Contribution of the charm quark to the $\Delta I = 1/2$ rule

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$\Delta I = 1/2$ rule (I)

$$K \longrightarrow \pi\pi$$

Particle	Isospin I
Kaon	1/2
Pion	1

$$K_{I=\frac{1}{2}} \nearrow (\pi\pi)_{I=2} \quad \Delta I = 3/2$$

$$K_{I=\frac{1}{2}} \rightarrow \cancel{(\pi\pi)_{I=1}}$$

$$K_{I=\frac{1}{2}} \searrow (\pi\pi)_{I=0} \quad \Delta I = 1/2$$

- T: Transition amplitude
- A: Amplitude
- δ: Scattering phase shift

$$T[K \to (\pi\pi)_I] \;=\; A_I e^{i\delta_I}$$

Experiments: $\frac{|A_0|}{|A_2|} \simeq 22.1$ "$\Delta I = 1/2$ rule"

Enhancement of amplitude changing Isospin by $\frac{1}{2}$

$\Delta I = 1/2$ rule (II)

$T(K \to (\pi\pi)_I) = A_I e^{i\delta_I}$, $\quad I = 0, 2$, $\quad |A_0|/|A_2| \simeq 22.1$ $\quad$ "$\Delta I = 1/2$ rule"

- "Rule": $\Delta I = \frac{1}{2}$ enhancement over $\Delta I = \frac{3}{2}$ transitions also in baryon sector, e.g. $\Lambda \to N\pi$ or $\Sigma \to N\pi$
- Weak interaction does not distinguish different isospin final states
  $\Longrightarrow$ Origin: Strong interaction effects
- Short-distance QCD effects and large N$_c$ arguments $\longrightarrow$ only small enhancement
  $\Longrightarrow$ Study long-distance, i.e. non-perturbative regime of QCD

$\Longrightarrow$ Lattice

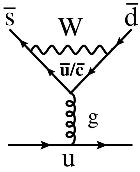
$\Delta I = 1/2$ rule (III)

$T(K \to (\pi\pi)_I) = A_I e^{i\delta_I}, \qquad I = 0, 2, \qquad |A_0|/|A_2| \simeq 22.1 \quad$ "$\Delta I = 1/2$ rule"

Several possible origins/contributions of long-distance QCD

- final state interactions
- physics at $E_{QCD} \approx 250$ MeV
- physics at $\mu = m_{charm} \sim 1$ GeV (via penguins)

  Role of charm quark unclear. Classic arguments suggest:
  Large up-charm quark mass difference may be important
  [Shifman et al. NPB 120 (1977)]



- all of the above (no dominating mechanism)

Separate intrinsic QCD effects from physics at m_c
[Giusti, Hernández, Laine, Weisz, Wittig, JHEP11 (2004)]

Outline

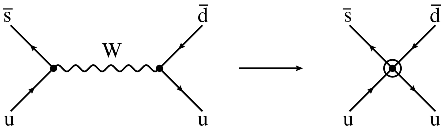
Theoretical approach to $\Delta I = 1/2$ rule
- Effective Hamiltonian
- Kaon decays on the lattice

$\Delta I = 1/2$ with an active charm quark
- Strategy
- $\mathcal{H}_{eff}^{\Delta S=1}$ with active charm

Lattice techniques
- Low-mode averaging (LMA)
- Stochastic volume sources (SVS)

Results

Effective Hamiltonian (I)



Length scale weak interaction
$\ll$
size of hadron
$\longrightarrow$ turn to effective theory

OPE: separates long-distance and short-distance effects via effective weak Hamiltonian

$$\mathcal{H}_w^{eff} = \frac{G_F}{\sqrt{2}} \sum_i k_i(V_{CKM}, M_W, \mu) \mathcal{Q}_i(\mu)$$

- $\mathcal{Q}$: (multi) quark field operators
- k: Wilson Coefficients, include all high-energy effects

