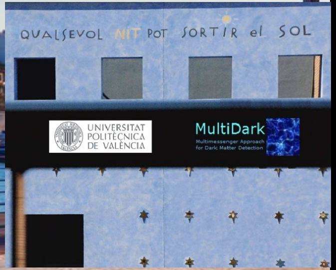
A Bayesian analysis with Machine Learning of EFT Operators in Direct Dark Matter Detection

Instituto de Física Teórica UAM-CSIC

Universidad Autónoma de Madrid



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In collaboration with David Cerdeño and Martín de los Rios

20th MultiDark Workshop
25-27 October 2023
Gandía, Spain

Outline

DM-nucleon interaction with NR-EFT

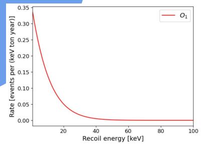
Outline

XENONnT 20ty

DM-nucleon interaction with NR-EFT

→

DM differential rate for a DD experiment



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DM-nucleon interaction with NR-EFT

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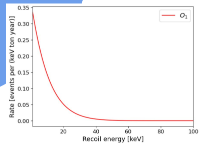
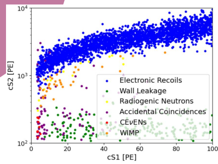


→

Simulate the signal reported by the experiment

XENONnT 20ty

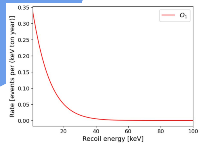


Outline

XENONnT 20ty

DM-nucleon interaction with NR-EFT

DM differential rate for a DD experiment



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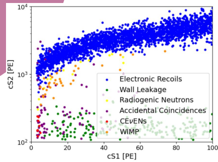
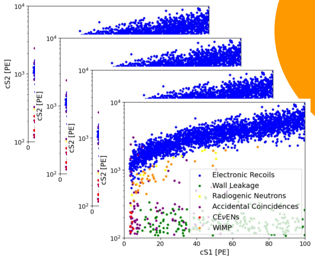
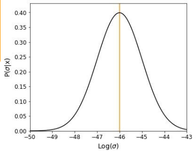
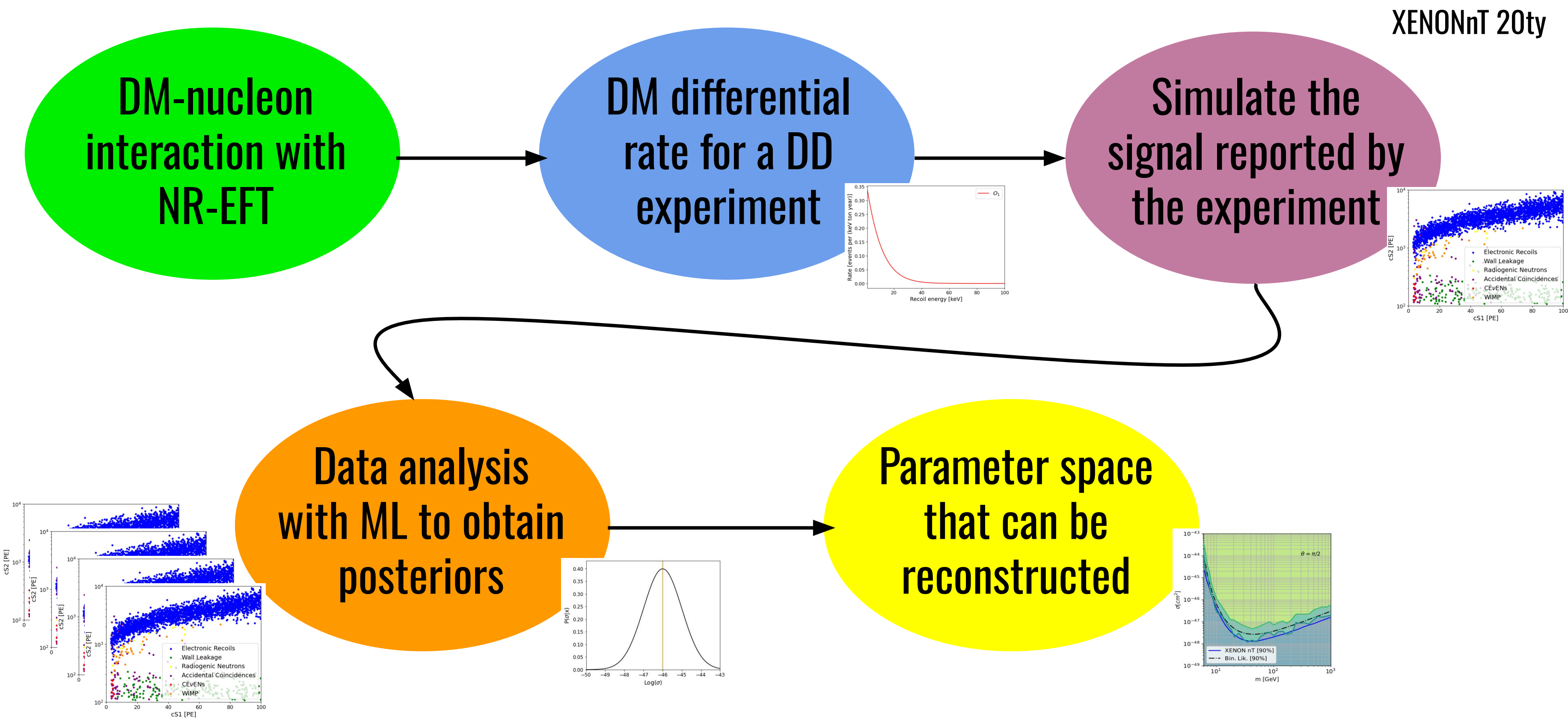


Data analysis with ML to obtain posteriors





Outline


XENONnT 20ty
DM-nucleon interaction with NR-EFT
DM differential rate for a DD experiment
Simulate the signal reported by the experiment
Data analysis with ML to obtain posteriors
Parameter space that can be reconstructed
Recoil energy [keV]
Rate [events per (keV ton year)]
cS1 [PE]
cS2 [PE]
Electronic Recoils
Wall Leakage
Radiogenic Neutrons
Accidental Coincidences
CEνENs
WIMP
Log(σ)
P(σ|x)
m [GeV]
XENON nT [90%]
Bin. Lik. [90%]

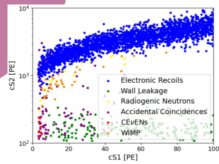
Outline

XENONnT 20ty

DM-nucleon interaction with NR-EFT

DM diff

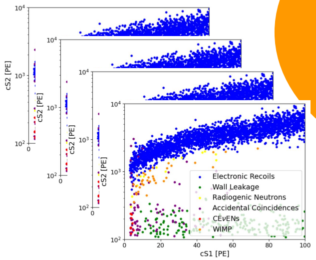
Simulate the signal reported by the experiment

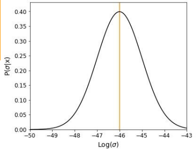


preliminary results

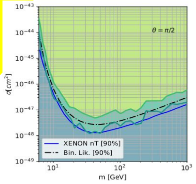
Data ana

with ML to obtain posteriors





Parameter space that can be reconstructed



Non-relativistic effective field theory (NR-EFT)

DM-nucleon non-relativistic effective field theory (NR-EFT)

For DM particles with spin up to ½ , the effective DM-nucleon scattering interaction Lagrangian

$$\mathcal{L}_{\mathrm{EFT}} = \sum_{\tau} \sum_{i} c_i^{\tau} \mathcal{O}_i \bar{\chi}\chi\bar{\tau}\tau$$

i=14 possible interactions

$$c_i^0 \mathbb{1}_{2\times 2} + c_i^1 \tau_3$$

isospin basis
c^0: isoscalar
c^1: isovector

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n)$$

$$c_i^1 = \frac{1}{2}(c_i^p - c_i^n)$$

nucleon basis
c^p: proton
c^n: neutron

O1: spin-independent (SI)
O4: spin-dependent (SD)

usually shown assuming isoscalar interactions

$c^p = c^n$ $\quad$ $c^0 = 1$ and $c^1 = 0$

DM-nucleon non-relativistic effective field theory (NR-EFT)

For DM particles with spin up to ½ , the effective DM-nucleon scattering interaction Lagrangian

$$\mathcal{L}_{\text{EFT}} = \sum_{\tau} \sum_{i} c_i^{\tau} \mathcal{O}_i \bar{\chi} \chi \bar{\tau} \tau$$

i=14 possible interactions

Change to polar coordinates:

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n) \; = A_i \sin(\theta_i)$$

$$c_i^1 = \frac{1}{2}(c_i^p - c_i^n) \; = A_i \cos(\theta_i)$$

Natural choice for the EFT parameter space because the interaction cross section:

$$\sigma_i \propto A_i^2$$

For SI (O1)

$$\sigma_{\chi N}^{\text{SI}} = \frac{A_1^2 \; \mu_{\chi N}^2}{\pi}$$

DM-nucleon reduced mass

DM-nucleon non-relativistic effective field theory (NR-EFT)

For DM particles with spin up to ½ , the effective DM-nucleon scattering interaction Lagrangian

$$\mathcal{L}_{\mathrm{EFT}} = \sum_{\tau} \sum_{i} c_i^{\tau} \mathcal{O}_i \bar{\chi} \chi \bar{\tau} \tau$$

i=14 possible interactions

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n) = A_i \sin(\theta_i)$$

$$c_i^1 = \frac{1}{2}(c_i^p - c_i^n) = A_i \cos(\theta_i)$$

For each operator

2 parameters

amplitude (cross-section)

phase

+ DM mass

(σ_i, θ_i, m_{DM})

Data sample generation

DM Differential rate

From NR-EFT operators to differential rate with WimPyDD

Inputs:

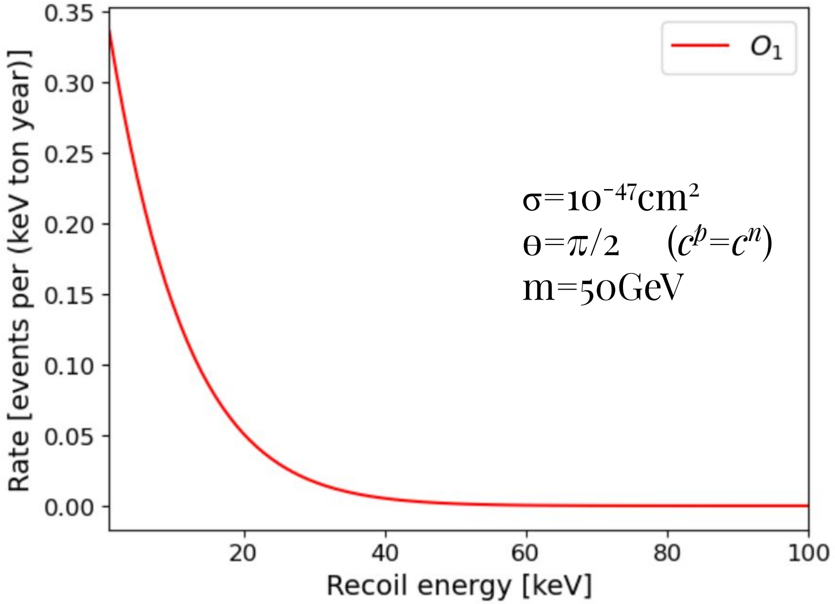
- Operator

- Parameters ⟶ amplitude (cross-section)
  phase
  DM mass

- DM halo model

- DD experiment (XENONnT)

