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Chiral separation effect: From high energy to Dirac & Weyl semimetals

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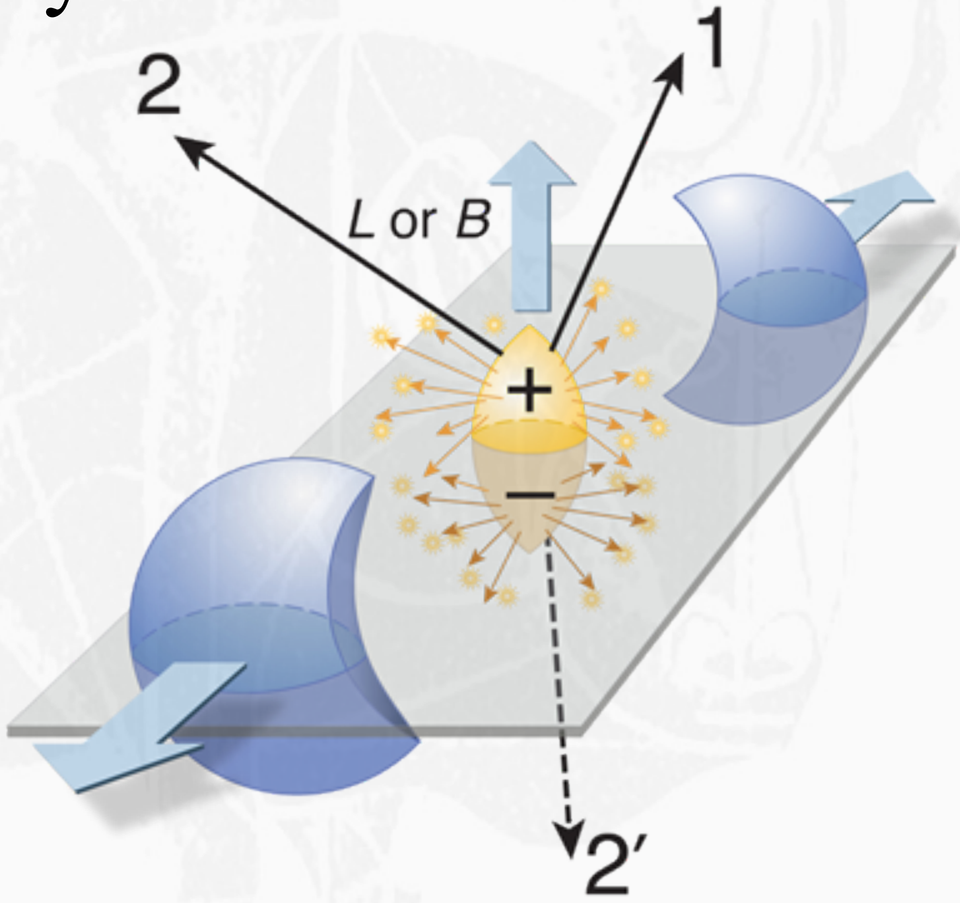
NSF

Chiral magnetic effect

- Dipole pattern of electric currents (charge correlations) in heavy ion collisions

$$\left\langle \vec{j} \right\rangle_{\text{free}} = -\frac{e^2\vec{B}}{2\pi^2}\mu_5$$



[Kharzeev, Zhitnitsky, Nucl. Phys. A **797**, 67 (2007)]

[Kharzeev, McLerran, Warringa, Nucl. Phys. A **803**, 227 (2008)]

[Fukushima, Kharzeev, Warringa, Phys. Rev. D **78**, 074033 (2008)]

Chiral separation effect

- Axial current induced by the chemical potential

$$\left\langle \vec{j}_5 \right\rangle_{\text{free}} = -\frac{e\vec{B}}{2\pi^2}\mu \qquad \text{(free theory!)}$$

[Vilenkin, Phys. Rev. D **22** (1980) 3067]

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

[Newman & Son, Phys. Rev. D **73** (2006) 045006]

- Exact result (is it?), which follows from chiral anomaly relation

- No radiative correction expected…

Possible implication

- Seed chemical potential (μ) induces axial current

$$\left\langle \vec{j}_5 \right\rangle_{\text{free}} = -\frac{e\vec{B}}{2\pi^2}\mu$$

- Leading to separation of chiral charges:

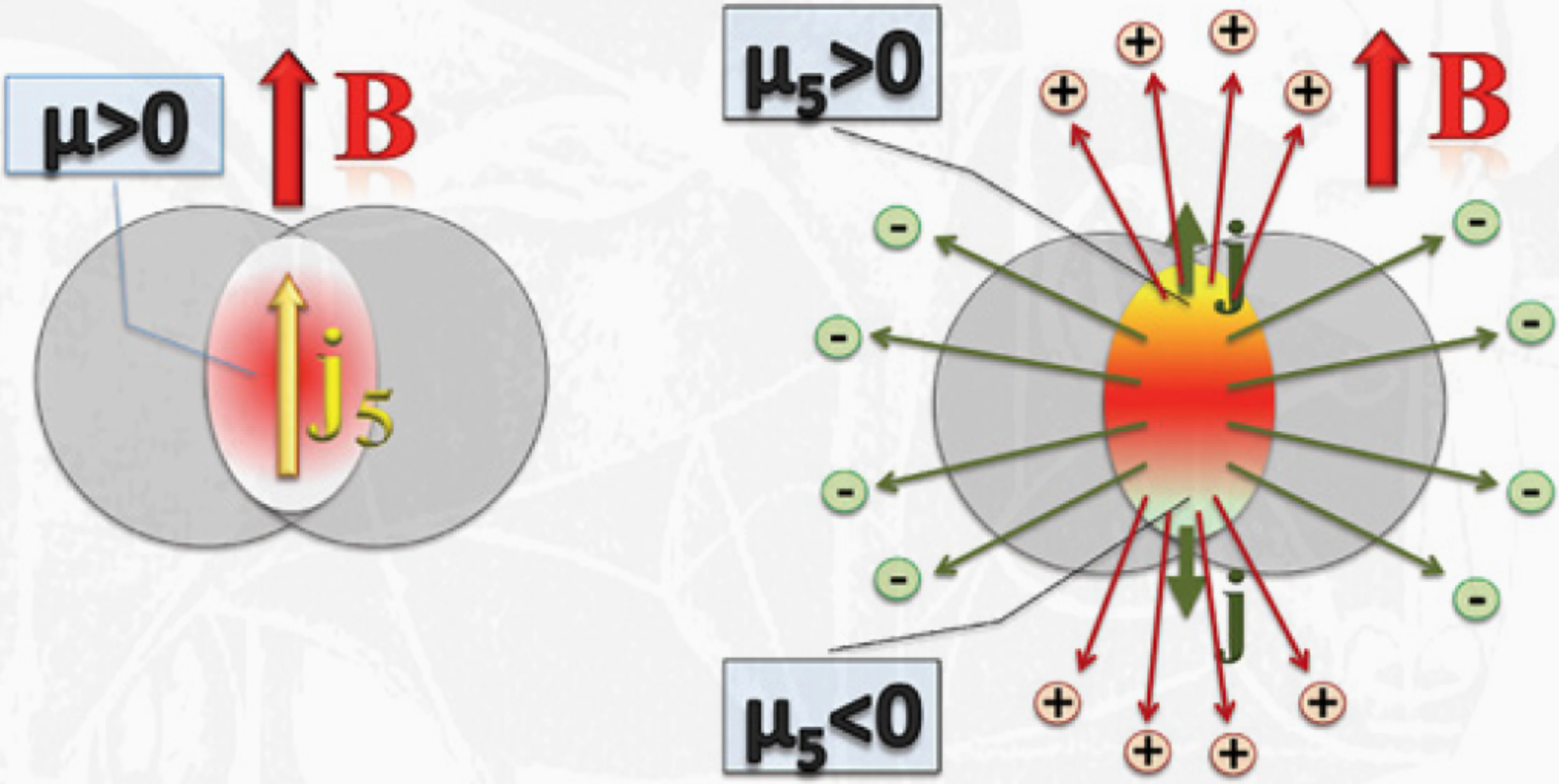
$\mu_5>0$ (one side) & $\mu_5<0$ (another side)

- In turn, chiral charges induce back-to-back electric currents through CME

$$\left\langle \vec{j} \right\rangle_{\text{free}} = -\frac{e^2\vec{B}}{2\pi^2}\mu_5$$

Quadrupole CME

- Start from a nonzero baryon density and B≠0



- Produce back-to-back electric currents

[Gorbar, Miransky, Shovkovy, Phys. Rev. D **83** (2011) 085003]

[Burnier, Kharzeev, Liao, Yee, Phys. Rev. Lett. **107** (2011) 052303]

Generation of chiral shift

- Any additional consequences of the CSE relation?

