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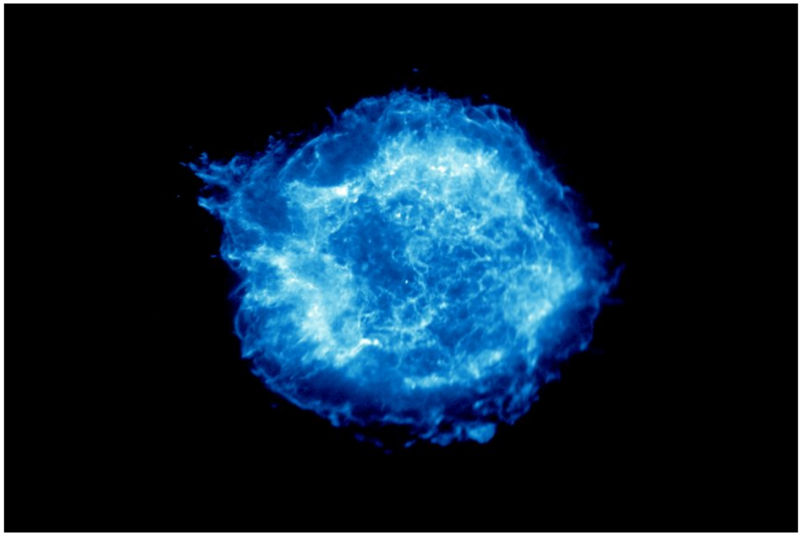
Sound modes and the two-stream instability in relativistic superfluids

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)
M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, arXiv:1310.5953 [hep-ph]
A. Schmitt, arXiv:1312.5993 [hep-ph]

- two-fluid picture of a superfluid
- role reversal in first and second sound
- two-stream instability

• Superfluid hydrodynamics: relevance for compact stars

- r-mode instability
- pulsar glitches
- precession
- asteroseismology
- superfluid turbulence (?)



Cas A, Chandra X-Ray Observatory

• Superfluidity in dense matter

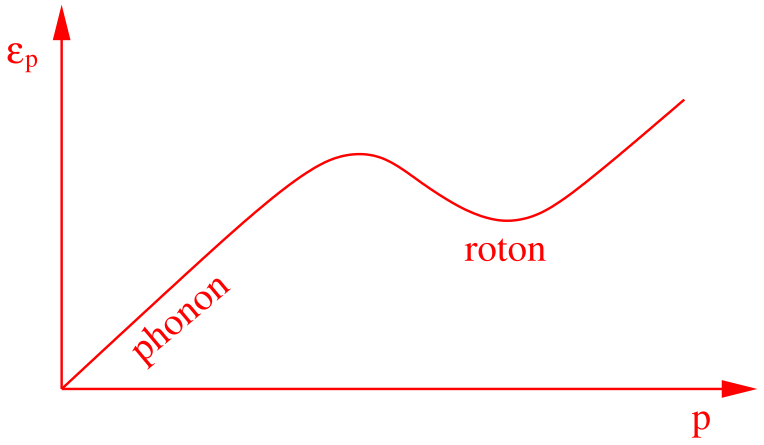
Nuclear matter	Quark matter
neutrons ($T_c \lesssim 10\,\text{keV}$)	color-flavor locked phase ($T_c \sim 10\,\text{MeV}$)
hyperons	color-spin locked phase ($T_c \sim 10\,\text{keV}$)

• Two-fluid picture of a superfluid (liquid helium)

London, Tisza (1938); Landau (1941)

relativistic: Khalatnikov, Lebedev (1982); Carter (1989)

- “superfluid component”: condensate, carries no entropy
- “normal component”: excitations (Goldstone mode), carries entropy



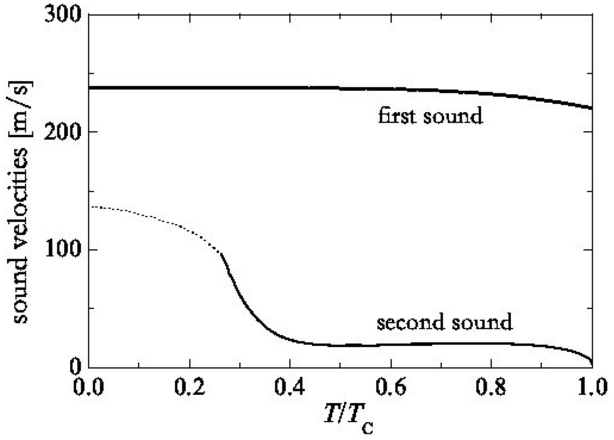
Hydrodynamic eqs. ⇒ **two sound modes**

1st sound	**2nd sound**
in-phase oscillation (primarily) density wave	out-of-phase oscillation (primarily) entropy wave

• First and second sound in non-relativistic systems

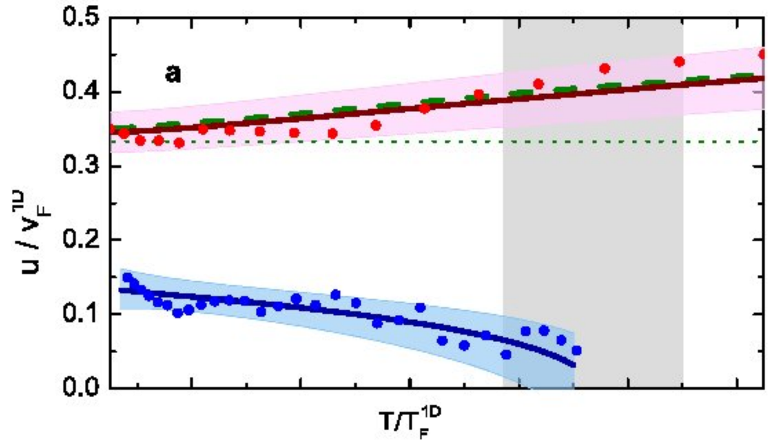
liquid helium

K.R. Atkins *et al.* (1953)



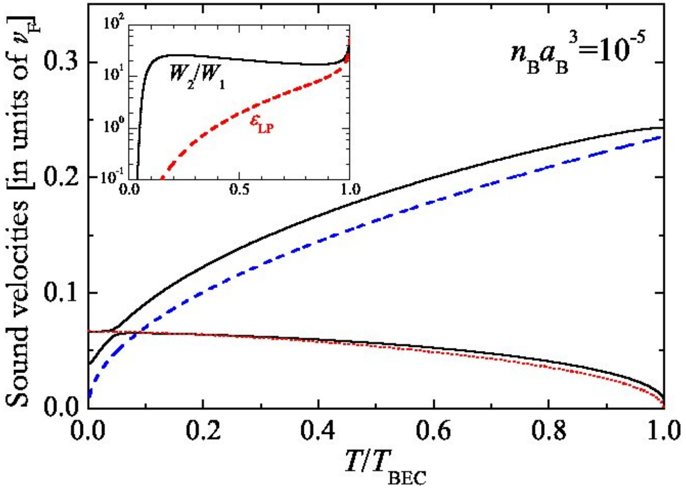
ultracold fermionic gas (exp.)

