Effective Field Theories and Transport Coefficients in Cold Superfluids

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with M.A. Escobedo, and M. Mannarelli, Phys. Rev. A79, 063623 (2009) arXiv:0904.3023 [cond-mat.quant-gas]

with F.J. Llanes-Estrada JCAP 0708, 001 (2007) arXiv:0705.3909 [hep-ph]

with A. Dobado and F. J. Llanes-Estrada. JHEP 0509,076(2005) hep-ph/0406058

Superfluidity

- Quantum phenomenon associated to the appearance of a quantum condensate = SSB of a global U(1)

$$\psi = |\psi| e^{-i\varphi} \qquad \qquad \langle\psi\psi\rangle = |\langle\psi\psi\rangle| e^{-2i\varphi}$$

$$\mathbf{j} = -\frac{i}{2m}\left(\psi^\dagger \nabla\psi - \nabla\psi^\dagger\psi\right)$$

$$\mathbf{j} = \rho_s \mathbf{v_s} \ , \qquad \mathbf{v_s} = \frac{\nabla\phi}{m}$$

- Hydrodynamics complicated: two-fluid model

superfluid component: no dissipation

normal component: dissipative processes are possible

$$\rho = \rho_n + \rho_s \qquad \mathbf{j} = \rho_n \mathbf{v_n} + \rho_s \mathbf{v_s}$$

- Hydrodynamical eqs: conservation laws + eq. for $\mathbf{v}_s$

$$\partial_t \mathbf{v_s} + \nabla \left(\mu + \frac{\mathbf{v}_s^2}{2} \right) = 0$$

- One can define more transport coefficients than in a standard fluid $(\kappa, \eta, \zeta_1, \zeta_2, \zeta_3)$ $\zeta_1^2 \leq \zeta_2 \zeta_3$

$$\begin{aligned} P &= P_{\rm eq} - \zeta_1 {\rm div}(\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \zeta_2 {\rm div}\, \mathbf{v_n} \\ \mu &= \mu_{\rm eq} - \zeta_3\, {\rm div}(\rho_s(\mathbf{v_n} - \mathbf{v_s})) - \zeta_4 {\rm div}\, \mathbf{v_n} \end{aligned}$$

$$\zeta_1 = \zeta_4$$

- Superfluids in rotation: appearance of quantized vortices

$$\oint \mathbf{v}_s \cdot d\mathbf{l} = n\kappa \ , \qquad \kappa = \frac{h}{m}$$

Superfluids we are interested in

- He4 (bosonic)
- Quark and neutron superfluid matter
  <qq> <nn>
- Ultracold Fermi atoms at unitarity

Fermions at unitarity

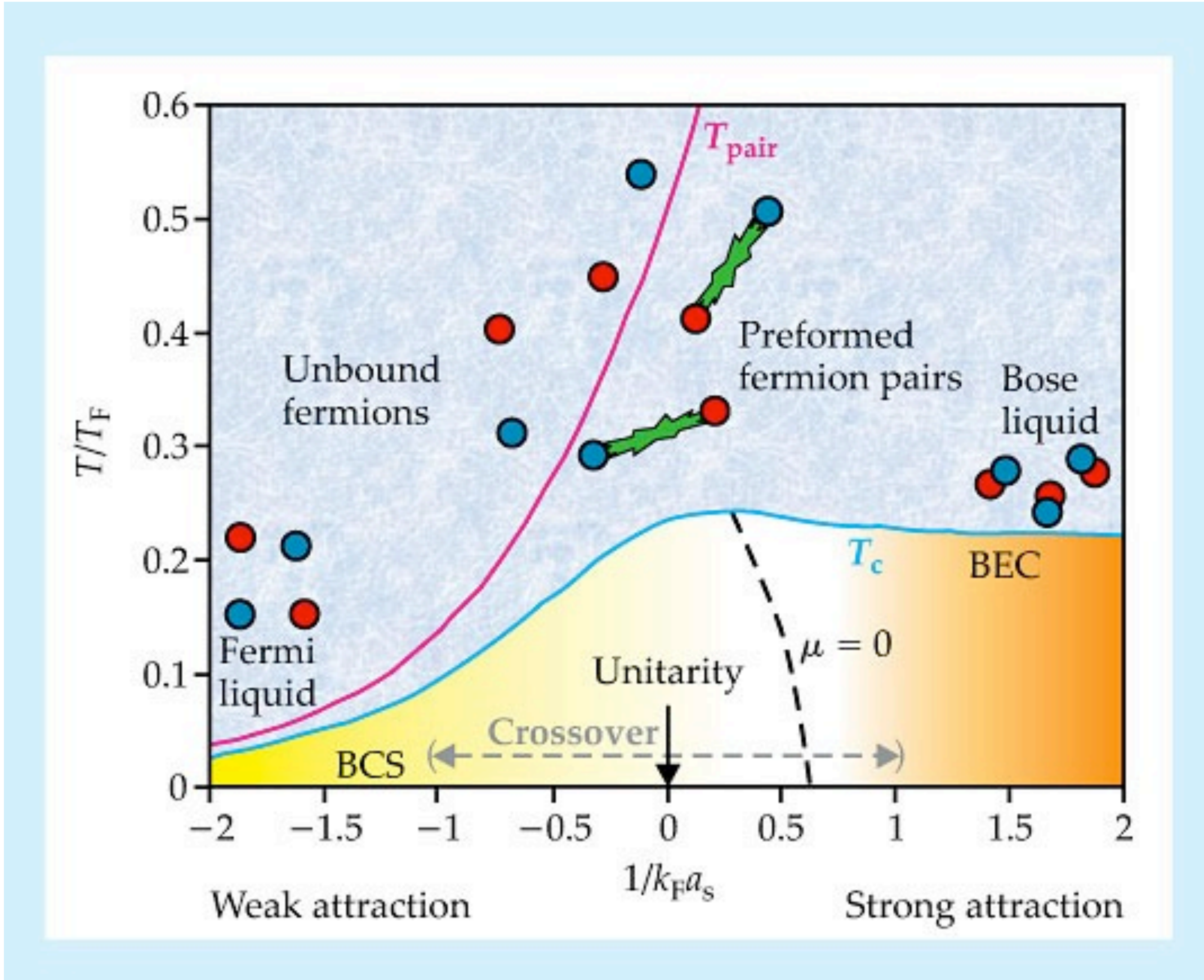
- Non-relativistic fermions at finite density, subject to two body (zero range) contact interactions

