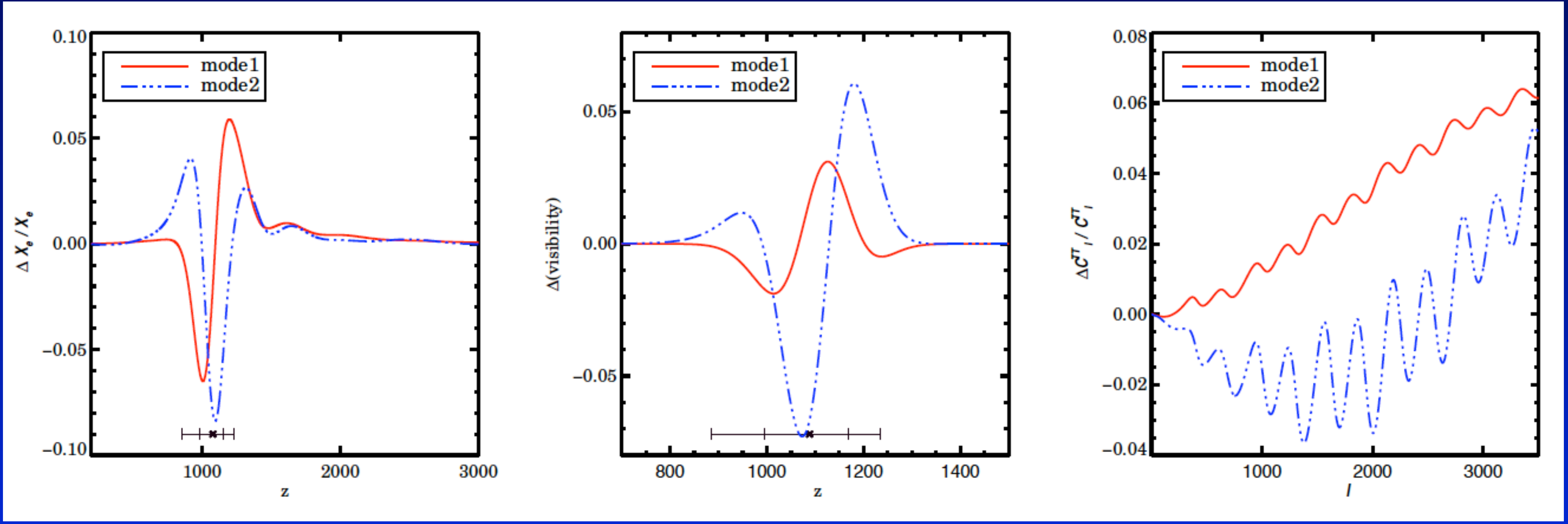


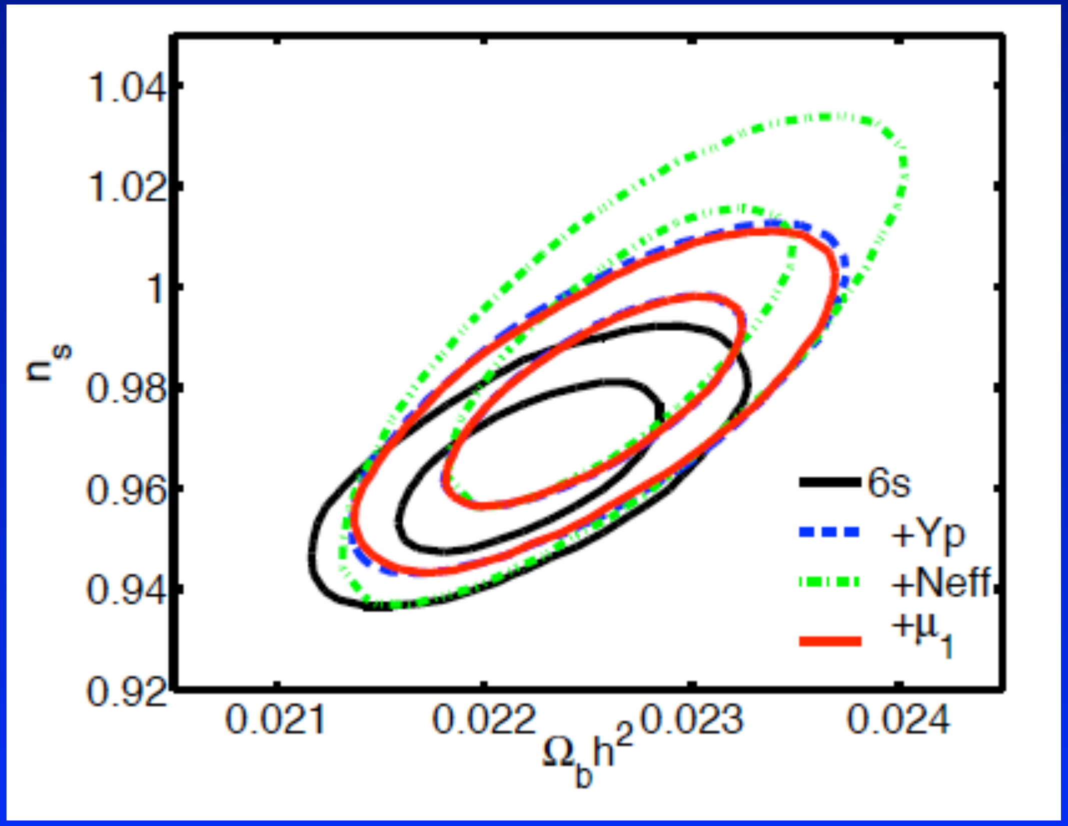
Farhang, Bond & JC, 2012

Measured mode amplitudes for ACT & SPT

parameters	SPT+WMAP7 6s	SPT+WMAP7 + mode 1	SPT+WMAP7 + mode 2	ACT+WMAP7 6s	ACT+WMAP7 + mode 1	ACT+WMAP7 + mode 2
$100\Omega_b h^2$	2.221 ± 0.042	2.253 ± 0.046	2.249 ± 0.047	2.219 ± 0.051	2.240 ± 0.050	2.236 ± 0.053
$\Omega_c h^2$	0.1110 ± 0.0048	0.1123 ± 0.0049	0.1118 ± 0.0052	0.1121 ± 0.0052	0.1155 ± 0.0056	0.1121 ± 0.0061
$100\theta_s$	1.041 ± 0.002	1.041 ± 0.002	1.040 ± 0.003	1.039 ± 0.002	1.039 ± 0.002	1.035 ± 0.004
τ	0.086 ± 0.015	0.089 ± 0.015	0.089 ± 0.015	0.086 ± 0.015	0.089 ± 0.015	0.0875 ± 0.015
n_s	0.964 ± 0.011	0.977 ± 0.013	0.975 ± 0.016	0.963 ± 0.013	0.976 ± 0.015	0.960 ± 0.019
$10^9 \Delta^2_{\mathcal{R}}$	2.43 ± 0.10	2.40 ± 0.10	2.40 ± 0.10	2.45 ± 0.11	2.43 ± 0.11	2.45 ± 0.11
