Fermilab

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Foreground Purification and Polarscopes

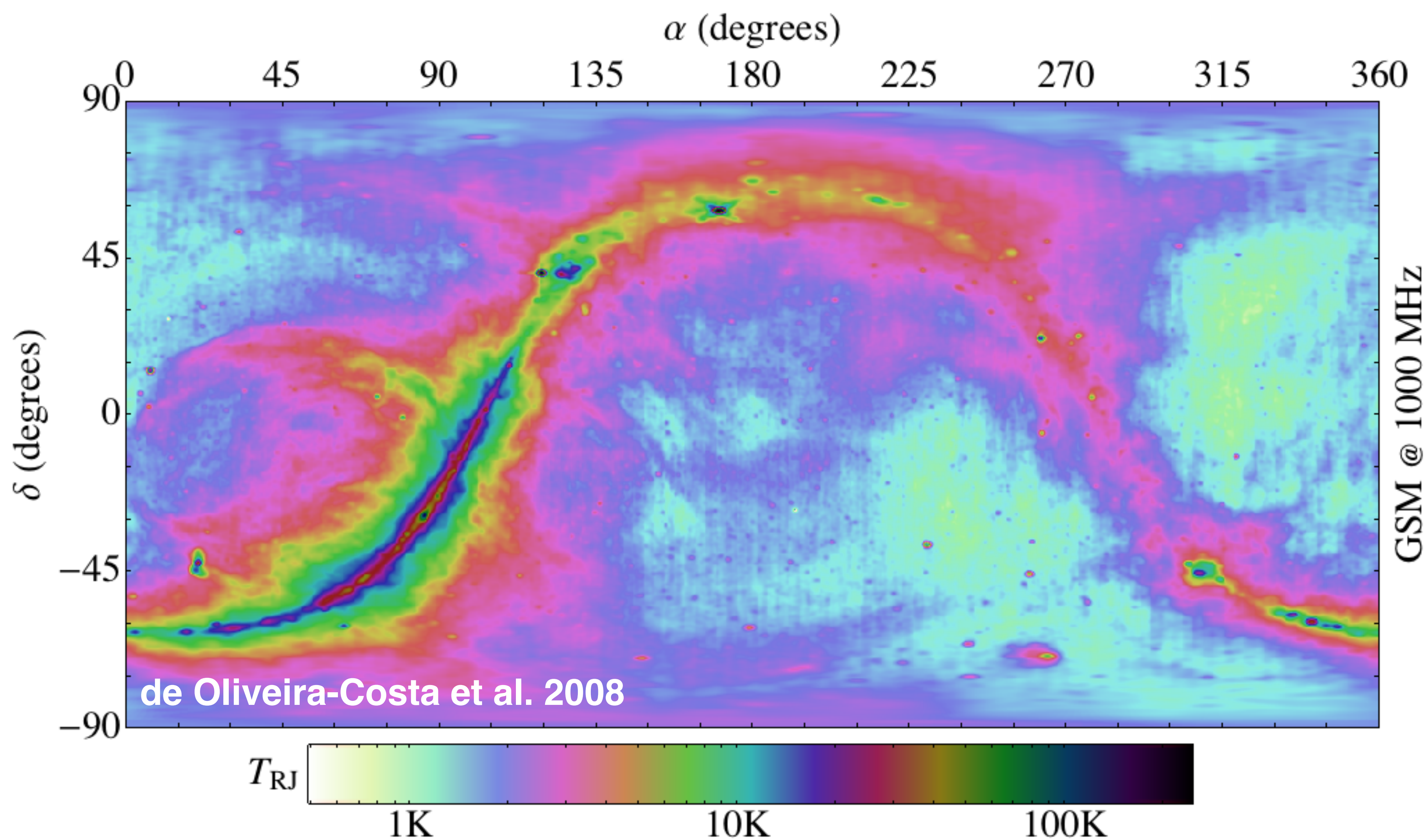
Albert Stebbins

CMB, LSS and 21 cm

Madrid, Spain

16 June 2016

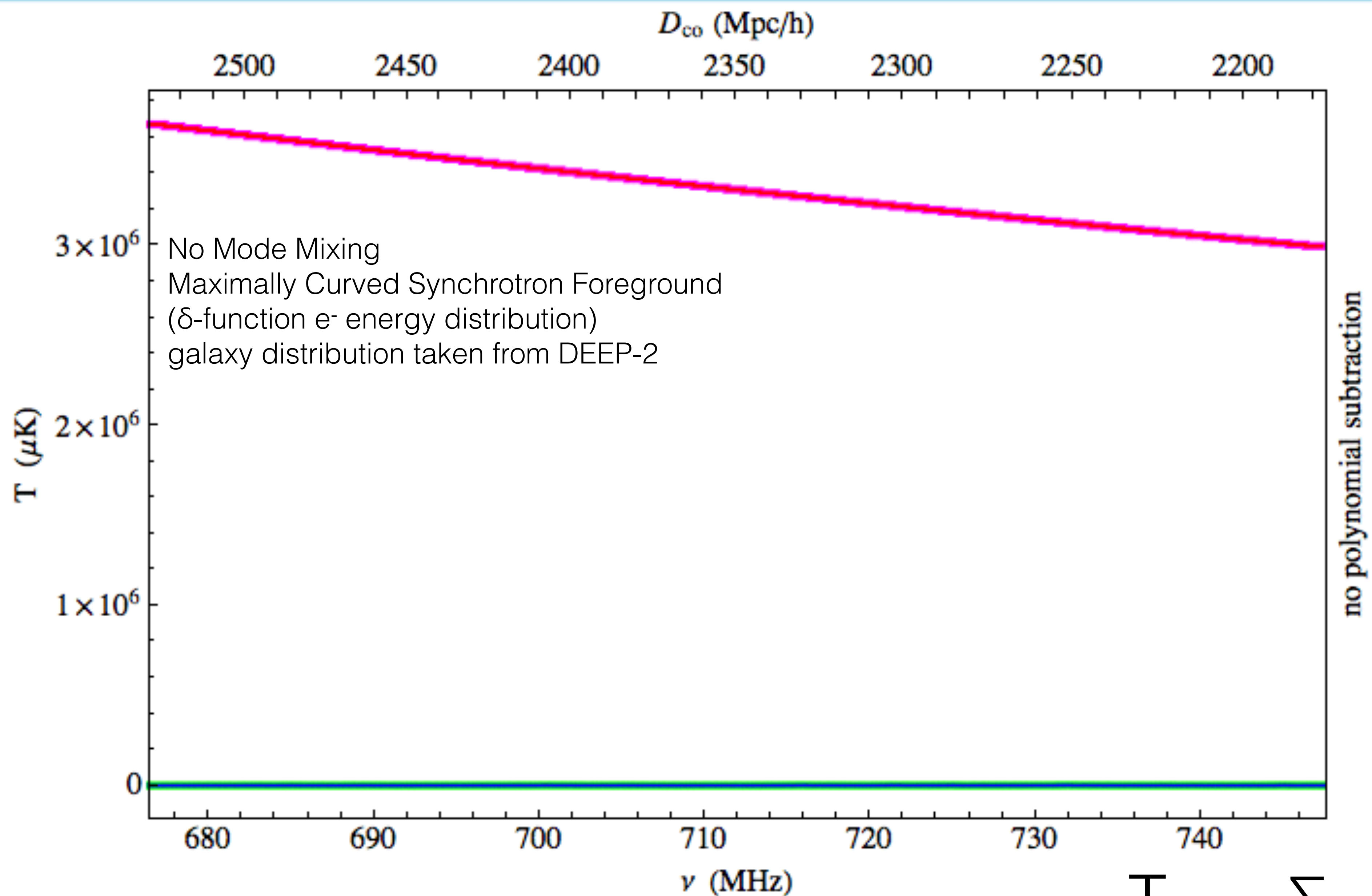
Foreground removal efficacy remains a significant issue



