# Determining $V_{us}$ from semi leptonic Kaon decays

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ITN STRONGnet

# AGENDA

To tell a story of


& hunt for
Cabibbo
Kobayashi
Maskawa

And if its "Malarkey" …

# CKM Matrix

- CKM Matrix contains information on the strength of flavour-changing weak decays.

Search deviations from unitarity $\rightarrow$ search for physics beyond the SM.

The most sensitive test of the unitarity of the CKM matrix is provided by the relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$$

In the above unitarity test, $|V_{ud}|$ is precisely determined, $|V_{ub}|$ is usually neglected as it is very small , so that leaves us with $|V_{us}|$

# $V_{us}$

- Semi-leptonic decays involve changes to flavors of quarks, or mixing of quarks, and from these processes we can determine CKM matrix elements

$K \rightarrow \pi l \nu (Kl3)$ semi-leptonic decay process leads to the determination of $|V_{us}|$

In a Kl3 decay, the decay rate can be written as

$$\Gamma = \frac{G_F{}^2 M_K{}^2 C^2}{192\pi^2} \quad S_{ew}(1+\delta_{em}) \quad |V_{us}|^2 \quad \left|f_+(0)\right|^2$$

Known constants Known corrections formfactor

The decay rate can be precisely estimated from experiments of semi-leptonic $K \rightarrow \pi$ decays and hence the value of the product $\left|f_+(0)\right|^2 |V_{us}|^2$

The determination of $f_+(0)$ using Lattice QCD is important in estimating $V_{us}$ .

# $f_+(0)$

- $f_+(0)$ is defined from the $K \to \pi$ matrix element of the weak vector current ($V_\mu$) at zero momentum transfer

$$\langle \pi(p') | V_\mu | K(p) \rangle = \left(p_\mu + p'_\mu\right) f_+(q^2) + \left(p_\mu - p'_\mu\right) f_-(q^2)$$

where $q^2 = (p - p')^2$

current conservation implies that $f_+(0)$ =1 [SU(3) flavour limit $m_\pi{}^2 = m_K{}^2$]

**Ademollo-Gatto Theorem :** SU(3) breaking effects in

$$f_+(0) = 1 + f_2 + f_4 + O(p^6)\ , \quad f_n = O(m_\pi{}^n, K, \eta)$$

- $\Delta f = 1 + f_2 - f_+(0)$

standard result from Leutwyler & Roos (1984) $\Delta f$=-0.016(8)

- $\langle \pi(\boldsymbol{p}_f) | V_\mu | K(\boldsymbol{p}_i) \rangle$

Construct ratios of correlation functions , such that we can extract the matrix element $\langle \pi(\boldsymbol{p}_f) | V_\mu | K(\boldsymbol{p}_i) \rangle$

$$\langle \pi(\boldsymbol{p}_f) | V_\mu | K(\boldsymbol{p}_i) \rangle = \begin{cases} R_{1,p_i,p_f} = 4\sqrt{E_i E_f}\sqrt{\dfrac{C_{K\pi}(p_i,p_f)C_{\pi K}(p_f,p_i)}{C_K(p_i)C_\pi(p_f)}} \\ \\ R_{2,p_i,p_f} = 2\sqrt{E_i E_f}\sqrt{\dfrac{C_{K\pi}(p_i,p_f)C_{\pi K}(p_f,p_i)}{C_{KK}(p_i,p_i)C_{\pi\pi}(p_f,p_f)}} \end{cases}$$

Where $C_K(p_i)$ is the Kaon two point functions

$C_{K\pi}(p_i,p_f)$ is the Kaon to Pion three point function and similarly …

# So far …

$V_{us}$ CKM Matrix

⬇

$f_+(0)$ K→ $\pi$ formfactor at zero momentum transfer

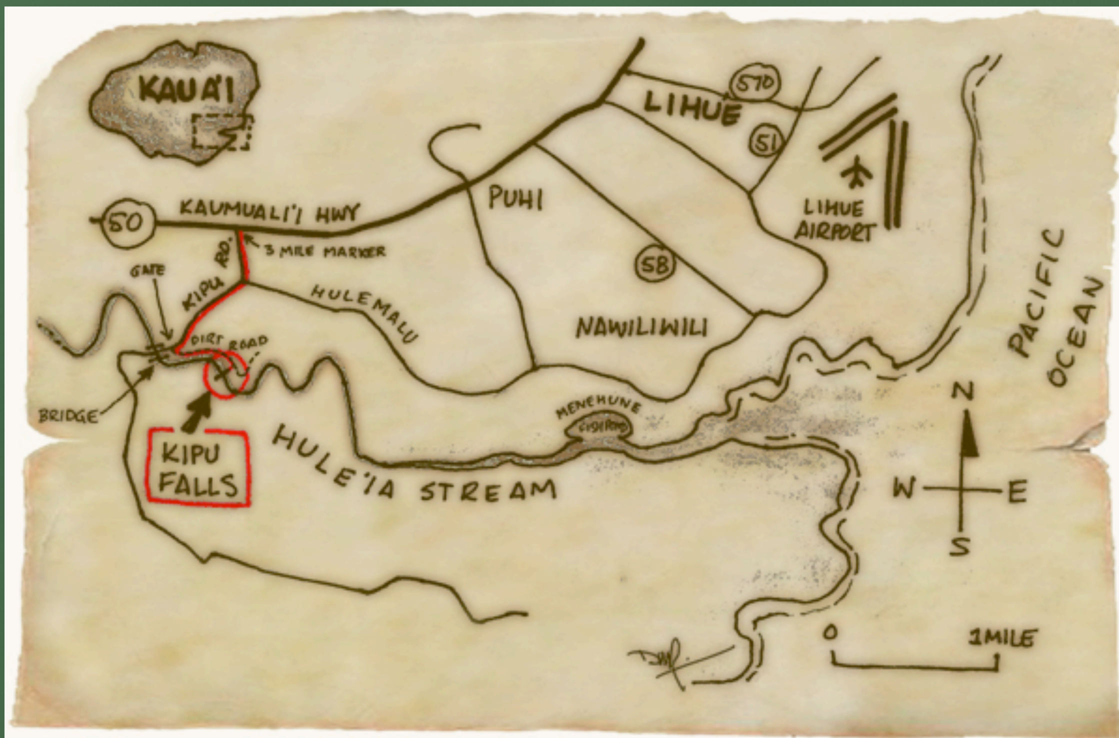
⬇

$\langle\pi(\boldsymbol{p}_f)|V_\mu|K(\boldsymbol{p}_i)\rangle$ $K \rightarrow \pi$ matrix element of the weak vector current ($V_\mu$)

⬇

$R_{1,K,\pi}$ , $R_{2,K,\pi}$ *Ratios of correlation functions*

# So far




We have a map!! That's it ???

# Existing Method

- Measure 2-pt and 3-pt correlation functions from Lattice QCD simulations

Periodic boundary condition results in hadronic momenta (p) **quantized** as $p_i = \frac{2\pi}{L} n_i$ where L is the cubic volume, $n_i$'s are integers.

