Impact of the Spin-Wave Interaction in Ferromagnets

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Outline

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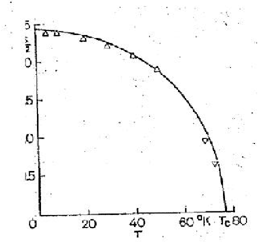
Order parameters as a measure of SSB

Spontaneous magnetization of the ferromagnet *EuO* as a function of temperature:



T_C : Curie temperature

Spontaneous magnetization Σ is an **order parameter**:

$\Sigma \neq 0$: System is in the broken phase ($T < T_C$)

$\Sigma = 0$: System is in the symmetric phase ($T > T_C$)

$\Sigma(T)\,/\,\Sigma(0) \;=\; 1 - \alpha_0 T^{\frac{3}{2}}$ *free magnons* (*Bloch*, 1932)

The problem of the spin-wave interaction in ferromagnets has a very long history

In 1955 I returned to Berkeley for a second summer to work with Charles Kittel. Kittel suggested that I clean up the theory of spin waves. The problem was to resolve a paradox which had arisen in the theory of ferromagnets. On the one hand, the old linear spin-wave theory of Felix Bloch agreed well with experiments. On the other hand, the coupling between the spins in a ferromagnet is strong and non-linear, and various more recent estimates of the effect of spin-coupling had predicted strong deviations from the linear Bloch theory. Three calculations of the spin-coupling effects not only disagreed with Bloch theory but also with one another.

Selected papers of Freeman Dyson with commentary (1996)

Manifestation of spin-wave interaction

- Suggested interaction terms before Dyson: $T^{7/4}$, T^2
- Dyson (1956): *"The method of the present paper settled the disagreement by showing that both calculations were wrong"*
  $\Sigma(T)\,/\,\Sigma(0) \;=\; 1 - \alpha_0 T^{\frac{3}{2}} - \alpha_1 T^{\frac{5}{2}} - \alpha_2 T^{\frac{7}{2}} - \alpha_{\mathbf{3}} \mathbf{T^4} + \mathcal{O}(T^{\frac{9}{2}})$
- Spin-wave **interaction** starts manifesting itself only at $\mathbf{T^4}$
- Terms of order $T^{3/2}$, $T^{5/2}$ and $T^{7/2}$ are related to the shape of the dispersion curve – or the discreteness of the lattice – and describe noninteracting magnons
- Motivation: Calculation within the systematic and universal effective Lagrangian method even beyond Dyson
- Manifestation of spin-wave **interaction** beyond order $\mathbf{T^4}$ unclear: $\mathbf{T^5}$, $\mathbf{T^{13/2}}$, $\mathbf{T^{15/2}}$

Outline

Spontaneous symmetry breaking in (anti)ferromagnets

Heisenberg model:

$$\mathcal{H}_0 \; = \; - J \sum_{n.n.} \vec{S}_m \cdot \vec{S}_n , \qquad J = const.$$

J : Exchange integral

$J > 0$: *Ferromagnetic* ground state

$J < 0$: *Antiferromagnetic* ground state (Néel state)

Symmetry of Hamiltonian $\neq$ **Symmetry of ground state**:

- Hamiltonian $\mathcal{H}_0$ is invariant under the group $\mathrm{G} = \mathrm{O}(3)$ describing rotations in the space of the spin variables
- Ground state is only invariant under the subgroup $\mathrm{H} = \mathrm{O}(2)$ describing rotations around the third axis

Spontaneous symmetry breaking $\mathrm{O}(3) \to \mathrm{O}(2)$

Goldstone theorem in the nonrelativistic domain

Goldstone theorem for nonrelativistic systems (Lange, 1965):
Spontaneous symmetry breaking in nonrelativistic systems leads to low-energy excitations in the spectrum whose energy tends to zero for large wavelengths: $\omega \to 0$ *for* $|\vec{k}| \to 0$

- Only the number of Goldstone boson **fields** is determined by $n_G - n_H$
- However, the **exact form of the dispersion relation** for these Goldstone bosons as well as the **number of** these **particles** depend on the properties of the nonrelativistic system

These low-energy excitations in magnetic systems are well-known:

Spin waves or **magnons** as collective fluctuations of the spins

Ferromagnet: $E \propto \vec{p}^2$ Antiferromagnet: $E \propto |\vec{p}|$

Explicit versus spontaneous symmetry breaking

Extension of the Heisenberg model:

$$\mathcal{H} \; = - J \sum_{n.n.} \vec{S}_m \cdot \vec{S}_n \; - \; \mu \sum_{n} \vec{S}_n \cdot \vec{H} \; = \; \mathcal{H}_0 + \mathcal{H}_{sb}$$

$\mathcal{H}_0$ is invariant under the rotation group $\mathrm{G} = \mathrm{O}(3)$

$\mathcal{H}_{sb}$ explicitly breaks the symmetry G

Magnetic field $\vec{H}$ is a symmetry breaking parameter, much like the quark masses in the QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2g^2} \mathrm{tr}_c G_{\mu\nu} G^{\mu\nu} + \bar{q} i \gamma^\mu D_\mu q - \bar{q} m q$$

Small m_q, weak $\vec{H}$: $\mathcal{H}_{sb}$ can be treated as a perturbation

Construction of effective Lagrangians

- Construction of effective theories via symmetry analysis, in particular of the spontaneously broken symmetry
- Weinberg (1979): If one writes down the most general possible Lagrangian, including all terms consistent with the assumed symmetries, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetries
- The degrees of freedom in the effective Lagrangian are the Goldstone bosons

Chiral perturbation theory (CHPT)

Expansion of scattering amplitudes in powers of **momentum** $\Leftrightarrow$ **derivative** expansion of the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} \;=\; \mathcal{L}_{\text{eff}}(\pi, \partial\pi, \partial^2\pi, \dots, m) \;=\; \mathcal{L}^2_{\text{eff}} + \mathcal{L}^4_{\text{eff}} + \mathcal{L}^6_{\text{eff}} + \dots$$

Power counting: $\partial_\mu \propto p, \;\; m \propto p^2 \;\; (M^2[\pi] \propto m)$

- Effective Lagrangian at *leading order* (order p^2):

$$\mathcal{L}^2_{\text{eff}} = \tfrac{1}{4} F^2_\pi \, \text{tr} \, \{\partial_\mu U \partial^\mu U^+ + \chi(U + U^+)\}, \qquad \chi = 2Bm$$

$$U = \exp(2i\pi(x)/F_\pi)$$

$\pi(x)$ contains the Goldstone boson octet

$\mathcal{O}(p^2)$: Two effective constants: F_π (Pion decay constant), B

Chiral perturbation theory (CHPT)

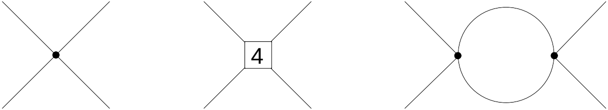
Effective Lagrangian at *next-to-leading order* (order p^4):

$$\mathcal{L}^4_{\text{eff}} = L_1 \langle D_\mu U D^\mu U^\dagger \rangle^2 + \dots + L_5 \langle D_\mu U D^\mu U^\dagger (\chi U^\dagger + U \chi^\dagger) \rangle + \dots$$

- $\mathcal{O}(p^4)$: 10 low-energy constants $L_1, L_2, \dots$
- Effective constants parametrize the physics of the underlying theory (QCD)
- Effective constants have to be determined by experiment (e.g. pion-pion scattering) or in numerical simulations

Momentum expansion and power counting in CHPT

- Derivative expansion of the effective Lagrangian corresponds to an expansion in the momenta or temperature
- Example: Pion-Pion scattering



1a 1b 1c

- Tree graph of order p^2 is finite
- Loops in d=3+1 are suppressed by two powers of momentum
- Divergences in one-loop graph 1c of order p^4 are absorbed into coupling constants of order p^4 graph 1b
- At a given order in the derivative expansion only a finite number of diagrams and coupling constants contribute

Loop suppression: Dispersion relation and space dimension

- Consider the Goldstone boson loop

$$\int \frac{\mathrm{d}E\,\mathrm{d}^{d_s}p}{E^2-\vec{p}^{\,2}} \;\propto\; \mathcal{P}^{d_s-1} \qquad\qquad \int \frac{\mathrm{d}E\,\mathrm{d}^{d_s}p}{E-\vec{p}^{\,2}} \;\propto\; \mathcal{P}^{d_s}$$