Even for dark radio sky ~1K foreground is ~10^4 larger than ~100μK signal

Foregrounds are expected to be smooth in frequency
… but are they?

Ideally smooth foreground subtraction should work well



$$T_{fit} = \sum_p a_p \ln[\nu]^p$$

- Many algorithms proposed to take advantage of this
- Would like to get noise down to ~50μK level to see real non-linear structures in lo-z (z<5) 21cm maps

Non-Smooth Spectrum Foregrounds

- While it is true that optically thin free-free or synchrotron emission is smooth in the optically thin limit for any electron energy distribute - yet it need not be so when self-absorption is present.

- There is evidence for synchrotron self-absorption in gigahertz peaked sources (GPS).

- Faraday rotation linearly polarized light can cause the linearly polarization to have oscillatory behavior which can leak into the inferred intensity.
  - e.g. GBT 21cm maps

- While it is unlikely that these could have spectral features as sharp as those expected in 21cm spectrum it can contaminate the low k modes which are important for measuring quantities like f_{NL}.

Mode Mixing: Abstract

- An interferometer with a finite number of elements will only “see” a finite number of “beams” on the sky.
- The Hilbert space of all linear combinations of beams we call the space of beams.
- This space of beams generally depends on frequency. This frequency dependence of the Hilbert spaces is called mode mixing because it irreducibly mixes frequency dependence and angle dependence.
- If we could eliminate mode mixing one could directly measure the (spatially averaged) frequency spectrum with no contamination from angular structure.
- Hi-Pass filtering out smooth spectrum foregrounds works better when the amount of mode mixing is minimized.
- Goal: to “purify” the spectrum from mode mixing contamination.
- Emphasis on optimizing interferometer design (not optimizing analysis).

Beam Projection and Purity

- Given a metric, $\cdot$, on the space of beam define the beam projection operator:

  $$\mathfrak{B}[\nu] \equiv \sum_{i,j} \mathbf{B}_i[\nu]\cdot(\mathbf{B}_i[\nu]\cdot\mathbf{B}_j[\nu])^{-1}\cdot\mathbf{B}_j[\nu]$$

  where $\mathbf{B}_i[\nu]$ are the frequency dependent beam in the Stokes $\times$ angle space. Each beam corresponds to a distinct inteferometric pair of feeds, distinct after considering rotational synthesis: $n_{beam} \leq \frac{1}{2} n_{feed} (n_{feed} - 1)$

- $\mathfrak{B}[\nu]$ has n_{beam} (number of beams) unit eigenvalues and the rest zero

- Define the purity operator by

  $$\boldsymbol{\mathcal{P}} \equiv \int d\nu\, W[\nu]\, \mathfrak{B}[\nu] = \sum_a p_a\ \boldsymbol{\wp}_a \otimes \boldsymbol{\wp}_a$$

  where $W[\nu]$ is a ν weight function (or purity band) such that: $\int d\nu\, W[\nu] = 1$

  the $\boldsymbol{\wp}_a$ (eigenvectors) are purity eigenbeams: $\boldsymbol{\wp}_i \cdot \boldsymbol{\wp}_j = \delta_{ij}$

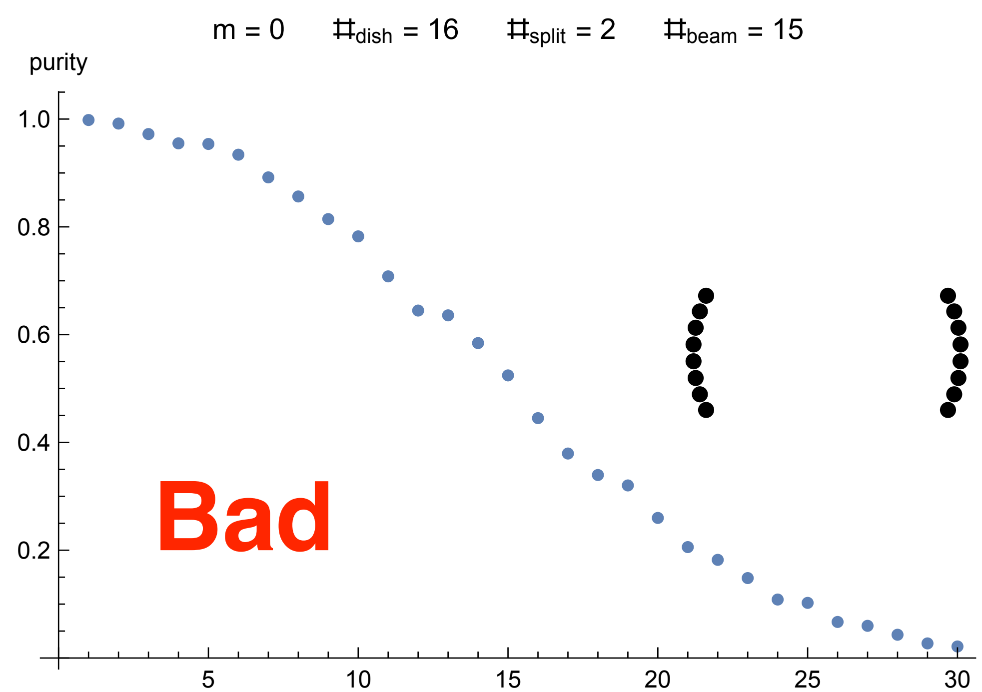
  the p_a (eigenvalues) are purities: $0 \leq p_a \leq 1$ and $\sum_a p_a = 1$

