TRIPLET CONDENSED NEUTRON MATTER AND ANGULONS

P. Bedaque, U. of Maryland
(A. Nicholson, S. Reddy, G. Rupak, M. Savage, S. Sen)

all nuclear fuel is spent:
star death

gravitational collapse

low mass

e$^-$ degeneracy pressure
stops collapse:
white dwarf

medium mass

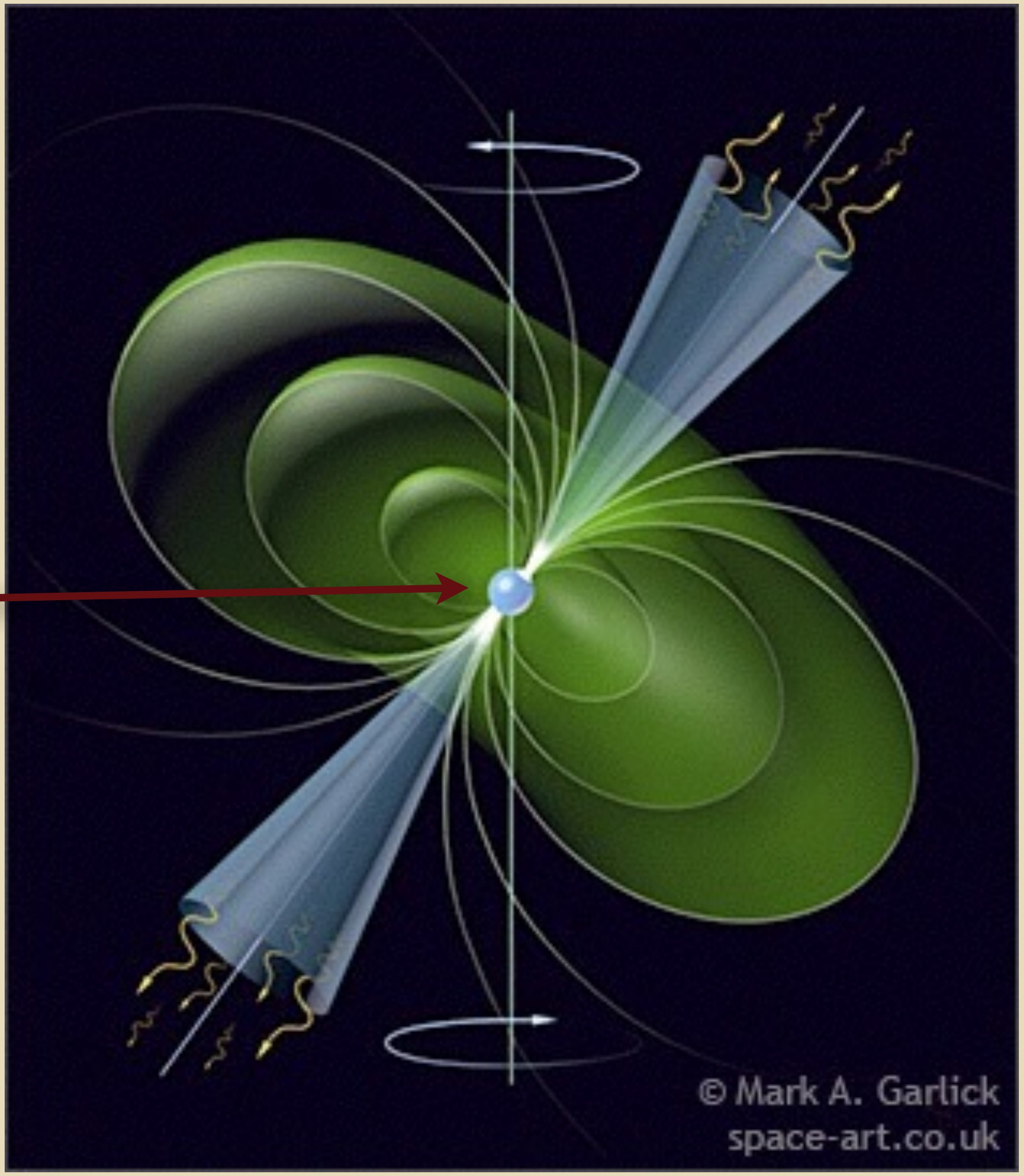
nuclear force
stops collapse:
neutron stars

high mass

nothing stops
collapse:
black hole

Pulsar = rotating, magnetized giant nucleus

neutrons at nuclear densitites and above



Early theoretical predictions:
Landau 1932
Baade & Zwicki 1933
Pacini 1965
...

Hard to imagine how an star can vibrate that fast
Fast spinning things have to be dense

$$\underbrace{\frac{v^2}{R}}_{\omega^2 R} = \frac{GM}{R^2} \Rightarrow \rho \approx \frac{M}{R^3} \approx \frac{\omega^2}{G}$$

$$\omega \approx 10^3 \; Hz \Rightarrow \rho \approx 10^{17} \frac{\text{Kg}}{\text{m}^3}$$

centripetal acceleration

gravity acceleration

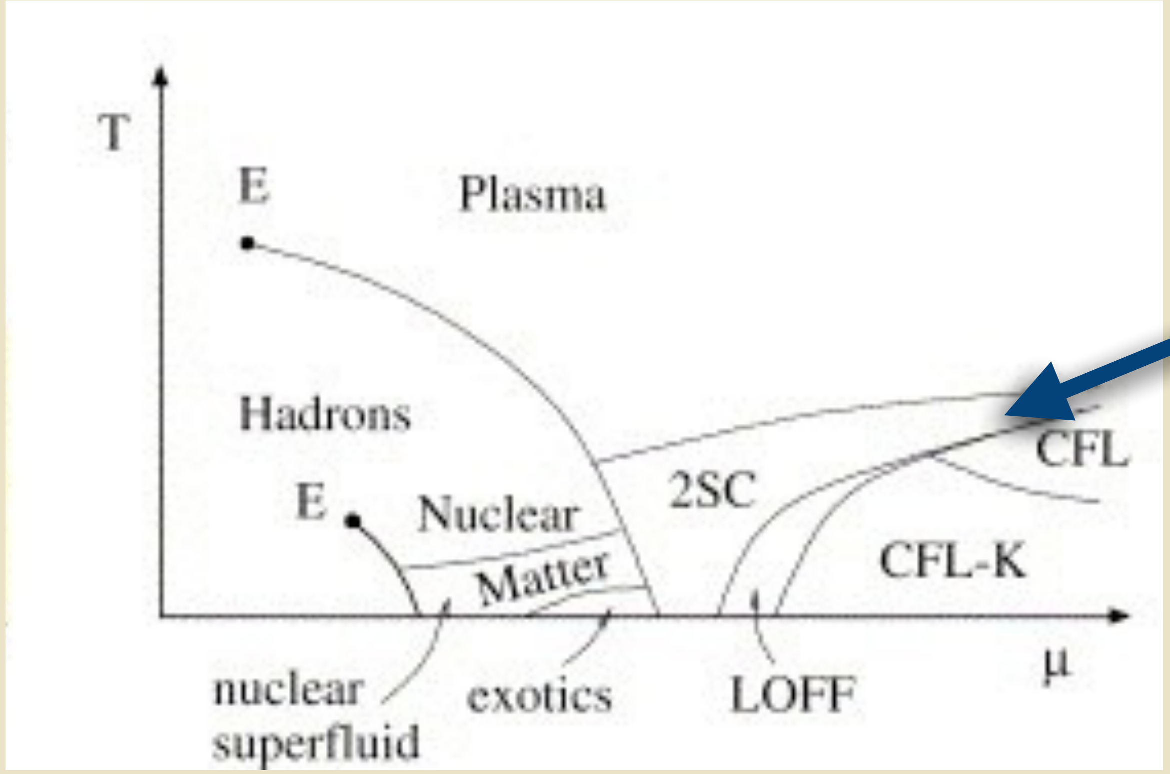
nuclear density

Why should I (a physicist) care?

density	$< 10^{15}\ \mathrm{g/cm^3}$	destroys atoms, nuclei and maybe hadrons
interparticle distance	< 0.5 fm	
temperature	$< 1\ \mathrm{MeV}\ (10^{10} K)$	<< Fermi energy, fairly cold, superfluid and superconductor
magnetic field	$< 10^{15}\ G$	disrupts atoms, but not nuclei/hadrons

