MGCAMB
CAMB works for Modified Gravity

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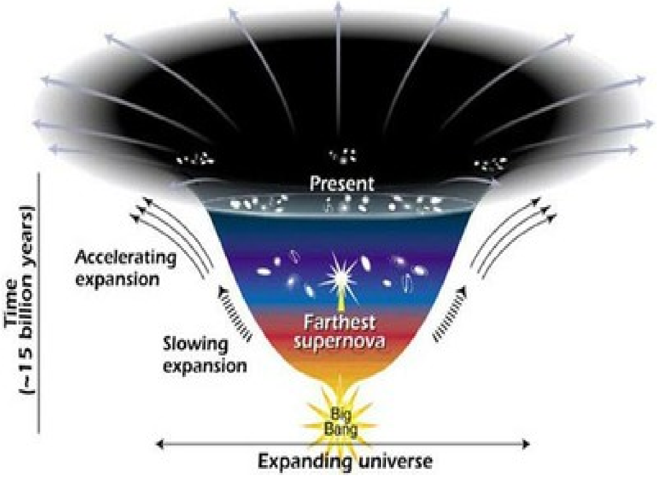
http://www.sfu.ca/~aha25/MGCAMB.html

A breakthrough discovery in physics

The **accelerating expansion** of the spacetime of Universe



Discovery

Expansion rate of the Universe measured using supernovae 1998-99 (Nobel prize 2011)

Confirmation

Cosmic Microwave Background measurements, 2003

Clustering of galaxies measurements, 2004

The accelerating Universe **challenges** Einstein's General Relativity (GR)

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Spacetime curvature = Matter distribution

General Relativity

↓

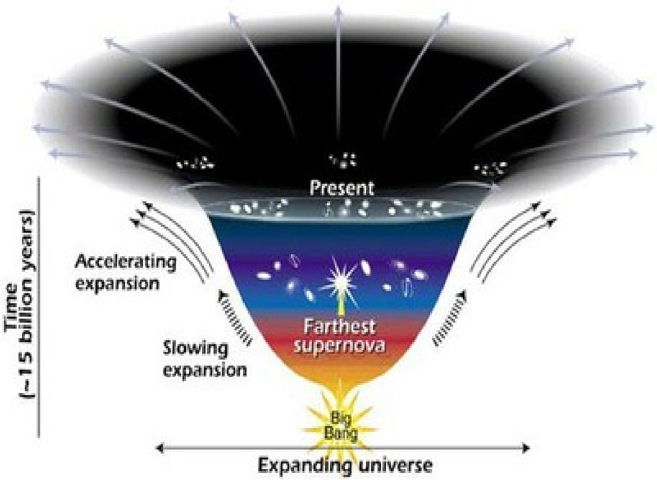
Attractive force among matter particles

↓

a **decelerating** expansion of spacetime

A breakthrough discovery in physics

The **accelerating expansion** of the spacetime of Universe



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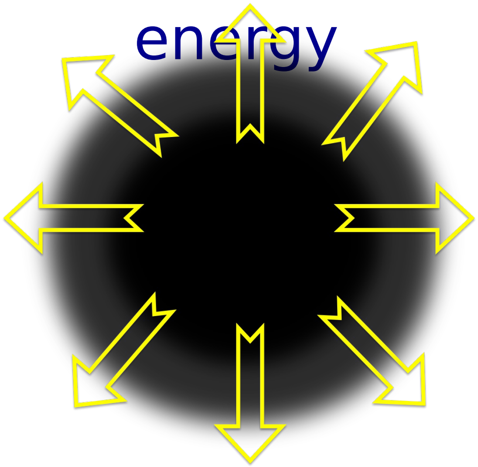
The accelerating Universe **challenges** Einstein's General Relativity (GR)

One of the most profound problems in science today!