Calculate $f_+(q^2)$ for different q by constructing ratios

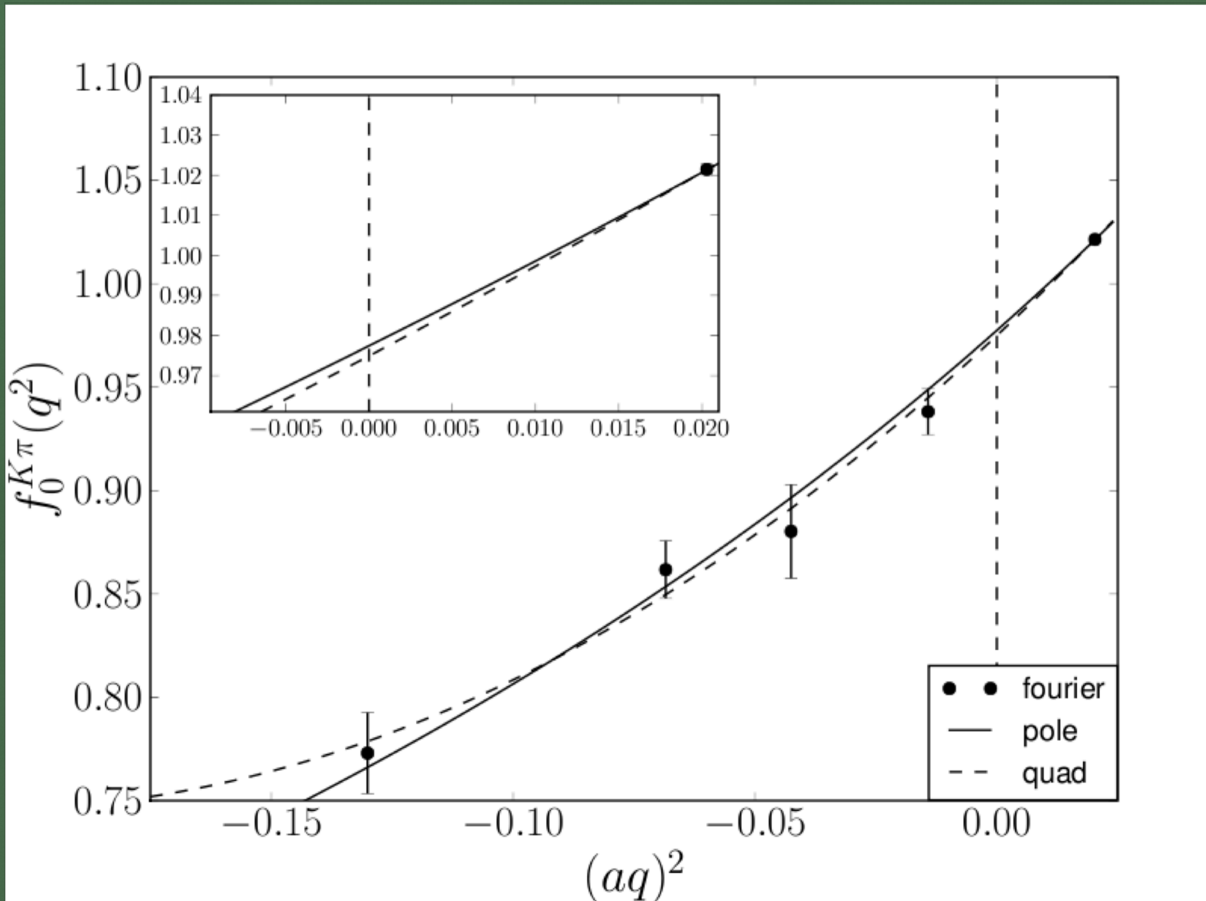
Find $f_+(0)$ by **interpolating** to $q^2 = 0$

Simulations performed at mass higher than the physical mass.

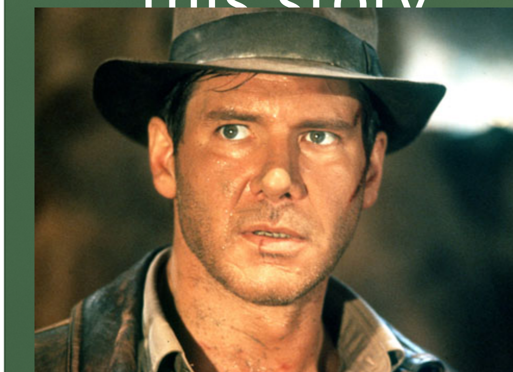
**Extrapolate** $f_+(0)$ result to that of actual physical mass of mesons. (chiral extrapolation)

Errors due to interpolation, extrapolation and finite volume effects.

# Interpolation to $q^2 = 0$




We may need some twist in this story



$$f_+(0)_{pole} = 0.9774(35)$$

$$f_+(0)_{quad} = 0.9749(59)$$

~ 0.2% Systematic error from difference in pole and quadratic fits

# Twisted boundary condition

Twisted boundary condition allows momenta smaller than $2\pi/L$ to be simulated

By partially twisting the boundary conditions of the quarks $(\theta_i,\theta_f)$, the momentum transfer $q^2$ can be modified as follows

