Exponentially improving lattice computations of the glueball spectrum

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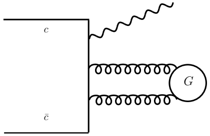
Outline

- Motivations for glueball studies
- Exponential growth of the noise to signal ratio in lattice QCD and YM theories
- Basic ideas for the Symmetry-Constrained Monte Carlo
  - The example of Parity (including results)
- Extension to other symmetries
- The strategy for the 0^{++} glueball (including results)
- Conclusions and outlook

- Self-interaction of gluon $\rightarrow$ Do bound states of gluon exist ?
- From the theoretical point of view the question is cleanly posen in the pure YM theory
- Experimentally: search of extra states (resonances) not in the quark model (could be glueballs, tetraquarks ... exotic)
  - no electric charge (no direct coupling to γ), no flavor (flavor blind decay mode)
- Most exploited channel, J/Ψ radiative decays

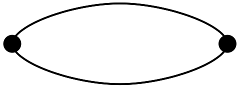
  BES, SLAC (MARK), FAIR ?



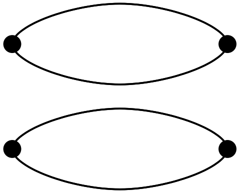
- Several resonances, f_0's for the 0^{++} and η's for the 0^{-+} between 1.5 and 2 GeV.
- Precise theoretical predictions are needed to clearly identify these states as glueballs.
- In the full theory glueballs mix with $q\bar{q}$ states !! [TχL, UKQCD]

Exponential growth of the signal to noise ratio (Parisi '84, Lepage '89)

Consider a point to point correlation function interpolating (eg) a meson. The signal is given by the expectation value of



while the a priori variance is given by the expectation value of

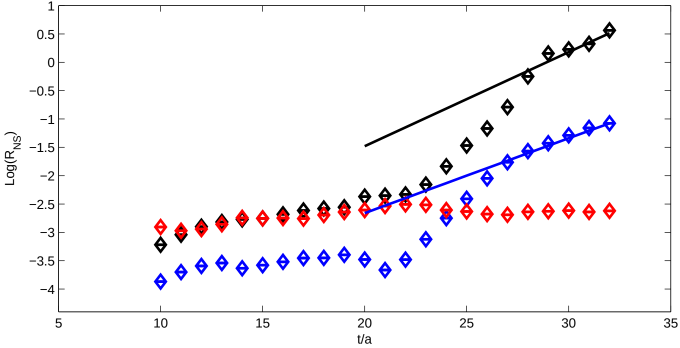


Luckily Wick-contractions are done *before squaring*, for the variance. Then a multi-pion state dominates, otherwise it would be the vacuum (as for YM).

pion $R_{NS} \propto const$

ρ $R_{NS} \propto \exp((m_\rho - m_\pi)t)$

N $R_{NS} \propto \exp((m_N - \frac{3}{2}m_\pi)t)$



O(2000) quenched confs ($\beta = 6.2,\ \kappa = 0.1526$) in APE, hep-lat/9611021

- For an operator interpolating a parity odd glueball

$$C_{O_G}(t) = \langle O_G(t) O_G(0) \rangle \to |\langle 0|O_G(0)|G^-\rangle|^2 \, e^{-M_{G^-} t} + \ldots$$

the variance can be estimated as

$$\sigma^2 = \langle O_G^2(t) O_G^2(0) \rangle - \langle O_G(t) O_G(0) \rangle^2 \to \langle 0|O_G^2(0)|0\rangle^2 + \ldots$$

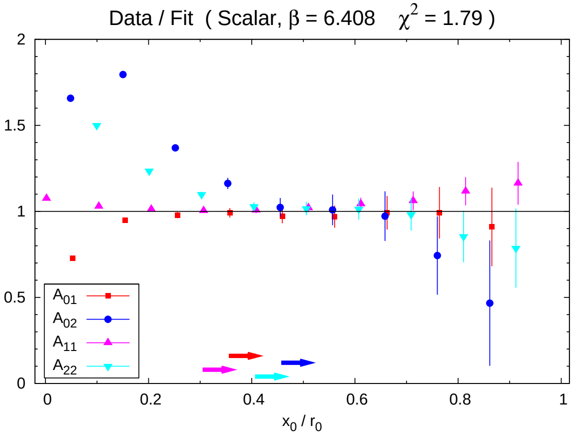
- The noise to signal ratio at large time separations is given by

$$R_{NS}(t) \to \frac{\langle 0|O_G^2(0)|0\rangle}{|\langle 0|O_G(0)|G^-\rangle|^2} \, e^{M_{G^-} t} + \ldots$$

$\Leftarrow$ On a given gauge configuration symmetries as parity are not preserved. All states are allowed to propagate despite the quantum numbers of the source.