$$\left\langle \vec{j}_5 \right\rangle_{\text{free}} = -\frac{e\vec{B}}{2\pi^2}\mu \quad \text{with} \quad \vec{B} = (0,0,B)$$

[Metlitski & Zhitnitsky, Phys. Rev. D **72**, 045011 (2005)]

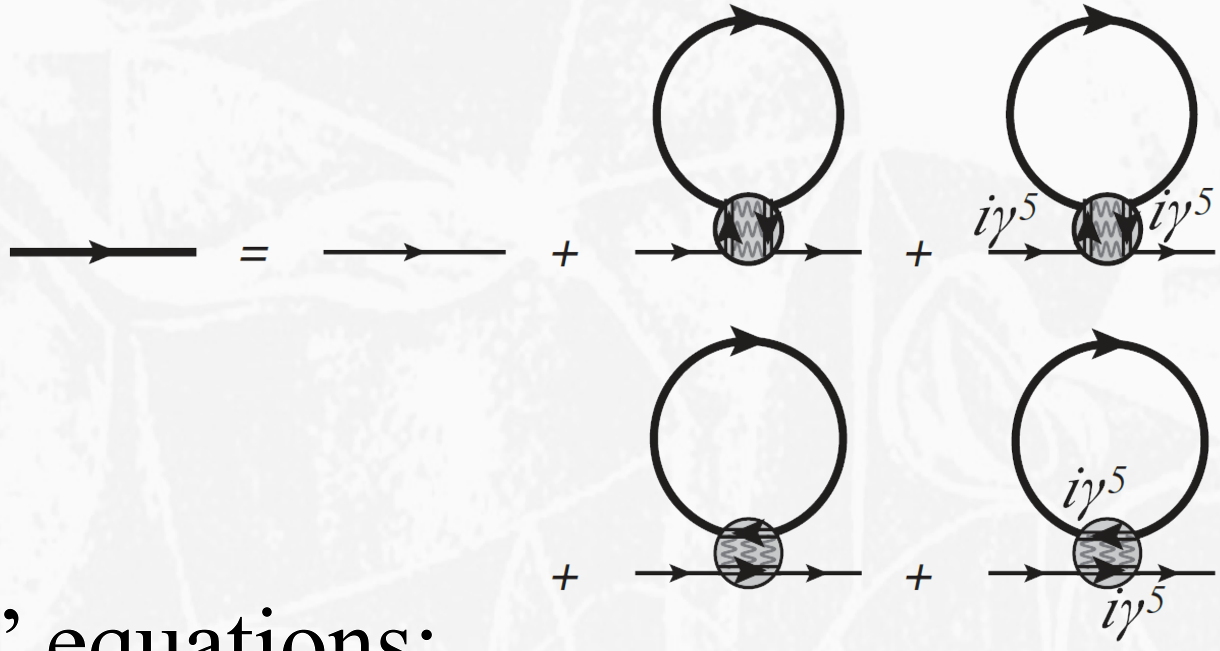
- Perhaps, a dynamical “chiral shift” parameter (Δ) associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \vec{\Delta}\cdot\vec{j}_5 \quad \text{with} \quad \vec{\Delta} = (0,0,\Delta)$$

- Note: Δ=0 is not protected by any symmetry

NJL model: YES

- NJL model (local interaction)



- “Gap” equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \left\langle j^0 \right\rangle$$ (“effective” chemical potential)

$$m = m_0 - G_{\text{int}} \langle \overline{\psi} \psi \rangle$$ (dynamical mass)

$$\Delta = -\frac{1}{2} G_{\text{int}} \left\langle j_5^3 \right\rangle$$ (chiral shift parameter)

Chiral shift @ Fermi surface

- Chirality is $\approx$ well defined at Fermi surface ($|\boldsymbol{k}^3| \gg \boldsymbol{m}$)
- L-handed Fermi surface:

$$n = 0: \qquad k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \qquad k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

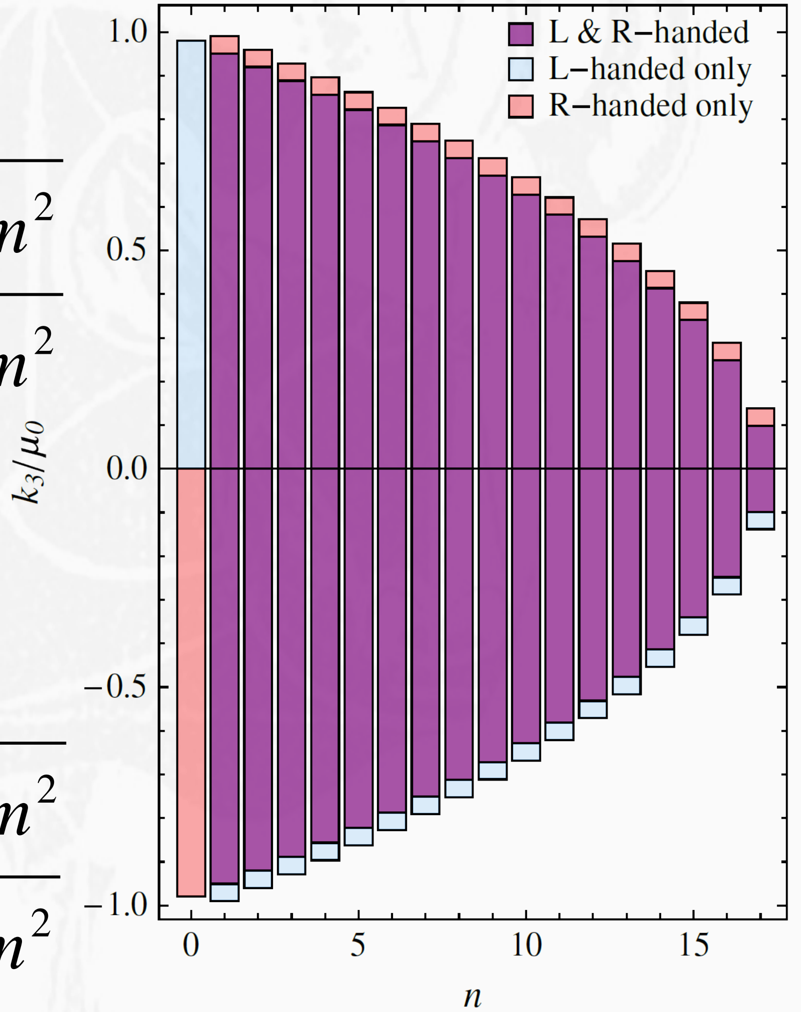
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \qquad k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \qquad k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



[Gorbar, Miransky, Shovkovy, PRD **83** (2011) 085003]

QED: YES

$$\bar{\Sigma}^{(1)}(p) = -4i\pi \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \bar{S}^{(1)}(k) \gamma^\nu D_{\mu\nu}(k-p)$$