Output:



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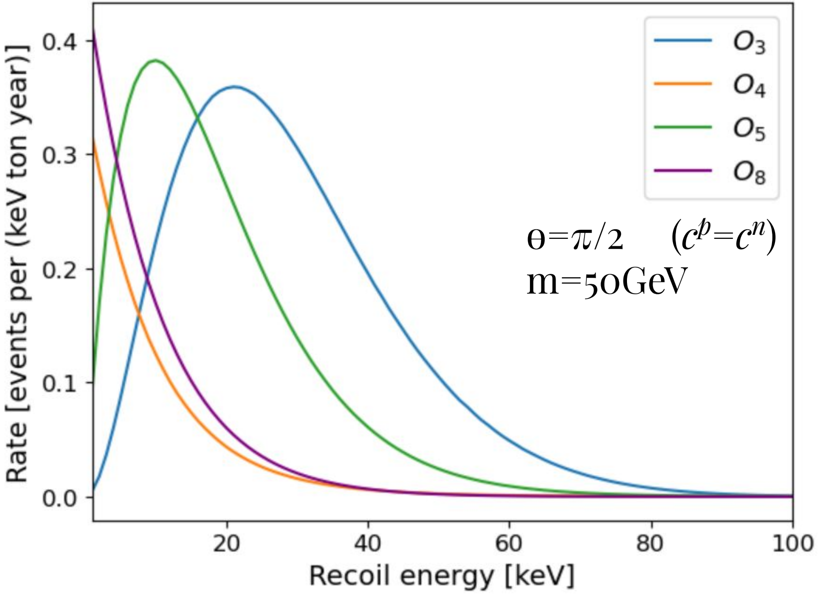
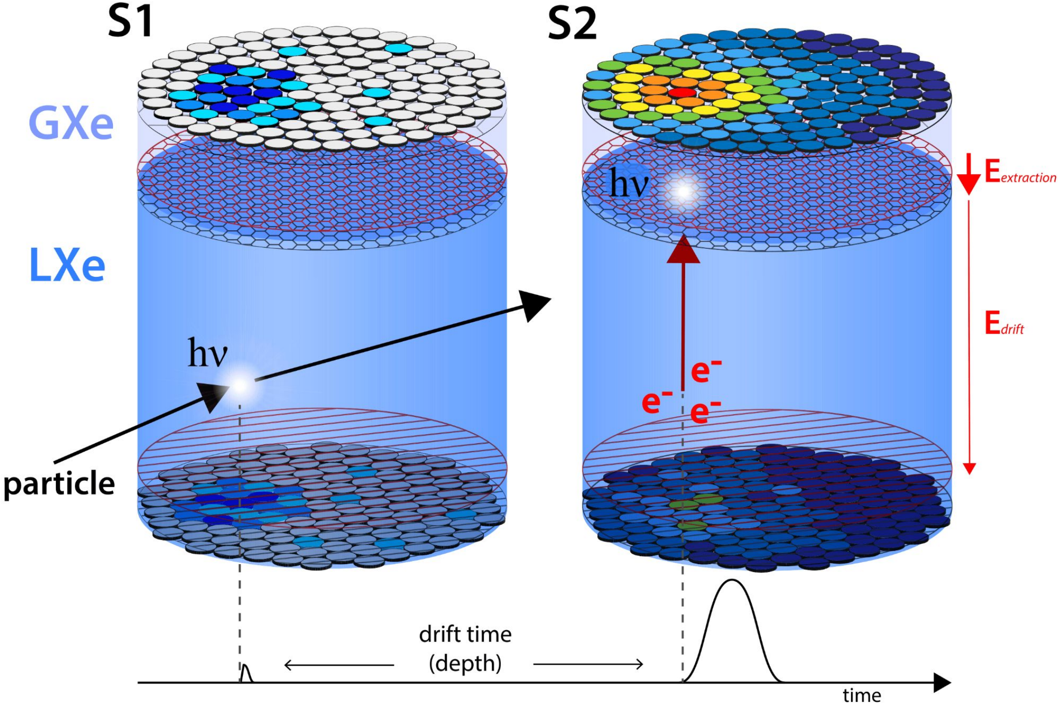


DM signal

XENONnT



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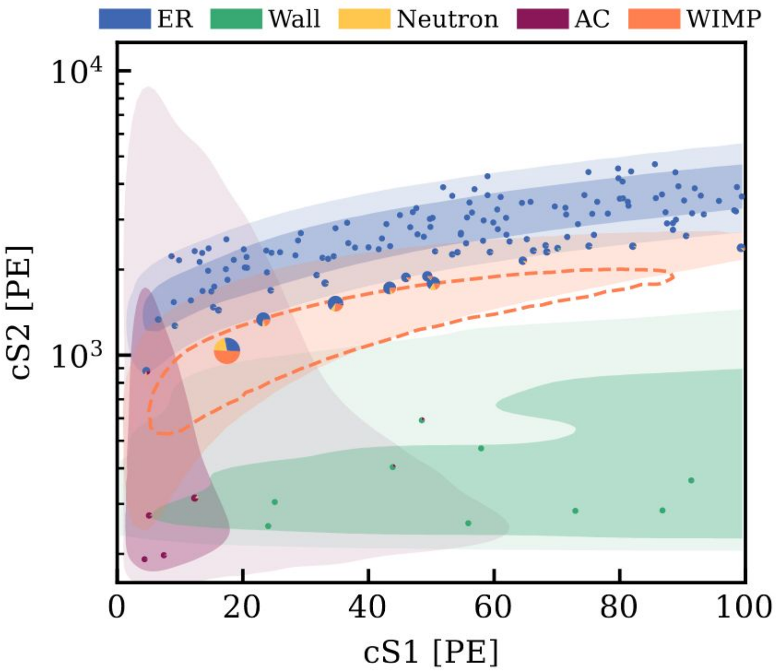
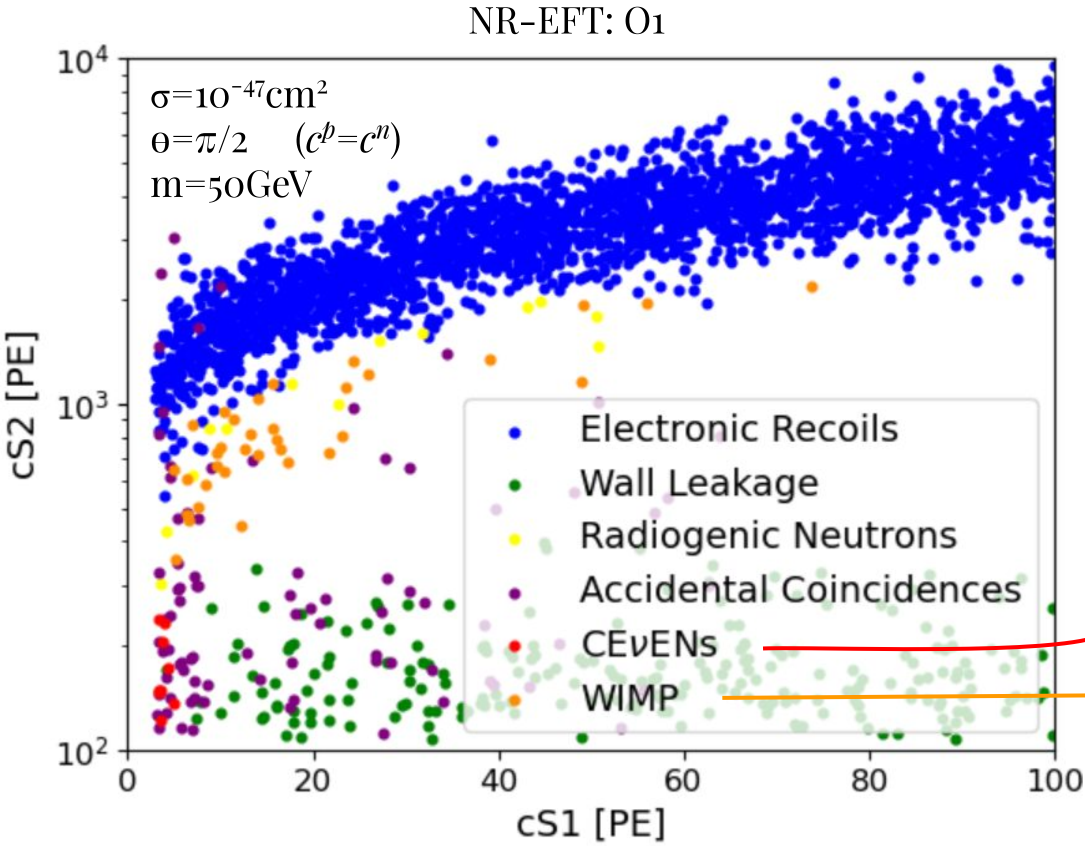


Phys. Rev. Lett. 131, 041003 (2023)

DM signal

XENONnT 20ty



XENONnT simulator

We specify background and signal characteristics

differential rate compute with SnuDD

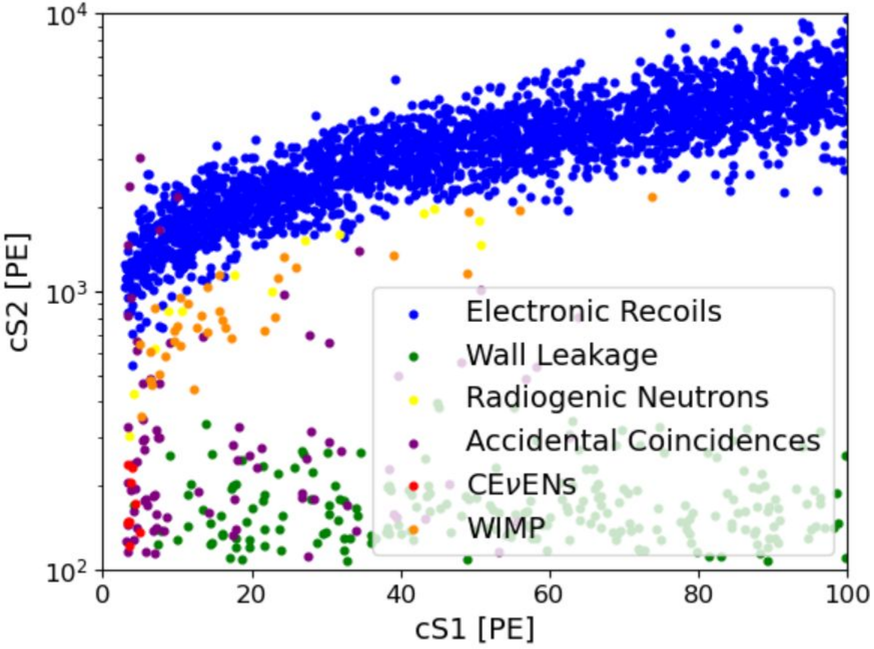
differential rate compute with WimPyDD for a particular operator, amplitud, phase and DM mass.