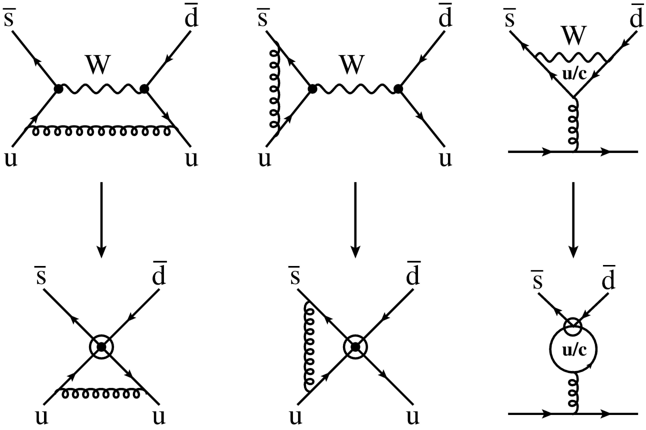
Continuum SM $\xrightarrow{OPE}$ $\mathcal{H}_w^{eff} \sim k(M_W)\mathcal{Q}^{cont}(M_W) \xrightarrow{RG} k(\mu)\mathcal{Q}^{cont}(\mu)$

matching $\updownarrow$

$k(\mu)Z(\mu a)\mathcal{Q}^{latt}(a)$

Effective Hamiltonian (II)

Lowest QCD corrections



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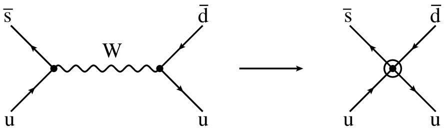

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Kaon decays on the lattice

Lattice: two approaches

$\swarrow \searrow$

$K \to \pi\pi$
Direct computation

[RBC/UKQCD collaboration]

Relate $K \to \pi\pi$ to $K \to \pi$, $K \to 0$ via ChPT

- Express $K \longrightarrow \pi\pi$ in terms of LECs
- Determine LECs in lattice simulation
- LECs defined in chiral limit $m \to 0$

Challenges/problems

- Evaluation of 4-point functions
- Large volume required

- Relies on ChPT

Strategy (I) [Giusti, Hernández, Laine, Weisz, Wittig, JHEP11 (2004)]

Approach:

- CP-conserving $\Delta S = 1$ effective weak Hamiltonian $\mathcal{H}_{eff}^{\Delta S=1}$ with active charm quark (both: in lattice QCD and Chiral Perturbation Theory)
- Bypass direct computation of $K \to \pi\pi$ [Bernard et al. PRD 32 (1985)]
  $\Longrightarrow$ Compute $K \to \pi$ and $K \to vac$ matrix elements in LQCD
  $\Longrightarrow$ relate to physical transition amplitude via ChPT
- Express ratio of kaon decay amplitudes A_I via LECs $[\hat{g}_1^{\pm}]$ in ChPT

$$\frac{|A_0|}{|A_2|} = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{3}{2}\frac{\hat{g}_1^-}{\hat{g}_1^+}\right) \qquad \text{(at LO)}$$

- Determine $\hat{g}_1^- / \hat{g}_1^+$ in lattice QCD

Step 1:	$m_c = m_u = m_d = m_s$	$SU(4)$-LQCD matched to $SU(4)$-ChPT
Step 2:	$m_c \gg m_u = m_d = m_s$	$SU(4)$-LQCD matched to $SU(3)$-ChPT

$\Longrightarrow$ monitor A_0, A_2 as a function of m_c

Strategy (II) [Giusti, Hernández, Laine, Weisz, Wittig, JHEP11 (2004)]

Ingredient:

Neuberger (overlap) fermions: $D_N = \frac{1+s}{a}\left\{1 - \frac{A}{(A^\dagger A)^{1/2}}\right\}, \qquad A = 1 - aD_W,$

[H. Neuberger, PL B417 (1998)]

- Renormalization & mixing patterns like in the continuum, provided

$$\psi \to \tilde{\psi} = \left(1 - \frac{1+s}{2a} D\right)\psi, \qquad \bar{\psi} \to \bar{\psi}$$

  No mixing with lower dimensional operators [Capitani, Giusti, PRD (2001) 014506]

- Allows simulating quark masses near chiral limit $\longrightarrow$ ChPT most reliable

- Numerical treatment expensive $\longrightarrow$ first results quenched
  Quantitative attempt to understand large $\Delta I = 1/2$ enhancement!

