Transport effects due to conformal anomaly

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(with many thanks to Karl Landsteiner for discussions!)

Fermions and axial anomaly

Massless Dirac fermions

Covariant formulation

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

$\not{D} = \gamma^\mu D_\mu$ $\qquad D_\mu = \partial_\mu + ieA_\mu$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$

Semimetals:

$$\bar{\psi}\left[i\gamma^0 \hbar \frac{\partial}{\partial t} + v_F \boldsymbol{\gamma}(i\hbar\boldsymbol{\nabla} - e\boldsymbol{A})\right]\psi$$

Currents

$j_V^\mu = \bar{\psi}\gamma^\mu\psi,$ $\qquad j_A^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi$

Vector $\qquad$ Axial

Axial anomaly

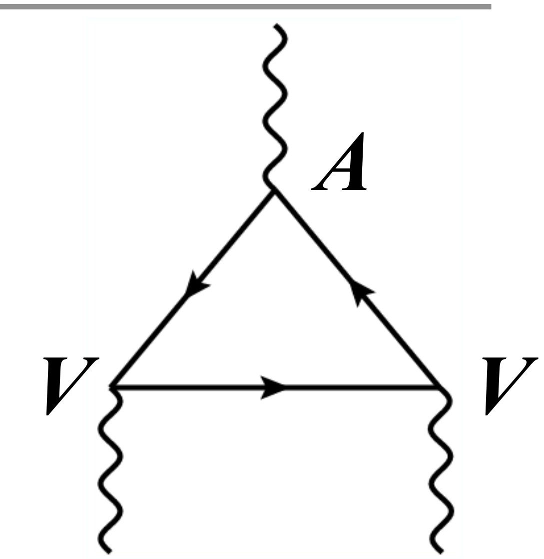
$$\partial_\mu j_A^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Usual (vector) chemical potential

Chiral (axial) chemical potential

Transport: $\boldsymbol{j}_A = \dfrac{\mu_V}{2\pi^2} e\boldsymbol{B},$ $\qquad \boldsymbol{j}_V = \dfrac{\mu_A}{2\pi^2} e\boldsymbol{B}$

Chiral separation effect $\qquad$ Chiral magnetic effect



AVV diagram

Classical symmetries

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

Vector

$$\psi \to e^{i\omega_V} \psi$$

vector current is classically conserved

$$j_V^\mu = \bar{\psi} \gamma^\mu \psi \qquad \partial_\mu j_V^\mu = 0$$

Axial

$$\psi \to e^{i\omega_5 \gamma^5} \psi$$

axial current is classically conserved

$$j_A^\mu = \bar{\psi} \gamma^5 \gamma^\mu \psi \qquad \partial_\mu j_A^\mu = 0$$

Conformal

$$x \to \lambda^{-1} x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2} \psi$$

Dilatation current is classically conserved

$$j_D^\mu = T^{\mu\nu} x_\nu \qquad \partial_\mu j_D^\mu \equiv T^\mu_{\ \mu} \equiv 0$$

$$(T^\mu_\mu)_{\rm cl} \equiv 0$$

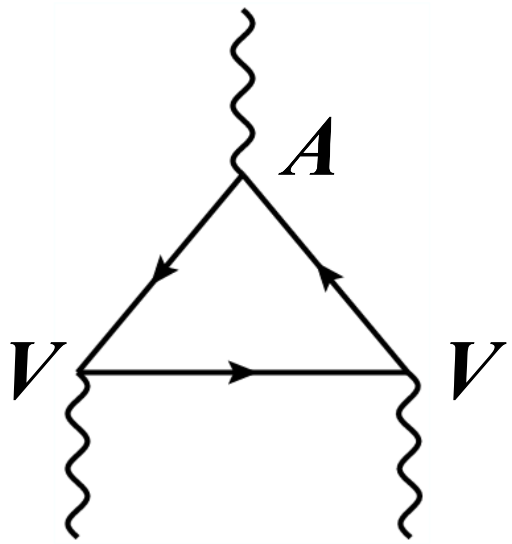
Energy-Momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha} F^\nu_{\ \alpha} + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} + \frac{i}{2} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi - \eta^{\mu\nu} \bar{\psi} i \not{D} \psi$$

Zoo of anomalies

(three out of six)

Axial

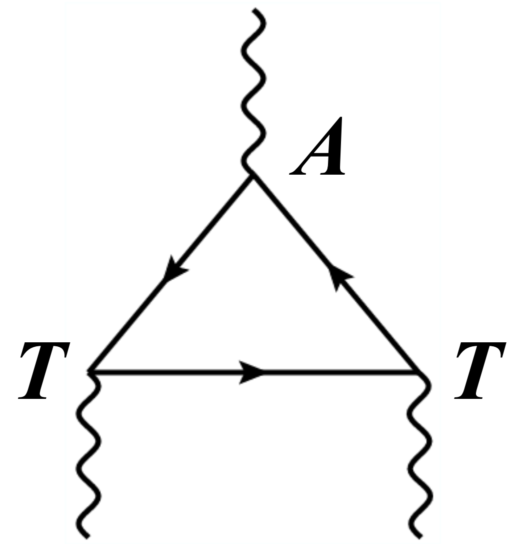


$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\tilde{F}^{\mu\nu} = \tfrac{1}{2}\,\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Mixed

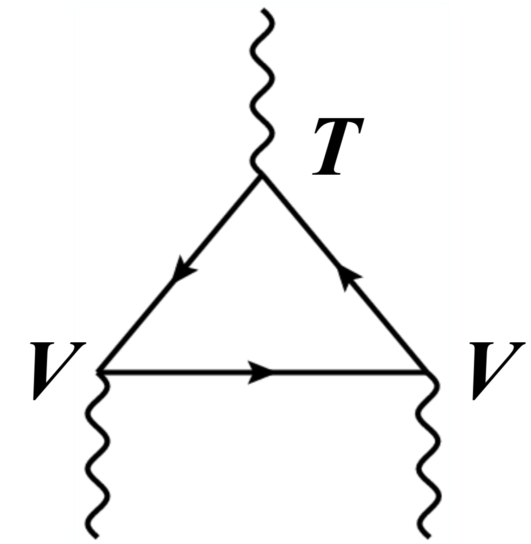
(axial-gravitational anomaly)



$$\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta}$$

$$\tilde{R}^{\mu\nu\alpha\beta} = \tfrac{1}{2}\,\varepsilon^{\mu\nu\gamma\lambda}R_{\gamma\lambda}{}^{\alpha\beta}$$

Conformal



$$\partial_\mu j^\mu_D = T^\alpha_\alpha$$

beta function

$$\langle T^\mu_{\ \mu}\rangle = \frac{\beta(e)}{2e} F_{\mu\nu}F^{\mu\nu}$$

Currents

$$j^\mu_V = \bar{\psi}\gamma^\mu\psi,$$

Vector

$$j^\mu_A = \bar{\psi}\gamma^5\gamma^\mu\psi$$

Axial

$$j^\mu_D = T^{\mu\nu}x_\nu$$

Dilatation

Energy-Momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha}F^\nu_\alpha + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + \frac{i}{2}\bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - \eta^{\mu\nu}\bar{\psi}i\not{D}\psi$$

Full list: AVV, AAA, ATT, TVV, TAA, TTT

Conformal anomaly and the beta function

Massless Dirac fermions $$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

are (classically) invariant under the conformal transformations:

$$x \to \lambda^{-1} x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2}\psi$$

But the quantum theory generates an intrinsic scale,
due to the renormalization (in this particular case) of the electric charge:

$$\beta(e) = \frac{de(\mu)}{d \ln \mu}$$

renormalization scale

In QED (for one Dirac fermion)

$$\beta_{\mathrm{QED}}^{1-\mathrm{loop}} = \frac{e^3}{12\pi^2}$$

→ conformal symmetry is broken at the quantum level

Quantum anomaly → anomalous transport

Axial anomaly

$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Transport

$$\boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}, \qquad \boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$$

Chiral separation and chiral magnetic effects

Mixed axial-gravitational anomaly

$$\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta}$$

Transport

$$\boldsymbol{j}_V = \frac{\mu_V \mu_A}{\pi^2} \boldsymbol{\Omega}, \qquad \boldsymbol{j}_A = \left(\frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}\right)\boldsymbol{\Omega}$$

Thermal contribution to chiral vortical effects

Conformal anomaly

Transport?

$$\partial_\mu j^\mu_D = T^\alpha_\alpha \qquad \langle T^\mu{}_\mu \rangle = \frac{\beta(e)}{2e} F_{\mu\nu}F^{\mu\nu}$$

Conformal anomaly →

Scale Electric Effect (SEE) and Scale Magnetic Effect (SME)

(Conformal Magnetic Effect = CME → interferes with Chiral Magnetic Effect … already taken, too late)

Simplified picture

Gravitational background: Weyl-transformed flat space

flat (Minkowski) metric

$$g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$$

scale factor (arbitrary function of coordinates)

The conformal anomaly leads to scale electromagnetic effects*:

$$\boxed{\langle T^{\mu}_{\ \mu}\rangle = \frac{\beta(e)}{2e} F_{\mu\nu}F^{\mu\nu}}$$

$$j^{\mu} \equiv \langle j^{\mu}_{V}\rangle = -\frac{2\beta(e)}{e} F^{\mu\nu}\partial_{\nu}\tau$$

"Generation of electric current in the background of electromagnetic and gravitational fields"

* Disclaimer: The names SME/SEE have been selected (out of a painful list) by Karl L. (many thanks from Maxim Ch.!)