- s-wave amplitude saturates the unitarity limit

$$f_0(k) = \frac{1}{ik - \frac{1}{a_0} + \frac{1}{2}r_0 k^2}$$

$$|f_0|^2 \leq \frac{1}{k^2} \qquad r_0 k_F^2 \ll 1 \,, \qquad \frac{1}{k_F a} \to 0$$

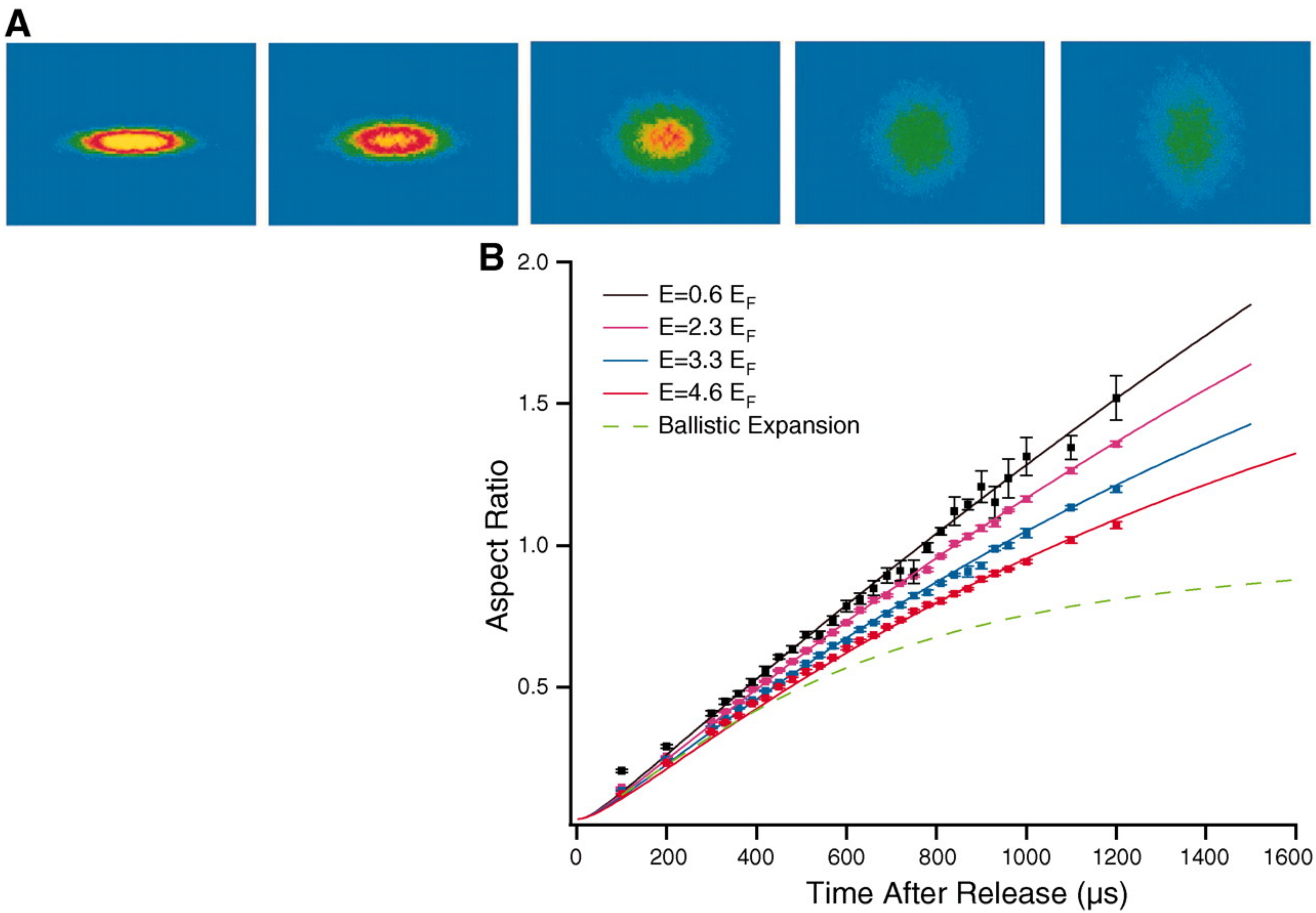
At unitarity the system is scale invariant!



from: Sa de Melo, Phys.Today (Oct. 2008)