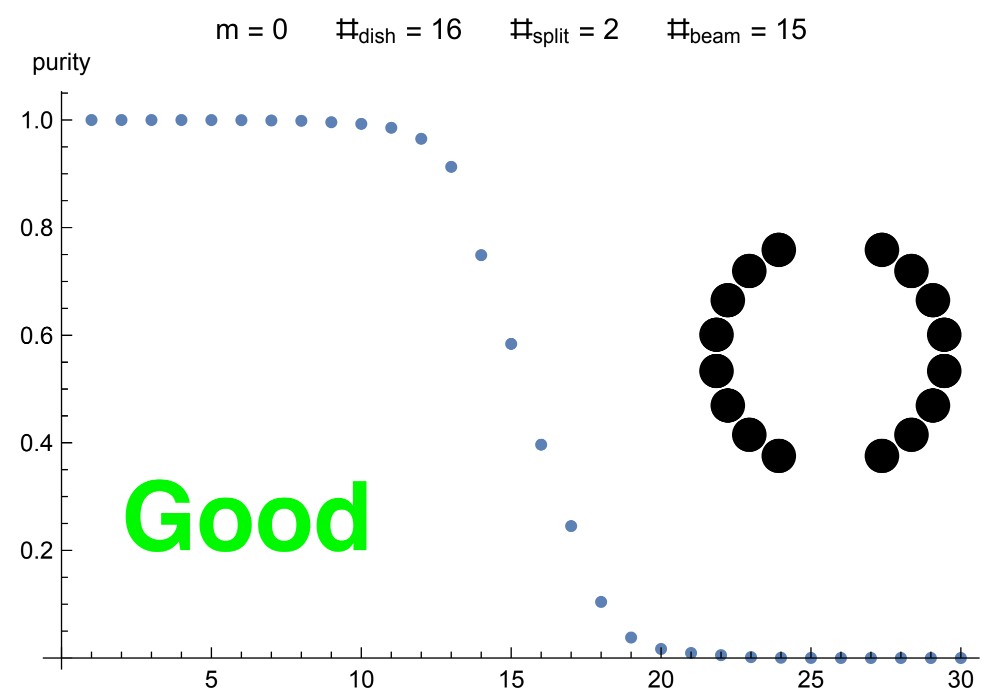
- The $\boldsymbol{\wp}_a$ with the largest purity has the least mode mixing!

- The $\boldsymbol{\wp}_a$ with $p_a \ll 1$ have large amounts of mode mixing and high pass filtering is less effective at removing smooth but highly anisotropic foregrounds.

Purity and Telescope Design

- There are at most n_{beam} very pure ($p_a \cong 1$) modes $\wp_a$.
- N.B. $p_a \rightarrow 1$ in the limit of zero bandwidth: $W[\nu] \rightarrow \delta[\nu-\nu_0]$
- A high purity interferometer is an one which for a given bandwidth has close to n_{beam} very pure modes. They are useful for understanding the underlying spectra of the emission.





- Define the purity number $=-\ln[1-p_a]$ which is large for very pure modes
- Dense arrays with large overlap, $\mathbf{B}_i[\nu]\cdot\mathbf{B}_j[\nu]$, do better than sparse arrays (see Reza's talk).
- One never does worse by adding an additional element to an existing array (but ¥ $)