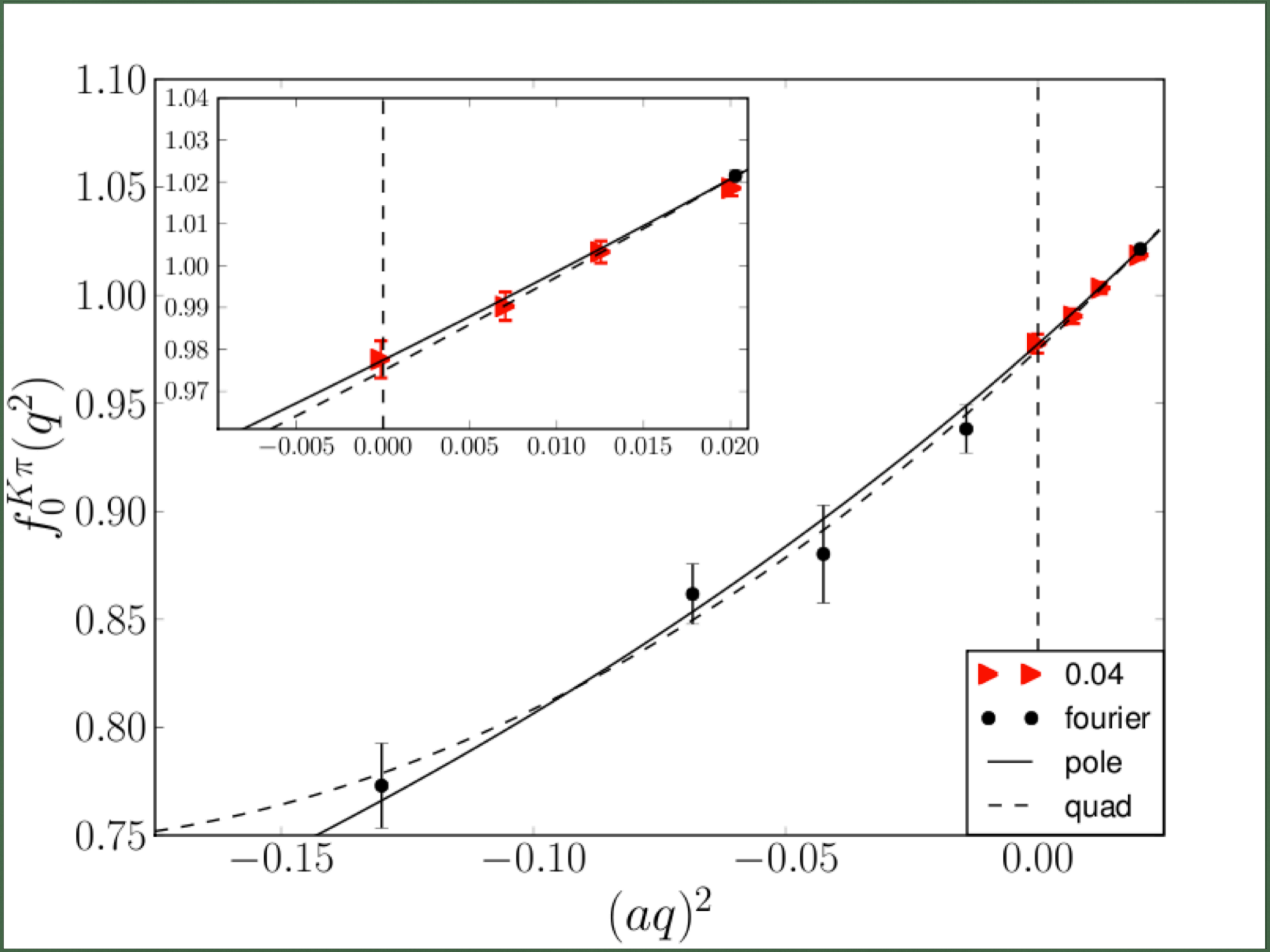
$$q^2 = \left\{\left[E_i(\vec{p}_i) - E_f(\vec{p}_f)\right]^2 - \left[\left(p_{\overrightarrow{FT},i} + \frac{\vec{\theta}_i}{L}\right) - \left(p_{\overrightarrow{FT},f} + \frac{\vec{\theta}_f}{L}\right)\right]\right\}$$

We can evaluate $K \to \pi$ formfactor by applying twist to Kaon or pion such that $q^2 = 0$. The twist angle can be evaluated as

$$|\vec{\theta}_K| = L\sqrt{\frac{({m_K}^2+{m_\pi}^2)}{2m_\pi} - {m_K}^2} \qquad |\vec{\theta}_\pi| = 0$$

$$|\vec{\theta}_\pi| = L\sqrt{\frac{({m_K}^2+{m_\pi}^2)}{2m_K} - {m_\pi}^2} \qquad |\vec{\theta}_K| = 0$$

# Twisted boundary results




$f_+(0) = 0.9757(44)$

[PA Boyle et al arXiv:1004.0886v1]

# Chiral Extrapolation

- Measurement performed on a variety of ensembles with different Lattice spacings, volumes, action and strange quark mass using the

I need serious speed to reach …

| Volume | $a^{-1}$ | Gauge action | $N^{ens}$ | $m_\pi$ (MeV) | # configs |
|---|---|---|---|---|---|
| $32^3 \times 64 \times 32$ | 1.35 | Iwasaki +DSDR | 2 | 170,250 | 196,162 |
| $32^3 \times 64 \times 16$ | 2.28 | Iwasaki | 3 | 300,355,405 | 136,153,12 |
| $24^3 \times 64 \times 16$ | 1.73 | Iwasaki | 4 | 330,420,560,670 | 90,90,45,45 |

With DWF action , we need some serious power to measure all that!!!

Our previous result - $|f_+(0)| = \mathbf{0.9599(34)_{stat}\left({}^{+31}_{-43}\right)_\chi(14)_a}$

dominated by uncertainty in chiral extrapolations

# Hardware

The measurement is don't using the following resources

STFC's DiRAC facility at

Swan

JUGENE at "Jülich Supercomputing Centre" (JSC)

(70 racks BG/P)

DiRAC/BGQ facility at Edinburgh

# BlueGene/Q Overview

16 × PowerPC 64 bit compute cores

16 KB L1 data cache, 4KB L1p prefetch engine, 32 MB L2 cache , 16GB DDR3

4 threads per core, 64 threads per chip

Quad double precision short vector (SIMD) fpu

Hu

| FP/Memory/ | Bandwidth GB/s |
|---|---|
| GFlops | 204.8 |
| L1 | 820 |
| L2 | 563 |
| DDR | 42.7 |
| Torus | 40 |

Most processors in the world spend 95% of their time idle stalled on

# Software

BAGEL is a QCD specific library that generates architecture specific assembly code

-written by P.A.Boyle et al.

Programmer specifies the register usage , loop unrolling strategies and declaring the memory access/prefetch pattern.

For each processor , BAGEL constructs a pipeline for plan of usage.

Algorithms : CG, Multi-shift CG, Mixed precision CG

Actions : Wilson, Wilson twisted mass, DWF, 5d Overlap, Clover

# Data Analysis

$f_+(\mathbf{0})$ is evaluated directly at $q^2 = 0$, by constructing ratios $R_{\alpha,K\pi}$ and solving the following simultaneous equations.

$$R_{\alpha,K\pi}(\overrightarrow{\theta_K},\vec{0},V_t) = (E_K + m_\pi)\, f_+{}^{K\pi}(\mathbf{0}) + (E_K - m_\pi)\, f_-{}^{K\pi}(\mathbf{0})$$
$$R_{\alpha,K\pi}(\vec{0},\overrightarrow{\theta_\pi},V_t) = (m_K + E_\pi)\, f_+{}^{K\pi}(\mathbf{0}) + (m_K - E_\pi)\, f_-{}^{K\pi}(\mathbf{0})$$
$$R_{\alpha,K\pi}(\overrightarrow{\theta_K},\vec{0},V_i) = \quad \theta_{K,i} \qquad f_+{}^{K\pi}(\mathbf{0}) \;+\; \theta_{K,i} \qquad f_-{}^{K\pi}(\mathbf{0})$$
$$R_{\alpha,K\pi}(\vec{0},\overrightarrow{\theta_\pi},V_i) = \quad \theta_{\pi,i} \qquad f_+{}^{K\pi}(\mathbf{0}) \;+\; \theta_{\pi,i} \qquad f_-{}^{K\pi}(\mathbf{0})$$

where i=x,y,z.

We expect $f_+(\mathbf{0})$ determined from twisting pion or Kaon to match!

But for ensembles with $m_\pi = 250MeV$, they are inconsistent, and for $m_\pi = 170MeV$,
It becomes so noisy that we cannot the Kaon ratios.

# $f_+(0)$ vs twist

- The simultaneous equations needs to be studied for dependence of $f_+(0)$ and $f_-(0)$ on twist choices.