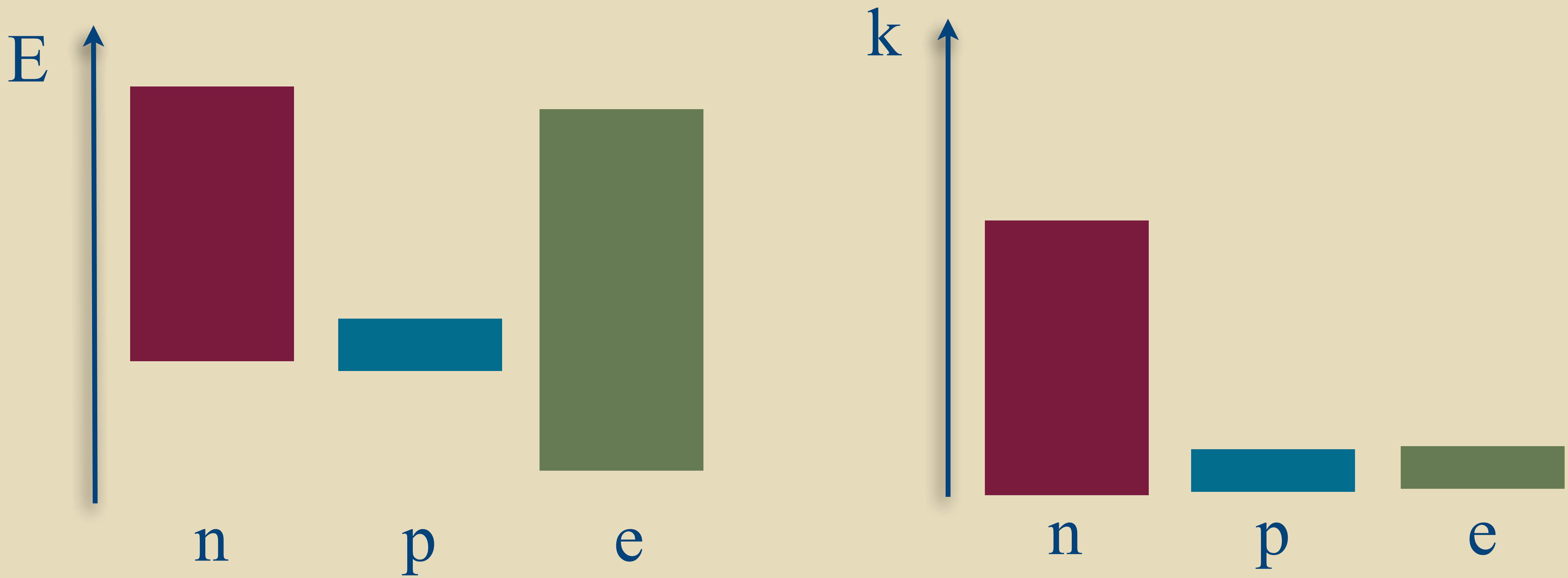
Nuclear forces (Quantum Chromodynamics) determines the properties of neutron matter but:

- energies not high enough for QCD to be perturbative
- lattice QCD fails due to sign problem



"made up stuff"

Dense matter is neutron matter



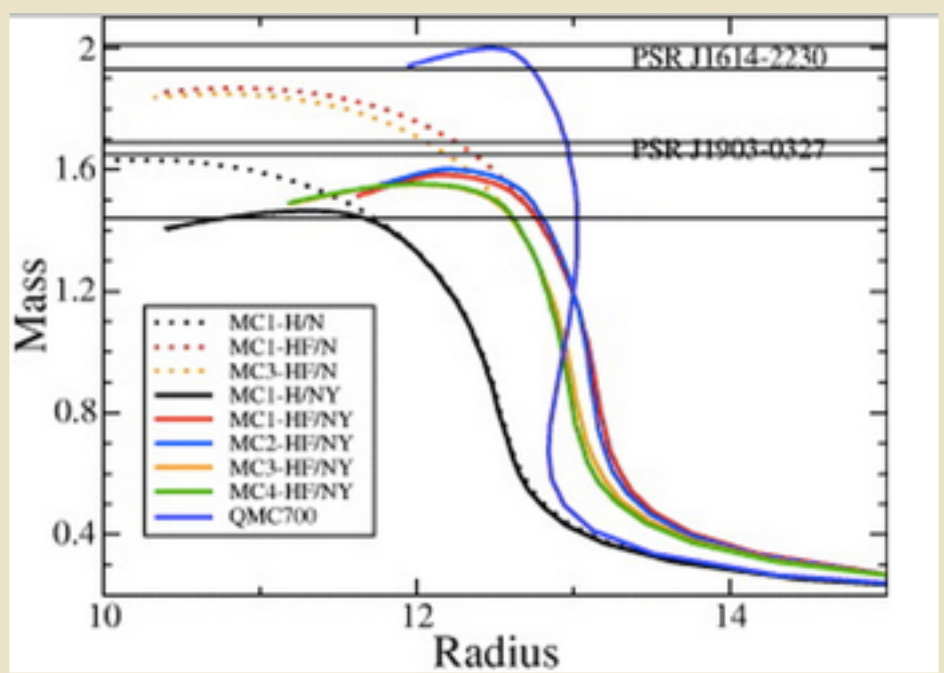
Neutron matter is universal: *anything* compressed to these densities turns into the same kind of matter

Equation of state: mass x radius

QCD

nucleon-nucleon scattering

→ nuclear forces → energy density x density → mass x radius



Other observables probe more subtle aspects of dense matter

cooling curves
(anti) glitches
flare oscillations
r-modes
gravitational waves from mergers
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neutrino emissivity
specific heat
viscosities
heat conduction
superfluidity/conductivity
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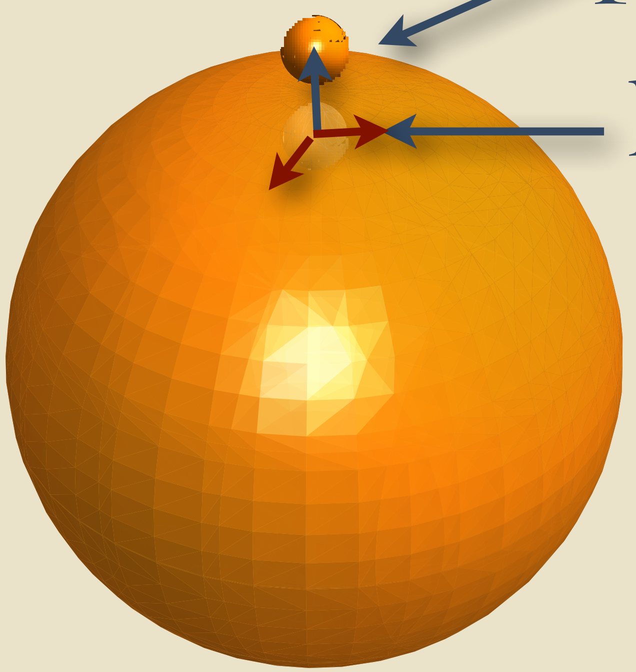
←

low lying excitations

Dense neutron matter is superfluid

The theory says so:

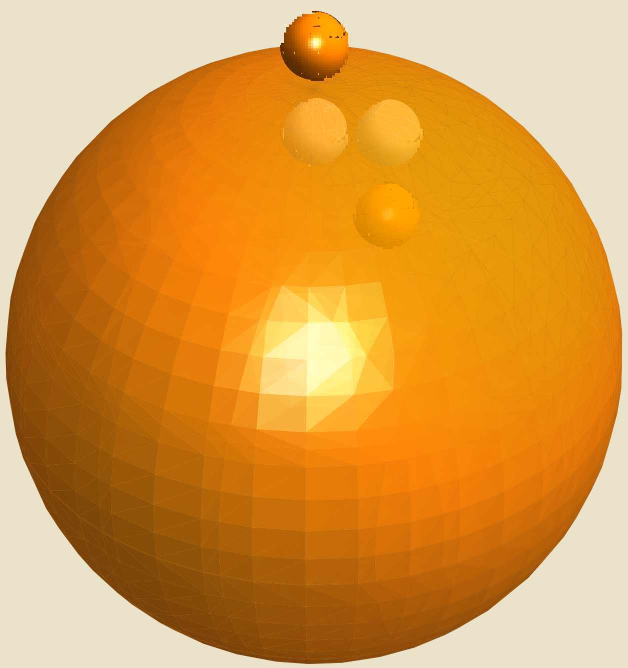
Effectively one dimensional



Dense neutron matter is superfluid

The theory says so:

scattering of nearby particles



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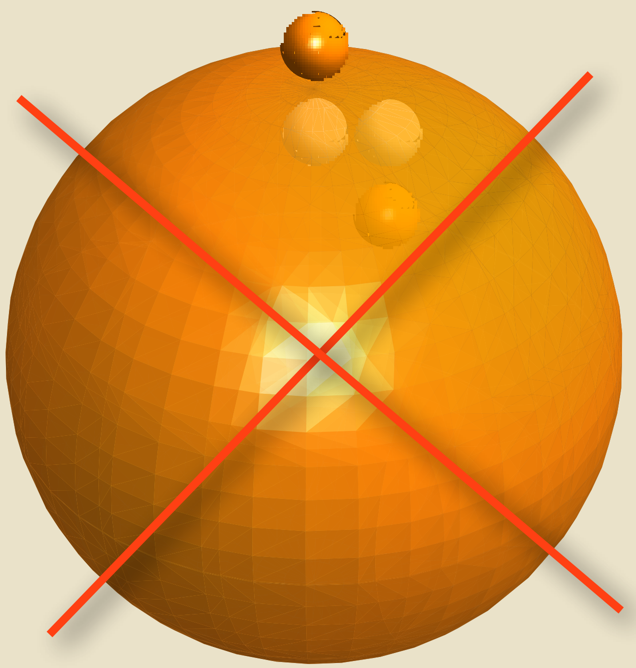
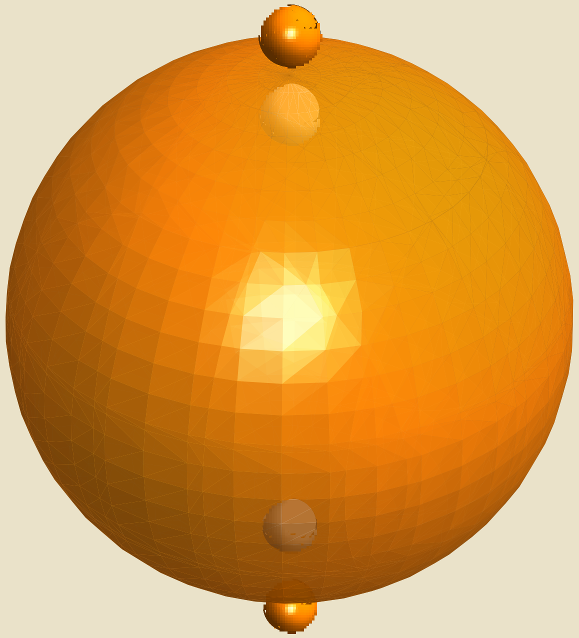


Dense neutron matter is superfluid

The theory says so:

scattering of antipodal particles

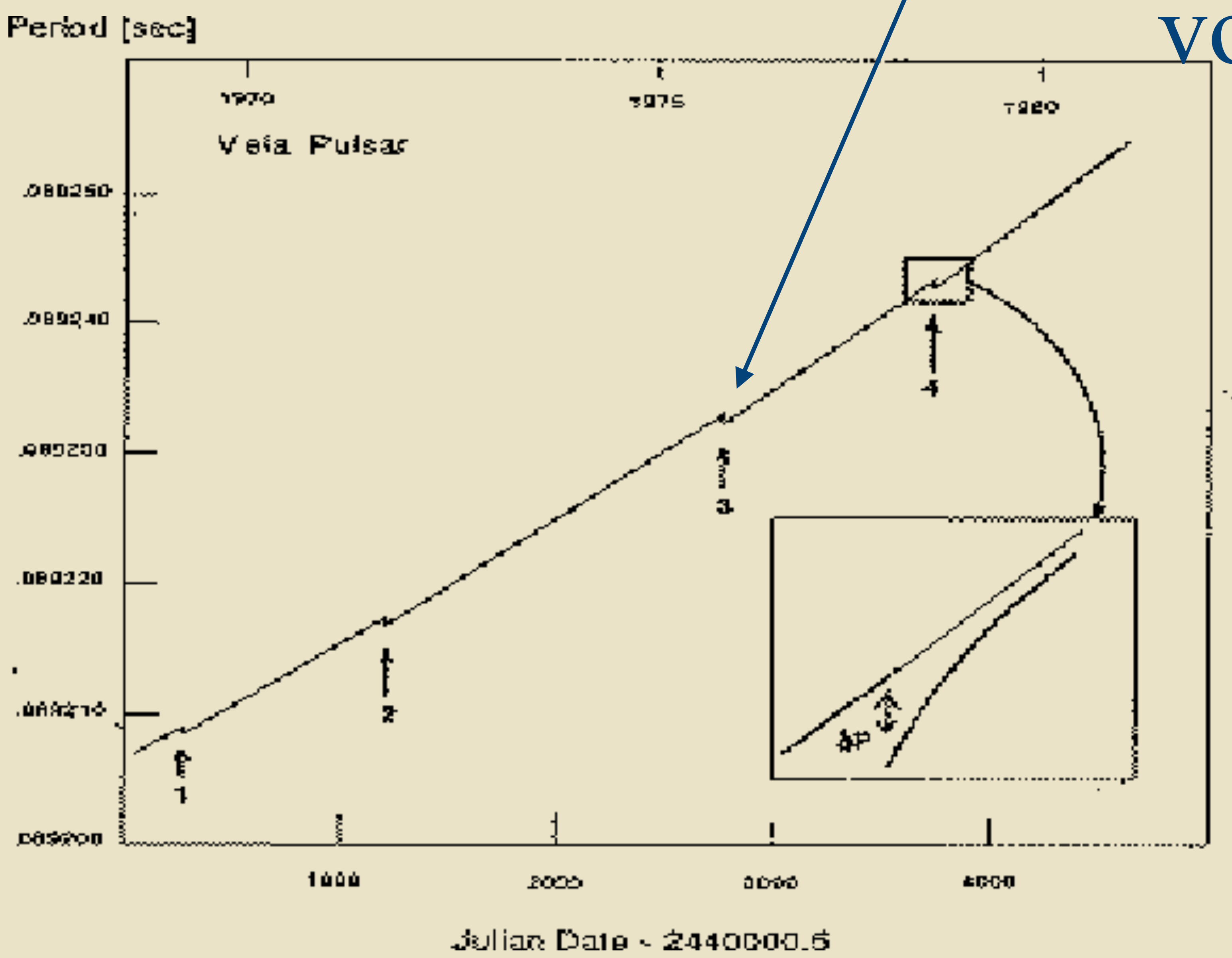


attraction leads to Cooper pairing

Dense neutron matter is superfluid

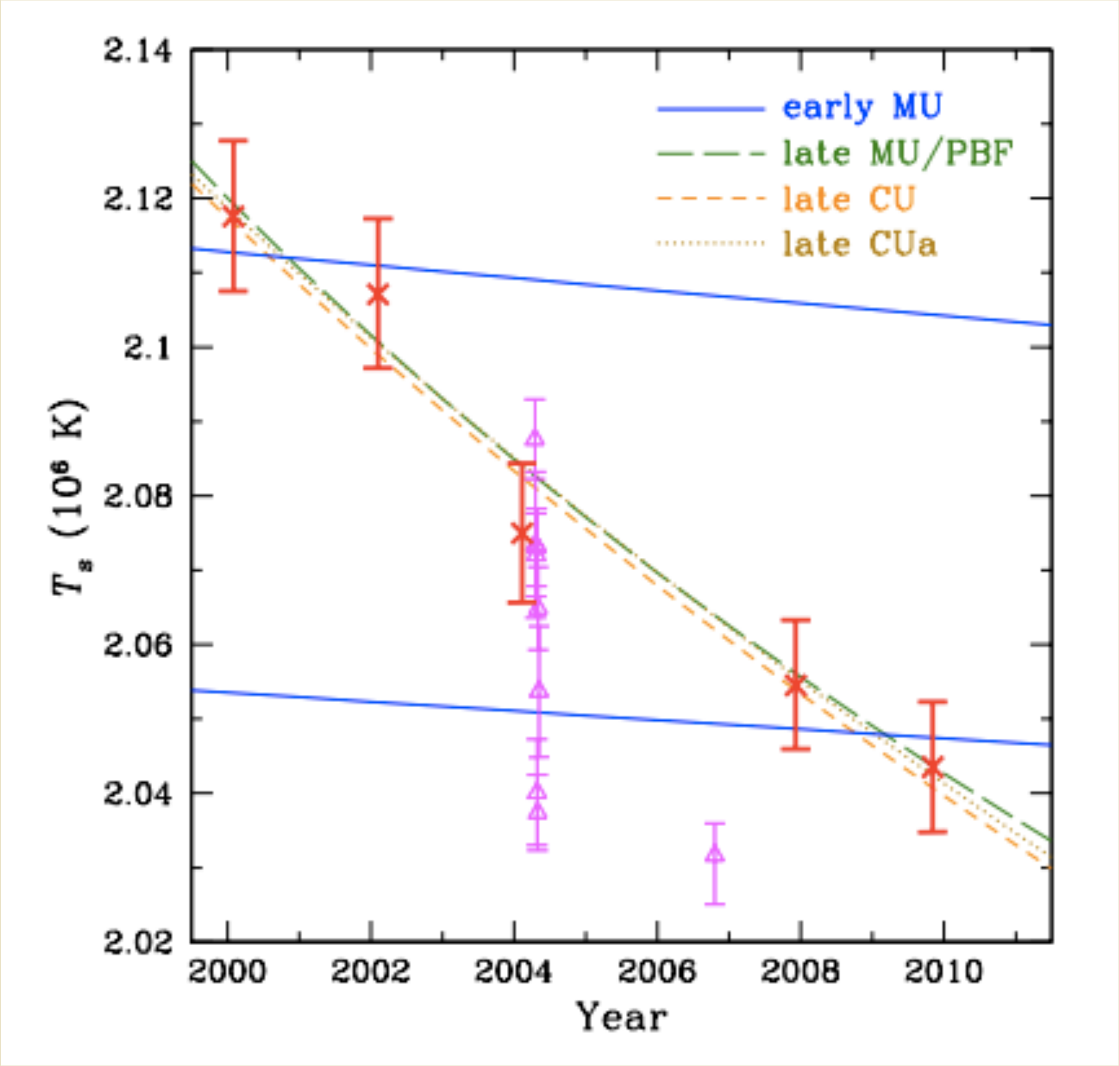
Glitches:

supposedly caused by unpinning of vortices



Dense neutron matter is superfluid

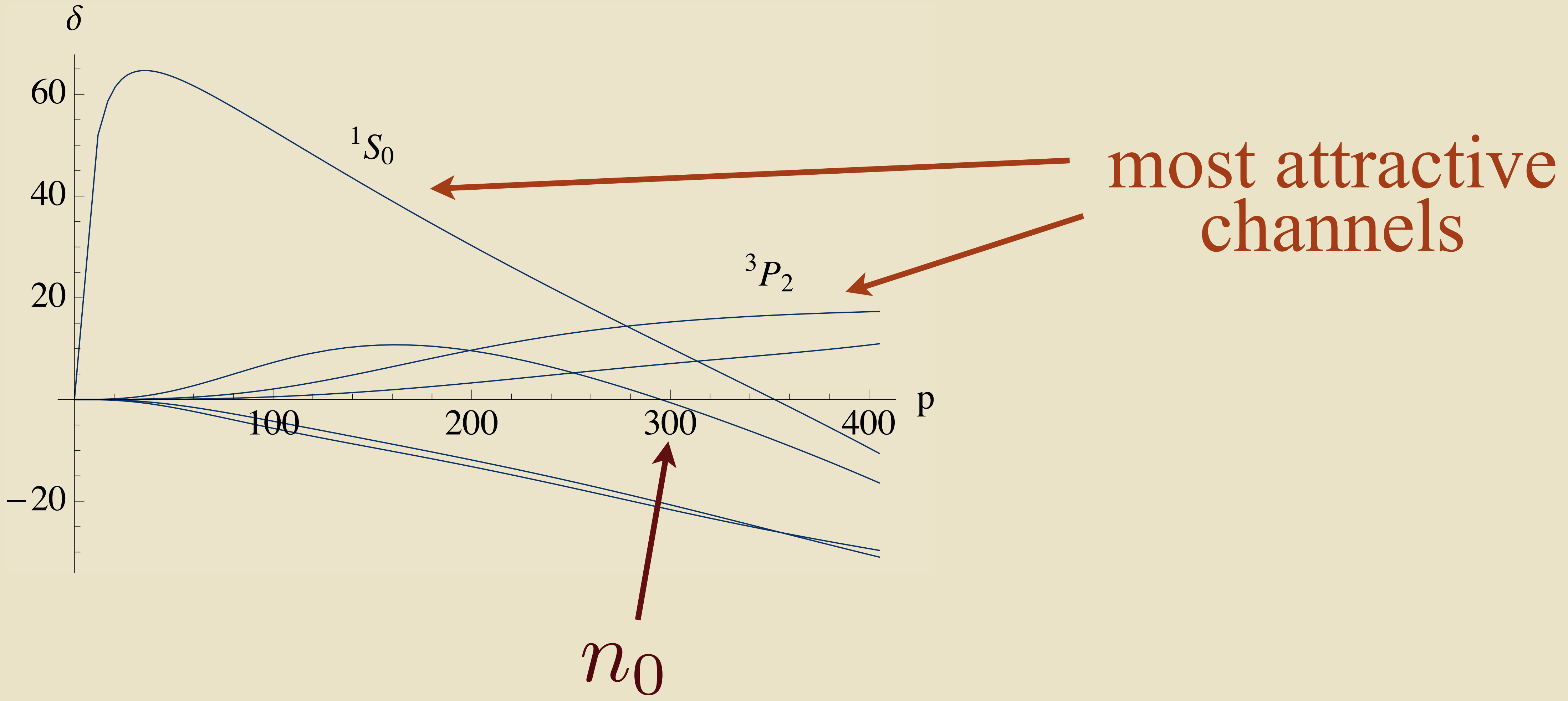
Cooling of Cassiopeia A:



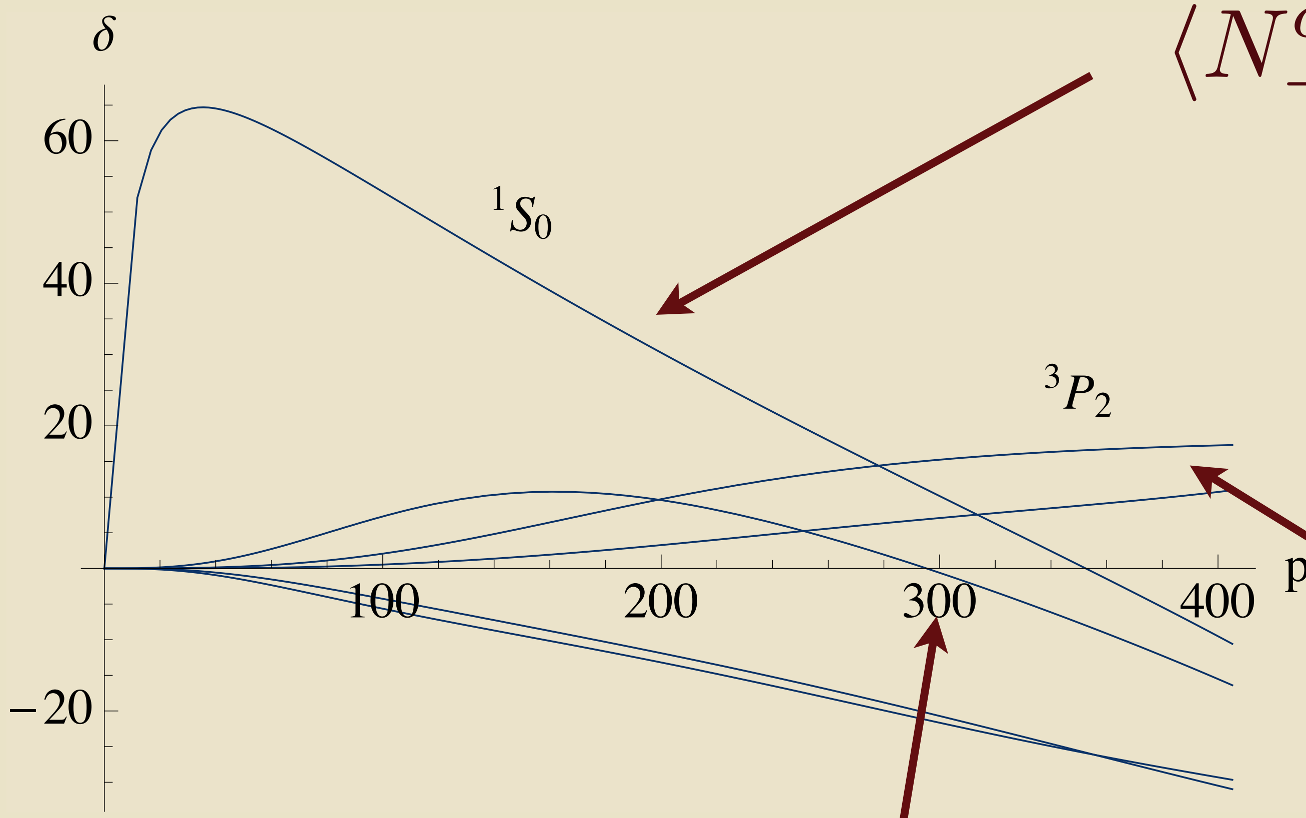
interpreted as the result of formation/ breaking of Cooper pairs

Heinke&Ho, 2007

What are the attractive forces?



What are the attractive forces?

