Nucleon sigma terms and dark matter searches

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NIC, DESY

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Dürr, Fodor, Frison, Hemmert, Hoelbling, Hoffman, Katz, Krieg, Kurth, Lellouch, Lippert, Portelli, Ramos, Schaefer, Szabo
[Budapest-Marseille-Wuppertal collaboration]

Dark matter

- Discrepancy between measurements of the mass of structures larger than galaxies made trough dynamical (GR) means and measurements based on the “luminous” matter these objects contains.

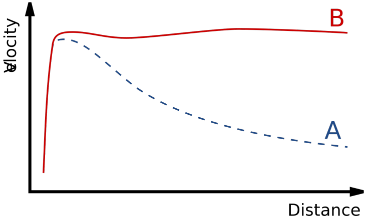

Figure: Rotation curve of a typical spiral galaxy: predicted (A) and observed (B). Dark matter can explain the velocity curve having a 'flat' appearance out to a large radius.

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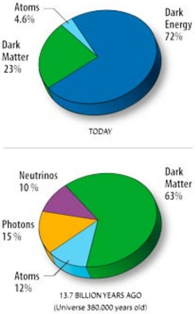
- Under standard interpretation (BB, FRW), "dark" matter accounts for 23% of the mass of the visible universe.



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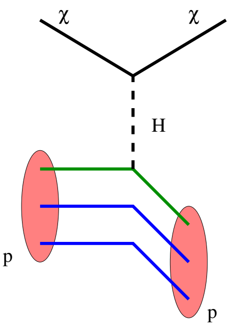
Dark matter detection

Dark mater detection/understanding is one of the challenges of the decade.

Direct DM detection



$$\mathcal{L}_{int} = \lambda_N \bar{n} n \bar{\chi}\chi \rightarrow \mathcal{L}_{int} = \lambda_q \bar{q} q \bar{\chi}\chi$$

$$\lambda_N \longrightarrow \sum_{q=1}^{6} f_q^N \lambda_q$$

Spin indep. WIMP-N X-section

$$\sigma_{SI} = \frac{4M^2}{\pi}\left[Z f_P + (A - Z) f_N\right]$$

with

$$\frac{f_N}{M_N} = \sum_q f_q^N \frac{\lambda_q}{m_q}$$

Sigma terms

$$f_{ud}^N M_N = \sigma_{\pi N} = \langle N(p)|(\bar{u}u + \bar{d}d)|N(p)\rangle \qquad f_s^N M_N = \sigma_{\bar{s}sN}/2 = \langle N(p)|\bar{s}s|N(p)\rangle$$

Phenomenological determination of the nucleon sigma terms

Scalar forma factors at zero momentum transfer $\sigma_{\pi N} = \sigma_{\pi N}(0)$

$$\sigma_{\pi N}(p - p') = \langle N(p)|(\overline{u}u + \overline{d}d)|N(p')\rangle$$

And a low energy theorem [Cheng, Dashen (1971)] relates the $\pi - N$ scattering amplitude at the (unphysical) Cheng-Dashen point

$$\Sigma = \sigma_{\pi N}(2M_\pi^2) + \Delta_R$$

with $\Delta_R \sim 2MeV$. Now we can use

$$\sigma_{\pi N} = \Sigma - \left[\sigma_{\pi N}(2M_\pi^2) - \sigma_{\pi N}\right] - \Delta_R$$

and use χPT to estimate $\Delta_\sigma = \sigma_{\pi N}(2M_\pi^2) - \sigma_{\pi N}$

$$\begin{aligned} \Delta_\sigma &= 15.2 \pm 0.4 MeV \quad \text{[Gasser, Leutwyler, Sainio, (1991)]} \\ &= 14.0 + 2M_\pi^4 \overline{e}_2 MeV \quad \text{[Becher, Leutwyler (2001)]} \end{aligned}$$

Use $SU(3)$ breaking and octet masses to determine $\sigma_{\overline{s}sN}$

$$m_{ud}\langle N|\overline{u}u + \overline{d}d - 2\overline{s}s|N\rangle \sim 36(7)MeV + \mathcal{O}(m_q^2)$$ [Borasoy, Meiner, (1997)]

Conclusion: How much is Σ?

$$\sigma_{\pi N} = \Sigma - \Delta_\sigma - \Delta_R \approx \Sigma - 16MeV$$

Scattering amplitude at CD point

Not a trivial thing

- There is no $\pi - N$ scattering data at the CD point.
- Dispersion relations (Unitarity, analiticity) allow to obtain information at the CD point from experimental data.
- Complex: hyperbolic dispersion relations, subtraction constants, partial wave analysis of $\pi - N$ scattering data

- Early (canonical) determination [Koch, (1982)]

$$\begin{aligned} \Sigma &= 64 \pm 8 MeV \\ \sigma_{\pi N} &= 45 \pm 8 MeV \\ f_{udN} &= 0.047 \pm 6 MeV \\ f_{sN} &= 0.10 \pm 0.10 MeV \end{aligned}$$

- Recent determination [Hite et al., (2005)] (but essentially same data)

$$\begin{aligned} \Sigma &= 81 \pm 6 MeV \\ \sigma_{\pi N} &= 66 \pm 6 MeV \\ f_{udN} &= 0.070 \pm 7 MeV \\ f_{sN} &= 0.39 \pm 0.11 MeV \end{aligned}$$