μ_1	(0)	-0.77 ± 0.46	-0.76 ± 0.47	(0)	-1.27 ± 0.74	-1.67 ± 0.86
μ_2	(0)	(0)	-0.39 ± 1.09	(0)	(0)	-3.5 ± 2.7
σ_8(derived)	0.807 ± 0.024	0.825 ± 0.027	0.818 ± 0.032	0.814 ± 0.028	0.841 ± 0.031	0.802 ± 0.040
$\delta z_{dec}/z_{dec}$ [a]	–	-0.6%	-0.7%	–	-1.0%	-1.7%
$\delta\sigma_{z,dec}/\sigma_{z,dec}$ [b]	–	1.5%	-0.5%	–	2.6%	-14.0%
$(\lvert\delta x_e\rvert/x_e)_{max}$ [c]	–	5% ($z \sim 1196$)	5% ($z \sim 1039$)	–	8% ($z \sim 1006$)	31% ($z \sim 1076$)
$\Delta\chi^2$	–	2.5	2.5	–	2.1	2.5

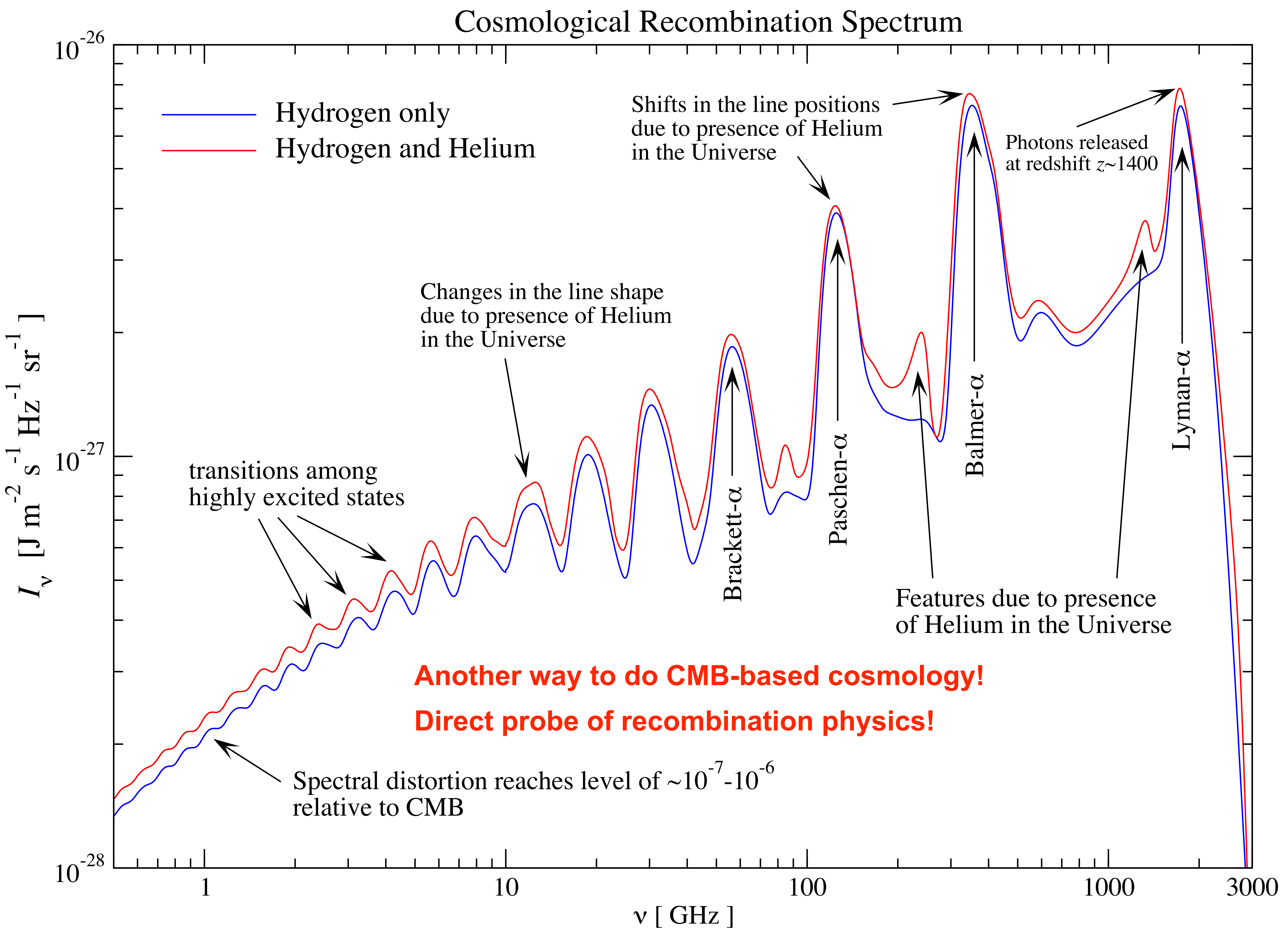
[a] relative change in the redshift of maximum visibility where $z_{dec} = 1088$ is the fiducial maximum visibility point.
[b] relative change in the width of the visibility function.
[c] maximum relative change in the ionization fraction. The redshift corresponding to this maximum change is also included.