M.Ch., PRL 117, 141601 (2016)

Scale electric effect (SEE)

Time-dependent background: $\tau = \tau(t)$

Metric: $g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$

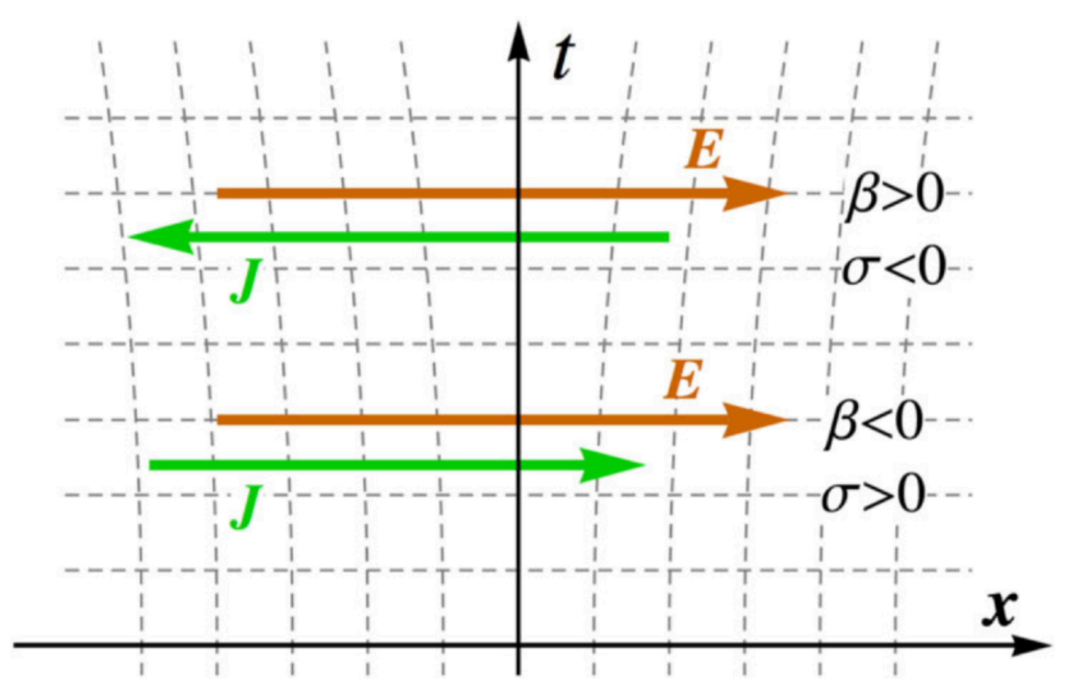
Scale Electric Effect:

$$\langle \boldsymbol{j}(t,\boldsymbol{x})\rangle_{\text{scale}} = \sigma(t)\boldsymbol{E}(t,\boldsymbol{x}) \quad \text{for } \boldsymbol{\nabla}\tau = 0$$

Conformal conductivity:

$$\sigma(t,\boldsymbol{x}) = -\frac{2\beta(e)}{e}\frac{\partial\tau(t,\boldsymbol{x})}{\partial t}$$

Negative in the expanding space-time!



Independently obtained in the de Sitter spacetime (a version of the Schwinger effect, both for fermions and bosons)

→ T. Hayashinaka, T. Fujita, and J. Yokoyama, Fermionic Schwinger effect and induced current in de Sitter space, J. Cosmol. Astropart. Phys. 07 (2016) 010; T. Hayashinaka and J. Yokoyama, Point splitting renormalization of Schwinger induced current in de Sitter spacetime, J. Cosmol. Astropart. Phys. 07 (2016) 012.

→ T. Kobayashi and N. Afshordi, Schwinger effect in 4D de Sitter space and constraints on magnetogenesis in the early Universe, J. High Energy Phys. 10 (2014) 166.

Scale magnetic effect (SME)

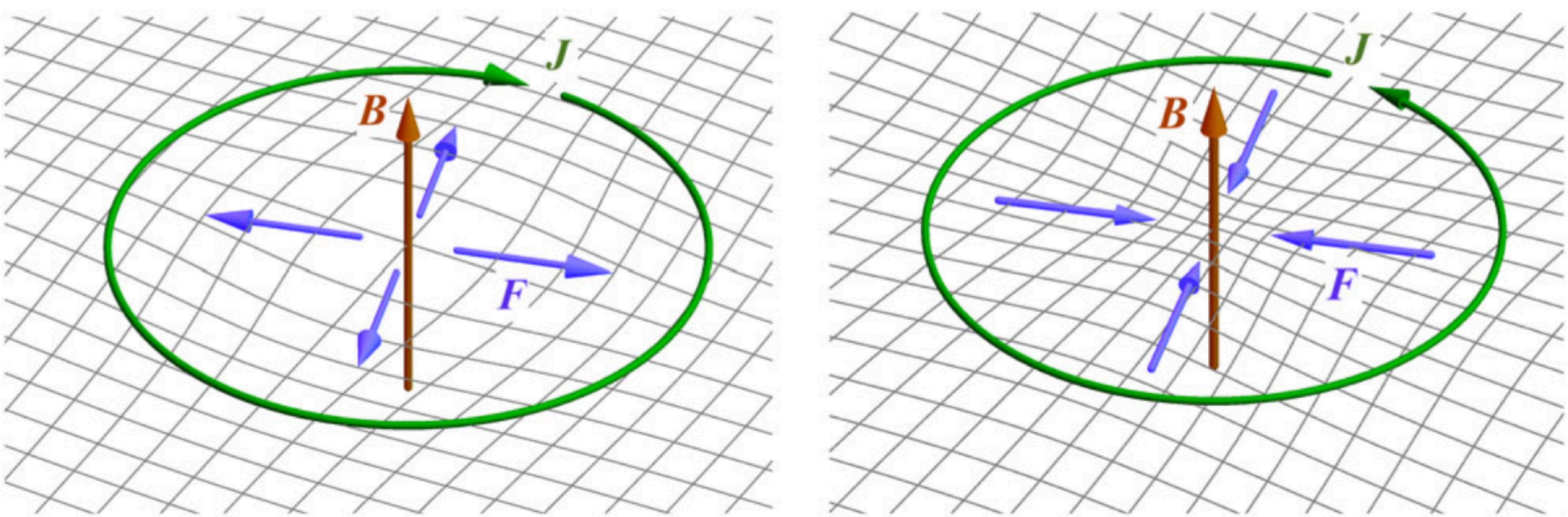
Space-dependent background: $$\tau = \tau(\boldsymbol{x})$$

Scale Magnetic Effect:

$$\langle \boldsymbol{j}(t,\boldsymbol{x})\rangle_{\text{scale}} = \boldsymbol{F}(\boldsymbol{x}) \times \boldsymbol{B}(t,\boldsymbol{x}) \quad \text{for } \partial_t \tau = 0$$

Gravitational deformation vector:

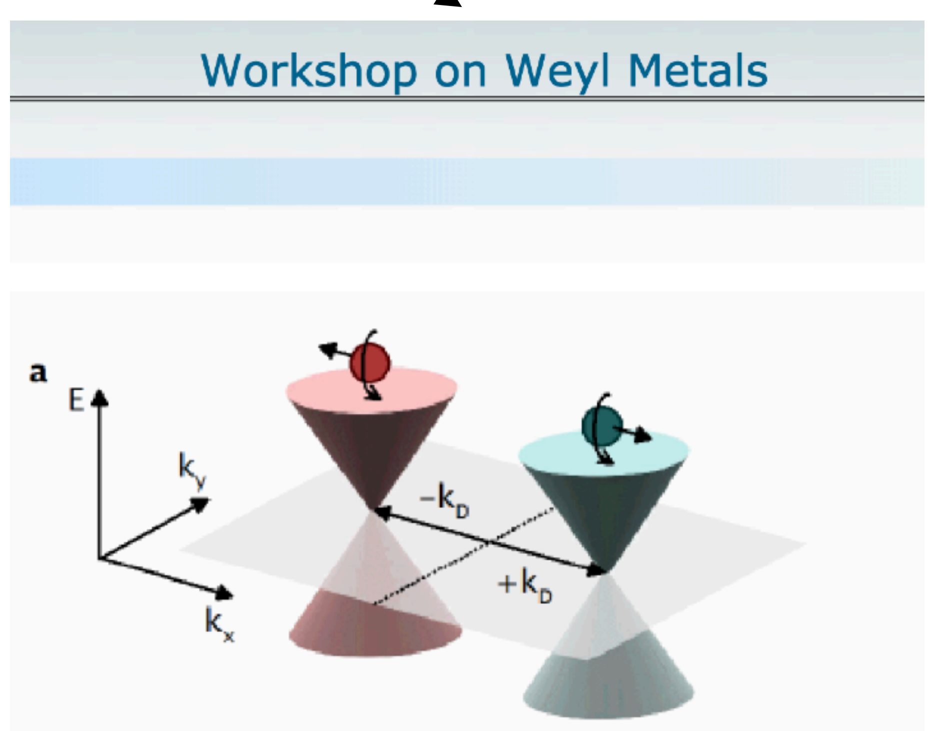
$$\boldsymbol{F}(t,\boldsymbol{x}) = \frac{2\beta(e)}{e} \boldsymbol{\nabla}\tau(t,\boldsymbol{x})$$



Similar to Hall effects

Gravity … de Sitter space …. deformed gravitational backgrounds …
and so what?….

Especially here, and in particular given the title of the workshop!



PHYSICAL REVIEW LETTERS **120,** 206601 (2018)

Generation of a Nernst Current from the Conformal Anomaly in Dirac and Weyl Semimetals

M. N. Chernodub,[1,2] Alberto Cortijo,[3] and María A. H. Vozmediano[3]

Temperature gradient drives system out of equilibrium, may be compensated by gravitational potential (Luttinger):

$$\frac{1}{T}\boldsymbol{\nabla} T = -\frac{1}{c^2}\boldsymbol{\nabla}\Phi \qquad\qquad g_{00} = 1 + \frac{2\Phi}{c^2}$$

1.) In a non-uniform thermal background we have an effective gravity.

2.) In magnetic-field background the SME generates an electric current.

→ Conformal anomaly generates thermoelectric transport!