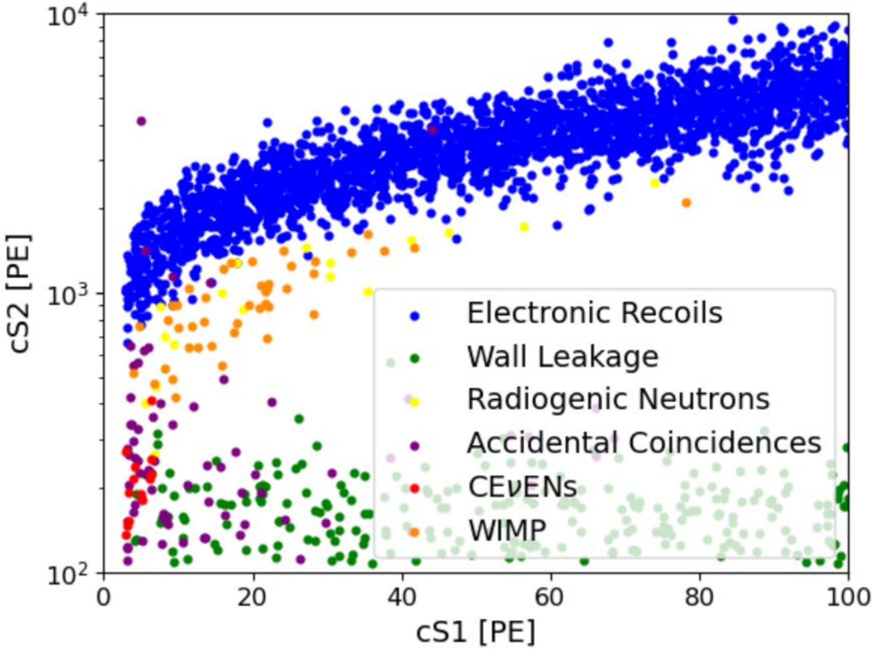
Data Representation: cS1 vs cS2 plane

XENONnT 20ty

NR-EFT: O1
$\sigma = 10^{-47} cm^2$
$\theta = \pi/2 \quad (c^p = c^n)$
m=50GeV

We generate a 10k pseudo experiments per operator varying σ, θ, and m_{DM}



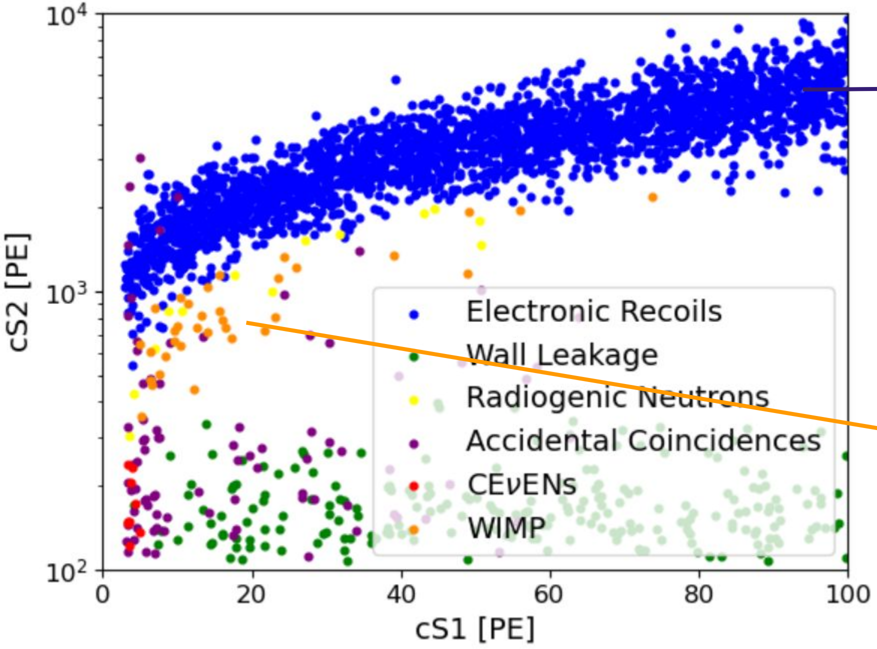


Data Representation: number of events

XENONnT 20ty

NR-EFT: O1
σ=10^{-47}cm²
θ=π/2 ($c^p=c^n$)
m=50GeV

We generate a 10k pseudo experiments per operator varying σ, θ, and m_{DM}



	name	pseudo_exp_events
0	er	2459
1	radiogenics	17
2	ac	71
3	wall	246
4	WIMP	43
5	CEVNS-SM	13

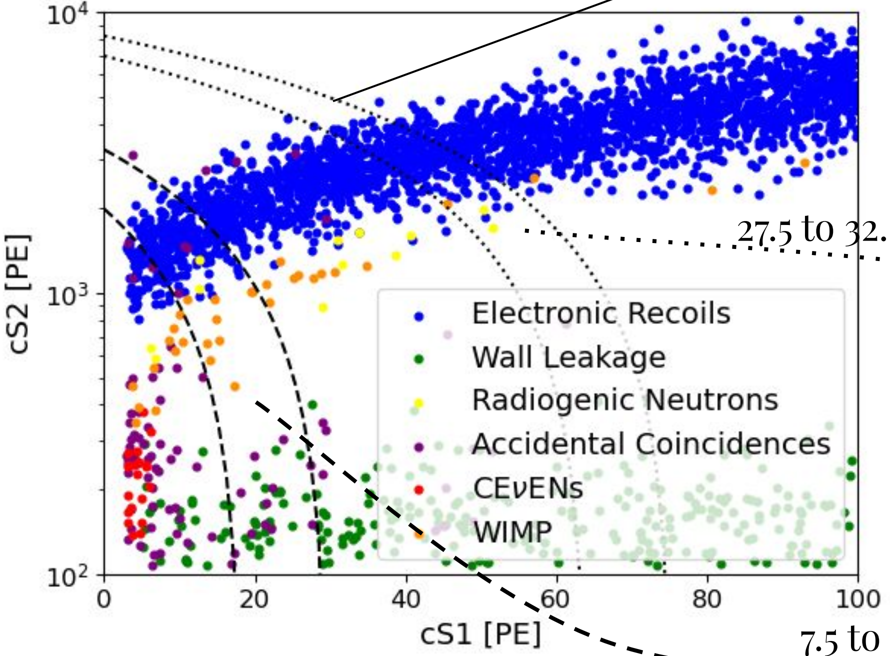
Data Representation: differential rate

XENONnT 20ty

NR-EFT: O1
$\sigma = 10^{-47}\text{cm}^2$
$\theta = \pi/2 \quad (c^p = c^n)$
m=50GeV

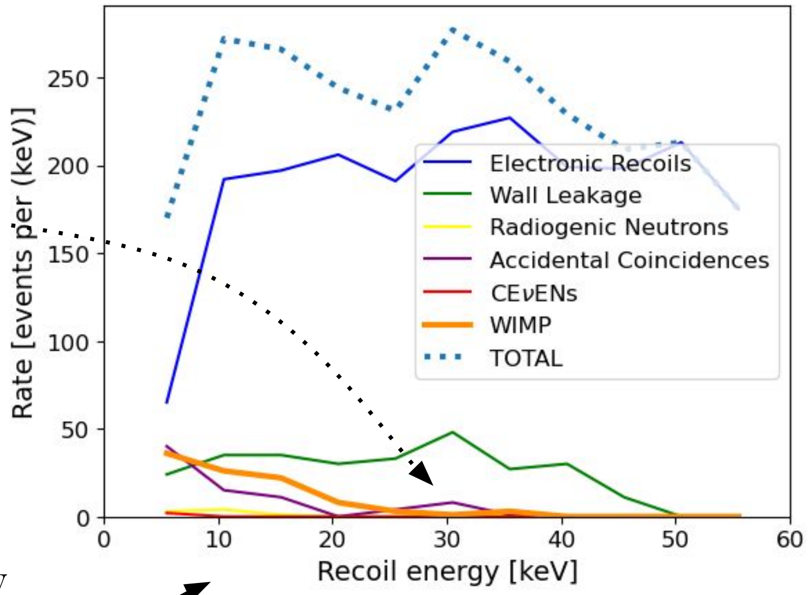
Nuclear Recoil isoenergy curves

We generate a 10k pseudo experiments per operator varying σ, θ, and m_{DM}



27.5 to 32.5 keV_{NR}

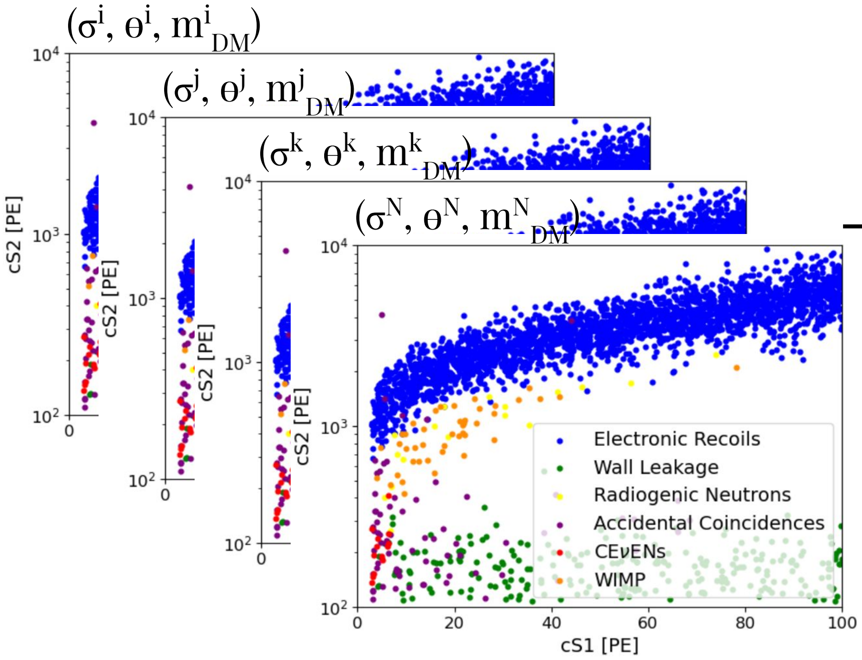
7.5 to 12.5 keV_{NR}



Analysis with SWYFT

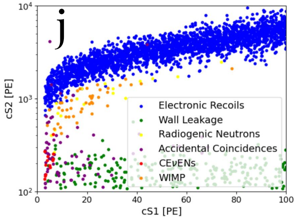
Data analysis to obtain posteriors

SWYFT → Sampling-based inference tool that estimates likelihood to evidence ratio with ML algorithms to obtain marginal and joint posteriors



Matching (parameter, data) → **label 1**

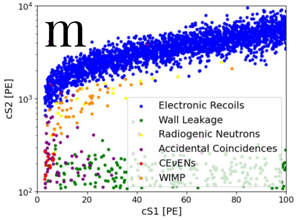
$\Big((\sigma^j, \theta^j, m^j_{DM})$,



$\Big)$

Scrambled (parameter, data) → **label 0**

$\Big((\sigma^k, \theta^k, m^k_{DM})$,



$\Big)$

Data analysis to obtain posteriors

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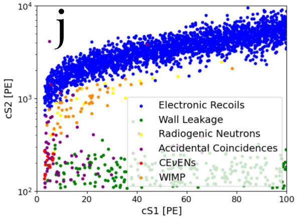
Matching (parameter, data) → **label 1**

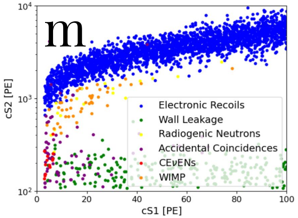
$\left((\sigma^j, \theta^j, m^j_{DM}) , \right.$



$\left. \right)$

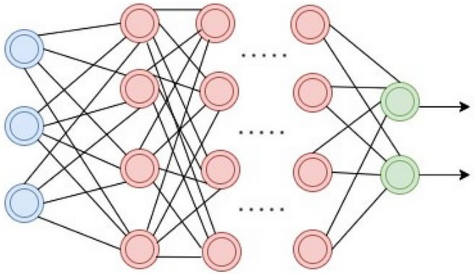
Scrambled (parameter, data) → **label 0**

$\left((\sigma^k, \theta^k, m^k_{DM}) , \right.$



$\left. \right)$

Binary classifier (DNN, CNN, ...)