- Lorentz-invariant framework
  - d=3+1: Loops are suppressed by two powers of momentum
  - d=2+1: Loops are suppressed by one power of momentum
- Nonrelativistic framework: Ferromagnet with $E \propto \vec{p}^{\,2}$
  - d=3+1: Loops are suppressed by three powers of momentum
  - d=2+1: Loops are suppressed by two powers of momentum
  - d=1+1: Loops are suppressed by one power of momentum
- **Analogy**: Two-dimensional ferromagnets correspond to QCD
  Difference: Derivative expansion fails in d=1+1 in the case of Lorentz-invariant theories, but ferromagnetic spin-chains are accessible by effective field theory

The nonrelativistic domain

Extension of the effective Lagrangian method to condensed matter is nontrivial since nonrelativistic systems single out a preferred frame of reference: the rest frame $\quad \rightarrow$ **Lorentz non-invariance**

Closer look at the relation: Symmetries of the underlying theory $\Longleftrightarrow$ Symmetries of the effective theory:

- *Underlying* theory: **Global** symmetries ($SU(3)_L \times SU(3)_R$, $O(3)$) $\rightarrow$ conserved currents (Noether theorem)
- *Effective* theory: Invariance under the same global symmetries $\rightarrow$ same conserved Noether currents

This **assumption**, however, is too strong: $\mathcal{L}_{\text{eff}}$ may not be invariant under the symmetry G, but pick up a *total derivative*; the action $\mathcal{S}_{\text{eff}} = \int d^4x \, \mathcal{L}_{\text{eff}}$ would still be invariant and lead to the required currents

Invariance theorem

Invariance theorem (Leutwyler, 1994):
For Lorentz-invariant systems, the effective Lagrangian shares the symmetries of the Lagrangian of the underlying theory

This statement is no longer true in the nonrelativistic domain: Nonrelativistic effective Lagrangians may violate gauge invariance

Rigorous analysis must rely on an analysis of the *Ward identities* obeyed by the Green functions of the currents, which represent the symmetry properties of the underlying theory on a **local** level

Generating functional $\Gamma\{f\}$ collects all the information of the Green functions of the currents $J^{\mu}_{i}(x)$ in compact form:

$$e^{i\,\Gamma\{f\}} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \dots d^4x_n \; f^{i_1}_{\mu_1}(x_1) \dots f^{i_n}_{\mu_n}(x_n) \\ \times <0|\; T\{J^{\mu_1}_{i_1}(x_1) \dots J^{\mu_n}_{i_n}(x_n)\}\;|0>$$

$f^{i}_{\mu}(x)$: External fields coupled to the currents $J^{\mu}_{i}(x)$

Systematic construction of effective Lagrangians

$$
\begin{aligned}
\mathcal{L}^{(0,1)}_{\text{eff}} &= c_a(\pi)\dot{\pi}^a + e_i(\pi) f^i_0 \\
\mathcal{L}^{(2,0)}_{\text{eff}} &= -\tfrac{1}{2} g_{ab}(\pi)\, \partial_r \pi^a \partial_r \pi^b + h_{ai}(\pi) f^i_r \partial_r \pi^a - \tfrac{1}{2} k_{ik}(\pi) f^i_s f^k_s \\
\mathcal{L}^{(0,2)}_{\text{eff}} &= \tfrac{1}{2} \bar{g}_{ab}(\pi)\, \dot{\pi}^a \dot{\pi}^b - \bar{h}_{ai}(\pi) f^i_0\, \dot{\pi}^a + \tfrac{1}{2} \bar{k}_{ik}(\pi) f^i_0 f^k_0
\end{aligned}
$$

$c_a(\pi), e_i(\pi), g_{ab}(\pi), h_{ai}(\pi), \dots$: Low-energy couplings

$a, b, c = 1, \dots, n_G - n_H$: Components of the effective field

$i, j, k = 1, \dots, n_G$: Generators of the group G

$r, s, t = 1, 2, 3$: Spatial coordinates

Main novelty in the nonrelativistic domain: Couplings $c_a(\pi)$, $e_i(\pi)$

Value of $e_i(\pi)$ at $\pi = 0$ corresponds to a term linear in f^i_0:

$$< 0 | J^0_i(x) | 0 > = e_i(0)$$

Magnetic systems: Third component of total spin operator is related to third component of the charge density operator by

$$\sum_n S_n^3 \;=\; Q_3 \;=\; \int d^3x J_3^0(x) \qquad \Longleftrightarrow \qquad NS \;=\; \langle 0| \, J_3^0 \, |0\rangle \; V$$

