The meson spectrum in large N QCD

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Outline

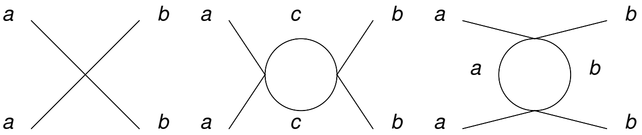
- Large N QCD: motivation
- Lattice simulation: details and techniques
- Quark and pion masses
- The meson spectrum
- Conclusions

Example

Consider a scalar field theory with N-component field ϕ^a, $a = 1, \ldots, N$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{2}\mu^2 \phi^a \phi^a - \frac{1}{8} g^2 (\phi^a \phi^a)^2$$

We define the 't Hooft coupling $\lambda = g^2 N$:



$O(g^2 = \frac{\lambda}{N})$ $\qquad$ $O(g^4 N = \frac{\lambda^2}{N})$ $\qquad$ $O(g^4 = \frac{\lambda^2}{N^2})$

What happens if $g^2 \to 0$ and $N \to \infty$ with fixed λ ('t Hooft limit)?

Large N QCD

Replacing,

$$A_\mu \to \sqrt{N} A_\mu, \quad \psi \to \sqrt{N} \psi$$

SU(N) Lagrangian:

$$\mathcal{L} = N \left[\frac{1}{4\lambda} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\not{D} + m)\psi \right]$$

Counting rules:

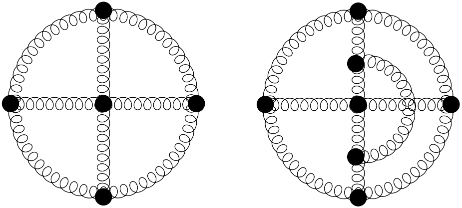
$$\left.\begin{array}{ccc} \text{corner} & \text{each vertices} & \propto N \\ \text{edge} & \text{each propagators} & \propto 1/N \\ \text{face} & \text{closed color loop} & \propto N \end{array}\right\} \quad \Rightarrow \quad \langle \quad \rangle \propto N^{V-E+F=\chi}$$

$\chi = V - E + F = 2 - 2H$(handles) $- B$(boundaries, holes) is the Euler characteristics.

sphere: $h = b = 0 \Rightarrow \chi = 2$, torus: $h = 1,\ b = 0 \Rightarrow \chi = 0$

Consequences of the counting rules

- Only “planar” diagrams survive:



planar non-planar

- The leading connected vacuum diagrams are of order N^2 (planar graphs made of gluons only)
- The leading connected vacuum diagrams with quark lines are of order N (planar graphs with just one quark loop at the boundary)
- Corrections down by factors of $1/N^2$ in the gauge theory and by $1/N$ in the theory with fermions

Properties of large N QCD

- Quark loop effects $\propto 1/N \Rightarrow$ The $N = \infty$ limit is quenched
- Mixing glueballs-mesons $\propto 1/\sqrt{N} \Rightarrow$ No mixing between glueballs and mesons at $N = \infty$
- Meson decay widths $\propto 1/N \Rightarrow$ mesons do not decay at $N = \infty$
- OZI rule exact at $N = \infty$

$N = \infty$ QCD is far from being solved!

Why lattice?

- Is $N = \infty$ close to $N = 3$ QCD? $\Rightarrow$ quantify $1/N$ effects
- AdS/QFT starts at $N = \infty$. $\Rightarrow$ comparison with lattice results.

Lattice parameters

- Number of colours $N = 2, 3, 4, 5, 6, 7, 17$
- Volumes:

N	vol
2,3	$16^3 \times 32$, $24^3 \times 48$, $32^3 \times 64$
4,5,6,7	$24^3 \times 48$
17	$12^3 \times 24$

- 200 configs for each N and volume (80 configs for $N = 17$)
- lattice spacing of $a \approx 0.093\,\text{fm}$
- pion mass as low as $m_\pi \approx 230\,\text{MeV}$
- Wilson gluon and quark action

Setting the scale

- Inverse coupling $\beta = 2N/g^2 = 2N^2/\lambda$ is fixed by imposing $a\sqrt{\sigma} = 0.2093$ for all $\mathrm{SU}(N)$. (Lattice spacing is kept constant in units of the string tension $\sigma \approx 1$ GeV/fm). Lattice spacing is the same for all groups
- Other possible choices can be made (e.g. $T_c = \text{const}$)
- The κ-parameter ($2am_q = \kappa^{-1} - \kappa_c^{-1}$) is adjusted such that a set of pseudoscalar masses matches between different N (achieved by exploratory simulations).