L.A. Sidorenkov *et al.*, Nature 498, 78 (2013)



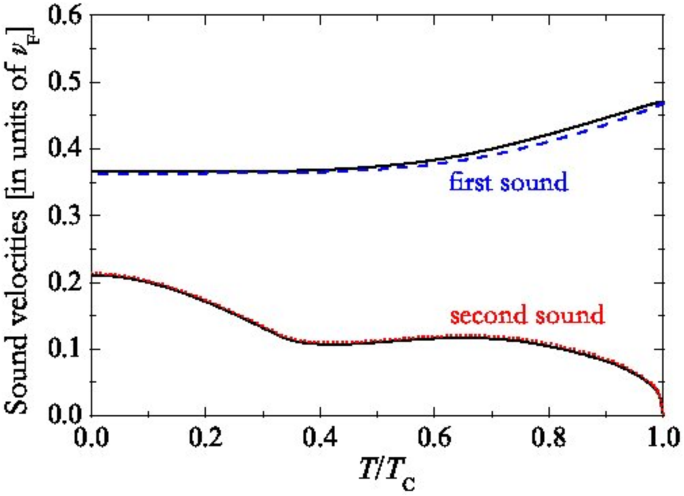
weakly interacting Bose gas

H.Hu, *et al.*, New Journ.Phys. 12, 043040 (2010)



unitary Fermi gas

E. Taylor *et al.*, PRA 80, 053601 (2009)



- **Goals**

How does the two-fluid picture
arise from a microscopic field theory?

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, PRD 87, 065001 (2013)

Compute sound modes in a relativistic superfluid
(and in the presence of a superflow)

M.G. Alford, S.K. Mallavarapu, A. Schmitt, S. Stetina, arXiv:1310.5953 [hep-ph]
A. Schmitt, arXiv:1312.5993 [hep-ph]

• Lagrangian and superfluid velocity

- starting point:
  complex scalar field

$$\boxed{\mathcal{L} = (\partial\varphi)^2 - m^2|\varphi|^2 - \lambda|\varphi|^4}$$

- Bose condensate $\langle\varphi\rangle = \rho\, e^{i\psi}$ spontaneously breaks $U(1)$

- zero temperature: single-fluid system

	Field theory	Hydrodynamics
current j^μ	$\frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi$	nv^μ
stress-energy tensor $T^{\mu\nu}$	$-g^{\mu\nu}\mathcal{L} + \frac{(\partial\psi)^2}{\lambda}\partial^\mu\psi\partial^\nu\psi$	$(\epsilon + P)v^\mu v^\nu - g^{\mu\nu}P$

- superfluid velocity

$$\boxed{v^\mu = \frac{\partial^\mu\psi}{\mu}}$$

$$\mu = |\partial\psi|$$

• Relativistic two-fluid formalism (page 1/2)

- write stress-energy tensor as

$$\boxed{T^{\mu\nu} = -g^{\mu\nu}\Psi + j^{\mu}\partial^{\nu}\psi + s^{\mu}\Theta^{\nu}}$$

- **"generalized pressure"** Ψ:
  - $\Psi = P_{\perp}$ in superfluid and normal-fluid rest frames,
  - Ψ depends on momenta $\partial^{\mu}\psi$, Θ^{μ}

$$\Psi = \Psi[(\partial\psi)^2, \Theta^2, \partial\psi\cdot\Theta]$$

- **"generalized energy density"** $\Lambda \equiv -\Psi + j\cdot\partial\psi + s\cdot\Theta$
  - Λ is Legendre transform of Ψ,
  - Λ depends on currents j^{μ}, s^{μ}

$$\Lambda = \Lambda[j^2, s^2, j\cdot s]$$

• Relativistic two-fluid formalism (page 2/2)

$$j^\mu = \frac{\partial\Psi}{\partial(\partial_\mu\psi)} = \mathcal{B}\,\partial^\mu\psi + \mathcal{A}\,\Theta^\mu$$

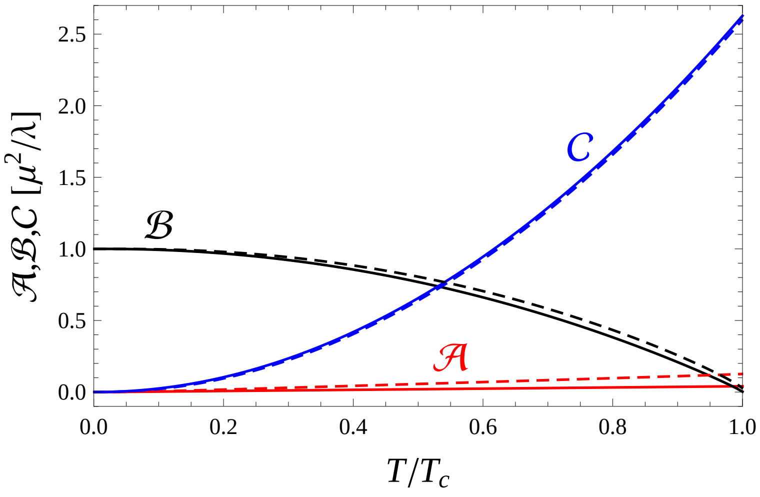
$$s^\mu = \frac{\partial\Psi}{\partial\Theta_\mu} = \mathcal{A}\,\partial^\mu\psi + \mathcal{C}\,\Theta^\mu$$

$$\mathcal{B} = 2\frac{\partial\Psi}{\partial(\partial\psi)^2}\,, \quad \mathcal{C} = 2\frac{\partial\Psi}{\partial\Theta^2}$$

