(multi-)Weyl Semimetals, QFT & chiral anomalies

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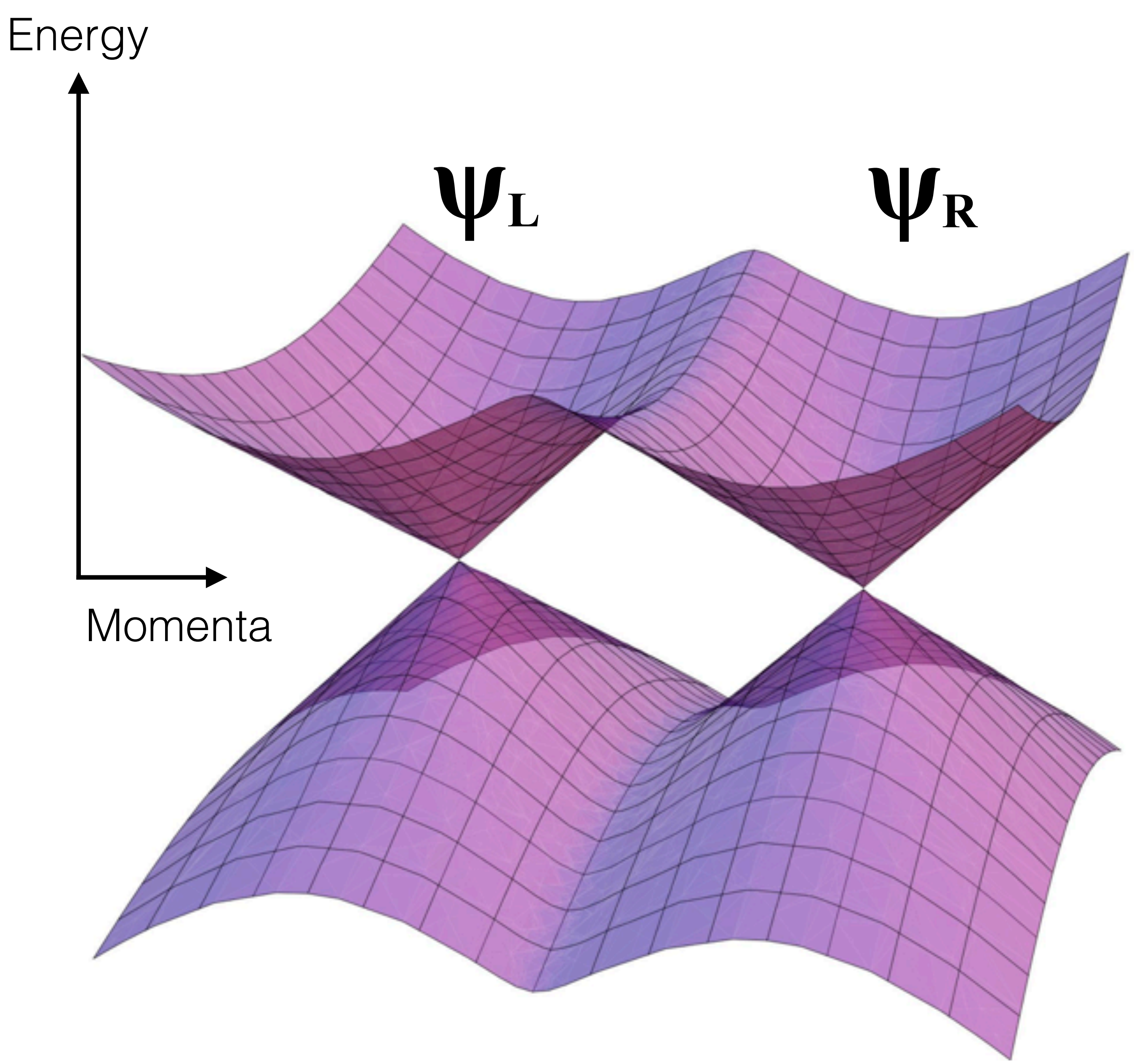
Based on arXiv:1802.07733 and arXiv:1902.?????

Weyl semimetals



- Weyl nodes always coming in pairs
- each node comes with a different chirality
- Breaking of time reversal, inversions or both are necessary

Weyl semimetals

Weyl semimetals are characterized by having a monopole Berry curvature in momentum space.

The best known examples correspond to monopole charge $n=1$.

$n=1$ Weyl semimetals have linear dispersion relations and suffer of chiral anomalies.

Chiral anomalies imply new non-dissipative transport coefficients and negative magnetoresistance.

Are there Weyl semimetals with higher monopole charge?

The answer is: **YES!**

multi-Weyl semimetals

$$H_n(\mathbf{p}) = \alpha_n p_\perp^n \left[\cos(n\phi_p)\,\sigma_x + \sin(n\phi_p)\,\sigma_y\right] + v p_z \sigma_z$$

$$\equiv \epsilon_\mathbf{p}\ (\mathbf{n}_\mathbf{p}\cdot\boldsymbol{\sigma})$$

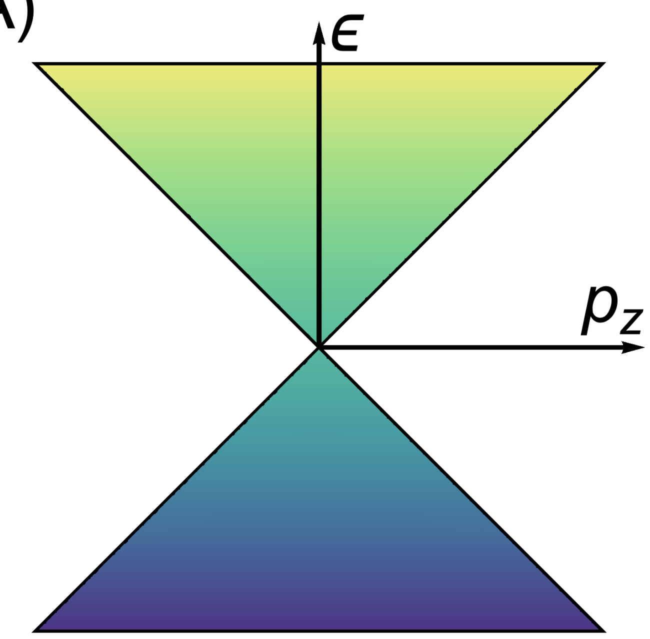
The Berry curvature around a Weyl point is:

$$\Omega^{\pm}_{\mathbf{p}\,a} = \pm\frac{1}{4}\epsilon_{abc}\mathbf{n}_\mathbf{p}\cdot\left(\frac{\partial \mathbf{n}_\mathbf{p}}{\partial p_b}\times\frac{\partial \mathbf{n}_\mathbf{p}}{\partial p_c}\right)$$