$\Delta S = 1$ weak Hamiltonian with an active charm quark (I)

Effective theory with approximate $SU(4)_L \times SU(4)_R$ chiral symmetry

$$\mathcal{H}_w^{eff} = \frac{g_w^2}{2M_W^2} V_{ud} V_{us}^* \sum_{\sigma=\pm} \{ k_1^\sigma \mathcal{Q}_1^\sigma + k_2^\sigma \mathcal{Q}_2^\sigma \}$$

Short distance QCD $\longrightarrow$ moderate enhancement $\frac{k_1^-}{k_1^+} \approx 2.8$ (2 loop PT)

Enhancement dominated by matrix elements of operators (long distance)

$$\mathcal{Q}_1^\pm = \{(\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d) \pm (\bar{s}\gamma_\mu P_- d)(\bar{u}\gamma_\mu P_- u)\} - (u \to c)$$
$$\mathcal{Q}_2^\pm = (m_u^2 - m_c^2)(m_d \bar{s} P_+ d + m_s \bar{s} P_+ d)$$

- $SU(4)_L \times SU(4)_R$ chiral group: 4-quark operator $\mathcal{Q}_1^+ : \ (84, 1)$, $\mathcal{Q}_1^- : \ (20, 1)$
  $\longrightarrow$ no mixing under renormalization for GW fermions
- $\mathcal{Q}_2^\pm$ don't contribute to phys. $K \longrightarrow \pi\pi$ decay, mix with $\mathcal{Q}_1^\pm$ for $m_c \neq m_u$

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$SU(4)$ vs. $SU(3)$ flavor approach. $SU(3)$ symmetry: charm integrated out

$$\mathcal{H}_{eff}^{\Delta S=1} \propto \sum_i^{10} k_i(V_{CKM}, M_W, \mu) \mathcal{Q}_i(\mu)$$

- 2 Current-current operators
- 4 QCD-penguin operators
- 4 EW-penguin operators

Nice: Active charm quark + GW-fermions $\longrightarrow$ only logarithmic divergences !

$\Delta S = 1$ weak Hamiltonian with an active charm quark (II)

Low-energy counterpart of weak effective Hamiltonian at LO

$$\mathcal{H}_w^{ChPT} = \frac{g_w^2}{2M_W^2} V_{ud} V_{us}^* \sum_{\sigma=\pm} \hat{g}_1^\sigma \left\{ \left[\hat{\mathcal{O}}_1^\sigma\right]_{suud} - \left[\hat{\mathcal{O}}_1^\sigma\right]_{sccd} \right\}$$

4-quark operator $\leftrightarrow$ $\left[\hat{\mathcal{O}}_1\right]_{\alpha\beta\gamma\delta} = \frac{1}{4} F^4 \left(U\partial_\mu U^\dagger\right)_{\gamma\alpha} \left(U\partial_\mu U^\dagger\right)_{\delta\beta}$

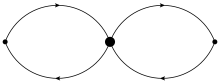
$U \in SU(4)$: Goldstone boson field, F: pion decay constant

$$\frac{|A_0|}{|A_2|} \propto \frac{\hat{g}_1^-}{\hat{g}_1^+}$$

Step 1: Degenerate charm [Giusti, Hernández, Laine, Pena, Wennekers, Wittig '07]

Study $K \to \pi$ transitions in the SU(4)-symmetric theory: $m_u = m_d = m_s = m_c$