The expansion of the Universe can **accelerate**

if

In GR, to add new 'repulsive matter', which contributes 70% total energy



To modify General Relativity

Dark Energy

$$G_{\mu\nu} = 8\pi G \widetilde{T}_{\mu\nu}$$

Modified Gravity

$$\widetilde{G}_{\mu\nu} = 8\pi G T_{\mu\nu}$$

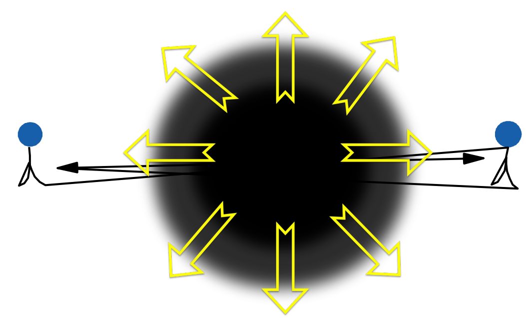
Modified Gravity

Resemble GR+L

Cosmological
scales
R>Gpc

Small scales

Large scales



Modified Gravity

Observationally testable feature

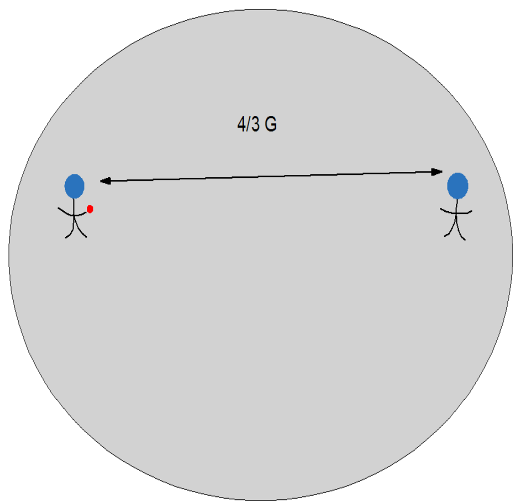
Resemble GR+L

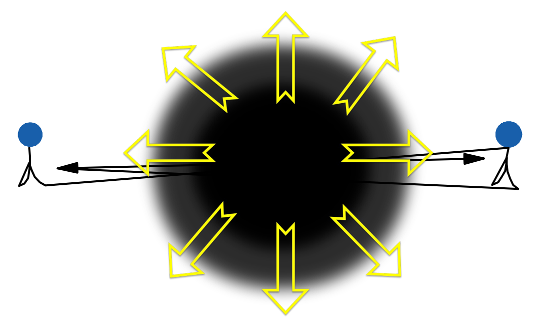
Structure formation scales
Mpc<R<Gpc

Cosmological scales
R>Gpc

Small scales

Large scales

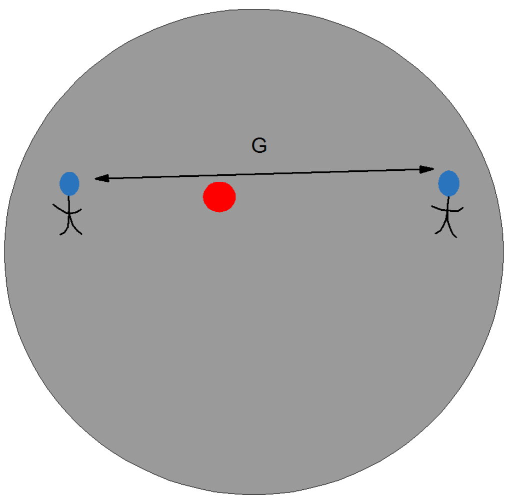


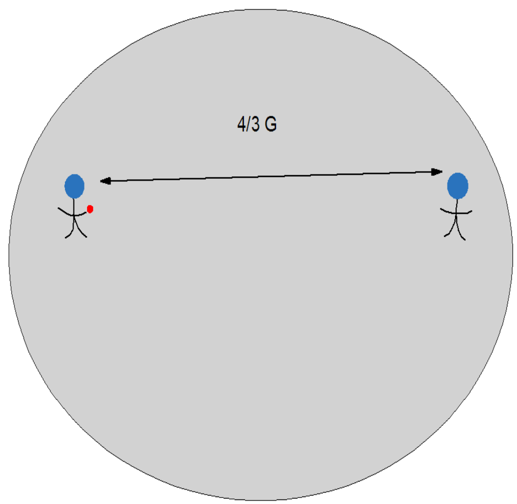


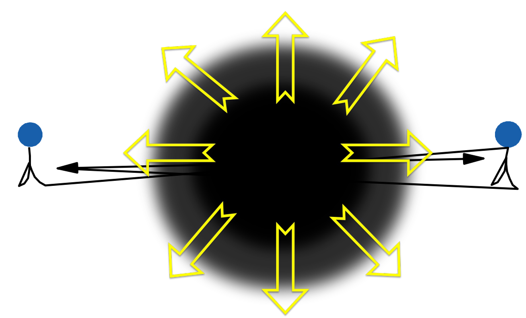
Modified Gravity

Recover GR	Observationally testable feature	Resemble GR+L
Galactic scales R<Mpc	Structure formation scales Mpc<R<Gpc	Cosmological scales R>Gpc

Small scales → Large scales







Modified Gravity

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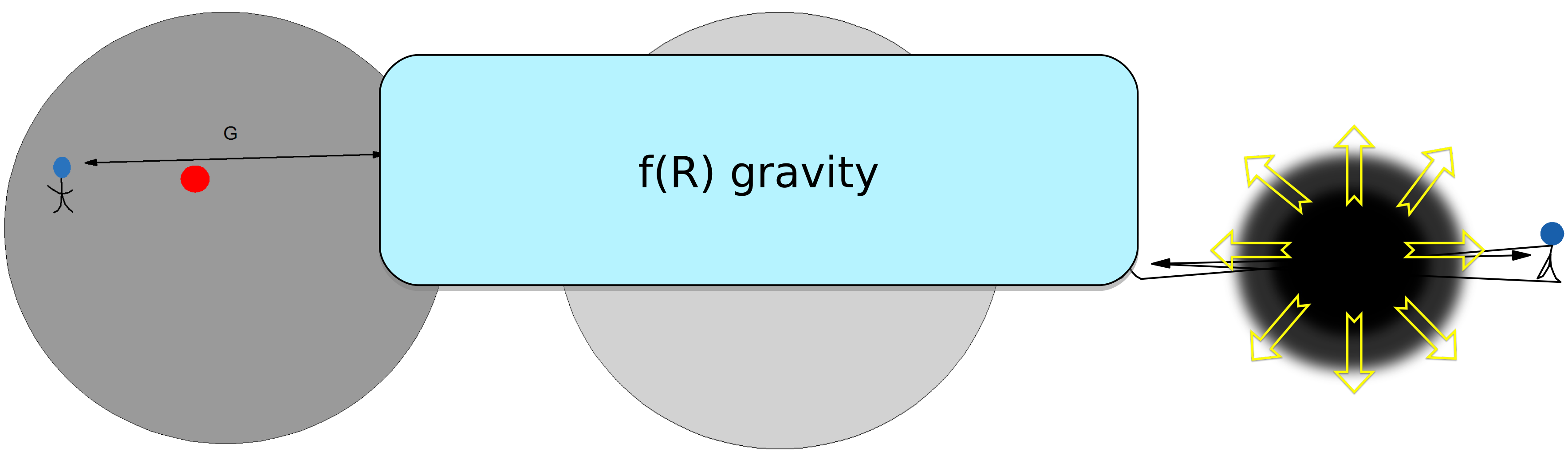

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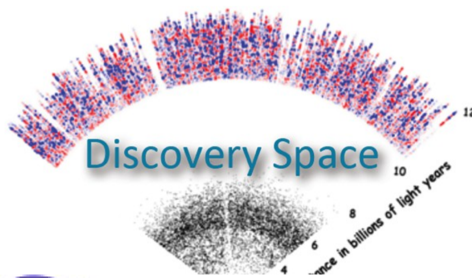
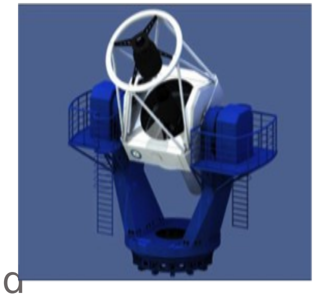
MANGA