- *First mode detected at ~ 2σ*
- *Similar for current Planck data*
- *Effect very similar to the one of helium*
- *In the future 2-3 modes detectable*
- *Can we break the degeneracies???*



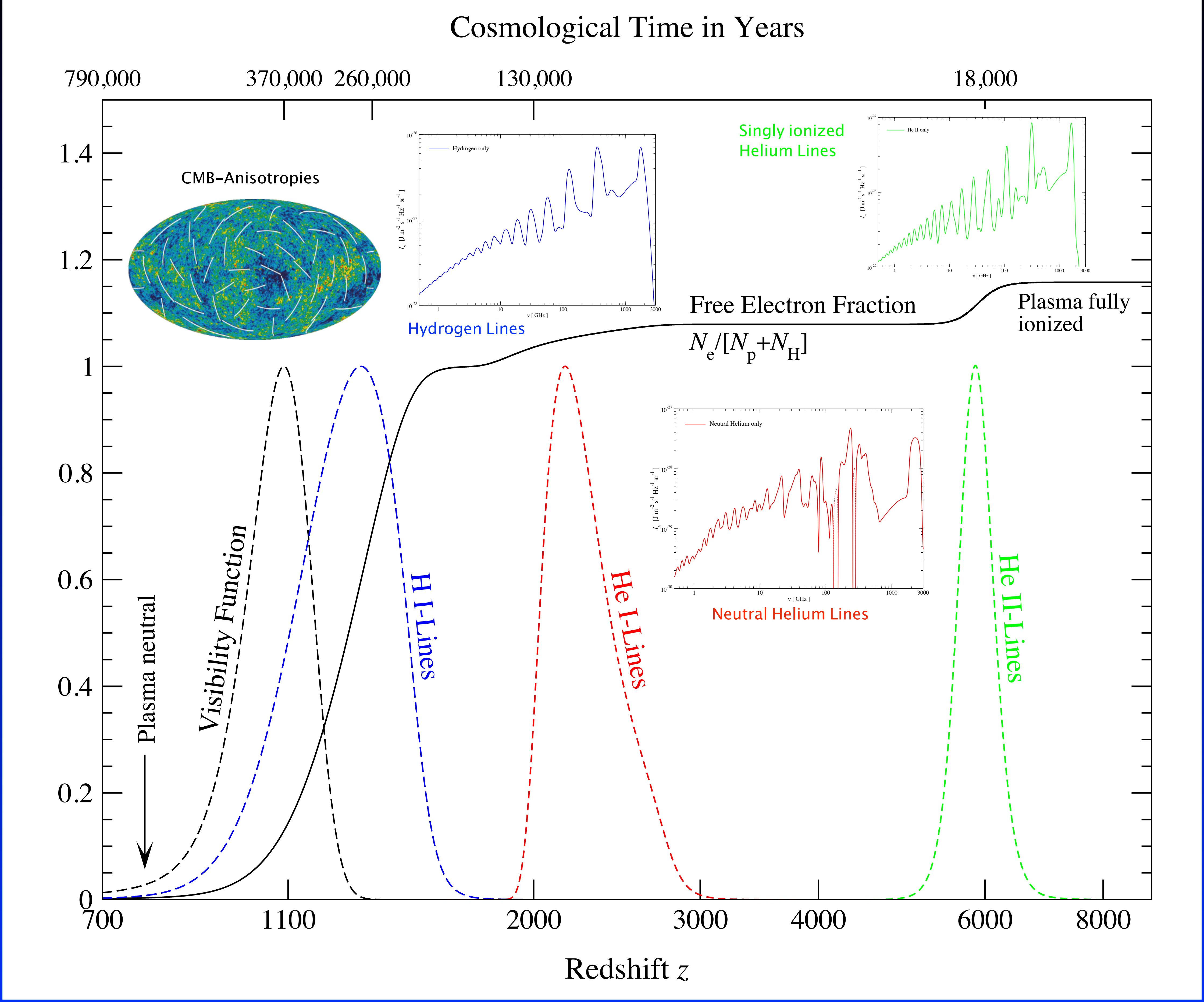
Farhang, Bond & JC, 2012

Can the Cosmological Recombination Radiation help us with this?



Another way to do CMB-based cosmology!

Direct probe of recombination physics!



Cosmological Time in Years
790,000
370,000
260,000
130,000
18,000
CMB-Anisotropies
Hydrogen Lines
Singly ionized
Helium Lines
Free Electron Fraction
$N_e/[N_p+N_H]$
Plasma fully
ionized
Neutral Helium Lines
Plasma neutral
Visibility Function
H I-Lines
He I-Lines
He II-Lines
1.4
1.2
1
0.8
0.6
0.4
0.2
0
700
1100
2000
3000
4000
6000
8000
Redshift z

What would we actually learn by doing such hard job?

