Dark Energy and CAMB

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Outline

- Background evolution
- Linear perturbation evolution
- Dark energy
  - Scalar field dark energy models.
  - Dark energy in default CAMB code and how to generalize.
- Parameterized post-Friedmann framework
  - PPF for DE, modified gravity.
  - How to use the published PPF module for CAMB?

Background Evolution

- Metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(-d\eta^2 + \gamma_{ij}dx^i dx^j)$$

$$\gamma_{ij}dx^i dx^j = dD^2 + D_A^2 d\Omega$$

$$D_A = K^{-1/2}\sin(K^{1/2}D) \qquad \Omega_K = -K/H_0^2$$

- Energy-momentum tensor

$$T^0{}_0 = -\rho, \qquad T_0{}^i = 0, \qquad T^i{}_j = p\delta^i{}_j$$

Background Evolution

Einstein equation $\quad\quad\quad\quad$ $T^{\mu\nu}$ Conservation

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \xrightarrow{\nabla_\mu G^{\mu\nu} = 0} \quad \nabla_\mu T^{\mu\nu} = 0$$

$$\left(\frac{\dot{a}}{a}\right)^2 + K = \frac{8\pi G}{3} a^2 \rho$$

$$\frac{d}{d\eta}\left(\frac{\dot{a}}{a}\right) = -\frac{4\pi G}{3} a^2 (\rho + 3p)$$

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$$

3 unknowns, 2 independent equations, need to specify $w=P/\rho$

Liner Perturbations

- Metric perturbations (scalar mode only)

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = a^2(-d\eta^2 + \gamma_{ij}dx^i dx^j)$$

$$\downarrow$$

$$\delta g_{00} = -a^2(2AY), \qquad \delta g_{0i} = -a^2 B Y_i,$$

$$\delta g_{ij} = a^2(2H_L Y \gamma_{ij} + 2H_T Y_{ij}).$$

unknowns = 4

$$\nabla^2 Y = -k^2 Y,$$

$$Y_i = (-k)\nabla_i Y, \qquad Y_{ij} = (k^{-2}\nabla_i\nabla_j + \gamma_{ij}/3)Y.$$

Liner Perturbations

- Energy-momentum perturbations

$$T^0{}_0 = -\rho, \qquad T_0{}^i = 0, \qquad T^i{}_j = p\delta^i{}_j$$

↓

$$\delta T^0{}_0 = -\delta\rho, \qquad \delta T_0{}^i = -(\rho + p)vY^i,$$
$$\delta T^i{}_j = \delta p Y \delta^i{}_j + p\Pi Y^i{}_j.$$

unknowns = 4

Einstein Equation:

$$H_L + \frac{1}{3}H_T + \frac{B}{k_H} - \frac{H_T'}{k_H^2} = \frac{4\pi G}{H^2 c_K k_H^2}\left[\delta\rho + 3(\rho + p)\frac{v - B}{k_H}\right],$$

$$A + H_L + \frac{H_T}{3} + \frac{B' + 2B}{k_H} - \left[\frac{H_T''}{k_H^2} + \left(3 + \frac{H'}{H}\right)\frac{H_T'}{k_H^2}\right] = -\frac{8\pi G}{H^2 k_H^2}p\Pi,$$

$$A - H_L' - \frac{H_T'}{3} - \frac{K}{(aH)^2}\left(\frac{B}{k_H} - \frac{H_T'}{k_H^2}\right) = \frac{4\pi G}{H^2}(\rho + p)\frac{v - B}{k_H},$$

$$A' + \left(2 + 2\frac{H'}{H} - \frac{k_H^2}{3}\right)A - \frac{k_H}{3}(B' + B) - H_L'' - \left(2 + \frac{H'}{H}\right)H_L' = \frac{4\pi G}{H^2}\left(\delta p + \frac{1}{3}\delta\rho\right)$$