The slope of $f_+(0)$ to $f_-(0)$ is given

for $V_t$
$$\frac{\partial f_+(\mathbf{0})}{\partial f_-(\mathbf{0})}_{\vec{\theta}_K=0} = -\frac{m_K - E_\pi}{m_K + E_\pi} \quad ;$$

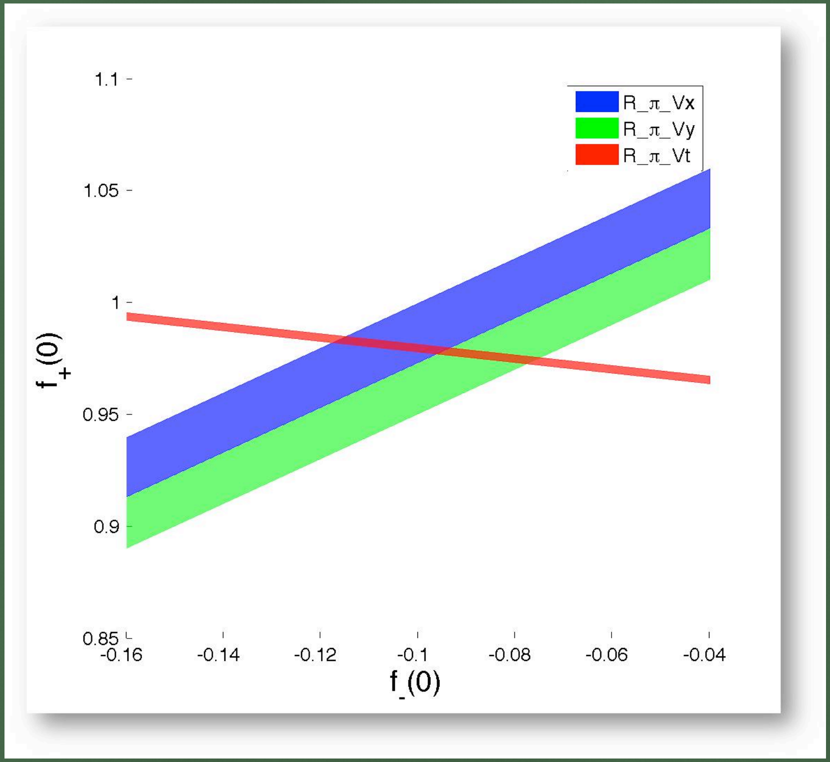
$$\frac{\partial f_+(\mathbf{0})}{\partial f_-(\mathbf{0})}_{\vec{\theta}_\pi=0} = -\frac{E_K - m_\pi}{E_K + m_\pi}$$

for $V_{x,y,z}$
$$\frac{\partial f_+(\mathbf{0})}{\partial f_-(\mathbf{0})}_{\vec{\theta}_K=0} = 1 \quad ;$$

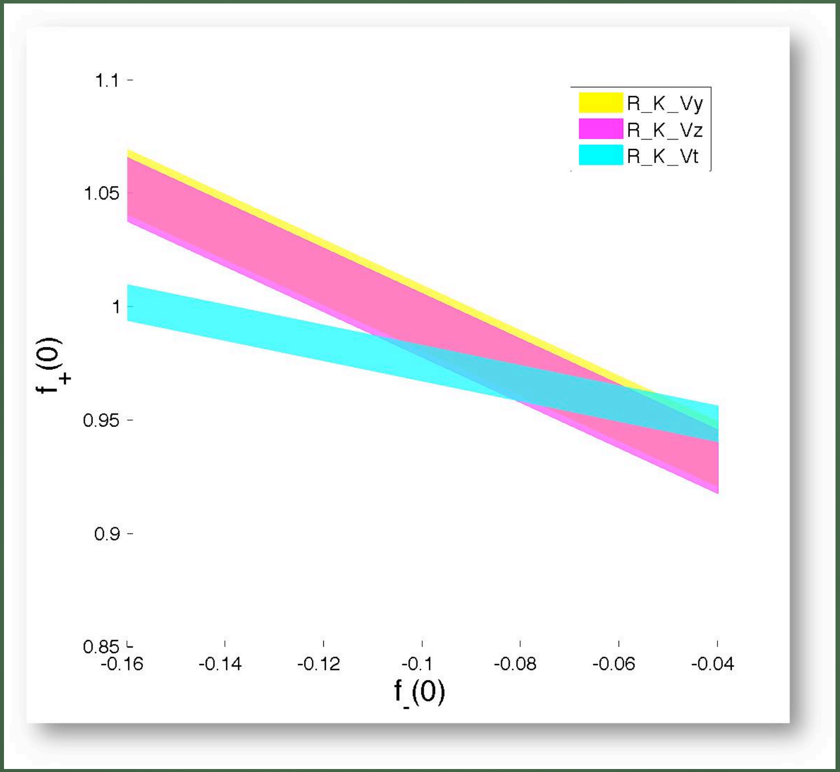
$$\frac{\partial f_+(\mathbf{0})}{\partial f_-(\mathbf{0})}_{\vec{\theta}_\pi=0} = -1$$