Need a clearer situation

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Hadronic Uncertainties in the Elastic Scattering of Supersymmetric Dark Matter

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(Dated: February 7, 2008)

Abstract

We review the uncertainties in the spin-independent and -dependent elastic scattering cross sections of supersymmetric dark matter particles on protons and neutrons. We propagate the uncertainties in quark masses and hadronic matrix elements that are related to the π-nucleon σ term and the spin content of the nucleon. By far the largest single uncertainty is that in spin-independent scattering induced by our ignorance of the $\langle N|\bar{q}q|N\rangle$ matrix elements linked to the π-nucleon σ term, which affects the ratio of cross sections on proton and neutron targets as well as their absolute values. This uncertainty is already impacting the interpretations of experimental searches for cold dark matter. *We plead for an experimental campaign to determine better the π-nucleon σ term.* Uncertainties in the spin content of the proton affect significantly, but less strongly, the calculation of rates used in indirect searches.

Need a clearer situation

m_u/m_d	0.553	$\pm$ 0.043	[4]
m_d	5	$\pm$ 2 MeV	[5]
m_s/m_d	18.9	$\pm$ 0.8	[4]
m_c	1.25	$\pm$0.09 GeV	[5]
m_b	4.20	$\pm$0.07 GeV	[5]
m_t	171.4	$\pm$ 2.1 GeV	[6]
σ_0	36	$\pm$ 7 MeV	[7]
$\Sigma_{\pi N}$	64	$\pm$ 8 MeV	[8, 9]
$a_3^{(p)}$	1.2695	$\pm$ 0.0029	[5]
$a_8^{(p)}$	0.585	$\pm$ 0.025	[10, 11]
$\Delta_s^{(p)}$	-0.09	$\pm$ 0.03	[12]

TABLE I: Hadronic parameters used to determine neutralino-nucleon scattering cross-sections, with estimates of their experimental uncertainties.

Higgs v.e.v.s $\tan\beta$ [14]. We illustrate our observations by studies of some specific CMSSM benchmark scenarios [15], and also by surveys along strips in the $(m_{1/2}, m_0)$ plane for $\tan\beta = 10, 50$ along which $\tilde{\tau}-\chi$ coannihilation maintains the relic neutralino density within the range favoured by WMAP and other experiments [16].

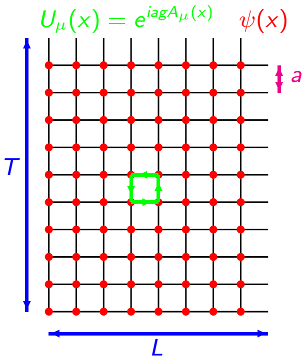
We find that the spin-independent cross section may vary by almost an order of magnitude for 48 MeV $< \Sigma_{\pi N} <$ 80 MeV, the ± 2-σ range according to the uncertainties in Table I. This uncertainty is already impacting the interpretations of experimental searches for cold dark matter. Propagating the ± 2-σ uncertainties in $\Delta_s^{(p)}$, the next most important parameter, we find a variation by a factor ~ 2 in the spin-dependent cross section. Since the spin-independent cross section may now be on the verge of detectability in certain models, and the uncertainty in the cross section is far greater, *we appeal for a greater, dedicated effort to reduce the experimental uncertainty in the π-nucleon σ term $\Sigma_{\pi N}$*. This quantity is not just an object of curiosity for those interested in the structure of the nucleon and non-perturbative strong-interaction effects: it may also be key to understanding new physics beyond the Standard Model.

II. SUPERSYMMETRIC FRAMEWORK

We briefly review in this Section the theoretical framework we use in the context of the MSSM; for more comprehensive reviews, see, *e.g.*, [17, 18]. The neutralino LSP is the lowest-mass eigenstate combination of the Bino $\tilde{B}$, Wino $\tilde{W}$ and Higgsinos $\tilde{H}_{1,2}$, whose mass matrix

Lattice QCD in one slide

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



$$\langle O \rangle = \int \mathcal{D}[U]\mathcal{D}\overline{\psi}\mathcal{D}\psi\, O(U,\overline{\psi},\psi)e^{-S_G[U]-S_F[U,\overline{\psi},\psi]}$$
$$= \int \mathcal{D}[U]\, O(U)_{\rm Wick}\, e^{-S_G[U]}\det(D)$$

- Compute the integral numerically $\to$ Monte Carlo sampling of $e^{-S_G[U]}\det(D) \geq 0$.
- Observable computed averaging over samples

$$\langle O \rangle = \frac{1}{N_{\rm conf}}\sum_{i=1}^{N_{\rm conf}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\rm conf}})$$

NOT A MODEL: Lattice QCD IS real world QCD ($a \to 0$, $L \to \infty$, ...)