Energy-Momentum Conservation

- For each component i

$$\delta\rho_i' + 3(\delta\rho_i + \delta p_i) = -(\rho_i + p_i)(k_H v_i + 3H_L'),$$

$$\frac{[a^4(\rho_i + p_i)(v_i - B)]'}{a^4 k_H} = \delta p_i - \frac{2}{3} c_K p_i \Pi_i + (\rho_i + p_i)A,$$

$${}' \equiv d/d\ln a \qquad {}^{\cdot} \equiv d/d\eta$$

$$k_H = (k/aH) \qquad c_K = 1 - 3K/k^2.$$

Gauge Transformation

- Under the transformation $\tilde{x}^\mu \to x^\mu$

$$\eta = \tilde{\eta} + T, \qquad x^i = \tilde{x}^i + LY^i$$

$$A = \tilde{A} - aH(T' + T), \qquad B = \tilde{B} + aH(L' + k_H T),$$

$$H_L = \tilde{H}_L - aH\left(T + \frac{1}{3}k_H L\right), \qquad H_T = \tilde{H}_T + aHk_H L,$$

$$\delta\rho = \tilde{\delta\rho} - \rho' aHT, \qquad \delta p = \tilde{\delta p} - p' aHT,$$

$$v = \tilde{v} + aHL', \qquad \Pi = \tilde{\Pi}.$$

Common Gauge Choices

- Conformal Newtonian gauge

$$\delta g_{00} = -a^2(2AY), \qquad \delta g_{0i} = -a^2 B Y_i,$$
$$\delta g_{ij} = a^2(2H_L Y \gamma_{ij} + 2H_T Y_{ij}).$$

$$B = H_T = 0$$
$$\Phi \equiv H_L, \qquad \Psi \equiv A.$$

$$T = -\frac{\tilde{B}}{k} + \frac{\tilde{H}_T'}{k k_H}, \qquad L = -\frac{\tilde{H}_T}{k}.$$

- Comoving (total matter) gauge

$$B = v_T, \qquad H_T = 0.$$
$$\zeta \equiv H_L, \qquad \xi \equiv A,$$

$$T = (\tilde{v}_T - \tilde{B})/k, \qquad L = -\tilde{H}_T/k.$$

Common Gauge Choices

- Synchronous gauge

used in CAMB

$$A = B = 0,$$

$$\eta_T \equiv -\frac{1}{3}H_T - H_L, \quad h_L = 6H_L,$$

Einstein

$$\begin{cases} \dfrac{\dot{h}_L}{2k} = c_K k_H \eta_T + \dfrac{4\pi G}{k_H H^2}\rho\delta \\ c_K k \dot{\eta}_T = 4\pi G a^2(\rho + P)v + K\dfrac{\dot{h}_L}{2k} \end{cases}$$

Conservation

$$\begin{cases} \dot{\delta} = -(1+w)\left(kv + \dfrac{\dot{h}_L}{2}\right) - 3\dfrac{\dot{a}}{a}\left(\dfrac{\delta P}{\delta\rho} - w\right)\delta \\ \dot{v} = -\dfrac{\dot{a}}{a}(1 - 3C_a^2)v + \dfrac{k}{1+w}\left[\left(\dfrac{\delta P}{\delta\rho}\right)\delta - \dfrac{2}{3}c_K w\Pi\right] \end{cases}$$

$$C_a^2 = \frac{\dot{P}}{\dot{\rho}}$$

Linear Perturbations

- Unkowns: 4 (metric) + 4 (matter)
- Constraints: 2 (Einstein) + 2 (Conservation) +2 (Gauge)
- Need to specify 2 d.o.f. to close the system, choose to be δP, Π.

Dark Energy

- Cosmological constant: w=-1, no perturbations.
- Scalar field: with perturbations.
  - Quintessence: $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

$$\rho = \frac{1}{2}a^{-2}\dot{\phi}^2 + V(\phi)$$

$$P = \frac{1}{2}a^{-2}\dot{\phi}^2 - V(\phi)$$

$\Rightarrow$ $w \geq -1$

$$\hat{C_s}^2 \equiv \left(\frac{\delta P}{\delta \rho}\right)_{\rm rest} = 1 \qquad \Pi = 0$$

  - Phantom: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$ $\Rightarrow$ $w \leq -1$