Details of derivation:

$$\begin{aligned} J^\mu(x) &= -\frac{1}{\sqrt{-g(x)}}\frac{\delta S_{\rm anom}}{\delta A_\mu(x)} \\ &= -\frac{1}{\sqrt{-g(x)}}\frac{\partial}{\partial x^\nu}\Big[\sqrt{-g(x)}\,F^{\mu\nu}(x) \\ &\quad \cdot \int d^4y\sqrt{-g(y)}G(x,y)\Big(E(y)-\frac{2}{3}\Box R(y)\Big)\Big], \end{aligned}$$

$$\begin{aligned} C^2 &= C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} \\ &\equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3} \end{aligned}$$

$$H = E - \frac{2}{3}\Box R$$

$$\begin{aligned} E &= {}^*R_{\mu\nu\alpha\beta}{}^*R^{\mu\nu\alpha\beta} \\ &\equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \end{aligned}$$

$${}^*R_{\mu\nu\alpha\beta} = \frac{1}{2}\epsilon_{\mu\nu\mu'\nu'}R^{\mu'\nu'}{}_{\alpha\beta}$$

$$\Delta_4 = \nabla_\mu\left(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \frac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu$$

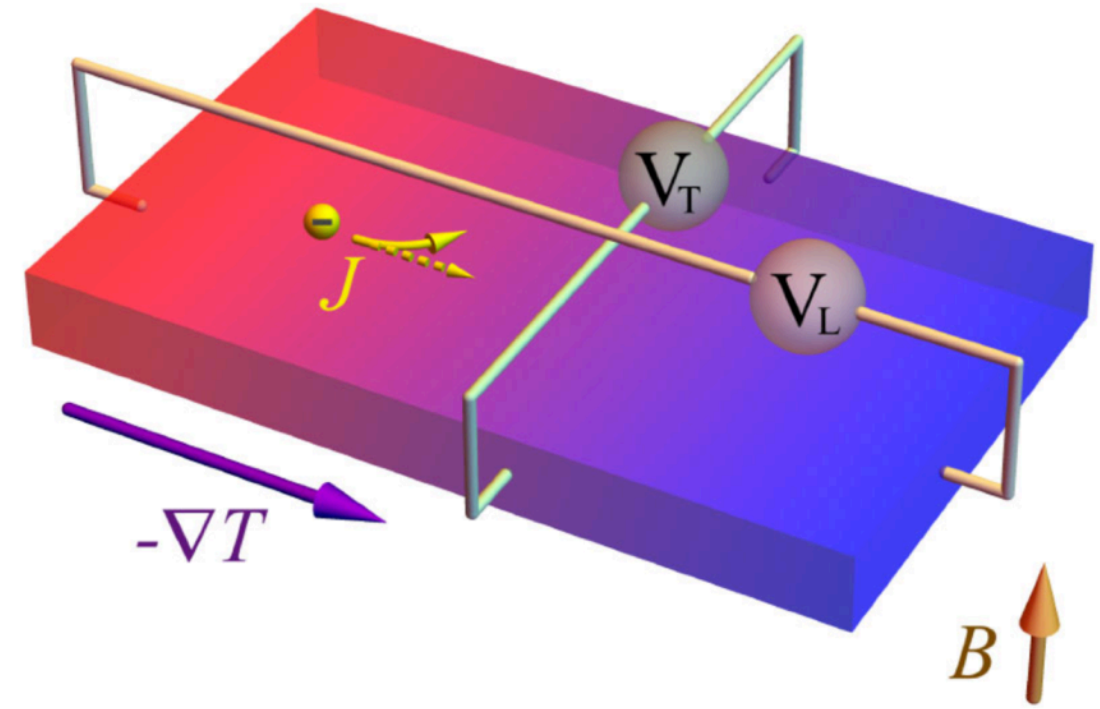
$$c = -\beta_{\rm QED}^{\rm 1loop}/(2e)$$

$$b = \frac{1}{320\pi^2}, \qquad b' = -\frac{11}{5670\pi^2}, \qquad c = -\frac{e^2}{24\pi^2}$$

Nernst-Ettingshausen Effect in Weyl semimetals

Giant Nernst Effect due to SME

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T\hbar} \boldsymbol{B} \times \boldsymbol{\nabla} T$$



The electric current is proportional to the beta function (conformal anomaly!)

Longitudinal anomalous transport in Weyl semimetals:

$$J_i = \sigma_{ij} E_j + \mathcal{L}^{12}_{ir}(-\nabla_r T) = 0$$

$$E_j = \rho_{ji} \mathcal{L}^{12}_{ir}(-\nabla_r T)$$

$$\mathcal{L}^{12}_{21} = \frac{e^2 v_F B_3}{18\pi^2 \hbar T}$$

$$S_{11} \equiv \frac{E_1}{B_3 \nabla_1 T} = \frac{\rho_{12}\mathcal{L}^{12}_{21}}{B_3} \sim \frac{v_F}{9|\boldsymbol{b}|T}$$

Estimations for an undoped Weyl semimetal ($v_F \sim 10^5$ m/s, $T \sim 10$ K, $|2\boldsymbol{b}| \sim 0.3$ Å^{-1})

$$S_{11}/T \sim 0.6\ \mu\mathrm{V/T\,K^{-2}}$$

Accessible experimentally!

works via the anomalous Hall current

$$\boldsymbol{J} = \frac{e^2}{2\pi^2\hbar} \boldsymbol{b} \times \boldsymbol{E}$$

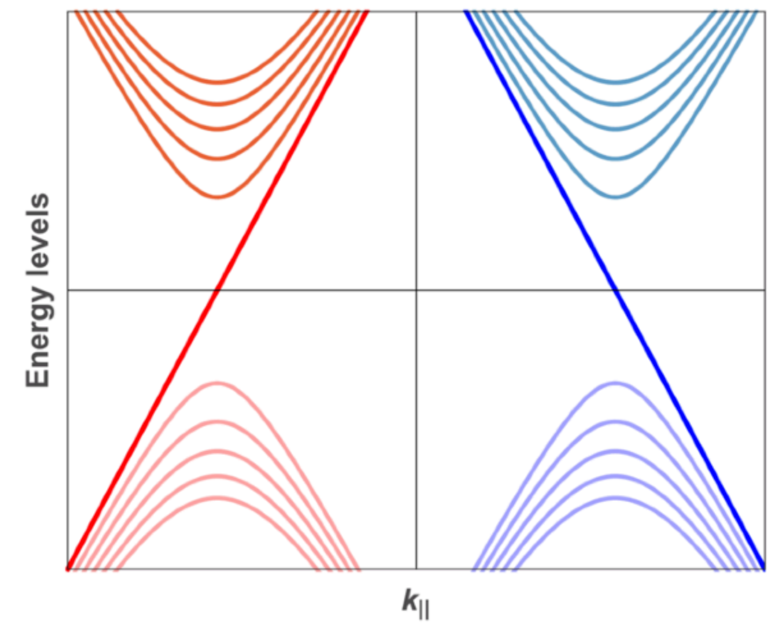
A. Cortijo, M. A. H. Vozmediano, M.Ch., PRL 120, 206601 (2018)

Thermoelectric transport from Kubo formula

Hamiltonian:

$$H_s = s v_{\rm F} \sigma^i (p_i + eA_i)$$

$$H_{pert}(t) = \Theta(t - t_0) T^{00}(t) g_{00}(t)$$



Thermal gradient:

$$\frac{1}{T}\boldsymbol{\nabla} T = -\frac{1}{c^2}\boldsymbol{\nabla}\Phi \qquad g_{00} = 1 + \frac{2\Phi}{c^2}$$

Transverse anomalous transport in Weyl semimetals:

$$\langle J^i \rangle(t, \mathbf{r}) = \int_{-\infty}^{\infty} \mathrm{d}t' \mathrm{d}\mathbf{r}' \underbrace{\left\{ \frac{-i}{\hbar} \Theta(t - t') \langle [J^i(t, \mathbf{r}), T^{00}(t', \mathbf{r}')] \rangle \right\}}_{\chi^i(t-t', \mathbf{r}-\mathbf{r}')} g_{00}(t', \mathbf{r}')$$

Transverse anomalous transport in Weyl semimetals:

$$\langle J^i \rangle(\omega, \mathbf{q}) = \chi^{ij}(\omega, \mathbf{q})(iq_j) g_{00}(\omega, \mathbf{q})$$

$$\chi^{ij}(\omega, \mathbf{q}) = (2\pi)^3 \int \mathrm{d}t\, e^{i\omega(t-t')} \int_{-\infty}^{t'} \mathrm{d}t'' \left\{ \frac{-iv_{\rm F}}{\mathcal{V}\hbar} \Theta(t - t') \langle [J^i(t, \mathbf{q}), T^{0j}(t'', -\mathbf{q})] \rangle \right\}$$

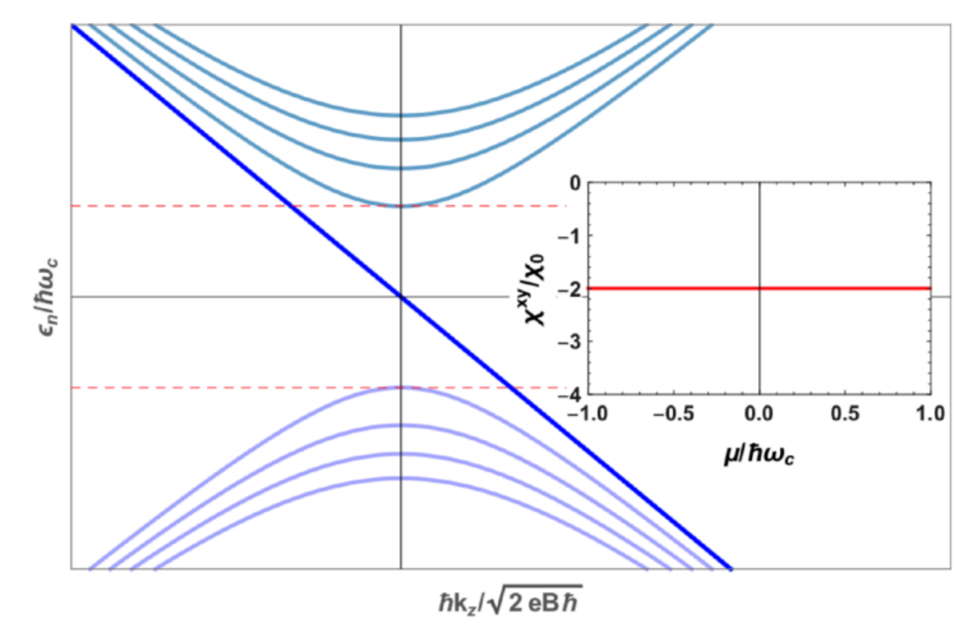
V. Arjona, M. A. H. Vozmediano, M.Ch., arXiv:1902.02358

Thermoelectric transport from Kubo formula

Lowest Landau Level gives for one chirality

$$\chi^{xy} \equiv \lim_{\mathbf{q}\to 0} \lim_{\omega\to 0} \chi^{xy}(\omega, \mathbf{q}) = \frac{-1}{2(2\pi)^2} \frac{v_{\mathrm{F}} e^2 B}{\hbar}$$

Higher LL's multiply the result by a factor of 2 (approx.)