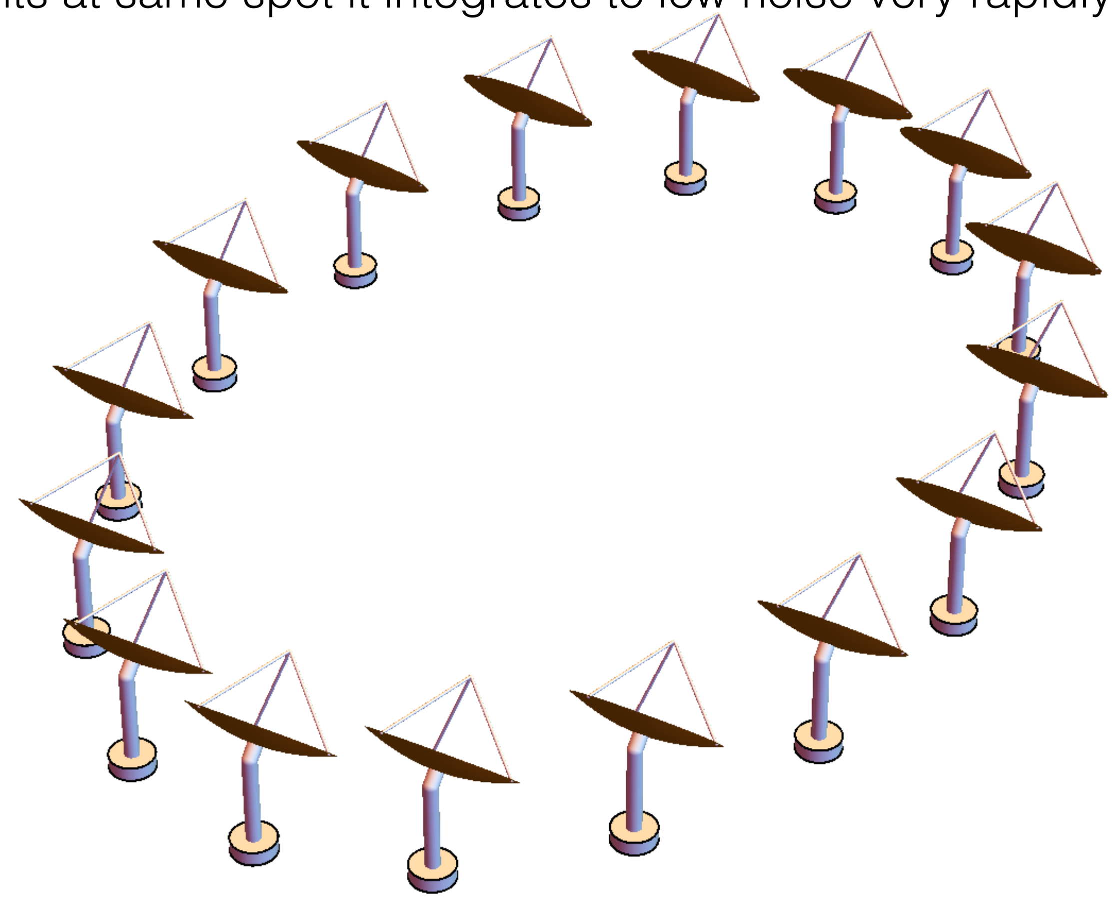
Polarscope

A polarscope is a transit interferometer consisting of a number antenna each pointing directly at a Celestial Pole, North or South.

Rotationally synthesized beam patterns depend only on magnitude of projected feed separation perpendicular to CP direction, i.e. diurnal orbits in UV plane are circles.

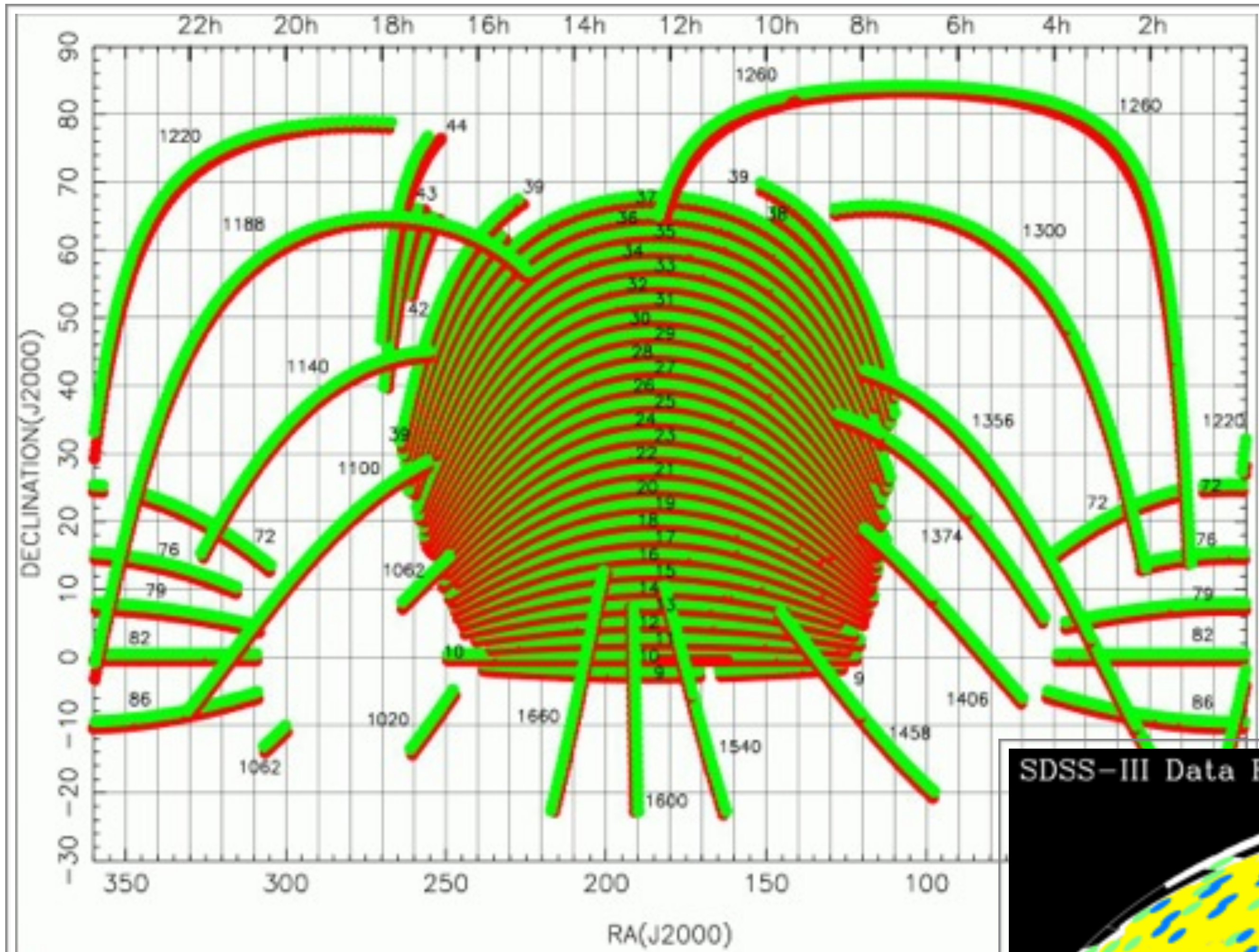
Like all transit telescopes the projection/purity operator is block diagonal in R.A. *m*-space