DE in Default CAMB

- Background specified by $w = \text{const}$
- Perturbations specified by $\hat{C}_s{}^2 = \text{const}$ and $\Pi = 0$
- Equations:

$$\frac{\dot\rho}{\rho} = -3(1+w)\frac{\dot a}{a} \quad \Rightarrow \quad \rho/\rho_0 = a^{-3(1+w)}$$

For general w, $$\rho/\rho_0 = \exp\left[\int_a^1 3(1+w)d\ln a\right]$$

e.g.

$$w = w_0 + w_a(1-a) \qquad \rho/\rho_0 = a^{-3(1+w_0+w_a)}e^{-3w_a(1-a)}$$

DE in Default CAMB

- Perturbation Equations:

$$\delta P = \hat{C_s}^2 \delta\rho + 3\mathcal{H}(1+w)(\hat{C_s}^2 - C_a^2)\frac{\rho v}{k}$$

$$C_a^2 = \frac{\dot{P}}{\dot{\rho}} = w - \frac{\dot{w}}{3\mathcal{H}(1+w)} \qquad \mathcal{H} = \frac{\dot{a}}{a}$$

$$\dot{\delta} = -(1+w)\left[kv + 9\mathcal{H}^2(\hat{C_s}^2 - C_a^2)\frac{v}{k} + \frac{\dot{h_L}}{2}\right] - 3\mathcal{H}(\hat{C_s}^2 - w)\delta$$

$$\dot{v} = -\mathcal{H}(1 - 3\hat{C_s}^2)v + \frac{k}{1+w}\hat{C_s}^2\delta$$

Generalize to Time-dependent E.O.S.

$$w : \text{const} \to w(a)$$

$$\rho/\rho_0 : a^{-3(1+w)} \to \exp\left[\int_a^1 3(1+w)d\ln a\right]$$

$$C_a^2 : w \to w - \frac{\dot{w}}{3\mathcal{H}(1+w)}$$

Exercise 1: modify default CAMB to solve for a quintessence DE model with (w0,wa)=(-0.8,0.1) (w>-1), and compare with calculations by using the PPF module.

Parameterized Post-Friedmann (PPF)

- Parameterized deviation from GR

$$\Phi_+ \equiv g(\eta, k)\Phi_- - \frac{4\pi G}{H^2 k_H^2} P_T \Pi_T$$

$$\Phi_- + \Gamma = \frac{4\pi G}{c_K k_H^2 H^2}(\Delta_T \rho_T + c_K P_T \Pi_T)$$

$$\Phi_+ \equiv (\Phi + \Psi)/2$$

$$\Phi_- \equiv (\Phi - \Psi)/2$$

$$\lim_{k_H \ll 1} \frac{\dot{\zeta}}{aH} = -\frac{\Delta P_T - \frac{2}{3} c_K P_T \Pi_T}{\rho_T + P_T} - \frac{K}{k^2} k_H V_T + \frac{1}{3} c_K f_\zeta(\eta) k_H V_T \sim \mathcal{O}(k_H^2)$$

$$\lim_{k_H \gg 1} \Phi_- = \frac{4\pi G}{c_K k_H^2 H^2} \frac{\Delta_T \rho_T + c_K P_T \Pi_T}{1 + f_G(\eta)}$$

Matter perturbations all in comoving gauge, T means total matter components.

PPF Framework

- To specify Γ:

$$(1 + c_\Gamma^2 k_H^2)\left[\frac{\dot{\Gamma}}{aH} + \Gamma + c_\Gamma^2 k_H^2 (\Gamma - f_G \Phi_-)\right] = S.$$

$$S = \frac{\dot{g}/(aH) - 2g}{g+1}\Phi_- + \frac{4\pi G}{(g+1)k_H^2 H^2} \times \left\{ g\left[\frac{(P_T \dot{\Pi}_T)}{aH} + P_T \Pi_T\right] - [(g + f_\zeta + g f_\zeta)(\rho_T + P_T) - (\rho_e + P_e)] k_H V_T \right\}.$$

PPF Parameterization:

3 functions of $g(\eta, k), f_\zeta(\eta), f_G(\eta)$
1 parameter of c_Γ

Modified Gravity viewed as GR + Effective DE

$$G^{\mu\nu} + F^{\mu\nu}(g^{\alpha\beta}) = 8\pi G T_T{}^{\mu\nu}$$

$$G^{\mu\nu} = 8\pi G(T_T{}^{\mu\nu} + T_{eff}^{\mu\nu})$$

$$\nabla_\mu G^{\mu\nu} = 0$$

$$\nabla_\mu T_T^{\mu\nu} = 0$$

$$\rightarrow \quad \nabla_\mu T_e^{\mu\nu} = 0$$

We can use standard linear perturbation tools based on GR, like CAMB, to solve for modified gravity, only need to specify the effective DE.

Effective DE Representation for PPF

- Two closure conditions:

$$P_e\Pi_e = -\frac{k_H^2 H^2}{4\pi G} g\Phi_- .$$

$$\rho_e\Delta_e + 3(\rho_e + P_e)\frac{V_e - V_T}{k_H} + c_K P_e \Pi_e = -\frac{k^2 c_K}{4\pi G a^2}\Gamma.$$

- Conservation of $T_e^{\mu\nu}$ gives V_e, no need to evolve equations.

$$\frac{V_e - V_T}{k_H} = -\frac{H^2}{4\pi G(\rho_e + P_e)}\frac{g+1}{F} \times \left[S - \Gamma - \frac{\dot{\Gamma}}{aH} + f_\zeta \frac{4\pi G(\rho_T + P_T)}{H^2}\frac{V_T}{k_H}\right],$$