Plan:

- Vary κ to study $m_A(m_q, N)$ for each particle A.
- Extrapolate to $N = \infty$ and study $1/N^2$ corrections.

Setting the scale for SU(17)

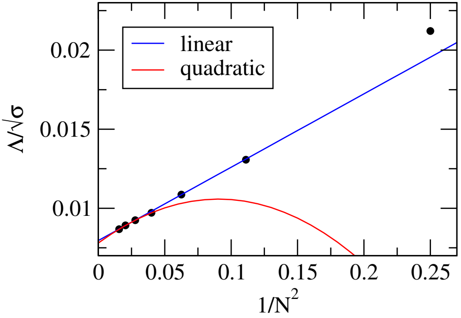
Fit estimated Λ_L parameter in $1/N^2$ to set the scale $\Rightarrow \beta_{17} = 208.45$

$$a\Lambda_L = \exp\left(-\frac{1}{2b_0\alpha}\right)(b_0\alpha)^{-\frac{b_1}{2b_0^2}}\left(1+\frac{1}{2b_0^3}\left(b_1^2 - b_2^L b_0\right)\alpha + \dots\right)$$

with $b_{0,1,2} = f(N)$

Other approaches are compatible within 0.3%:

- linear fit vs. quadratic fit
- fit of $\Lambda_{\overline{\mathrm{MS}}}$
- fit of 't Hooft λ



Meson interpolators

- A generic meson interpolator is a bilinear:

$$O(x,t) = \bar{\psi}(x,t)\Gamma\psi(x,t)$$

  with Γ product of Dirac matrices

- Zero momentum projected two point function:

$$C_{\Gamma,\Gamma}(t) = \sum_x \left\langle O(x,t)\overline{O}(0) \right\rangle = -\sum_x \mathrm{Tr}\,\langle \Gamma G(x,t)\Gamma G(x,0)\rangle$$

$$C(t) \propto e^{-mt} \text{ for } t \to \infty$$

- We study mesons with $m_u = m_d$:

Particle	π	ρ	a_0	a_1	b_1
Bilinear	$\bar{u}\gamma_5 d$	$\bar{u}\gamma_i d$	$\bar{u}d$	$\bar{u}\gamma_5\gamma_i d$	$\frac{1}{2}\epsilon_{ijk}\bar{u}\gamma_i\gamma_j d$
J^{PC}	0^{-+}	1^{--}	0^{++}	1^{++}	1^{+-}

Variational method

- APE smearing on the links and Wuppertal smearing on sources/sinks.
- Study of the cross-correlation matrix

$$C_{i,j} = \langle O_i(t)\overline{O}_j(0)\rangle$$

  where i, j correspond to a different number of smearing iterations $(0, 20, 80$ and 180 steps).
- Solve the generalized eigenvalue problem:

$$C(t)\mathbf{v}^{\alpha} = \lambda^{\alpha}(t)C(t_0)\mathbf{v}^{\alpha}$$

  and extract the masses from the large t behaviour of the eigenvalues:

$$\lambda(t) = A\left(e^{-mt} + e^{-m(N_t a - t)}\right)$$

- $\Rightarrow$ Ground and first excited states.

PCAC relation

Partially conserved axial current (PCAC):

$$\sum_x \partial_4 \langle 0|A_4(x,t)|\pi\rangle = 2m_{\mathrm{PCAC}} \sum_x \langle 0|j_5(x,t)|\pi\rangle \quad \text{where} \quad \begin{cases} A_\mu(x) & = & \bar{u}(x)\gamma_\mu\gamma_5 d(x) \\ j_5(x) & = & \bar{u}(x)\gamma_5 d(x) \\ m_{\mathrm{PCAC}} & = & Z\, m_q \end{cases}$$