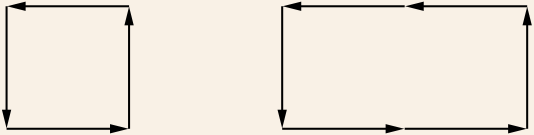
Action details

Gauge action

Tree level improved Lüscher-Weisz action [Lüscher et al (1985)]

$$S_g = \beta \left\{ \frac{c_0}{3} \sum_{\Box} \mathrm{ReTr}(1 - U_{\Box}) + \frac{c_1}{3} \sum_{\mathrm{Rec.}} \mathrm{ReTr}(1 - U_{\mathrm{Rec}}) \right\}$$

with $c_0 = 5/3$ and $c_1 = -1/12$



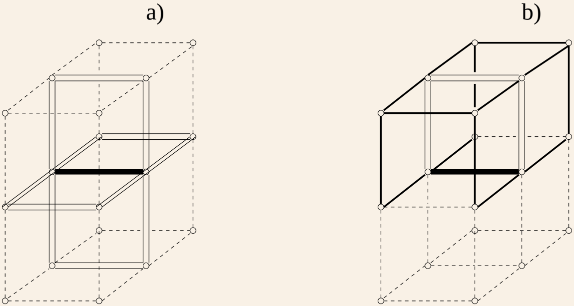
Action details

Fermion action

Tree level $\mathcal{O}(a)$ improved Wilson fermions [Sheikholeslami et al (1985)]

$$S_f = S_W[U^{(2)}] - \frac{c_{SW}}{2} \sum_x \sum_{\mu<\nu} (\overline{\psi}\sigma_{\mu\nu} F_{\mu\nu}[U^{(2)}]\psi)(x)$$

Coupled to smeared links $U^{(2)}(x)$



Action details

Fermion action

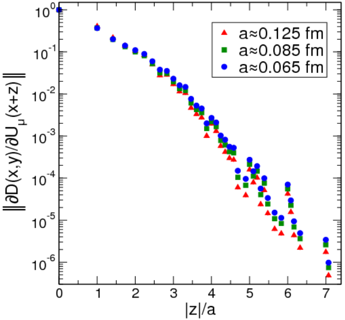
Combination of HYP setup and EXP recipe [Capitani, Dürr, Hoelbling (2006)].

$$
\begin{aligned}
\Gamma^{(1)}_{\mu;\nu\rho} &= \sum_{\pm\sigma\neq(\mu,\nu\rho)} U_\sigma(x) U_\mu(x+\sigma) U^+_\sigma(x+\mu) \\
V^{(1)}_{\mu;\nu\rho} &= \exp\left\{ \frac{\alpha_3}{2} \mathcal{P}\left(\Gamma^{(1)}_{\mu;\nu\rho} U^+_\mu(x) \right) \right\} U_\mu(x) \\
\Gamma^{(2)}_{\mu;\nu} &= \sum_{\pm\sigma\neq(\mu,\nu)} V^{(1)}_{\sigma;\mu\nu}(x) V^{(1)}_{\mu;\nu\sigma}(x+\sigma) V^{(1)+}_{\sigma;\mu\nu}(x+\mu) \\
V^{(2)}_{\mu;\nu} &= \exp\left\{ \frac{\alpha_2}{4} \mathcal{P}\left(\Gamma^{(2)}_{\mu;\nu} U^+_\mu(x) \right) \right\} U_\mu(x) \\
\Gamma^{(3)}_{\mu} &= \sum_{\pm\sigma\neq(\mu)} V^{(2)}_{\sigma;\mu}(x) V^{(2)}_{\mu;\nu}(x+\sigma) V^{(2)+}_{\sigma;\mu}(x+\mu) \\
U^{(1)}_{\mu} &= \exp\left\{ \frac{\alpha_3}{6} \mathcal{P}\left(\Gamma^{(3)}_{\mu} U^+_\mu(x) \right) \right\} U_\mu(x)
\end{aligned}
$$

Smearing and locality [Dürr et al Science (2008)]

Our Dirac operator is ultralocal

- $\overline{\psi}(x)D(x,y)\psi(y)$, $D(x,y) \equiv 0$ for $|x-y| > a$.
- $D(x,y)$ depends on $U_\mu(x+z)$ for $|z| > a$, but



$$\left\| \frac{\partial D(x,y)}{\partial U_\mu(x+z)} \right\| \equiv 0 \quad \text{for} \quad |z| > 7.1a$$

and

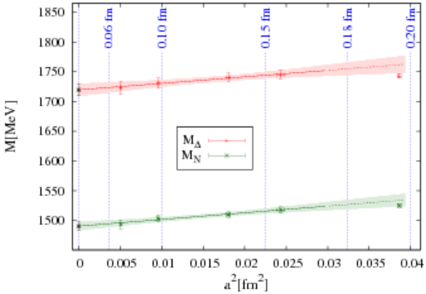
$$\left\| \frac{\partial D(x,y)}{\partial U_\mu(x+z)} \right\| \propto e^{-2.2|z|/a}$$

Not a problem

$2.2a^{-1}$ is much larger than any scale of interests (masses, ...)

Smearing and the continuum limit

Detailed scaling study: $N_f = 3$ with our preferred action(s). 5 lattice spacings, $M_\pi L \gtrsim 4$ and $m_q \sim m_s^{\text{phys}}$. [Dürr et al. Phys.Rev. D79 (2009)].



M_N and M_Δ linear in a^2 for $a \in [0.065, 0.16]$ fm.

Very good scaling properties

Looks non-perturbatively $\mathcal{O}(a)$ improved.

Properties of the action

- The action differs from the usual one by terms that are both ultralocal and irrelevant.
- Action in the same universality class as QCD (true for any number of smearing steps).
- Chiral symmetry breaking is reduced.
- One can reach smaller quark masses before one runs into the problem of "exceptional" configurations.
- Action with tree level C_{SW} is close to be non perturbatively $\mathcal{O}(a)$ improved.

