Type II Goldstone Bosons

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Cosmology (or: how I got into this)

- Strong indication for a primordial inflation phase of quasi-de Sitter expansion

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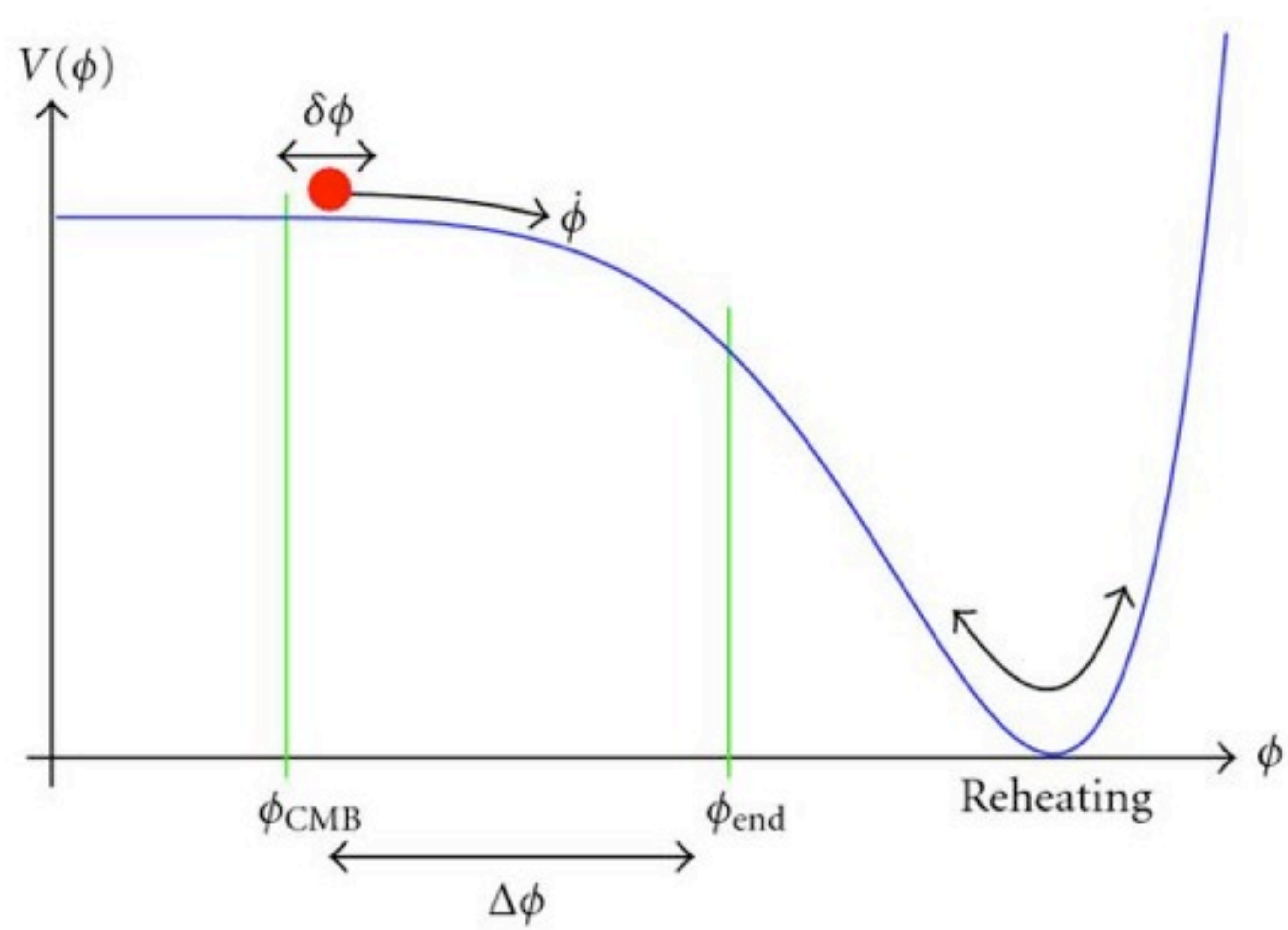
Flat to good approximation

$$\frac{V'}{V} \ll 1, \ \frac{V''}{V} \ll 1$$

Generalization: moving in a symmetry direction



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Spontaneous Symmetry Probing

