Targeting precision in lattice determinations of hadronic contributions to muonic $g-2$.

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Work with: Peter Boyle, Luigi Del Debbio and James Zanotti

hep-lat/1107.1497

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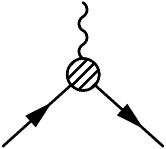
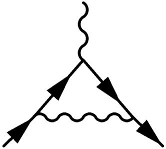
Anomalous Magnetic Moment

$$\vec{\mu} = g\frac{e}{2m}\vec{S} \qquad\qquad a = \frac{g-2}{2}$$



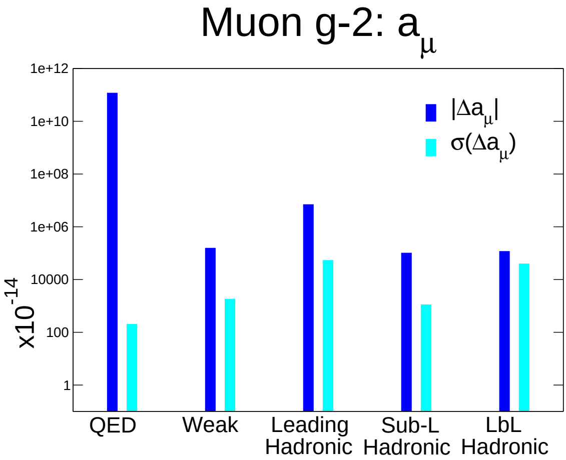
$=$



$+ \quad \cdots$

$$= \frac{\alpha}{2\pi} + \cdots$$

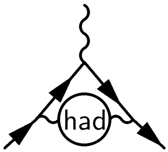
Contributions



- Difficult, but theory now ∼as accurate as experiment.
- Largest uncertainty is from hadronic contributions.

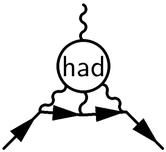
Hadronic Contributions

Leading order vacuum polarisation



$690.3(5.3) \times 10^{-10}$

Light-by-light

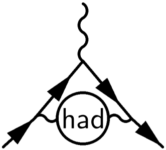


$11.6(3.9) \times 10^{-10}$

[Jegerlehner and Nyffeler, 2009]

Hadronic Vacuum Polarisation

Leading order vacuum polarisation



$690.3(5.3) \times 10^{-10}$

- Estimated by relating to $e^+ e^-$ decay to hadrons, and a dispersive integral over data.

$$a_\mu^{(2)had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m^2_{\pi^0}}^{\infty} ds \frac{R_{had}(s)\hat{K}(s)}{s^2}$$

$$R_{had}(s) = \frac{4\pi\alpha(s)^2\sigma(e^+e^- \to hadrons)}{3s}$$

Consequences

- a_μ discrepancy is among strongest suggestions of NP.
- Widely used in constraining models of BSM physics.
- $a_\mu^{(2)had}$ contributes large uncertainties.
- First principles calculation clearly desired.
- Accurate determination would have large impact on search for new physics.

$a_\mu^{(2)had}$ on the lattice

- First quenched calculations:
  - DWF [Blum, 2003]
  - Improved Wilson [Göckeler, M. and others, 2004]
- Followed by dynamical calculations:
  - 2+1f Staggered [Aubin and Blum, 2007]
  - 2f Twisted-Mass [Feng et al., 2011a]
  - 2f, 2+1+1f Wilson [Della Morte et al., 2012]
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- Activity and progress accelerating [Blum, 2012, Blum et al., 2012]

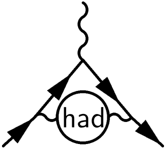
Outline

Introduction

Framework

Fitting the data

Calculation Approach



$$a_\mu^{(2)had} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 \, \hat{\Pi}(Q^2) \times f(Q^2)$$

[Blum, 2003]

Hadronic Vacuum Polarisation

$$= \int d^4x \, e^{iq\cdot(x-y)} \langle J_\mu(x) J_\nu(y) \rangle$$

$$= (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

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On the lattice:

$$\widetilde{\Pi}_{\mu\nu}(x) = Z_V \sum_i Q_i^2 a^6 \langle \mathcal{V}_\mu^i(x) V_\nu^i(0) \rangle$$