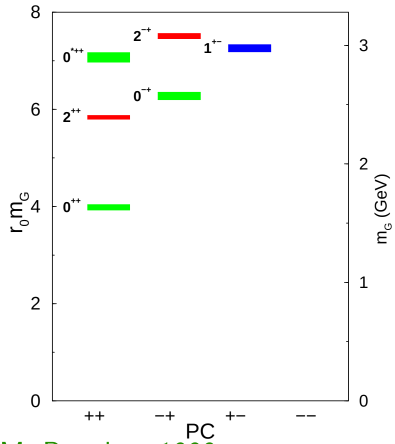
$\Rightarrow$ For every gauge-field configuration the vacuum dominates. The signal emerges due to large cancellations in the gauge average.

$\Rightarrow$ In the standard approach glueball masses are extracted at rather short separations.



H. Meyer, JHEP 0901:071,2009.

Signal up to 0.5 fm at most.



C. Morningstar and M. Peardon, 1999 ....

Nice results, which however we believe need to be checked concerning systematic effects. In particular a single state dominance in the correlation function for large x_0 (in fm.) is not always observed. Rather, compromises between excited states contributions at short time- and large errors at large time-separations.

with periodic boundary conditions $Z = \int D_3[V]\langle V|e^{-T\hat{H}}\hat{P}_G|V\rangle$

$$Z = Z^+ + Z^- \,, \quad Z^\pm = e^{-E_0\,T}\left[\frac{1\pm 1}{2} + \sum_{n=1}\omega_n^\pm\, e^{-E_n^\pm T}\right]$$

We introduce a parity transformation

$$\hat{\wp}\,|V\rangle = |V^\wp\rangle \,, \qquad V_k^\wp(\mathbf{x}) = V_k^\dagger(-\mathbf{x}-\hat{k}) \,,$$

with $\hat{\mathrm{V}}_k(\mathbf{x})|V\rangle = V_k(\mathbf{x})|V\rangle$ and

$$Z^{tw} = \int D_3[V]\langle V|e^{-T\hat{H}}\hat{P}_G|V^\wp\rangle =$$

$$\sum_G \int D_3[V]\langle V|G\rangle\langle G|e^{-T\hat{H}}\hat{\wp}|G\rangle\langle G|V\rangle = Z^+ - Z^-$$

- We want to compute $\frac{Z^-}{Z}(T) = \frac{1}{2}\left(1 - \frac{Z^{tw}}{Z}\right)(T)$ where, compared to Z, the boundary conditions in Z^{tw} are parity twisted. At large T we should be able to extract the lightest parity odd glueball.
- We aim at a hierarchical integration scheme [Lüscher and Weisz, '01] and divide the system in thick time-slices of size d with boundaries updated at different rates wrt the internal dof.
- We start from the factorized expression for $Z(T)$

$$Z(T) = \int \prod_{i=0}^{T/d-1} \mathbf{D}_3[V_{id}]\, T^d[V_{(i+1)d}, V_{id}]\ , \quad \text{with}$$

$$T^d[V_{x_0+d}, V_{x_0}] = \langle V_{x_0+d} | \hat{T}^d | V_{x_0} \rangle$$

and by introducing

$$(T^-)^d[V_{x_0+d}, V_{x_0}] = \frac{1}{2}\left\{ T^d[V_{x_0+d}, V_{x_0}] - T^d[V_{x_0+d}, V_{x_0}^{\wp}] \right\}$$

we generalize it to Z^-/Z.

Relevant property: orthogonality

Using the invariance of the Hamiltonian under parity it is easy to show

$$T^d[V_{x_0+d}, V_{x_0}^{\wp}] = T^d[V_{x_0+d}^{\wp}, V_{x_0}]$$

and therefore

$$\int \mathbf{D}_3[V_{id}]\,(T^+)^d[V_{(i+1)d}, V_{id}]\,(T^-)^d[V_{id}, V_{(i-1)d}] = 0$$

- The basic quantity to be computed for each *sub*-lattice of time extent d with Dirichlet boundary conditions is the ratio of partition functions

$$\frac{T^d\left[V^{\wp}_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]}$$