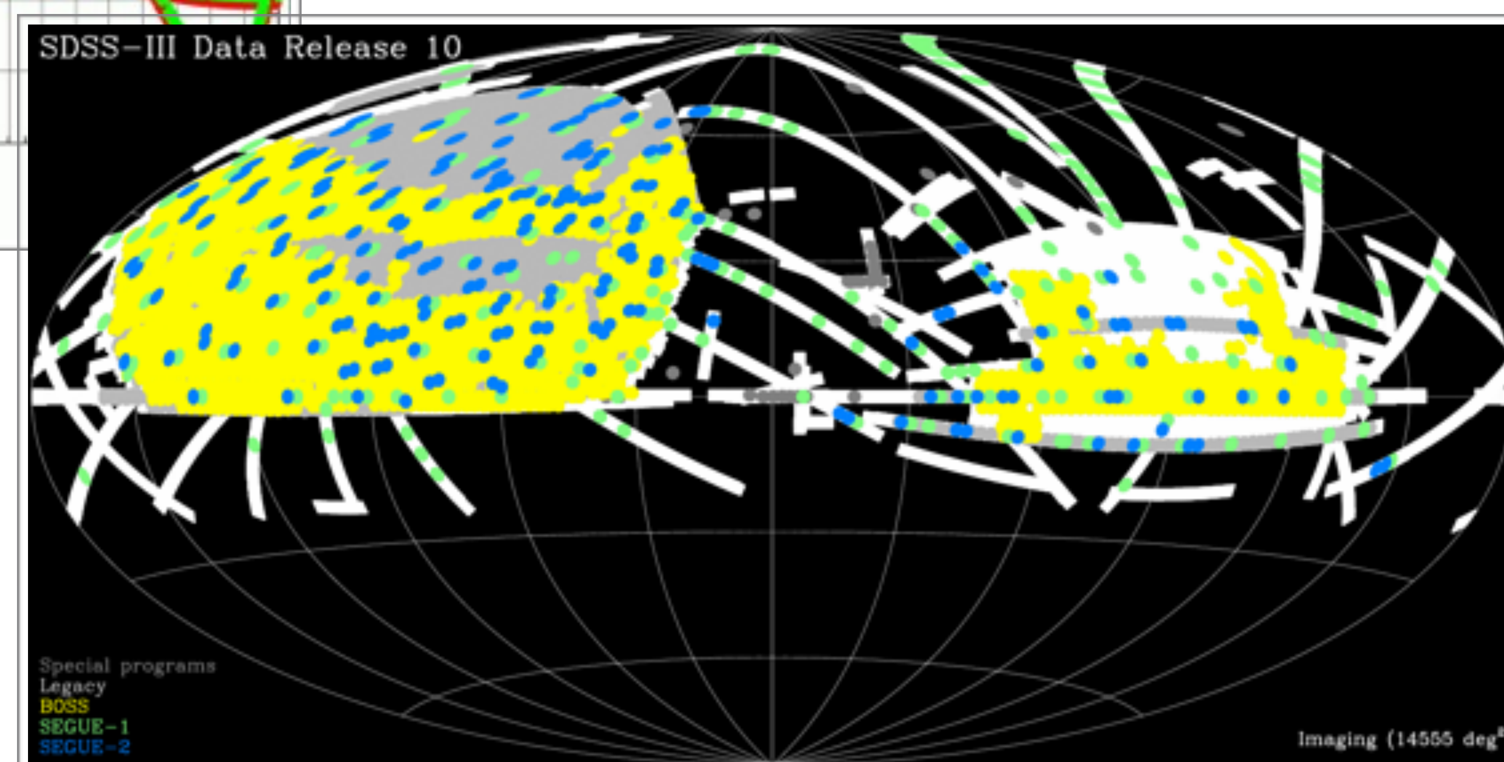
good: Since it always points at same spot it integrates to low noise very rapidly



bad: sources near celestial poles move slowly so a polarscope has very little handle on diurnal timescale transients, *e.g*. ground pickup. N.B. Signals repeat every half day not every day.

Unfortunately Other Surveys Have Avoided NCP

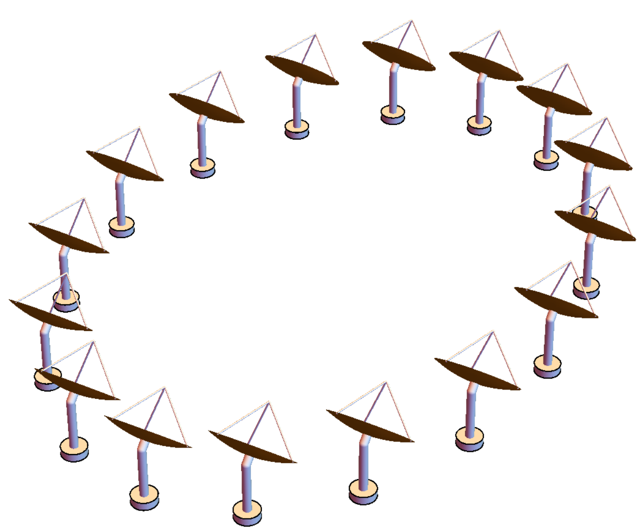




Tianlai dish array could operate as a Polarscope

Spherical Cow Polarscope

- 16 identical dishes
- analyze only intensity (no polarization)
- assume Gaussian intensity primary beam (allows fast analytic computation)
- this is not too bad a representation of Tianlai dish/feed configuration according to EM simulations
- graphically represent beam pattern by dish pattern as seen from the Celestial Pole (not patten on ground as seen from zenith)