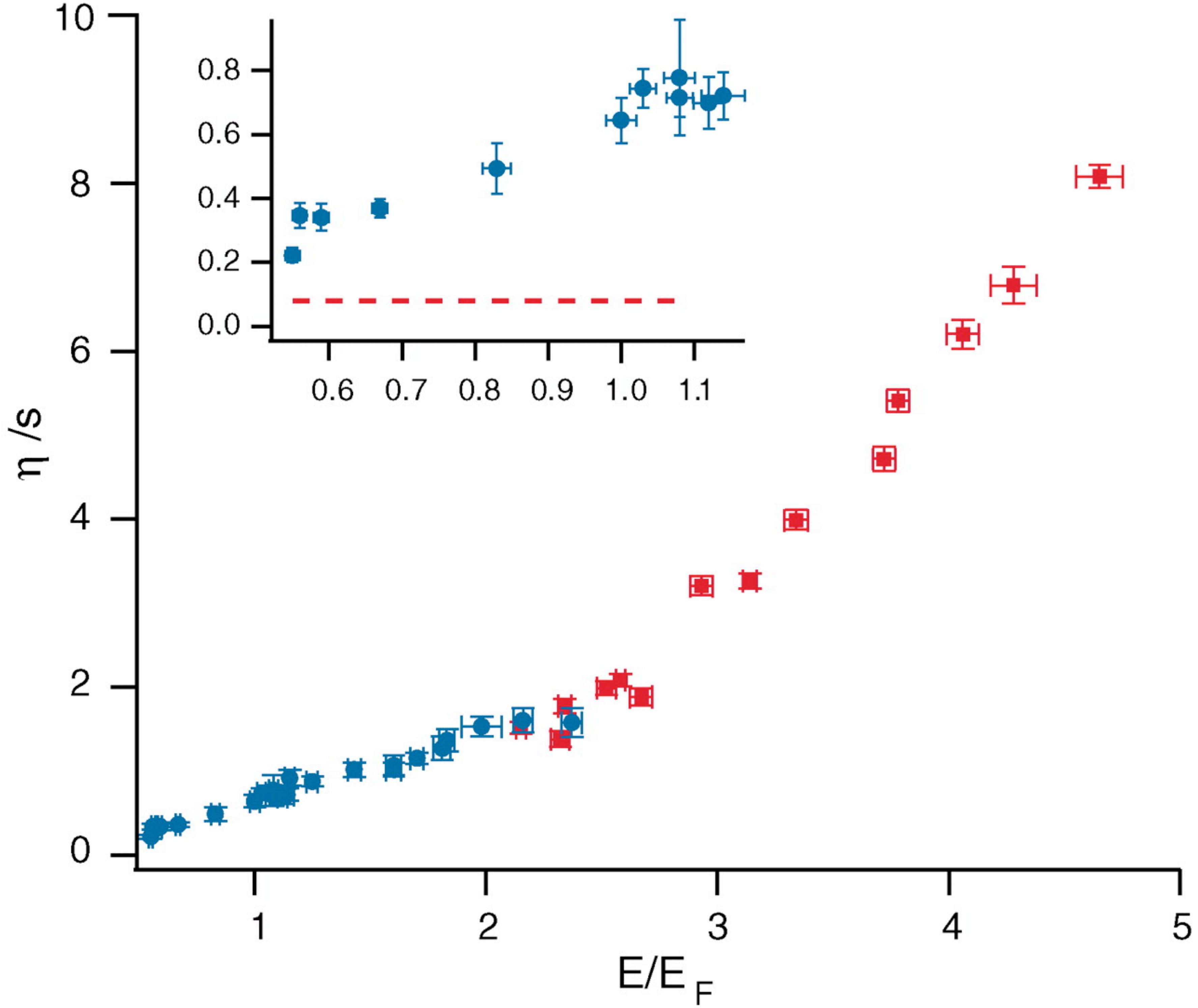
Experimental success

- Ultracold: Fermi T in the microKelvin range
- Feshbach resonance techniques allow to tune $k_F a$ at will, and reach the unitarity limit
- We are particularly interested in the measure of transport coefficients at unitarity



C Cao et al. Science 2011;331:58-61

Fig. 3 Estimated ratio of the shear viscosity to the entropy density.



C Cao et al. Science 2011;331:58-61

Shear viscosity = momentum/area

$$\eta \sim \frac{k}{\frac{4\pi^2}{k^2}}$$

At high T

$$k \sim p_T = \sqrt{2mT}$$

$$\eta \propto T^{3/2}$$

which agrees with the experimental measures

Still to understand the low T behavior

Low energy EFT for the phonons

Son 2002, Son and Wingate 2006

$$< \psi\psi >= | < \psi\psi > | e^{-i\theta} \qquad \theta = \mu t - \varphi$$

Symmetry considerations

$$\mathcal{L}_{\rm LO} = P(X), \qquad X = \partial_t \theta - \frac{(\partial_i \theta)^2}{2m}$$

Introduce artificial U(1) gauge field

$$X = D_t \theta - \frac{(D_i \theta)^2}{2m}$$

Density at ground state

$$\theta = \mu t \;, \qquad X = \mu$$

$$n = P'(X = \mu)$$

Eos fixes the LO Lagrangian

$$\mathcal{L}_{\rm LO} = \frac{1}{2}\left((\partial_t\phi)^2 - v_{\rm ph}^2(\nabla\phi)^2\right) - g\left((\partial_t\phi)^3 - 3\eta_g\,\partial_t\phi(\nabla\phi)^2\right)$$
$$+ \quad \lambda\left((\partial_t\phi)^4 - \eta_{\lambda,1}(\partial_t\phi)^2(\nabla\phi)^2 + \eta_{\lambda,2}(\nabla\phi)^4\right) \; + \cdots$$

$$E_p = c_s p$$

$$g = \frac{1}{6\sqrt{m\rho}\; c_s}\left(1 - 2\frac{\rho}{c_s}\frac{\partial c_s}{\partial\rho}\right) \,, \qquad \eta_g = \frac{c_s}{6\sqrt{m\rho}\; g}$$

$$\lambda = \frac{1}{24\; m\rho\; c_s^2}\left(1 - 8\frac{\rho}{c_s}\frac{\partial c_s}{\partial\rho} + 10\frac{\rho^2}{c_s^2}\left(\frac{\partial c_s}{\partial\rho}\right)^2 - 2\frac{\rho^2}{c_s}\frac{\partial^2 c_s}{\partial\rho^2}\right) \;,$$
$$\eta_{\lambda_2} = \frac{c_s^2}{8\; m\rho\;\lambda} \,, \qquad \eta_{\lambda,1} = 2\frac{\eta_{\lambda,2}}{\eta_g}$$

Escobedo and CM, 2010

Phonons in the cold Fermi gas at unitarity

$$P = \frac{2^{5/2}}{15\pi^2\xi^{3/2}} m^{3/2}\mu_0^{5/2}$$

EoS fermi gas at unitarity

$$\mu_0 = \xi E_F \ , \qquad \xi \sim 0.4$$

$$\mathcal{L}_{\rm NLO} = c_1 m^{1/2}\frac{(\nabla X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}}(\nabla^2\varphi)^2\sqrt{X}$$

$$\gamma = -\left(c_1 + \frac{3}{2}c_2\right)\frac{\pi^2\sqrt{2\xi}}{k_F^2} \simeq \frac{0.18}{k_F^2}$$

Rupak and Schafer, 2009

$$E_k = c_s k(1+\gamma k^2)$$

and at higher orders

$$E_k = c_s k(1+\psi(k))$$

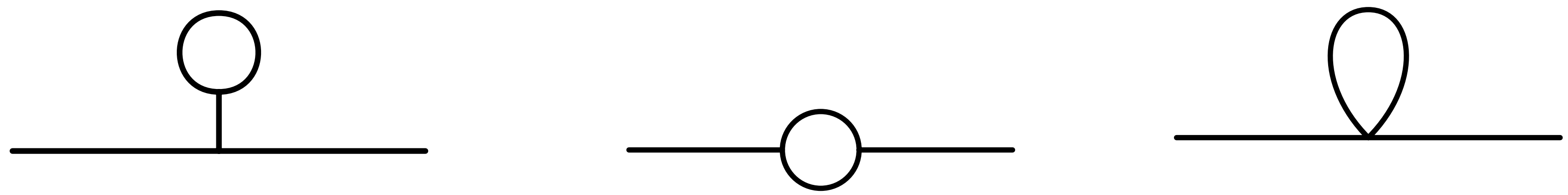
$$\psi(k) = \gamma\, k^2 + \delta\, k^4 + \mathcal{O}\left(\frac{k^6}{k_F^6}\right)$$