• $m_\pi = 250 MeV$

PION ONLY

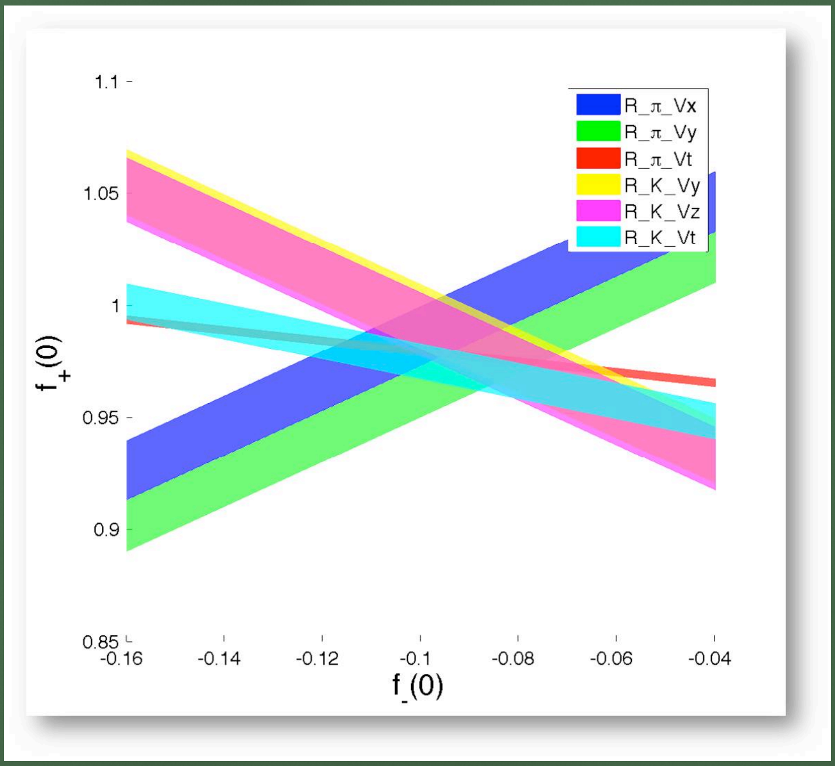



Kaon only

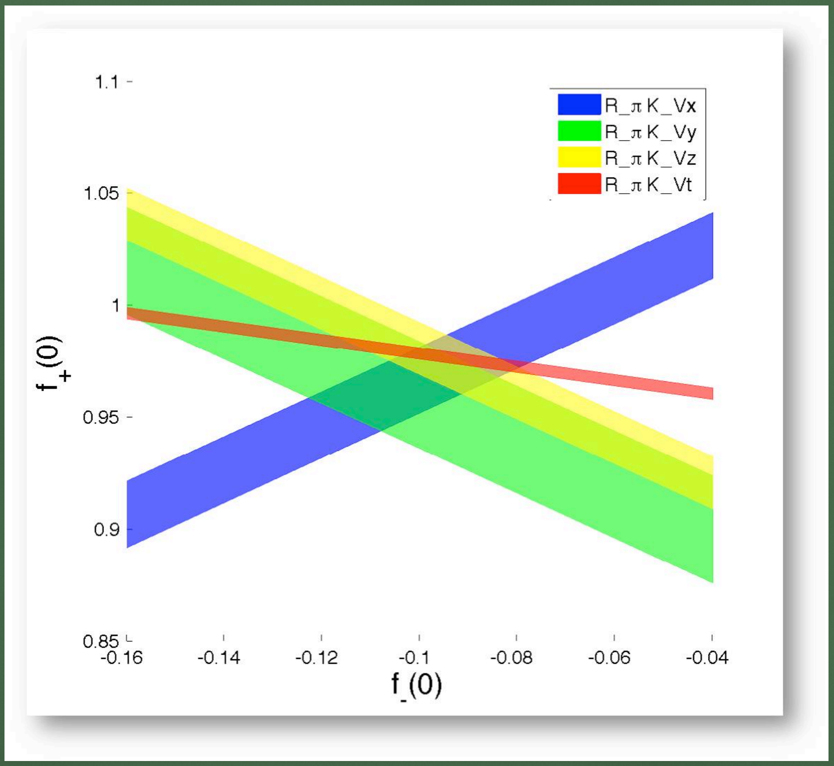


$$m_\pi = 250MeV$$

PION ONLY + KAON ONLY




Pion and Kaon



# Global fit

We can define scalar form factor $f_0(0)$

$$f_0(0) = f_+(0) + \frac{q^2}{m_K{}^2 - m_\pi{}^2}\ f_-(0)$$

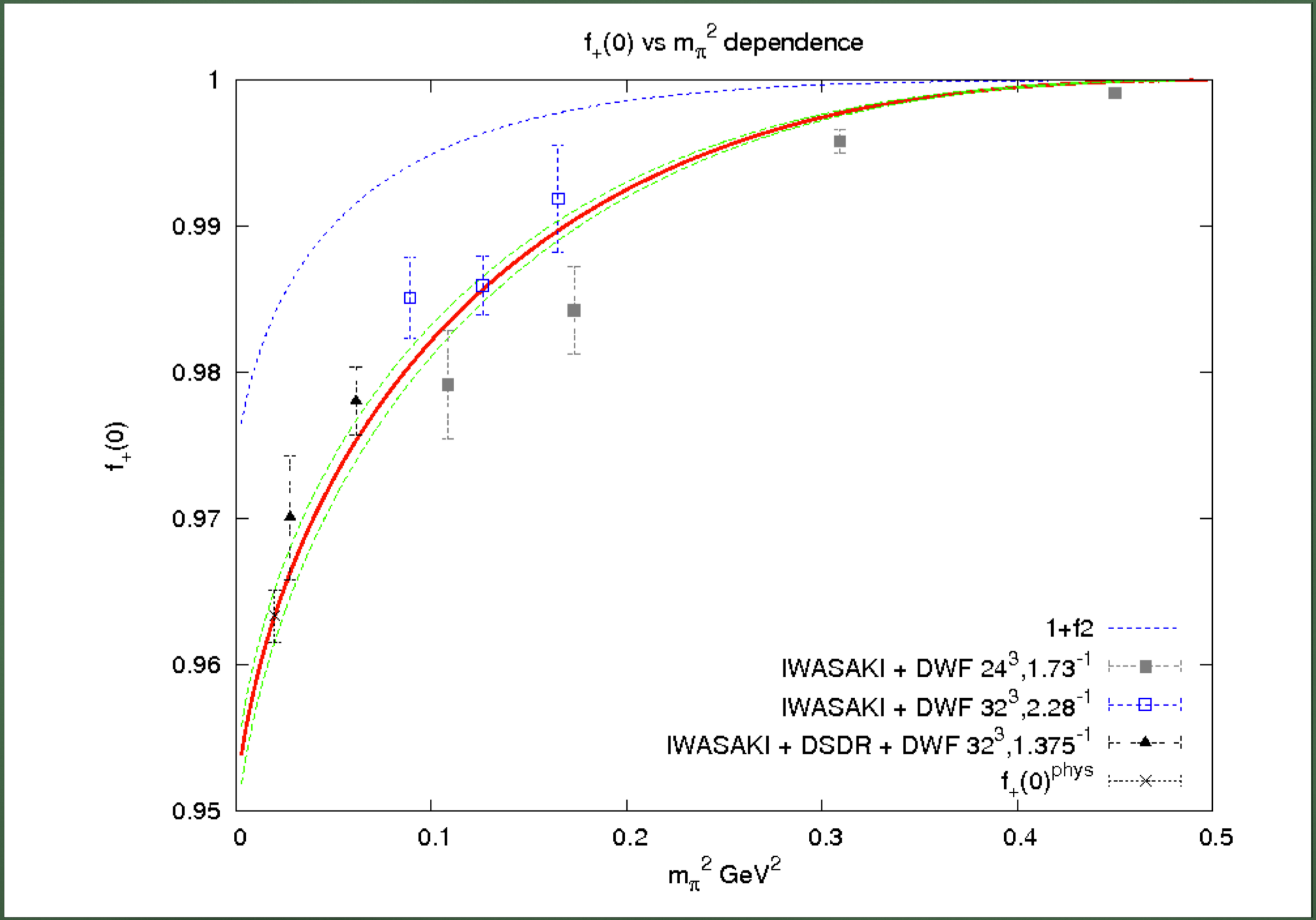
Simultaneous fit all data to $q^2$ , $m_\pi{}^2$, $m_K{}^2$ using the ansatz

$$f_0(q^2, m_\pi{}^2, m_K{}^2) = \frac{1 + f_2 + (m_K{}^2 - m_\pi{}^2)^2 \{A_0 + A_1(m_K{}^2 + m_\pi{}^2)\}}{1 - q^2/(M_0 + M_1(m_K{}^2 + m_\pi{}^2))^2}$$