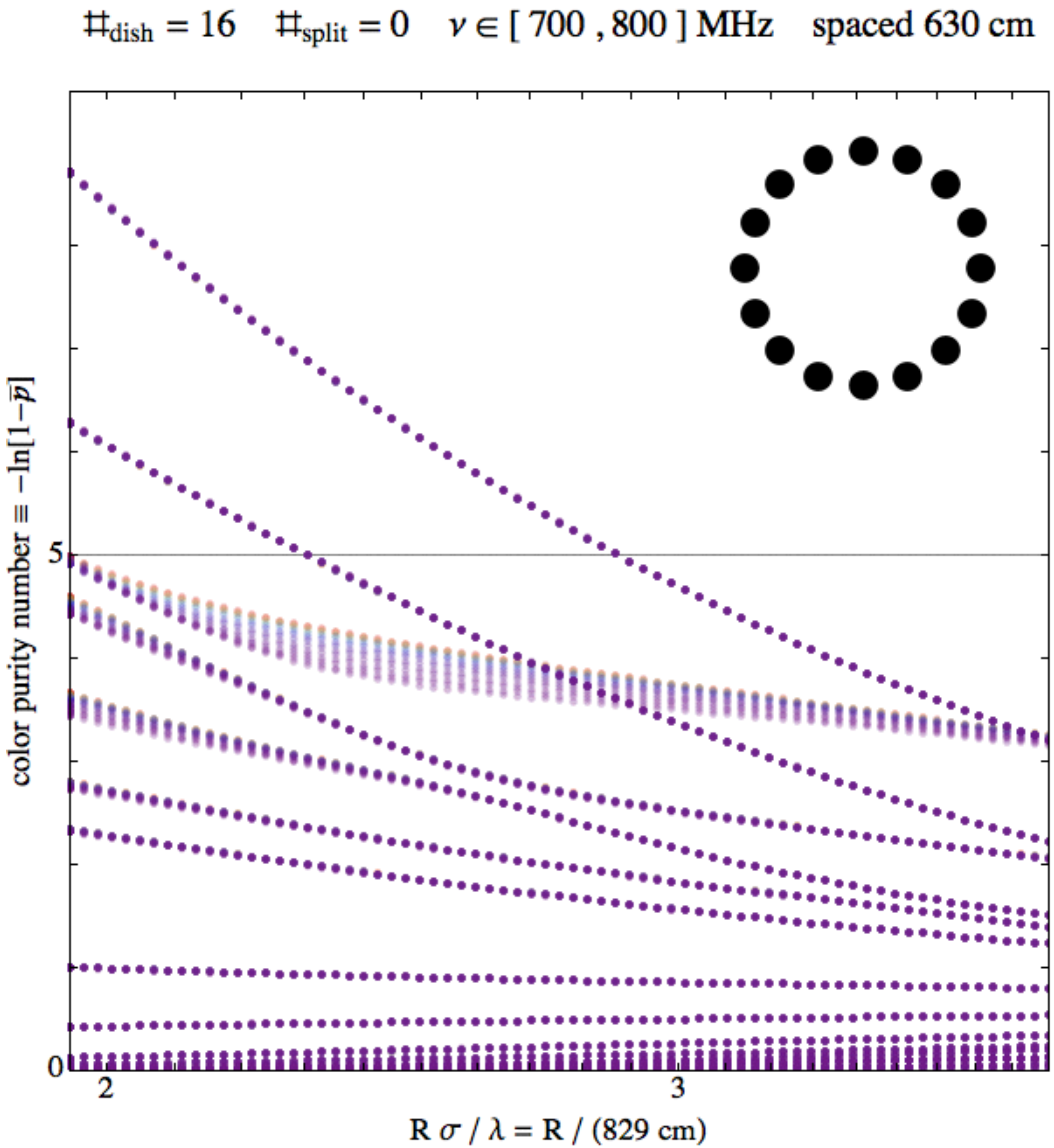
uniformly distributed on projected circle

each purity eigenmode has fixed *m* (R.A. dependence)

different *m* are colored differently

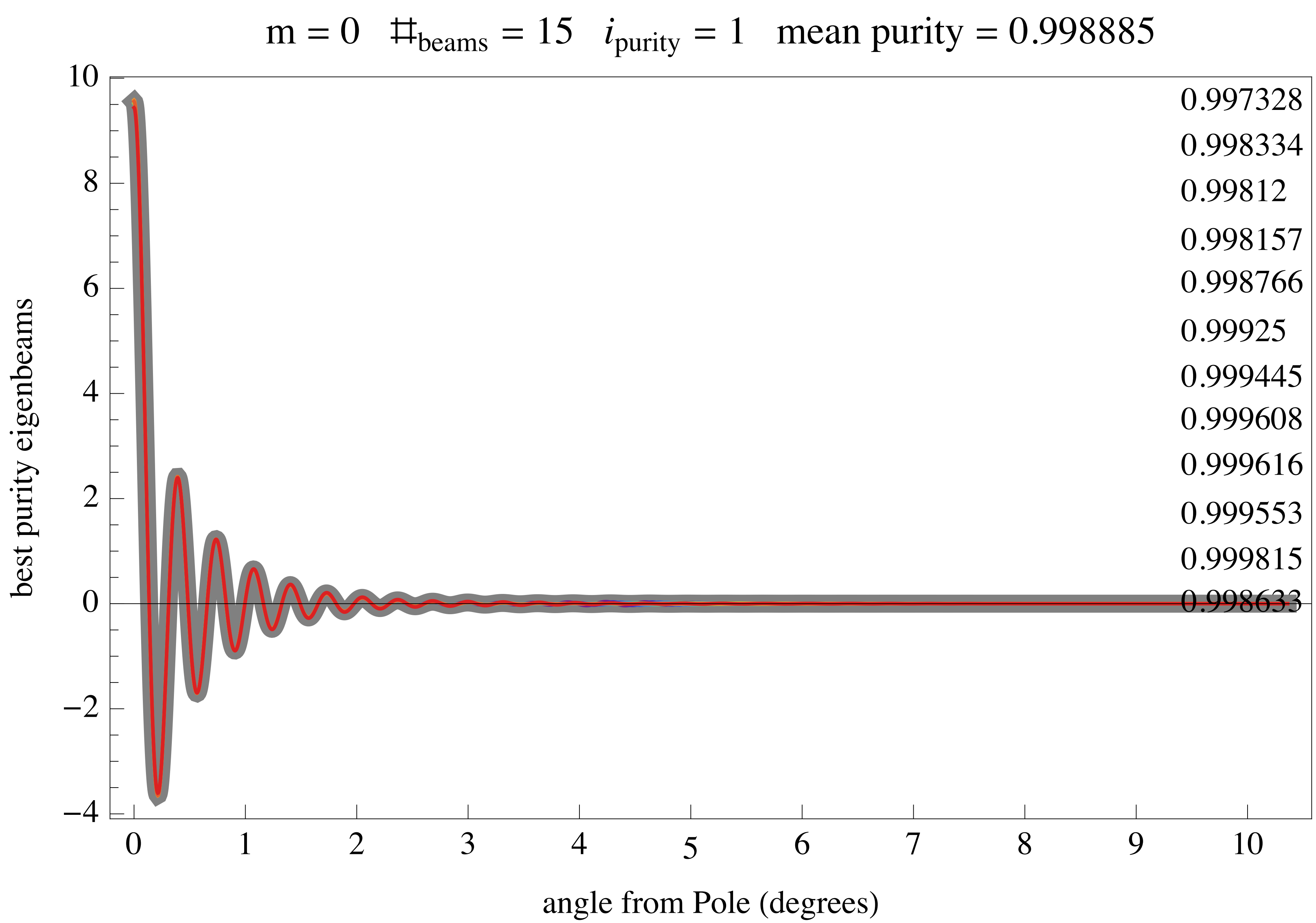
however purity is largely degenerate with *m* so only see one color (purple)