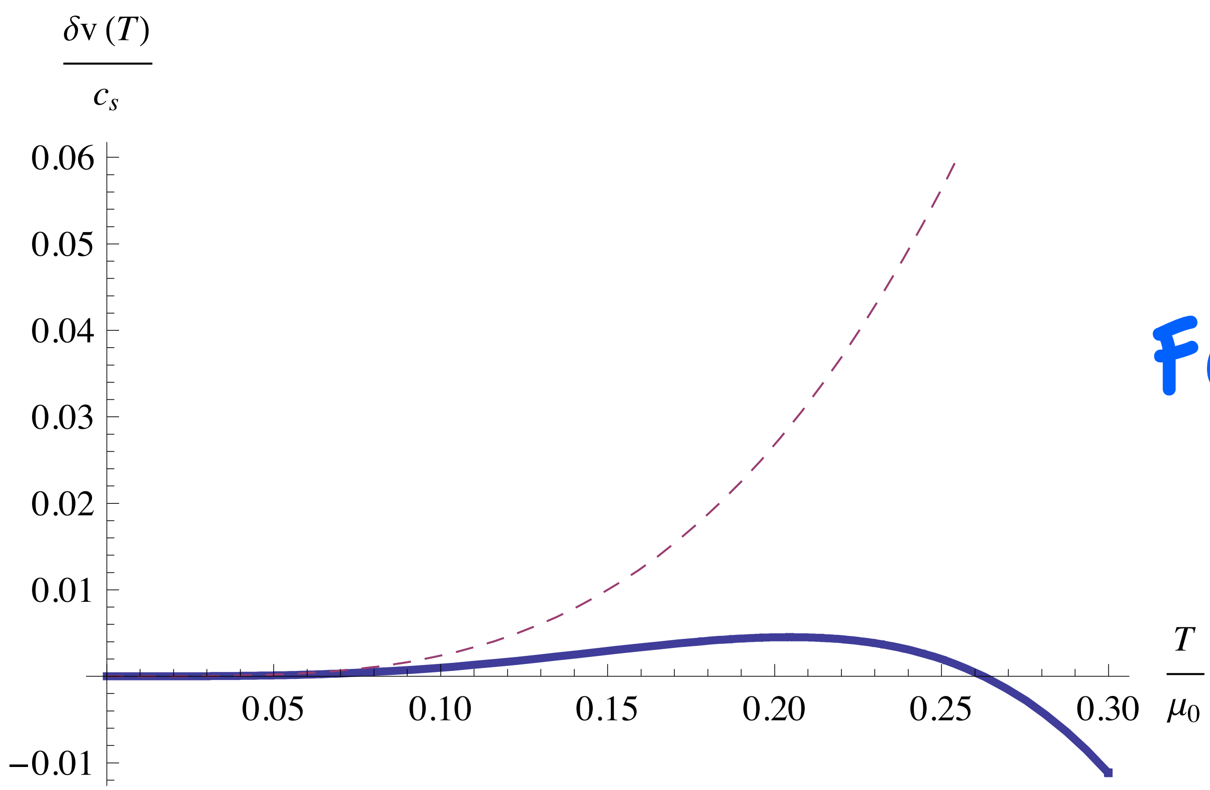
One-loop physics with the LO

$$p_0^2 - p^2 c_s^2 - \Pi(p_0, \mathbf{p}) = 0$$

$$\delta v(T) \approx -\frac{\pi^2}{15\rho_0}\left(\frac{T}{c_s}\right)^4 \left[-\frac{1}{4}\frac{\rho_0^2}{c_s}\frac{\partial^2 c_s}{\partial \rho_0^2} + \frac{1}{2} - \frac{1}{2}\frac{\rho_0}{c_s}\frac{\partial c_s}{\partial \rho_0} + \frac{2\rho_0^2}{c_s^2}\left(\frac{\partial c_s}{\partial \rho_0}\right)^2 \right.$$
$$\left. + \left(1 + \frac{\rho_0}{c_s}\frac{\partial c_s}{\partial \rho_0}\right)^2 \left(1 + \frac{1}{2}\log\frac{27|\gamma|T^2}{c_s^2}\right)\right] ,$$

$$\alpha(p, T) = \Theta(\gamma)\frac{T^4 p \pi^3}{30\rho_0 c_s^4}\left(1 + \frac{\rho_0}{c_s}\frac{\partial c_s}{\partial \rho_0}\right)^2$$





For Fermi gas at unitarity

$$\frac{\delta v(T)}{c_s} = -\frac{\pi^4\sqrt{3}\xi^{3/2}}{160}\left(\frac{T}{\mu_0}\right)^4\left\{43 + 16\log\left(\frac{243\lambda}{32}\frac{T^2}{\mu_0^2}\right)\right\}$$

$$\frac{\gamma}{c_s^2} = -\frac{\lambda}{8m^2c_s^4} = -\frac{9\lambda}{32\mu_0^2}$$

Phonon contribution to transport coefficients in a superfluid

Naively once could expect that the phonon contribution to transport coefficients can be obtained once the EoS is known

but with a LO phonon dispersion law, $\kappa, \zeta_i = 0$

further, the NLO dictates whether some processes are kinematically allowed or not

η, κ, ζ_i

Phonon contribution to the bulk viscosities

$$\zeta_i = \frac{T}{\Gamma_{ph}} C_i \ , \qquad i = 1, 2, 3$$

$$C_1 = -I_1 I_2 \ , \qquad C_2 = I_2^2 \ , \qquad C_3 = I_1^2 .$$

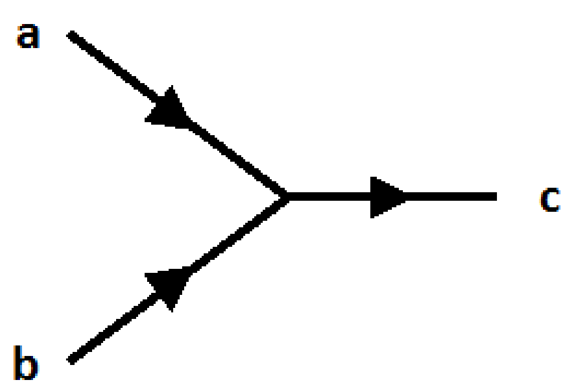
$$I_1 = \frac{\partial N_{ph}}{\partial \rho} \qquad I_2 = N_{ph} - S \frac{\partial N_{ph}}{\partial S} - \rho \frac{\partial N_{ph}}{\partial \rho} \ .$$

For a LO dispersion law, the bulk viscosities vanish!