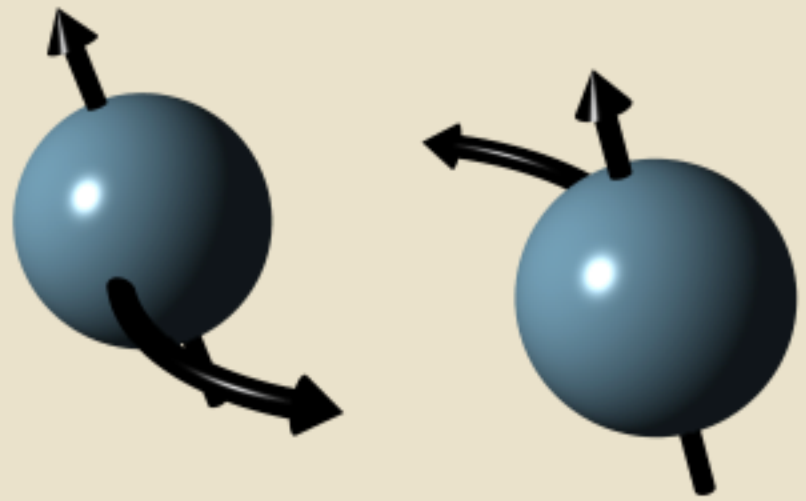


$$\langle N^{\alpha}_{-p} N^{\beta}_{p} \rangle = \Delta \ \delta_{\alpha\beta}$$

$$\langle N^{\alpha}_{-p} N^{\beta}_{p} \rangle = \Delta^{ij} \ (\sigma_2 \sigma^i)_{\alpha\beta} p^j$$

nuclear density

I want to concentrate on the $^3P_2$ phase(s):



$$\langle N^T \sigma_2 \sigma_i \nabla_j N \rangle = \Delta_{ij}$$

traceless,
symmetric tensor,
5 complex numbers
corresponding to
m=-2, ..., 2.

Which ground state is favored?

Near T~ Tc there is Ginsburg-Landau:

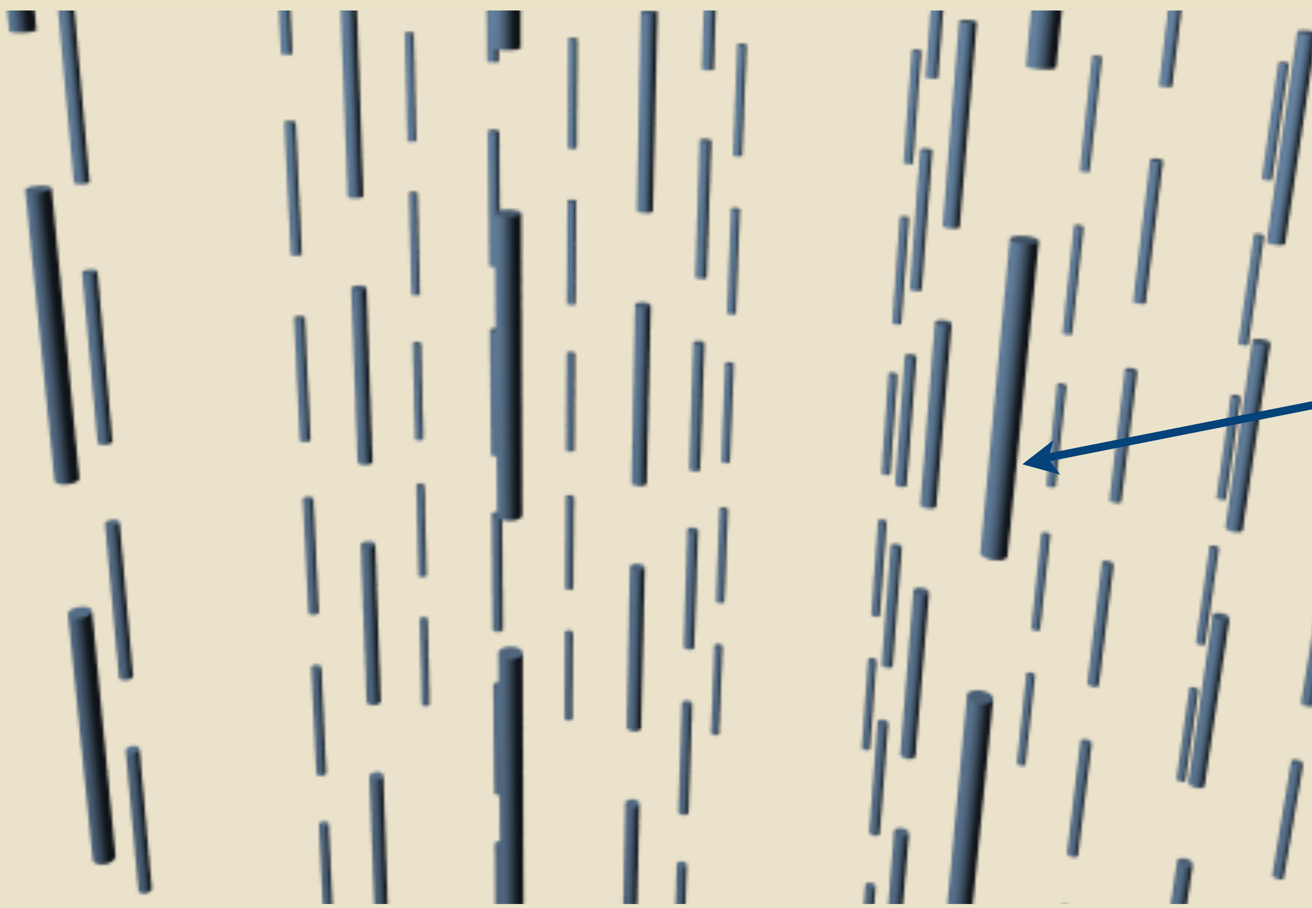
the winner is:

$$\Delta \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \qquad \Delta \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

looses out at order Δ^6

For $T << T_c$ the pattern is likely to continue

visual representation of $^3P_2$ ground state



point in the
directions of the -2
direction

The main point:

↓

Goldstone bosons (angulons) P.B., Rupak,Savage, 2003

In the (1,1,-2) phase there are two GB:

$$\frac{SO(3)}{U(1)\times\mathbb{Z}_2} = \mathbb{R}P_2$$

a rotation by π does nothing

The effective theory of angulons

Microscopic theory:

$$\mathcal{L} = N^\dagger(i\partial_0 + \epsilon(i\nabla))N + \frac{g^2}{4}(N^\dagger\sigma^2\sigma^k\nabla^l N^*)\chi^{ij}_{kl}(N^T\sigma^2\sigma^i\nabla^j N)$$

contact interaction
in the $^3P_2$ channel

The effective theory of angulons

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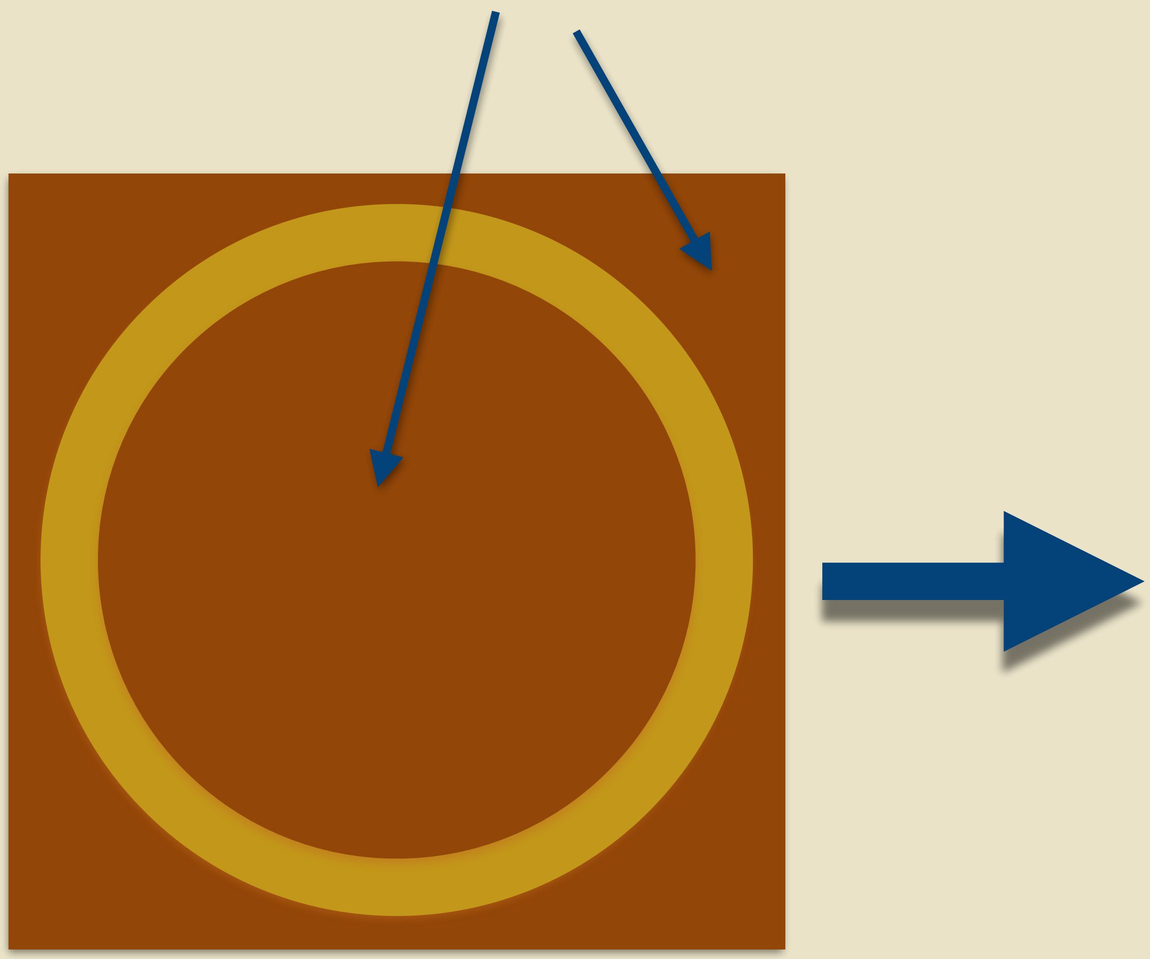
- Simple model, qualitatively right
- Quantitatively correct if "Landau Fermi Liquid Effective Theory" is perturbative (and it is, otherwise gaps would be large, …). The effects of interaction on the dispersion relation can be large.