Estimates the density ratio

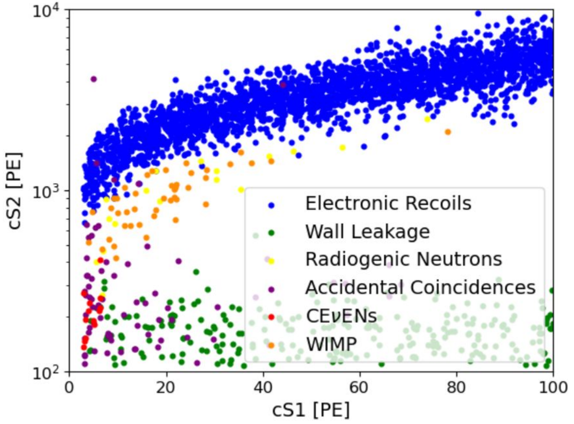
(likelihood ratio trick)

$$r(\mathbf{x}; \mathbf{z}) \equiv \frac{p(\mathbf{x} \mid \mathbf{z})}{p(\mathbf{x})} = \frac{p(\mathbf{z} \mid \mathbf{x})}{p(\mathbf{z})} = \frac{p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x})p(\mathbf{z})}$$

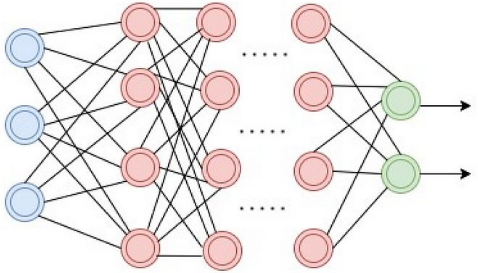
Data analysis to obtain posteriors

SWYFT → Sampling-based inference tool that estimates likelihood to evidence ratio with ML algorithms to obtain marginal and joint posteriors

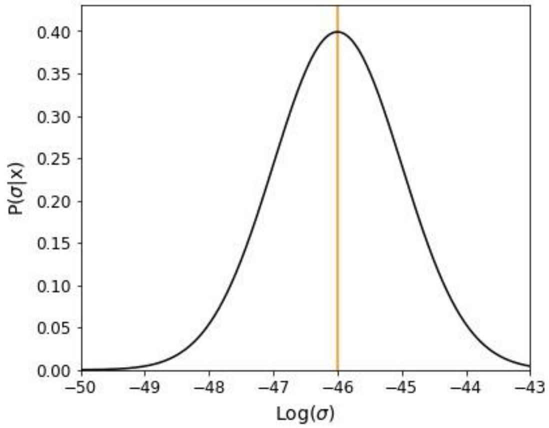
New data sample x^{new}



Trained binary classifier



Posterior $P(\sigma|x^{new})$



For another data sample → we do **not** need to train everything again, use the same classifier

Results

Posteriors

this is a gaussian as an example, not the actual posterior!

Once we trained SWYFT we can compute the posterior for any new pseudo experiment

For example $P(\sigma|x)$

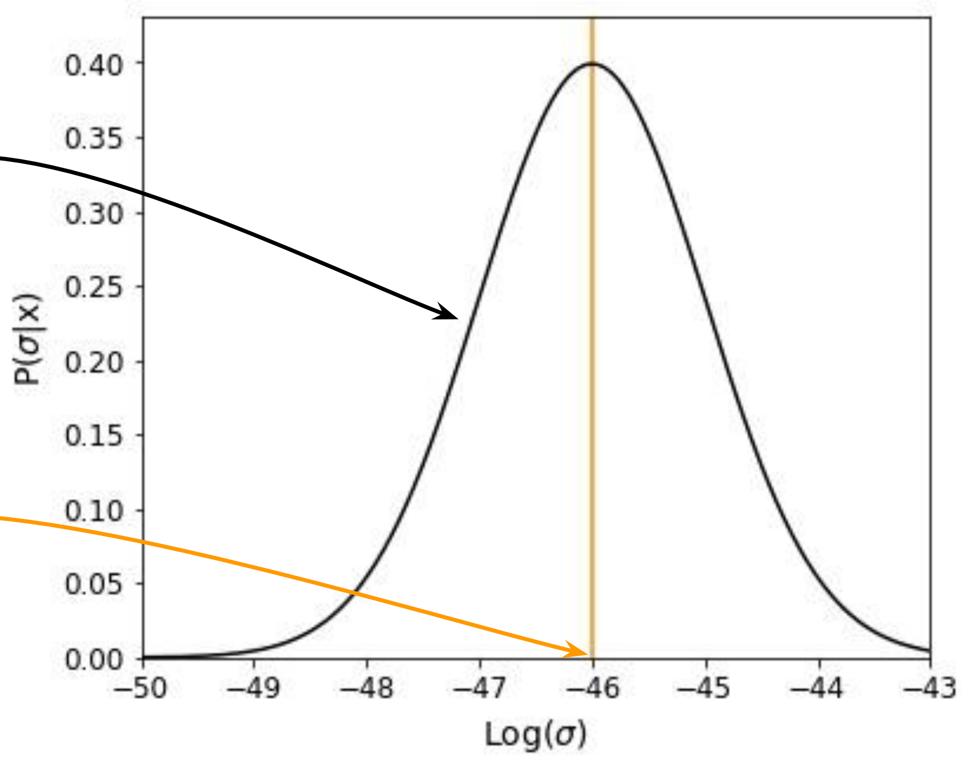
x: a data generated with

O1 (SI)

$m_{DM} \simeq 100 GeV$

$\theta = \pi/2$

$\sigma = 10^{-46} cm^2$



Reconstruction of parameters

this is a gaussian as an example, not the actual posterior!

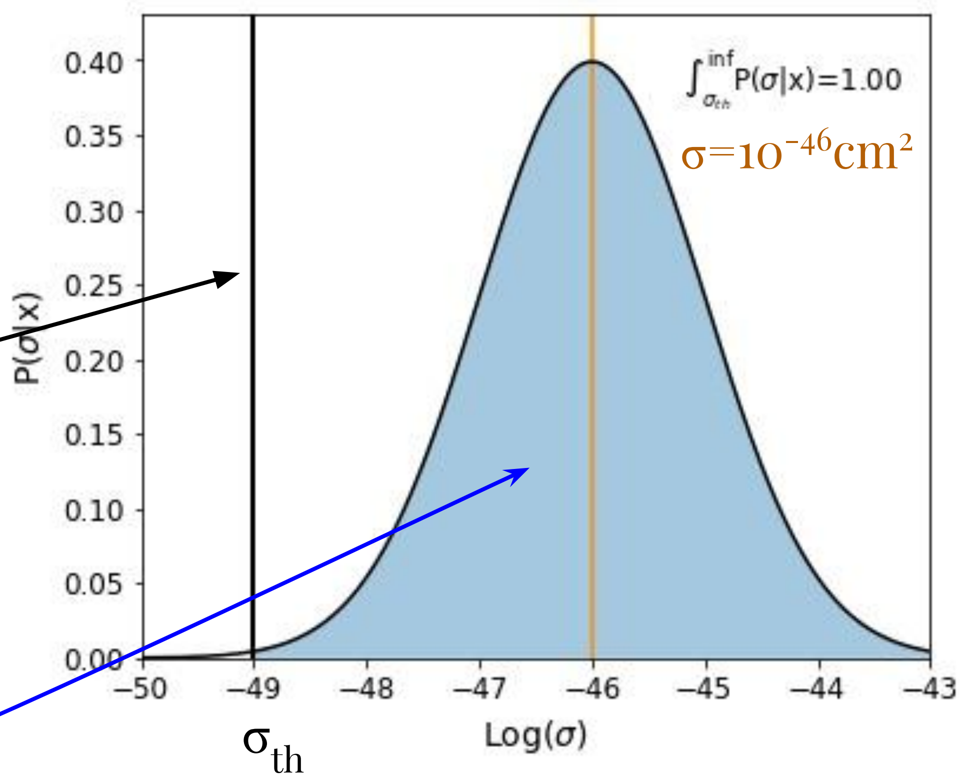
Once we trained SWYFT we can compute the posterior for any new pseudo experiment

We define a σ_{th} threshold:

$\sigma_{th}=10^{-49}\mathrm{cm}^2 \longrightarrow$ NO SIGNAL!