Cosmological Recombination Spectrum opens a way to measure:

→ **the specific *entropy* of our universe (related to $\Omega_b h^2$)**

→ **the CMB *monopole* temperature T_0**

→ ***the pre-stellar abundance of helium Y_p***

→ ***If recombination occurs as we think it does, then the lines can be predicted with very high accuracy!***

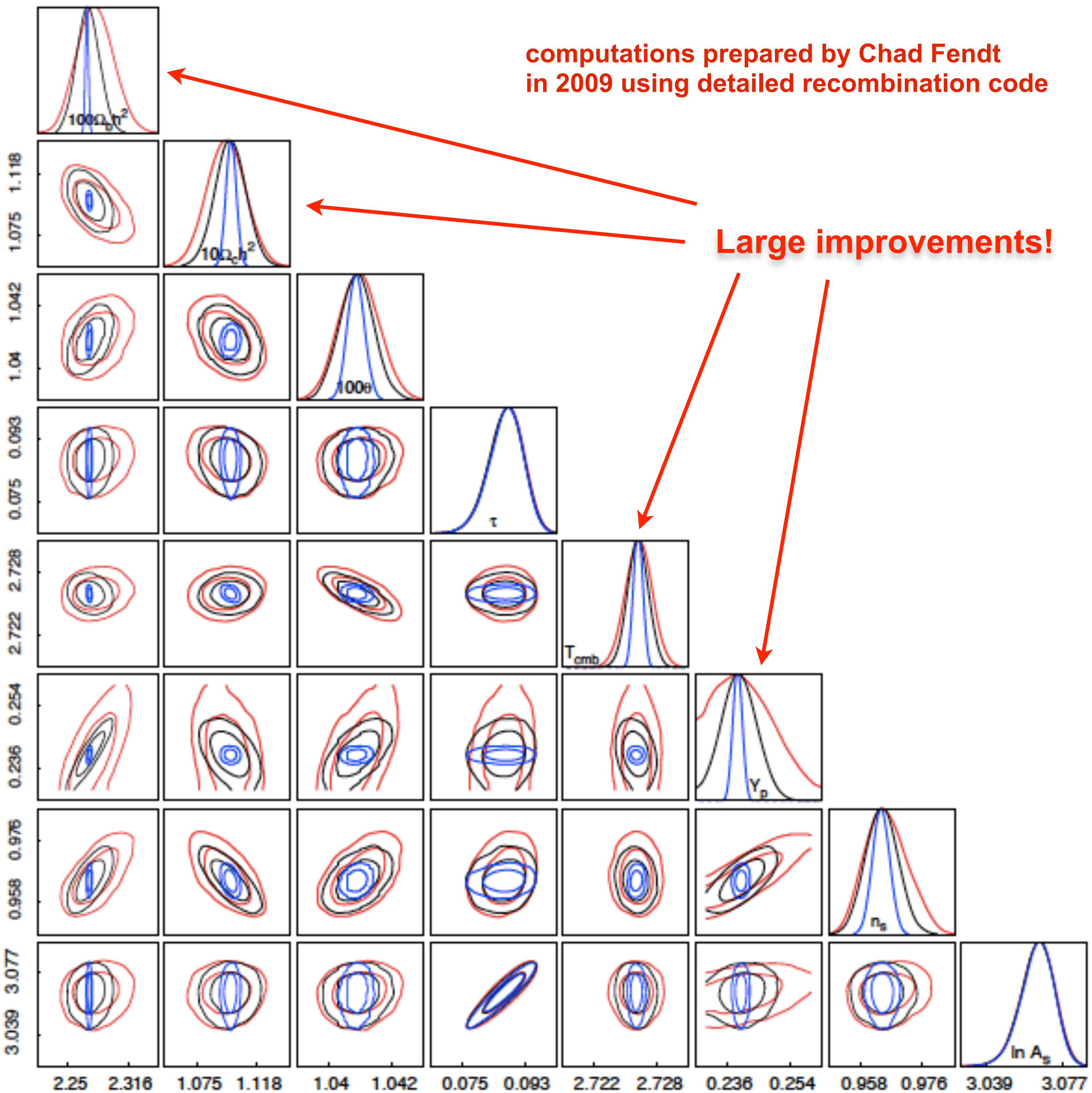


Figure 7.3: The 1 and 2 dimensional marginalized parameter posterior using the CMB spectral distortions. All three cases constrain the CMB power spectrum using a Gaussian likelihood based on Planck noise levels. The black line adds constraints due to a 10% measurement of the spectral distortions, while the blue line assumes a 1% measurement. The red line does not include the data from the spectral distortions.

- CMB based cosmology alone
- Spectrum helps to break some of the parameter degeneracies
- Planning to provide a module that computes the recombination spectrum in a fast way
- detailed forecasts: which lines to measure; how important is the absolute amplitude; how accurately one should measure; best frequency resolution;