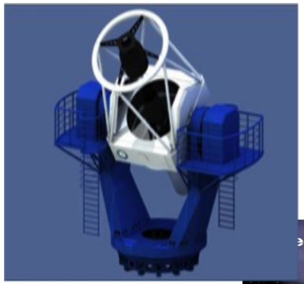
DARK ENERGY SURVEY



SDSS III

DARK ENERGY SURVEY





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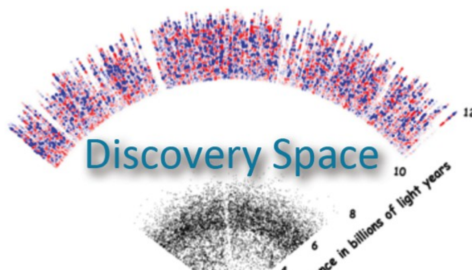
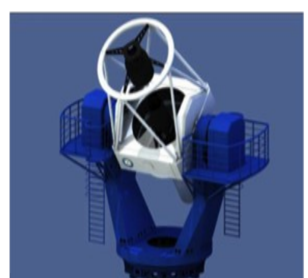
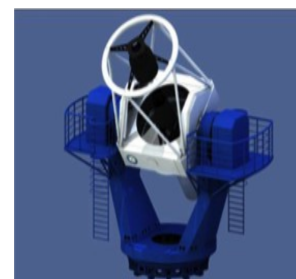


SDSS III

DARK ENERGY SURVEY





Cosmological tests of GR on **linear** scales

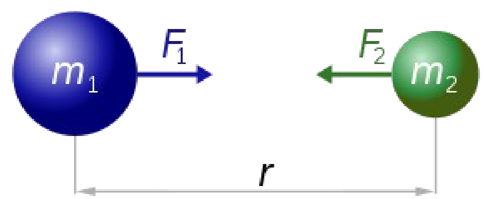
Galaxy scales | Structure formation scales | Cosmological scales

Small scales → Large scales

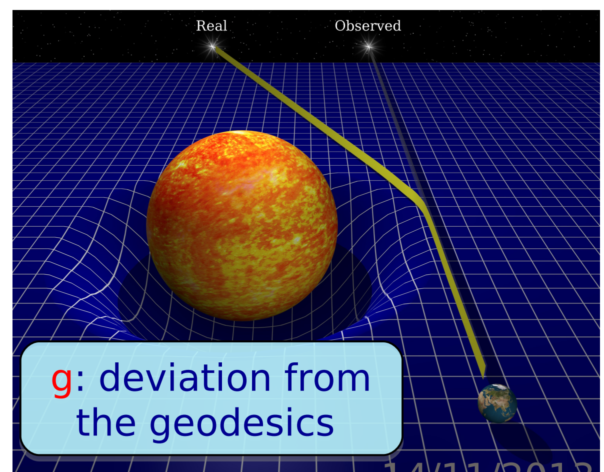
The deviation from GR is encoded in

$$k^2\Phi = -\mu(a,k)4\pi G a^2\rho\delta$$

$$\frac{\Phi}{\Psi} = \gamma(a,k)$$



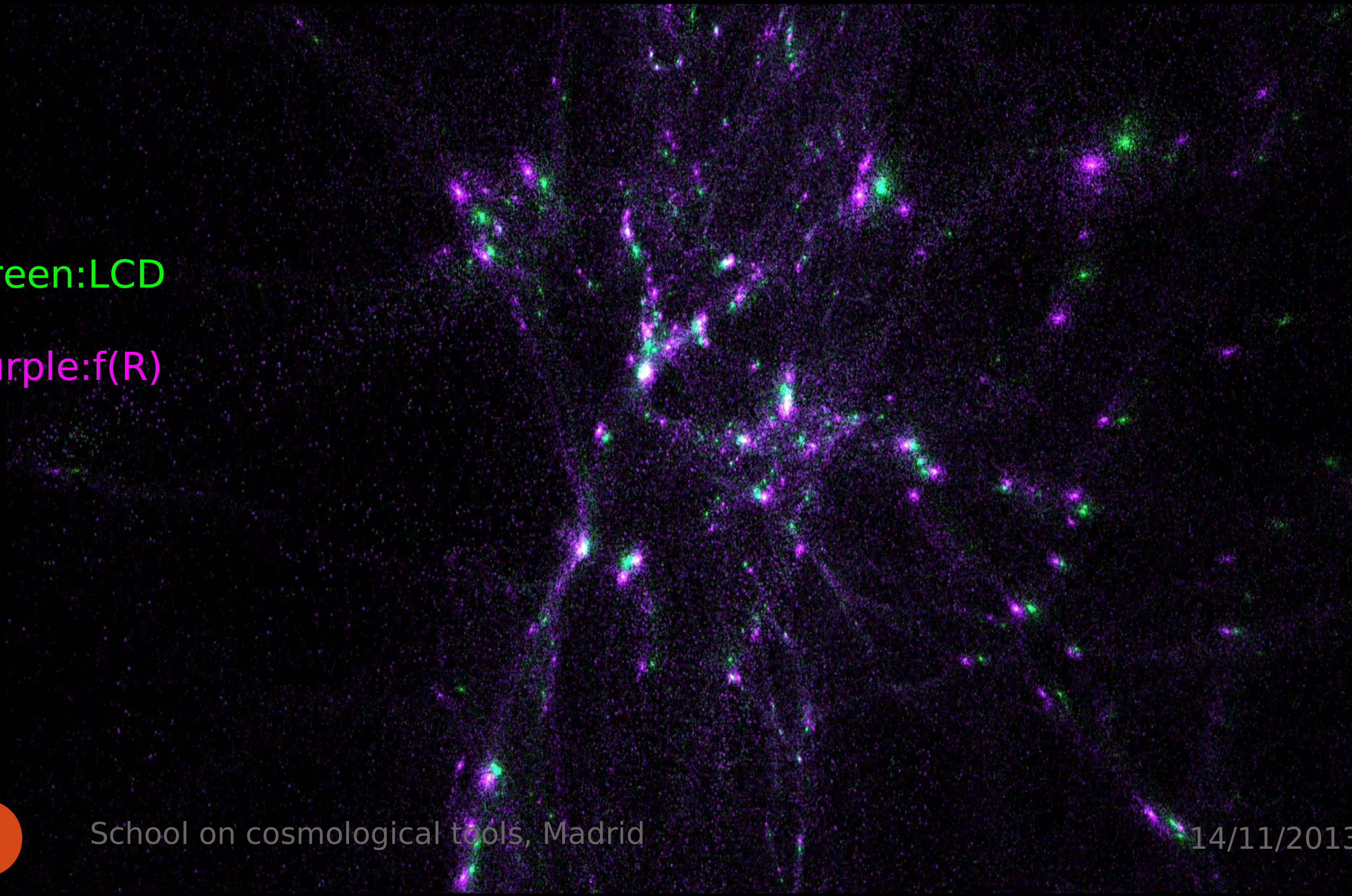
$$F_1 = F_2 = \mu G\frac{m_1 \times m_2}{r^2}$$