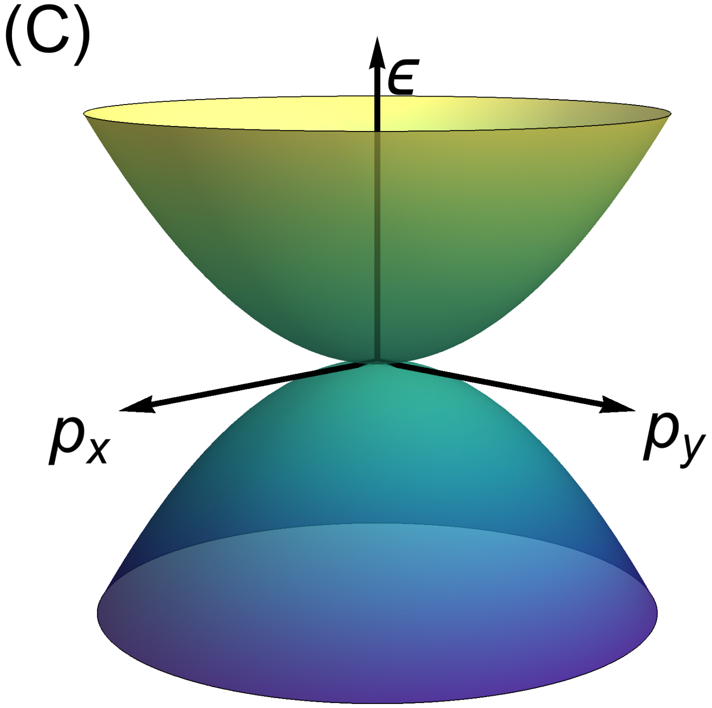
$$\boldsymbol{\Omega}_\mathbf{p} = \frac{1}{2}\frac{n v \alpha_n^2 (p_x^2+p_y^2)^{n-1}}{\left[\alpha_n^2(p_x^2+p_y^2)^n + v^2 p_z^2\right]^{3/2}}\ (p_x, p_y, n p_z)$$

$$n = \frac{1}{2\pi}\oint_\Sigma \Omega_\mathbf{p}\cdot d\mathbf{S}.$$

(A)



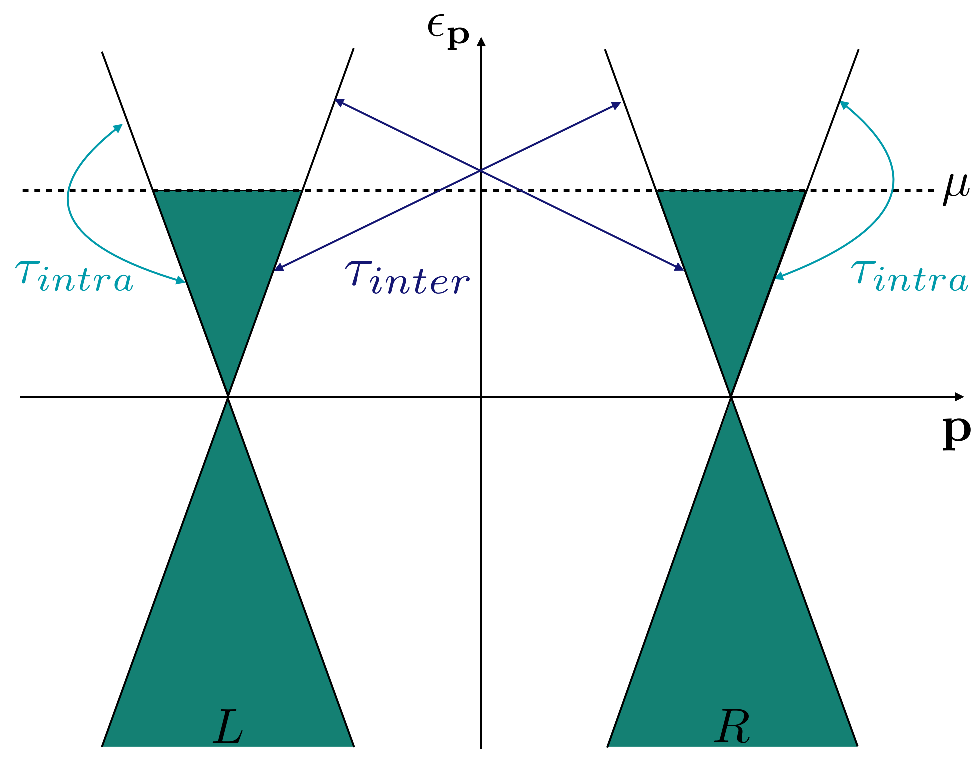
(C)



Proposed to exist for example in: $\mathrm{SrSi_2}$ $\mathrm{HgCr_2Se_4}$

multi-Weyl semimetals

$$\partial_t f^{(s)} + \nabla_{\mathbf{x}} f^{(s)} \cdot \dot{\mathbf{x}}^{(s)} + \nabla_{\mathbf{p}} f^{(s)} \cdot \dot{\mathbf{p}}^{(s)} = C[f^{(s)}]$$



$$\rho^{(s)} = e \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(s)}\right) f^{(s)}$$

$$C_2[f^{(s)}] = \frac{\bar{f}^{(s)} - f^{(s)}}{\tau_{intra}} + \frac{\bar{f}^{(\bar{s})} - f^{(s)}}{\tau_{inter}}$$

$$\mathbf{J}^{(s)} = e \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(s)}\right) \dot{\mathbf{x}} f^{(s)}$$