Then, we can ***reconstruct*** σ if:

$$\int_{\sigma_{th}}^{inf} P(\sigma|x) > 0.90$$



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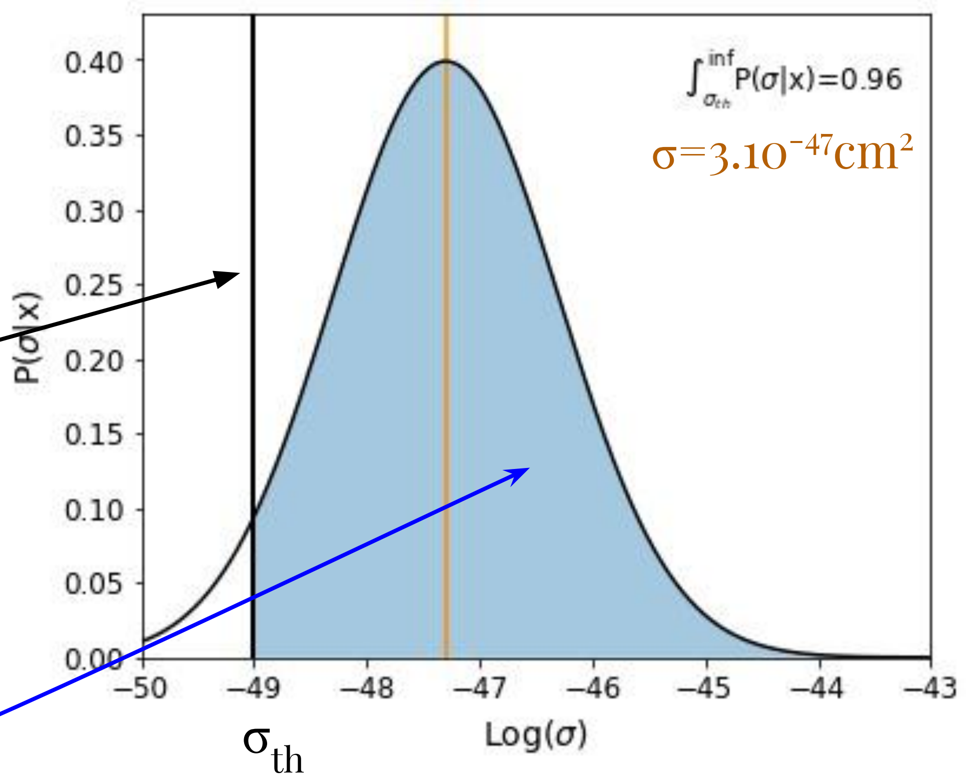

Reconstruction of parameters

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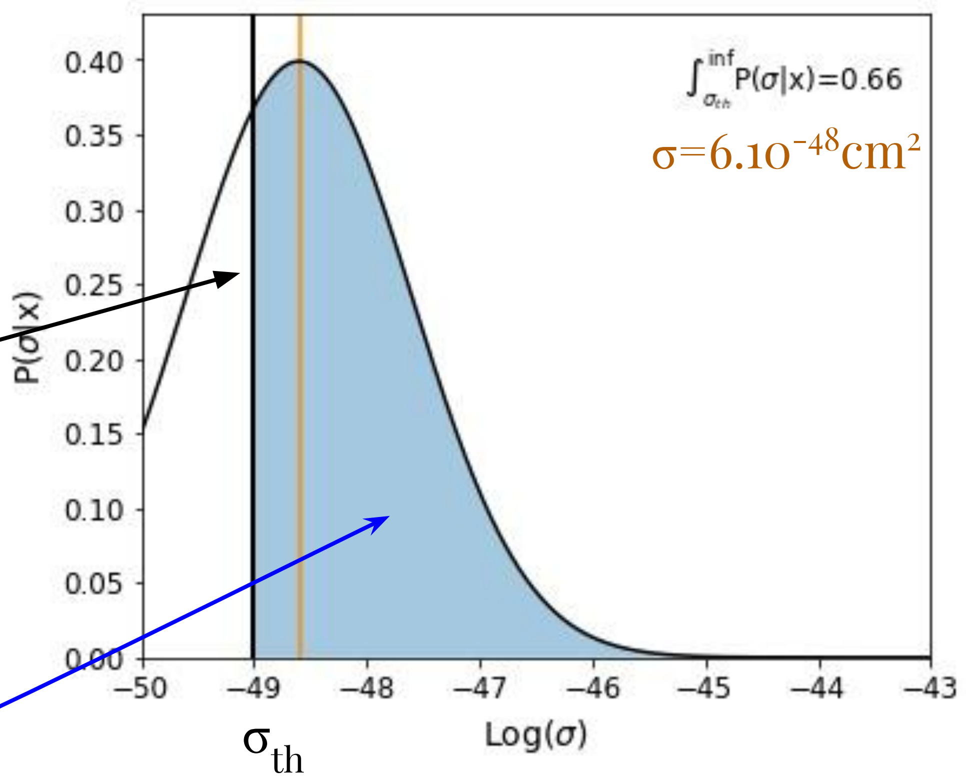


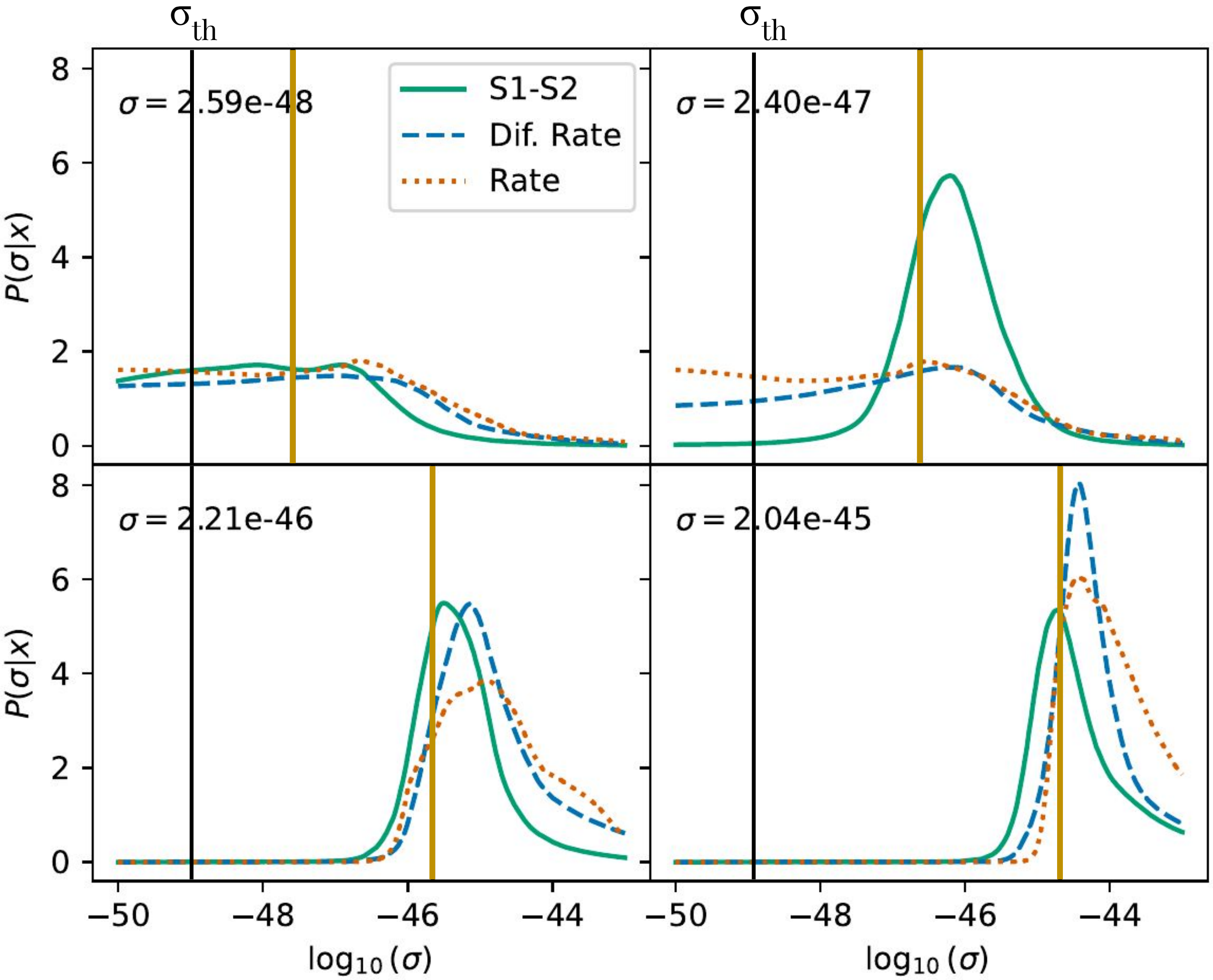
Results

Data:

entire cS1 vs cS2 plane
differential rate
total number of events

examples with
O1 (SI)
$m_{DM} \simeq 100\text{GeV} \longrightarrow$ fixed
$\theta = \pi/2 \longrightarrow$ fixed

threshold:
$\sigma_{th} = 10^{-49} \text{cm}^2$



Results

Data:

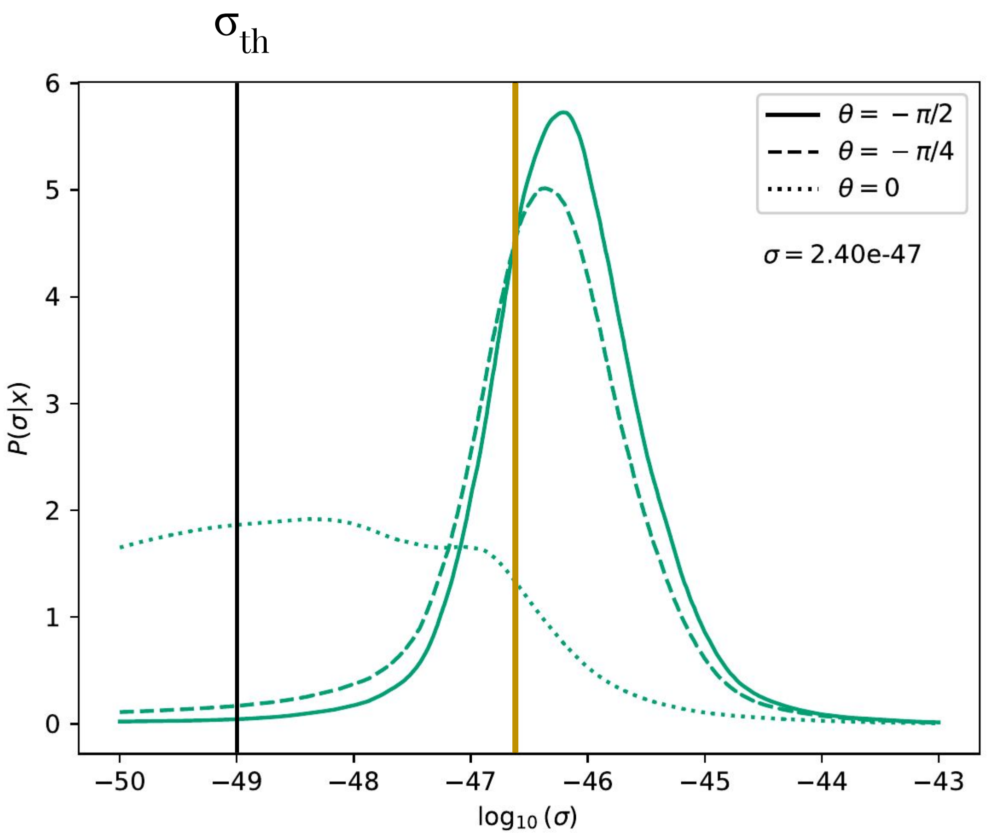
entire cS1 vs cS2 plane

These are all the posteriors for

O1 (SI)
$m_{DM} \simeq 100 GeV \longrightarrow$ fixed
$\sigma = 2.4\ 10^{-47} cm^2 \longrightarrow$ fixed