N : Total number of lattice sites
S : Highest eigenvalue of the spin operator S_n^3
V : Volume of the entire crystal

$$\rightarrow \; \langle 0| \, J_i^0 \, |0\rangle \;=\; \delta_i^3 \, (NS/V) \;=\; \delta_i^3 \, \Sigma$$

Σ : Spontaneous magnetization

Ward identities and equation of motion have to be compatible with the derivative expansion of the effective Lagrangian:

Ferromagnet $(\Sigma \neq 0) : e_i(0) \;\rightarrow\; e_i(\pi) \;\rightarrow\; c_a(\pi)$

Antiferromagnet $(\Sigma = 0) : e_i(0) = 0 \;\rightarrow\; \underline{e_i(\pi), \; c_a(\pi) = 0}$

Leading order effective Lagrangians

Spin-waves are fluctuations of the magnetization vector $\vec{U}$:
$\vec{U} = (U^1, U^2, U^3) = (\pi^1, \pi^2, \sqrt{1-\pi^a\pi^a}), \qquad |\vec{U}| = 1$

$$\mathcal{L}^F_{\text{eff}} = \Sigma \frac{\partial_0 U^1 U^2 - \partial_0 U^2 U^1}{1 + U^3} + \Sigma H^i U^i - \tfrac{1}{2} F^2 D_r U^i D_r U^i$$

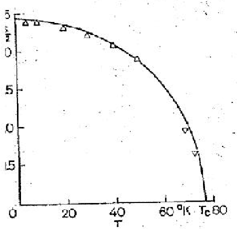
$$\mathcal{L}^{AF}_{\text{eff}} = \tfrac{1}{2} F_1^2 D_0 U^i D_0 U^i - \tfrac{1}{2} F_2^2 D_r U^i D_r U^i, \quad D_\mu U^i = \partial_\mu U^i + \varepsilon_{ijk} f^j_\mu U^k$$

- Ferromagnet: Spontaneous magnetization Σ shows up as a leading-order effective constant of a term involving one time derivative only which dominates the dynamics
- *Dispersion relation* for spin waves:
  Quadratic for a ferromagnet: $\omega = \gamma \vec{k}^2, \qquad \gamma = F^2/\Sigma$
  Linear for an antiferromagnet: $\omega = v|\vec{k}|, \qquad v = F_2/F_1$

Outline

Spontaneous magnetization

Spontaneous magnetization of the three-dimensional ferromagnet *EuO* as a function of temperature:



$\Sigma(T) \,/\, \Sigma(0) \; = \; 1 - \alpha_0 T^{\frac{3}{2}}$ *free magnons* (*Bloch*, 1932)

At what order does the **spin-wave interaction** manifest itself in the low-temperature expansion of the spontaneous magnetization $\Sigma(T)$?

Finite temperature effective field theory

Partition function is represented as Euclidean functional integral

$$\mathrm{Tr}\,[\exp(-\mathcal{H}/T)] \,=\, \int [\mathrm{d}U] \, \exp\Big(- \int_{\mathcal{T}} \mathrm{d}^4 x \, \mathcal{L}_{eff} \Big) \, ,$$

where the integration is performed over all field configurations which are periodic in the Euclidean time direction:
$U(\vec{x}, x_4 + \beta) = U(\vec{x}, x_4)$, with $\beta \equiv 1/T$