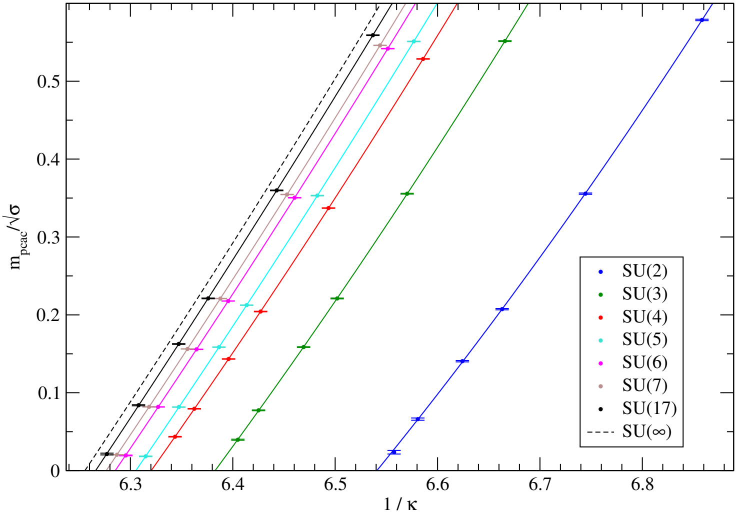
$m_q = (\kappa^{-1} - \kappa_c^{-1})/(2a)$ is the quark mass and $Z(\lambda)$ is a renormalization constants.

We fit

$$\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}} = A\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right) + B\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^2$$

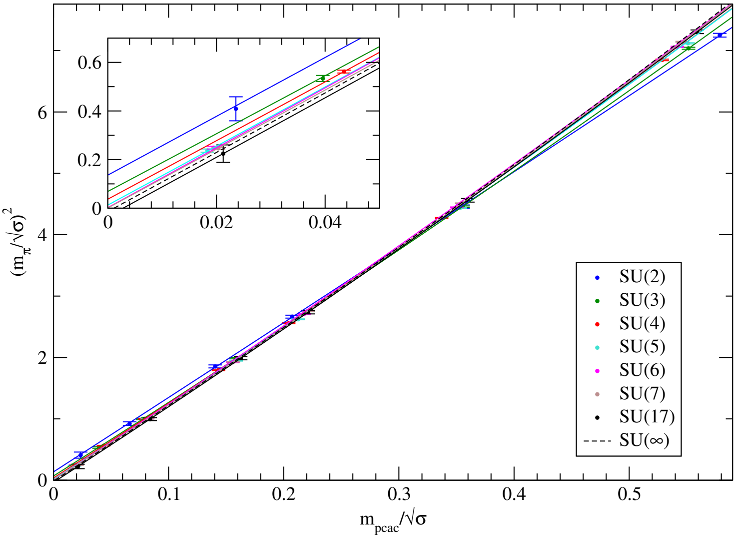
with dimensionless fit parameters A, B, κ_c that are expected to have $\mathcal{O}(1/N^2)$ corrections.

Determination of $\kappa_c(N)$



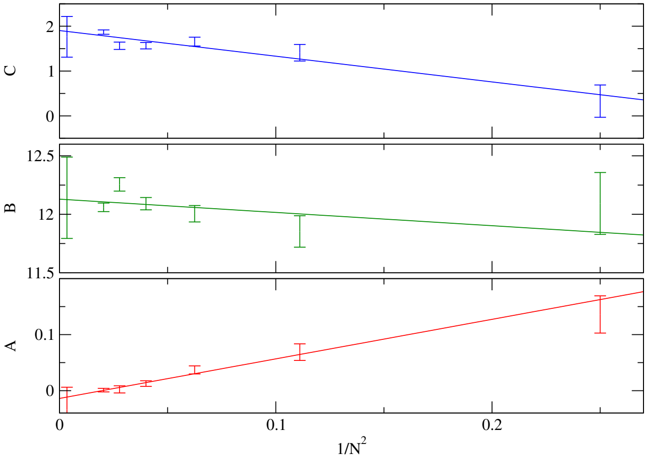
$$\frac{m_{\text{PCAC}}}{\sqrt{\sigma}} = A\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right) + B\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^2, \quad \kappa_c = 0.15980(4) - 0.0275(4)\frac{1}{N^2}$$