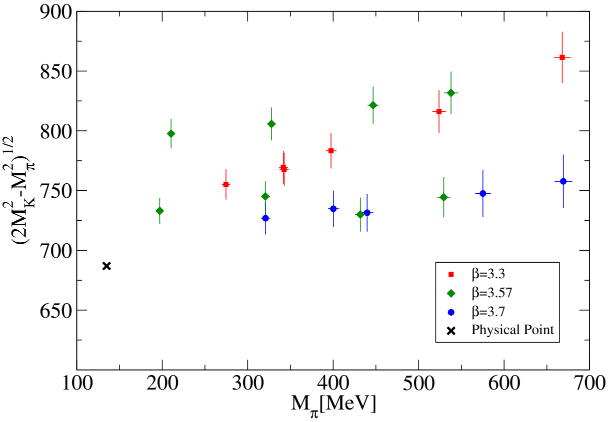
Nice properties for phenomenological studies.

Reaching the physical point

β	am_{ud}	am_s	$L^3 \times T$	traj.	aM_π	aM_K
3.3	-0.0960	-0.057	$16^3 \times 32$	10000	0.4115(6)	0.4749(6)
	-0.1100	-0.057	$16^3 \times 32$	1450	0.322(1)	0.422(1)
	-0.1200	-0.057	$16^3 \times 64$	4500	0.2448(9)	0.3826(6)
	-0.1233	-0.057	$24^3 \times 64$	2000	0.2105(8)	0.3668(6)
	-0.1233	-0.057	$32^3 \times 64$	1300	0.211(1)	0.3663(8)
	-0.1265	-0.057	$24^3 \times 64$	2100	0.169(1)	0.3500(7)
3.57	-0.0318	0,-0.010	$24^3 \times 64$	1650,1650	0.2214(7),0.2178(5)	0.2883(7),0.2657(5)
	-0.0380	0,-0.010	$24^3 \times 64$	1350,1550	0.1837(7),0.1778(7)	0.2720(6),0.2469(6)
	-0.0440	0,-0.007	$32^3 \times 64$	1000,1000	0.1348(7),0.1320(7)	0.2531(6),0.2362(7)
	-0.0483	0,-0.007	$48^3 \times 64$	500,1000	0.0865(8),0.0811(5)	0.2401(8),0.2210(5)
3.7	-0.007	0.0	$32^3 \times 96$	1100	0.2130(4)	0.2275(4)
	-0.013	0.0	$32^3 \times 96$	1450	0.1830(4)	0.2123(3)
	-0.020	0.0	$32^3 \times 96$	2050	0.1399(3)	0.1920(3)
	-0.022	0.0	$32^3 \times 96$	1350	0.1273(5)	0.1882(4)
	-0.025	0.0	$40^3 \times 96$	1450	0.1021(4)	0.1788(4)

Table: 20 ensembles. $a = 0.125, 0.085, 0.065$ fm. $M_\pi L \gtrsim 4$.

Reaching the physical point



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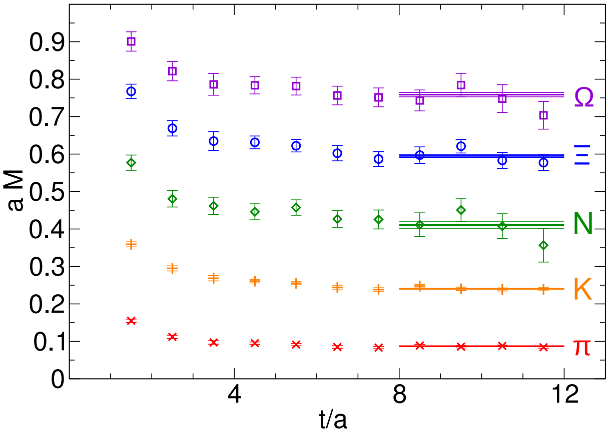

Figure: Effective masses for $\beta = 3.57$ and $M_\pi = 190$ MeV

Nucleon sigma terms

Definitions

$$\begin{aligned}
\sigma_{\pi N} &= \hat{m}\langle N(p) \mid (\overline{u}u + \overline{d}d)(0) \mid N(p)\rangle \\
\sigma_{\bar{s}sN} &= m_s\langle N(p) \mid (\bar{s}s)(0) \mid N(p)\rangle \\
y &= \frac{2\langle N \mid \bar{s}s \mid N\rangle}{\langle N \mid \overline{u}u + \overline{d}d \mid N\rangle}
\end{aligned}$$

Important for

- Hadron spectrum
- The quark mas ratio $m_s/\hat{m}$
- $\pi - N$ and $K - N$ scattering amplitudes
- Counting rates in searches of the Higgs boson

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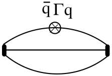
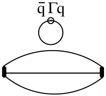
Nucleon sigma terms

Direct computation...



Difficult [R. Gupta talk.]



Very difficult [A. Vaquero talk.]

Feynman-Hellman th: The nucleon mass is given by

$$M_N = \langle N \mid T_{\mu\mu} \mid N \rangle = \sum_q m_q \langle N \mid \overline{q}q \mid N \rangle + \text{Gluonic contribution}$$

then

$$m_{ud}\frac{\partial M_N}{\partial m_{ud}} = m_{ud}\langle N \mid \overline{u}u + \overline{d}d \mid N \rangle = \sigma_{\pi N}$$

$$m_s\frac{\partial M_N}{\partial m_s} = m_s\langle N \mid \overline{s}s \mid N \rangle = \sigma_{\overline{s}sN}$$

- The sigma terms measures how much the nucleon mass changes when you change quark masses...
- ...and we routinely change m_q in lattice simulations!