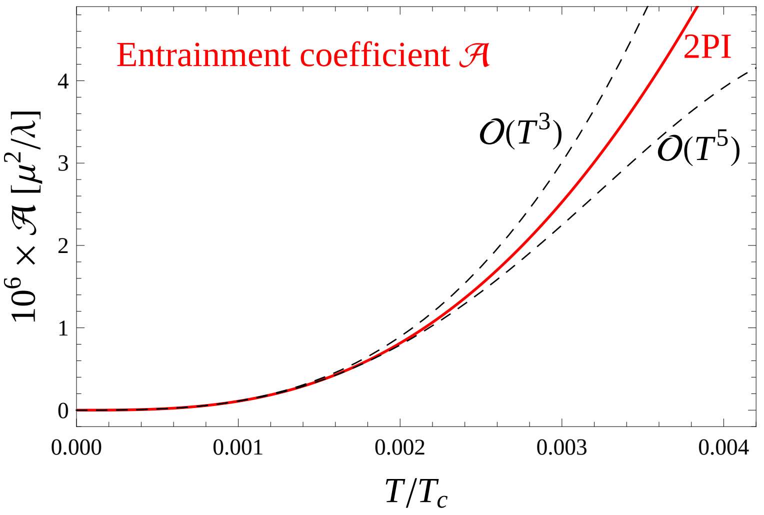
$$\mathcal{A} = \frac{\partial\Psi}{\partial(\partial\psi\cdot\Theta)}$$

"entrainment coefficient"

- compute $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ from microscopic physics



all temperatures



(very) small temperatures

• Microscopic calculation for arbitrary T (page 1/2)

- effective action density in the 2PI formalism (CJT)

$$\boxed{\Gamma[\rho, S] = -U(\rho) - \frac{1}{2}\mathrm{Tr}\ln S^{-1} - \frac{1}{2}\mathrm{Tr}[S_0^{-1}(\rho)S - 1] - V_2[\rho, S]}$$

- $V_2[\rho, S]$: two-loop two-particle irreducible (2PI) diagrams
- use Hartree approximation
- impose Goldstone theorem by hand
- solve self-consistency equations for condensate ρ and M, δM

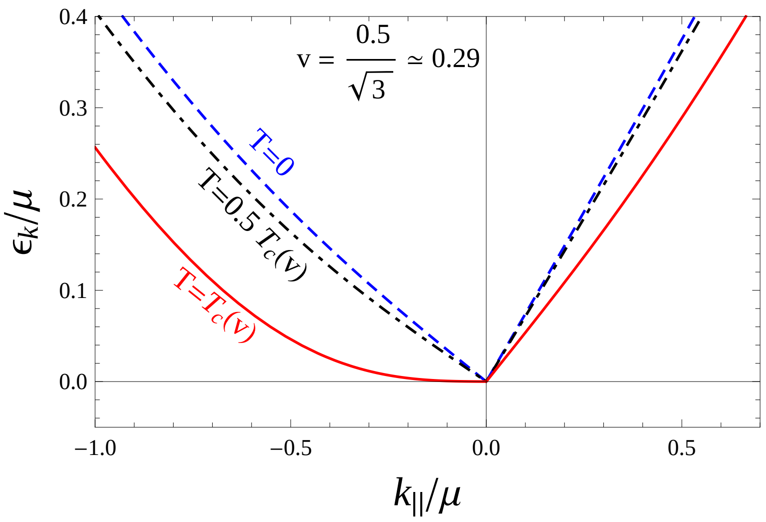
• Microscopic calculation for arbitrary T (page 2/2)

- microscopic calculation done in normal-fluid rest frame
- identify effective action density with generalized pressure

$$\boxed{\Gamma[\mu, T, \nabla\psi] = \Psi}$$

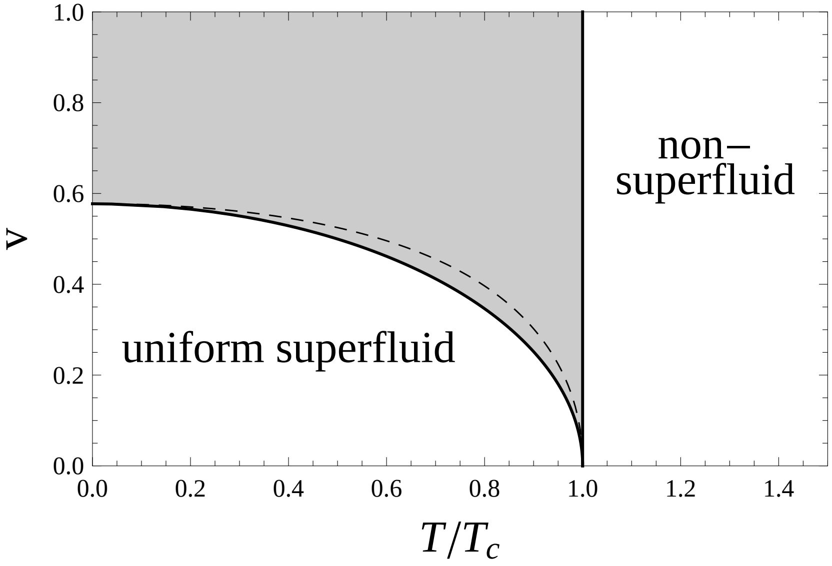
- restrict to weak coupling → no dependence on renormalization scale
- consider uniform superflow $\mathbf{v}$
- neglect dissipation → thermodynamics with $(\mu, T, \mathbf{v})$
- compute entrainment coefficient, sound velocities etc.

- # Results I: critical velocity



  - instability at $v = v_c$
  - negative energies in Goldstone dispersion $\epsilon_{\mathbf{k}}(\mathbf{v}) < 0$

- generalization to Landau's original argument $\epsilon_{\mathbf{k}} - \mathbf{k} \cdot \mathbf{v} < 0$