Bulk viscosities

Phonon number changing processes are needed

$\varphi \leftrightarrow \varphi\varphi$ only if E curves upward



$$\delta\theta_{bc} = \sqrt{6\gamma}\,(p_b + p_c)\ .$$

(similar conclusion for 1 <-> N phonons)

For the Fermi gas at unitarity

$$\zeta_3 \simeq 3695.4 \left(\frac{\xi}{\mu_0}\right)^{9/2} \frac{(c_1 + \frac{3}{2}c_2)^2}{m^8} T^3 \qquad\qquad \zeta_1 = \zeta_2 = 0$$

away from the conformal limit

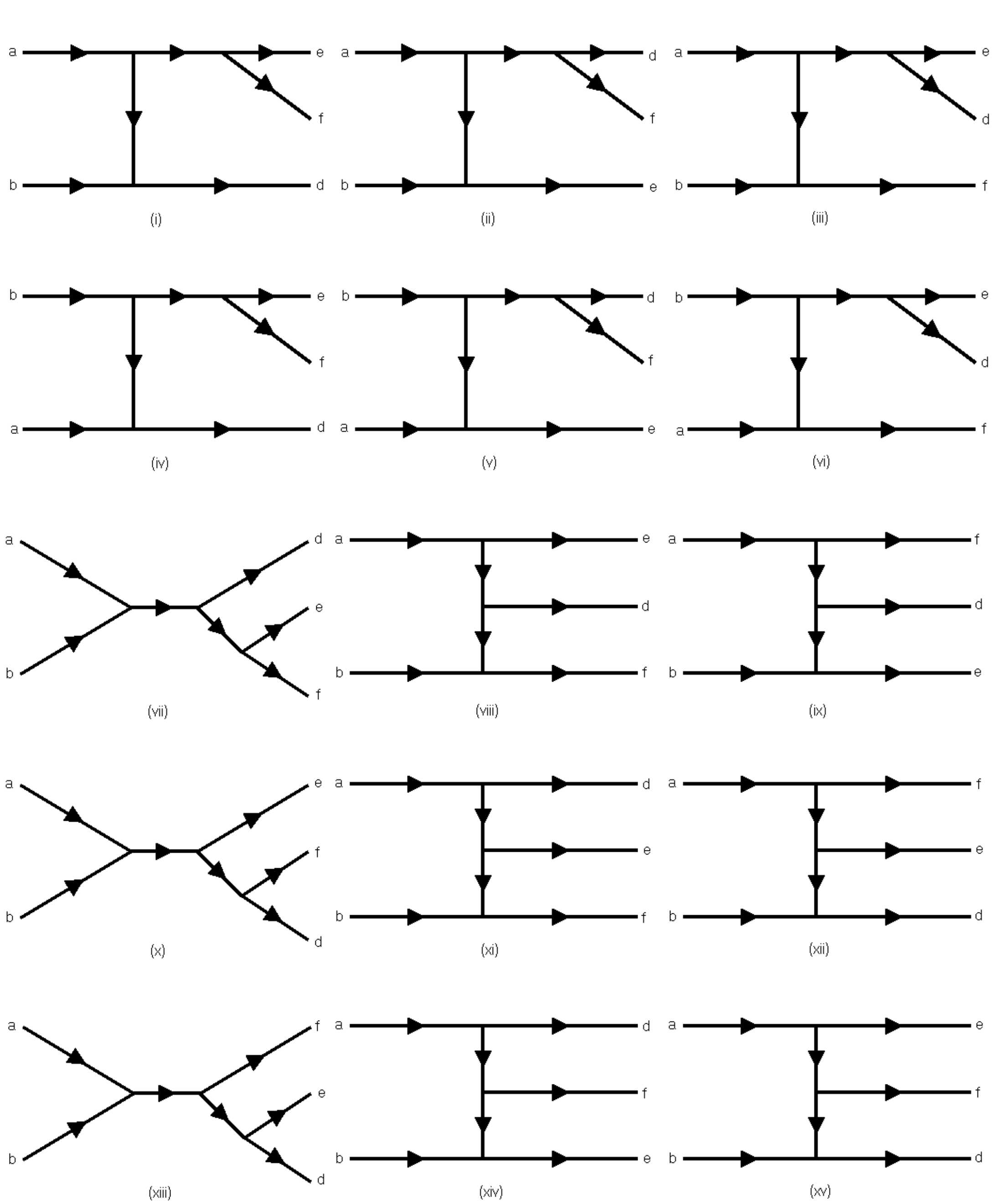
$$P = P_0 + P_{\rm CB} = c_0 m^4 \mu_0^{5/2} + \frac{d_0 m^3 \mu_0^2}{a}$$