In GR, m=g=1
A smoking gun of modified gravity:
m and/or g deviates from 1

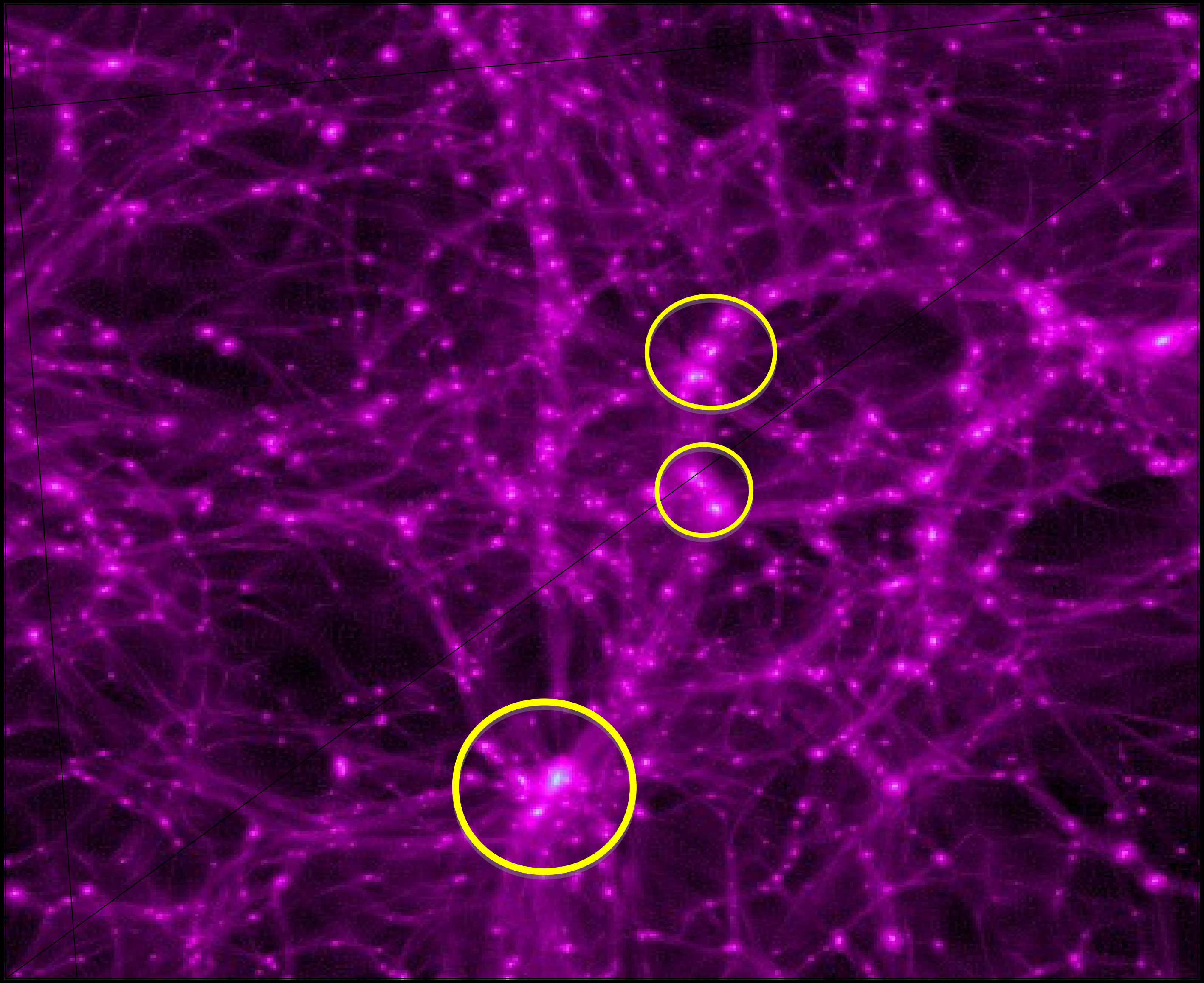
Galaxy scales | Structure formation scales | Cosmological scales