Regular extrapolations

Any physical quantity is analytic in the quark masses if you do not expand around $m_q = 0$.

Expansion variables

$$
\begin{aligned}
(M_\pi^{\text{exp}})^2 &= \frac{1}{2}[(M_\pi^{\Phi})^2 + (M_\pi^{\text{max}})^2] \\
M_{\bar{s}s}^2 &= 2M_K^2 - M_\pi^2
\end{aligned}
$$

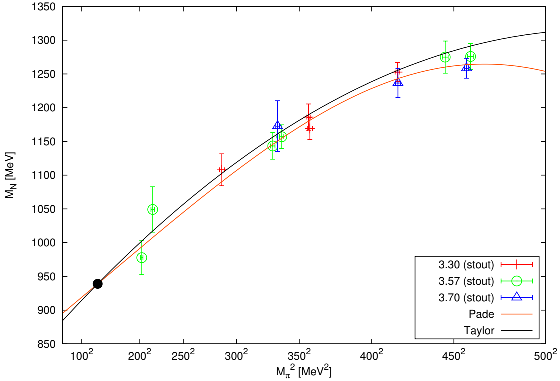
Nucleon mass dependence

$$
M_N = M_0 + \sum_{i=1}^{N_\pi} \alpha_i' \left[M_\pi^2 - (M_\pi^{\text{exp}})^2\right]^i + \sum_{i=1}^{N_{\bar{s}s}} \beta_i' \left[M_{\bar{s}s}^2 - \left(M_{\bar{s}s}^{\Phi}\right)^2\right]^j
$$

$$
M_N = \frac{M_0}{1 + \sum_{i=1}^{N_\pi} \alpha_i' \left[M_\pi^2 - (M_\pi^{\text{exp}})^2\right]^i + \sum_{i=1}^{N_{\bar{s}s}} \beta_i' \left[M_{\bar{s}s}^2 - (M_{\bar{s}s}^{\Phi})^2\right]^j}
$$

One example fit

Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.



$$\sigma_{\pi N} = 53(10)_{\text{stat}}\,\mathrm{MeV} \qquad \text{Taylor}$$
$$\sigma_{\pi N} = 44(6)_{\text{stat}}\,\mathrm{MeV} \qquad \text{Pade}$$

χPT

Nucleon mass as a function of quark masses

$$M_N = M_0 + \alpha M_\pi^2 + \text{Higher order terms}$$

Higer order terms

- HBχPT: $\propto g_A M_\pi^3$
- CBχPT: $\propto g_A h(M_\pi)$

$$h(M_\pi) = -\frac{M_\pi^3}{4\pi^2}\left\{\sqrt{1-\left(\frac{M_\pi}{2M_0}\right)^2}\arccos\frac{M_\pi}{2M_0} + \frac{M_\pi}{2M_0}\log\frac{M_\pi}{M_0}\right\}$$

Three flavour CBχPT [S. Dürr @ LAT10]

$$M_X = M_0 - 4c_X^\pi M_\pi^2 - 4c_X^s M_{\bar{s}s}^2 + \sum_{\alpha=\pi,K,\eta} \frac{g_X^\alpha}{F_\alpha^2} M_0^3 h\left(\frac{M_\alpha}{M_0}\right) + d^\pi M_\pi^4 + d^s M_{\bar{s}s}^4$$

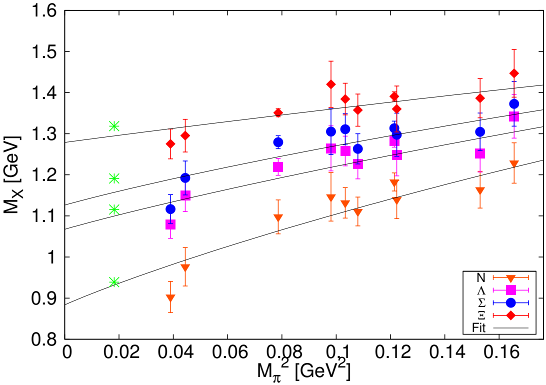
The good thing: All the constants $c_X^{\pi,s}, g_X^\alpha$ depend only on 5 independent parameters: b_0, b_D, b_F, g_A, ξ

How to fit lattice data

- First we set the scale (using Ω).
- Fit data in physical units using the formula above.

CBχPT fits

Figure: $M_\pi < 420$ MeV; χ^2/points $\approx 9/10$.



$$\sigma_{\pi N} = 47(9)_{\text{stat}} \qquad \text{CB}\chi\text{PT}$$

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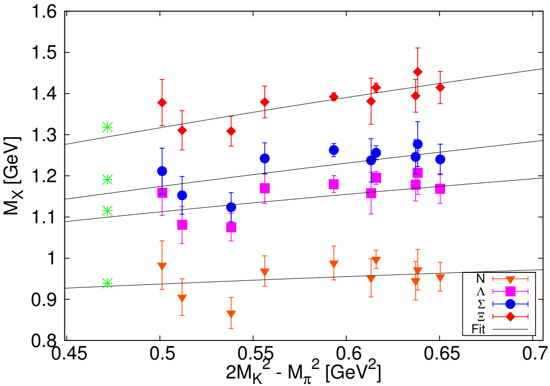

Complete analysis of the sigma term

Analysis

Simultaneously analyse all four octect members: N, Λ, Σ, Ξ.