- The result has the form

$$\bar{\Sigma}^{(1)}(p) = \gamma^3\gamma^5\Delta + \gamma^0\gamma^5\mu_5(p)$$

where

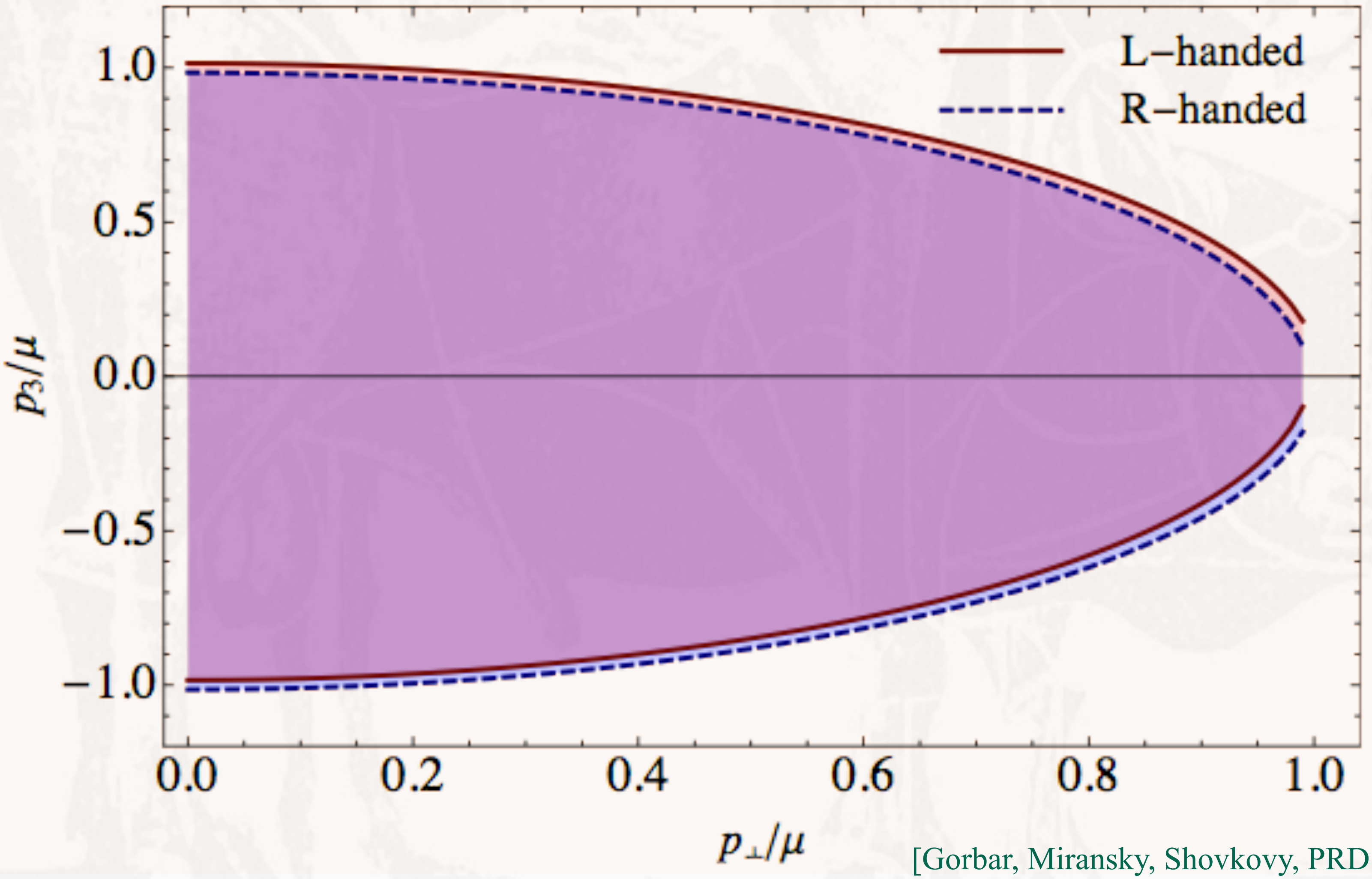
$$\Delta \approx \frac{\alpha eB\mu}{\pi m^2}\left(\ln\frac{m^2}{2\mu(|\mathbf{p}|-p_F)} - 1\right)$$

$$\mu_5(p) \approx -\frac{\alpha eB\mu}{\pi m^2}\frac{p_3}{p_F}\left(\ln\frac{m^2}{2\mu(|\mathbf{p}|-p_F)} - 1\right)$$

Dispersion relations in QED

- Let us use the condition (for a small *B*)

$$\mathrm{Det}\left[i\bar{S}^{-1}(p) + \bar{\Sigma}^{(1)}(p) \right] = 0$$



[Gorbar, Miransky, Shovkovy, PRD **88** (2013) 025043]

Chiral shift vs. axial anomaly

- Does the chiral shift modify the axial anomaly relation?

- Using point splitting method, one derives

$$
\begin{aligned}
\langle \partial_\mu j_5^\mu(u) \rangle &= -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right) \\
&\to -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \to 0
\end{aligned}
$$

[Gorbar, Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

Axial current

- However, the chiral shift does give a contribution to the axial current
- In the point splitting method, one has

$$\left\langle j_5^\mu \right\rangle_{\text{singular}} = -\frac{\Delta}{2\pi^2\varepsilon^2}\delta_\mu^3 \cong \frac{\Lambda^2\Delta}{2\pi^2}\delta_\mu^3$$

[Gorbar, Miransky, I.A.S., Phys. Lett. B **695** (2011) 354]

- This is consistent with the NJL calculations
- Since $\Delta \sim g\mu\, eB/\Lambda^2$, the correction to the axial current is finite

Axial current in QED

- Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \overline{\psi}\left(i\gamma^{\mu}D_{\mu} + \mu\gamma^{0} - m\right)\psi + \text{(counterterms)}$$

- Axial current

$$\left\langle j_5^3 \right\rangle = -Z_2\,\mathrm{tr}\left[\gamma^3\gamma^5 G(x,x)\right]$$

- To leading order in coupling α=e^2/(4π)

$$G(x,y) = S(x,y) + i\int d^4u\, d^4v\, S(x,u)\,\Sigma(u,v)\,S(v,y)$$

[Gorbar, Miransky, Shovkovy, PRD **88** (2013) 025025]

Expansion in external field

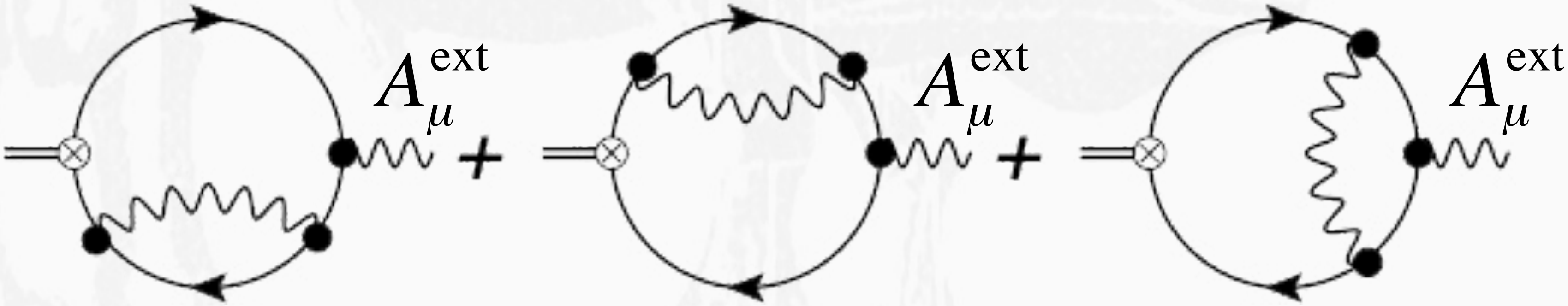
- Expand $S(x,y)$ in powers of gauge field A_μ^{ext}

- To leading order in coupling,

$$\left\langle j_5^3 \right\rangle_0 =$$

- Next-order radiative corrections are

$$\left\langle j_5^3 \right\rangle_\alpha =$$



Alternative form of expansion

- Expand $S(x,y) = e^{i\Phi(x,y)}\overline{S}(x-y)$ as follows:

$$S(x,y) = \underbrace{\overline{S}^{(0)}(x-y) + \overline{S}^{(1)}(x-y)}_{\boxed{\text{Translation invariant part}}} + i\,\underbrace{\Phi(x,y)}_{\boxed{\text{Schwinger phase}}}\,S^{(0)}(x-y)$$

- The Schwinger phase (in Landau gauge)

$$\Phi(x,y) = -\frac{eB}{2}(x_1+y_1)(x_2-y_2)$$

- Note: the phase is not translation invariant

Translation invariant parts

- Fourier transforms of translation invariant parts:

$$\bar{S}^{(0)}(k) = i\frac{(k_0+\mu)\gamma^0 - \mathbf{k}\cdot\gamma + m}{\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2 - \mathbf{k}^2 - m^2}$$

$$\bar{S}^{(1)}(k) = -\gamma^1\gamma^2 eB\frac{(k_0+\mu)\gamma^0 - k_3\gamma^3 + m}{\left[\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2 - \mathbf{k}^2 - m^2\right]^2}$$

- Note the singularity near the Fermi surface…

[Gorbar, Miransky, Shovkovy, PRD **88** (2013) 025025]

Fermi surface singularity

- “Vacuum” + “matter” parts

$$\frac{1}{\left[\left(k_0+\mu+i\varepsilon\,\mathrm{sign}(k_0)\right)^2-\mathbf{k}^2-m^2\right]^n}=\text{"Vac."}+\text{"Mat."}$$

where

$$\text{"Vac."}=\frac{1}{\left[\left(k_0+\mu\right)^2-\mathbf{k}^2-m^2+i\varepsilon\right]^n}$$

$$\text{"Mat."}=\frac{2\pi i(\text{-}1)^{n\text{-}1}}{(n\text{ - }1)!}\theta\left(|\mu|-|k_0|\right)\theta\left(-k_0\mu\right)\delta^{(n-1)}\left[\left(k_0+\mu\right)^2-\mathbf{k}^2-m^2\right]$$

Axial current (0th order)

- From definition

$$\langle j_5^3 \rangle_0 = -\int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\gamma^3\gamma^5 \bar{S}^{(1)}(k)\right]$$

- After integrating over energy

$$\langle j_5^3 \rangle_0 = -\frac{eB\,\mathrm{sign}(\mu)}{4\pi^3} \int d^3\mathbf{k}\, \underbrace{\delta\left(\mu^2 - \mathbf{k}^2 - m^2\right)}_{\boxed{\text{Matter part}}}$$

and finally

$$\langle j_5^3 \rangle_0 = -\frac{eB\,\mathrm{sign}(\mu)}{2\pi^2} \sqrt{\mu^2 - m^2}$$

- Note the role of the Fermi surface (!)