The periodicity condition manifests itself in the thermal propagator

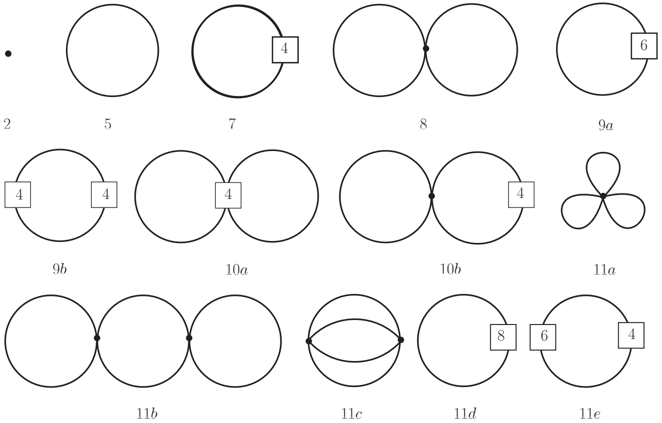
$$G(x) \,=\, \sum_{n=-\infty}^{\infty} \Delta(\vec{x}, x_4 + n\beta)$$

We work in the infinite volume limit

$$z \,=\, - \, T \lim_{L \to \infty} L^{-3} \ln \, [\mathrm{Tr} \exp(-\mathcal{H}/T)]$$

3D ferromagnets: Partition function diagrams

Loops are suppressed by three powers of momentum:



- Communication of graphs (UV divergences), LEC's, T-powers (Ferromagnet: $p^2 \propto E \propto T$)

Effective Lagrangian for ferromagnets

Pieces of the effective Lagrangian for the ferromagnet required up to three-loop order:

$$
\begin{aligned}
\mathcal{L}^2_{eff} &= \Sigma \frac{\varepsilon_{ab} \partial_0 U^a U^b}{1+U^3} + \Sigma \mu H U^3 - \tfrac{1}{2} F^2 \partial_r U^i \partial_r U^i \\
\mathcal{L}^4_{eff} &= l_1 (\partial_r U^i \partial_r U^i)^2 + l_2 (\partial_r U^i \partial_s U^i)^2 + l_3 \Delta U^i \Delta U^i \\
\mathcal{L}^6_{eff} &= c_1 \, U^i \Delta^3 U^i \\
\mathcal{L}^8_{eff} &= d_1 \, U^i \Delta^4 U^i
\end{aligned}
$$

- Idealization: Space rotation invariance also at subleading orders of the effective Lagrangian

3D Ferromagnet: Spontaneous magnetization

$$\frac{\Sigma(T)}{\Sigma} = 1 - \alpha_0 T^{\frac{3}{2}} - \alpha_1 T^{\frac{5}{2}} - \alpha_2 T^{\frac{7}{2}} - \alpha_3 \mathbf{T^4} - \alpha_4 \mathbf{T^{\frac{9}{2}}} + \mathcal{O}(\mathbf{T^5})$$

$$\alpha_4 = \frac{945}{64\pi^{\frac{3}{2}}\Sigma^2\gamma^{\frac{11}{2}}} \left(d_1 - 11 l_3 c_1 F^{-2}\right) \zeta(\tfrac{9}{2}) - \frac{\Sigma^{\frac{5}{2}}}{2F^9} j_2$$

$$j_2 = -0.000167$$

- Half-integer T-powers in odd space dimensions
- No logarithmic terms in this series
- This series has a very long history: Impact of spin-wave interaction: Bloch (1932), Dyson (1956), CPH (2011)
- Earlier attempts:
  $\mathbf{T^4}$: $T^{7/4}$, T^2, $T^{9/4}$, T^3 $\quad$ $\mathbf{T^{9/2}}$: T^5, $T^{13/2}$, $T^{15/2}$

Spurious cubic term in the spontaneous magnetization

There is no *interaction* term of order T^3 in the low-temperature expansion for the spontaneous magnetization of the three-dimensional ferromagnet:

- $\mathcal{O}(T^4)$ in the free energy density: Two-loop graph 8:

$$z_8 \propto \Big[\partial_r G(x)\Big]_{x=0} \Big[\partial_r G(x)\Big]_{x=0} = 0$$

- Space rotation invariance of the thermal propagator (and the effective Lagrangian) excludes a T^4-term in the free energy density or a T^3-term in the spontaneous magnetization

Scales in QCD and 3D ferromagnets

Low-temperature series of the order parameters:

$$\langle \bar{q}q \rangle(T, m_q)/\langle 0|\bar{q}q|0\rangle = 1 - \beta_0 T^2 - \beta_1 \mathbf{T^4} - \beta_2 \mathbf{T^6 \ln T} + \mathcal{O}(\mathbf{T^8})$$