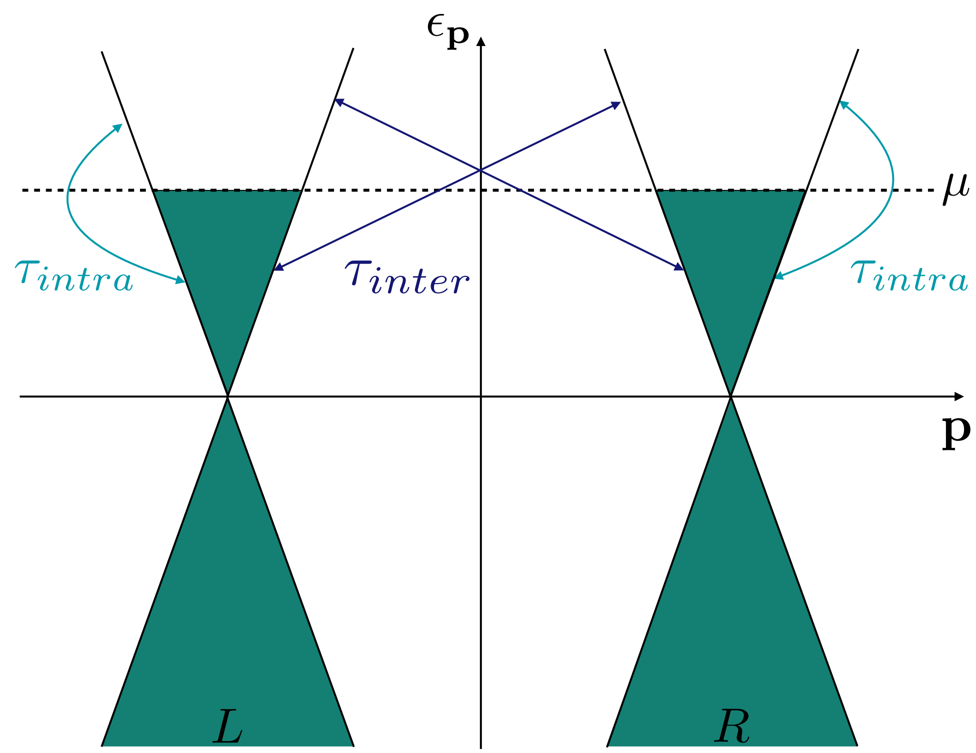
$$\partial_t \rho + \nabla \cdot \mathbf{J} = 0$$

$$\partial_t \rho_5 + \nabla \cdot \mathbf{J}_5 = \frac{e^3 n}{4\pi^2} \mathbf{E} \cdot \mathbf{B} - \frac{\rho_5}{2\tau_{inter}}$$

For a QFT analysis of the pure gauge anomaly see also 1705.04576, 1803.01684

multi-Weyl semimetals

$$\partial_t f^{(s)} + \nabla_{\mathbf{x}} f^{(s)} \cdot \dot{\mathbf{x}}^{(s)} + \nabla_{\mathbf{p}} f^{(s)} \cdot \dot{\mathbf{p}}^{(s)} = C[f^{(s)}]$$



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$$\mathbf{J}^{(s)} = e \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(s)}\right) \dot{\mathbf{x}} f^{(s)}$$

$$\dot{\mathbf{x}}^{(s)} = \left(1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(s)}\right)^{-1} \left[\mathbf{v}_{\mathbf{p}} + e\mathbf{E} \times \mathbf{\Omega}_{\mathbf{p}}^{(s)} + e\left(\mathbf{v}_{\mathbf{p}} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(s)}\right)\mathbf{B}\right],$$
$$\dot{\mathbf{p}}^{(s)} = \left(1 + e\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(s)}\right)^{-1} \left[e\mathbf{E} + e\mathbf{v}_{\mathbf{p}} \times \mathbf{B} + e^2 (\mathbf{E} \cdot \mathbf{B})\, \mathbf{\Omega}_{\mathbf{p}}^{(s)}\right].$$

Magnetoconductance

$$\sigma_{jj} = \tau_{inter} \frac{e^4 n^3 v \Gamma(\frac{1}{2} + \frac{1}{n})}{4\pi^{5/2}\Gamma(\frac{1}{n})} \left(\frac{\alpha}{\mu}\right)^{2/n} B^2$$

Now we take a different point of view

Let's start with the Hamiltonian $$H_{L,R} = \pm \vec{k} \cdot \vec{\tau} \otimes \mathbb{I}_{2\times 2}$$

And find the perturbations which are invariant under C_4 $$R_z = e^{-i\alpha\tau_z} \otimes e^{-i\alpha s_z}$$

$\tau_z \otimes s_z$ **Two Weyl points with the same chirality**

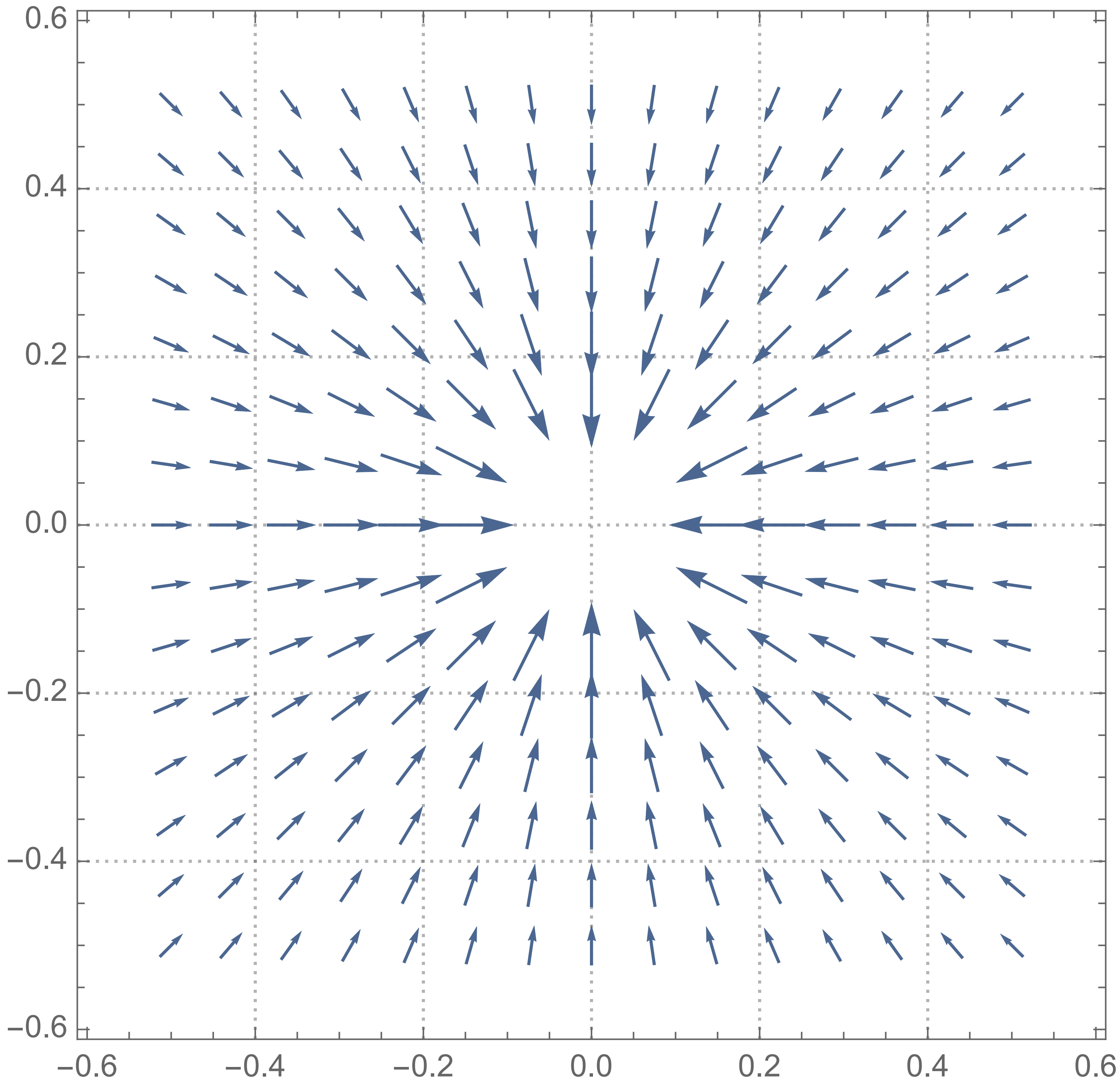
$$\tau_x \otimes s_x + \tau_y \otimes s_y$$