$\Longrightarrow$ only "Figure-8" graphs contribute



$$C_1^{\pm}(x,y) = \left\langle Tr\left[\gamma_0 P_- S(x,0)^\dagger \gamma_0 P_- S(x,0)\right] Tr\left[\gamma_0 P_- S(y,0)^\dagger \gamma_0 P_- S(y,0)\right]\right\rangle$$
$$\mp \left\langle Tr\left[\gamma_0 P_- S(x,0)^\dagger \gamma_0 P_- S(y,0) \gamma_0 P_- S(y,0)^\dagger \gamma_0 P_- S(x,0)\right]\right\rangle$$

	$\hat{g}_1^+$	$\hat{g}_1^-$
Lattice	0.51(3)(5)(6)	2.6(1)(3)(3)
"Exp"	~ 0.5	~ 10.4
large N_c	1	1

errors
-1st: statistical
-2nd: matching to ChPT
-3rd: renormalization

- $\Delta I = 3/2$ amplitude A_2: close to "experiment"
- $\Delta I = 1/2$ amplitude A_0: factor ~ 4 too small
- Significant enhancement in $SU(4)$-symmetric limit: $\dfrac{|A_0|}{|A_2|} \sim 6$ (Exp: $\sim$22)

Low-mode averaging (LMA)

- Strong statistical fluctuations as $m \to 0$: “spikes” in MC history
- Spectral representation of quark propagator:

$$S(x,y) = \frac{1}{V} \sum_i \frac{v_i(x) \otimes v_i^\dagger(y)}{\lambda_i + m}$$

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$m > 1/\Sigma V$ - low-lying spectrum of D_m dense near m ($m \gg \lambda_{low}$)
- contributions from v_i averaged with same weight ($\sim 1/\mathrm{m}$)

$m \leq 1/\Sigma V$ - low-lying spectrum of D_m discrete: $m \approx \Delta\lambda = 1/\Sigma V$
- sizeable contributions come from a few v_i
- space-time fluctuations of wave-functions can be amplified significantly for individual eigenmodes $\longrightarrow$ "spikes"

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$\Longrightarrow$ Treat a number of low-lying modes, n_{low}, exactly:
Averaging over $\vec{x}, \vec{y}$ in low-lying contribution reduces local fluctuations
[Giusti et al. JHEP04 (2004),DeGrand, Schaefer CPC159 (2004)]

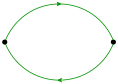
Low-mode averaging (LMA)

$$S(x,y) = \frac{1}{V}\sum_i^{n_{low}} \frac{v_i(x)\otimes v_i^\dagger(y)}{\lambda_i + m} \; + \; S^h(x,y) \qquad \text{("low" part + "high" part)}$$

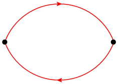
$\Longrightarrow$ Decomposition of CF:

$$C^{2pt} = -\sum_{\vec{x}} \left\langle Tr\{S(x,y)\gamma_0 P_- S(y,x)\gamma_0 P_-\}\right\rangle = C^{ll}(t) + C^{lh}(t) + C^{hh}(t)$$

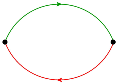
$$C^{hh}(t) = -\sum_{\vec{x}} \left\langle Tr\{\gamma_0 P_- S^h(x,0)^\dagger \gamma_0 P_- S^h(x,0)\}\right\rangle$$



$$C^{ll}(t) \propto -\frac{1}{V}\sum_{k,l=1}^{n_{low}}\sum_{x,y}\delta_{t,t_x\text{-}t_y}\left\langle [v_k^\dagger\gamma_0 P_- v_l](x)[v_l^\dagger\gamma_0 P_- v_k](y)\right\rangle$$



$$C^{hl}(t) \propto -\frac{1}{L^3}\sum_{k=1}^{n_{low}}\sum_{x,\vec{y}}\delta_{t,t_x\text{-}t_y}\left\langle v_k^\dagger(x)\gamma_0 P_- S^h(x,y)\gamma_0 P_- v_k(y)\right\rangle + (x\leftrightarrow y)$$