With four fit parameters $A_0, A_1$ , $M_0, M_1$

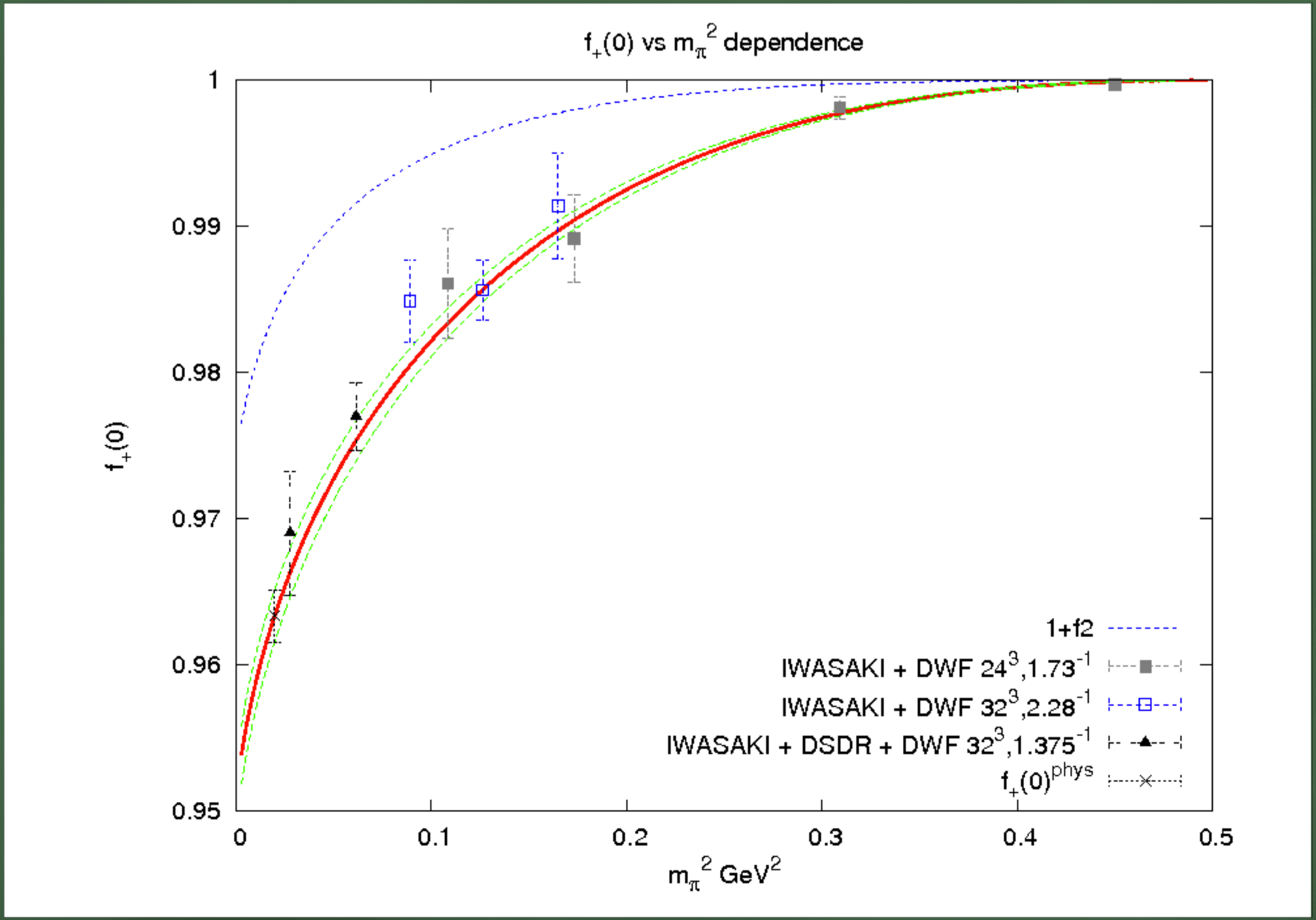
Expression motivated from the Ademollo-Gatto Theorem