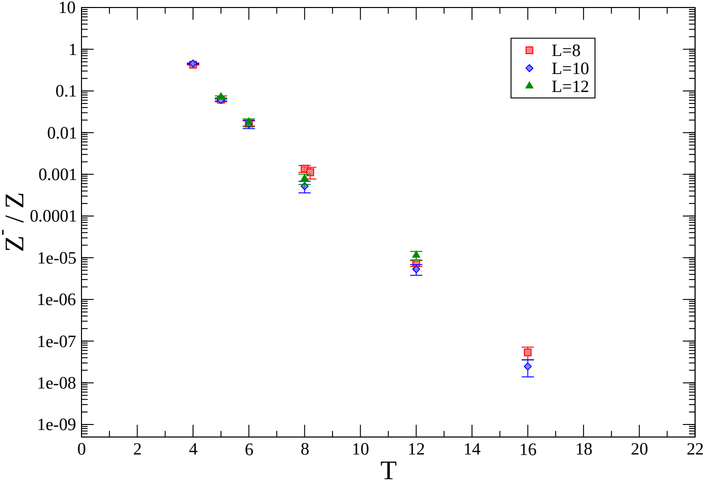
The product (over the thick-slices) of a simple linear function of that is then integrated numerically on the boundary configurations $V_{x_0=id}$.

- We need $\mathrm{O}((L/a)^3)$ MC simulations to estimate the ratio above becauase we interpolate among the two systems using a telescopic procedure:

$$\frac{Z_P}{Z} = \frac{Z_1}{Z}\frac{Z_2}{Z_1}\dots\frac{Z_P}{Z_{V-1}}$$

We have a $V^2 = (L/a)^6$ algorithm but we get rid of the exponential (in time) degradation of the signal, if we choose $d \geq 1/T_c$, such that the ratio above is of the right size $\mathrm{O}(e^{-M_G - d})$ and its fluctuations are reduced to the same level.

Wilson action $\beta = 5.7$ ($a \simeq 0.17$ fm) and O(50) meas at each T/a.



- The algorithm works as expected. We see a clear signal up to a separation of about 3 fm.
- There is no strong dependence of the results from L for $1.4\ \mathrm{fm} < L < 2\ \mathrm{fm}$ ($\Rightarrow$ negligible "torelon" contribution)
- However, by using parity only it is difficult to correctly identify the dominating state. For example, a rather light parity odd state (maybe lighter than the lightest 0^{+-} glueball) could be

$$\frac{1}{\sqrt{2}}\left(|0^{++},\vec{p}\rangle - |0^{++},-\vec{p}\rangle\right) \ , \quad |\vec{p}| = 2\pi/L$$

- We want to consider the lattice YM symmetry groups
    - C and P, $g = 2$
    - spatial translations, $g = L^3$
    - central charge conjugations, Z_3^3, $g = 27$
    - spatial rotations, octahedral group, $g = 24$

Fixing an irreducible representation (quantum numbers) [DM and Giusti, 2010]

- The phase space of the theory can be factorized into regular representations of the group. In the partition function

$$Z(T) = \mathrm{Tr}\left[\hat{T}^{T}\right]$$

one inserts the identity I written as

$$I = \frac{1}{g}\sum_{i=1}^{g}\int \mathbf{D}_3[V]|V^{\Gamma^i}\rangle\langle V^{\Gamma^i}|$$

eg on the boundaries of our thick-slices.

- Then group theory tells us how to project on an irreducible representation μ

$$\hat{P}^{(\mu)} = \frac{n_\mu}{g}\sum_{i=1}^{g}\chi_i^{(\mu)^*}\hat{\Gamma}^i$$

- So, one has to compute

$$\frac{T^d\left[V^{\Gamma^i}_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]}, \quad i = 1 \dots g$$

and then form linear combinations of them.

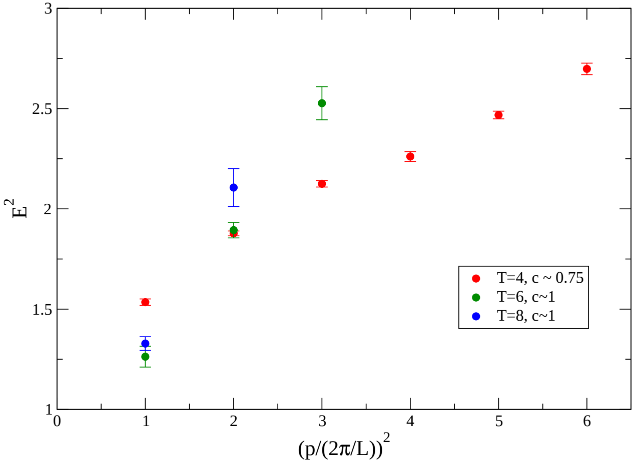
For example: The relative contribution of states with momentum $\vec{p}$ in the system with Dirichlet bc is ($\hat{P}(\vec{x})$ representing translations by $\vec{x}$)

$$\frac{(T^{\vec{p}})^d\left[V_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]} = \frac{1}{\sqrt{L^3}} \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \frac{T^d\left[V^{P(\vec{x})}_{x_0+d}, V_{x_0}\right]}{T^d\left[V_{x_0+d}, V_{x_0}\right]}$$