Vacuum polarisation

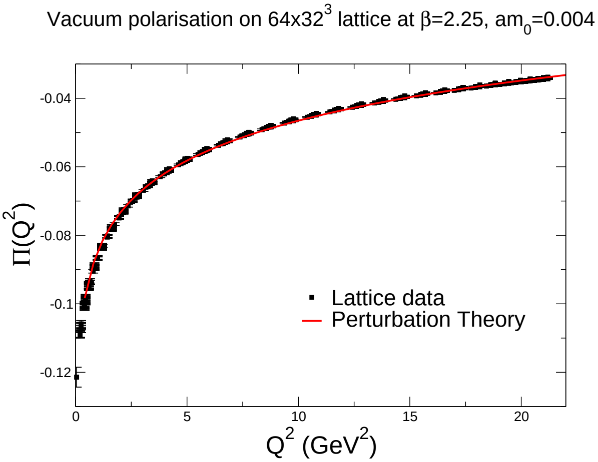

Figure: $\widetilde{\Pi}(\hat{q}^2) = \frac{\widetilde{\Pi}_{\mu\mu}(\hat{q})}{\hat{q}^2}$

Setting up integral

$$a_{\mu}^{(2)had} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$$

- Split integral into low and high momentum regions.
  - High momentum ($Q^2 > Q_C^2$) region is calculated from perturbation theory [Chetyrkin et al., 1996].
  - Low momentum ($Q^2 > Q_C^2$) region must be interpolated from data.

Outline

Low momentum fit

- Procedure for fitting data has attracted focus.
- Many forms attempted:
  * Polynomials
  * Pade approximants
  * Vector Dominance Model ansatze
  * χPT expressions (vector model)

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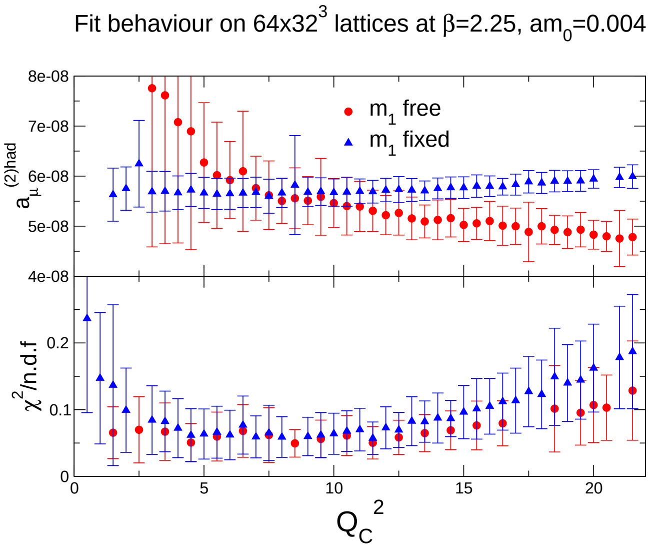
- Fit range must also be chosen.

Fit behaviour



Kernel

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$$f(Q^2) = \frac{m_\mu^2 Q^2 Z(Q^2)^3 (1 - Q^2 Z(Q^2))}{1 + m_\mu^2 Q^2 Z(Q^2)^2}$$

$$Z(Q^2) = -\frac{Q^2 - \sqrt{Q^4 + 4 m_\mu^2 Q^2}}{2 m_\mu^2 Q^2}$$

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$$f(Q^2) \xrightarrow{Q^2 \to 0} \frac{1}{\sqrt{Q^2}}$$

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Weighted χ^2?

Weighted χ^2 fit

- First guess?

$$\chi^2 = \sum_i f(x_i) \times \frac{(y(x_i) - y_i)^2}{\sigma_i^2}$$

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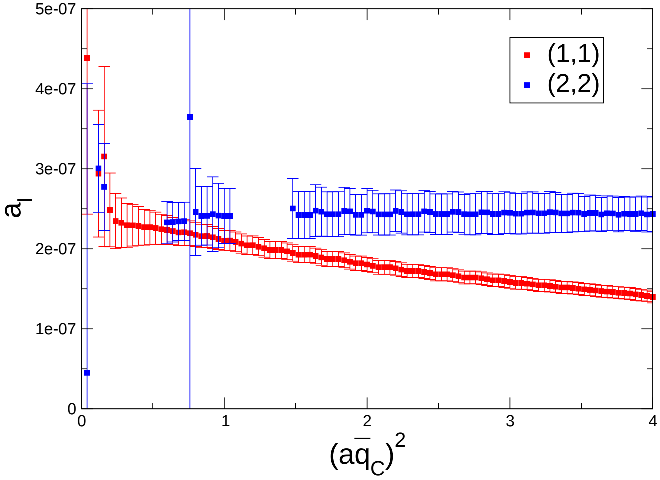
- Recall kernel behavior:

$$f(x) \xrightarrow{x \to 0} \frac{1}{2m_\mu \sqrt{x}}$$

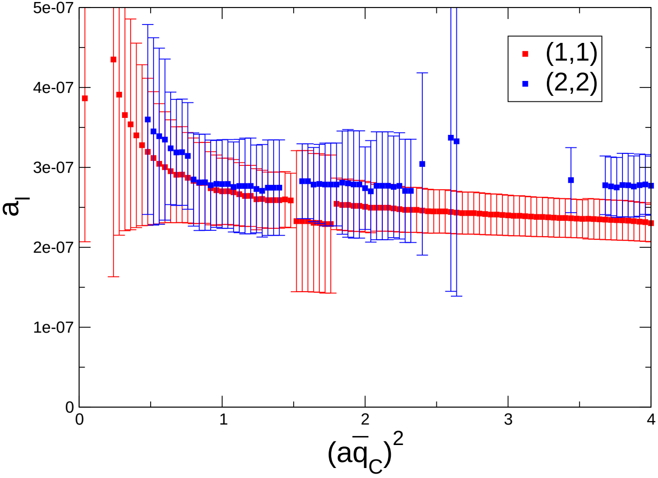
- Better choice:

$$\chi^2 = \sum_i f(x_i) \times 2m_\mu \sqrt{x_i} \times \frac{(y(x_i) - y_i)^2}{\sigma_i^2}$$

Original Pade fits

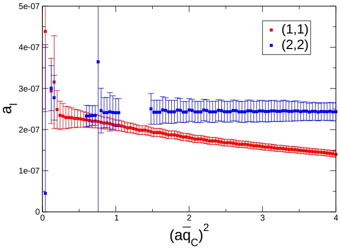


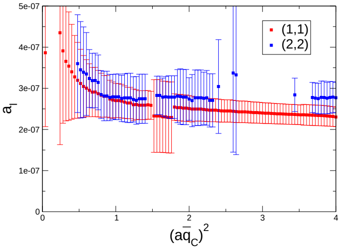
Weighted Pade fits



Diagnosis

- Weighted fit does not allow further data to over constrain fit.
- Should allow for simpler identification of correct fit range.
- Also reduces discrepancies between different ansatze.





Outlook

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Thank you.

Lattice details

- RBC & UKQCD, 2+1f, DWF.

V	β	a^{-1} GeV	$\hat{q}^2_{min}$ GeV^2	am_{PS} MeV
$24^3 \times 64$	2.13	1.73(2)	0.028	330, 420, 560
$32^3 \times 64$	2.25	2.28(3)	0.05	290, 345, 395
$32^3 \times 64$	1.75	1.375(9)	0.018	170, 250

Table: Parameters of the lattice ensembles used in our study.

- $\beta = 1.75$ use Iwasaki + DSDR & $L_5 = 32$.
- $\beta = 2.25, 2.13$ simply Iwasaki & $L_5 = 16$ [Aoki et al., 2010].
- Compare $\hat{q}^2_{min}$ to $m^2_\mu \simeq 0.011$ GeV^2.

Lattice correlators

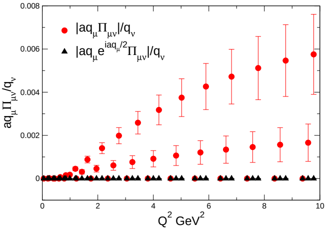
$$\widetilde{\Pi}_{\mu\nu}(x) = Z_V \sum_i Q_i^2 a^6 \langle \mathcal{V}_\mu^i(x) V_\nu^i(0) \rangle$$

- We omit the flavour non-diagonal terms, because they are "disconnected".
- We also omit the disconnected contraction in the diagonal term.
- The disconnected contraction, has been shown in χPT to be of the order of 10% of the connected
- This will be our largest systematic effect.

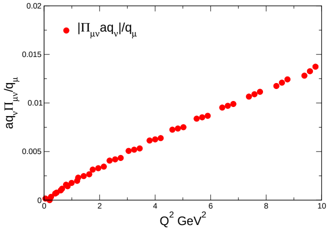
Ward Identities

- Because we use conserved-local, we have a Ward identity:

$$\Delta_\mu \widetilde{\Pi}^i_{\mu\nu}(x) = 0 \qquad \Rightarrow e^{\frac{iaq_\mu}{2}} \hat{q}_\mu \widetilde{\Pi}^i_{\mu\nu} = 0$$