Pion mass vs. PCAC mass



$$\left(\frac{m_\pi}{\sqrt{\sigma}}\right)^2 = A + B\,\frac{m_{\rm PCAC}}{\sqrt{\sigma}} + C\left(\frac{m_{\rm PCAC}}{\sqrt{\sigma}}\right)^2$$

Pion mass: $1/N^2$ fit of the parameters



$$\left(\frac{m_\pi}{\sqrt{\sigma}}\right)^2 = \left(-0.0139(40) + 0.706(97)\frac{1}{N^2}\right) + \left(12.129(39) - 1.13(87)\frac{1}{N^2}\right)\frac{m_{\text{PCAC}}}{\sqrt{\sigma}}$$
$$+ \left(1.904(54) - 5.7(12)\frac{1}{N^2}\right)\left(\frac{m_{\text{PCAC}}}{\sqrt{\sigma}}\right)^2$$

Pion mass - chiral logs

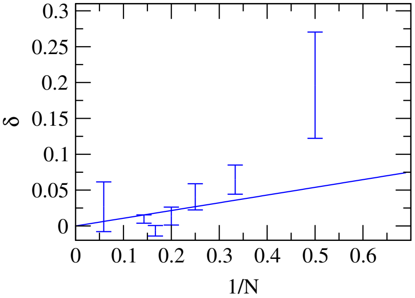
In the **quenched** theory the pion behaviour should be modified as,

$$\left(\frac{m_\pi}{\sqrt{\sigma}}\right)^2 = A\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^{\frac{1}{1+\delta}} + B\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^2$$

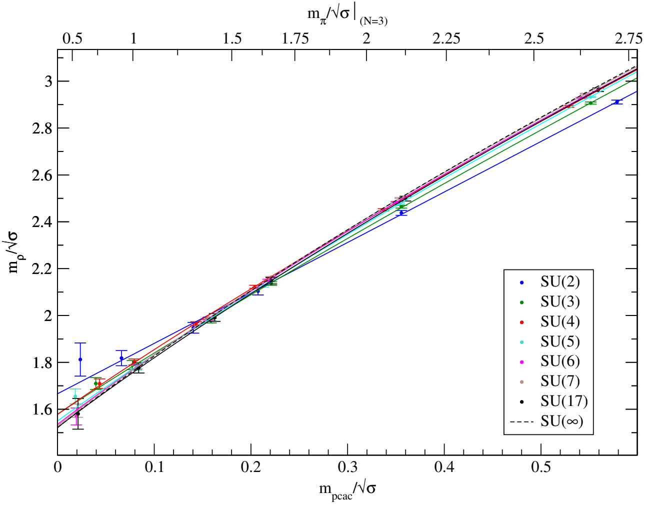
where $\delta \propto 1/N$ goes to zero at large N (Sharpe - hep-lat/9205020)

- δ compatible with zero for $N \geq 5$
- not sensible to chiral logs at $m_\pi > \sqrt{\sigma}$
- linear fit with to $3 \leq N \leq 5$:

$$\delta = 0.108(27)\frac{1}{N}$$

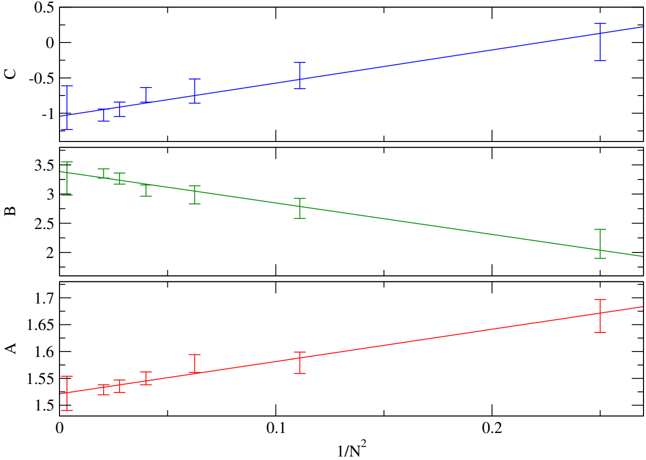


ρ: fit vs. PCAC mass



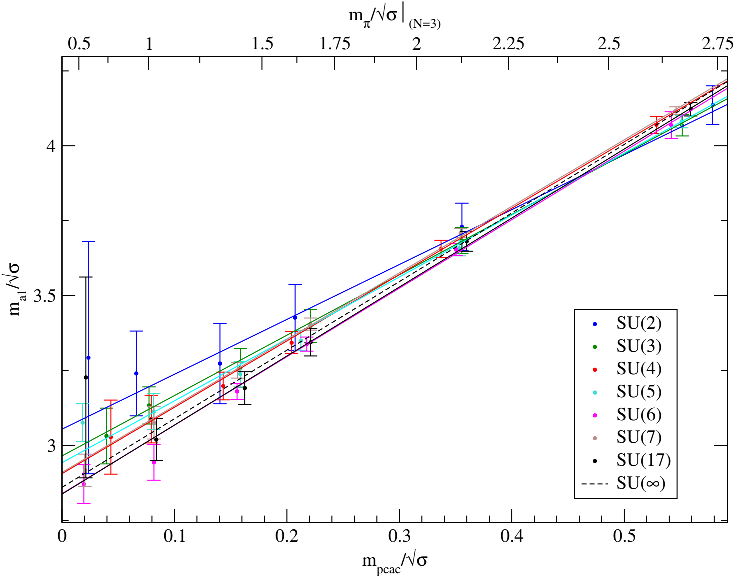
$$\frac{m_\rho}{\sqrt{\sigma}} = A + B\,\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}} + C\left(\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}}\right)^{\frac{3}{2}}$$

ρ: $1/N^2$ fit of the parameters



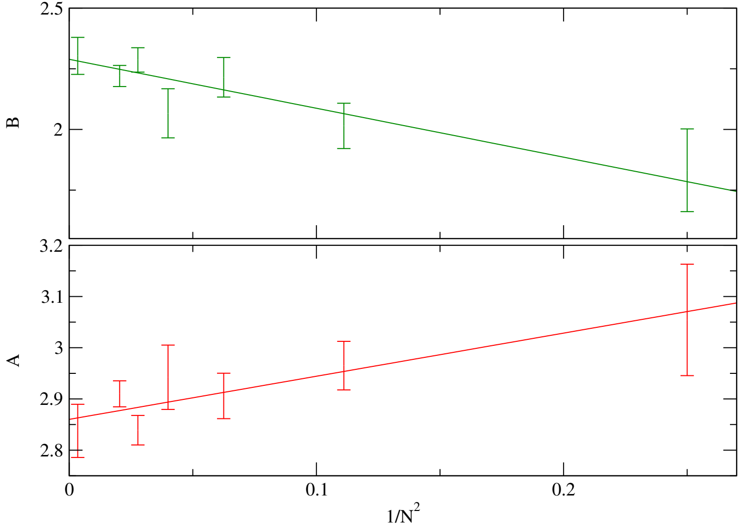
$$\frac{m_\rho}{\sqrt{\sigma}} = \left(1.5212(76) + 0.60(12)\frac{1}{N^2}\right) + \left(3.387(63) + -5.4(10)\frac{1}{N^2}\right)\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}}$$
$$+ \left(-1.043(68) + 4.7(11)\frac{1}{N^2}\right)\left(\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}}\right)^{\frac{3}{2}}$$

a_1: fit vs. PCAC mass



$$\frac{m_{a1}}{\sqrt{\sigma}} = A + B\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}}$$

a_1: $1/N^2$ fit of the parameters



$$\frac{m_{a1}}{\sqrt{\sigma}} = \left(2.860(21) + 0.84(36)\frac{1}{N^2}\right) + \left(2.289(35) - 2.02(61)\frac{1}{N^2}\right)\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}}$$

The ρ mass issue

Discrepancies between L Del Debbio et al JHEP 03 (08) 062, G Bali, F Bursa JHEP 09 (08) 110, T DeGrand PRD 86 (12) 034508 vs. A Hietanen et al PLB 674 (09) 80