the most compact array (most overlapping beams) have highest purity



vary overall radius of configuration

declination dependence of purity eigenbeams

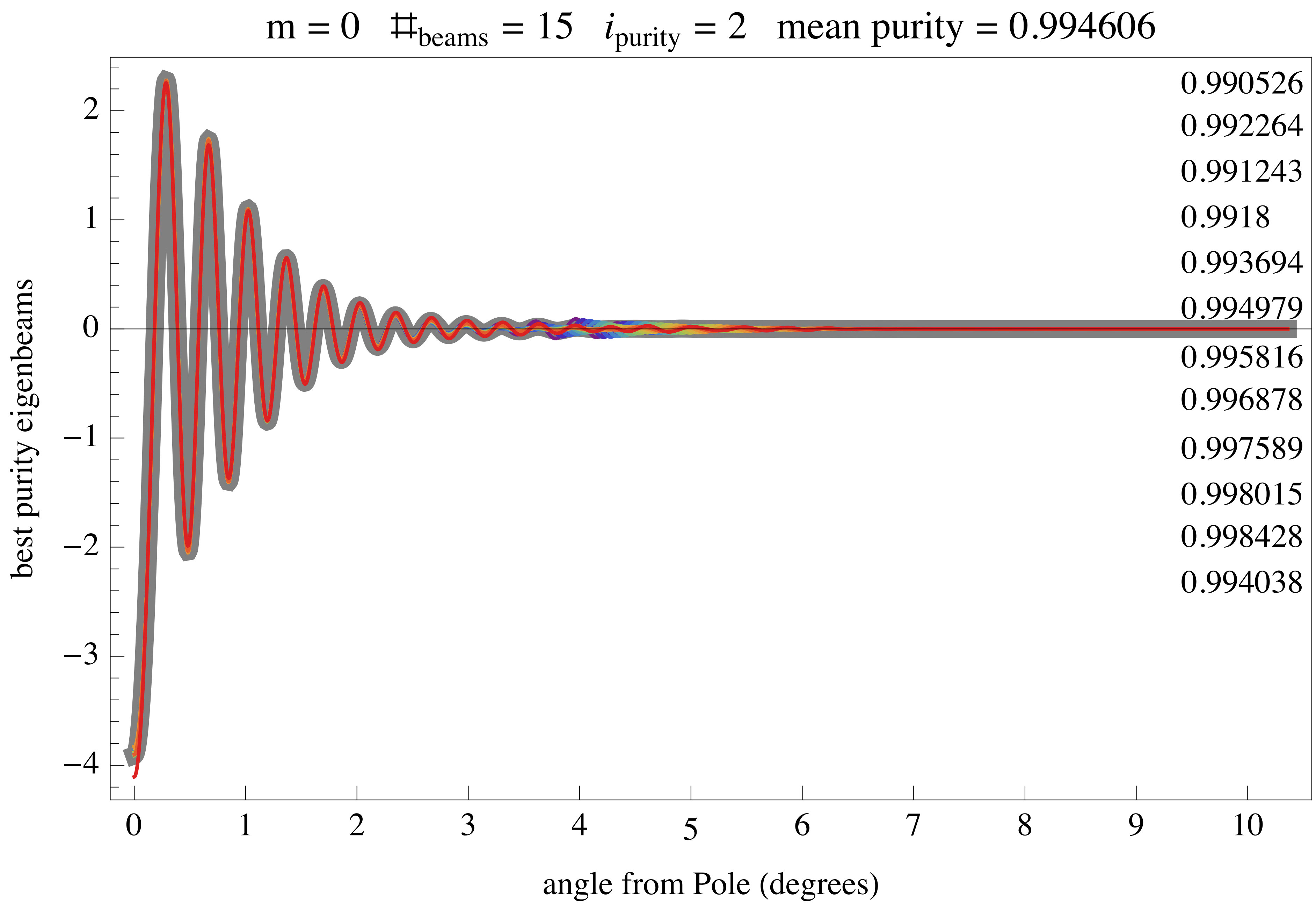


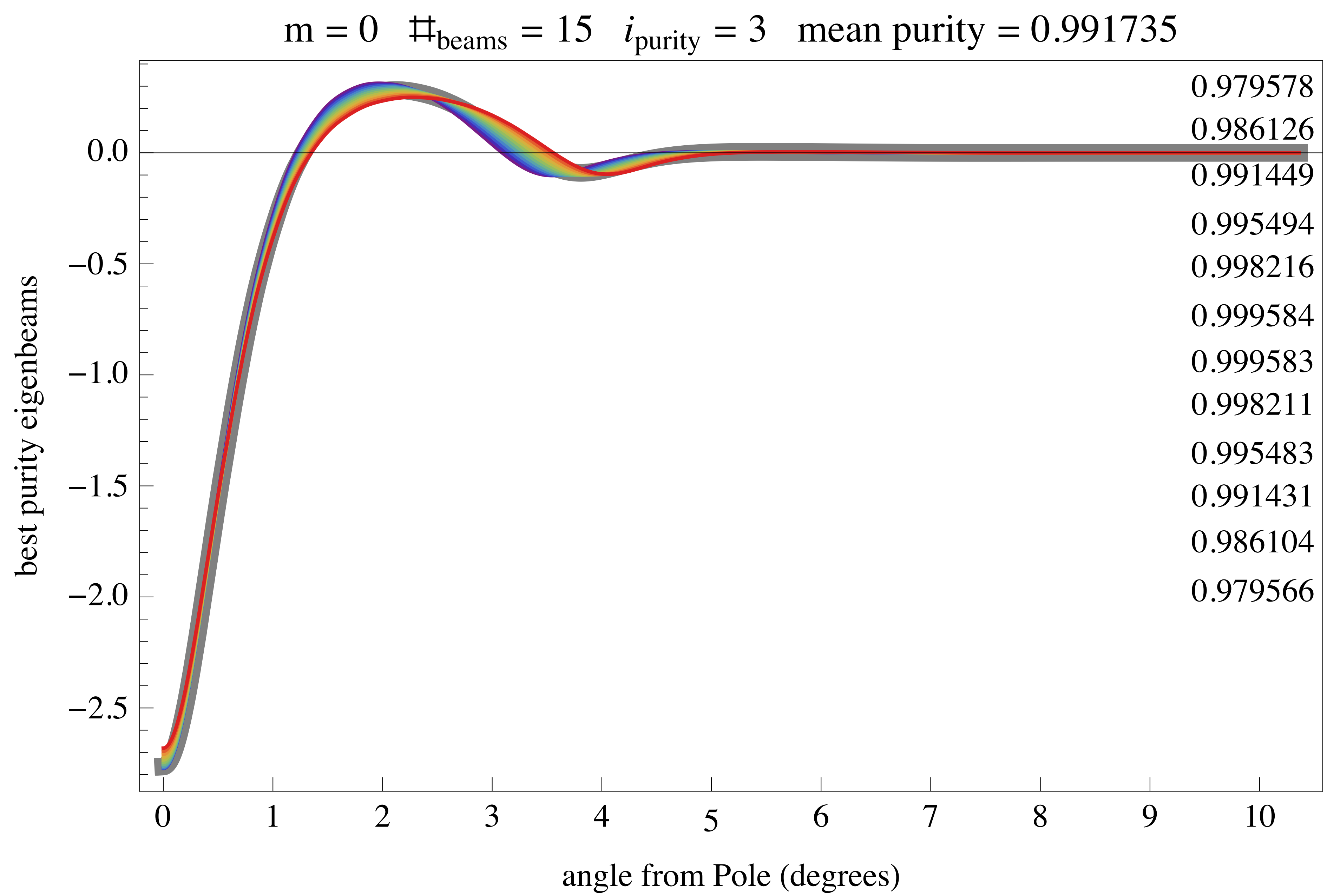
brown curve gives the eigenbeam in the full (full bandwidth) space of beams.