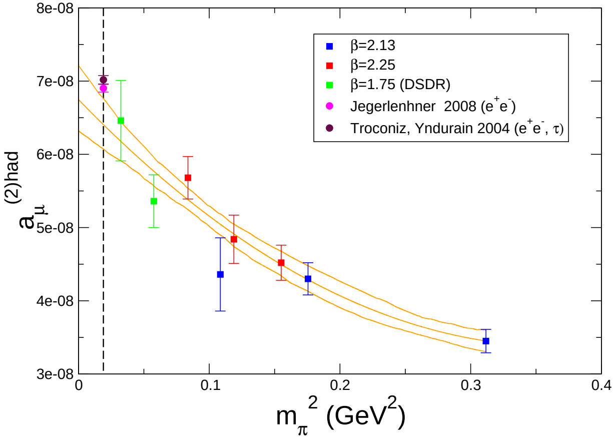


$$\Delta_\mu \widetilde{\Pi}^i_{\mu\nu}(x) = 0$$



$$\Delta_\nu \widetilde{\Pi}^i_{\mu\nu}(x) \neq 0$$

Lattice results

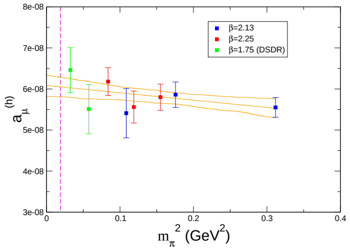


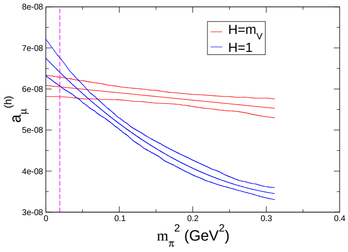
Our result: $a_\mu^{(2)had} = 641(33)(32) \times 10^{-10}$

Improving the chiral behaviour?

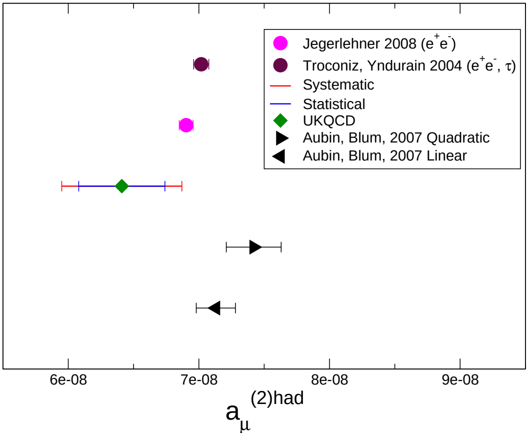
$$f(Q^2) \simeq \frac{w(\frac{Q^2}{m_\mu^2})}{Q^2} \rightarrow \frac{w(\frac{Q^2}{m_\mu^2} \times \frac{H_{phys}^2}{H^2})}{Q^2}$$

- $H = m_V$ a good choice. [Feng et al., 2011b]





Results Comparison



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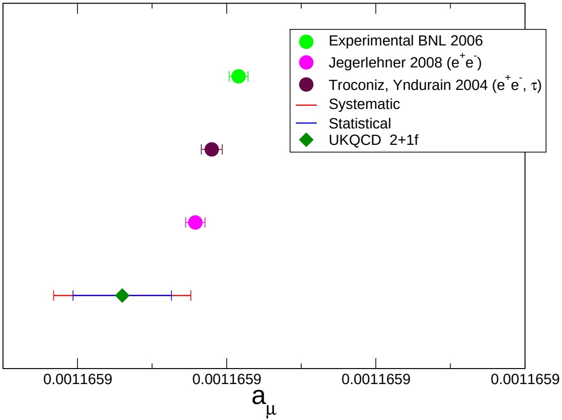
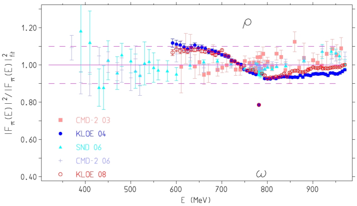
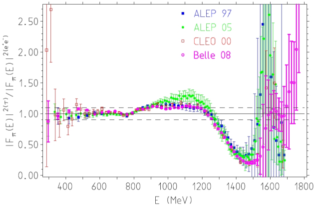


Scattering data





- Possible systematics in e^+e^- data?

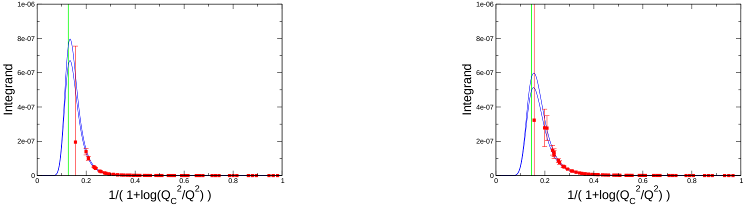
- More pronounced when e^+e^- is compared to τ.

Integrand

- After fitting $\Pi(Q^2)$, can perform low momentum integral.
- Rescale to more precisely sample relevant region

$$t = \frac{1}{1 + \log\left(\frac{Q_C^2}{Q^2}\right)}$$

$$\int_0^{Q_C^2} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2) \longrightarrow \int_0^1 dt\, f(Q^2) \times \hat{\Pi}(Q^2) \times \frac{Q^2}{t^2}$$



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