Integrate out modes away from Fermi surface



EFT for quasi-neutrons with momentum restricted around the Fermi surface

relevant: kinetic term
marginally relevant: back-to-back interactions
irrelevant: everything else

- Quantitatively correct if “Landau Fermi Liquid Effective Theory” is perturbative (and it is, otherwise gaps would be large, …). The effects of interaction on the dispersion relation can be large.

introduce auxiliary field Δ

$$S = \int d^4x \left[\frac{1}{4g^2} \Delta^\dagger_{ij} \Delta_{ji} + \frac{1}{2} \begin{pmatrix} \psi^\dagger & \psi \end{pmatrix} \begin{pmatrix} i\partial_0 - \epsilon(-i\nabla) & -\Delta_{ji}\sigma_i\sigma_2\nabla_j \\ \Delta^\dagger_{ij}\sigma_2\sigma_i\nabla_j & i\partial_0 + \epsilon(-i\nabla) \end{pmatrix} \begin{pmatrix} \psi \\ \psi^* \end{pmatrix} \right]$$

integrate over ψ

$$S = \int d^4x \left[\frac{1}{4g^2} \Delta^\dagger_{ij} \Delta_{ji} - iTr \log \begin{pmatrix} i\partial_0 - \epsilon(-i\nabla) & -\Delta_{ji}\sigma_i\sigma_2\nabla_j \\ \Delta^\dagger_{ij}\sigma_2\sigma_i\nabla_j & i\partial_0 + \epsilon(-i\nabla) \end{pmatrix} \right]$$

saddle point: $\frac{\delta S}{\delta \Delta} = 0$ ⟶ gap equation

expand on the number of derivatives:

$$S_0 = -\int d^4x \; V_{eff}(\hat{\Delta}^\dagger \hat{\Delta})$$

$$\begin{aligned} S_2 \;=\; & \frac{M k_F}{12\pi^2 \bar{\Delta}^2} \int d^4x \left[\mathcal{I}^{(1)}_{ij}(\hat{\Delta}^\dagger \hat{\Delta}) \left[\partial_0 \Delta \cdot \partial_0 \Delta^\dagger\right]_{ij} - v_F^2 \mathcal{I}^{(1)}_{ijkl}(\hat{\Delta}^\dagger \hat{\Delta}) \left[\partial_k \Delta \cdot \partial_l \Delta^\dagger\right]_{ij} \right. \\ & + \frac{1}{2} \mathcal{I}^{(2)}_{ijkl}(\hat{\Delta}^\dagger \hat{\Delta}) \left(-2 \left[\hat{\Delta} \cdot \partial_0 \Delta^\dagger\right]_{ij} \left[\hat{\Delta} \cdot \partial_0 \Delta^\dagger\right]_{kl} + \left[\partial_0 \Delta^\dagger \cdot \partial_0 \Delta^*\right]_{ij} \left[\hat{\Delta} \cdot \hat{\Delta}^T\right]_{kl} \right) \\ & + \frac{v_F^2}{2} \mathcal{I}^{(2)}_{ijklmn}(\hat{\Delta}^\dagger \hat{\Delta}) \left(2 \left[\hat{\Delta} \cdot \partial_k \Delta^\dagger\right]_{ij} \left[\hat{\Delta} \cdot \partial_l \Delta^\dagger\right]_{mn} - \left[\partial_k \Delta^\dagger \cdot \partial_l \Delta^*\right]_{ij} \left[\hat{\Delta} \cdot \hat{\Delta}^T\right]_{mn} \right) + \text{h.c.} \end{aligned}$$

with $$\mathcal{I}^{(\alpha)}_{ij\dots}(\hat{\Delta}^{0\dagger} \hat{\Delta}^0) \equiv \int \frac{d\hat{p}}{4\pi} \frac{\hat{p}_i \hat{p}_j \cdots}{\left(\hat{p} \cdot \hat{\Delta}^{0\dagger} \hat{\Delta}^0 \cdot \hat{p}\right)^\alpha}$$ ← determined by the eigenvalues of Δ

everything depends on integrals like

$$\mathcal{I}^{(\alpha)}_{ij\cdots}(\hat{\Delta}^{0\dagger}\hat{\Delta}^{0}) \equiv \int \frac{d\hat{p}}{4\pi}\frac{\hat{p}_i\hat{p}_j\cdots}{\left(\hat{p}\cdot\hat{\Delta}^{0\dagger}\hat{\Delta}^{0}\cdot\hat{p}\right)^{\alpha}}$$

codifying the geometry of symmetry breaking.

Expand around Δ_0

$$\Delta = e^{-i(\alpha_1(x)J_1+\alpha_2(x)J_2)/f}\,\Delta^0 e^{i(\alpha_1(x)J_1+\alpha_2(x)J_2)/f}$$

angulons

Standard arguments imply

- leading order in p/f = tree level w/ p^2 vertices
- next-to-leading order in p/f = one loop w/ p^2 vertices + tree level w/ p^4 vertices
- …
- terms suppressed by $p^2/M\Delta$

angulon velocities

P.B., A. Nicholson, 2013

$$
\begin{aligned}
v_{x,y}^{(1)} &= \frac{v_F}{3}\sqrt{\frac{117}{18+2\sqrt{3}\pi}-2} \approx 0.477 v_F, \\
v_{x,y}^{(2)} &= 2v_F\sqrt{\frac{\pi}{9\sqrt{3}+3\pi}} \approx 0.709 v_F, \\
v_z^{(1,2)} &= \frac{v_F}{3}\sqrt{\frac{99}{18+2\sqrt{3}\pi}-1} \approx 0.519 v_F
\end{aligned}
$$

specific heat

$$
\begin{aligned}
c_v &= \sum_{a=1,2} \frac{d}{dT}\int \frac{d^3p}{(2\pi)^3}\frac{\epsilon_a(p)}{e^{\epsilon_a(p)/T}-1} \\
&\approx 16.16\frac{T^3}{v_F^3} = 1.44\times 10^{-13}\left(\frac{T/^\circ K}{v_F/c}\right)^3 \frac{\text{erg}}{^\circ K\,\text{cm}^3}
\end{aligned}
$$

smaller than electrons

P.B., A. Nicholson, 2013

angulon interactions

$$\sim \frac{1}{f^2}(\alpha\partial\alpha)^2 \quad \text{with} \quad f^2 = \frac{Mk_F}{6\pi^2}$$

anisotropic

~density of states (mild model dependence)

coupling to neutral currents

$$\frac{G_F M_Z^2}{\sqrt{8}}\left(-Z_0^0\psi^\dagger\psi + (g_A + g_{As})Z_0^i\psi^\dagger\sigma_i\psi\right)$$

↓

$$\frac{G_F M_Z^2}{\sqrt{8}}(g_A + g_{As}) \quad \left(9f(Z_2^0\partial_0\alpha_2 - Z_1^0\partial_0\alpha_1) + 9Z_3^0(\alpha_2\partial_0\alpha_1 - \alpha_1\partial_0\alpha_2)\right)$$

Coupling to magnetic fields

$$\frac{egB_i}{2M}\psi^\dagger\sigma_i\psi$$

neutron anomalous magnetic moment

$$\frac{9egk_F}{12\pi^2}B_z(\alpha_2\partial_0\alpha_1 - \alpha_1\partial_0\alpha_2)$$