$$\tau_x \otimes s_y - \tau_y \otimes s_x$$

Two gapped bands and two gapless

$$\epsilon = \pm\sqrt{2\Delta^2 - 2\Delta\sqrt{\Delta^2 + k_\perp^2} + k_\perp^2 + k_\parallel^2} \longrightarrow \epsilon_{\mathbf{p}} = \sqrt{\alpha_n^2 p_\perp^{2n} + v^2 p_z^2}$$

Berry Curvature on the kx-ky plane



Previous Hamiltonian can be written in terms of a covariant Lagrangian

$$\mathcal{L}_L = i\psi_L^\dagger \tau^\mu \left[\partial_\mu - i\Delta \left(\delta_\mu^1 s_1 + \delta_\mu^2 s_2\right)\right] \psi_L \longrightarrow \mathcal{L}_L = i\psi_L^\dagger \tau^\mu \left[\partial_\mu - iA_\mu^a s_a\right] \psi_L$$

Allow me for a while simplify the picture removing the gauge field and remain classical

$$\mathcal{L} = i\psi^\dagger \tau^\mu \partial_\mu \psi$$

This system has the following symmetry $$\begin{array}{ccccccc} \mathbf{R}^{3,1} & \times & SO(3,1) & \times & U(1) & \times & SU(2) \\ \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\ T^{\mu\nu} & & L^{\mu\nu\rho} & & J^\mu & & J^\mu_a \end{array}$$

Noether theorem implies

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu = 0$$

$$\partial_\mu L^{\mu\nu\rho} = 0 \qquad \partial_\mu J^\mu_a = 0$$

But we know that this theory has the following anomaly

$$(\mathcal{D}_\mu \tilde{J}^\mu)_a \;=\; \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma} tr\left[s_a \partial_\mu \left(A_\nu \partial_\rho A_\sigma + \frac{1}{2} A_\nu A_\rho A_\sigma\right)\right] \;+ \frac{b_a}{768\pi^2}\epsilon^{\mu\nu\rho\lambda} R^{\alpha}{}_{\beta\mu\nu} R^{\beta}{}_{\alpha\rho\lambda}$$

And we also know that will have some anomaly induced transport, within linear response the conductivities read

$$J^\mu_a \;=\; \sigma^B_{ab} B^\mu_b + \sigma^V_a \omega^\mu$$

$$\sigma^B_{ab} \;=\; \frac{1}{4\pi^2} d_{abc}\mu^c$$

$$\sigma^V_a \;=\; \sigma^{\epsilon,B}_a = \frac{1}{8\pi^2} d_{abc}\mu_b\mu_c + \frac{T^2}{24} b_a$$

$$d_{abc} \;=\; \tfrac{1}{2}\mathrm{Tr}\left[\{s_a, s_b\}\, s_c\right]$$

$$b_a \;=\; \mathrm{Tr}\left[s_a\right]$$

In the definition of magnetic field the commutator is not included

For example we can apply previous formulas to the well known case

$$U(1)_L \times U(1)_R$$

$$\begin{aligned}
\boxed{\rho_e \;=\; \frac{1}{2\pi^2}\vec{b}\cdot\vec{B}} \\
\vec{j}_e \;&=\; \frac{1}{2\pi^2}\mu\vec{B}_5 + \frac{1}{2\pi^2}\mu\mu_5\vec{\omega} \boxed{+\frac{1}{2\pi^2}\vec{E}\times\vec{b}} \\
\boxed{\rho_5 \;=\; \frac{1}{6\pi^2}\vec{b}\cdot\vec{B}_5} \\
\vec{j}_5 \;&=\; \frac{1}{2\pi^2}\mu\vec{B} + \frac{1}{3\pi^2}\mu_5\vec{B}_5 + \left(\frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\omega} \boxed{+\frac{1}{6\pi^2}\vec{E}_5\times\vec{b}}
\end{aligned}$$

And the chiral magnetic effect in the electric current cancel!

Now we play the same game for the theory

$$U(1)_L \times U(1)_R \times SU(2)_L \times SU(2)_R$$

$$\mathcal{L} = i\psi_L^\dagger \tau^\mu \partial_\mu \psi_L + i\psi_R^\dagger \bar{\tau}^\mu \partial_\mu \psi_R$$

The abelian currents are

$$\begin{aligned}
\rho_e &= \frac{n}{2\pi^2}\vec{b}\cdot\vec{B} && (4 \\
\vec{j}_e &= \frac{n}{2\pi^2}\mu\vec{B}_5 + \frac{c(n)}{2\pi^2}\mu_3\vec{B}_{3_5} + \frac{n}{2\pi^2}\vec{E}\times\vec{b} \\
\rho_5 &= \frac{n}{6\pi^2}\vec{b}\cdot\vec{B}_5 && (4 \\
\vec{j}_5 &= \frac{n}{2\pi^2}\mu\vec{B} + \frac{n}{3\pi^2}\mu_5\vec{B}_5 + \frac{c(n)}{2\pi^2}\mu_3\vec{B}_3 + \frac{c(n)}{3\pi^2}\mu_{3_5}\vec{B}_{3_5} + \frac{n}{6\pi^2}\vec{E}_5\times\vec{b}
\end{aligned}$$