Complete analysis of the sigma term

- Use a total of 8 different functional forms for the chiral extrapolation.
  - 4 Based of χPT. Fitting or not g_a, ξ, fitting or not h.o.t. $d_\pi, d_{\bar{s}s}$.
  - 2 General regular expansions. 1 Taylor, 1 Padé.
  - 2 $SU(3)$ constrained regular expansions. 1 Taylor, 1 Padé.
- Impose two pion mass cuts $M_\pi < 410 \text{ MeV}, 550 \text{ MeV}$
- Do a correlated fit for all four channels.
- Formally cutoff effects are $\mathcal{O}(\alpha_s a)$, but they are small in our data,and compatible with them being absent or $\mathcal{O}(a^2)$. $M_X \to M_X(1 + \eta a^p)$, with $p = 0, 1, 2$.
- Finite volume effects below the 1% in our data.
- Use 18 different fitting intervals for the correlators.

Idea of the analysis

Use all the 864 different method to obtain the physical quantity:

- Weight them by the fit quality and build a distribution.
- The median is our final result (typical result of our analysis).
- The 68% confidence interval gives the systematic error.

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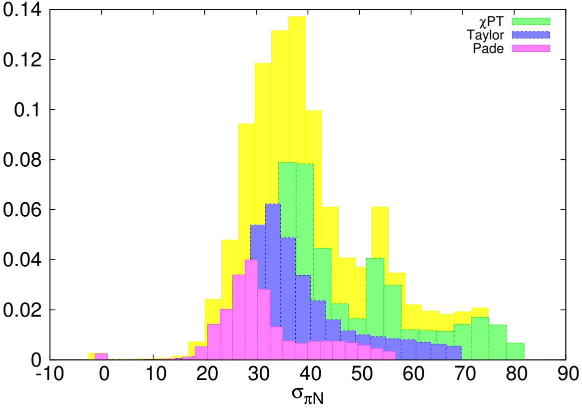


Figure: Final result $\sigma_{\pi N} = 39(4)_{\mathrm{stat}}(^{+18}_{-7})_{\mathrm{sys}}$

Complete analysis of the sigma term

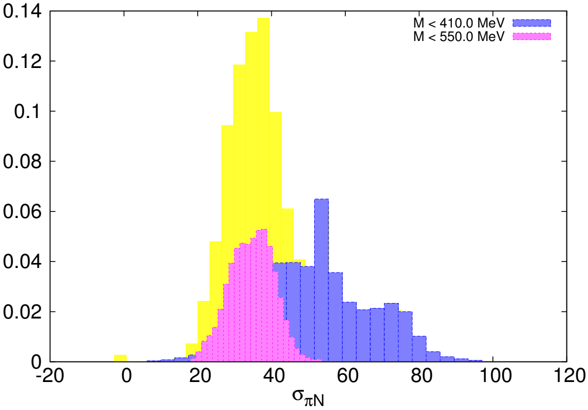


Figure: Final result $\sigma_{\pi N} = 39(4)_{\rm stat}(^{+18}_{-7})_{\rm sys}$

Complete analysis of the sigma term

Source of systematic error	error on $\sigma_{\pi N}$ [MeV]
Chiral Extrapolation:	
- Pion mass range	9.0
- Functional form	5.5
Continuum extrapolation	1.9

Table: Different sources of systematic error.

Complete analysis of the sigma term

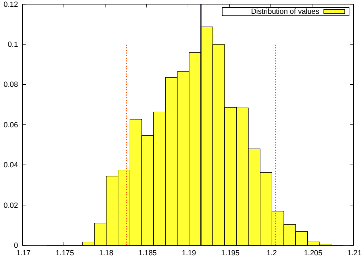
	$\sigma_{\pi X}$ [MeV]	$\sigma_{\bar{s}sX}$ [MeV]	y_X	f_{udX}	$f_{\bar{s}sX}$
N	$39(4)(^{+18}_{-7})$	$67(27)(^{+55}_{-47})$	$0.20(7)(^{+13}_{-17})$	$0.042(5)(^{+21}_{-4})$	$0.036(14)(^{+30}_{-25})$
Λ	$29(3)(^{+11}_{-5})$	$180(26)(^{+48}_{-77})$	$0.51(15)(^{+48}_{-27})$	$0.027(3)(^{+5}_{-10})$	$0.083(12)(^{+23}_{-31})$
Σ	$23(3)(^{+19}_{-3})$	$245(29)(^{+50}_{-72})$	$0.82(21)(^{+87}_{-39})$	$0.019(3)(^{+17}_{-3})$	$0.104(12)(^{+23}_{-31})$
Ξ	$16(2)(^{+8}_{-3})$	$312(32)(^{+72}_{-77})$	$1.7(5)(^{+1.9}_{-0.7})$	$0.0116(18)(^{+59}_{-22})$	$0.120(13)(^{+30}_{-30})$

Table: Final results. All quantities, all octet members.

2010 BMWc dataset

Main source of uncertainty

Chiral extrapolation being the main source of uncertainty is a general characteristic of lattice QCD computations.