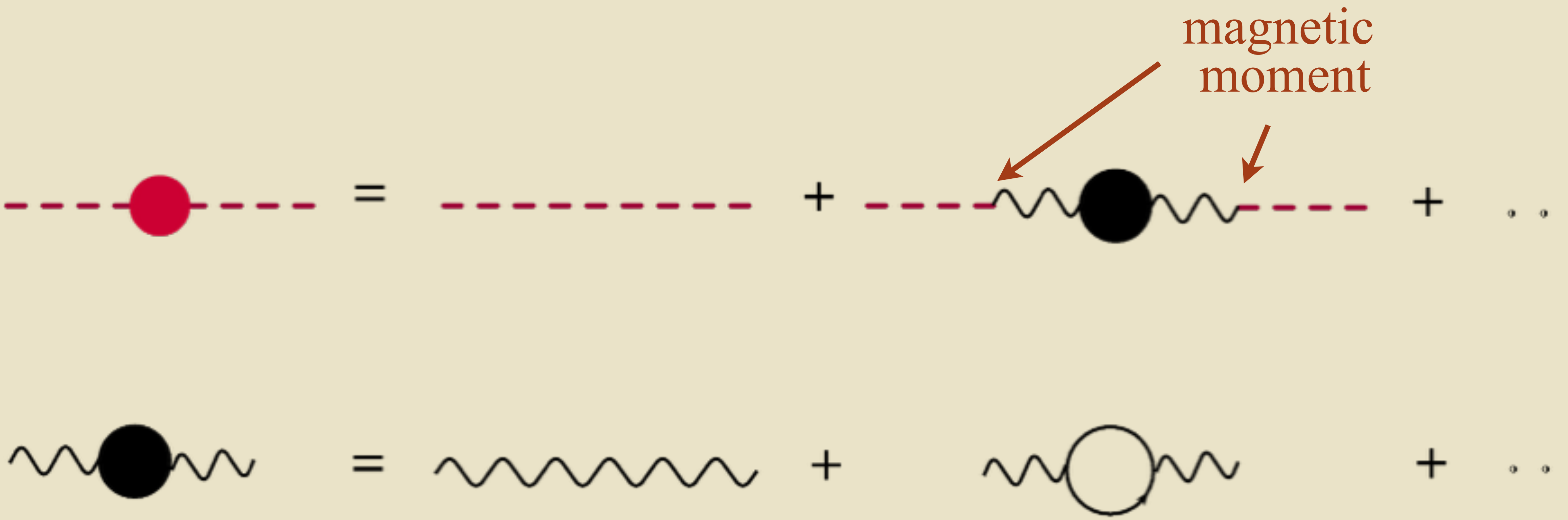
for non-zero B:
one gapped and one quadratic angulon

Transporte "should" be dominated by angulons because angulons mean free path is huge

angulon-angulon: $\lambda \sim \frac{1}{n\sigma} \sim \frac{c^3}{T^3}\frac{f^4}{T^2} \approx 3 \times 10^{10} m \left(\frac{M}{GeV}\right)^2 \left(\frac{k_F}{400\ MeV}\right)^2 \left(\frac{KeV}{T}\right)^5$

Enormous: forget it!

angulon-electron
(mediated by the neutron mag. moment):



$$\lambda \sim \frac{16 M^2 k_e^2 c^4}{\pi f^2 g^2 \omega^3} \sim \frac{c^4}{v_F} \left(\frac{k_e}{100 \text{ MeV}} \right)^2 \left(\frac{10 \text{ KeV}}{\omega} \right)^3 \text{ cm}$$

steep dependence on ω

Transporte coefficients

I should solve the Boltzmann eq. but I should estimate first

$$\kappa \sim \frac{1}{3} C_V v \langle \lambda \rangle$$

divergent

$$\sim \frac{1}{3} \int \frac{d^3 q}{(2\pi)^3} \frac{d}{dT} \left(\frac{vq}{e^{\beta v q} - 1} \right) v \lambda(q)$$

$$\approx 2.5 \times 10^{19} \left(\frac{k_e}{100 MeV} \right)^2 \left(\frac{v_\beta}{v_F} \right)^2 v_F (1 - \log(\beta v q_{star})) \frac{\text{erg}}{\text{cm s K}}$$

cutoff where $\lambda \sim R_{star}$

Unfortunately (for me):

Still:

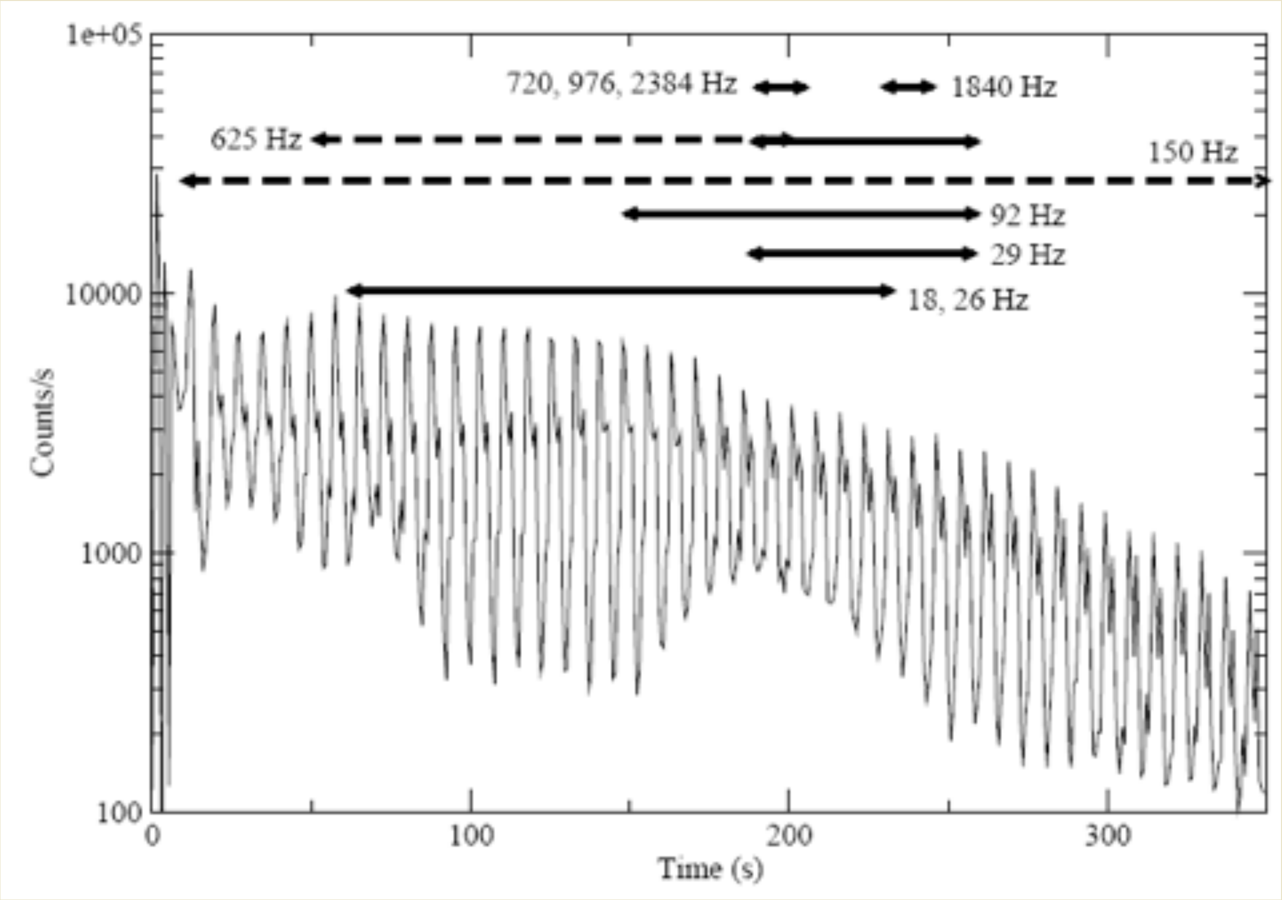
$$\kappa_e \gg \kappa_{ang}$$

$$\nu_e \gg \nu_{ang}$$

There are too many electrons: $n_e \sim k_e^2 T, \qquad n_{ang} \sim \frac{T^3}{v^3}$