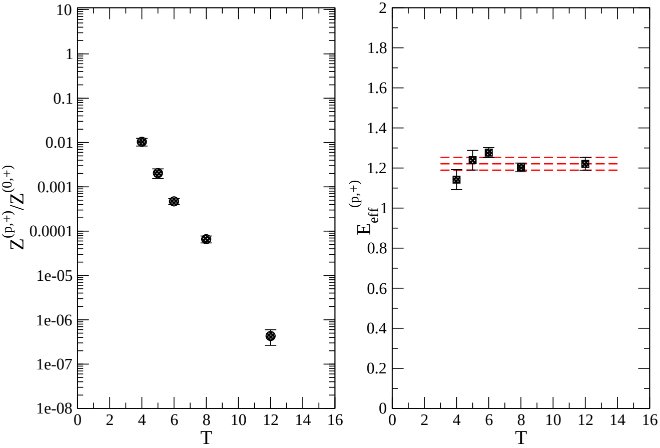
We will use this setup to extract the mass of the lightest 0^{++} glueball through the dispersive relation. By selecting non-zero momentum we get rid of the vacuum.

Results for the dispersion relation (fixing in addition C parity to be even)

$\beta = 5.7,\ L/a = 8$ $\quad E^2 = \left(\frac{\ln(Z^{C,\mathbf{p}}(T)/Z(T))}{T} \right)^2$



As T grows the signal at large $|\vec{p}|$ becomes smaller. We concentrate on $\vec{p} = 2\pi/L, 0, 0$ and C$= +$ and project only stochastically on $p_y = p_z = 0$ and central charge $\mathbf{e} = 0$



We [MDM and Giusti, 2010] fit the last 4 points with a function

$$\ln\left[\frac{Z^{(\mathbf{p},+)}}{Z^{(\mathbf{0},+)}}\right] = A - BT$$

and obtain $A = \ln(\omega) = -0.6(4)$ and $B = E_{\rm eff}^{(\mathbf{p},+)} = 1.15(6)$. The rightmost plot is produced by setting the multiplicity to 1 and defining

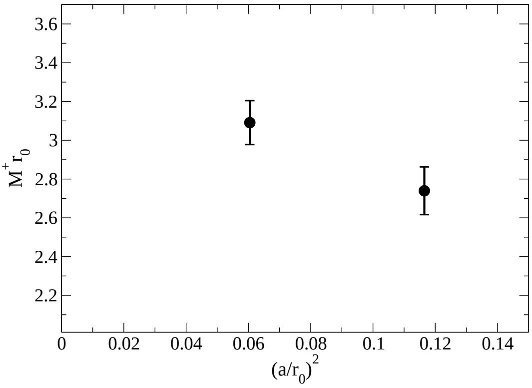
$$E_{\rm eff}^{(\mathbf{p},+)} = -\frac{1}{T}\left[\frac{Z^{(\mathbf{p},+)}}{Z^{(\mathbf{0},+)}}\right] \; .$$

We use the $T = 12$ result $E_{\rm eff}^{(\mathbf{p},+)} = 1.22(3)$ to finally get

$$M^{+} = 0.935(42)\;, \quad \rightarrow \quad M^{+} = 1.08(5)\;\mathrm{GeV}\;\; [r_0 = 0.5\;\mathrm{fm}]$$

through the continuum dispersion relation. The result agrees with the 0.955(15) computed in the standard way with the same a and action in [Vaccarino and Weingarten, 1999].

We have repeated the computation at $a = 0.012$ fm $[\beta = 5.85]$ on a $14^3 \times 10$ lattice (corresponding to $T = 7$ at $\beta = 5.7$)



The signal is still good.

Conclusions and outlook

- In the YM theory the noise to signal problem can be solved by enforcing the propagation in time of states with the desired quantum numbers only.
- We have shown that all quantum numbers can be fixed in this approach.
- We are now exploring the stochastic projection on the singlet component (eg zero momentum in order to avoid another L^3 factor in the scaling of the algorithm).
- In the near future we will concentrate on the 0^{++} in the continuum limit but also the 2^{++} glueball masses.
- Matter fields ?
- The inclusion of fermions appears difficult, due to the non-locality of the resulting gauge action after integrating out the quarks.
- On the other hand the method can be used for scalar gauge theories (Higgs), which suffer of the same exponential noise to signal problem as pure gauge theories or to two dimensional systems as CP(N).