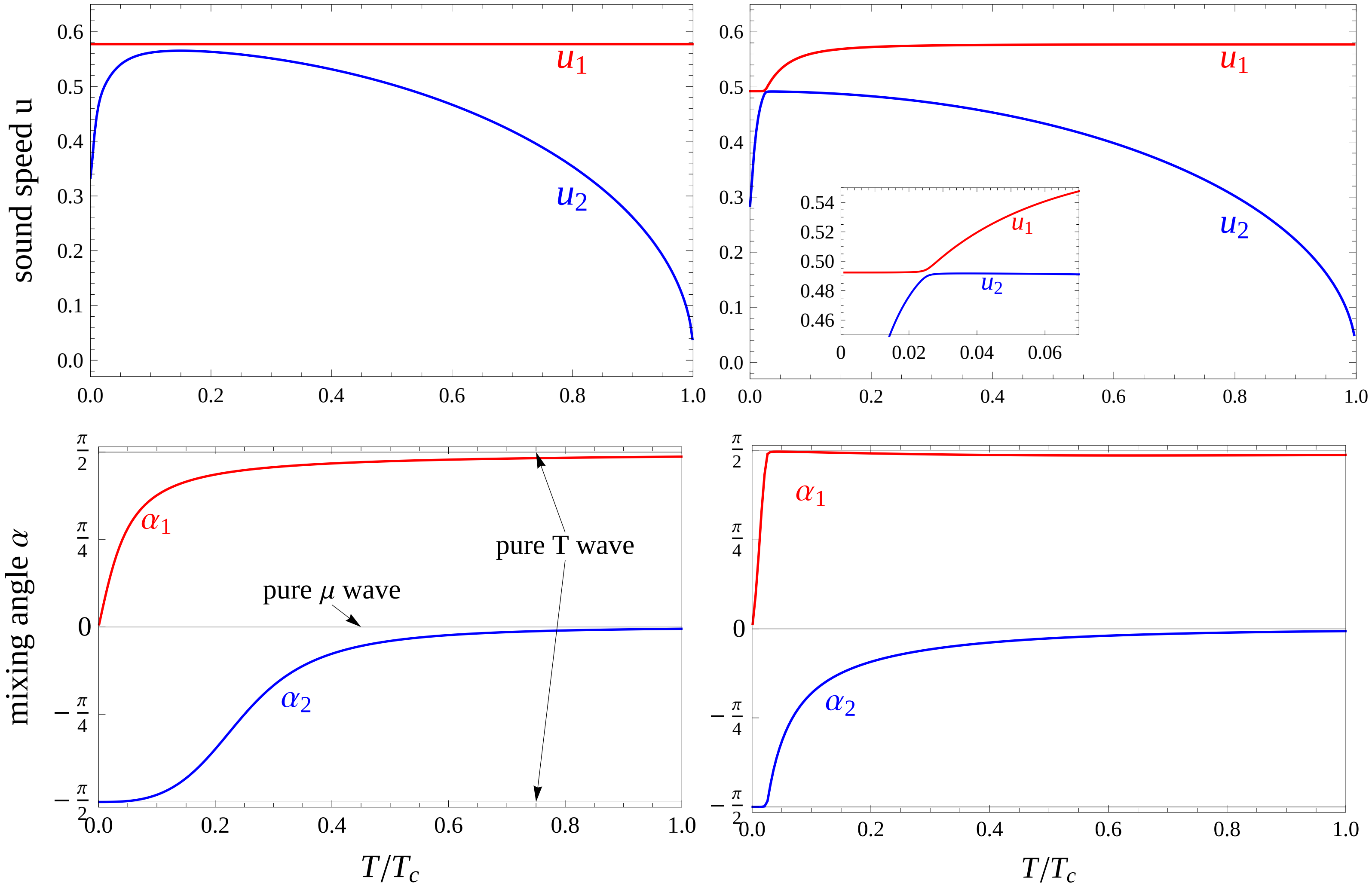
- dashed line: without backreaction of condensate
- shaded region: dissipation, turbulence?

- similar phase diagram for holographic superfluid I. Amado, D. Arean, A. Jimenez-Alba, K. Landsteiner, L. Melgar and I. S. Landea, arXiv:1307.8100 [hep-th]

• Results II: sound speeds and mixing angle

ultra-relativistic

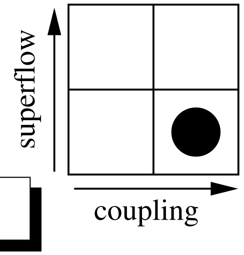
(towards) non-relativistic

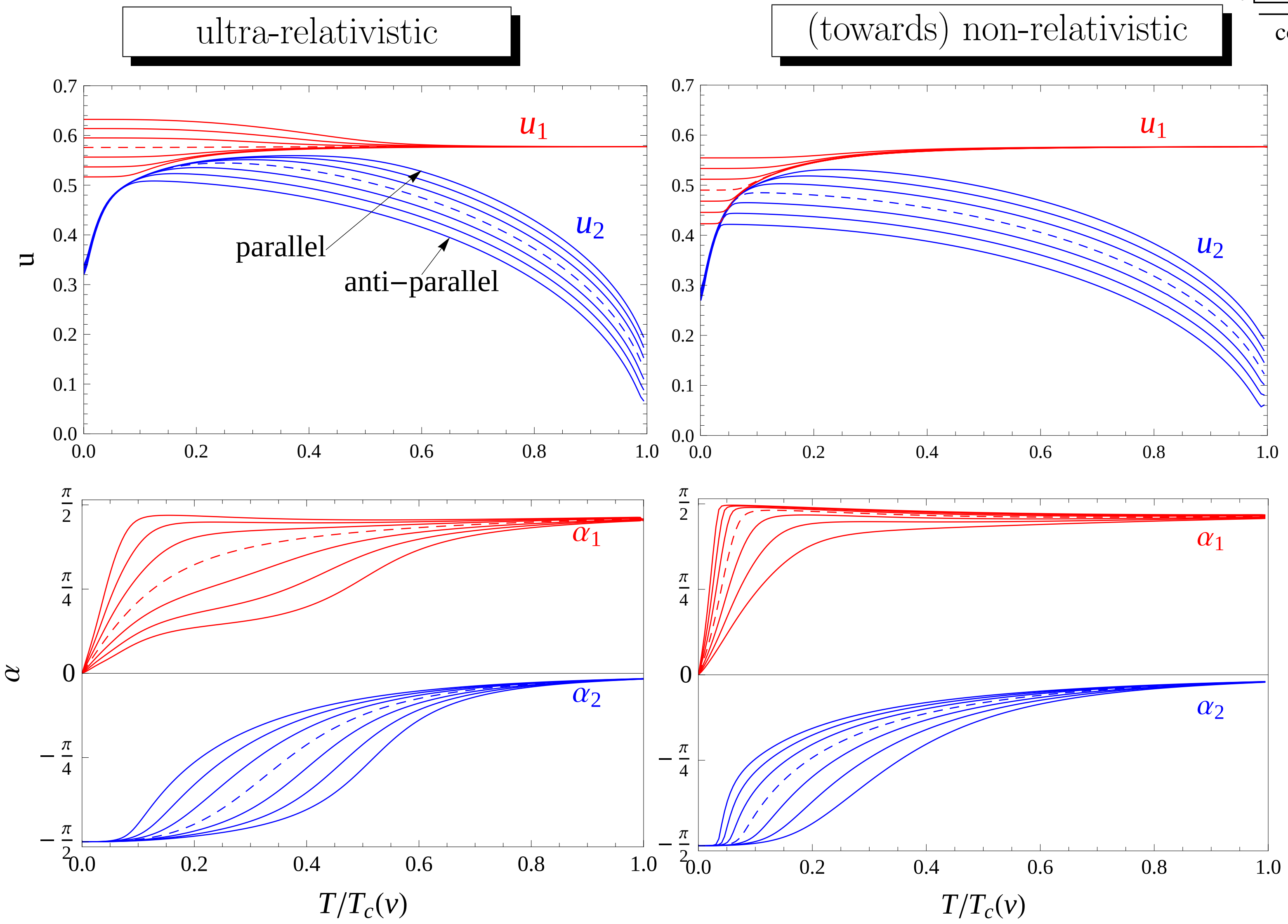


$$\alpha = \arctan \frac{\delta T}{\delta \mu}$$

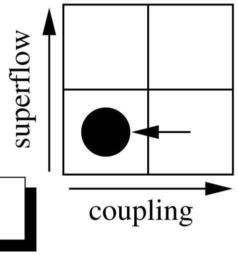
role reversal in first and second sound!