$$F = 1 + \frac{12\pi G a^2}{k^2 c_K}(g+1)(\rho_T + P_T).$$

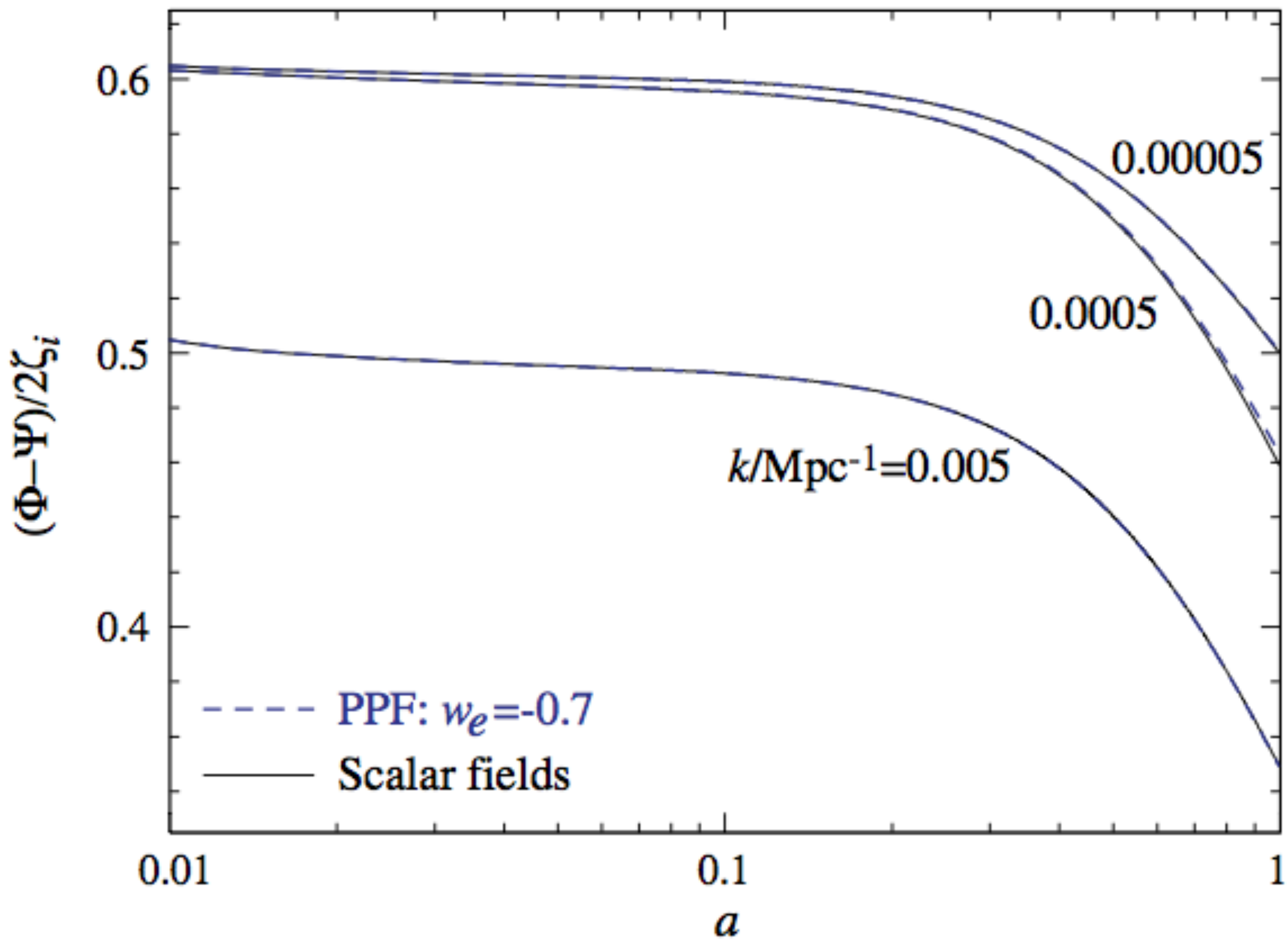
PPF for True DE

- PPF parameterization for dark energy, based on scalar field models, single or multiple.