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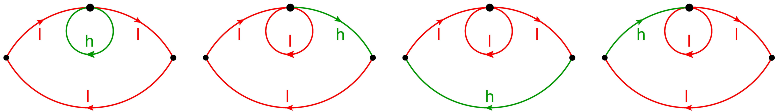
$$C^{3pt} = C^{llll} + C^{hlll} + C^{hhll} + C^{hhhl} + C^{hhhh}$$

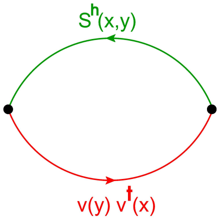
# of diagrams: 1, 4, 6, 4, 1 = **16**

Example: C^{hlll}



Extended propagators

$$C^{hl}(t) \propto \left\langle v_k^\dagger(x)\gamma_0 P_- \, S^h(x,y)\gamma_0 P_- v_k(y) \right\rangle + (x \leftrightarrow y)$$



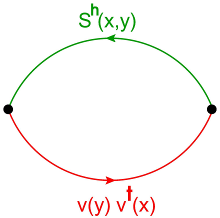
Use eigenmode as source for inversion

$$\eta(y; t_0) = \delta_{ty_0}\gamma_0 P_- v_k(y)$$

Solution at fixed timeslice y_0:

Extended propagators

$$C^{hl}(t) \propto \left\langle v_k^\dagger(x)\gamma_0 P_- \underbrace{S^h(x,y)\gamma_0 P_- v_k(y)}_{S^{ext}(x,y)} \right\rangle + (x \leftrightarrow y)$$



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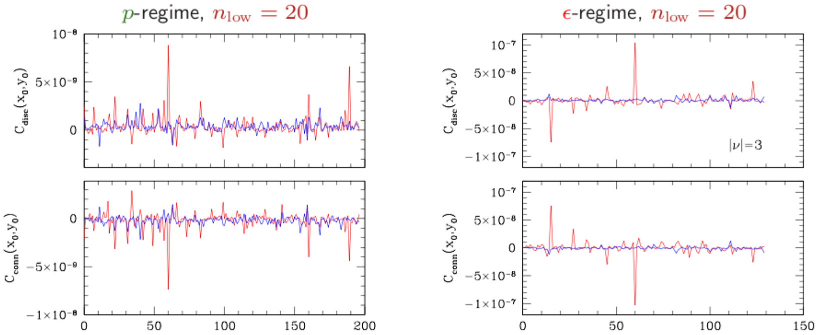
Solution at fixed timeslice y_0:

$$S^{ext}(x,y) = \sum_{\vec{y}} S^h(x,y)\gamma_0 P_- v_k(y)$$

PRO: Implicit volume averaging
CON: y_0 fixed and n_{low} extra inversions

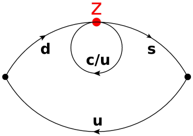
Low-mode averaging (LMA)



- with LMA: signal improves
- without LMA: signal lost

Downside: main part of CPU time spent on manipulations involving low-modes

Step 2: Decouple charm quark $m_c > m_u$



$$S(z,z) = \frac{1}{V}\sum_{i}^{n_{low}} \frac{v_i(z) \otimes v_i^\dagger(z)}{\lambda_i + m} + S^h(z,z)$$

- "EYE"-diagram: Signal still lost despite LMA
- Problem: Use of point-to-all propagators $S^h(z,0)$ does not allow for averaging over $\vec{z}$, the position of 4-quark operator insertion
- Approach: Estimate entire propagator $S^h(z,z)$ ("high part") stochastically; not only single column! ["Hybrid-approach", Peardon et al. CPC 172 (2005)]
- Not only loop requires stochastic all-to-all propagator

Reminder: Stochastic volume sources (SVS)

[Bernardson et al. CPC 78 (1993) '93, Dong, Liu PLB 328 (1994)]

- Source method: Quark propagator by solving the linear system

$$D\Phi = \eta$$

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$\langle \ldots \rangle_{src}$: expectation value over the distribution of noise vectors