Thermoelectric coefficient from Kubo approach

$$\alpha^{xy} = \frac{2}{T}\chi^{xy} = -\frac{e^2 v_{\mathrm{F}} B}{4\pi^2 T\hbar}$$

Agrees, up to a coefficient, with purely conformal result:

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T\hbar} \boldsymbol{B} \times \boldsymbol{\nabla} T$$

Valid irrespective of the relative positions of the Weyl cones

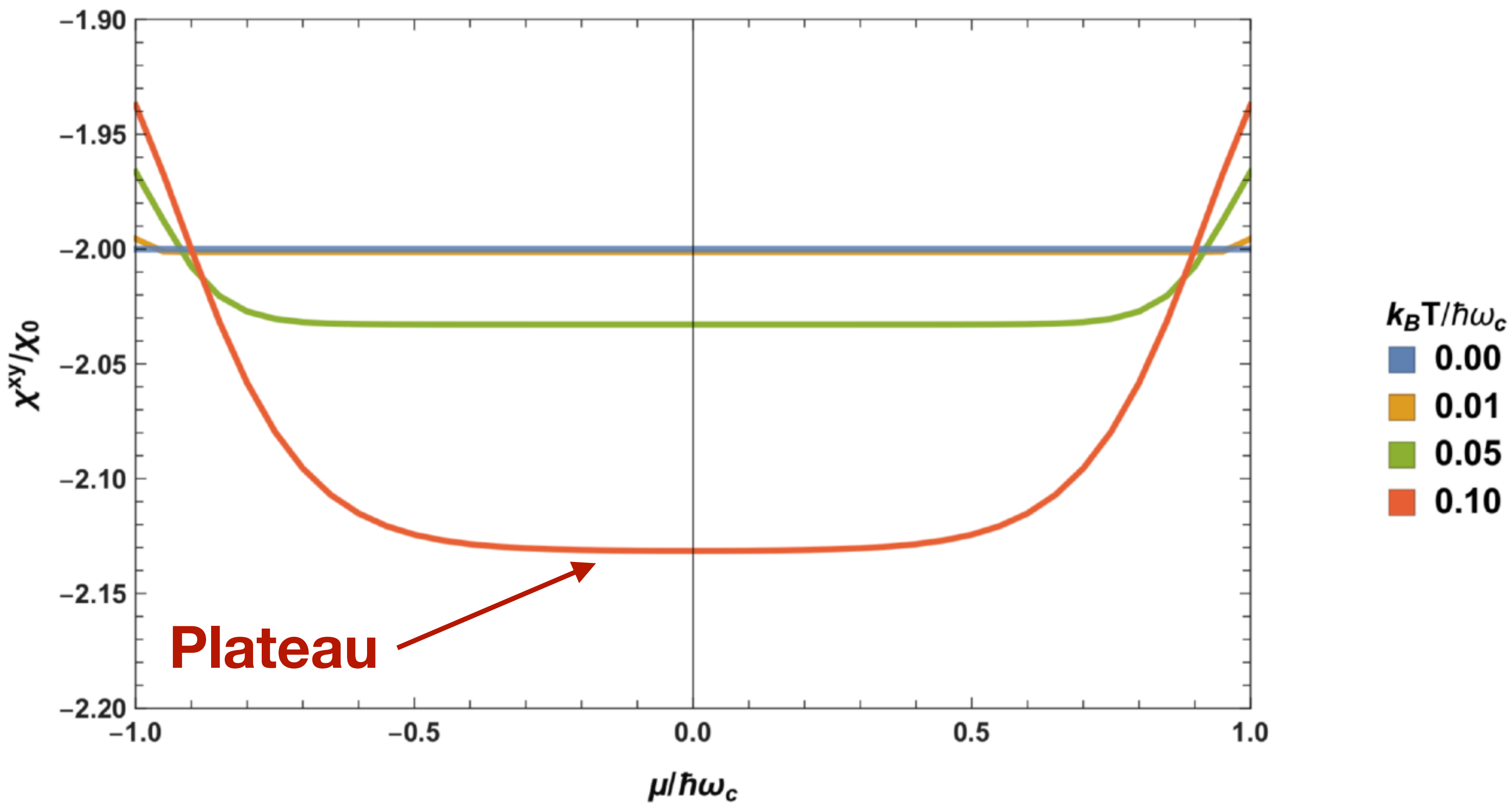
No anomalous contribution coming from the Berry phase!

V. Arjona, M. A. H. Vozmediano, M.Ch., arXiv:1902.02358

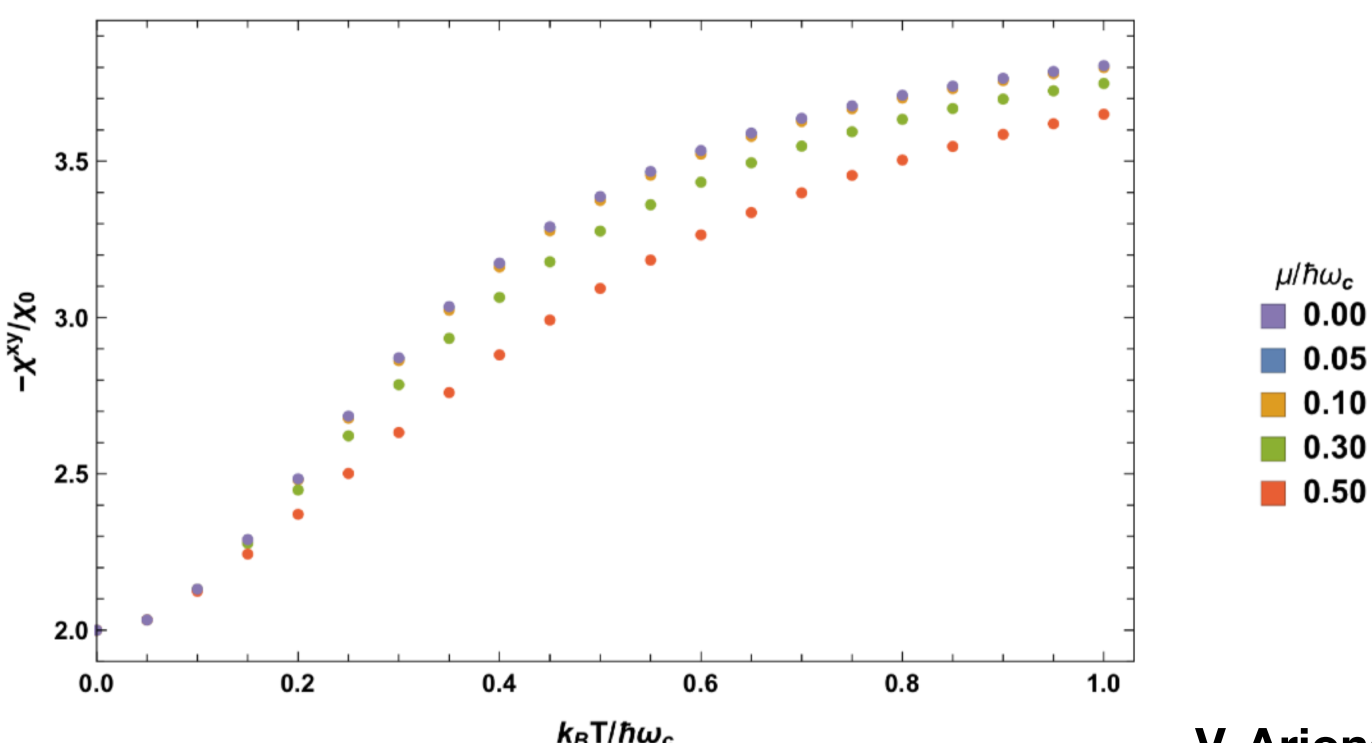
Thermoelectric transport at Fermi surface

Calculation from Kubo formula

- **Plateau at small μ and nonzero T**
- **Increases with T at μ=0**
- **Decreases with T at $\mu>\omega_c$**



$$\chi_0 = v_F e^2 B/4(2\pi)^2\hbar$$



V. Arjona, M. A. H. Vozmediano, M.Ch., arXiv:1902.02358

Remarks on the Scale Magnetic Effect (SME)

– Appears due to conformal anomaly $\langle T^{\mu}{}_{\mu} \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$

– Bulk phenomenon, works at zero chemical potential

– Leads to a giant Nernst effect in Dirac/Weyl semimetals

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T \hbar} \boldsymbol{B} \times \boldsymbol{\nabla} T$$

– Is not related to axial or axial-gravitational anomalies

– Strength is given by the beta function $\beta(e) = \frac{de(\mu)}{d\ln\mu}$

– Universal: works both in fermionic and bosonic systems

Scale Electric Effect: negative conductivity in time-dependent backgrounds

Scale electromagnetic effects at the edge

What is about the boundaries?

electric current

beta function

normal to the boundary

$$J^\mu = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}n_\nu}{x}$$

distance to the boundary

D.M. McAvity, H. Osborn, Class. Quantum Gravity 8, 603 (1991).
C.-S. Chu and R.-X. Miao, JHEP 07, 005 (2018), PRL 121, 251602 (2018).