If we switch on again the gauge field

$$A^a_\mu = \Delta \left(\delta^x_\mu \delta^1_a + \delta^y_\mu \delta^2_a \right)$$

We explicitly break the symmetry

$$\mathbf{R}^{3,1} \times SO(3,1) \times U(1) \times SU(2)$$

$$\mathbf{R}^{3,1} \times SO(1,1) \times U(1) \times U(1)_3$$

The lattice $C_{4,6}$ is included here

For the holographers

$$S \;=\; -\int \mathrm{Tr}\left[\frac{1}{2n}F\wedge{}^{\star}F+\frac{1}{2c(n)}G\wedge{}^{\star}G+\;\lambda\left(\mathcal{A}\wedge(d\mathcal{A})^2+\frac{3}{2}\mathcal{A}^3\wedge\mathrm{d}\mathcal{A}+\frac{3}{5}\mathcal{A}^5\right)\right]$$

$$A=A^{(0)}s_0 \quad , \quad \mathbb{A}=\mathbb{A}^{(a)}s_a \quad , \quad \mathcal{A}=A+\mathbb{A} \qquad\qquad F=dA \quad , \quad G=d\mathbb{A}-i\mathbb{A}^2$$

$$\mathcal{A} \;=\; \left(\mathcal{A}_t(r)s_0+\mathcal{A}_t^3(r)s_z\right)\mathrm{d}t+\mathcal{Q}(r)\left(s_x\mathrm{d}x+s_y\mathrm{d}y\right)+\left(\mathcal{A}_z(r)s_0+\mathcal{A}_z^3(r)s_z\right)\mathrm{d}z+xBs_0\mathrm{d}y$$

For the non holographers

$$\mathcal{L}=\mathcal{L}_{CFT}+\mu Q+\mu_3Q^3+\Delta\left(\delta_x^\mu J_\mu^1+\delta_y^\mu J_\mu^2\right)$$

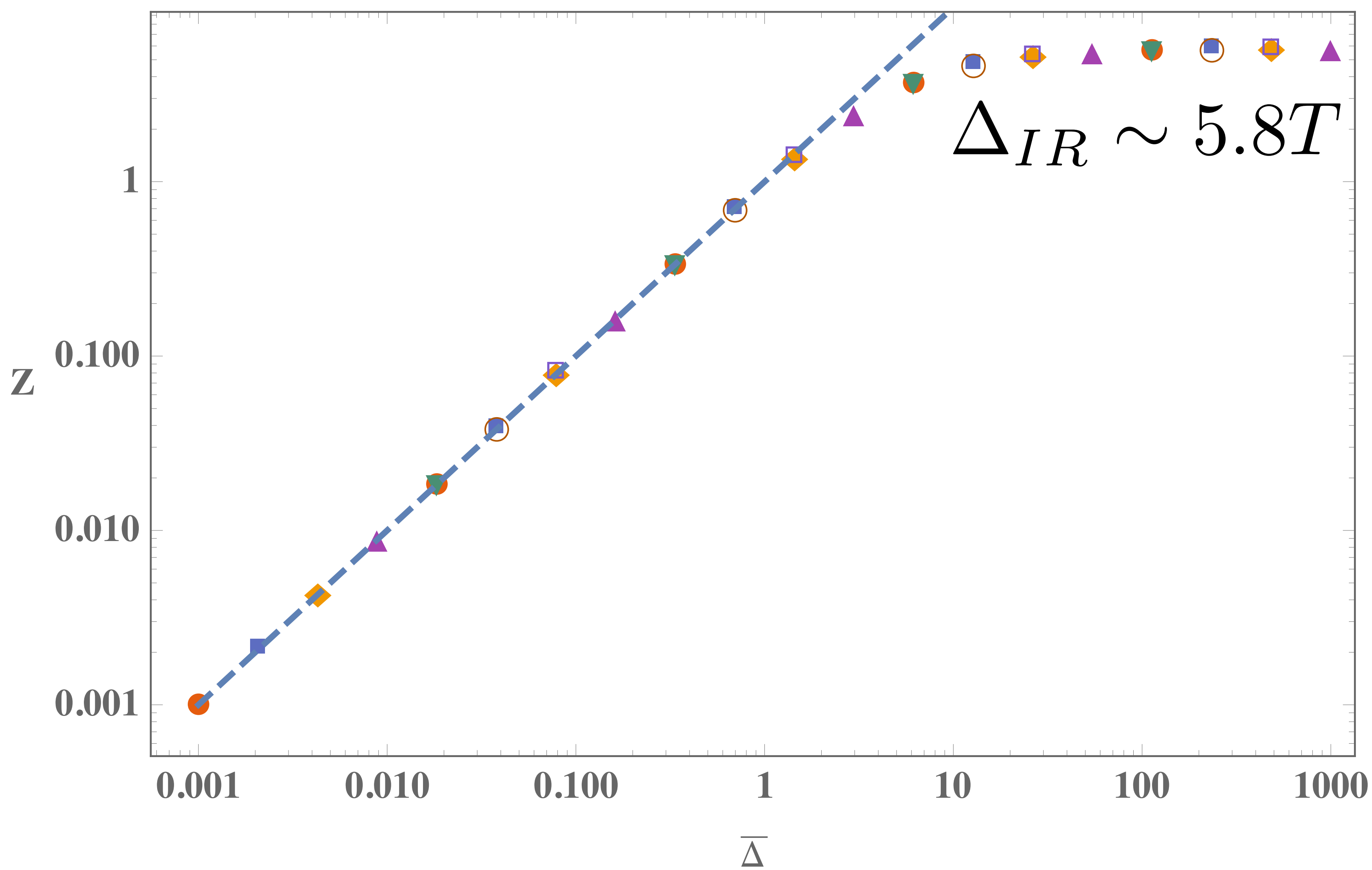
The equations of motion automatically imply