- Time dependent field states in the presence of a continuous symmetry

- In particular:

$$\dot{\phi}_j \;\propto\; \delta\phi_j$$

time evolution

symmetry action

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time evolution symmetry action

- Equivalently,

$$H'|\mu\rangle \;\equiv\; (H - \mu Q)|\mu\rangle = 0$$

+ $|\mu\rangle$ **breaks** Q

Systems at finite charge density

$$H' = H - \mu Q$$

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Non-relativistic Hamiltonian

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Non-relativistic Hamiltonian

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Relativistic $\sim \int d^3x\, T^{00}$

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Non-relativistic Hamiltonian

Lagrange Multiplier,
chemical potential

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$$H' = H - \mu Q$$

Relativistic $\sim \int d^3x\, T^{00}$

Conserved charge,
e.g. particles number $\sim \int d^3x\, J^0$

Systems at finite charge density

At first sight: explicit breaking of Lorentz and all non-commuting charges

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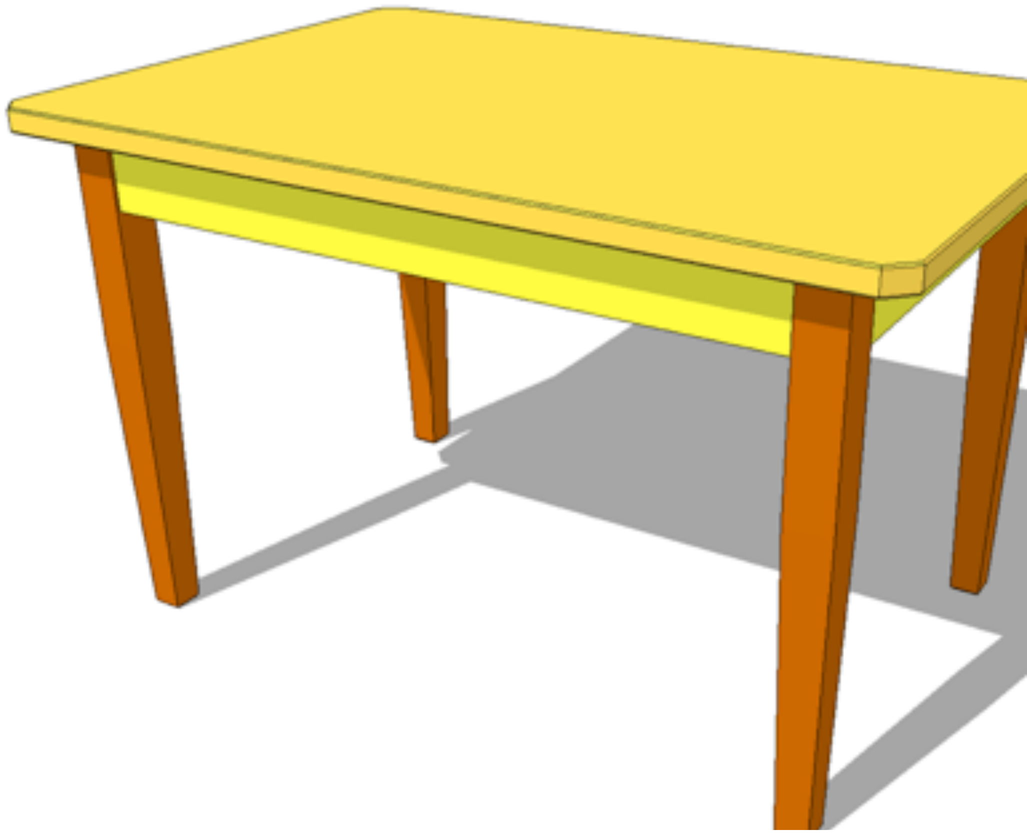
Lorentz is always broken spontaneously in the real world!