Conventional wisdom

- Only the lowest (n=0) Landau level contributes

$$\langle j_5^3 \rangle_0 = \frac{eB}{4\pi^2}\int dk_3 \left[\theta\left(-\mu - \sqrt{k_3^2 + m^2}\right) - \theta\left(\mu - \sqrt{k_3^2 + m^2}\right)\right]$$

giving same answer

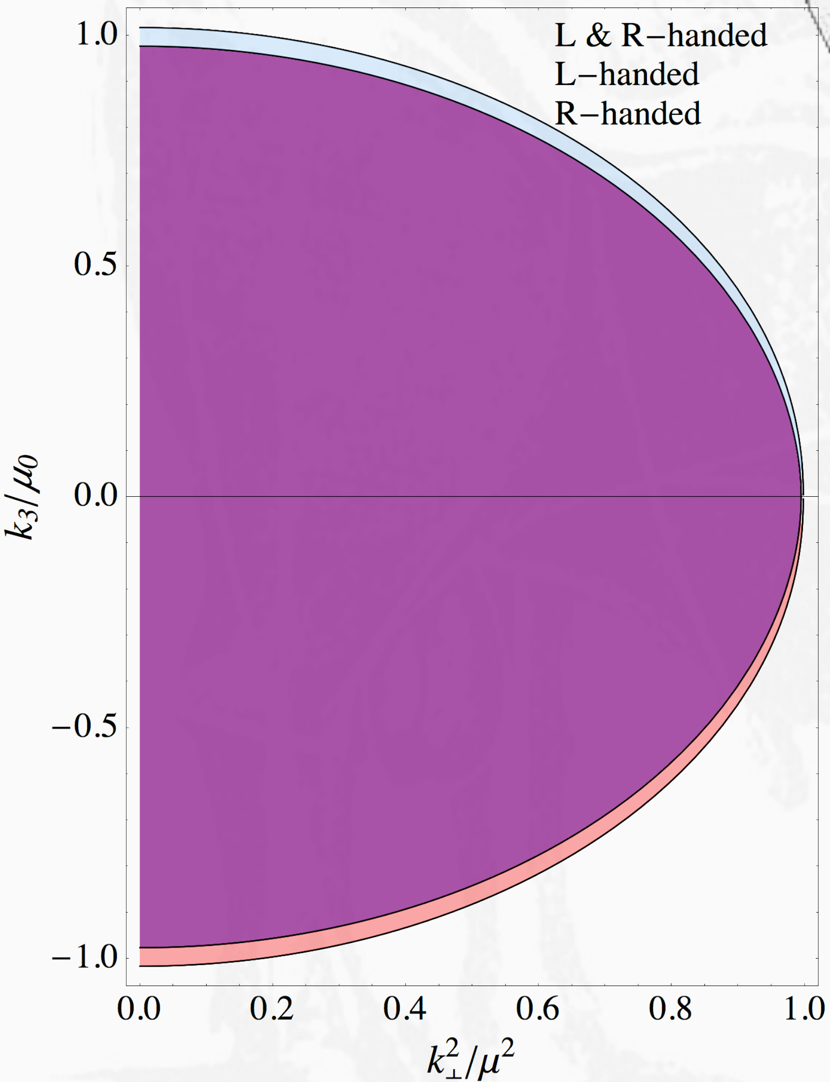
$$\langle j_5^3 \rangle_0 = -\frac{eB\,\mathrm{sign}(\mu)}{2\pi^2}\sqrt{\mu^2 - m^2}$$

- There are no contributions from higher Landau levels (n≥1)
- There is a connection with the index theorem

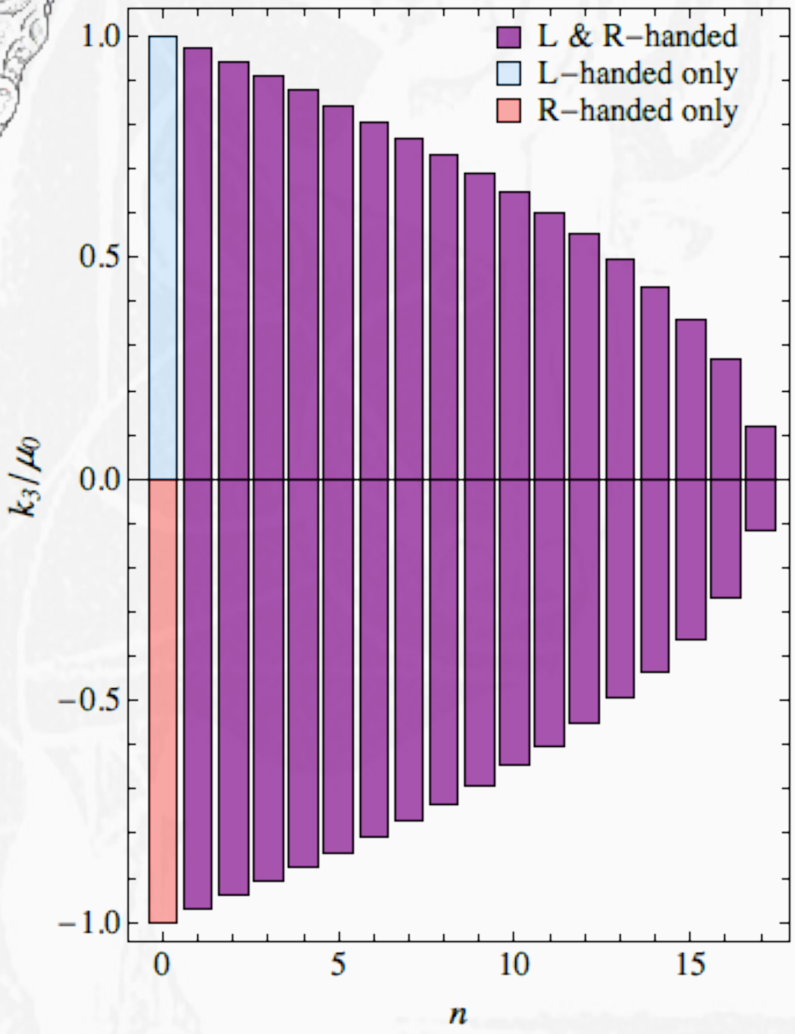
Two facets

- Two ways to look at the same result

$B \rightarrow 0$



$B \neq 0$



Radiative correction

- Original two-loop expression

$$\langle j_5^3\rangle_\alpha = 32\pi\alpha e B \int \boxed{\frac{d^4p\, d^4k}{(2\pi)^8}}\, \frac{1}{(P-K)_\Lambda^2}\left[\frac{(k_0+\mu)[3(p_0+\mu)^2+\mathbf{p}^2+m^2]-4(p_0+\mu)(\mathbf{p}\cdot\mathbf{k}+2m^2)}{(P^2-m^2)^3(K^2-m^2)} - \frac{(k_0+\mu)[3(p_0+\mu)^2-\mathbf{p}^2+3m^2]-2(p_0+\mu)(\mathbf{p}\cdot\mathbf{k})}{3(P^2-m^2)^2(K^2-m^2)^2}\right] + \langle j_5^3\rangle_{\mathrm{ct}}.$$

- After integration by parts

$$\langle j_5^3\rangle_\alpha = 64 i\pi^2\alpha e B \int \frac{d^4p d^4k}{(2\pi)^8}\left[\frac{(k_0+\mu)(p_0+\mu)-\mathbf{p}\cdot\mathbf{k}-2m^2}{(P-K)_\Lambda^2(K^2-m^2)}\boxed{\delta'\left[\mu^2-m^2-\mathbf{p}^2\right]\delta(p_0)} + \frac{3(p_0+\mu)^2-3(k_0+\mu)(p_0+\mu)+\mathbf{p}^2-\mathbf{p}\cdot\mathbf{k}+3m^2}{3(P-K)_\Lambda^2(P^2-m^2)^2}\boxed{\delta\left(\mu^2-m^2-\mathbf{k}^2\right)\delta(k_0)}\right] + \langle j_5^3\rangle_{\mathrm{ct}}$$

Result (m<<μ)

- Loop contributions:

$$f_1 + f_2 + f_3 = \frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{\Lambda}{2\mu} + \frac{11}{12}\right) + \frac{\alpha eB m^2}{2\pi^3\mu}\left(\ln\frac{\Lambda}{2^{3/2}\mu} + \frac{1}{6}\right)$$

- Counterterms:

$$\left\langle j_5^3 \right\rangle_{\mathrm{ct}} = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{\Lambda}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{9}{4}\right) - \frac{\alpha eB m^2}{2\pi^3\mu}\left(\ln\frac{\Lambda}{m_\gamma} - \frac{3}{4}\right)$$

- The final result:

$$\left\langle j_5^3 \right\rangle_{\alpha} = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha eB m^2}{2\pi^3\mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

Sign of nonperturbative physics

- Unphysical dependence on photon mass

$$\left\langle j_5^3 \right\rangle_\alpha = -\frac{\alpha eB\mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha eB m^2}{2\pi^3 \mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

- Infrared physics with

$$m_\gamma \le |k_0|, |k_3| \le \sqrt{|eB|}$$

not captured properly

- Note: similar problem exists in calculation of Lamb shift

[Gorbar, Miransky, Shovkovy, PRD **88** (2013) 025025]

Nonperturbative effects (?)