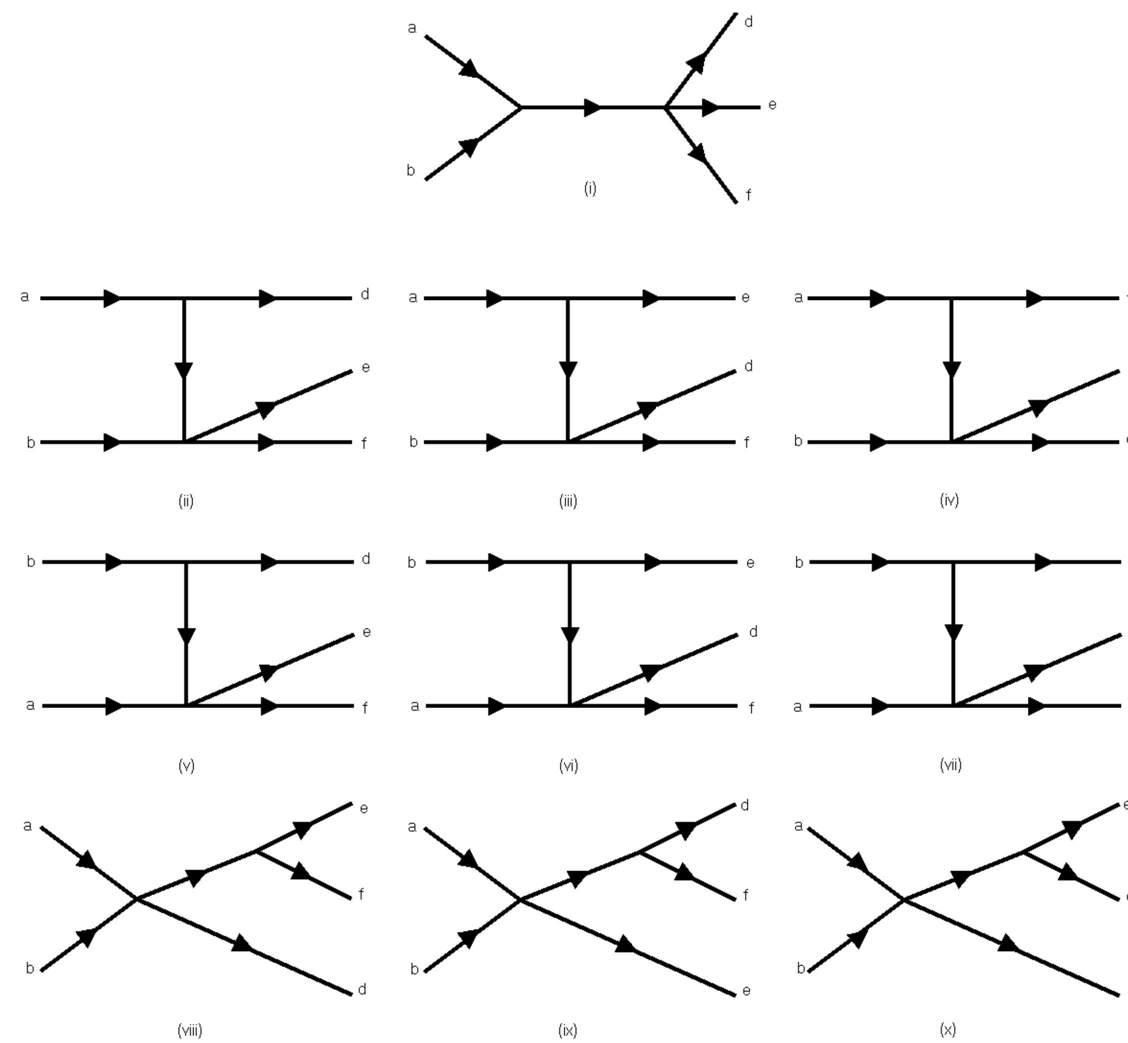
$$\zeta_1 \simeq -264.7\, c_2 \left(c_1 + \frac{3}{2}c_2\right) \frac{T^3 \xi^3}{m^4 \mu_0^3} y\,, \qquad \zeta_2 \simeq 19.0\, c_2^2 \frac{T^3 \xi^{3/2}}{\mu_0^{3/2}} y^2\,,$$

$$\zeta_3 \simeq 3695.4 \left(\frac{\xi}{\mu_0}\right)^{9/2} \frac{(c_1 + \frac{3}{2}c_2)^2}{m^8} T^3 \left(1 - \frac{66c_1 + 135c_2}{8c_1 + 12c_2} y\right)$$

$$y \equiv \frac{d_0 \pi^2 \xi^{3/2}}{am\sqrt{2\mu_0}}$$

For E that curves downward, one has to consider $\varphi\varphi \leftrightarrow \varphi\varphi\varphi$





Type I **Type II**

Large angle collisions

$$\Gamma \sim T^{16}$$

Small angle collisions: collinear singularities arise if the phonon propagator at LO is used

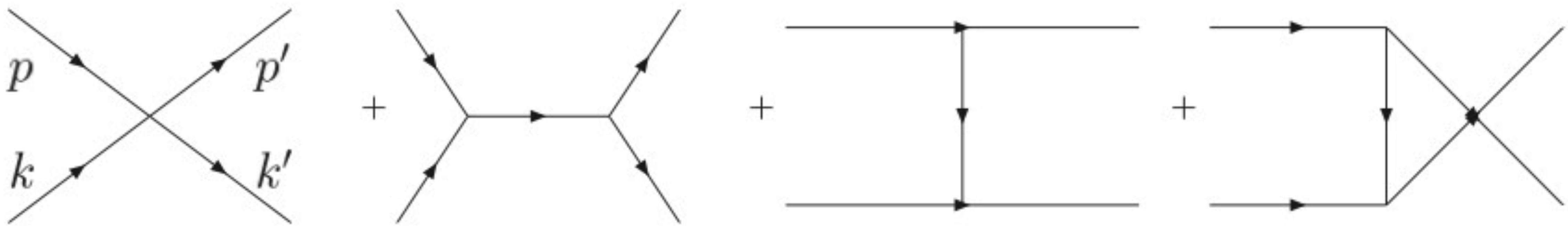
using the propagator at NLO

$$\Gamma_{\text{Type I}} \sim T^{12}\,, \quad \Gamma_{\text{Type II}} \sim T^{8}\,, \quad \Gamma_{\text{Cross}} \sim T^{10}$$

Analysis for neutron matter for different EoS and gaps demands numerical analysis