- **Sound speeds and mixing angle with superflow**

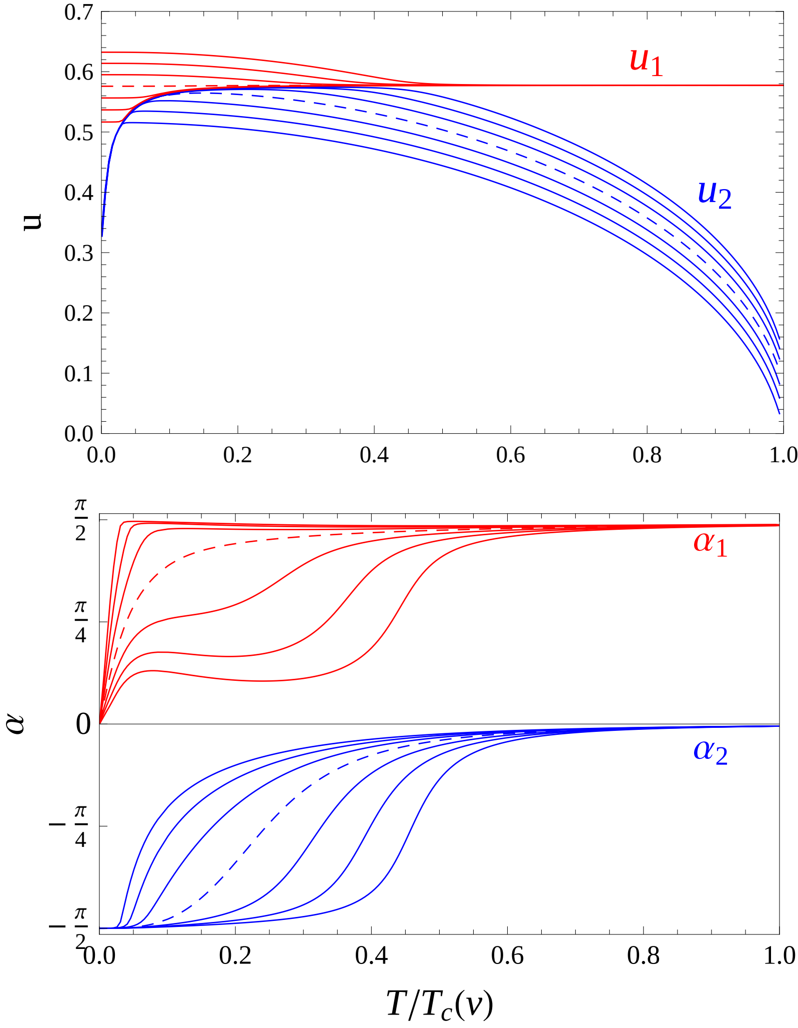




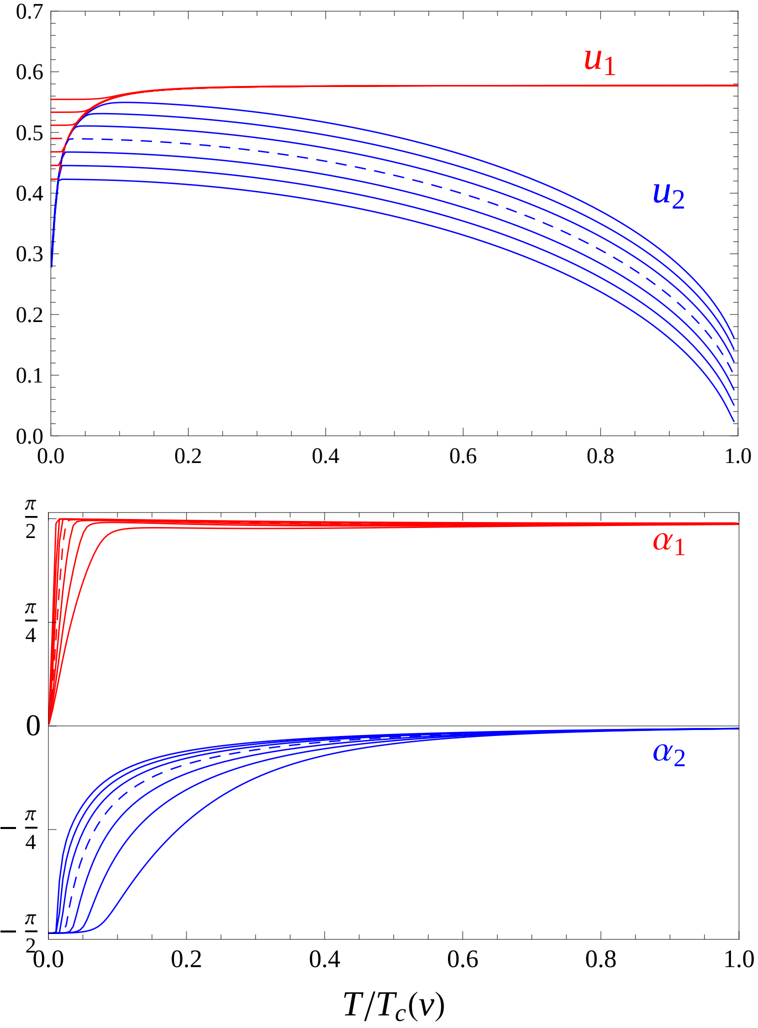
- # Sound speeds and mixing angle with superflow