High-resolution numerical simulations, **GBZ et al, Phys. Rev. D 2010**

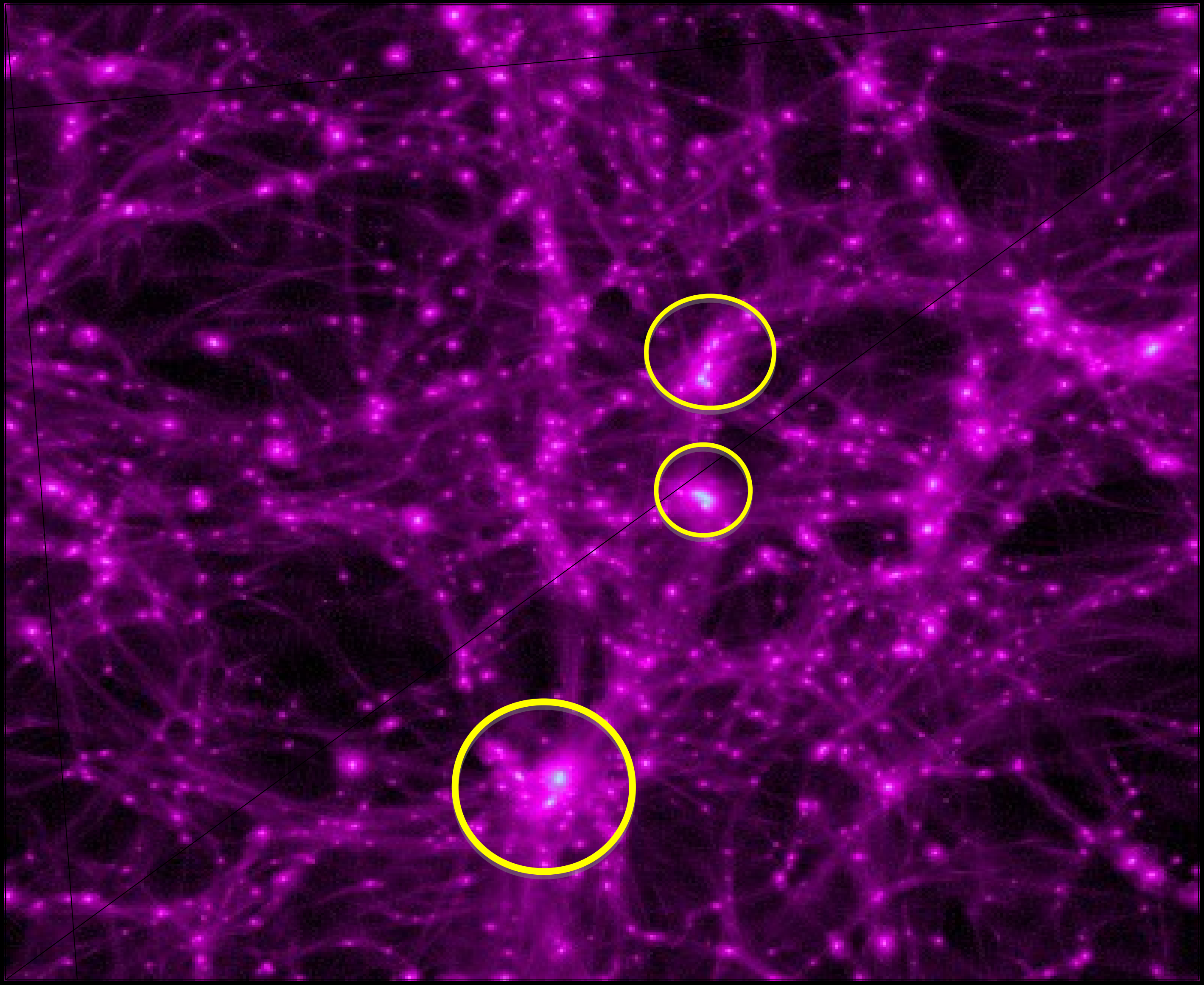


Green:LCDM
Purple:f(R)

GR



f(R)



Linear perturbation in FRW universe

$$ds^2 = -a^2(\eta)[(1+2\Psi(\vec{x},\eta))d\eta^2 - (1-2\Phi(\vec{x},\eta))d\vec{x}^2]$$

$$\nabla_\mu T^{\mu\nu} = 0 \Longrightarrow \begin{aligned} \Phi' &= \frac{1}{3}(\delta' + \frac{k}{aH}v) \\ \Psi &= \frac{aH}{k}(v' + v) \end{aligned}$$

General Relativity / **Modified Gravity** $\Longrightarrow$

$$\begin{aligned} k^2\Phi &= -\mu(a,k)4\pi G a^2 \rho\delta \\ \frac{\Phi}{\Psi} &= \gamma(a,k) \end{aligned}$$

In default CAMB (GR)

CAMB code	M+B ' 96
	astro-ph/9506072
$\eta' k = \mathrm{dgq}/2$	$\eta' k^2 = 4\pi G a^2(\bar{\rho} + \bar{P})\theta$
$\mathrm{clxcdot} = -kz$	$\delta'_c = -\frac{1}{2}h'$
$z = (0.5\mathrm{dgrho}/k + \eta k)/\mathrm{adotoa}$	$k^2\eta - \frac{1}{2}\frac{a'}{a}h' = 4\pi G a^2 \delta T_0^0$
$\mathrm{sigma} = z + 1.5\mathrm{dgq}/k^2$	$\sigma = \frac{h' + 6\eta'}{2k}$

Differential equations to evolve in CAMB

Constraint equations (algebraic)

➔

In MG (arXiv:1106.4543)

$$k^2\Psi = -\mu(k,a)4\pi G a^2\{\rho\Delta + 3(\rho + P)\sigma\}\,,$$
$$k^2[\Phi - \gamma(k,a)\Psi] = \mu(k,a)12\pi G a^2(\rho + P)\sigma\,,$$

$$\Psi = \dot{\alpha} + \mathcal{H}\alpha\,,$$
$$\Phi = \eta - \mathcal{H}\alpha\,,$$

$$\alpha = (\dot{h} + 6\dot{\eta})/2k^2$$

$$k^2(\dot{\alpha} + \mathcal{H}\alpha) = -\frac{\kappa}{2}\mu(k,a)\{\rho\Delta + 3(\rho + P)\sigma\}\,,$$
$$\eta - \mathcal{H}\alpha - \gamma(\dot{\alpha} + \mathcal{H}\alpha) = \frac{3\kappa}{2k^2}\mu(\rho + P)\sigma\,,$$

$$\alpha = \left\{ \eta + \frac{\mu\kappa}{2k^2} \left[\gamma\rho\Delta + 3(\gamma - 1)(\rho + P)\sigma \right] \right\} / \mathcal{H} \, ,$$

$$\eta = \mathcal{H}\alpha - \frac{\mu\kappa\rho}{2k^2}\Gamma \, , \qquad \Gamma = \gamma\Delta + 3(1+w)\sigma(\gamma - 1) \, .$$

$$\dot{\eta} = \dot{\mathcal{H}}\alpha + \mathcal{H}\dot{\alpha} - \frac{\mu\kappa\rho}{2k^2} \{ 2\mathcal{H}\Gamma - 3\mathcal{H}(1+w)\Gamma + \frac{\dot{\mu}}{\mu}\Gamma + \dot{\Gamma} \} \, .$$

$$\dot{\eta} = \frac{\kappa\rho}{2\mathcal{D}}\{(1+w)\left[\mu\gamma\theta\left(1+\frac{3\kappa\rho}{2k^2}(1+w)\right)+k^2\alpha(\mu\gamma-1)\right]+\Delta\left[\mu(\gamma-1)\mathcal{H}-\dot{\mu}\gamma-\dot{\gamma}\mu\right]$$
$$+\ 3\dot{\sigma}(1+w)(1-\gamma)\mu+3\sigma(1+w)\left[3w\mu(\gamma-1)\mathcal{H}-(\gamma-1)\dot{\mu}-\mu\dot{\gamma}\right]\}\ ,$$