- Perpendicular momenta cannot be defined with accuracy better than

$$\left|\Delta \mathbf{k}_{\perp}\right|_{\min} \sim \sqrt{|eB|}$$

(In contrast to the tacit assumption in using expansion in powers of *B*-field)

- Screening effects provide a natural infrared regulator

$$m_{\gamma} \Rightarrow \sqrt{\alpha}\,\mu$$

(Formally, this goes beyond the leading order in coupling)

Nonperturbative result (?)

- Conjectured nonpertubative modifications

(1) If non-conservation of momentum dominates

$$\left\langle j_5^3 \right\rangle_\alpha = -\frac{\alpha\, eB\,\mu}{2\pi^3}\left(\ln\frac{\mu|eB|}{m^3} + O(1)\right) - \frac{\alpha\, eB\, m^2}{2\pi^3\mu}\left(\ln\frac{\mu}{\sqrt{|eB|}} + O(1)\right)$$

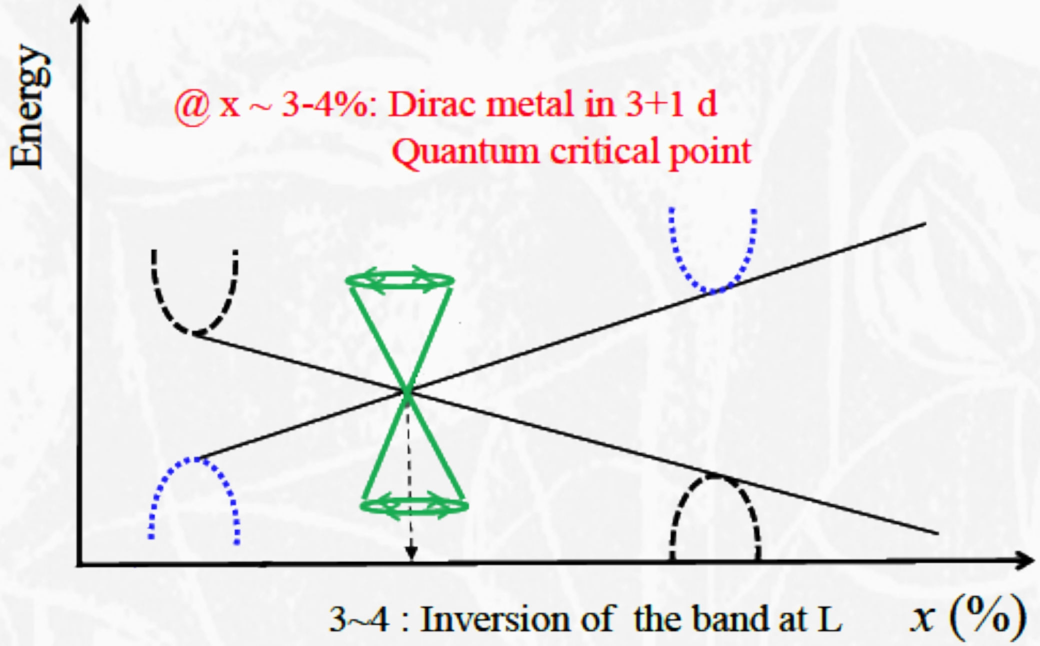
(2) If photon screening is more important

$$\left\langle j_5^3 \right\rangle_\alpha = -\frac{\alpha\, eB\,\mu}{2\pi^3}\left(\ln\frac{\alpha\,\mu^3}{m^3} + O(1)\right) - \frac{\alpha\, eB\, m^2}{2\pi^3\mu}\left(\ln\frac{1}{\sqrt{\alpha}} + O(1)\right)$$