# $f_+(0)$ vs $m_\pi{}^2$ dependence
-no strange quark mass correction

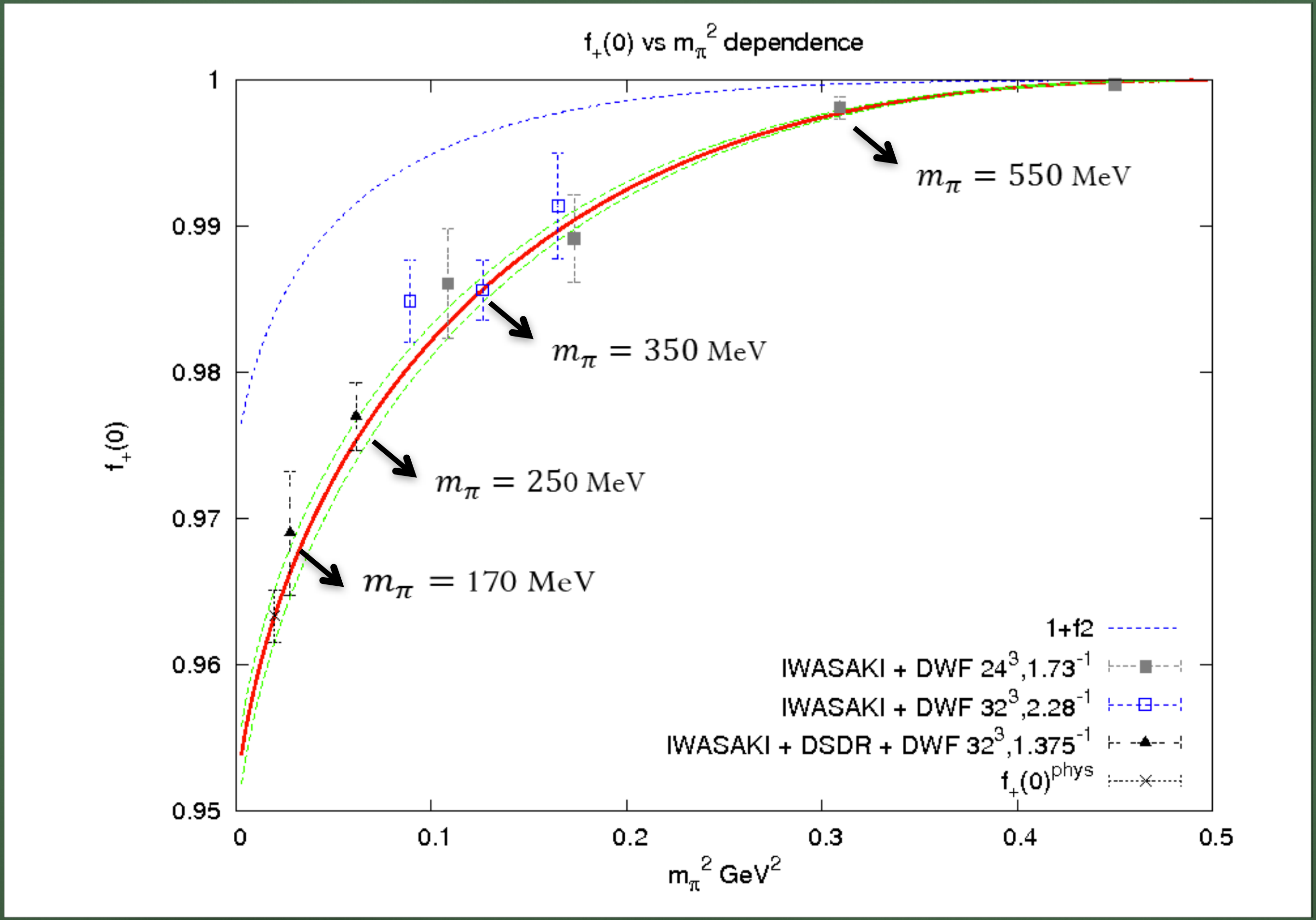


## $f_+(0)$ vs $m_\pi{}^2$ dependence with correction for strange quark mass

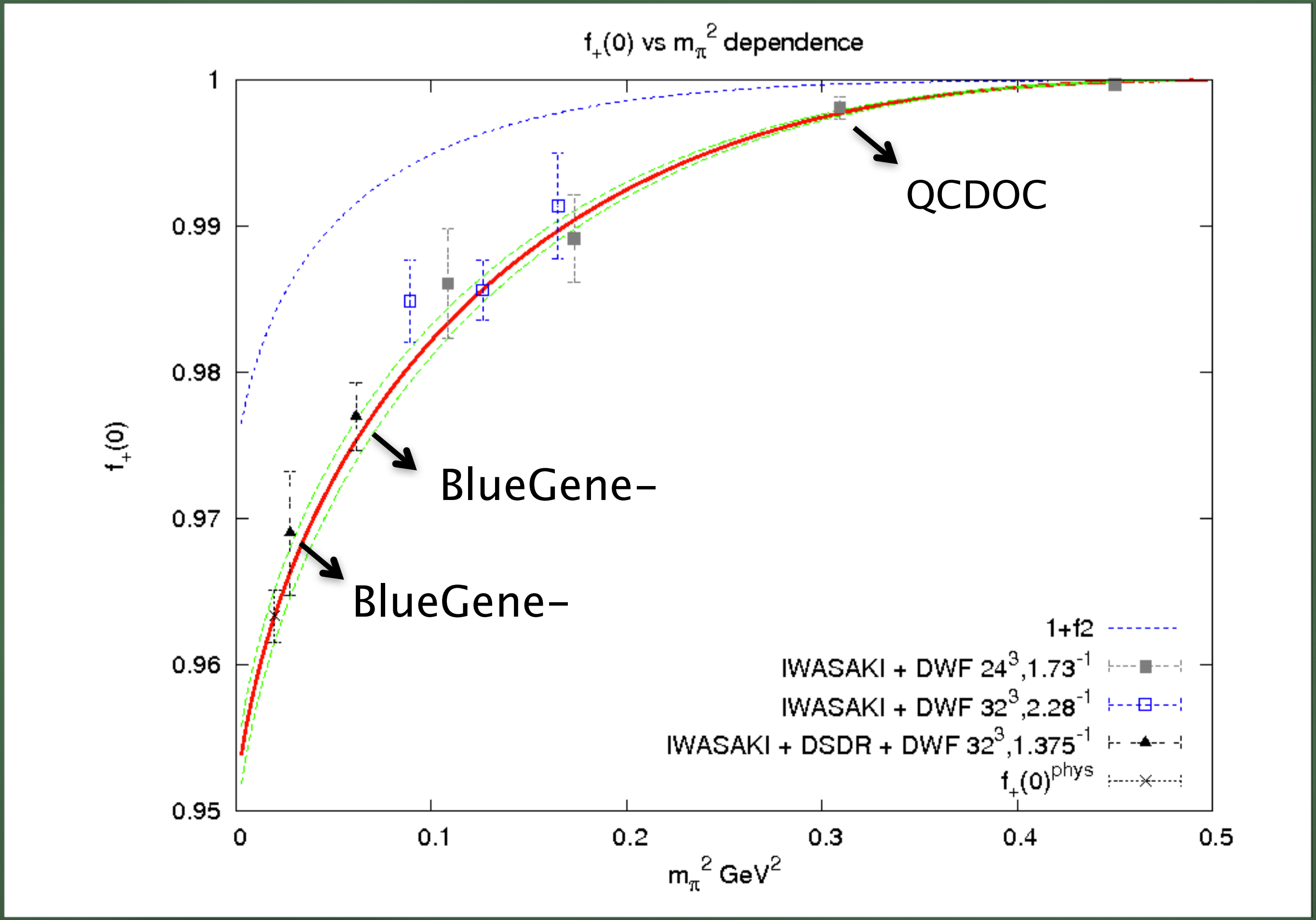
## $f_0(q^2, m_\pi{}^{latt}, m_K{}^{latt})$ - $f_0(q^2, m_\pi{}^{phys}, m_K{}^{phys})$