$$\mathcal{D} = k^2+\frac{3\kappa}{2}\gamma\mu\rho(1+w)\ .$$ (MGCAMB)

$$\dot{\eta}=\frac{\kappa\rho}{2k^2}(1+w)\theta$$ (CAMB)

$$(\mu=\gamma=1)$$

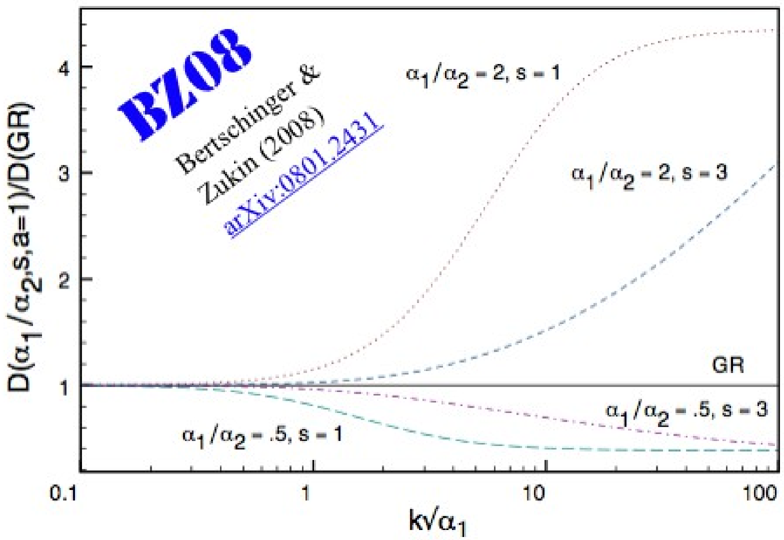
Example 1:
(model=1, 0809.3791)

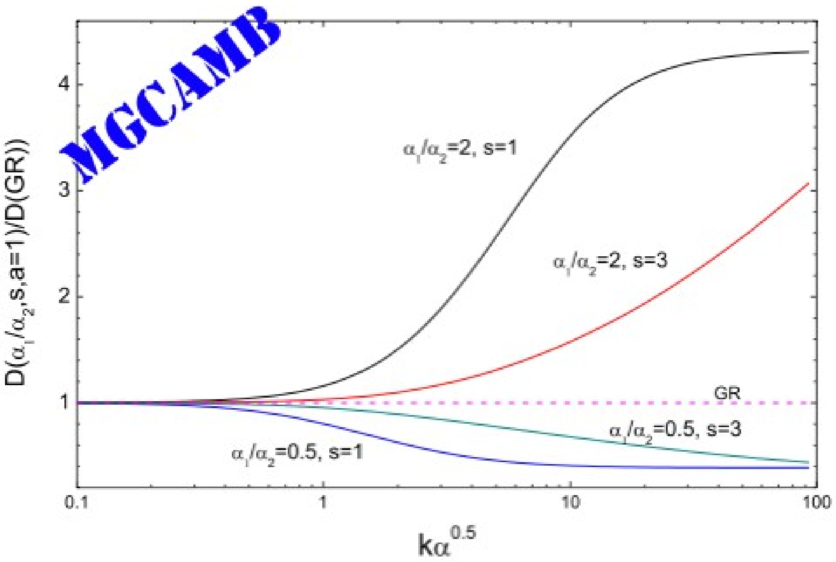
$$\mu(a,k)=\frac{1+\alpha_1 k^2 a^s}{1+\alpha_2 k^2 a^s},\quad \gamma(a,k)=\frac{1+\beta_1 k^2 a^s}{1+\beta_2 k^2 a^s}$$

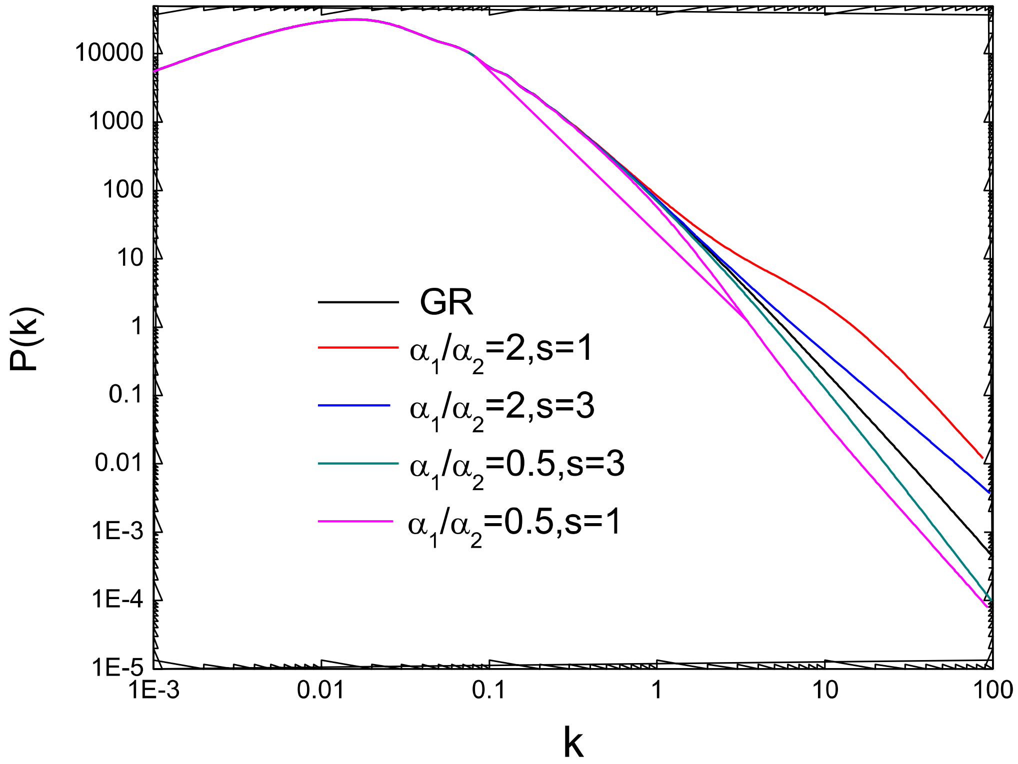
$$S_E=\int d^4x\sqrt{-\tilde{g}}\left[\frac{M_P^2}{2}\tilde{R}-\frac{1}{2}\tilde{g}^{\mu\nu}(\tilde{\nabla}_\mu\phi)\tilde{\nabla}_\nu\phi-V(\phi)\right]+S_i\left(\chi_i,e^{-\kappa\alpha_i(\phi)}\tilde{g}_{\mu\nu}\right).$$