Some challenging work remains…

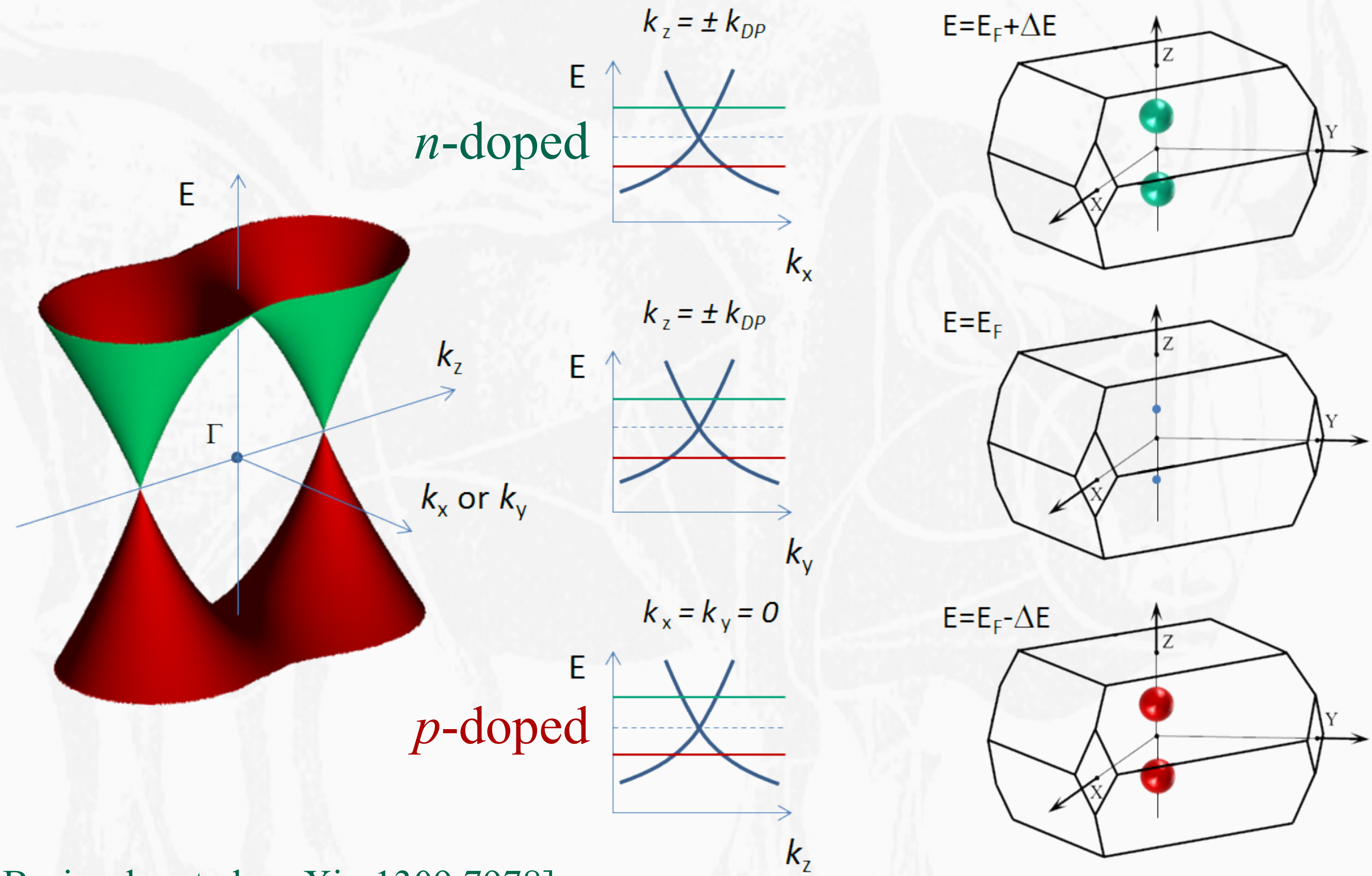
Dirac semimetals

- Solid state materials with Dirac quasiparticles:
  - – $Bi_{1-x}Sb_x$ alloy



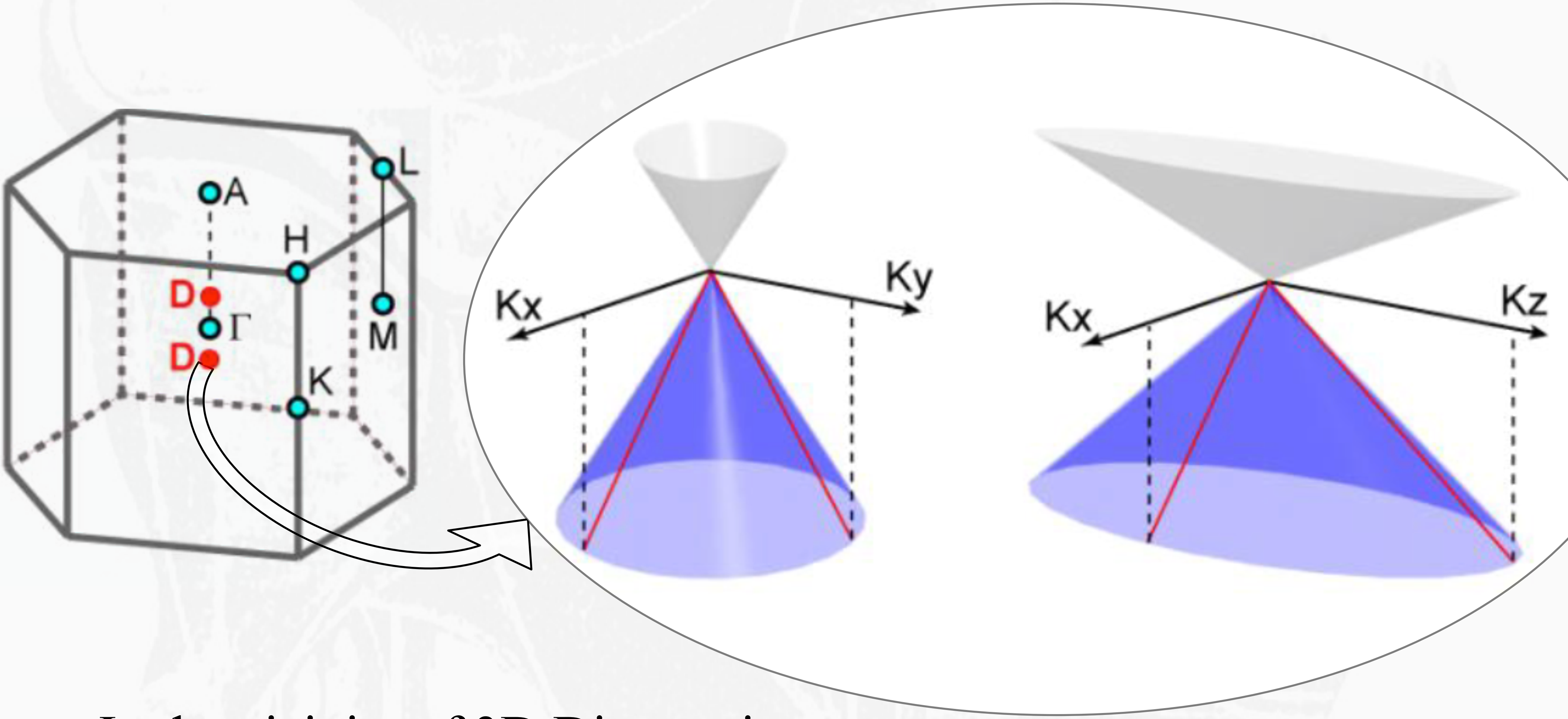
- "New" 3D Dirac materials (ARPES):
  - – Na_3Bi [Z. K. Liu et al., arXiv:1310.0391]
  - – Cd_3As_2 [M. Neupane et al., arXiv:1309.7892] [S. Borisenko et al., arXiv:1309.7978]

Cadmium arsenide


3D Dirac semimetal Cd_3As_2 | Dispersion | Fermi surface

[S. Borisenko et al., arXiv:1309.7978]

Potassium bismuthide



In the vicinity of 3D Dirac points:

$$E = v_x k_x + v_y k_y + v_z k_z$$

[Z. K. Liu et al., arXiv:1310.0391]

Dirac into Weyl semimetal

- Hamiltonian of a Dirac semimetal

$$H^{(D)} = \int d^3r\,\overline{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\nabla}\right) - \mu_0\gamma^0\right]\psi + H_{\text{int}}$$

cf. Weyl semimetal

“chiral shift”

$$H^{(W)} = \int d^3r\,\overline{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\nabla}\right) - \left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5 - \mu_0\gamma^0\right]\psi + H_{\text{int}}$$

- In a Dirac semimetal, a nonzero chiral shift $\vec{b}$ will be induced when $B\neq0$, i.e.,

$$\vec{b} \propto -\frac{g}{v_F^2 c}\mu_0 e\vec{B}$$

[Gorbar, Miransky, Shovkovy, Phys. Rev. B **88**, 165105 (2013)]