Shear viscosity

It typically requires large angle collisions (= transport of p orthogonal to the flow)



$$\eta \propto \frac{1}{T^5}$$

For the Fermi gas at unitarity

$$\eta = 9.3 \cdot 10^{-6} \frac{\xi^5}{c_s^3} \frac{T_F^8}{T^5}$$

Rupak and Schafer, 2007

Shear viscosity

When E curves upward, small angle collisions contribute, as a large-angle collision can be achieved by the addition of many small angle

$$\varphi \leftrightarrow \varphi\varphi$$

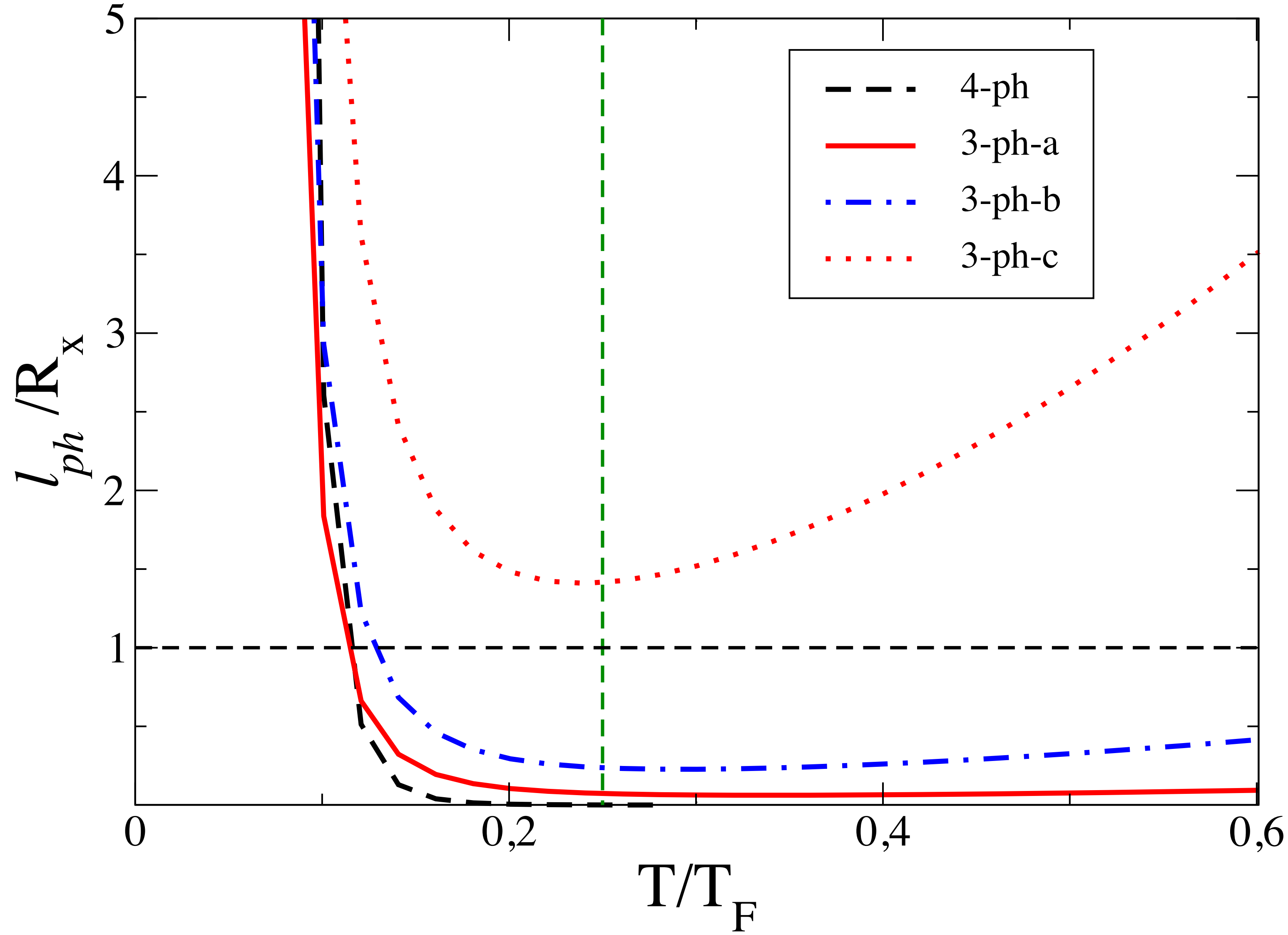
$$E_p = c_s p \qquad \eta \sim \infty$$

$$E_p = c_s p(1+\gamma^2) \qquad \eta \propto \frac{1}{T^5}$$

$$E_p = c_s p(1+\gamma p^2 - \delta p^4) \qquad \eta \propto \frac{1}{T}$$

Knudsen number

$R_x \sim 7\mu\mathrm{m}$



$$\psi_{\max} = -\frac{\gamma^2}{4\delta}$$

$$\psi_{\max} = 0.2, 0.3, 0.4$$

Hydro needs Kn << 1

For Kn > 1 phonons collide more often with the boundary

Data for the phonons of He4

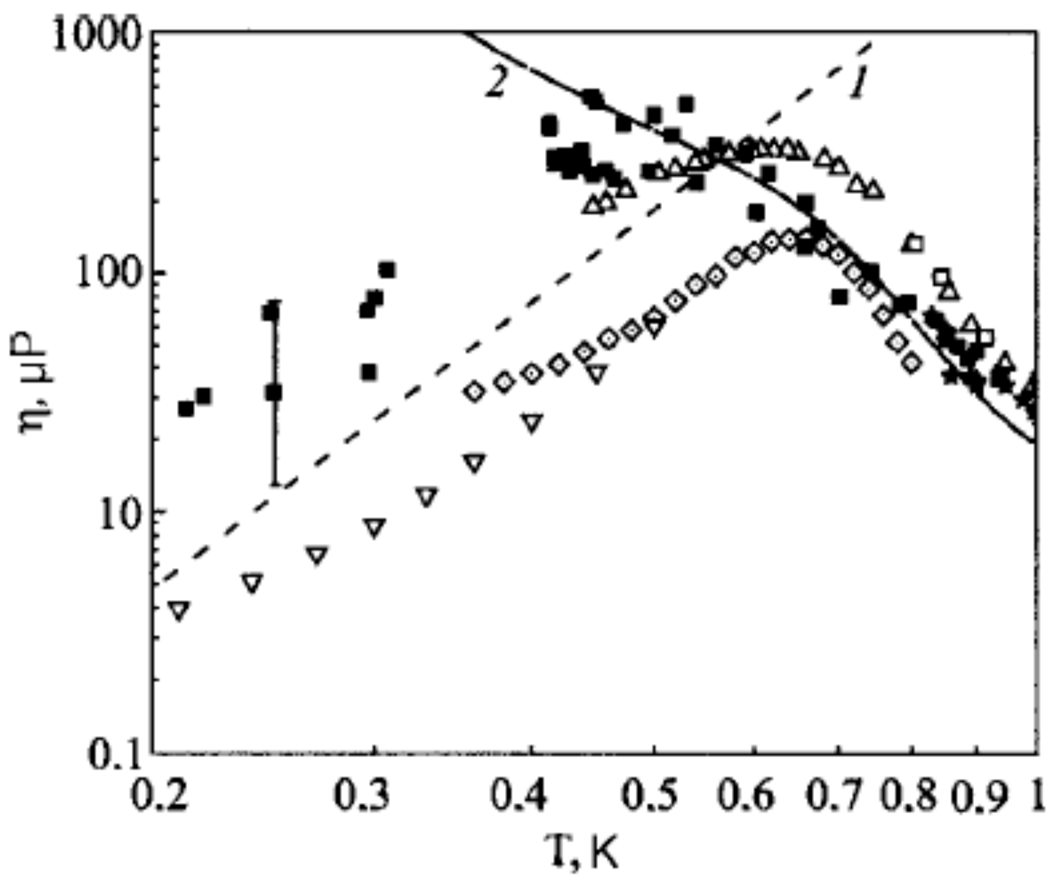


FIG. 3. Temperature dependence of the coefficient of effective viscosity near the transition from the hydrodynamic to the ballistic regime: this work (■); Ref. 11 (△); Ref. 10 (◇), (▽); Ref. 15 (☆). Line 1—calculation using Eq. (22); 2—temperature dependence of the coefficient of viscosity of the normal component.