$$\mu(a,k)=\frac{1+\left(1+\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}{1+\frac{k^2}{a^2m^2}}$$

$$\gamma(a,k)=\frac{1+\left(1-\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}{1+\left(1+\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}.$$


BZ08
Bertschinger &
Zukin (2008)
arXiv:0801.2431
$\alpha_1/\alpha_2 = 2, s = 1$
$\alpha_1/\alpha_2 = 2, s = 3$
GR
$\alpha_1/\alpha_2 = .5, s = 3$
$\alpha_1/\alpha_2 = .5, s = 1$
$D(\alpha_1/\alpha_2, s, a=1)/D(GR)$
$k\sqrt{\alpha_1}$


MGCAMB
$\alpha_1/\alpha_2=2, s=1$
$\alpha_1/\alpha_2=2, s=3$
GR
$\alpha_1/\alpha_2=0.5, s=3$
$\alpha_1/\alpha_2=0.5, s=1$
$D(\alpha_1/\alpha_2, s, a=1)/D(GR))$
$k\alpha^{0.5}$



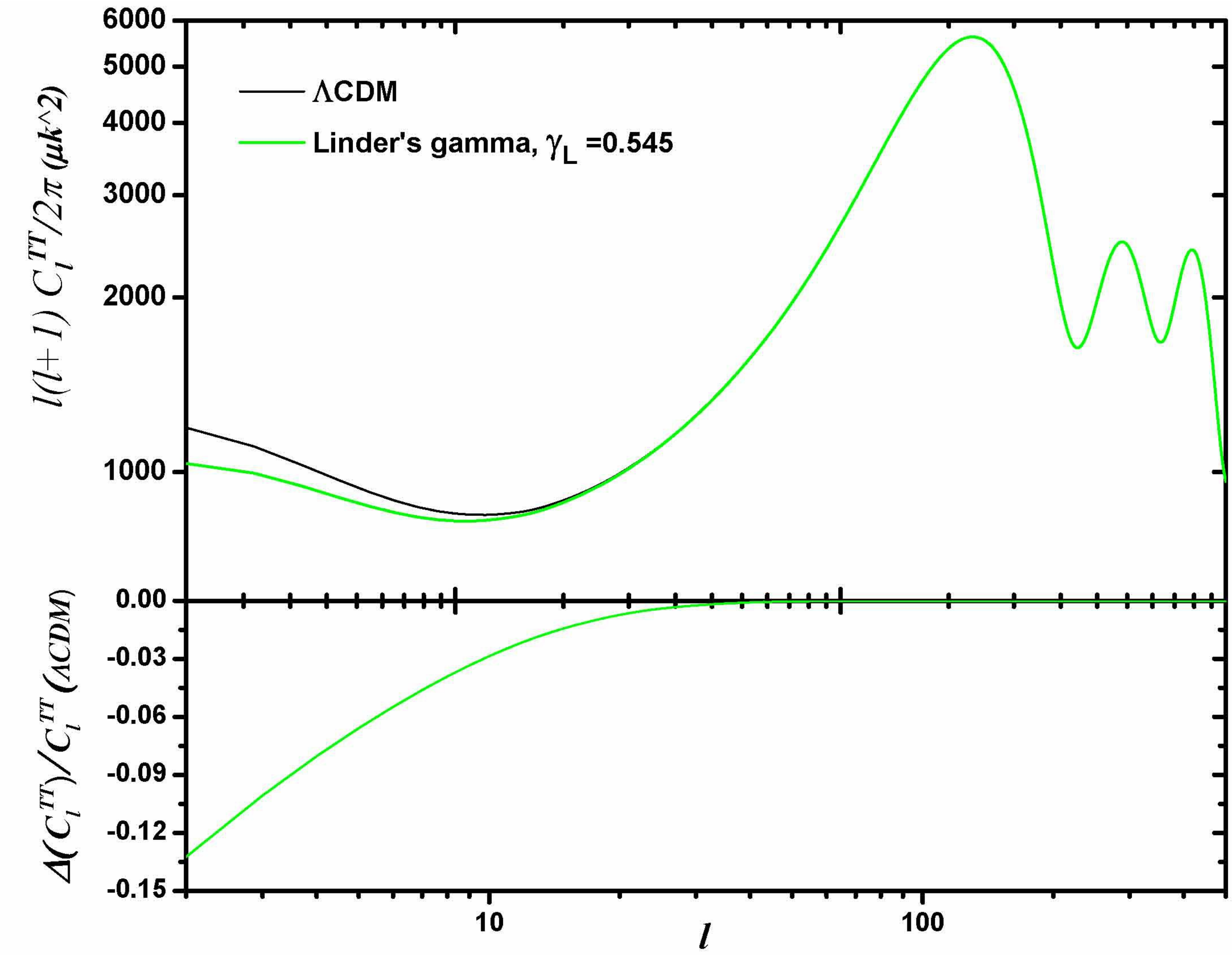
10000
1000
100
10
1
0.1
0.01
1E-3
1E-4
1E-5
P(k)
1E-3
0.01
0.1
1
10
100
k
GR
α1/α2=2,s=1
α1/α2=2,s=3
α1/α2=0.5,s=3
α1/α2=0.5,s=1