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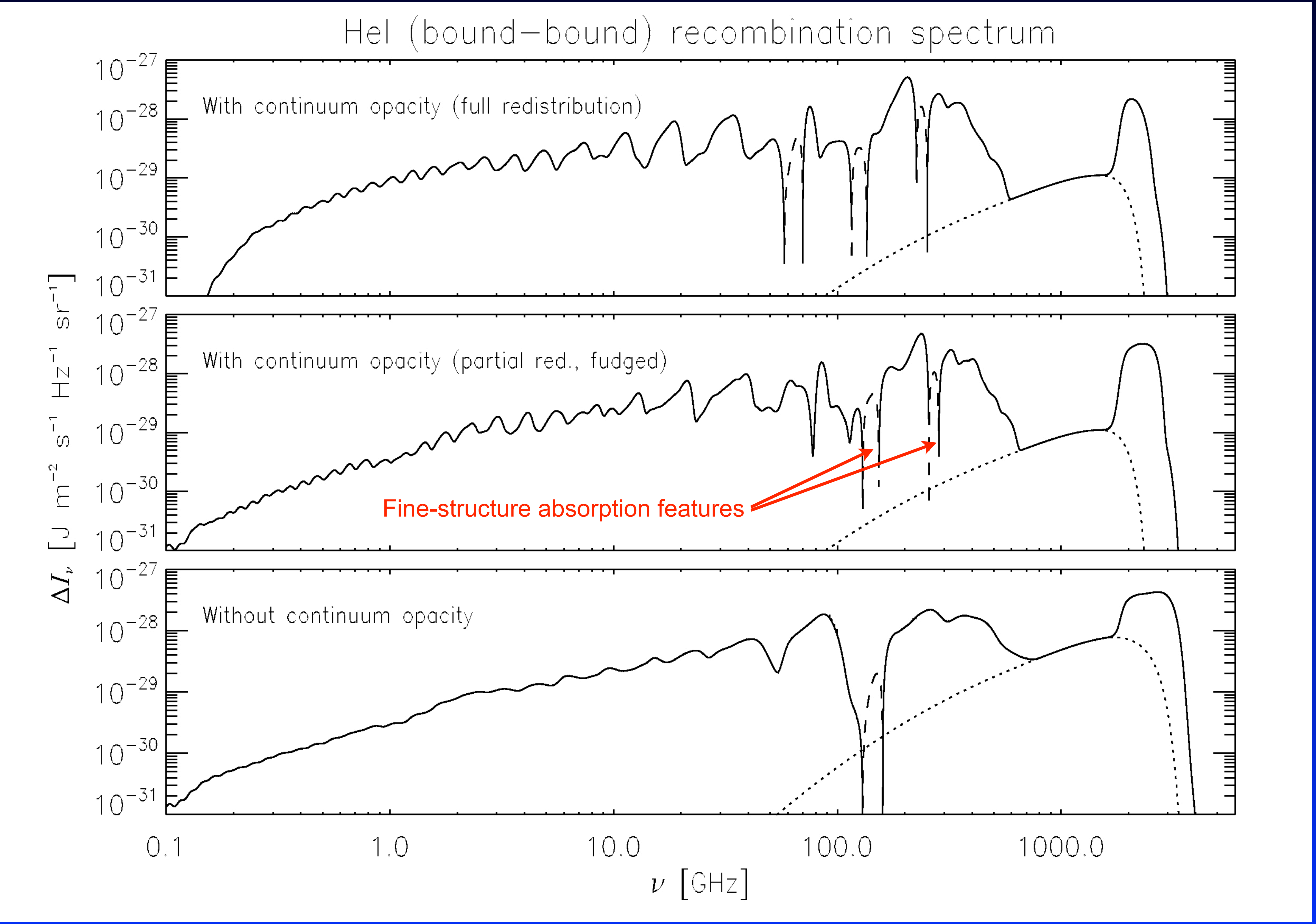
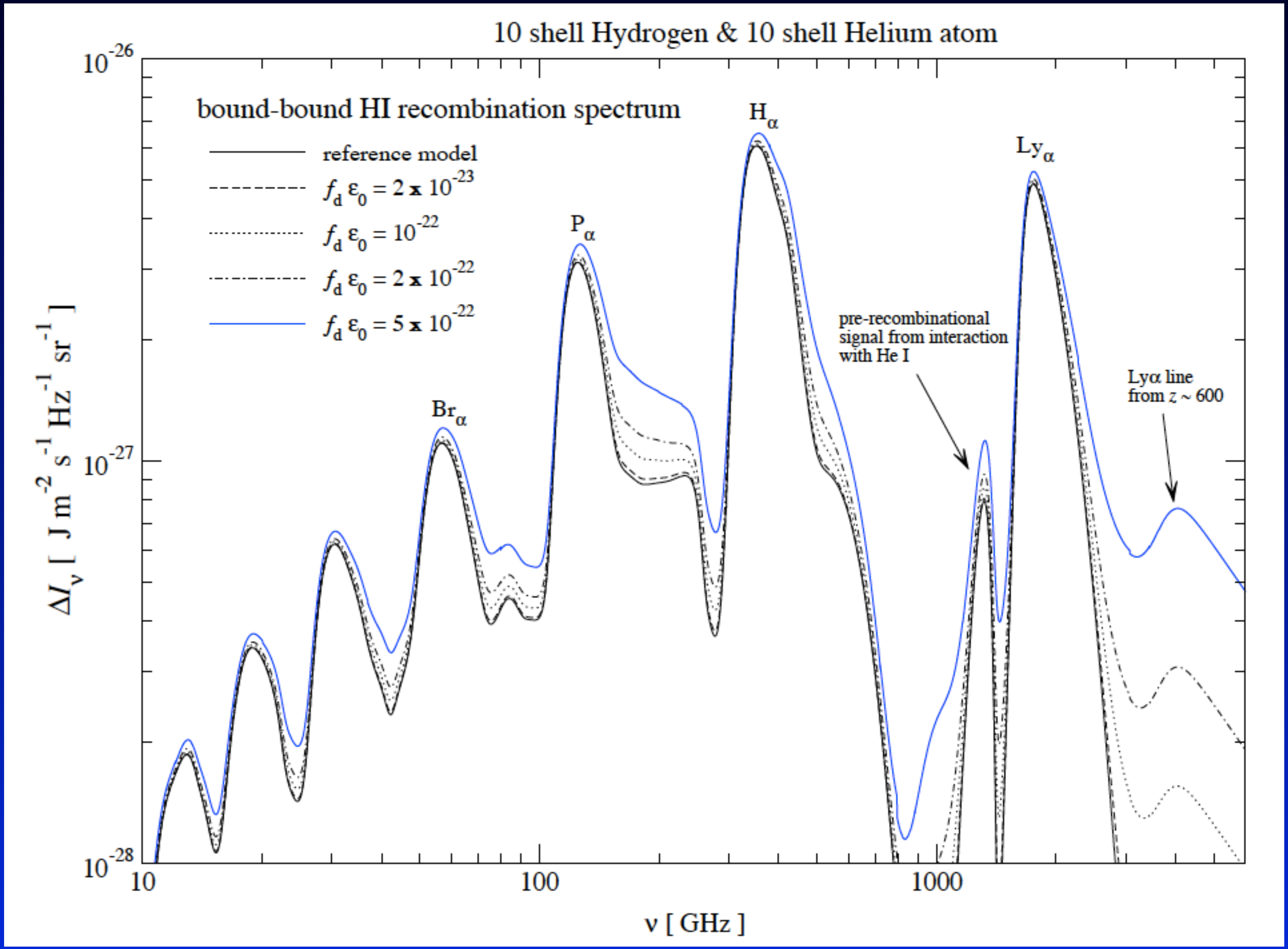
→ ***In principle allows us to directly check our understanding of the standard recombination physics***