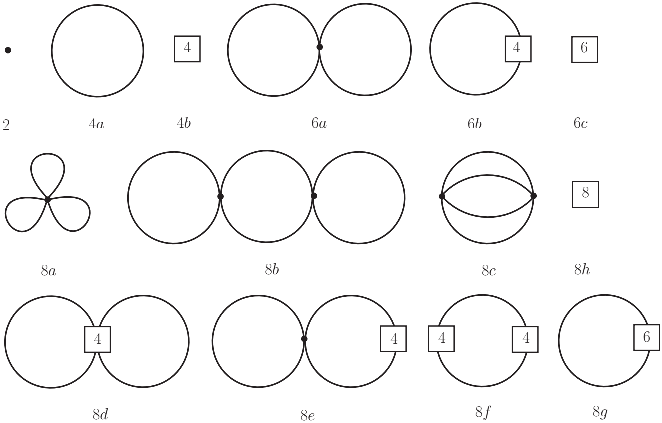
$$\Sigma(T)/\Sigma = 1 - \alpha_0 T^{\frac{3}{2}} - \alpha_1 T^{\frac{5}{2}} - \alpha_2 T^{\frac{7}{2}} - \alpha_3 \mathbf{T^4} - \alpha_4 \mathbf{T^{\frac{9}{2}}} + \mathcal{O}(\mathbf{T^5})$$

In the chiral limit ($m_q \to 0$) and in zero magnetic field, the leading coefficients read $\beta_0 = 1/8\mathcal{F}^2$ and $\alpha_0 = \zeta(\frac{3}{2})/8\pi^{\frac{3}{2}}\Sigma\gamma^{\frac{3}{2}}$

- The corresponding temperature scales differ in more than ten orders of magnitude:
  $\Lambda^T_{\mathrm{QCD}} = \sqrt{8}\mathcal{F} \approx 250\mathrm{MeV} \qquad \Lambda^T_{\mathrm{F}} = {\alpha_0}^{-2/3} \approx 10\mathrm{meV}$
- For temperatures small compared to Λ^T_{QCD} and Λ^T_{F}, the low-temperature series are perfectly valid
- The situation in 2D and 1D is more subtle (Mermin-Wagner)

2D ferromagnet and QCD: Partition function

Loops are suppressed by two powers of momentum:



- Communication of graphs (UV divergences), LEC's, T-powers (QCD: $p \propto E \propto T$, Ferromagnet: $p^2 \propto E \propto T$)

2D ferromagnet and QCD: Pressure and order parameter

Pressure: (All interaction contributions are boldfaced)

$$P_{\mathrm{QCD}} = b_0 T^4 + \mathbf{b_1 T^6} + \mathbf{b_2 T^8 \ln T} + \mathcal{O}(\mathbf{p^{10}})$$

$$P_{Ferro2D} = \hat{a}_0 T^2 + \hat{a}_1 T^3 + \mathbf{\hat{a}_2^A T^4} + \mathbf{\hat{a}_2^B T^4 \ln T} + \mathcal{O}(\mathbf{p^{10}})$$

Order parameters: Quark condensate and magnetization:

$$\langle \bar{q}q \rangle (T, m_q) = \partial z / \partial m_q \qquad \Sigma(T, H) = -\partial z / \partial(\mu H)$$

$$\langle \bar{q}q \rangle (T, m_q) / \langle 0 | \bar{q}q | 0 \rangle = 1 - \beta_0 T^2 - \boldsymbol{\beta_1 T^4} - \boldsymbol{\beta_2 T^6 \ln T} + \mathcal{O}(\mathbf{T^8})$$

$$\Sigma(T, H)/\Sigma_{Ferro2D} = 1 - \hat{\alpha}_0 T - \hat{\alpha}_1 T^2 - \boldsymbol{\hat{\alpha}_2^A T^3} + \boldsymbol{\hat{\alpha}_2^B T^3 \ln T} + \mathcal{O}(\mathbf{T^4})$$

- **Analogy**: Structure of T-powers, chiral logarithms, logarithms in the low-temperature series
- **Difference**: Magnitude of the corresponding low-energy scales