$$\eta_{\rm bulk} = \frac{1}{5}\rho_{\rm ph} c_s l_{\rm ph}$$

$$\eta_{\rm ball} = \frac{1}{5}\rho_{\rm ph} c_s a$$

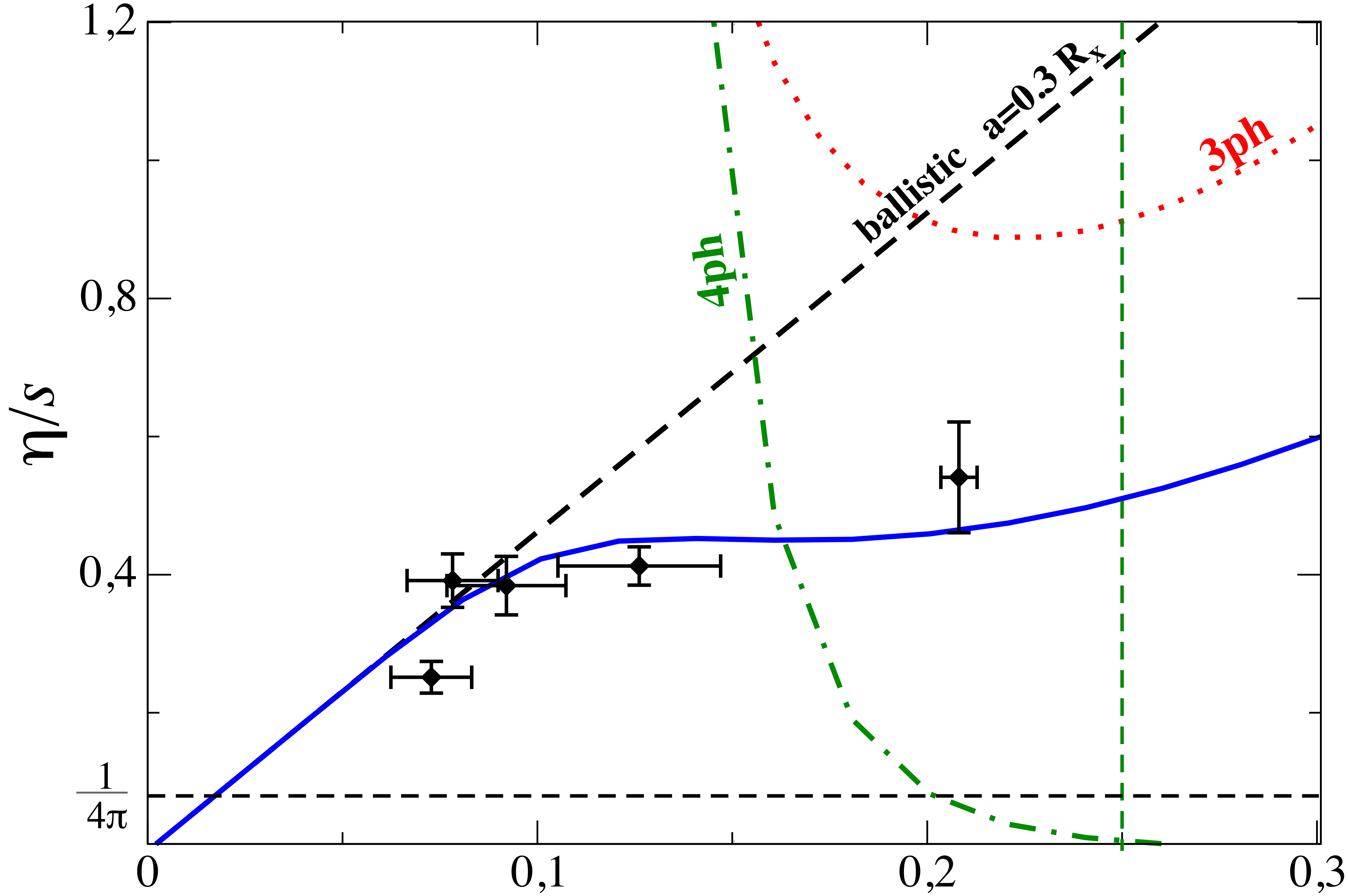
$$\eta_{\rm eff} = \left(\eta_{\rm bulk}^{-1} + \eta_{\rm ball}^{-1}\right)^{-1}$$

A. Zadorozhko et al , Low Temperature Physics 35, 100 (2009)

Experimental values of viscosity can be explained if

- Viscosity dominated by the center of the trap
- Superfluid fermions dont contribute at low T
- The outer fermionic corona is too dilute
- Phonons collide with the superfluid-normal interface in a diffusive way

Fermi gas at unitarity



$\psi_{\max} = 0.3$

Predictions

- The viscosity should still decrease if the T is decreased
- The viscosity should correlate with the size of the trap
- Possibility of reaching the bound η/s

but it is not in a hydrodynamical regime!

(no conceptual problem with the bound)

Conclusions

The physics of the phonon of a superfluid can be conveniently described with EFT

- Phonons can explain the low T values of the shear viscosity for the unitarity Fermi gas.
- We used EFT to determine the phonon interactions; an anomalous dispersion law and finite size effects are needed to explain the data