Global angulons are essentially undamped

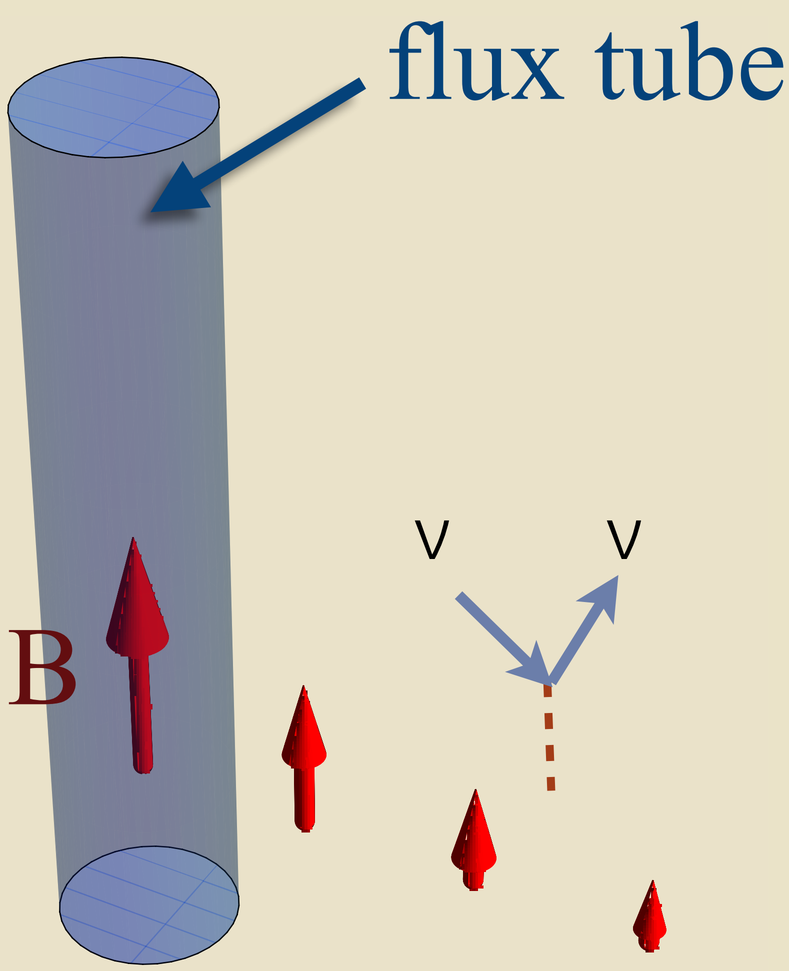
- couple to magnetic fields
- may be excited in giant flares
- may be observed as QPO (seismology)



Watts and Strohmayer (2005)

Massive angulon decay and cooling

P.B., S. Sen, 2013



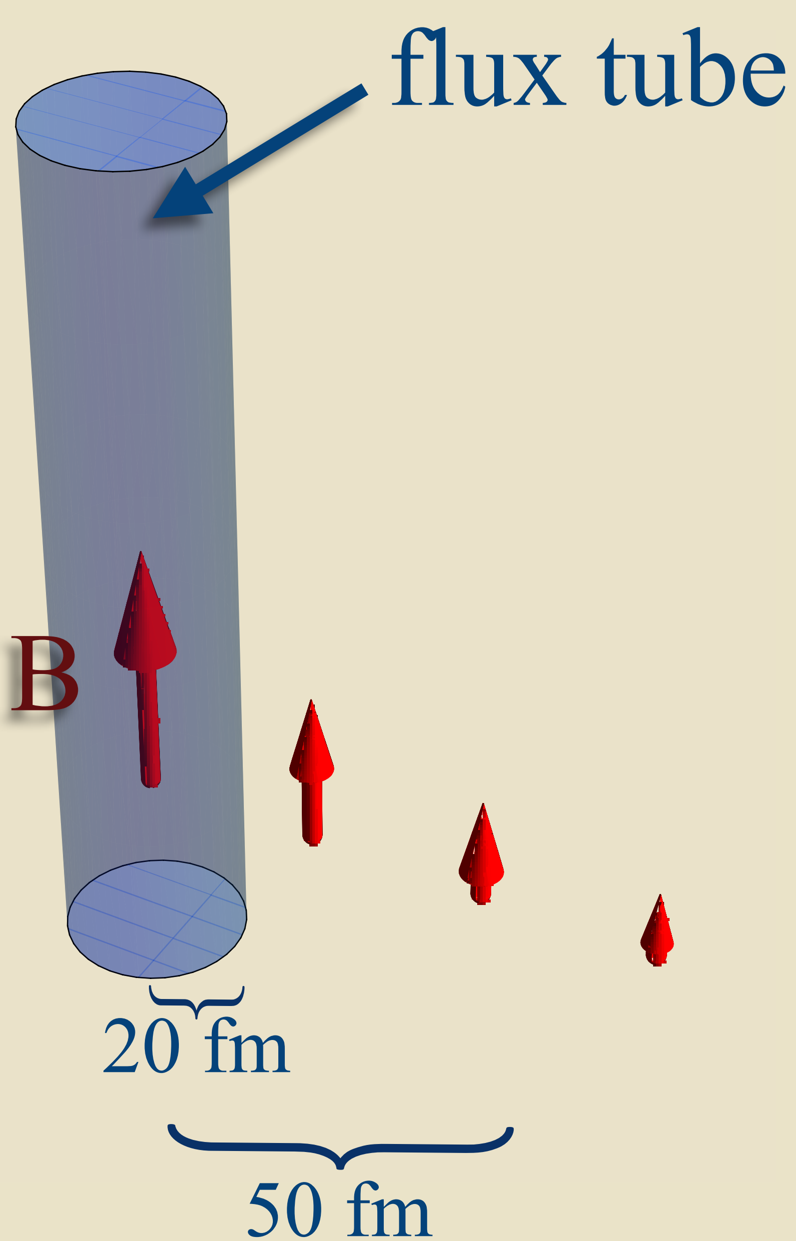
angulons are massive around flux tubes (m~eB/M)

↓

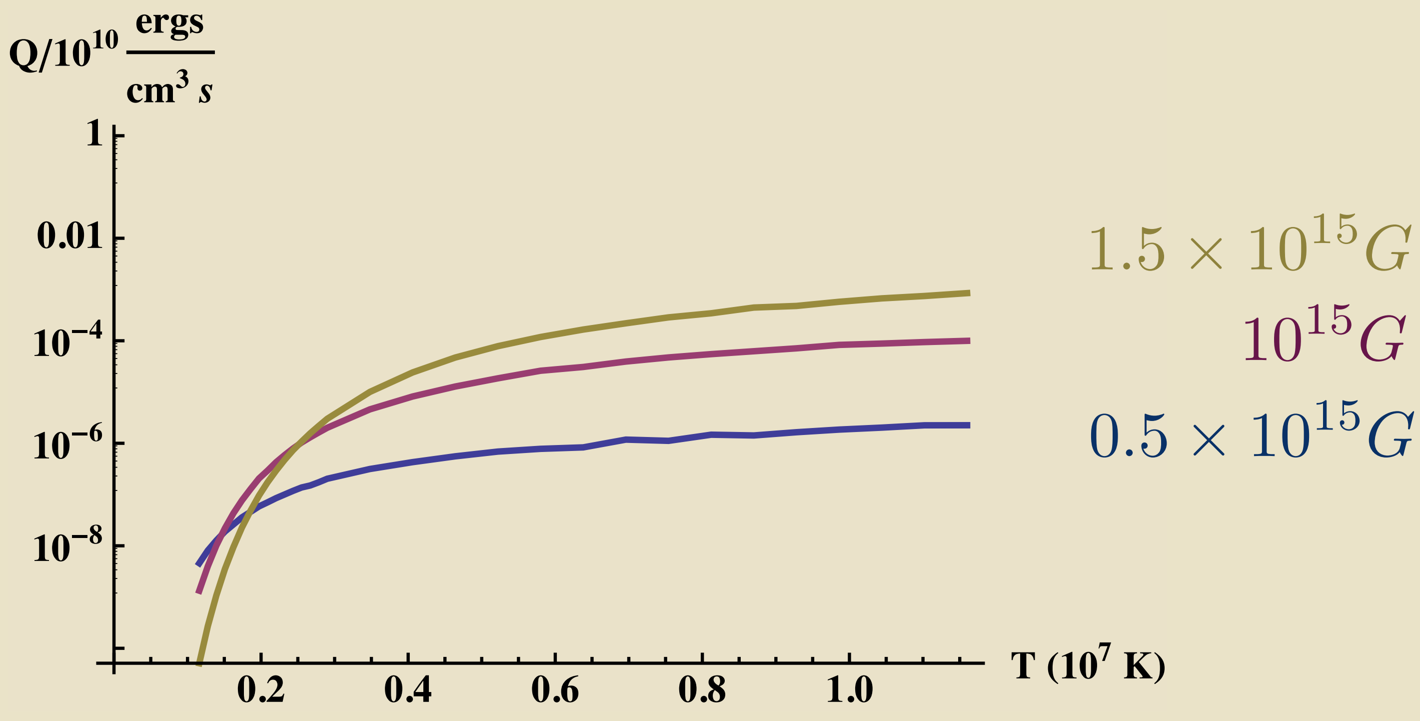
can decay into ν

Massive angulon decay and cooling

(P.B., S. Sen,2013)



$$Q \sim G_F^2 M k_F T^7 \, g\left(\frac{eB}{MT}\right)$$



Some open questions:

- are angulons massive at finite T ?
- can angulons dominate some transport property?
- are "giant angulons" related to QPOs?
- is it the "same" as pion condensation?
- ungapped $^3P_2$ phases: B_{crit} and properties