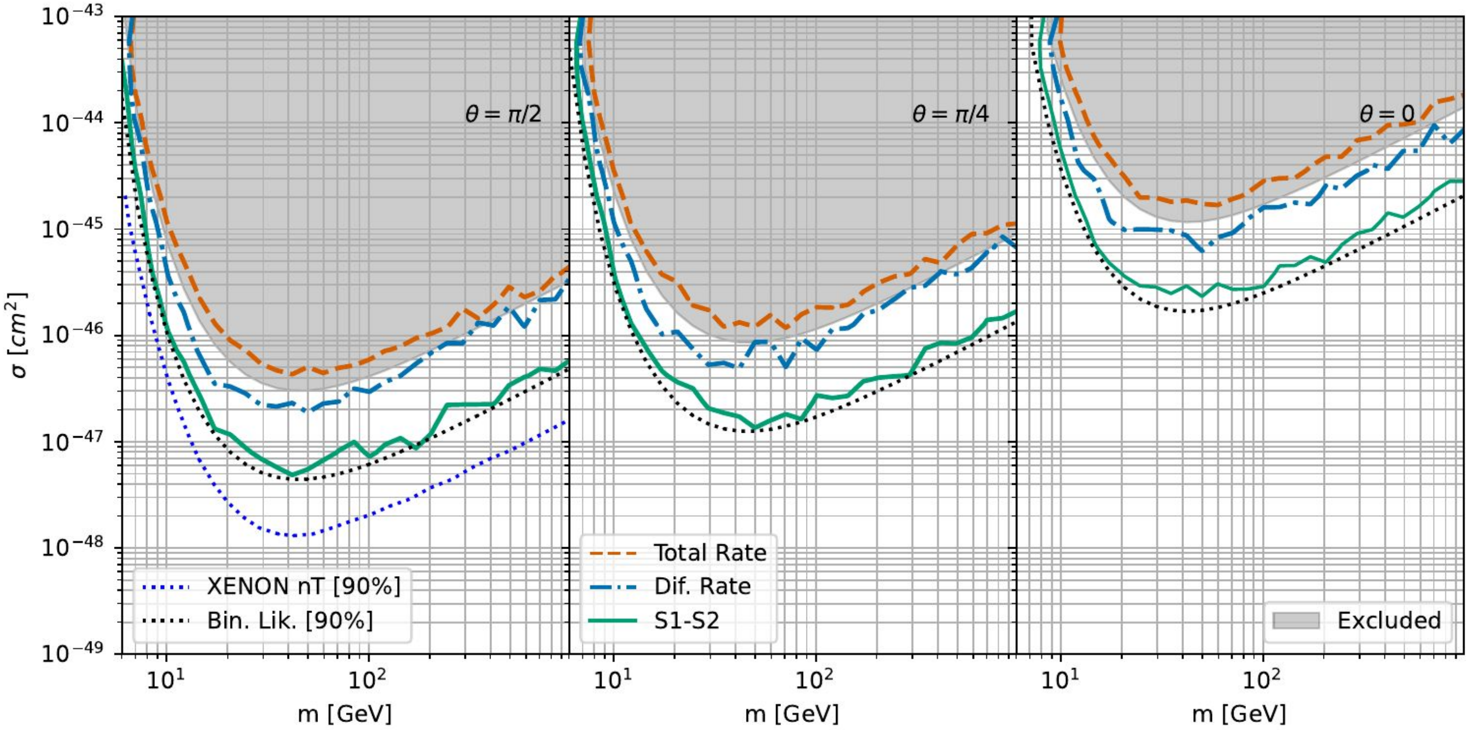
threshold:
$\sigma_{th} = 10^{-49} cm^2$

cS1 vs cS2



Results: σ reconstruction plot

O1 operator
XENONnT 20ty

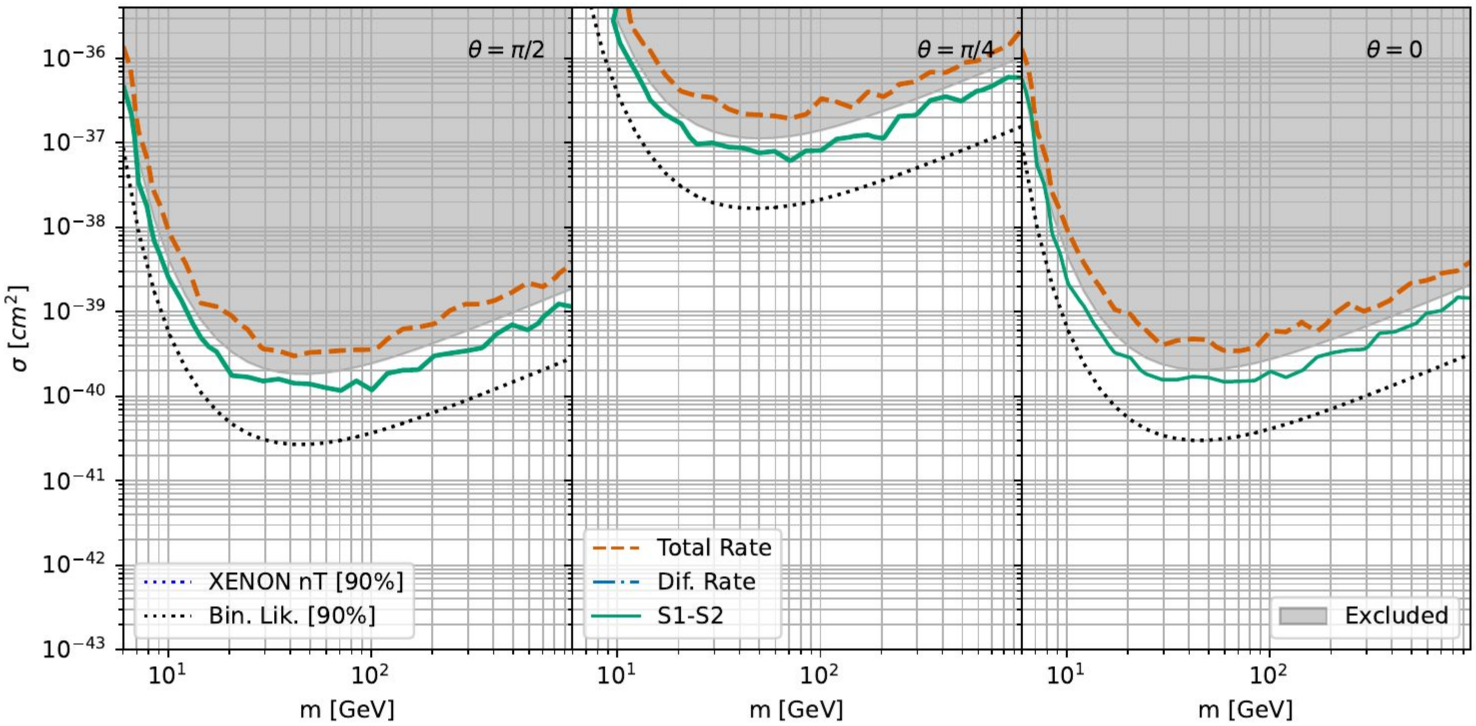


This panel is the usually shown **SI** parameter space

Data:
total number of events
differential rate
entire cS1 vs cS2 plane

Results: σ reconstruction plot

O4 operator
XENONnT 20ty



Data: total number of events
differential rate
entire cS1 vs cS2 plane

Conclusions

preliminary results

- A Bayesian analysis to explore the reach of direct detection experiments that can be applied to any DM model (translate it into NR-EFT)

  - O_1 (SI) and O_4 (SD) presented here as examples,
  - SWYFT, a data driven tool, allows a really fast estimation of posteriors,
  - we computed the parameter space where σ that can be **reconstructed**,
  - we compared:
    total number of events vs the differential rate vs the full cS_1,cS_2 space.

- **Next:**
  Apply to other NR-EFT operators → combine operators
  Different DD experiments → combine experiments
  Reconstruct σ, m, Θ at the same time


Thank you!

Back-up

DM-nucleon non-relativistic effective field theory (NR-EFT)

For DM particles with spin up to ½ , the effective DM-nucleon scattering interaction Lagrangian

$$\mathcal{L}_{\rm EFT} = \sum_{\tau} \sum_{i} c_i^{\tau} \mathcal{O}_i \bar{\chi}\chi\bar{\tau}\tau$$

i=14 possible interactions

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^{\perp}, \quad \vec{S}_{\chi}, \quad \vec{S}_N.$$

$$\vec{v} \equiv \vec{v}_{\chi,\rm in} - \vec{v}_{N,\rm in}$$

$$\vec{v}^{\perp} = \vec{v} + \frac{\vec{q}}{2\mu_N}$$

momentum transfer, spin operators, relative velocity

$\mathcal{O}_1 = 1_{\chi} 1_N$	$\mathcal{O}_9 = i\vec{S}_{\chi} \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^{\perp})$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_4 = \vec{S}_{\chi} \cdot \vec{S}_N$	$\mathcal{O}_{11} = i\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_5 = i\vec{S}_{\chi} \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^{\perp})$	$\mathcal{O}_{12} = \vec{S}_{\chi} \cdot (\vec{S}_N \times \vec{v}^{\perp})$
$\mathcal{O}_6 = (\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	$\mathcal{O}_{13} = i(\vec{S}_{\chi} \cdot \vec{v}^{\perp})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$
$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^{\perp}$	$\mathcal{O}_{14} = i(\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^{\perp})$
$\mathcal{O}_8 = \vec{S}_{\chi} \cdot \vec{v}^{\perp}$	$\mathcal{O}_{15} = -(\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^{\perp}) \cdot \frac{\vec{q}}{m_N})$

DM-nucleon non-relativistic effective field theory (NR-EFT)

For DM particles with spin up to ½ , the effective DM-nucleon scattering interaction Lagrangian

$$\mathcal{L}_{\mathrm{EFT}} = \sum_{\tau} \sum_{i} c_i^{\tau} \mathcal{O}_i \bar{\chi}\chi\bar{\tau}\tau$$

i=14 possible interactions

$$c_i^0 \mathbb{1}_{2\times 2} + c_i^1 \tau_3$$

isospin basis
c^0: isoscalar
c^1: isovector

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n)$$

$$c_i^1 = \frac{1}{2}(c_i^p - c_i^n)$$

nucleon basis
c^p: proton
c^n: neutron

O1: spin-independent (SI)
O4: spin-dependent (SD)

usually shown assuming isoscalar interactions

$c^p = c^n$ $c^0 = 1$ and $c^1 = 0$

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i=14 possible interactions

Change to polar coordinates:

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n) = A_i \sin(\theta_i)$$

$$c_i^1 = \frac{1}{2}(c_i^p - c_i^n) = A_i \cos(\theta_i)$$

Natural choice for the EFT parameter space because the interaction cross section:

$$\sigma_i \propto A_i^2$$

For SI (O1) $\sigma_{\chi N}^{\rm SI} = \frac{A_1^2 \ \mu_{\chi N}^2}{\pi}$

DM-nucleon reduced mass

DM-nucleon non-relativistic effective field theory (NR-EFT)

Contact interaction between a spin ½ DM and nucleon

$$\mathcal{L}^{\rm SI}_{\rm int}(\vec{x}) = c_1\ \bar{\Psi}_\chi(\vec{x})\Psi_\chi(\vec{x})\ \bar{\Psi}_N(\vec{x})\Psi_N(\vec{x})$$

$$U_\chi(p) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \xi_\chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m_\chi}\xi_\chi \end{pmatrix} \sim \begin{pmatrix} \xi_\chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{2m_\chi}\xi_\chi \end{pmatrix}$$

at low momenta.