Scale Magnetic Effect at the Edge (SMEE):

Electric current along the edge due to tangential magnetic field

$$\boldsymbol{j}(\boldsymbol{x}) = -f(\boldsymbol{x})\,\boldsymbol{n}\times\boldsymbol{B}$$

$$f(\boldsymbol{x}) = \frac{2\beta(e)}{e^2}\frac{1}{x_\perp}$$

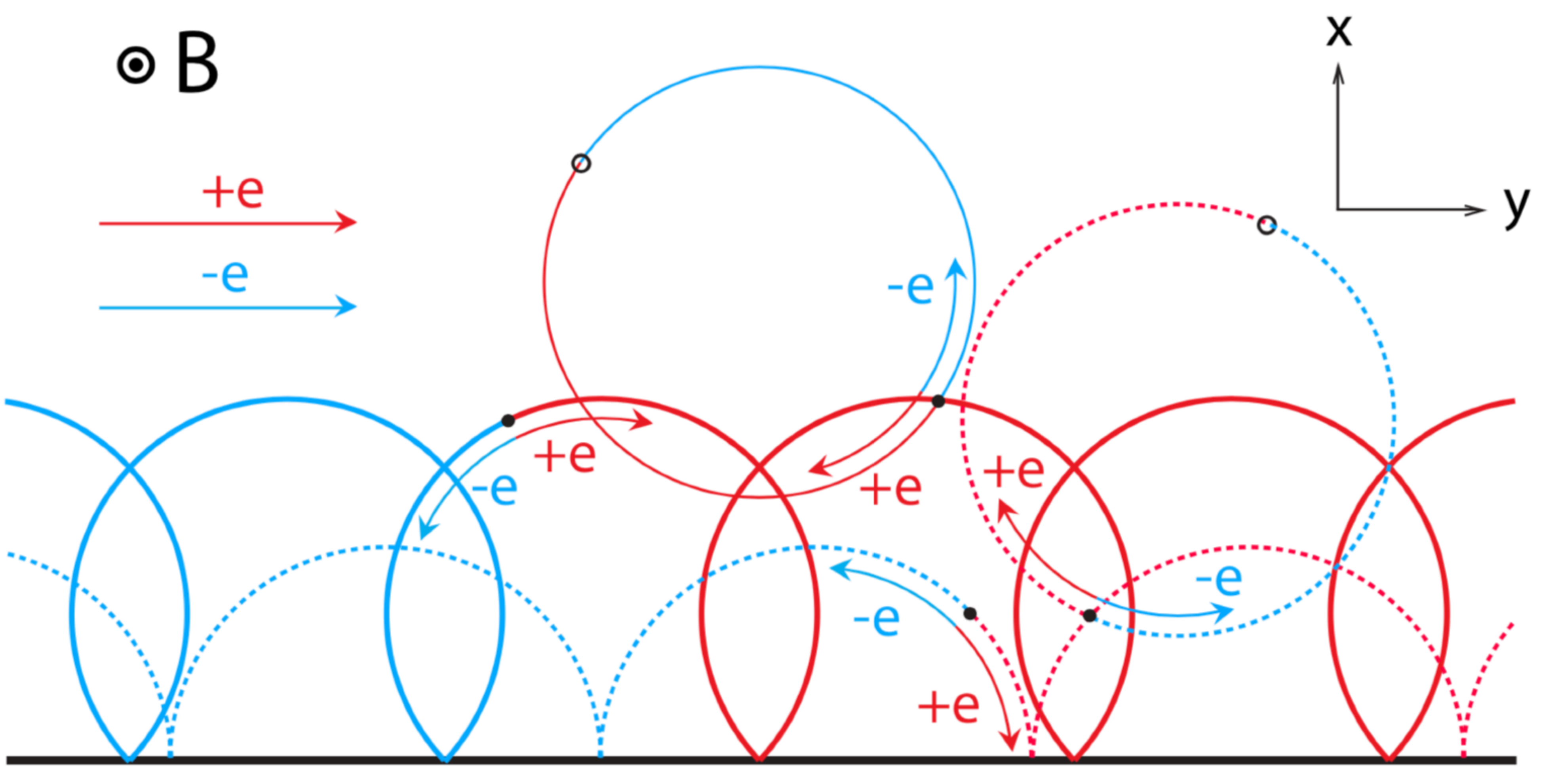
diverges at the boundary!

– Effect due to conformal anomaly
– No topology (Berry, Chern etc)

Scale Magnetic Effect at the Edge (SMEE)

A physical picture

Ingredients: vacuum, edge and magnetic field



Skipping orbits (like in the Hall effect, but now in the vacuum)

Absent: No Fermi surface, no temperature.

SMEE: numerical check

Generates the current at the boundary?

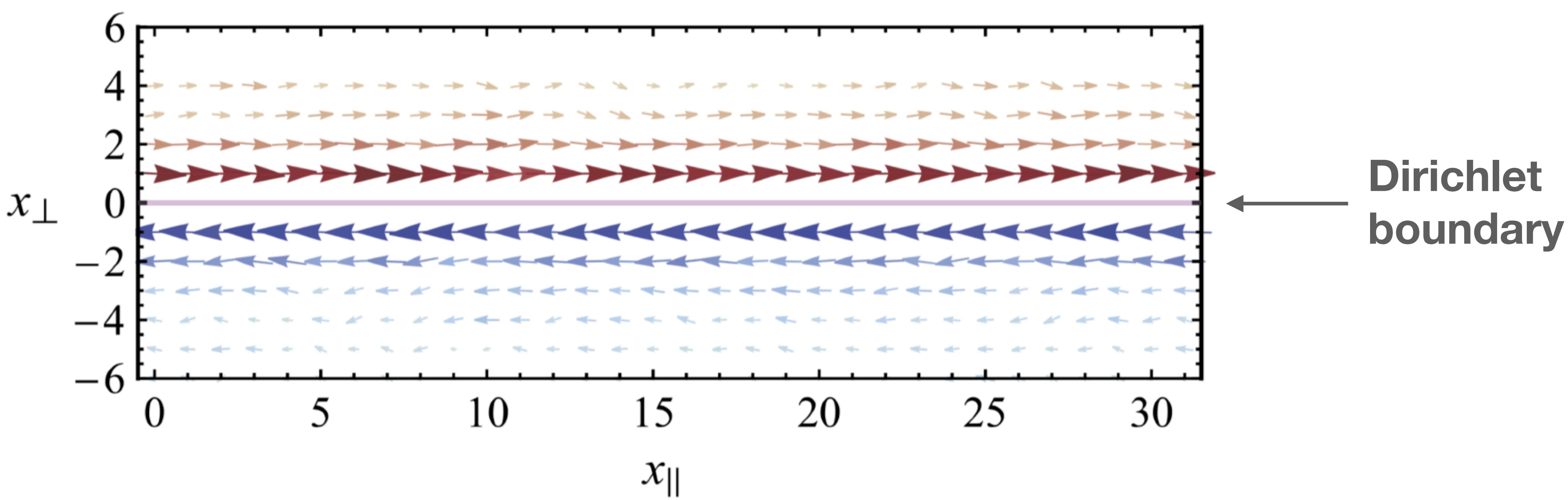
Scalar electrodynamics at a conformal point:

$$\mathcal{L}_{\mathrm{sQED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[\left(\partial_\mu - ieA_\mu \right) \phi \right]^* \left(\partial^\mu - ieA^\mu \right) \phi$$

Massless one-component electrically-charged scalar field

Numerical Monte-Carlo simulations

We see the generated electric current!