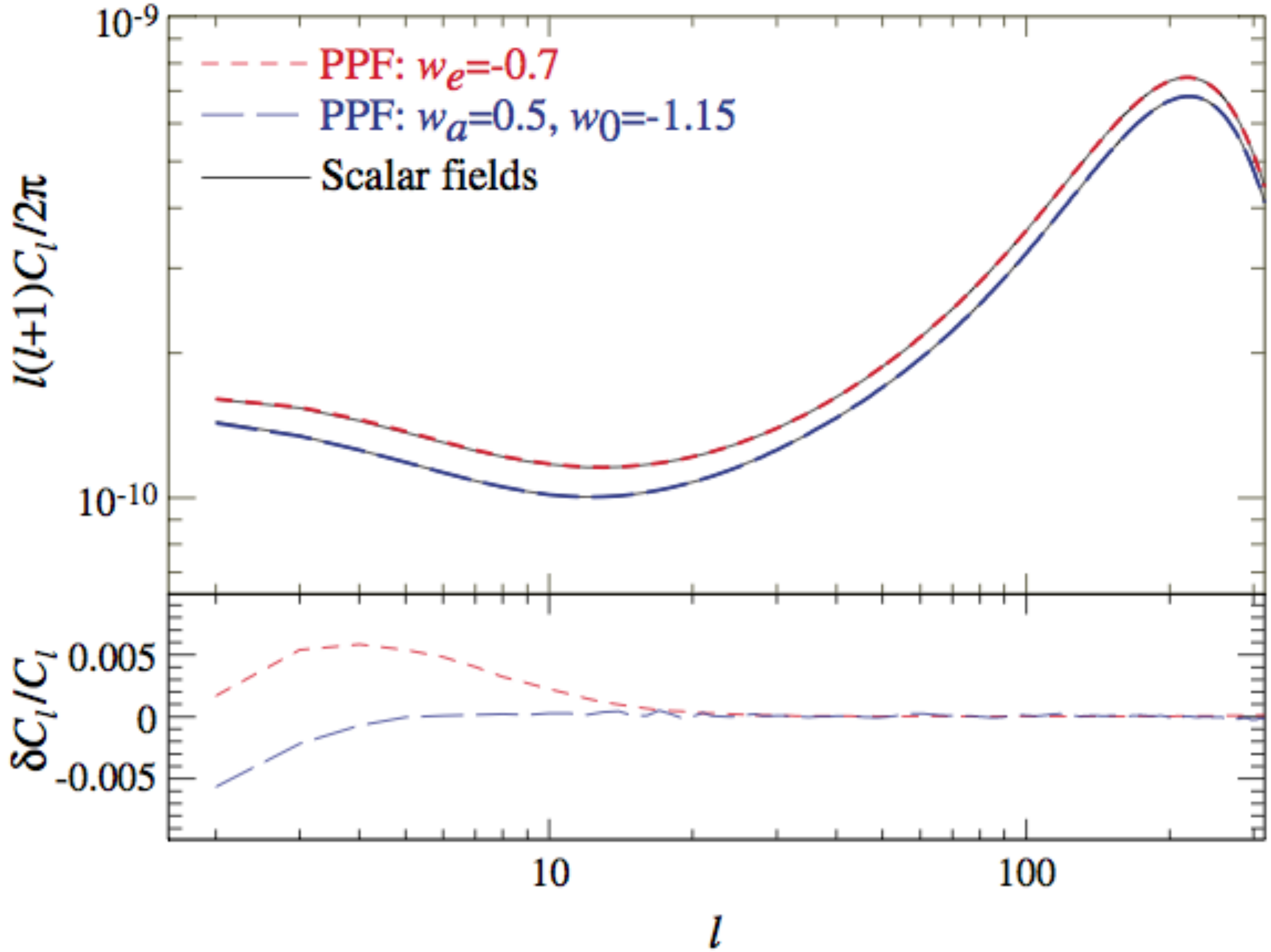
$$
\begin{aligned}
g(\eta, k) &= 0 \\
f_\zeta(\eta) &= 0 \\
f_G(\eta) &= 0 \\
c_\Gamma &= 0.4\hat{C}_s
\end{aligned}
$$