$$\boxed{J^z=\frac{n}{4\pi^2}\mu B+\frac{c(n)}{4\pi^2}\mu_3\Delta^2}$$

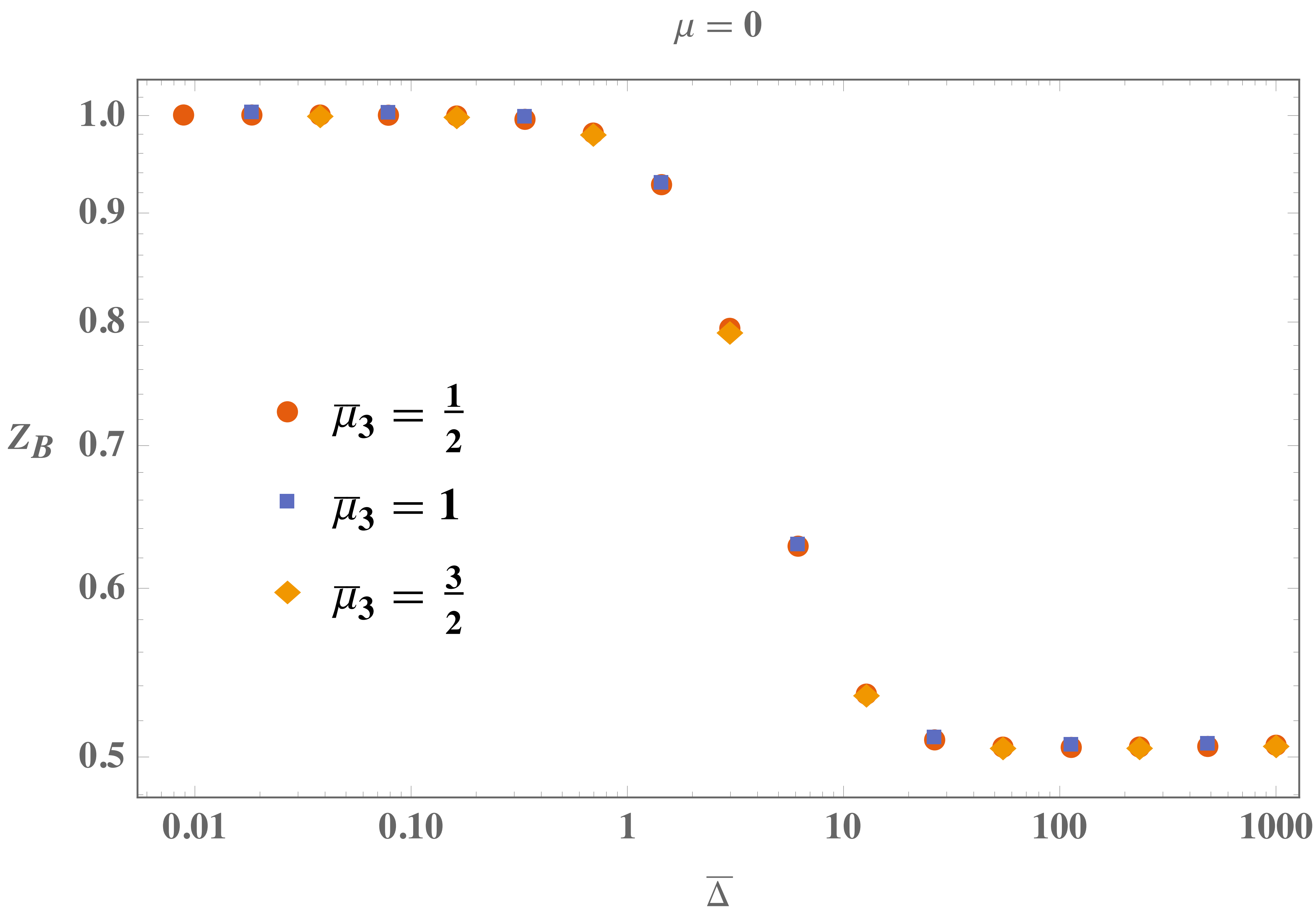
The non abelian sector has to be solved numerically