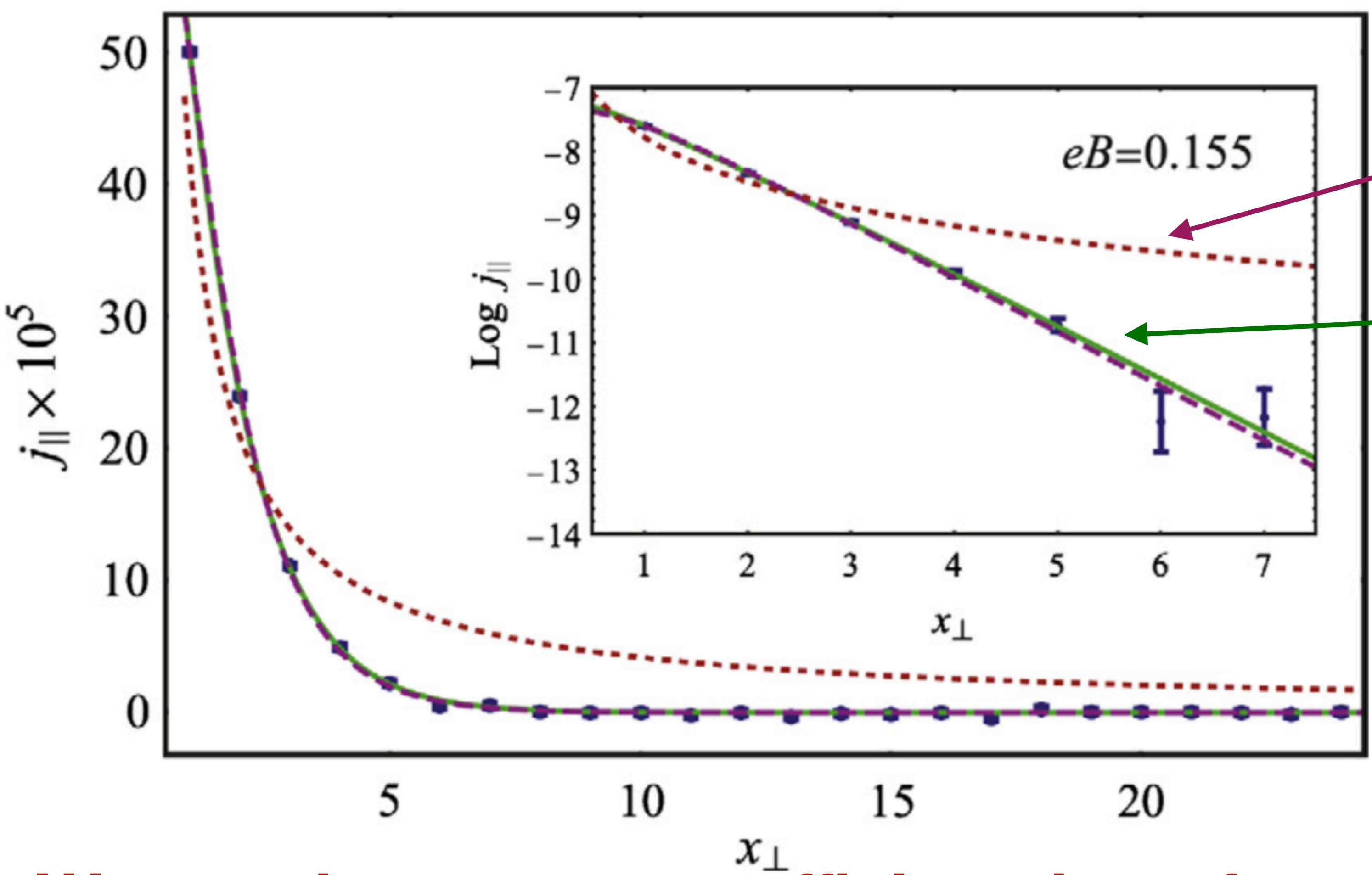


V.A. Goy, A.V. Molochkov, M.Ch., Phys. Lett. B 789, 556 (2019)

SMEE: numerical check

Diverges at the boundary?

We see the *1/x* divergence of the current at the boundary



non-conformal point
(the scalar is massive)

conformal point
(the massless scalar)

We see the correct coefficient, beta function:

$$\beta^{\text{1-loop}}_{\text{sQED}} = \frac{e^3}{48\pi^2}$$

The beta function of scalar QED is four times smaller than the beta function in usual QED

V.A. Goy, A.V. Molochkov, M.Ch., Phys. Lett. B 789, 556 (2019)

Scale magnetic effect at the edge and superconductivity

Scalar electrodynamics at a conformal point (massless scalar):

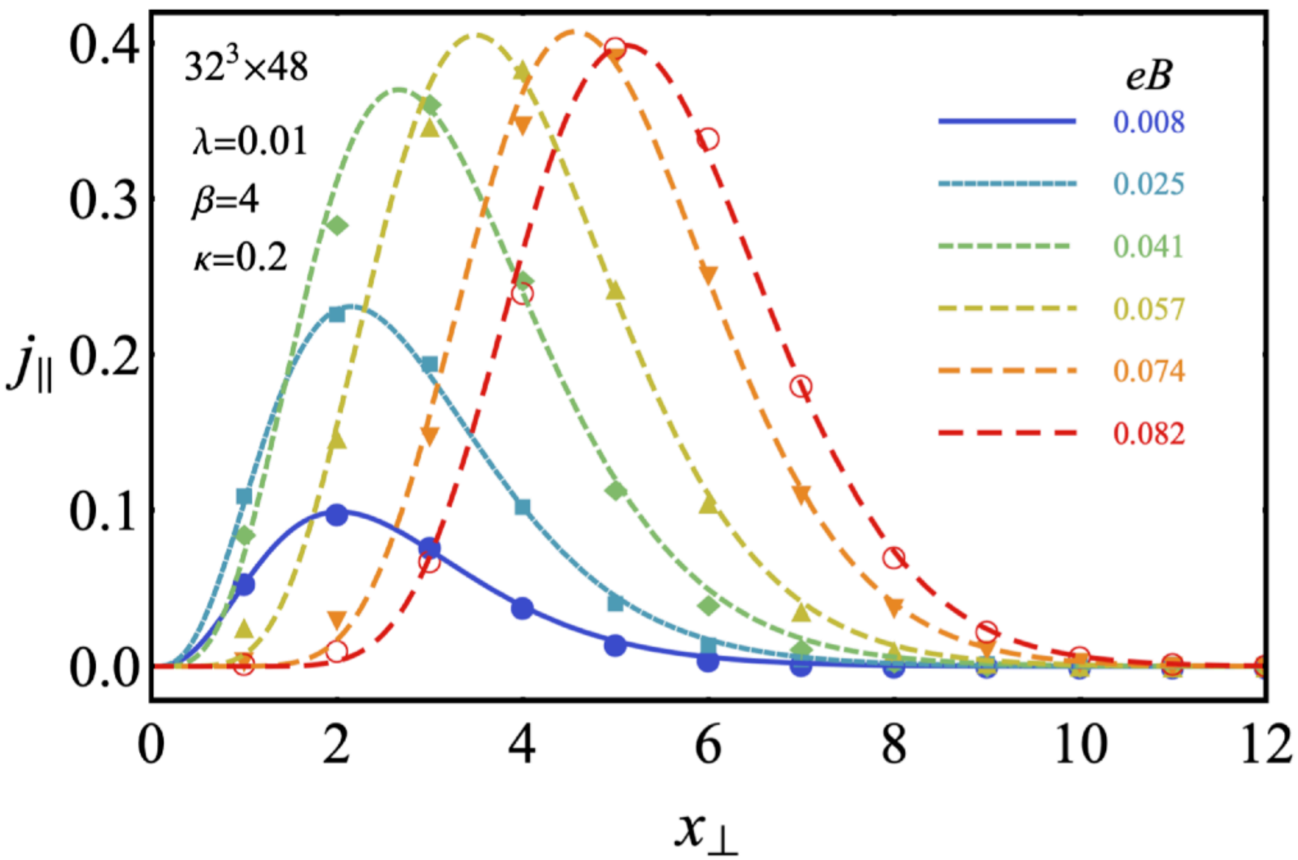
$$\mathcal{L}_{\rm sQED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[\left(\partial_\mu - ieA_\mu\right)\phi\right]^* \left(\partial^\mu - ieA^\mu\right)\phi$$

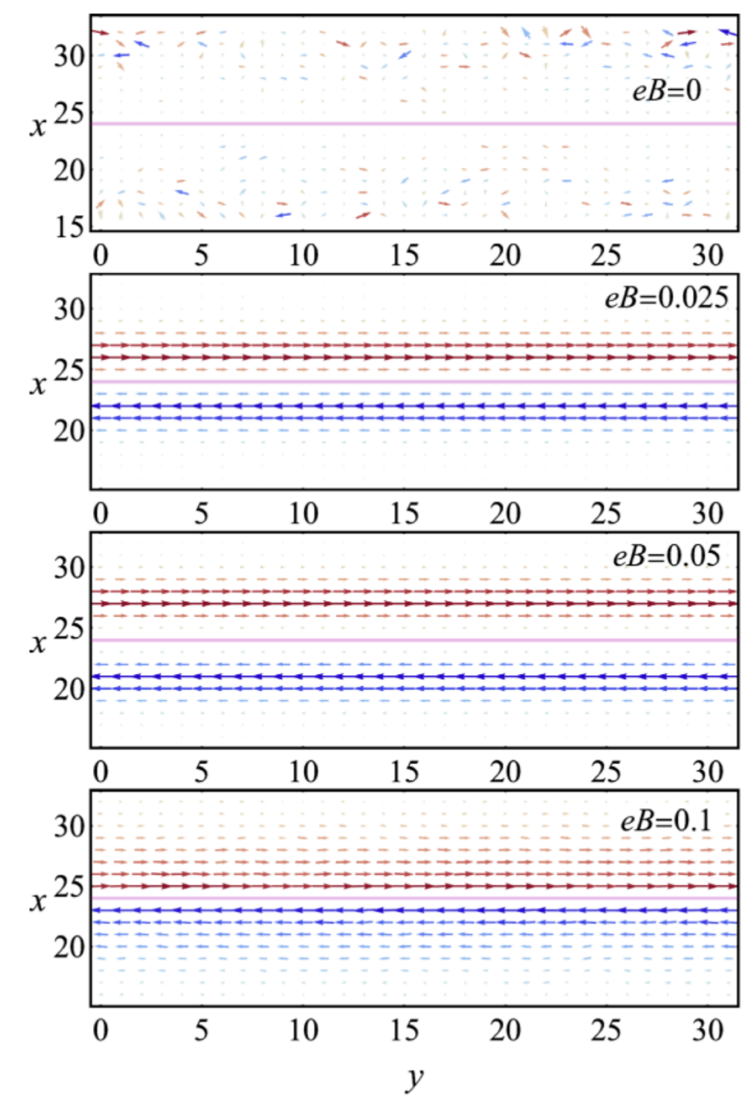
Charged fields may condense!

potential

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* D^\mu \varphi - V(\varphi)$$

What happens to the SEEE current in the superconducting phase?