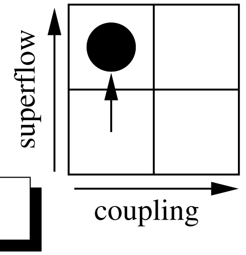
ultra-relativistic



(towards) non-relativistic

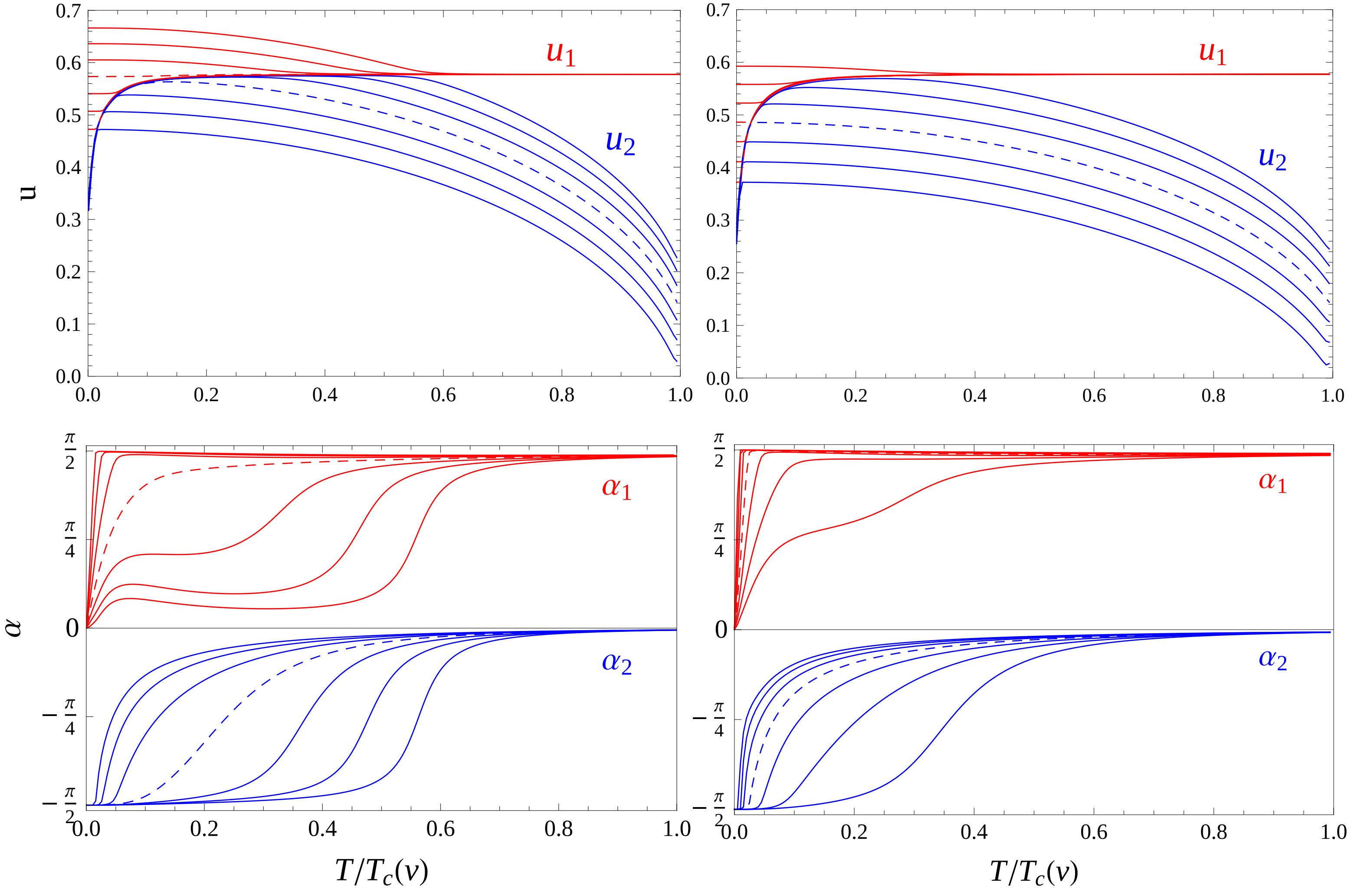


• Sound speeds and mixing angle with superflow

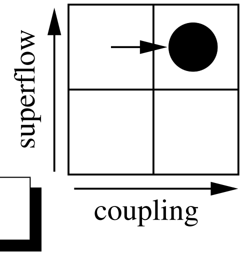


ultra-relativistic

(towards) non-relativistic

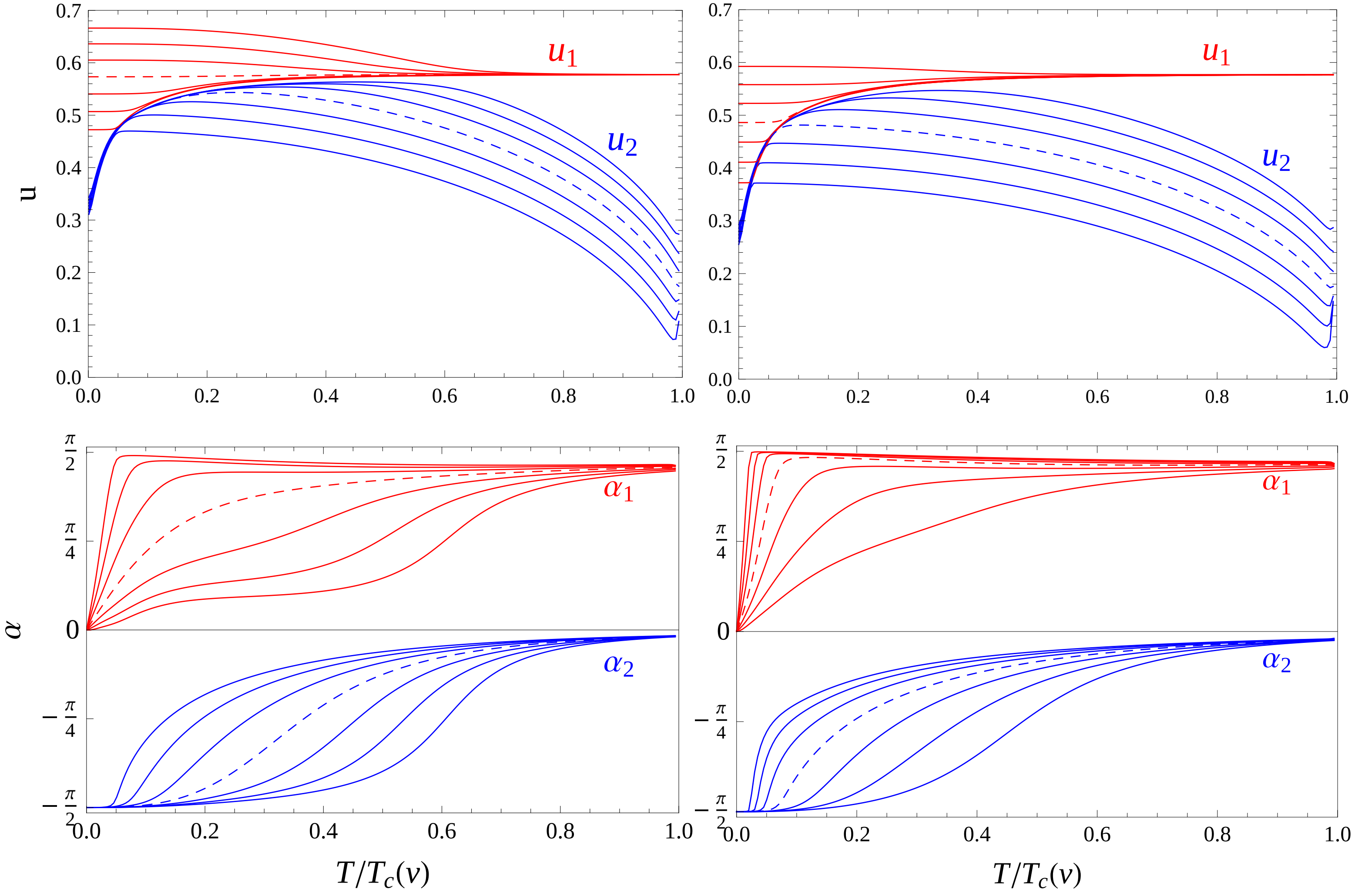


- **Sound speeds and mixing angle with superflow**

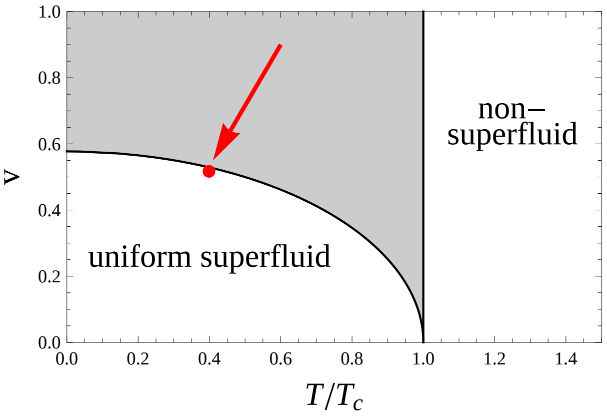


ultra-relativistic

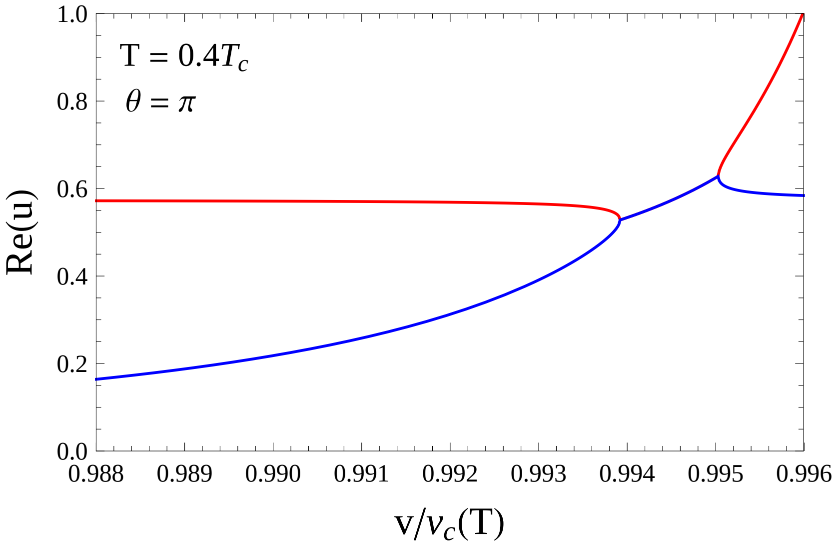
(towards) non-relativistic

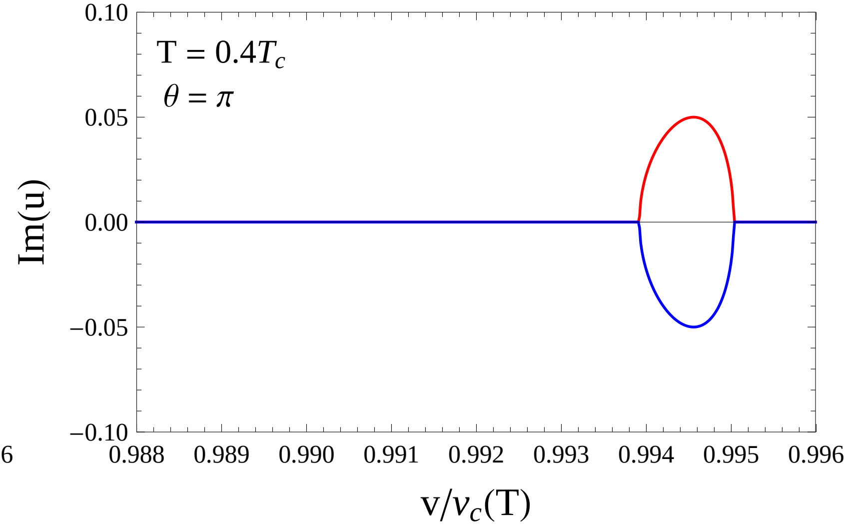


• Results III: two-stream instability



- compute sound speed close to Landau's critical velocity





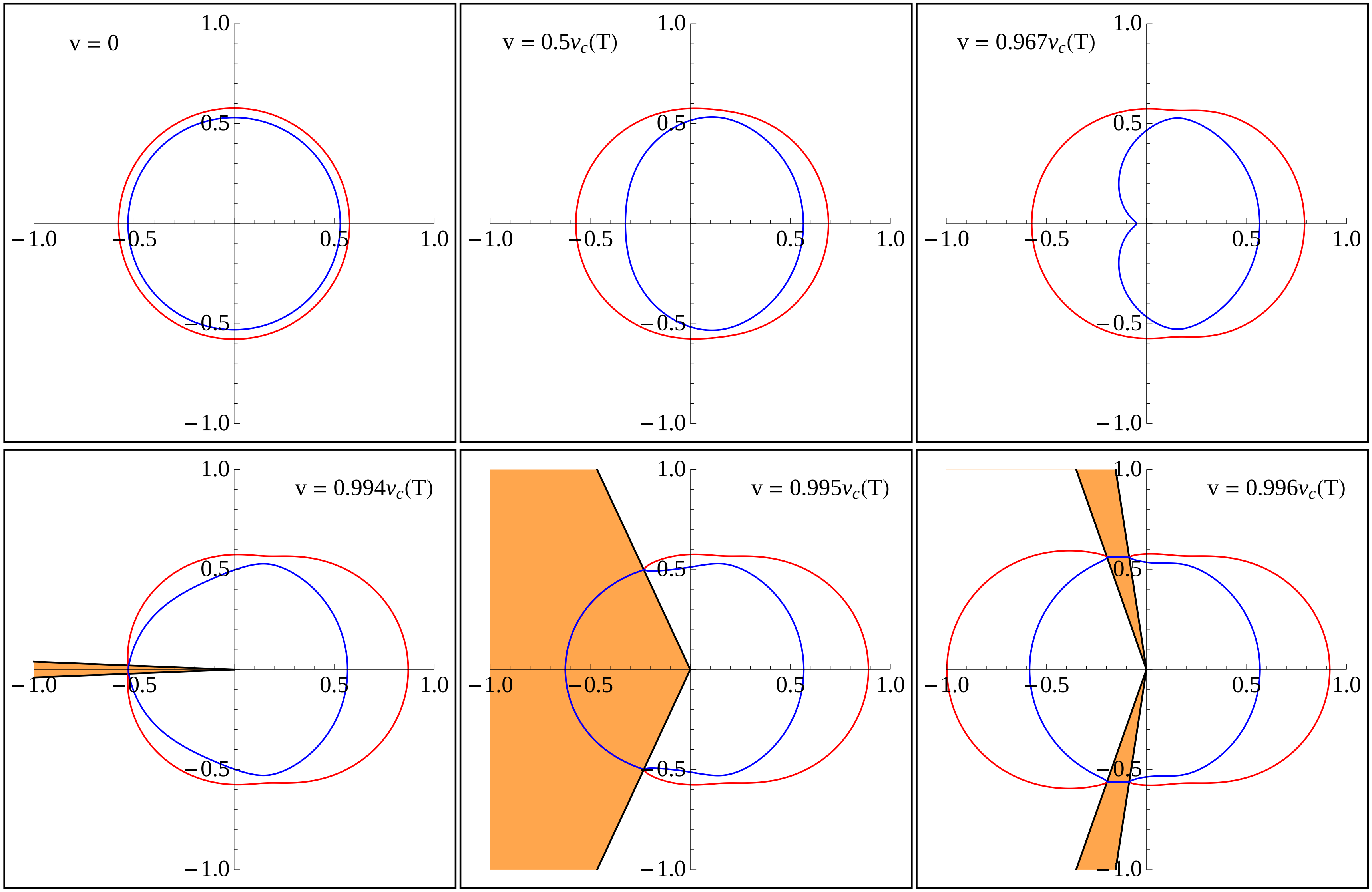
- complex sound speeds → one mode damped, one mode explodes

plasma physics: O. Buneman, Phys.Rev. 115, 503 (1959); D.T. Farley, PRL 10, 279 (1963)