The importance of HI continuum absorption



Rubino-Martin, JC & Sunyaev, A&A, 2008

Dark matter annihilations / decays



JC, 2009, arXiv:0910.3663

- Additional photons at all frequencies
- Broadening of spectral features
- Shifts in the positions

What would we actually learn by doing such hard job?

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→ ***In principle allows us to directly check our understanding of the standard recombination physics***

If something unexpected or non-standard happened:

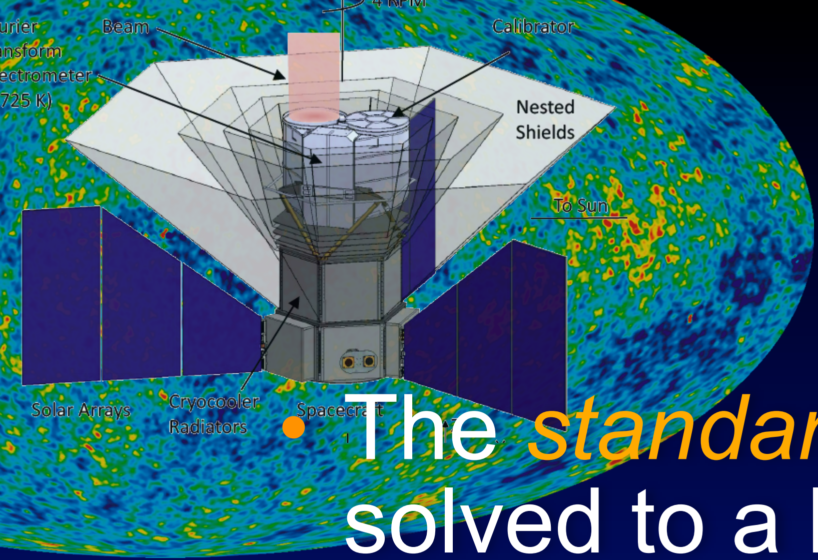
→ ***non-standard thermal histories should leave some measurable traces***

→ ***direct way to measure/reconstruct the recombination history!***

→ **possibility to distinguish *pre-* and *post-recombination y*-type distortions**