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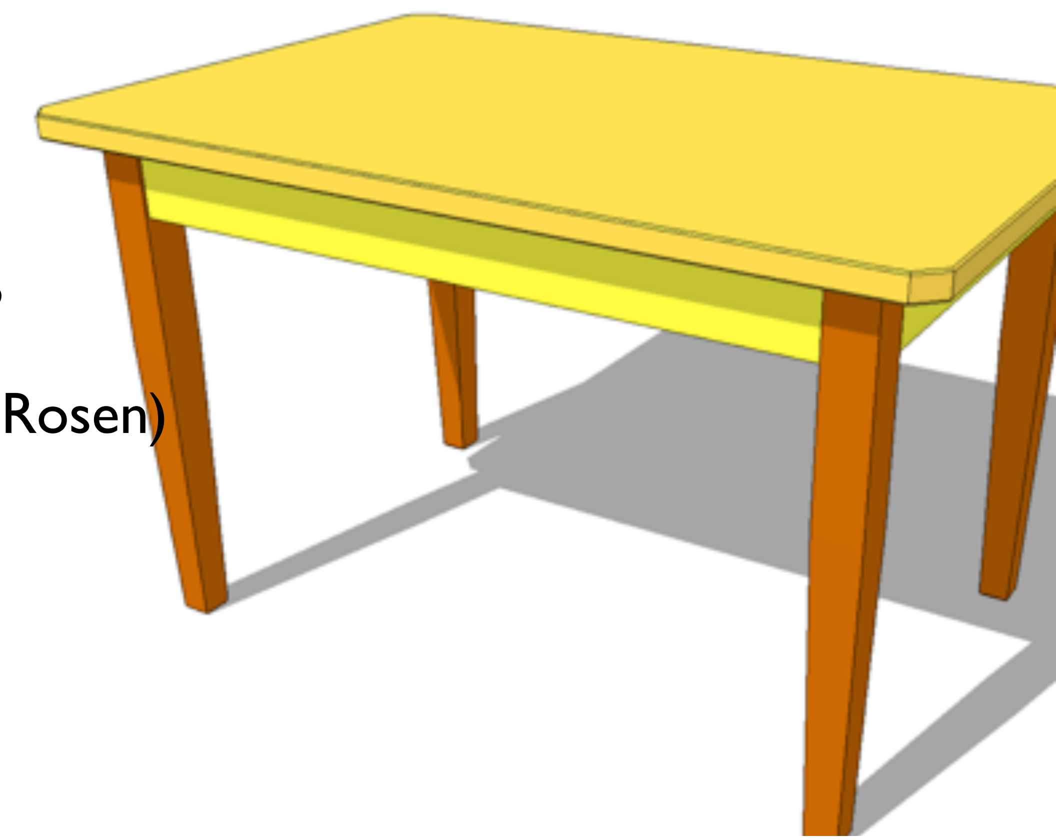

1) Classification of ``condensed matter systems''
(Alberto's talk and in preparation with Nicolis, Penco, Rattazzi, Rosen)

2) Exact results in this case (gapped Goldstones)

Spontaneous Symmetry Breaking: Generalities

$$\langle 0|[Q(t), A(0)]|0\rangle$$

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More precisely,

$$\begin{aligned} 0 &= \int d^3x \langle 0|[\partial_\mu J^\mu(\vec{x}, t), A]|0\rangle \\ &= \int d^3x \langle 0|[\dot{J}^0(\vec{x}, t), A]|0\rangle + \int d^3x \langle 0|[\partial_i J^i(\vec{x}, t), A]|0\rangle \end{aligned}$$

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Because commutator
of local operators