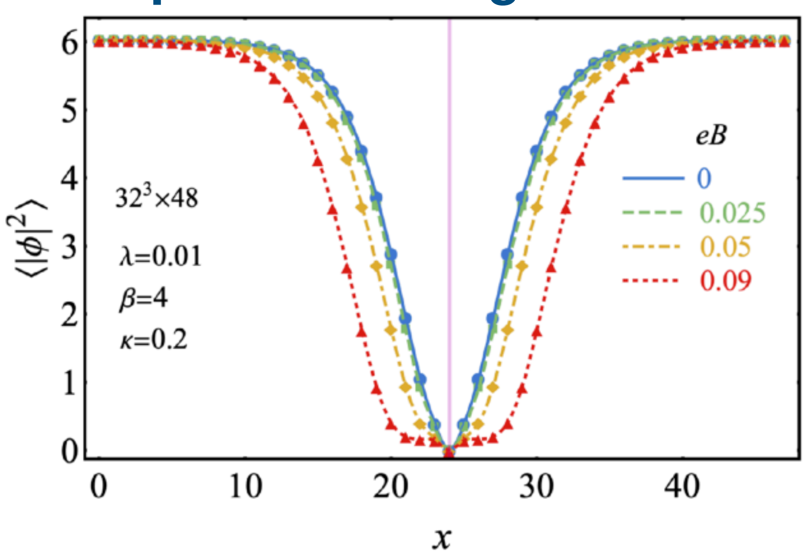
V.A. Goy, A.V. Molochkov, M.Ch., in preparation

Scale magnetic effect at the edge and superconductivity

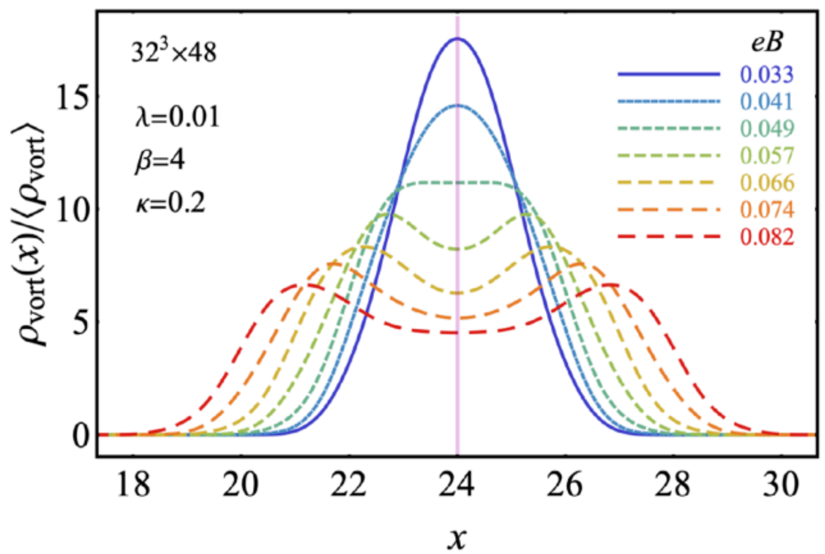
What happens to the SEEE boundary current in superconducting phase?

It becomes the Meissner current!

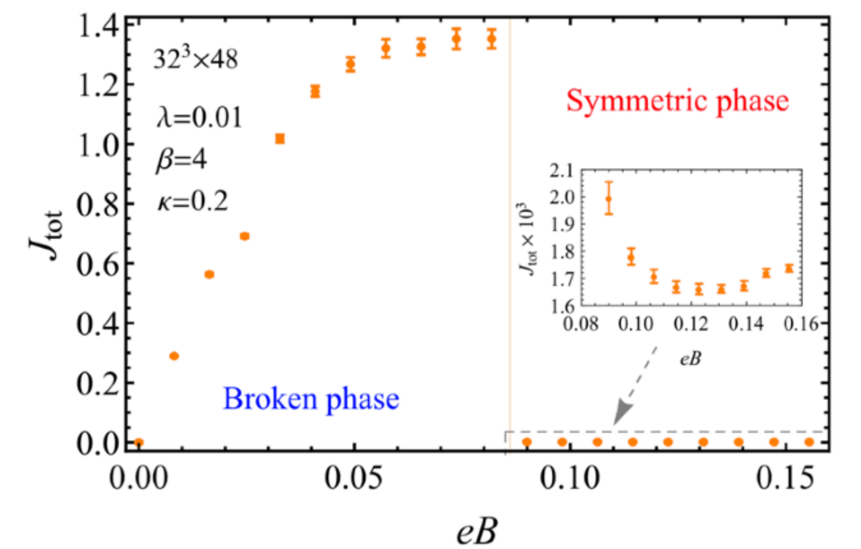
Superconducting condensate

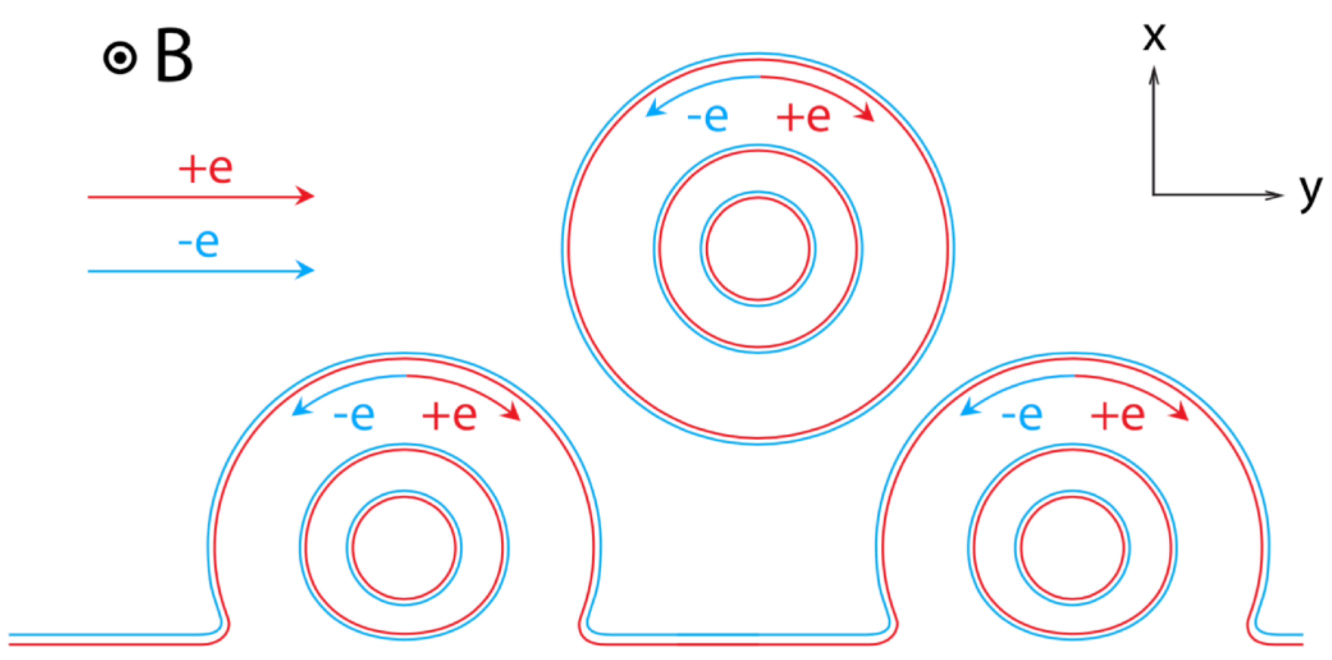


Vortex density



Total (integrated) electric current





Meissner: total current is large, but it vanishes at the boundary

SMEE: total current is small, but it diverges at the boundary

Scale electric effect at the edge: conformal screening

Screening of electrostatic field in metals:

$$E(x) \sim E(0)e^{-x/\lambda}$$

Fermi momentum

$$p_F = (2\pi^2 n)^{1/3}\hbar$$

Density of carriers n

Screening lengths:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\varepsilon_0 k_B T}{ne^2}}, \qquad \lambda_{\mathrm{FT}} = \sqrt{\frac{\varepsilon_0 \pi^2 \hbar^3}{me^2 p_F}}$$

Debye **Fermi-Thomas**

What if the medium is totally conformal and possess no dimensionful parameters?

For example, take a Dirac semimetal at particle-hole symmetric point.

– We have the mobile carriers (massless fermionic quasiparticles)

– Classically, there is no dimensionful scale.

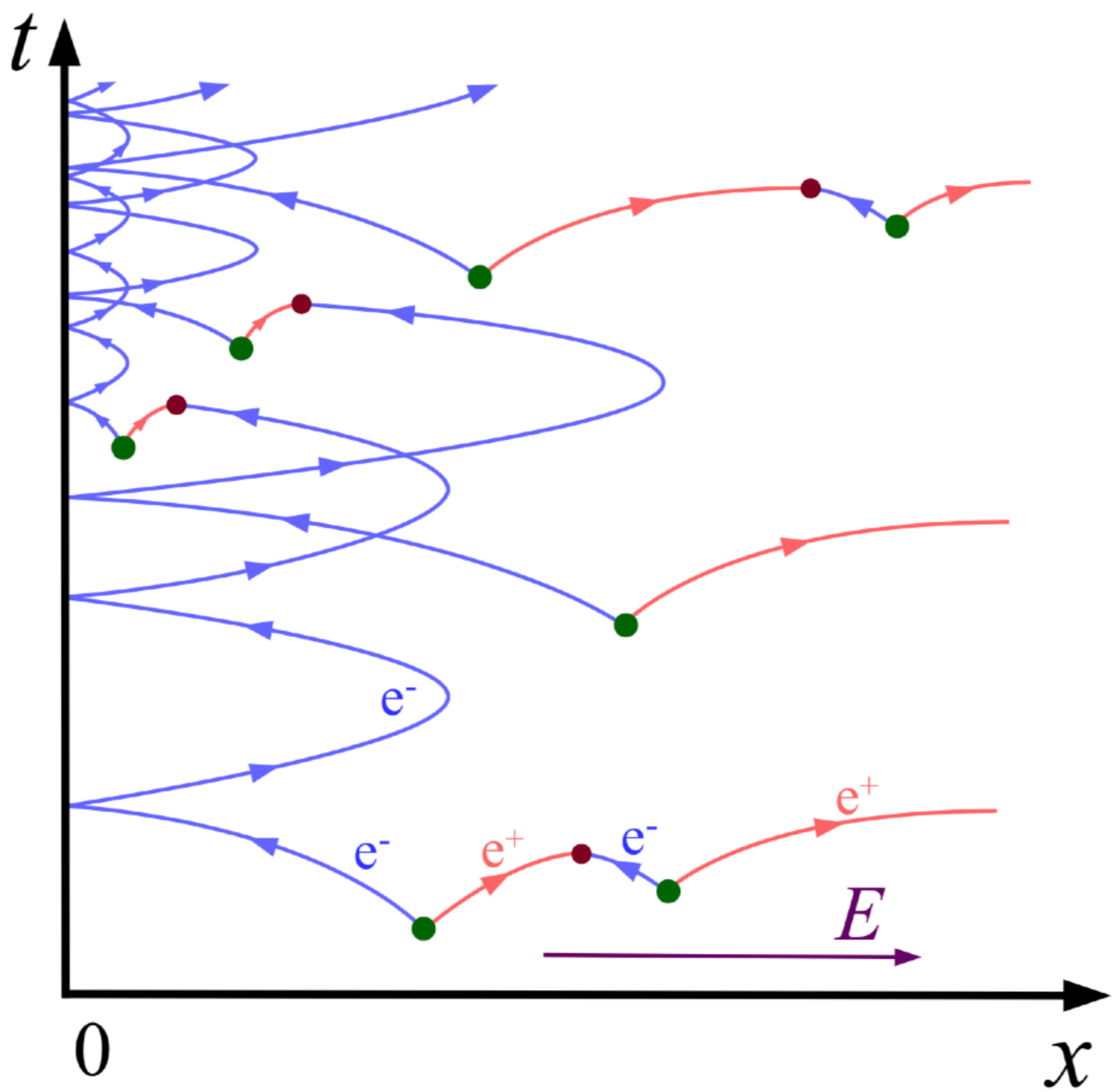
No quantity to construct the screening length from!