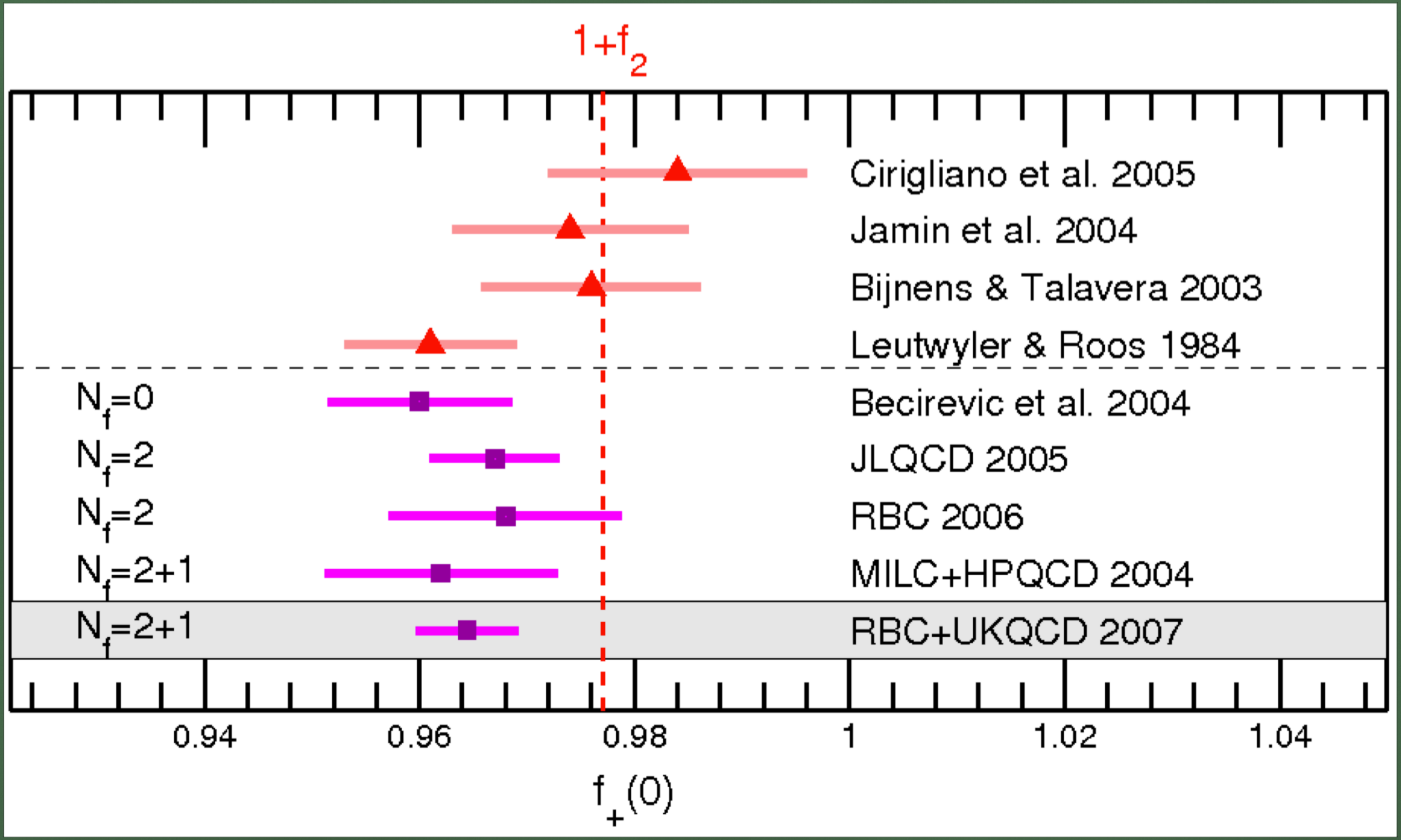
# $f_+(0)$ vs $m_\pi{}^2$ dependence with correction $f_0(q^2, m_\pi{}^{latt}, m_K{}^{latt})$ - $f_0(q^2, m_\pi{}^{phys}, m_K{}^{phys})$



# $f_+(0)$ vs $m_\pi{}^2$ dependence with correction
# $f_0(q^2, m_\pi{}^{latt}, m_K{}^{latt})$ - $f_0(q^2, m_\pi{}^{phys}, m_K{}^{phys})$



Comparison of Lattice results (blue squares) with various model estimates based on $\chi$PT (red triangles)




PA Boyle et al PRL 100,141601 (2008)

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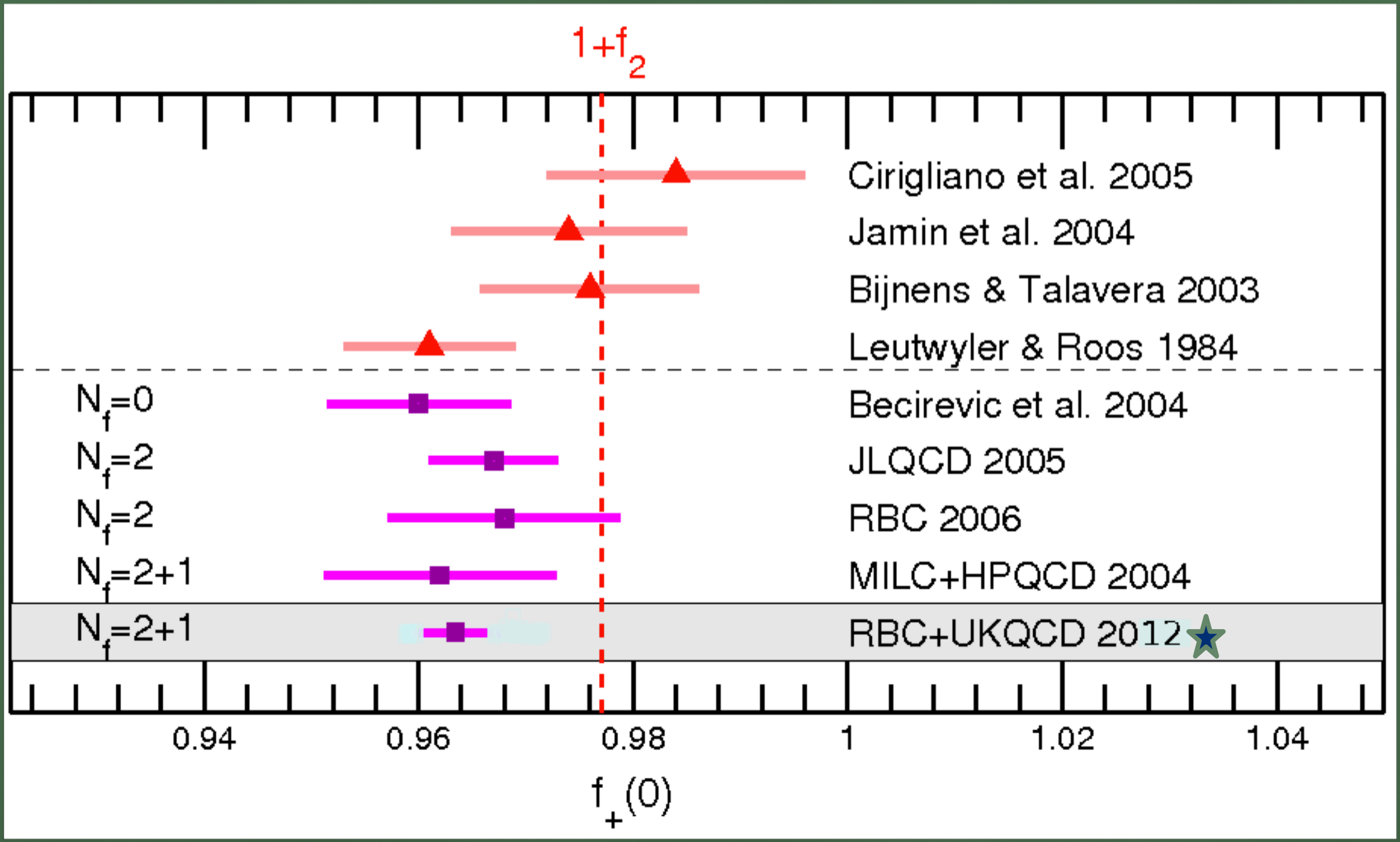


★RBC+UKQCD 2012 – Expected Preliminary result

# Conclusion

- ✓ We can determine CKM Matrix element $V_{us}$ precisely using kl3 semi-leptonic decays
- ✓ Measurement made on different lattice spacing and volume with $m_{\pi}$ as low as 170 MeV
- ✓ Twisting Kaon only to get $q^2$=0 is unreliable and twisting pion and Kaon together offers better solution.
- ✓ Chiral extrapolation error for $f_+^{K\pi}(0)$ can be almost be halved
- ✓ Future simulations at physical pion mass.


धन्यवाद
Obrigado!
Gracias
תודה
THANK YOU
Ευχαριστώ
Merci
Vielen Dank
شكراً