The non-abelian symmetry is explicitly broken, therefore Δ will renormalise

$$\Delta_{IR} = Z(\Delta/T)T$$

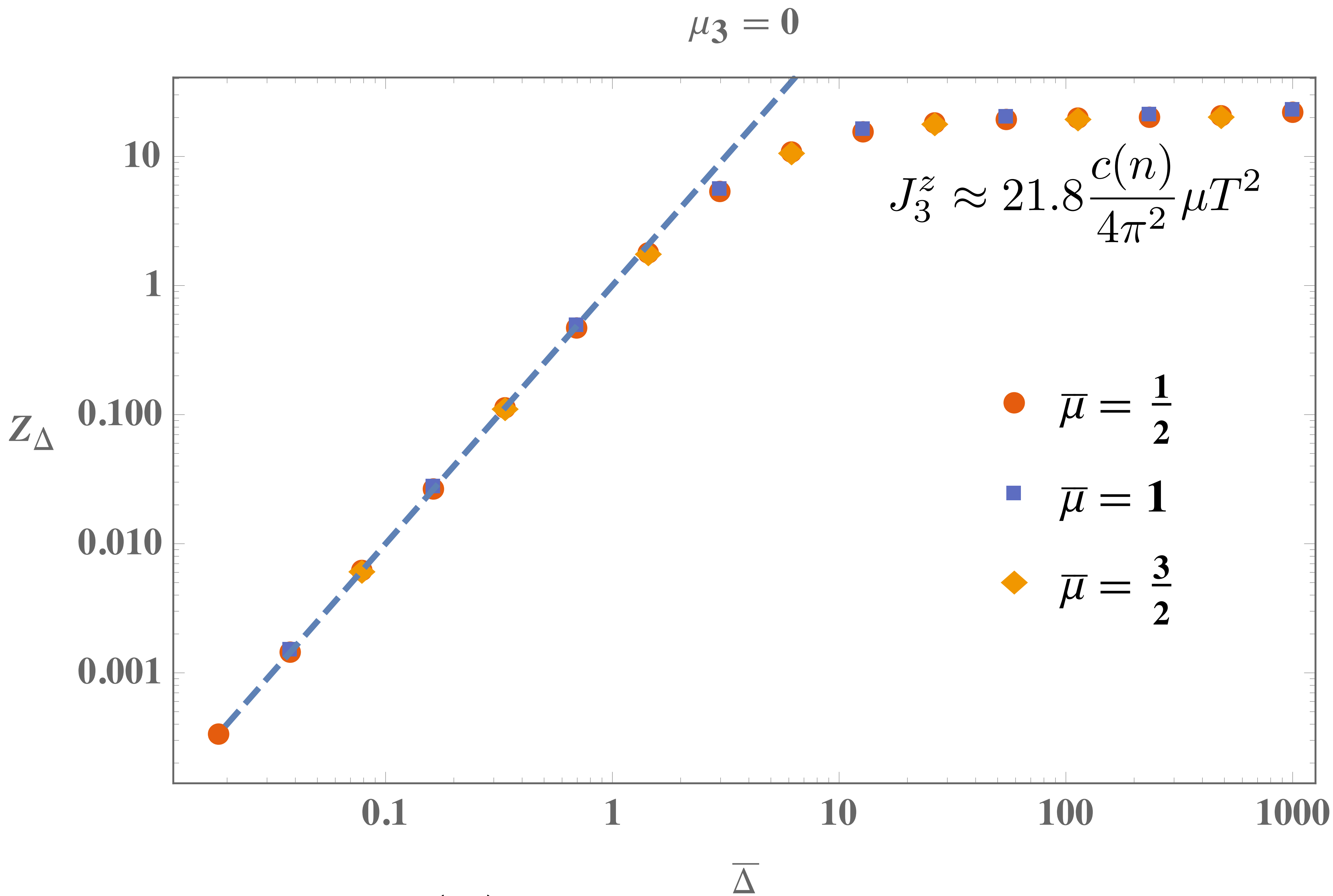


Same thing happens with the current



$$\vec{J}_3 = Z_B(\Delta/T)\frac{n\mu_3}{4\pi^2}\vec{B}$$

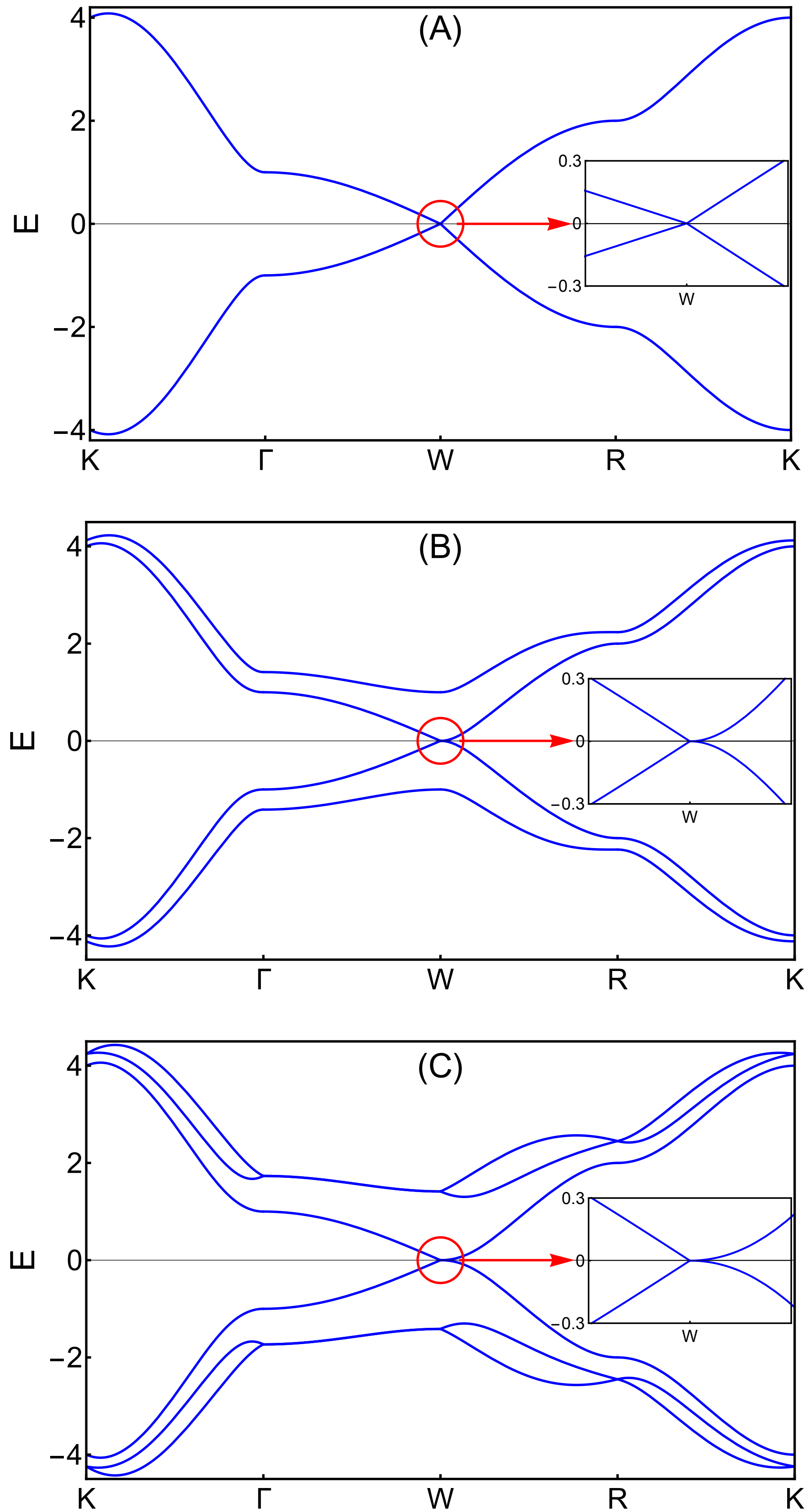
Same thing happens with the current



$$J_3^z = Z_\Delta(\Delta/T) \frac{c(n)\mu}{4\pi^2} T^2$$

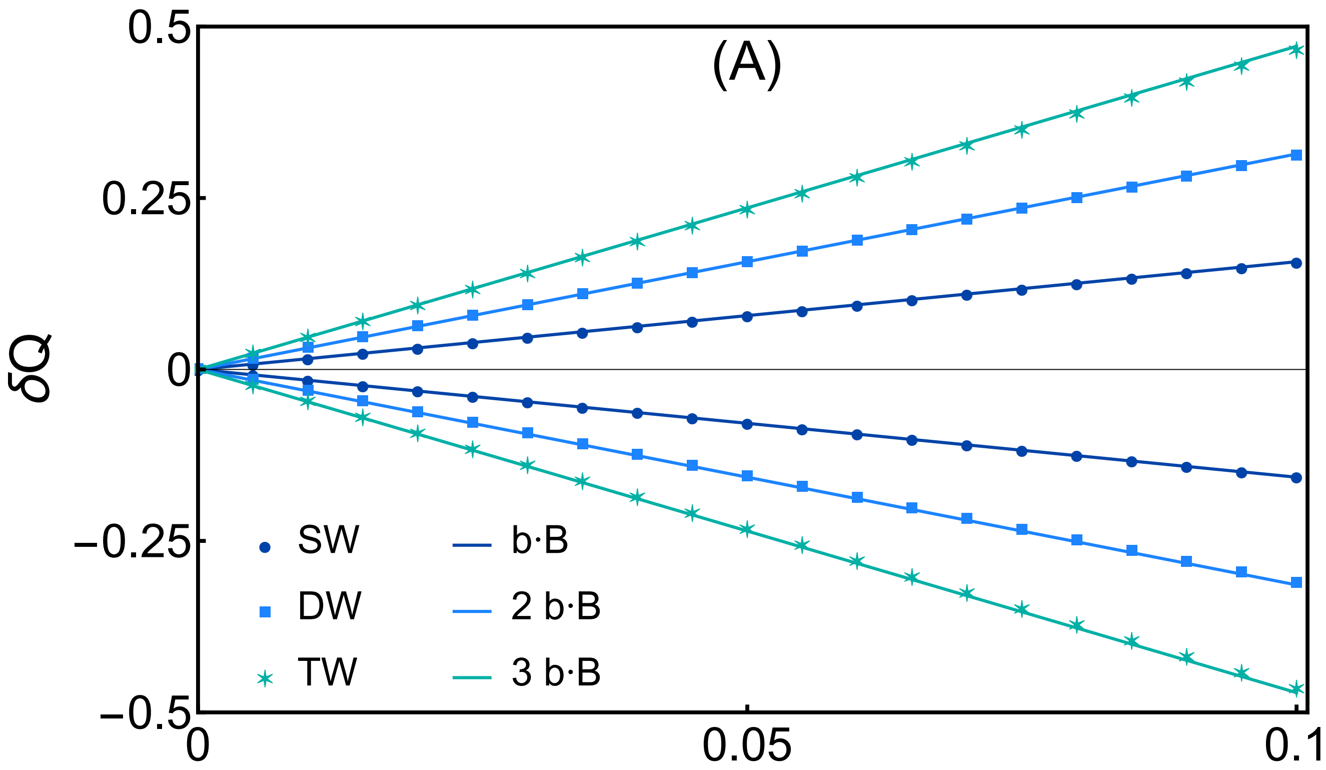
Let’s be “realistic” again!

To do so we computed anomalous Hall effect on a tight binding model for simple and multi-Weyl semimetals

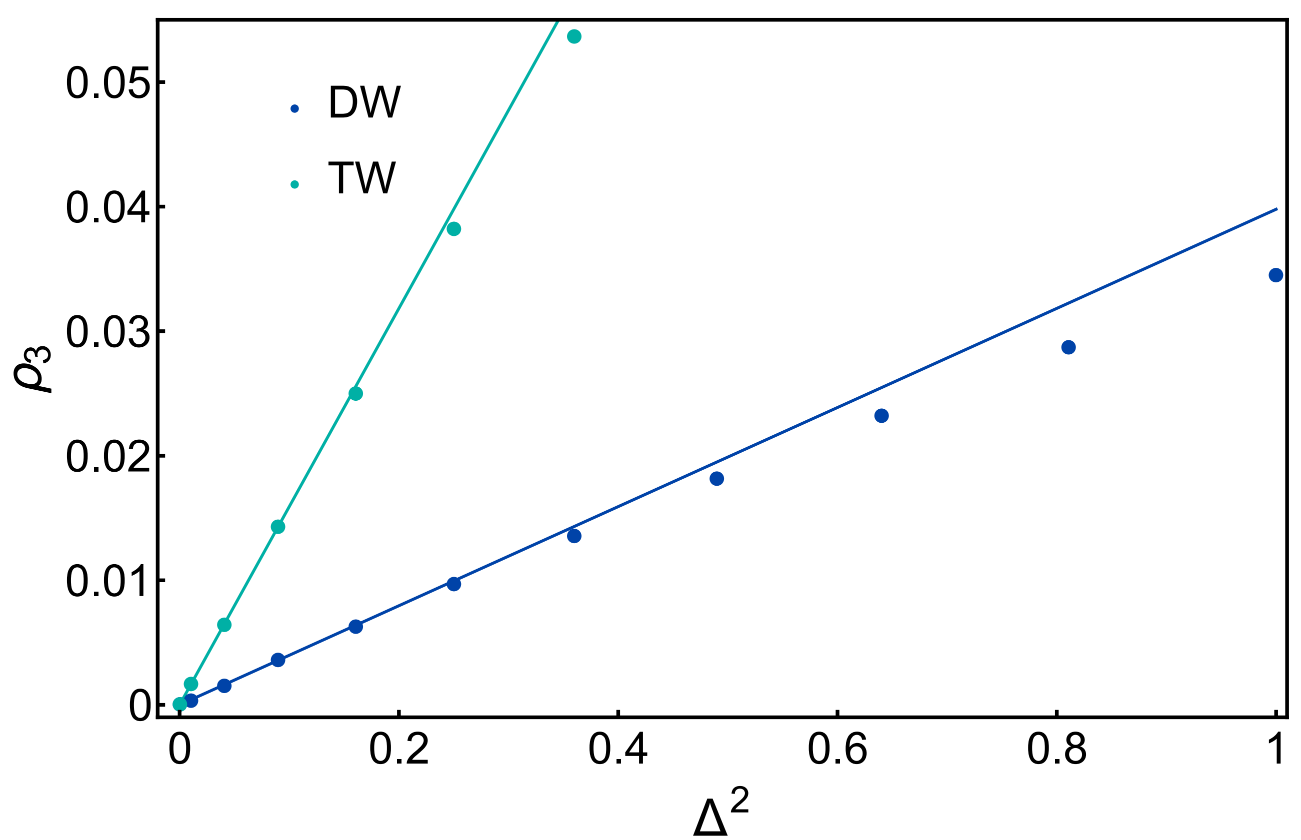


$$H_{SW} = t\left[\sin k_x\, \sigma_1 + \sin k_y\, \sigma_2 + (\cos k_z - m_z)\, \sigma_3\right] \\ + t_0\left[2 - \cos k_x - \cos k_y\right],$$

$$H_{MW} = H_{SW} \otimes \mathbb{I}_n + \Delta\left(\sigma_1 \otimes s_1 + \sigma \otimes s_2\right)$$



$$\rho_e \;=\; \frac{n}{2\pi^2}\vec{b}\cdot\vec{B}$$



$$\rho_3 \;=\; \frac{c(n)}{2\pi^2}\vec{b}\cdot\vec{B}_3$$

Summary

- Anomalies are present in multi-Weyl systems
- Negative magnetoresistance is enhance with the monopole charge
- Hidden non-abelian anomaly in the system
- Some of our predictions tested with a tight-binding model