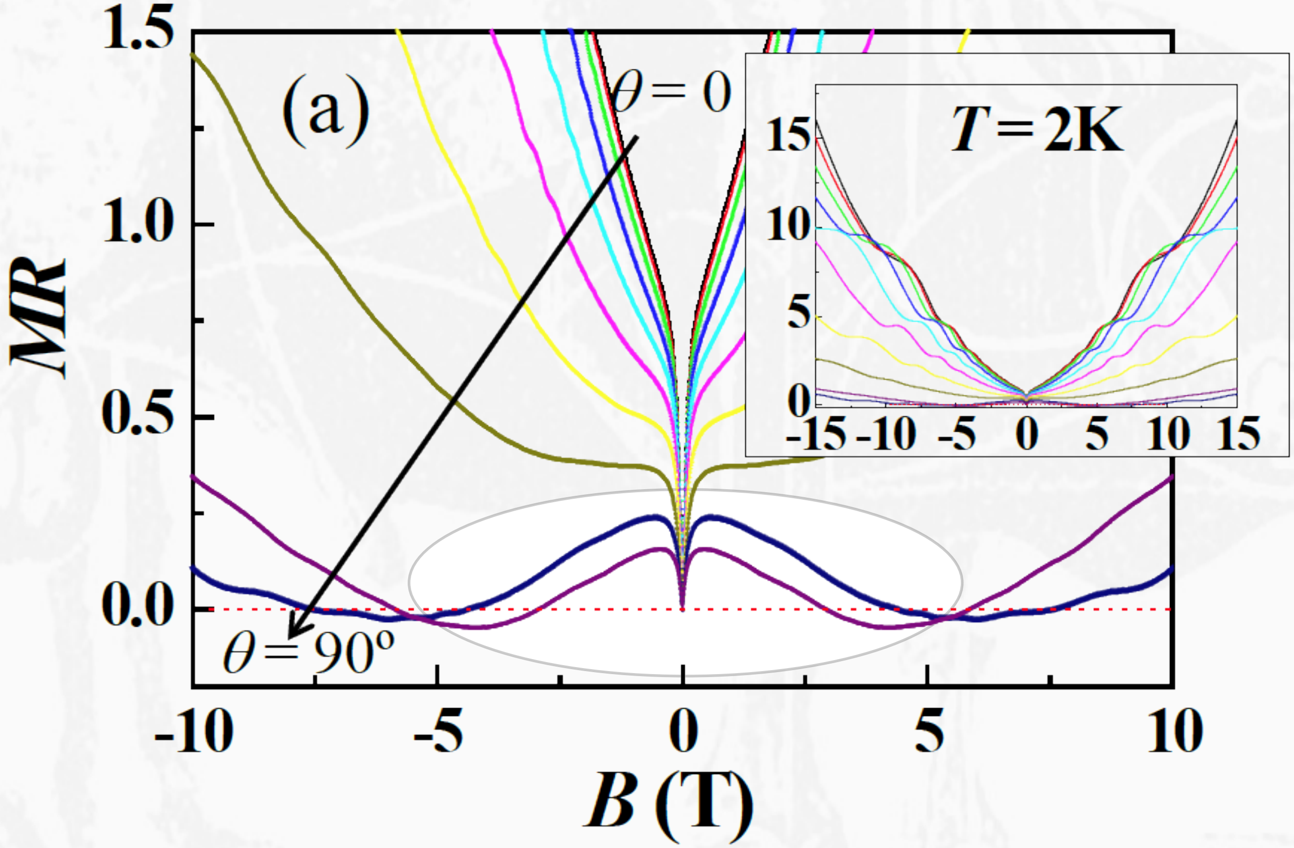
Negative magnetoresistance

- ρ_{33} is expected to decrease with B because

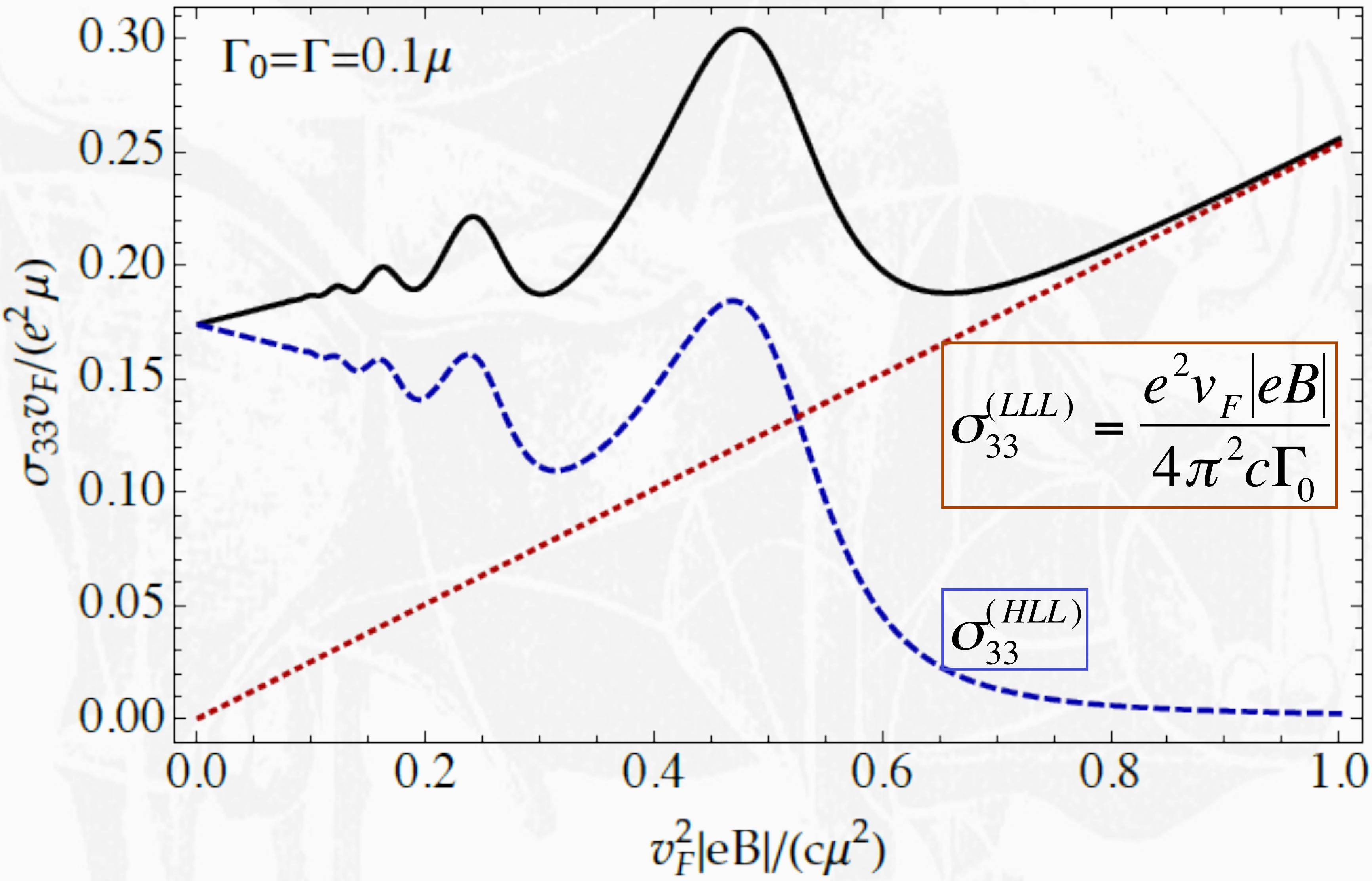
  $\sigma_{33} \propto B^2$ (weak B) [Son & Spivak, Phys. Rev. B 88, 104412 (2013)]

  $\sigma_{33} \propto B$ (strong B) [Nielsen & Ninomiya, Phys. Lett. 130B, 390 (1983)]

- Experimental confirmation (?) [Kim, et al., arXiv:1307.6990]



Longitudinal conductivity

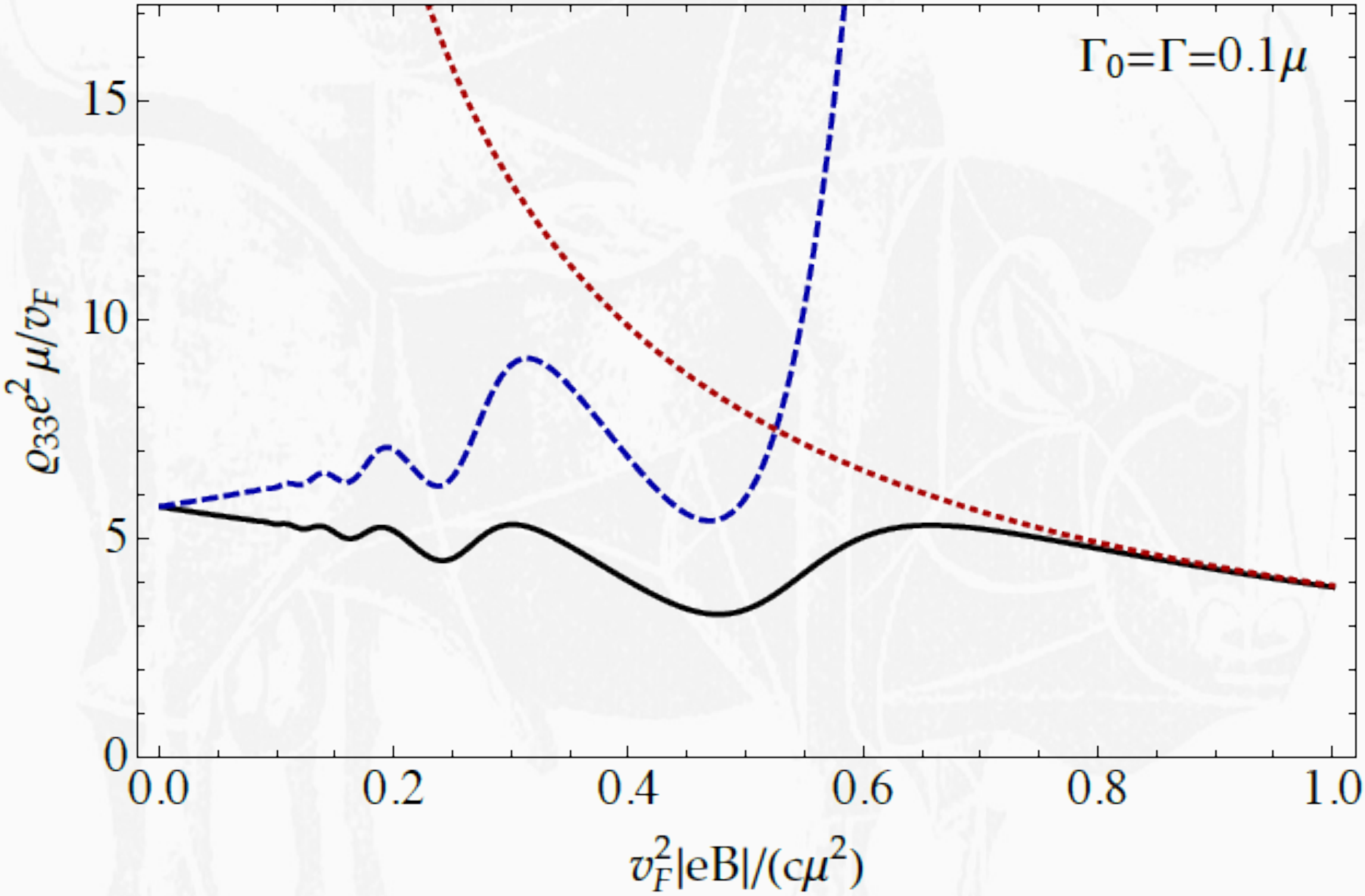


$$\sigma_{33}^{(LLL)} = \frac{e^2 v_F |eB|}{4\pi^2 c\Gamma_0}$$

- σ_{33} grows with *B* even in Dirac semimetals (*b*=0)