general two-fluid system: L. Samuelsson, C. S. Lopez-Monsalvo, N. Andersson, G. L. Comer, Gen. Rel. Grav. 42, 413 (2010)

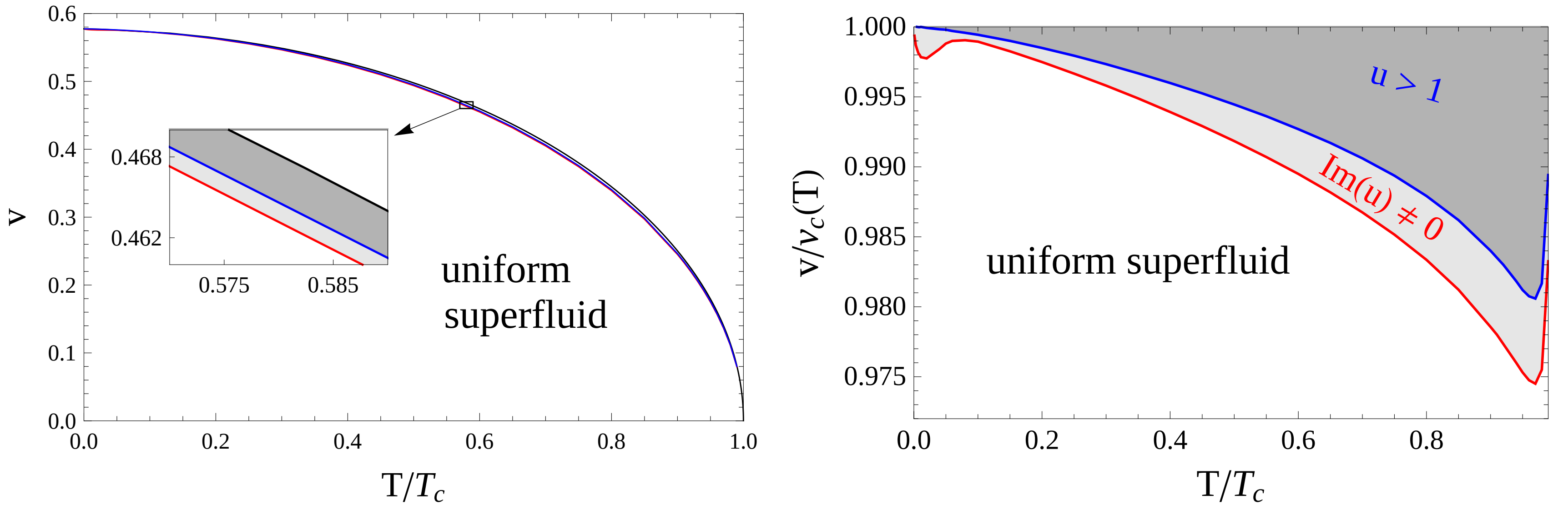
relevance for superfluids: N. Andersson, G. L. Comer, R. Prix, MNRAS 354, 101 (2004)

• All directions



(superflow pointing to the right)

• Instability window in phase diagram



- tiny window for weak coupling $\lambda = 0.05$
  (varying λ shows that the window grows with λ)
- region with $u > 1$: problem in the formalism?
  (Hartree? enforced Goldstone theorem?)
- very small T: qualitatively different angular structure of instability

• Summary

- a superfluid is a two-fluid system, and this can be derived from microscopic physics
- the two sound modes in a (weakly coupled, relativistic) superfluid can reverse their roles (in terms of density and entropy waves)
- at large relative velocities of the two fluids, there is a dynamical instability ("two-stream instability")

• Outlook

- start from fermionic theory
  D. Müller, A. Schmitt, work in progress
- behavior beyond critical velocity
- sound modes (role reversal):
  - predictions for ^{4}He or ultracold gases?
  - apply to compact stars
    neutron superfluid & ion lattice: N. Chamel, D. Page and S. Reddy, PRC 87, 035803 (2013)
- two-stream instability:
  - instability more prominent at strong coupling?
    holographic approach: C.P.Herzog and A.Yarom, PRD 80, 106002 (2009); I.Amado, D.Arean, A.Jimenez-Alba, K.Landsteiner, L.Melgar, I.S.Landea, arXiv:1307.8100 [hep-th]
  - time evolution of instability
    I. Hawke, G. L. Comer and N. Andersson, Class. Quant. Grav. 30, 145007 (2013)
  - relevance for compact stars, e.g., pulsar glitches
    N. Andersson, G. L. Comer, R. Prix, MNRAS 354, 101 (2004)