- Taking the solution vector $\Phi_{[r]} = D^{-1}\eta_{[r]} \longrightarrow$ estimate of entire propagator matrix

$$\left\langle \Phi^a_\alpha(x)_{[r]} \right\rangle_{\mathrm{src}} = \left\langle (D^{-1})^{ac}_{\alpha\gamma}(x,z)\eta^c_\gamma(z)_{[r]} \right\rangle_{\mathrm{src}}$$

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- Source method: Quark propagator by solving the linear system

$$D\Phi = \eta$$

- Generate set of N_r volume-filling source vectors $\{\eta_{[1]}, \dots, \eta_{[N_r]}\}$ by assigning random numbers, e.g. $\in Z(2) = \{\pm 1\}$ satisfying
  - $\left\langle \eta^a_\alpha(x)_{[r]} \right\rangle_{\mathrm{src}} = 0$
  - $\left\langle \eta^a_\alpha(y)_{[r]} (\eta^\dagger)^b_\beta(x)_{[r]} \right\rangle_{\mathrm{src}} = \delta_{yx}\delta_{\alpha\beta}\delta^{ab}$

  $\langle \dots \rangle_{src}$: expectation value over the distribution of noise vectors

- Taking the solution vector $\Phi_{[r]} = D^{-1}\eta_{[r]} \longrightarrow$ estimate of entire propagator matrix

$$\left\langle \Phi^a_\alpha(x)_{[r]} (\eta^\dagger)^b_\beta(y)_{[r]} \right\rangle_{\mathrm{src}} = \left\langle (D^{-1})^{ac}_{\alpha\gamma}(x,z)\, \overbrace{\eta^c_\gamma(z)_{[r]} (\eta^\dagger)^b_\beta(y)_{[r]}}^{\longrightarrow \delta_{zy}\delta_{\gamma\beta}\delta^{cb}} \right\rangle_{\mathrm{src}}$$

$$= S^{ab}_{\alpha\beta}(x,y)$$

Dilution (I) [Peardon et al. CPC 172 (2005)]

Stochastic volume sources (SVS):

NICE: Access to entire propagator $\longrightarrow$ increase in statistics
$\longrightarrow$ reduced sensitivity to local fluctuations

BAD: Explicit introduction of stochastic noise (exact only limit $N_r \to \infty$)

Technique for reducing intrinsic stochastic noise: "Dilution"

$$\begin{matrix} \eta \\ \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ \vdots \\ Z_n \end{pmatrix} \end{matrix}$$

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$$\mathrm{n} \times \overset{\eta}{\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ \vdots \\ Z_n \end{pmatrix}} \xrightarrow{dilution} \overset{\eta_1}{\begin{pmatrix} Z_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}, \overset{\eta_2}{\begin{pmatrix} 0 \\ Z_2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}, \overset{\eta_3}{\begin{pmatrix} 0 \\ 0 \\ Z_3 \\ \vdots \\ 0 \end{pmatrix}}, \ldots, \overset{\eta\,\mathrm{n}}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ Z_n \end{pmatrix}}$$

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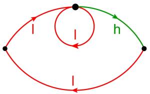
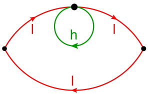
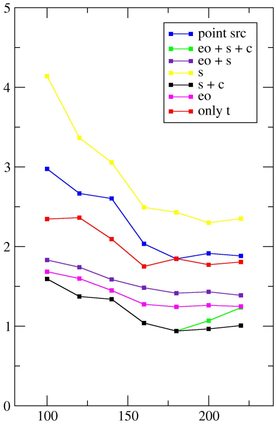
Examples: time, spin, color, even-odd space-time

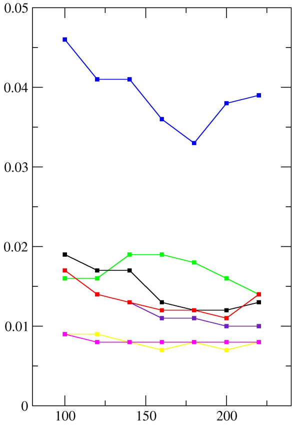
- Experience: Additional inversions (often) outperform application of multiple hits
- Full time dilution mandatory for connected correlators