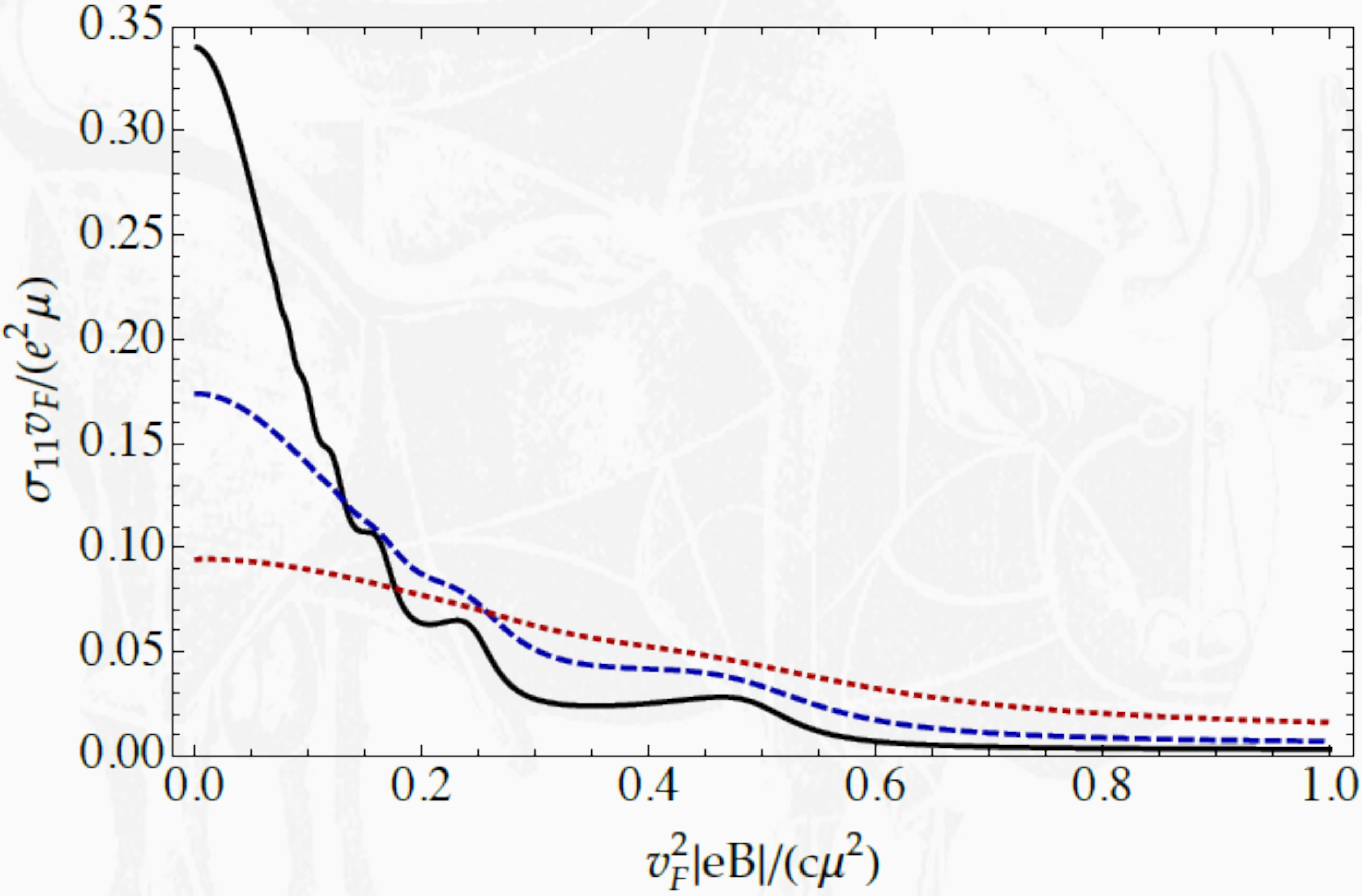
[Gorbar, Miransky, Shovkovy, arXiv:1312.0027]

Longitudinal resistivity



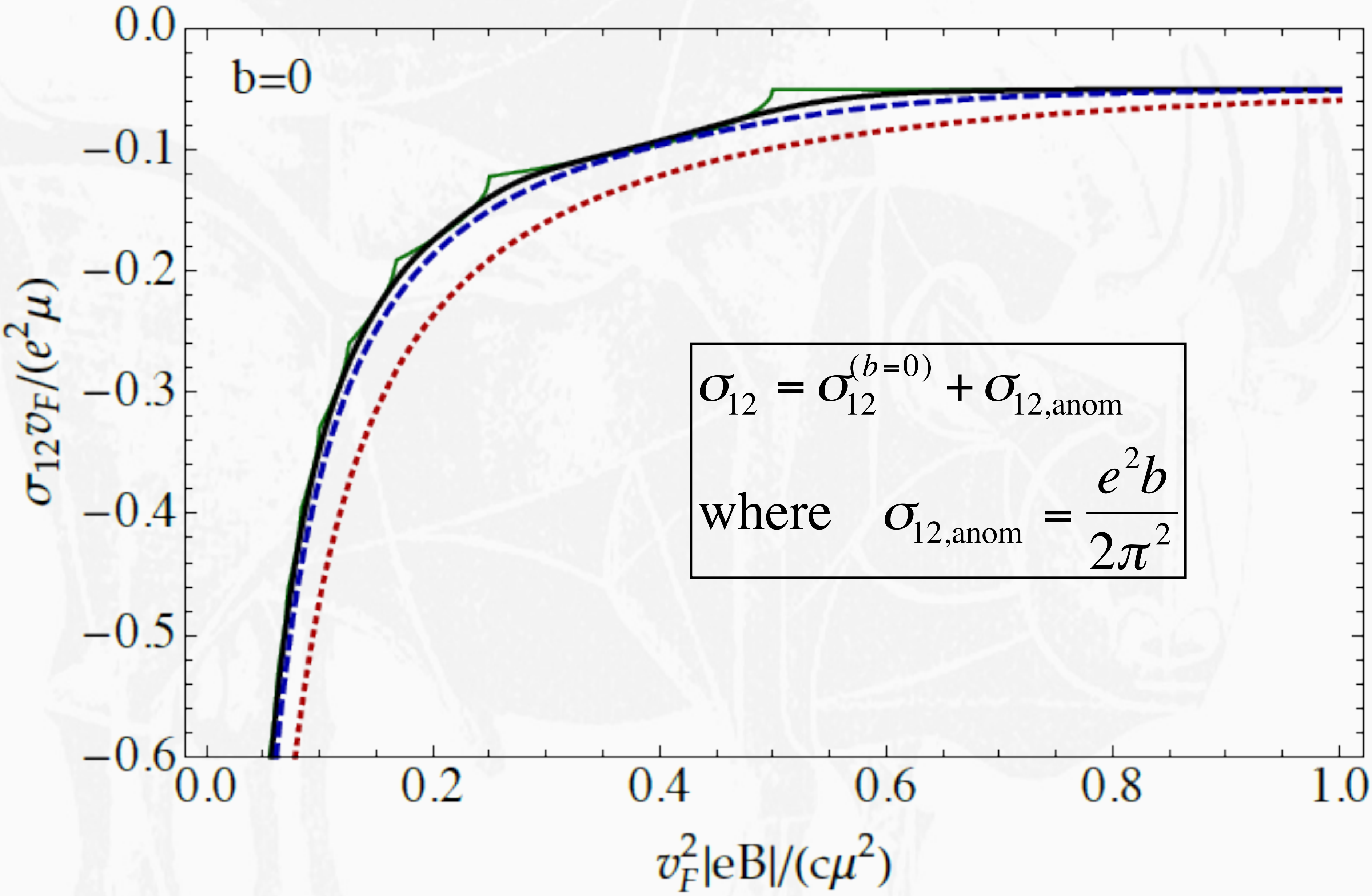
[Gorbar, Miransky, Shovkovy, arXiv:1312.0027]

Transverse diagonal σ_{11}



[Gorbar, Miransky, Shovkovy, arXiv:1312.0027]

Transverse off-diagonal σ_{12}



$$\sigma_{12} = \sigma_{12}^{(b=0)} + \sigma_{12,\mathrm{anom}}$$

where $$\sigma_{12,\mathrm{anom}} = \frac{e^2 b}{2\pi^2}$$

[Gorbar, Miransky, Shovkovy, arXiv:1312.0027]

Summary (1)

- Weak *B*-field limit: **new interpretation** of the topological contribution to CSE relation
- Radiative corrections are **nonzero**
- Radiative corrections vanish without "matter" part with **singularity on Fermi surface**
- **Nonperturbative** physics complicates the infrared contribution
- With **logarithmic accuracy**, the result can be conjectured

Summary (2)

- Chiral shift is generated in magnetized matter (evidence from renormalizable model now)

- The magnitude of chiral shift scales as

$$\Delta \propto \frac{\alpha\, eB\, \mu}{m^2} \ln\alpha$$

- Chiral shift induces a chiral asymmetry at the Fermi surface

- Chiral shift contributes to the axial current

Summary (3)

- Chiral shift can be realized in condensed matter (Dirac into Weyl semimetals)
- Some features may indicate the appearance of Weyl semimetals
- Magneto-transport is quite involved
  - Negative longitudinal magnetoresistance
  - Anomalous off-diagonal transverse conductivity
  - Shubnikov-de Haas oscillations may complicate the interpretation