- $N \to \infty$ from bigger lattices and smaller N
- $\mathrm{SU}(17)$ volume: $12^3 \times 24$
- $\beta = 208.45$ ($b = \frac{1}{g^2 N} = 0.3606$)
- Wilson fermions
- Zero momentum correlators:

$$C(t) = e^{-m_\rho t}$$

- ρ mass for chiral limit

$$\frac{m_\rho(N=17)}{\sqrt{\sigma}} = 1.54(3)$$

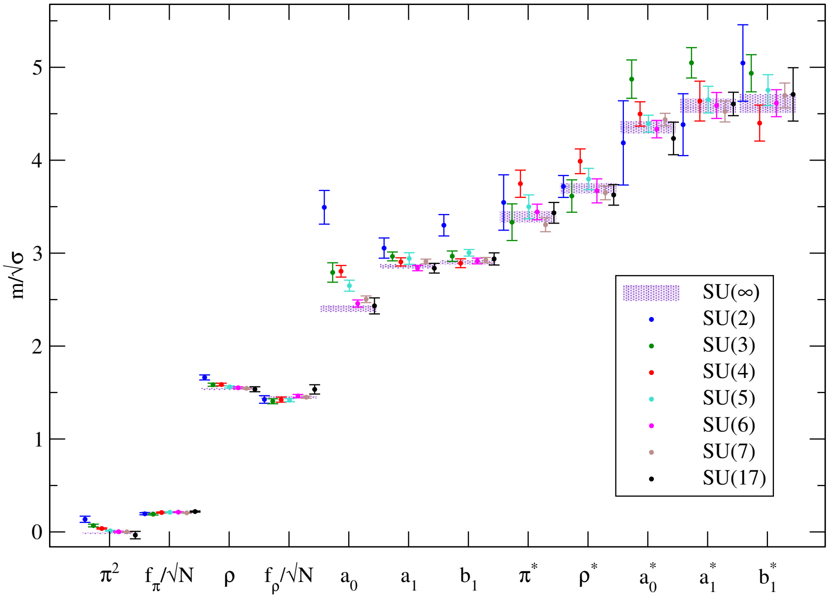
- $N \to \infty$ from smaller lattices and bigger N
- $\mathrm{SU}(17)$ volume: 11^4
- $\beta = 208.08$ ($b = \frac{1}{g^2 N} = 0.360$)
- Overlap fermions
- Propagator in momentum space:

$$M_{\mu\nu} = \frac{A(p_\mu p_\nu - p^2 \delta_{\mu\nu})}{p^2 + m_\rho^2}$$

- ρ mass for chiral limit

$$\frac{m_\rho(N=17)}{\sqrt{\sigma}} = 3.50(22)$$

The meson spectrum in the chiral limit



Comparison with AdS/QFT

AdS/CFT

AdS/CFT calculation by Babington et al. (hep-th/0306018)

$$\frac{M_\rho(M_\pi)}{M_\rho(0)} \simeq 1 + 0.307 \left[\frac{M_\pi}{M_\rho(0)}\right]^2$$

Holographic model by Sakai - Sugimoto (hep-th/0412141)

$$\frac{M^2_{a_1(1260)}}{M^2_\rho} \simeq 2.4$$
$$\frac{M^2_{\rho(1450)}}{M^2_\rho} \simeq 4.3$$
$$\frac{M^2_{a_0(1450)}}{M^2_\rho} \simeq 4.9$$

Results from the lattice

$$\frac{M_\rho(M_\pi)}{M_\rho(0)} \simeq 1 + 0.369(4) \left[\frac{M_\pi}{M_\rho(0)}\right]^2$$

$$\frac{M^2_{a_1(1260)}}{M^2_\rho} \simeq 3.3$$
$$\frac{M^2_{\rho(1450)}}{M^2_\rho} \simeq 5.4$$
$$\frac{M^2_{a_0(1450)}}{M^2_\rho} \simeq 7.5$$

Conclusion

- We computed the quenched meson spectrum of SU(N) for degenerate quark masses.
- The isovector SU(3) masses as well as decay constants are close to the $N = \infty$ limit. Even $N = 2$ is qualitatively well described.
- $1/N^2$ corrections are small.
- Differences between the $N_f = 2 + 1$ theory and the quenched approximation at $N = 3$ indicate that N_f/N corrections may be small too.
- Results may help improving AdS backgrounds or holographic models.
- Results may constrain large N based models and EFTs of low energy strong interactions.