Dilution (II)









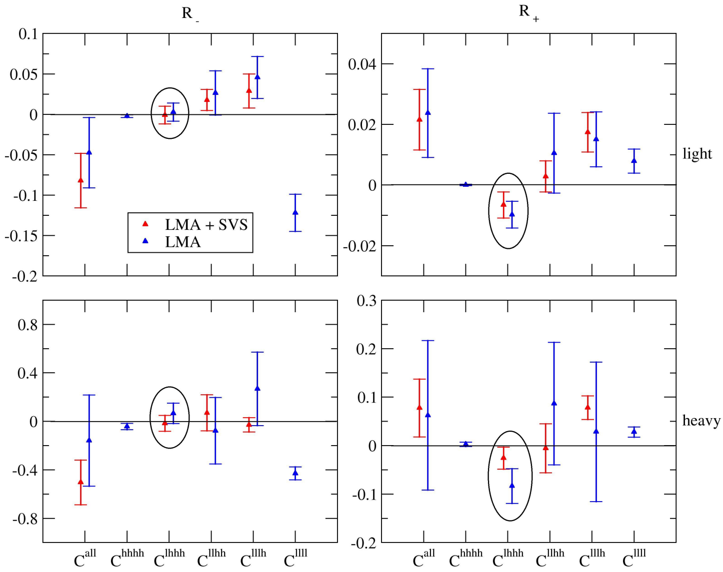
abs. error $\times \sqrt{N_{cfg}}$

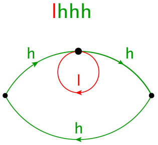
vs.

$\sqrt{N_{cfg}}$

Preliminary results: Eye-diagram

$R_- = C^-_{EYE}/C^2_{2pt}$ $\quad$ $R_+ = C^+_{EYE}/C^2_{2pt}$





- 140 cnfgs, $32x16^3$
- Quenched
- $\beta = 5.8485$
- $n_{low} = 20$
- Spin-dilution
- $m_\pi = 322$ MeV
- light: $m_c = 2m_u$
- heavy: $m_c = 10m_u$

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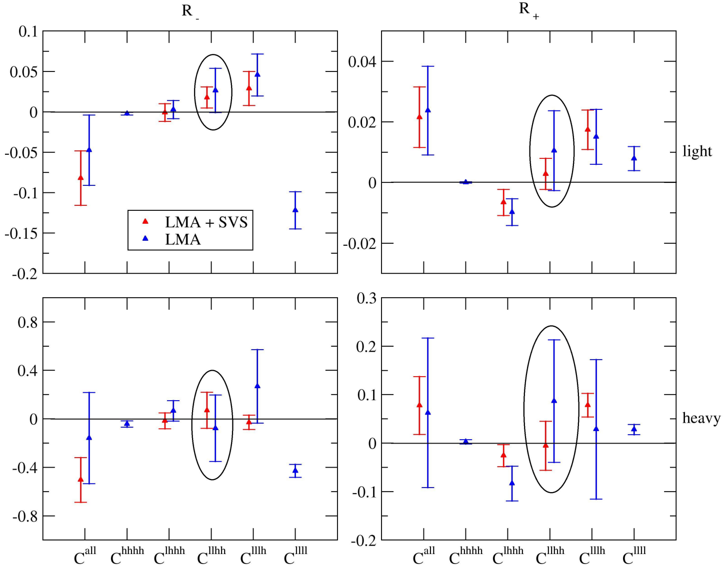