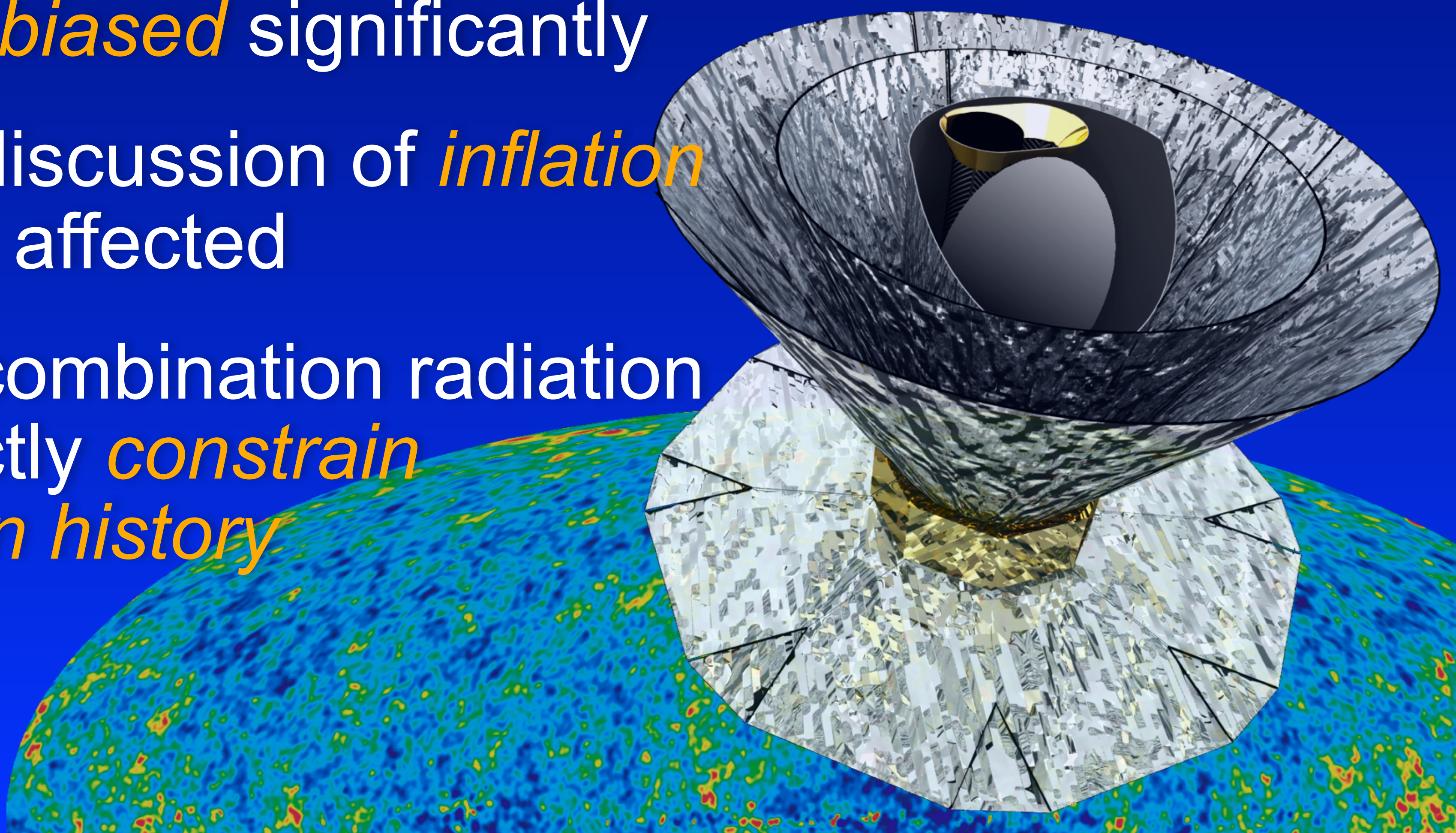
→ ***sensitive to energy release during recombination***

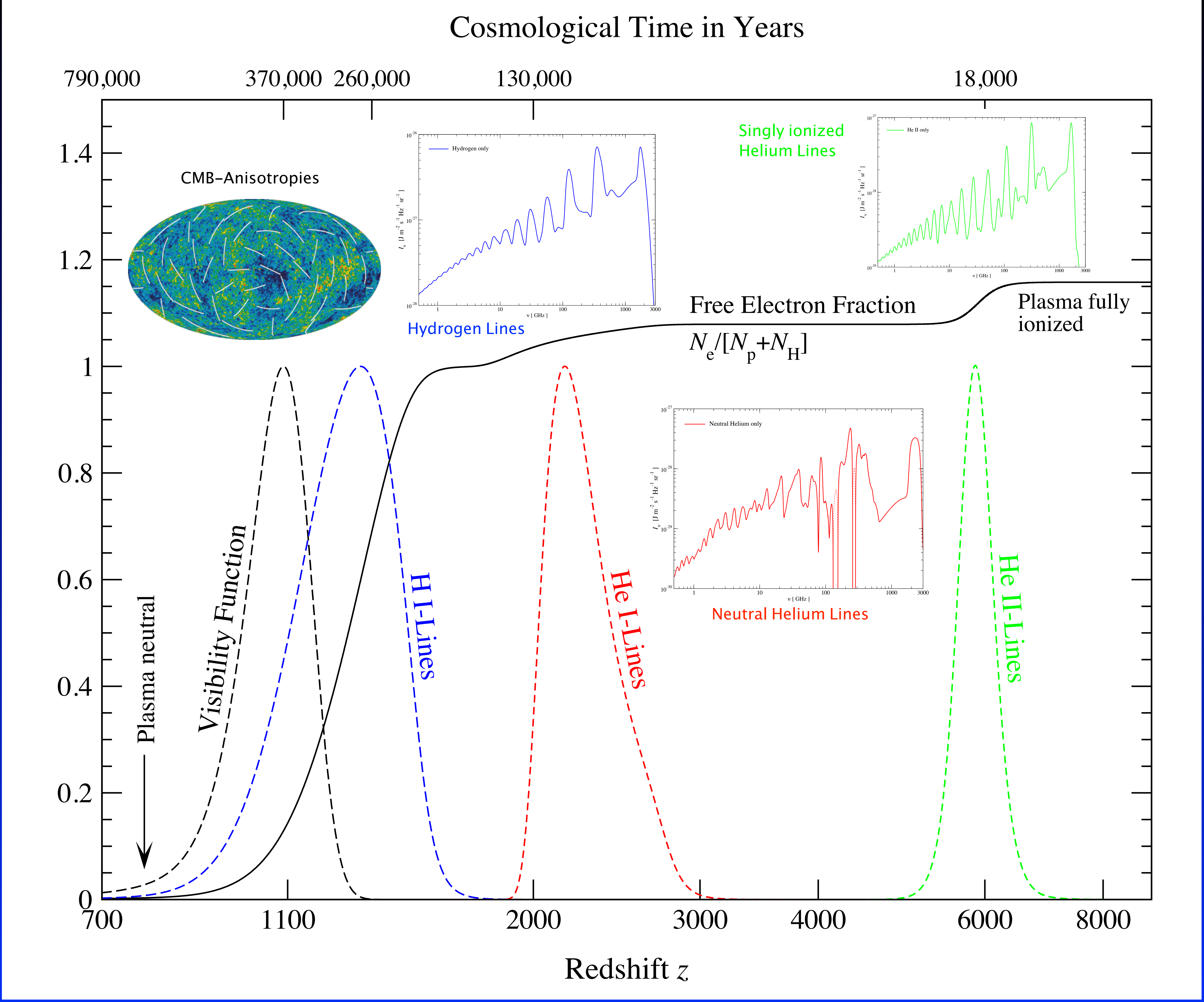
→ **variation of fundamental constants**