Idem for the nucleon spinor

ξ Pauli spinors

at leading order in p/m $\quad c_1\ 1_\chi 1_N\ \equiv\ c_1\ \mathcal{O}_1$

DM-nucleon non-relativistic effective field theory (NR-EFT)

Another interaction

$$\mathcal{L}_{\rm int}^{\rm SD} = c_4 \; \bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$$

the dominant contribution in the non-relativistic limit comes from the spatial indices

$$\bar{\chi}\gamma^i\gamma^5\chi \sim \xi_\chi^\dagger\sigma^i\xi_\chi$$

Since $\hat{S}^i = \sigma^i/2$

$$-\,4c_4 \; \vec{S}_\chi \cdot \vec{S}_N \;\equiv\; -\,4c_4 \; \mathcal{O}_4$$

Data analysis to obtain posteriors

SWYFT → Sampling-based inference tool that estimates likelihood to evidence ratio with ML algorithms to obtain marginal and joint posteriors

	MCMC	**SWYFT**
Forward Model	x=f(parameters)	x=f(parameters)
Likelihood	L(x, f(parameters))	Data Driven
Samples	All parameters space > # samples	Only Interesting parameters
Amortization	NO	YES

Motivation

Bayes' Rule: determine a probability distribution over model parameters **θ** given an observation **x**

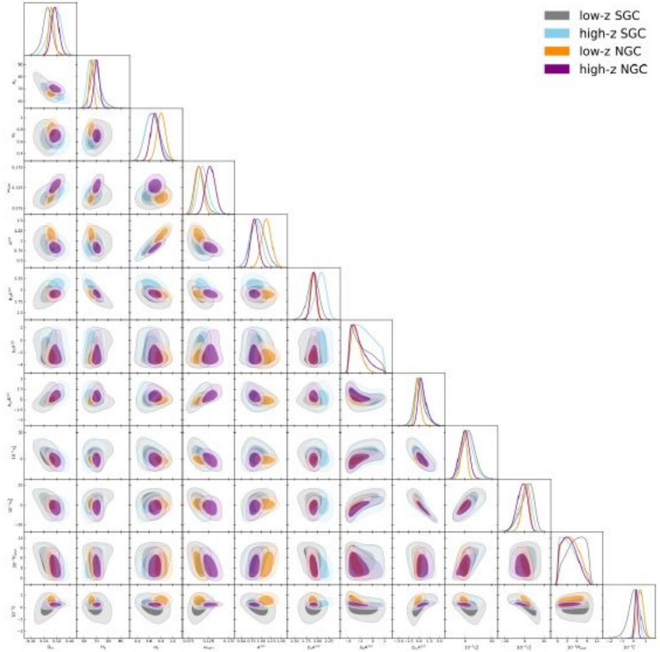
Likelihood of **x** given **θ**

$$p(\boldsymbol{\theta} \mid \boldsymbol{x}) = \frac{p(\boldsymbol{x} \mid \boldsymbol{\theta})}{p(\boldsymbol{x})} p(\boldsymbol{\theta})$$

Posterior

Evidence of the data

Prior



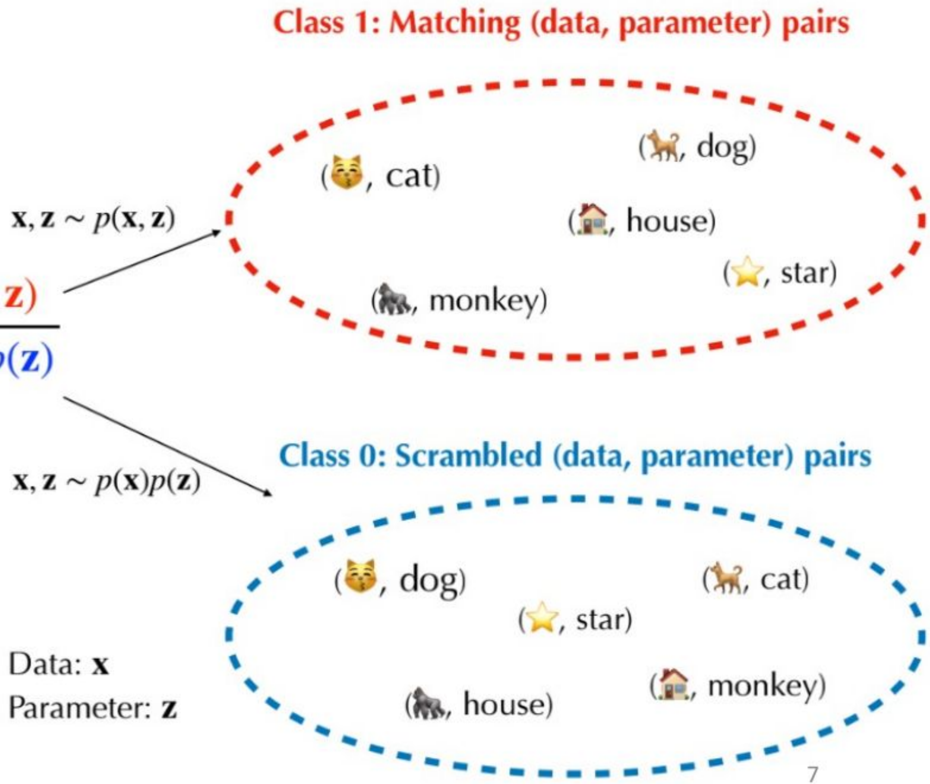
BOSS, Ivanov+ 1909.05277

Samples tipically generated with *Markov Chain Monte Carlo* (MCMC) or *Nested sampling*

Neural Ratio Estimation (NRE)

Approximate density ratios.

$$r(\mathbf{x};\mathbf{z}) \equiv \frac{p(\mathbf{x}\,|\,\mathbf{z})}{p(\mathbf{x})} = \frac{p(\mathbf{z}\,|\,\mathbf{x})}{p(\mathbf{z})} = \frac{p(\mathbf{x},\mathbf{z})}{p(\mathbf{x})p(\mathbf{z})}$$



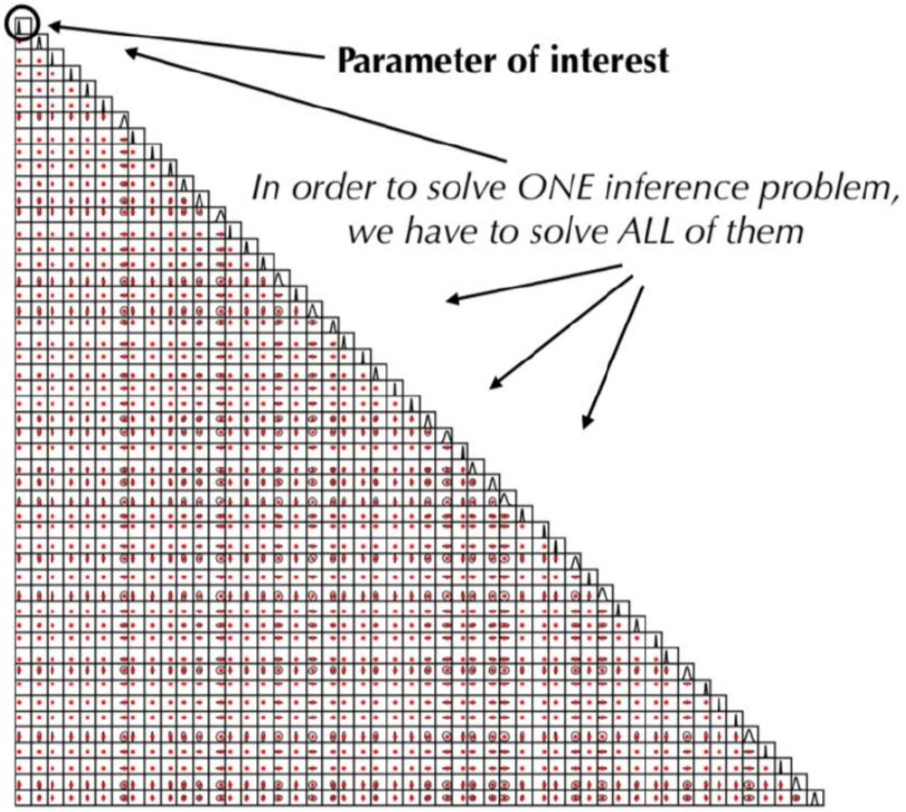
Strategy: *We estimate posteriors-to-prior ratio by training a binary classifier to discriminate between matching and scrambled (data, parameter) pairs.*

MARGINAL

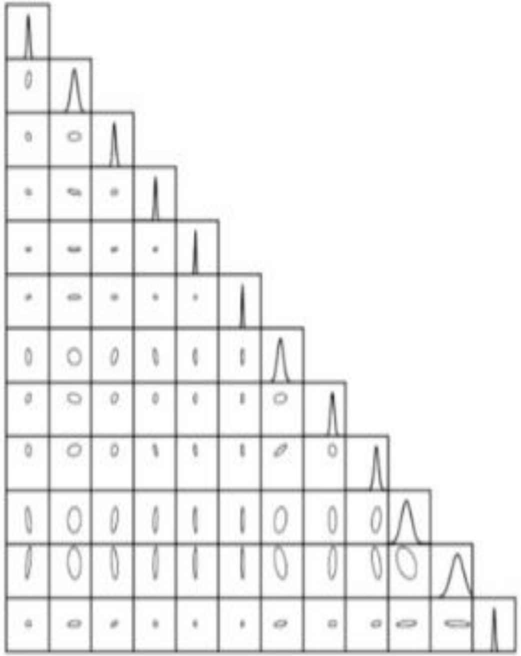
- MCMC or Nested sampling methods produce samples from the **posterior distribution**.

- Classical methods require sampling the **full joint posterior**, so that they are slow to converge.

- Novel approaches in the field of *simulation-based inference* (SBI) are starting to overcome these obstacles.