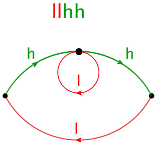

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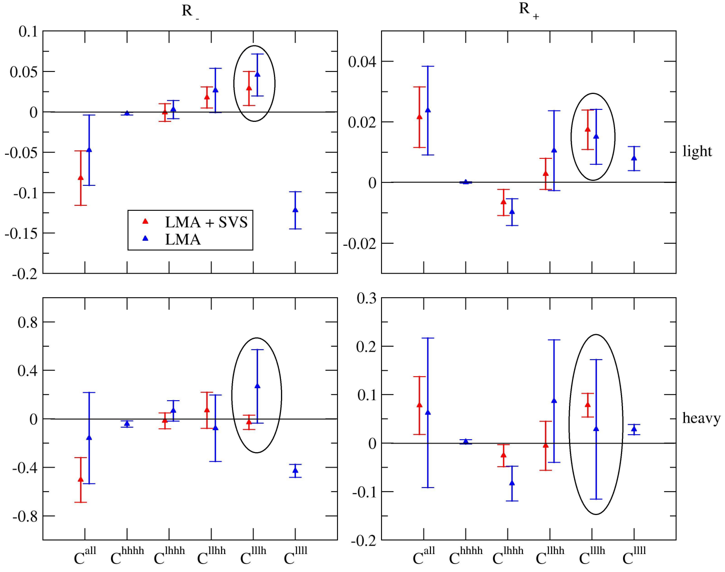

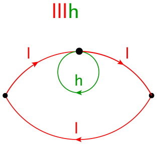

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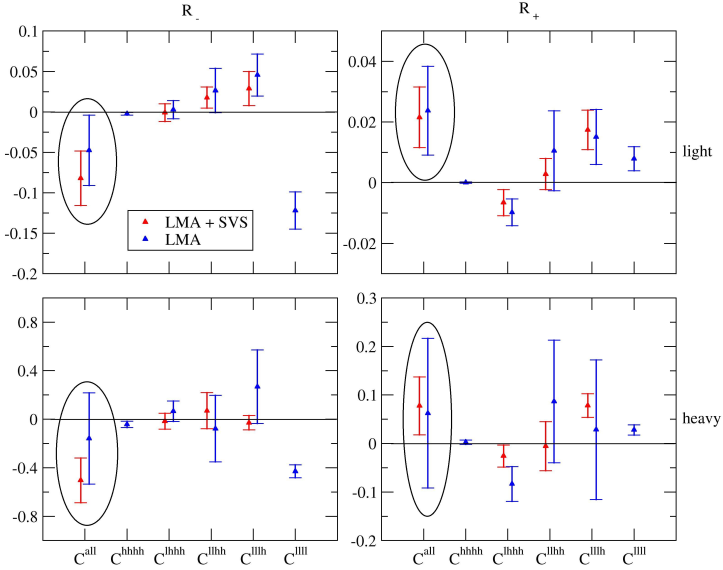

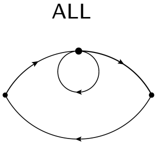

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Summary and Outlook

- Computational framework to quantify different sources of $\Delta I = 1/2$ enhancement: $\Delta S = 1$ weak effective Hamiltonian with active charm and Ginsparg-Wilson fermions
- Current stage: Decouple charm quark mass, i.e. $m_c > m_u$, to monitor dependence of amplitudes on m_c
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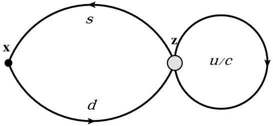
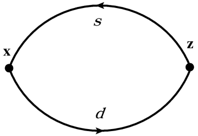
  - Still bare quantities $\rightarrow$ renormalization
  - Dynamical configurations with Wilson sea quarks (Mixed action)

Diagrams required for renormalization

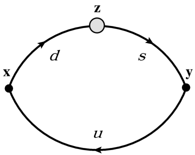
$\left\langle 0|\mathcal{O}_1^{\pm}(z)|K\right\rangle$:



$\langle 0|\bar{s}P_{\pm}d|K\rangle$:



$\left\langle \pi|\mathcal{O}_2^{\pm}(z)|K\right\rangle$:



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    [F. Bernardoni, N. Garron, P. Hernandez, S. Necco and C. Pena et al. Phys.Rev D83 (2011)]