– No classical scale → no screening?

Scale electric effect at the edge

$$J^\mu = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}n_\nu}{x} \longrightarrow \rho = -\frac{2\beta_e}{e\hbar}\frac{\boldsymbol{nE}}{x}$$

electric charge density
at the boundary



Physics: the screening is due to
the Schwinger effect (skipping orbits in time)

Works efficiently due to
the absence of the gap

Generated by the conformal anomaly!

Mechanism in semimetals: creation of electron-hole pairs in the
presence of a uniform electric field (the Zener effect)

Scale electric effect at the edge

Consider QED:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{a=1}^{N_f} \bar{\psi}_a i\gamma^\mu D_\mu \psi_a$$

Charge density due to conformal anomaly: $$\rho = -\frac{2\beta_e}{e\hbar}\frac{\boldsymbol{n}\boldsymbol{E}}{x}$$

Solve the Maxwell equation $$\partial_x E_x(x) = \frac{1}{\varepsilon_0}\rho(x)$$

At the boundary the conformal screening is polynomial:

$$E_x(x) = \frac{C}{x^\nu}$$

Electric field

$$\rho(x) = -\frac{C\varepsilon_0\nu}{x^{1+\nu}}$$

Charge density

$$\phi(x) = \phi_0 - \frac{Cx^{1-\nu}}{1-\nu}$$

Electrostatic potential

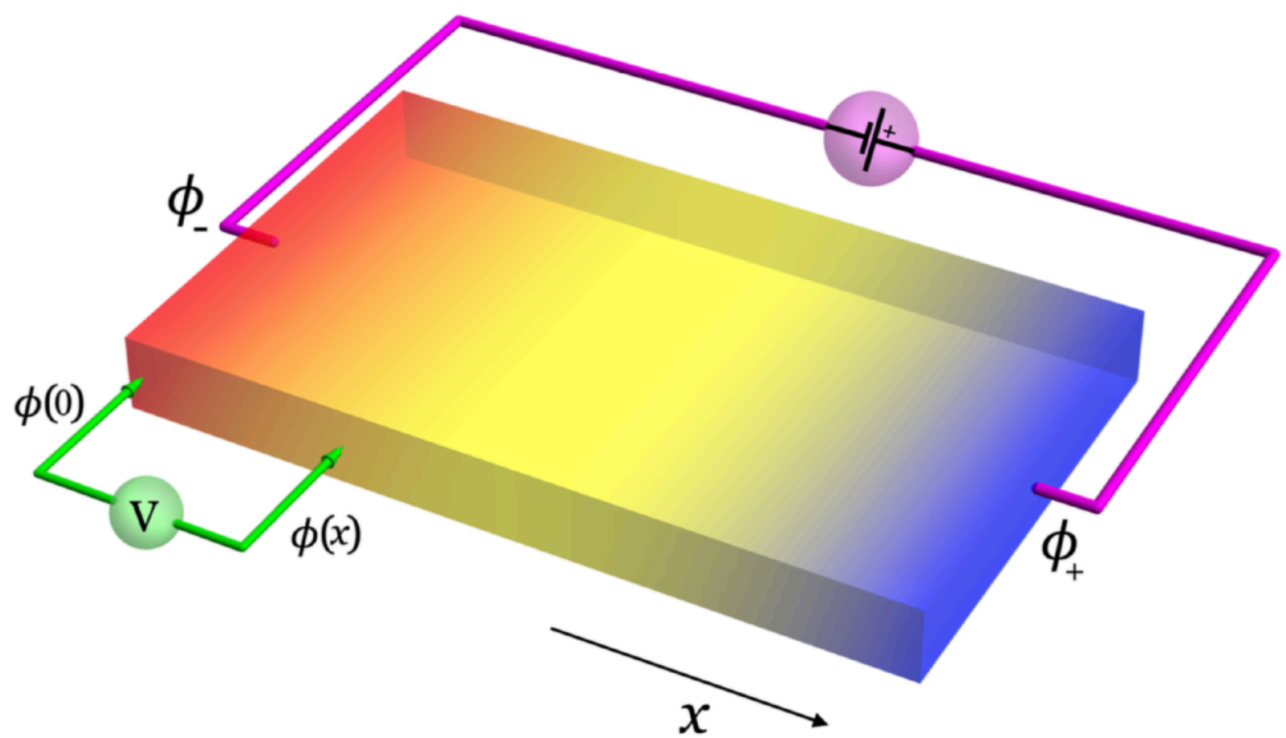
Conformal screening exponent: $$\nu = \frac{2\beta_e}{ec\hbar\varepsilon_0}$$

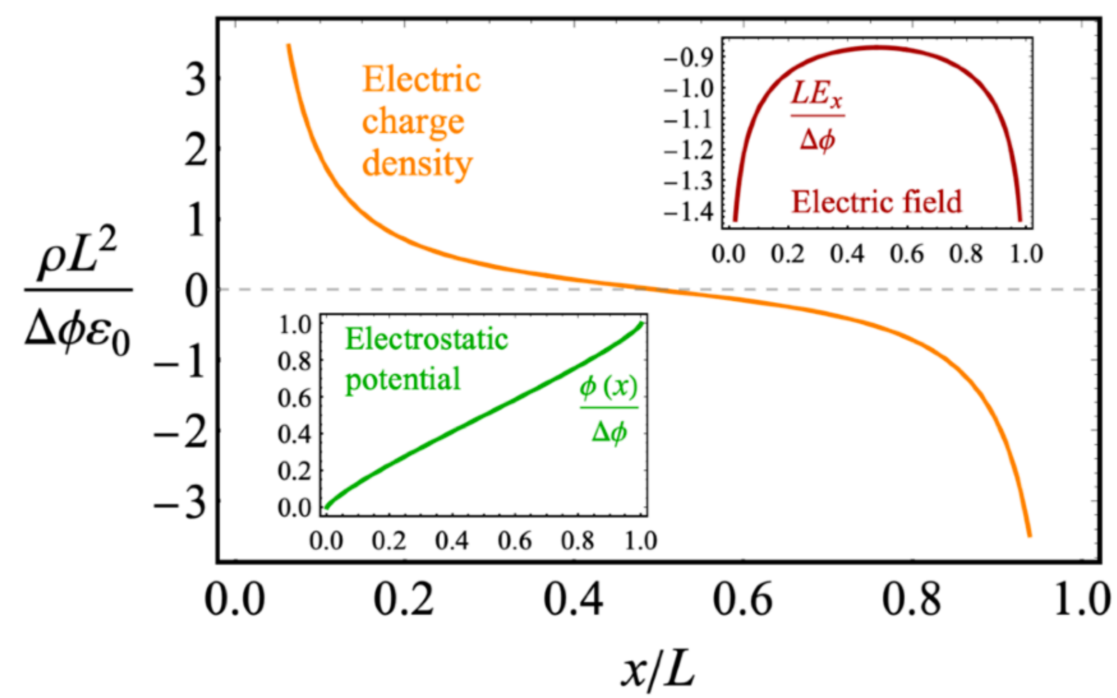
Scale electric effect at the edge

Conformal exponent in a Dirac semimetal: $$\nu = \frac{e^2}{6\pi^2 \hbar v_F \varepsilon \varepsilon_0}$$

Particle density in a finite sample with two boundaries:

$$\rho(x) = \frac{\Delta\phi}{L^2} \varepsilon_0 \nu h(\nu) \left(1 - \frac{2x}{L}\right) \left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{-1-\nu}$$





Direct measurement of the beta function. Indirect evidence of the Schwinger effect.

M. A. H. Vozmediano, M.Ch., arXiv:1902.02694

Accessible experimentally

- direct measurement of the beta function
  associated with the renormalization of the electric charge
  (never done in solid state)

- evidence of the elusive Schwinger effect
  (particle-antiparticle production by electric field)

Conformal exponent in a Dirac semimetal: $$\nu = \frac{e^2}{6\pi^2 \hbar v_F \varepsilon \varepsilon_0}$$

In typical Dirac/Weyl materials $v_F \sim 10^{-3} c$ and $\varepsilon \sim 10$

→ large, experimentally accessible conformal exponent: $\nu \sim 10^{-1}$

Electrostatic screening potential vs. distance from the boundary of a Dirac material (at the Lifshitz point)