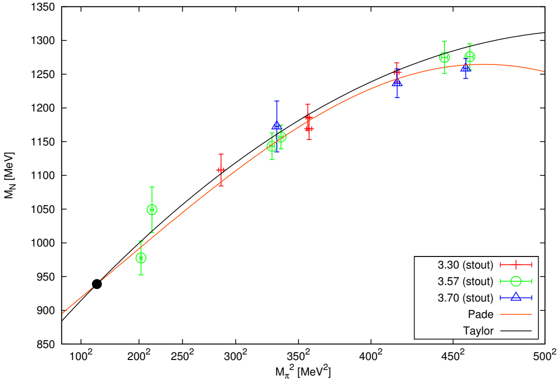
Source of systematic error	error on F_K/F_π
Chiral Extrapolation:	
- Functional form	3.3×10^{-3}
- Pion mass range	3.0×10^{-3}
Continuum extrapolation	3.3×10^{-3}
Excited states	1.9×10^{-3}
Scale setting	1.0×10^{-3}
Finite volume	6.2×10^{-4}

Result for F_K/F_π [Dürr et al. Phys. Rev. D81 (2010)]

Result for the ratio $F_K/F_\pi = 1.192(7)_{\rm st}(6)_{\rm sy}$

2010 BMWc dataset

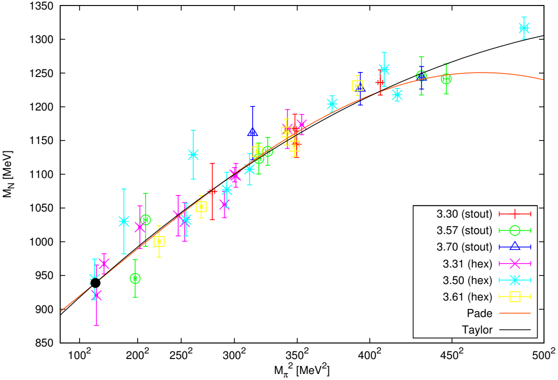
Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.



$$\sigma_{\pi N} = 53(10)_{\text{stat}}\,\mathrm{MeV} \qquad \text{Taylor}$$
$$\sigma_{\pi N} = 44(6)_{\text{stat}}\,\mathrm{MeV} \qquad \text{Pade}$$

2010 BMWc dataset

Figure: $M_\pi < 420$ MeV; $\chi^2/dof \approx 4.9/7$.



$$\sigma_{\pi N} = 42(5)_{\text{stat}}\text{MeV} \qquad \text{Taylor}$$
$$\sigma_{\pi N} = 46(3)_{\text{stat}}\text{MeV} \qquad \text{Pade}$$

2010 BMWc dataset

Impact DM searches

A precise determination (uncertainty $\lesssim 8$ MeV) can have a significant impact in DM searches.

Is this possible?

- 5 values of the lattice spacing: $a \approx 0.115$ fm, $a \approx 0.093$ fm, $a \approx 0.077$ fm, $a \approx 0.065$ fm, $a \approx 0.054$ fm.
- Reaching the physical point, and even below ($M_\pi = 120$ MeV).
- Big volumes (up to $L = 6$ fm). All ensembles $M_\pi L > 4$ fm.
- Good statistics. More than 47 ensembles. 35 ensembles with $M_\pi < 400$ MeV. 18 ensembles with $M_\pi < 300$ Mev. 6 ensembles with $M_\pi < 200$ MeV.

Proof of concept

Proof of concept

Use all available information

- Use M_N^{phys} to set scale.
- 5 values of β. 33 ensembles. Subset of BMW 2010 dataset.
- Only Taylor and Pade to interpolate both in m_{ud} and m_s.
- $M_\pi < 410$ MeV and $M_\pi < 350$ MeV
- Cutoff for $\sigma_{\pi N}$: Absent, $\mathcal{O}(a)$, $\mathcal{O}(a^2)$.
- 32 time intervals to estimate excited state contributions.
- Total 384 analysis.

$$
\begin{aligned}
(aM_N) &= (aM_N^\Phi)\left\{1 + \sum_{i=1}^{N_\pi} \alpha_i \left[\left(\frac{aM_\pi}{aM_N^\Phi}\right)^2 - \left(\frac{M_\pi^\Phi}{M_N^\Phi}\right)^2\right]^i + \right. \\
&+ \left. \sum_{j=1}^{N_K} \beta_j \left[\left(\frac{aM_{ks}}{aM_N^\Phi}\right)^2 - \left(\frac{M_{ks}^\Phi}{M_N^\Phi}\right)^2\right]^j \right\}
\end{aligned}
$$

Proof of concept

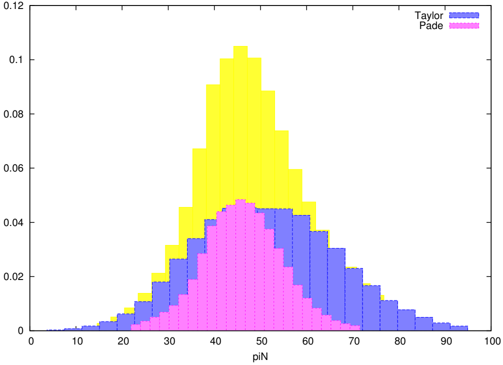
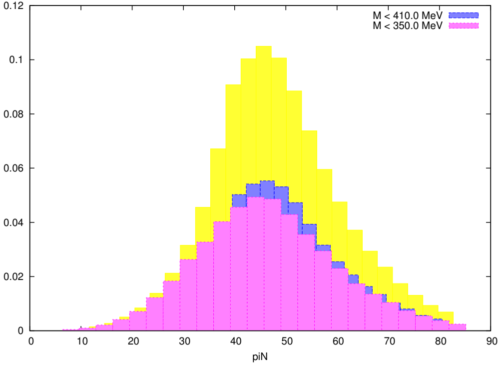