Conclusions

- The *standard recombination* problem has been solved to a level that is sufficient for the analysis of current and future CMB data (<0.1% precision!)
- Many people helped with this problem!
- Without the improvements over the original version of Recfast cosmological parameters derived from Planck would be *biased* significantly
- In particular the discussion of *inflation* models would be affected
- Cosmological recombination radiation allows us to directly *constrain* the *recombination history*





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Neutral Helium Lines
Visibility Function
Plasma neutral
H I-Lines
He I-Lines
He II-Lines
Redshift z
700
1100
2000
3000
4000
6000
8000

Recombination exercises

1 Warmup

a) Run Recfast v1.5.2, Recfast++ (with/without correction function) and CosmoRec v1.4.2 for $\Omega_c = 0.216$, $\Omega_b = 0.044$, $\Omega_k = 0$, $T_0 = 2.726\,\mathrm{K}$, $h = 0.7$, $Y_p = 0.24$ and $N_{\rm eff} = 3.046$ (default settings otherwise). Plot the resulting free electron fraction, X_e, and their relative difference, $\Delta X_e/X_e$, (relative to Recfast++ without correction function) as a function of redshift for $z = [0, 3000]$.

b) Repeat a) but now plot the CMB power spectra (TT and EE) and their relative differences for $\ell = [2, 4000]$. Are the differences between Recfast v1.5.2 and CosmoRec v1.4.2 relevant for the cosmological parameters?

c) Repeat a) and b) but change $\Omega_b = 0.02$. Are the differences between Recfast v1.5.2 and CosmoRec v1.4.2 relevant? Any idea what the main cause for the difference is?

2 Exploring some standard CosmoRec options

a) Run CosmoRec v1.4.2 for the cosmology given in 1a) and change the effective number of shells that is included for the hydrogen atom. Plot the free electron fraction, X_e, for some examples and briefly explain the physical reason for the differences in the freeze-out tail.

b) Plot the relative difference in the free electron fraction when including (i) stimulated 2s-1s transitions, (ii) the 1s-2s feedback effect and (iii) both effects. Do you understand the physics behind the differences?

c) Plot the free electron faction around helium recombination with (i) none of the helium corrections, (ii) the spin-forbidden lines on, (iii) H I absorption and diffusion correction included and (iv) feedback among the helium lines included.

d) Plot the relative difference in the TT and EE power spectra with and without all the helium corrections switched on. Are the helium corrections significant?

3 Exploring some non-standard CosmoRec features

a) The heart of the CosmoRec radiative transfer module resides in `./PDE_Problem/` with the main driver `./PDE_Problem/Solve_PDEs.cpp`. Can you plot the high frequency distortion at a few redshifts ($x^3\Delta n$ as a function of x is fine)? Does the figure make sense to you? Can you change the number of outputs in redshift and the frequency resolution?

b) What about the 2s-1s two-photon profile? Do you know how to access it? Also, how about the 5d-1s two-photon profile and the 4s-1s Raman profile? [Hint: have a closer look at `./PDE_Problem/Solve_PDEs.cpp` and be clever with uncommenting things. Also, make sure you included enough hydrogen shells]

c) The main setup for the hydrogen and helium atom models can be found in `./Modules/HI_routines.cpp` and `./Modules/He_routines.cpp`, respectively. If you were interested in atomic transitions rates and recombination rates for certain levels, this would be a good place to start. Can you setup a 30-shell hydrogen atom (make it 100 if you are brave) and compute the vacuum dipole transition rate for $(27, 5) \rightarrow (22, 4)$? How about the recombination rates for $T_e = 3500\,\mathrm{K}$ in a blackbody radiation field at $T_\gamma = 3000\,\mathrm{K}$ to each of these levels? Why does T_γ enter the problem? [Hint: if you want to know how to access those rates check the `./Development/Hydrogenic/Atom.h` header-file. The recombination rate setup also has to be activated]

4 Dark matter annihilation and decay with CosmoRec

a) Run CosmoRec v1.4.2 switching the annihilation efficiency to $f_{\rm ann} = 10^{-23}\,\mathrm{eV\,s^{-1}}$. Illustrate the effect on the free electron faction and CMB power spectra. What happens when you set $f_{\rm ann} = 10^{-22}\,\mathrm{eV\,s^{-1}}$? Any idea how to solve the problem?

b) Repeat a) but using Recfast++ and compare the results. How large are the effects for $f_{\rm ann} = 10^{-22}\,\mathrm{eV\,s^{-1}}$?

c) The dark matter annihilation terms are defined in the file `./Modules/DM_annihilation.cpp`. Can you modify the code to include decaying particles instead? Argue why the effective heating rate for decaying particles can be parametrized as $\mathrm{d}E/\,\mathrm{d}t = f_X \Gamma_X N_H(z)\,\mathrm{e}^{-\Gamma_X t}$, where $\Gamma_X = 1/t(z_X)$ sets the lifetime of the particle and f_X the energy-release efficiency. Plot the free electron fraction for some reasonable values of f_X (estimate the best values or try a bit starting really small) and $z_X = 900$. [Hint: you will need the function `cosmos.t(z)` from the Cosmology-object to obtain the cosmological time as a function of redshift]

d) Plot the final shape of the high frequency distortion for the decaying particle model of 4c) and dark matter annihilation with $f_{\rm ann} = 5 \times 10^{-23}\,\mathrm{eV\,s^{-1}}$. Do you understand the differences?