Example 2: Linder's g (model=6, astro-ph/0701317)

$$f \equiv \frac{d}{d\ln a}\left(\ln\frac{\Delta(k,a)}{\Delta(k,a_i)}\right) = [\Omega_m(a)]^{\gamma_L}$$

$$\mu = \frac{2}{3}\Omega_m^{\gamma-1}\left[\Omega_m^{\gamma} + 2 + \frac{H'}{H} + \gamma\frac{\Omega_m'}{\Omega_m} + \gamma'\ln(\Omega_m)\right]$$

$$\mu = \frac{2}{3}\Omega_m^{\gamma-1}\left[\Omega_m^{\gamma} + 2 - 3\gamma + 3\left(\gamma - \tfrac{1}{2}\right)\Omega_m + \gamma'\ln(\Omega_m)\right] \quad \text{(LCDM)}$$


ΛCDM
Linder's gamma, γ_L =0.545
$l(l+1)\,C_l^{TT}/2\pi\,(\mu k^2)$
6000
5000
4000
3000
2000
1000
0.00
-0.03
-0.06
-0.09
-0.12
-0.15
$\Delta(C_l^{TT})/C_l^{TT}(\Lambda CDM)$
10
100
l

Example 3: f(R) gravity [Hu-Sawicki model] (model=8, 0705.1158)

$$S = \int d^4x\sqrt{-g}\left[\frac{R+f(R)}{2\kappa^2} + \mathcal{L}_{\rm m}\right]$$

$$f(R) = -m^2\frac{c_1(R/m^2)^n}{c_2(R/m^2)^n+1},$$

$$G_{\alpha\beta} + f_R R_{\alpha\beta} - \left(\frac{f}{2} - \Box f_R\right)g_{\alpha\beta} - \nabla_\alpha\nabla_\beta f_R = \kappa^2 T_{\alpha\beta}$$

$$3\Box f_R - R + f_R R - 2f = -\kappa^2\rho\,.$$

$$\nabla^2 \delta f_R = -\frac{a^2}{3}[\delta R(f_R) + 8\pi G \delta\rho_{\rm M}],$$

$$\nabla^2 \Phi = \frac{16\pi G}{3} a^2 \delta\rho_{\rm M} + \frac{a^2}{6}\delta R(f_R),$$

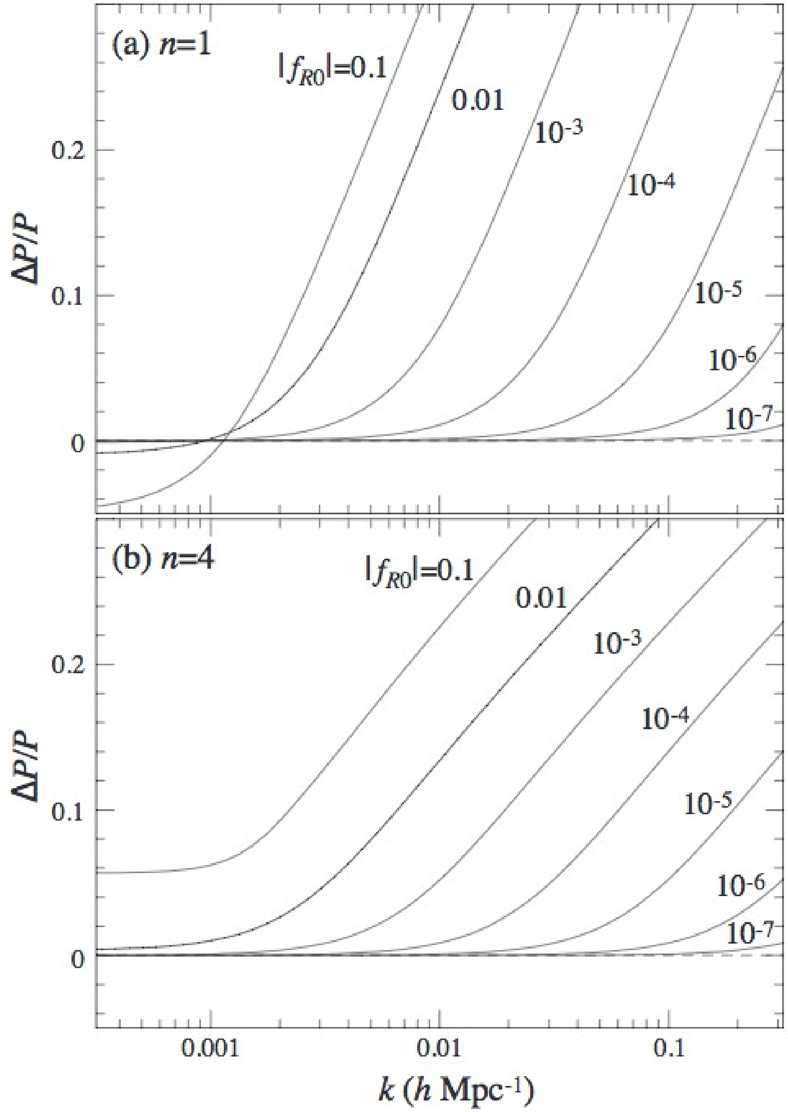
$$f_R = -\frac{c_1}{c_2^2}\frac{n(-R/m^2)^{n-1}}{[(-R/m^2)^n + 1]^2} \approx -\frac{n c_1}{c_2^2}\left(\frac{m^2}{-R}\right)^{n+1}.$$

$$\nabla^2 \delta f_R = a^2 \bar{\mu}^2 \delta f_R - \frac{8\pi G}{3} a^2 \delta\rho_{\rm M},$$

$$\bar{\mu} = \lambda_c^{-1} = \left(\frac{1}{3(n+1)}\frac{\bar{R}}{|\bar{f}_{R0}|}\left(\frac{\bar{R}}{\bar{R}_0}\right)^{n+1}\right)^{1/2}.$$

$$\mu = \frac{4}{3} - \frac{(\bar{\mu}a)^2}{3[k^2 + (\bar{\mu}a)^2]}$$

$$\gamma = 1 - \frac{2k^2}{[3(\bar{\mu}a)^2 + 4k^2]}$$


(a) n=1
|f_R0|=0.1
0.01
10-3
10-4
10-5
10-6
10-7
(b) n=4
|f_R0|=0.1
0.01
10-3
10-4
10-5
10-6
10-7
ΔP/P
0.2
0.1
0
0.001
0.01
0.1
k (h Mpc-1)

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