Figure: ONLY PROOF OF CONCEPT: $\sigma_{\pi N} = 50.1(6.1)_{\rm stat}(9.9)_{\rm sys}$

Not enough for strange content

$$y_N = 0.54(2.2)_{\rm stat}(0.71)_{\rm sys}$$

Proof of concept



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Not enough

$$y_N = 0.54(2.2)_{\rm stat}(0.71)_{\rm sys}$$

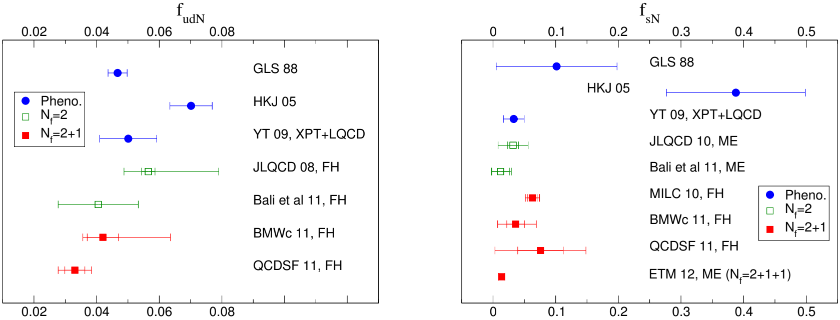
State of lattice computations

- Mixed approach: Feynman-Hellmann th. applied to nucleon correlator [Toussaint, Freeman (2009)]. $\sigma_{\pi N} = 57^{+7}_{-5} MeV$.
- "only" BχPT [Shanahan-Thomas-Young (2010)]. $\sigma_{\pi N} = 21 \pm 6 MeV$.
- Many direct computations....
  - QCDSF [1111.1600]. Wilson Fermions.
  - ETMC [Lat '12].
  - χQCD [Lat '12]. Overlap on DW.
  - ...
- And via Feynman-Hellmann
  - QCDSF. "fan plots".
  - RBC-UKQCD. m_s reweighting.

Not exhaustive list

Recent focus from the lattice community.

State of lattice computations

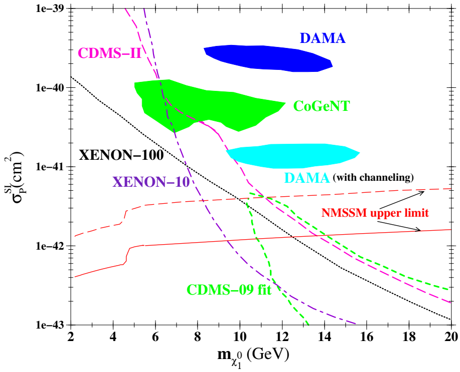


Personal opinion

A *precise* *and* *model independent* computation is still missing, but

- Lattice results seem to "prefer" low scenario.
- Mixed techniques might be necessary for a precise determination.

Impact in DM searches



	f_{udN}	f_{sN}
dashed red	0.078	0.63
solid red	0.059	0.29
Our result	$0.042(5)(^{+21}_{4})$	$0.036(14)(^{+30}_{25})$

Conclusions: A success for the lattice

- Still a model independent and precise determination of σ_N is interesting.
- Lattice have shown to be the superior tool to investigate σ_N
- Being at the physical point always desirable, but not enough!
- Probably a "mixed" direct/F-H-th strategy would be more successful (i.e. evaluating the sigma terms at some ensembles to constrain the fit).

With (lot, lot of) caution

Lattice computations have "ruled-out" (/made life difficult for) the high scenario.

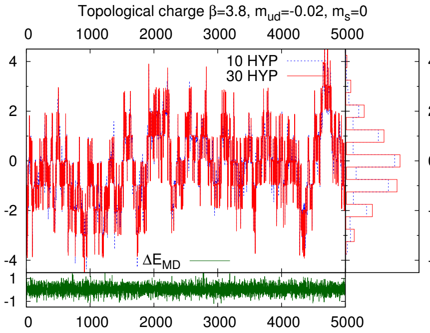
Topological charge sampling on finest lattice

Long autocorrelations in top. charge as $a \to 0$ have been observed [(Schaefer et al (2010))]

5000 trajectory autocorrelation-check run

Bad case: $a \simeq 0.054\ fm$ and $M_\pi \simeq 260\ MeV$ on $48^3 \times 64$ lattice

$$Q_{\rm naive} = \frac{a^4}{(4\pi)^2} \sum_x Tr[F^{\rm HYP}_{\mu\nu} \tilde{F}^{\rm HYP}_{\mu\nu}](x)$$



- Q fluctuates and evolves: $\tau_{\rm int} \sim 30$
- Q falls into integer centered bins

- Q distribution is reasonably symmetric

$\Rightarrow$ no obvious ergodicity problem