Spontaneous Symmetry Breaking: Generalities

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$$\neq 0 \qquad \textbf{for some } A$$

by definition of SSB $\langle 0|\delta A|0\rangle \neq 0$

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Goldstone Theorem: both $J^\mu(x)$ and $A(x)$ interpolate a massless state

$$\langle 0|J^\mu(x)|\pi(p)\rangle = i\, v\, e^{ip_\mu x^\mu} p^\mu$$

Alert! $\mu \to c$ in the next 3 slices

Spontaneous Symmetry Probing

$$\langle c|[H, A(x)]|c\rangle \;=\; c\,\langle c|[Q, A(x)]|c\rangle$$

$Q = Q_1, Q_2, \ldots Q_N$ Conserved charges of a symmetry group

1) Conserved currents evolve with the relativistic Hamiltonian H

$$J_a^\mu(\vec{x}, t) = e^{i(Ht - P\cdot\vec{x})t}\, J_a^\mu(0)\, e^{-i(Ht - P\cdot\vec{x})}$$

2) We study the spectrum of the unbroken combination

$$H' = H - cQ; \quad H'|c\rangle = 0$$

Good old-fashion demonstration, revisited...

$$
\begin{aligned}
\kappa_{aI} &= \langle c|[Q_a(t), A_I]|c\rangle \\
&= \int d^3x \langle c|J_a^0(\vec{x},t) A_I|c\rangle - \text{c.c.} \\
&= \int d^3x \langle c|e^{i(Ht-P\cdot\vec{x})t}\, J_a^0(0)\, e^{-i(Ht-P\cdot\vec{x})} A_I|c\rangle - \text{c.c.} \\
&= \int d^3x \langle c|e^{icQt}\, J_a^0(0)\, e^{-i(Ht-P\cdot\vec{x})} A_I|c\rangle - \text{c.c.} \\
&= \int d^3x \sum_{n,p} e^{i\vec{p}\cdot\vec{x}} \langle c|e^{icQt} J_a^0(0) e^{-icQt} e^{-i\tilde{H}t}|n,\vec{p}\rangle\langle n,\vec{p}|\, A_I|c\rangle - \text{c.c.} \\
&= \sum_n \delta^3(\vec{p})\langle c|e^{icQt}\, J_a^0(0)\, e^{-icQt} e^{-i\tilde{H}t}|n,0\rangle\langle n,0|A_I|c\rangle - \text{c.c.} \\
&= \sum_n e^{-iE_n(0)t}\langle c|e^{icQt} J_a^0(0) e^{-icQt}|n,0\rangle\langle n,0|A_I|c\rangle - \text{c.c.}
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Explicit the space-time dependence (1)

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SSP state ground state of $H - cQ$ (2)

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Two cases: either J_a and Q commute or they do not.

A. Nicolis, F.P. 1204.1570

Non-Commuting case: massive Goldstones

$$\kappa_{Ia} = e^{-iE_n t}\langle c|e^{icQt} J^0_a(0) e^{-icQt}|n,0\rangle\langle n,0|A_I|c\rangle - \text{c.c.}$$

Say, $$[Q_a, J^0_b(x)] = i f_{ab}^{\ \ c}\, J^0_c(x)$$

Then, $$e^{ic\,Qt} J_a e^{-ic\,Qt} = (e^{-f_1\,ct})_a^{\ b} J_b$$

The interpolator is a time-dependent combination of conserved currents

Take $f_{1a}^{\ \ b}$ in `normal form': block diagonal with pieces $\begin{pmatrix} 0 & +q_\alpha \\ -q_\alpha & 0 \end{pmatrix}$

Each block: one massive Goldstone state

$$m = c\, q_\alpha$$

Example: SO(3) - one triplet

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\vec{\phi}\,\partial^\mu\vec{\phi} - \frac{1}{2}m^2\vec{\phi}^{\,2} - \frac{1}{4}\lambda(\vec{\phi}^{\,2})^2$$

radial and angular coordinates for 1-2

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}\sigma^2\partial_\mu\theta\partial^\mu\theta - \frac{1}{2}\partial_\mu\phi_3\partial^\mu\phi_3 - \frac{1}{2}m^2\left(\sigma^2+\phi_3^2\right) - \frac{1}{4}\lambda\left(\sigma^2+\phi_3^2\right)^2$$