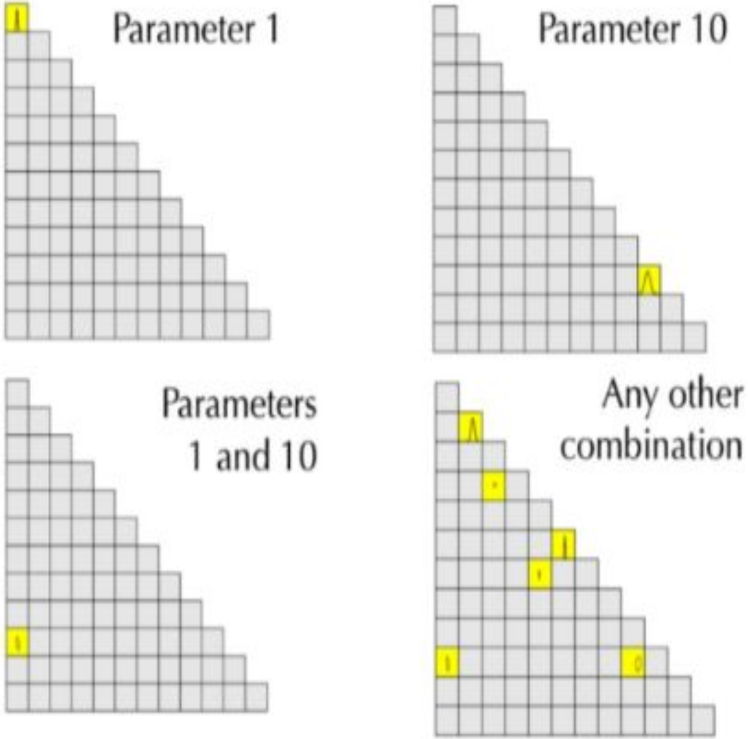


Instead of estimating all parameters…



50 parameters ~ 100 Million simulations

…we can choose what we care about



Depending on which parameter is scrambled

Results

Data:

entire cS1 vs cS2 plane

These are all the posteriors for

$m_{DM} \simeq 100\text{GeV} \longrightarrow$ fixed

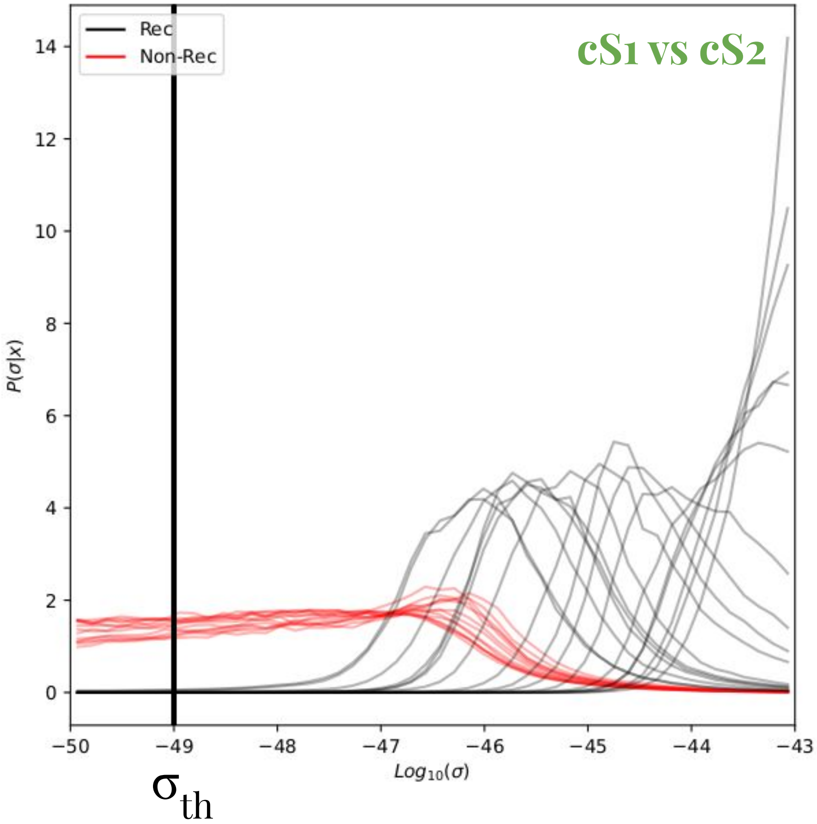
$\theta = \pi/2 \longrightarrow$ fixed

red $\longrightarrow \sigma$ not reconstructed

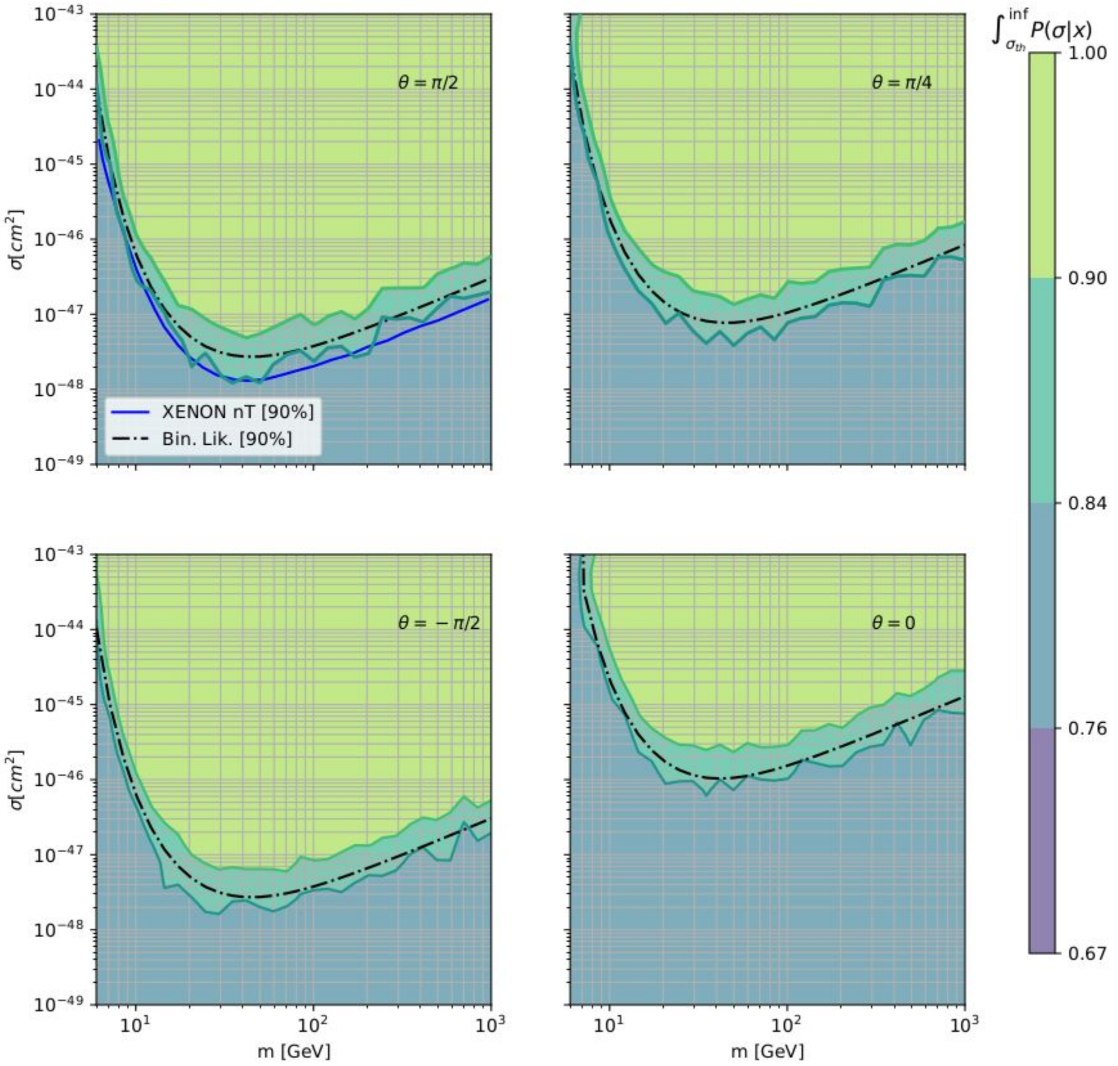
~no signal,

~similar posteriors

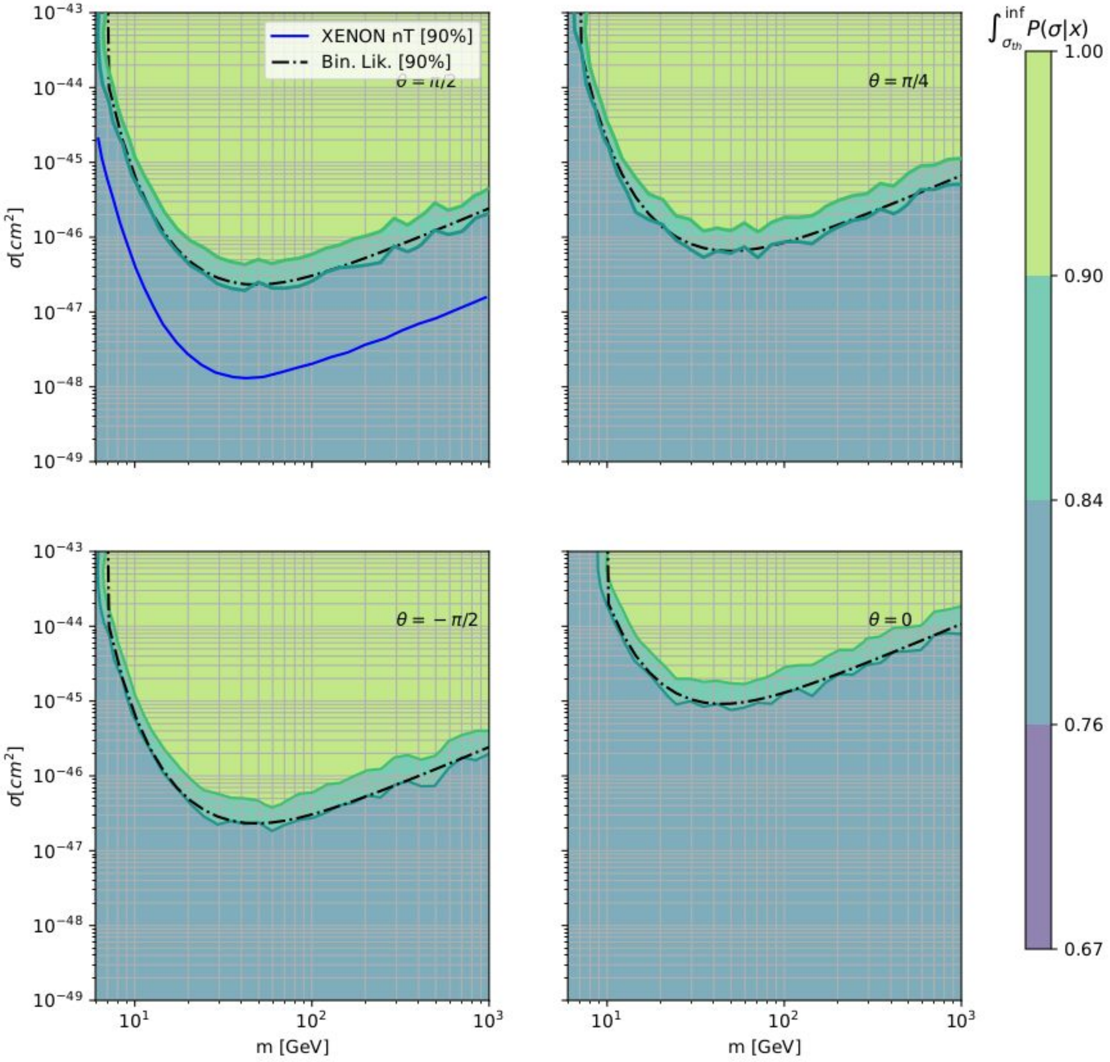
black $\longrightarrow \sigma$ reconstructed



Data:
the entire cS1 vs cS2 plane



Data:
the total number of events



Results

Testing with:
m_{DM}=84.6GeV ⟶ fixed
θ=π/2 ⟶ fixed