Occurrence of logarithmic terms

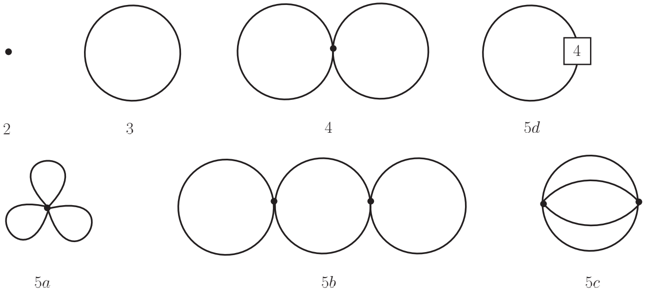
As in QCD, the cateye-graph for 2D ferromagnets is logarithmically divergent:

$$T^{d_s+2} (\mu H)^{\frac{d_s-2}{2}} \left\{ \sum_{n=1}^{\infty} \frac{e^{-\mu H n/T}}{n^{\frac{d_s+2}{2}}} \right\}^2 \Gamma(1-\frac{d_s}{2})$$

- Dimensionally regularized in the spatial dimension d_s
- UV-singularity of cateye-graph 8c is absorbed by the LEC's l_1 and l_2 from $\mathcal{L}^4_{eff}$ contained in the two-loop graphs 8d and 8e
- Logarithmic renormalization of effective constants implies *chiral logarithms*: $\log(H/\mu)$ and $\log(M/\mu)$
- In the limit $\vec{H} \to 0$ (much like in the chiral limit $m_q \to 0$), the cateye graph diverges logarithmically. As a consequence terms $\propto T^n \, lnT$ emerge in the low-temperature series

Ferromagnetic spin chains: Partition function

Loops are still suppressed by one power of momentum:



- Only very few effective constants are required
- Effective loop expansion in one space dimension does not work in a Lorentz-invariant framework

Ferromagnetic spin chains: Pressure and Magnetization

$$P = \tilde{a}_0 T^{\frac{3}{2}} + \mathbf{\tilde{a}_1 T^{\frac{5}{2}}} + \mathcal{O}(\mathbf{T^3})$$

$$\frac{\Sigma(T)}{\Sigma} = 1 - \tilde{\alpha}_0 T^{\frac{1}{2}} - \tilde{\alpha}_{\mathbf{1}} \mathbf{T^{\frac{3}{2}}} + \mathcal{O}(\mathbf{T^2})$$

- No logarithmic terms in odd space dimensions
- Half-integer T-powers in odd space dimensions
- Spin-wave interaction already at next-to-leading order
- Spin-wave interaction in the pressure is repulsive ($\mathbf{\tilde{a}_1 > 0}$)
- The impact of the spin-wave interaction in ferromagnetic spin chains was conclusively solved only very recently [CPH (2013)]

Outline

1. Motivation
2. From the underlying to the effective field theory
   - Symmetry breaking and Goldstone's theorem
   - Relativistic effective Lagrangians
   - Nonrelativistic effective Lagrangians
3. Low-temperature properties of ferromagnets
   - Ferromagnets in three space dimensions
   - Analogies between 2D ferromagnets and QCD
   - Ferromagnetic spin chains
4. Conclusions

Summary

- The low-energy dynamics of ferromagnets is governed by a term which involves one time derivative only
- The structure of the low-temperature series is an immediate consequence of the symmetries inherent in the underlying theory (Heisenberg Hamiltonian, QCD Lagrangian)
- Although ferromagnets follow nonrelativistic kinematics, 2D ferromagnets also exhibit logarithmic terms in T like QCD
- The scales in QCD and ferromagnets differ in about ten orders of magnitude. Still the effective field theory captures the low-T behavior of both systems in a systematic way
- In the nonrelativistic domain the effective Lagrangian method even works in one space dimension

Virtues of the effective Lagrangian method

- The effective Lagrangian method addresses the problem from a model-independent point of view based on symmetry. Any microscopic information is contained in the numerical values of a few LEC's which parametrize the microscopic details
- The effective calculation has been systematically worked out up to three-loop order for ferromagnets, i.e., beyond the reach of any other approach involving conventional condensed matter methods (spin-wave theory, Schwinger boson mean field theory, Green's function approach, etc.)