SSP solution:

$$\dot{\theta} = c;\ \ \sigma^2 = \frac{c^2-m^2}{\lambda};\ \ \phi_3 = 0$$

Perturbations:

$$\mathcal{L}^{(2)} = -\frac{1}{2}\partial_\mu\delta\sigma\partial^\mu\delta\sigma - \frac{1}{2}\sigma^2\partial_\mu\pi\partial^\mu\pi - \frac{1}{2}\partial_\mu\phi_3\partial^\mu\phi_3 + 2c\sigma\dot{\pi}\delta\sigma - (c^2-m^2)\delta\sigma^2 - \frac{1}{2}c^2\phi_3^2$$

However: SO(3) - symmetric traceless rep.

$$\mathcal{L} = -\frac{1}{2}\,\partial_\mu \Phi^i{}_j \partial^\mu \Phi^j{}_i - \lambda \left(\Phi^i{}_j \Phi^j{}_i - v^2\right)^2$$

SSP solution: $$\langle \Phi \rangle = e^{i\mu t L_3} \begin{pmatrix} \Phi_0 & 0 & 0 \\ 0 & -\Phi_0 & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{-i\mu t L_3}$$

1) Fixed gap Goldstone $\quad m = \mu$

2) ``Un-fixed gap Goldstone $\quad m = 3\mu$

Other ex: SO(3) - two triplets. Etc.

1306.1240, A. Nicolis, R. Penco, F.P., R. Rosen

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Finite charge density: coset construction

- Full symmetry group: Q_I
- Unbroken generators T_A (subgroup)
- Broken generators X_{a}
- Charge at finite density $\mu Q = \mu_X X + \mu_T T$

1) maximum number of unbroken generators
2) completely antisymmetric in (X_{a}, T_A)

Unbroken

$$\begin{cases} \bar{P}^0 \equiv H - \mu Q \\ \bar{P}^i \equiv P^i \\ J_i \\ T_A \end{cases}$$

Broken

$$\begin{cases} Q \\ X, X_{\mathsf{a}} \\ K_i \end{cases}$$

Finite charge density: coset construction

$$\Omega = e^{ix^\mu \bar{P}_\mu} e^{i\pi(x)X} e^{i\pi^{\mathsf{a}}(x)X_{\mathsf{a}}} e^{i\eta^i(x)K_i}$$

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- Internal Goldstones further classified: commuting vs. non-commuting

$$[Q, X_{\mathsf{a}}] = i M_{\mathsf{ab}} X^{\mathsf{b}} \qquad M_{\mathsf{ab}} = \mathrm{diag}\left\{ \underbrace{0, \cdots, 0}_{\text{commuting}}, \underbrace{\begin{pmatrix} 0 & q_1 \\ -q_1 & 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 & q_k \\ -q_k & 0 \end{pmatrix}}_{\text{non-commuting}} \right\}.$$

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- This defines a new inverse Higgs constraint! $\quad [\bar{P}_0, X_a^\pm] = \pm i\mu q_a X_a^\mp$

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- For each fixed-mass Goldstone an ``optional” non-fixed-mass one

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- Commuting Goldstones $\pi - \pi^\alpha$ only appear with derivatives

- One derivative mixing is important: $M_{\alpha\beta}\pi^\alpha\dot{\pi}^\beta$

$$M = \operatorname{diag}\left\{\underline{0,\cdots,0},\underline{\begin{pmatrix} 0 & M_1 \\ -M_1 & 0 \end{pmatrix},\cdots,\begin{pmatrix} 0 & M_k \\ -M_k & 0 \end{pmatrix}}\right\}$$

- Linear dispersion relations + massless quadratic <-> gapped $m \sim \mu$

Nielsen and Chada `76, Watanabe and Brauner `11

Summary

- n_1 massless linear (from commuting sector)
- n_2 massless quadratic (from commuting sector)
- n_3 fixed gap (from non-commuting sector)
- n_4 unfixed gap (from both sectors)

$$n_2 \leq n_4 \leq n_2 + n_3$$