- Accurate description for the phenomenology of "smooth" dark energy.

PPF for True DE



PPF for True DE:

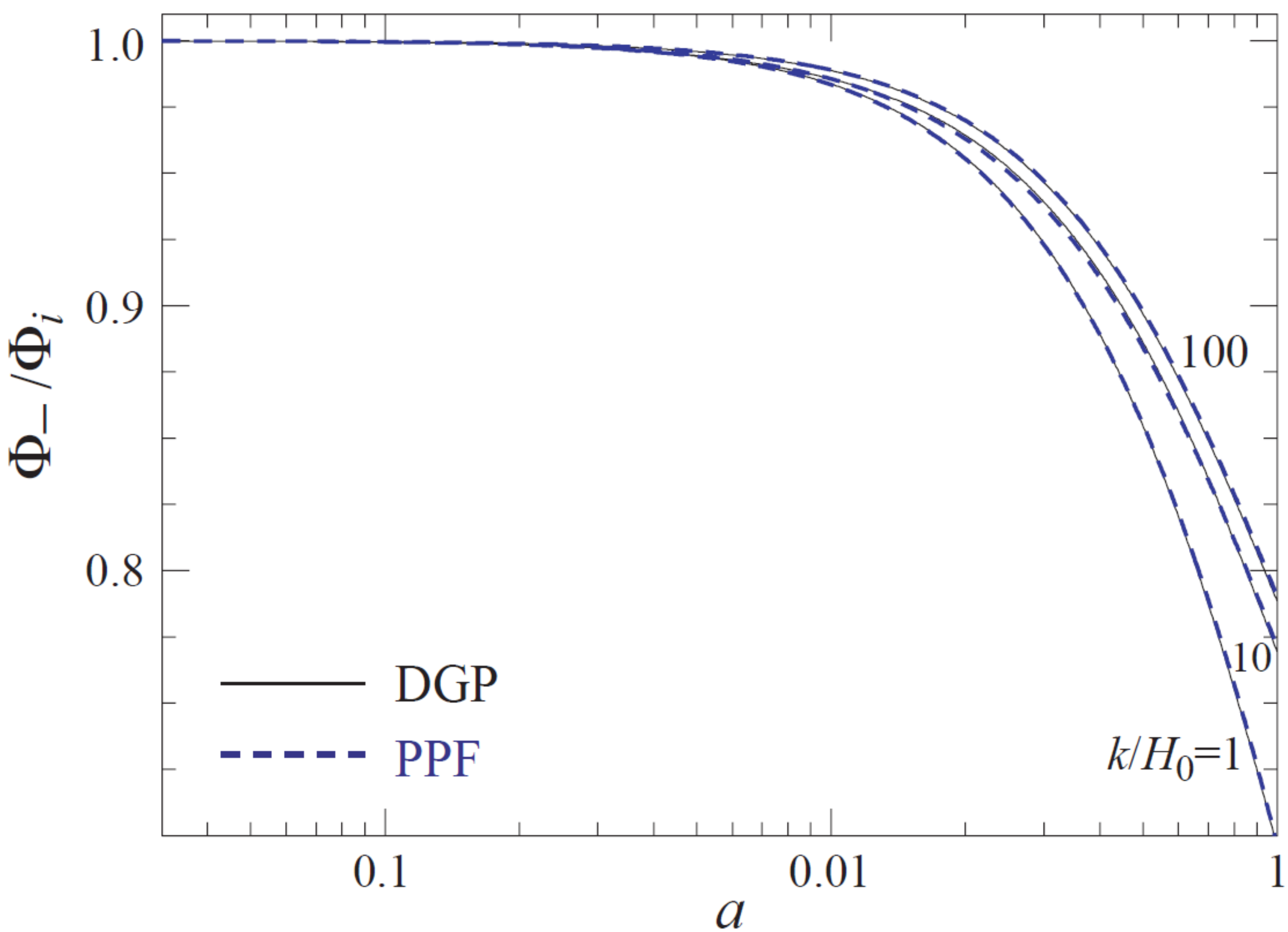


Advantage: 1. general w can be t-dependent, and can evolve across w=-1.
2. easy to implement in a likelihood analysis.

PPF (DE) Module for CAMB

- Download at http://camb.info/ppf/
- Default code supports two options for DE:
  - (w_0, w_a) parameterization.
  - read $w(a)$ from a user-supplied file.
- Modify module LambdaGeneral to specify any analytical form for $w(a)$, $\rho_{\mathrm{DE}}(a)$
- Modify modules LambdaGeneral and ppf to solve linear perturbations in modified gravity.

PPF for Modified Gravity: DGP



PPF Parameterization for DGP

$$g_{\rm SH}(\eta) = \frac{9}{8Hr_c - 1}\left(1 + \frac{0.51}{Hr_c - 1.08}\right),$$

$$g_{\rm QS}(\eta) = -\frac{1}{3}\left[1 - 2Hr_c\left(1 + \frac{\dot{H}}{3aH^2}\right)\right]^{-1},$$

$$g(\eta, k) = \frac{g_{\rm SH} + g_{\rm QS}(c_g k_H)^{n_g}}{1 + (c_g k_H)^{n_g}}$$

$$f_\zeta(\eta) = 0.4 g_{\rm SH}(\eta),$$

$$f_G(\eta) = 0 \qquad c_\Gamma = 1,$$

Exercise 2: Modify PPF to Calculate DGP

- 2.1: modify LambdaGeneral to include background evolution with

$$w_e = \frac{\sum_i(\rho_i + P_i)}{3(H^2 + a^{-2}K)/8\pi G + \sum_i \rho_i} - 1.$$

$$\rho_e \equiv \frac{3}{8\pi G}\left(H^2 + \frac{K}{a^2}\right) - \sum_i \rho_i.$$

Sum is over all true components, no effective DE.

- 2.2: modify ppf module to include ppf functions for DGP.

Reference: Fang et al 2008, arXiv:0808.2208

References

- DE perturbations:
  - Hu 1998
  - Weller & Lewis 2003
  - Bean & Dore 2004
- PPF framework
  - Hu & Sawicki 2007
  - Hu 2008
- Incorporate PPF into CAMB: Fang et al 2008
- PPF DE: Fang, Hu & Lewis 2008