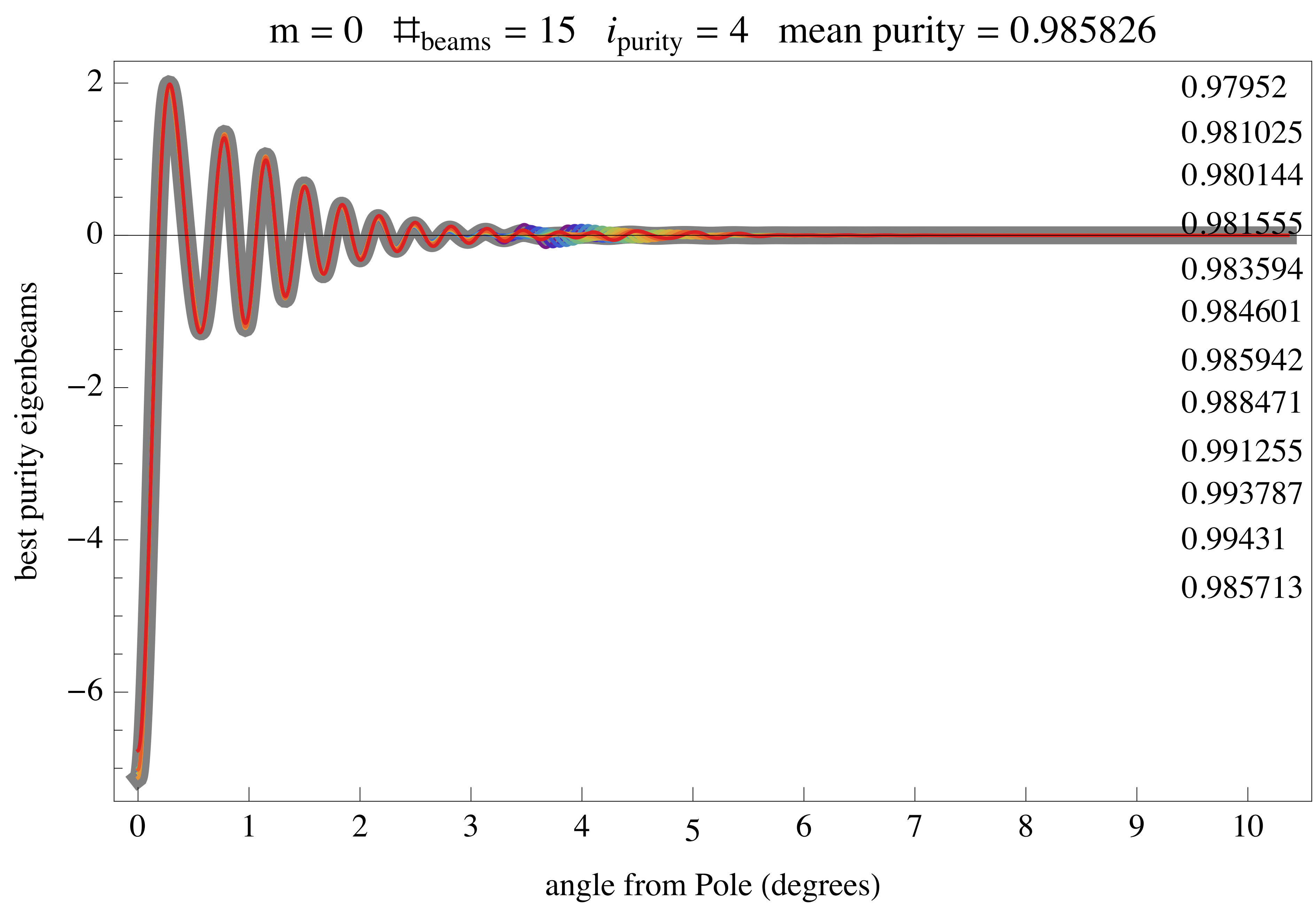
colored curves are projection of a single channel space of beams for 12 different frequency channels.

different channels are colored differently

however for this purest eigenbeam the patterns are largely identical so only see one channel (red)


m = 0 ♯beams = 15 ipurity = 2 mean purity = 0.994606
0.990526
0.992264
0.991243
0.9918
0.993694
0.994979
0.995816
0.996878
0.997589
0.998015
0.998428
0.994038
best purity eigenbeams
angle from Pole (degrees)
2
1
0
−1
−2
−3
−4
0 1 2 3 4 5 6 7 8 9 10


m = 0   ♯beams = 15   ipurity = 3   mean purity = 0.991735
0.979578
0.986126
0.991449
0.995494
0.998216
0.999584
0.999583
0.998211
0.995483
0.991431
0.986104
0.979566
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)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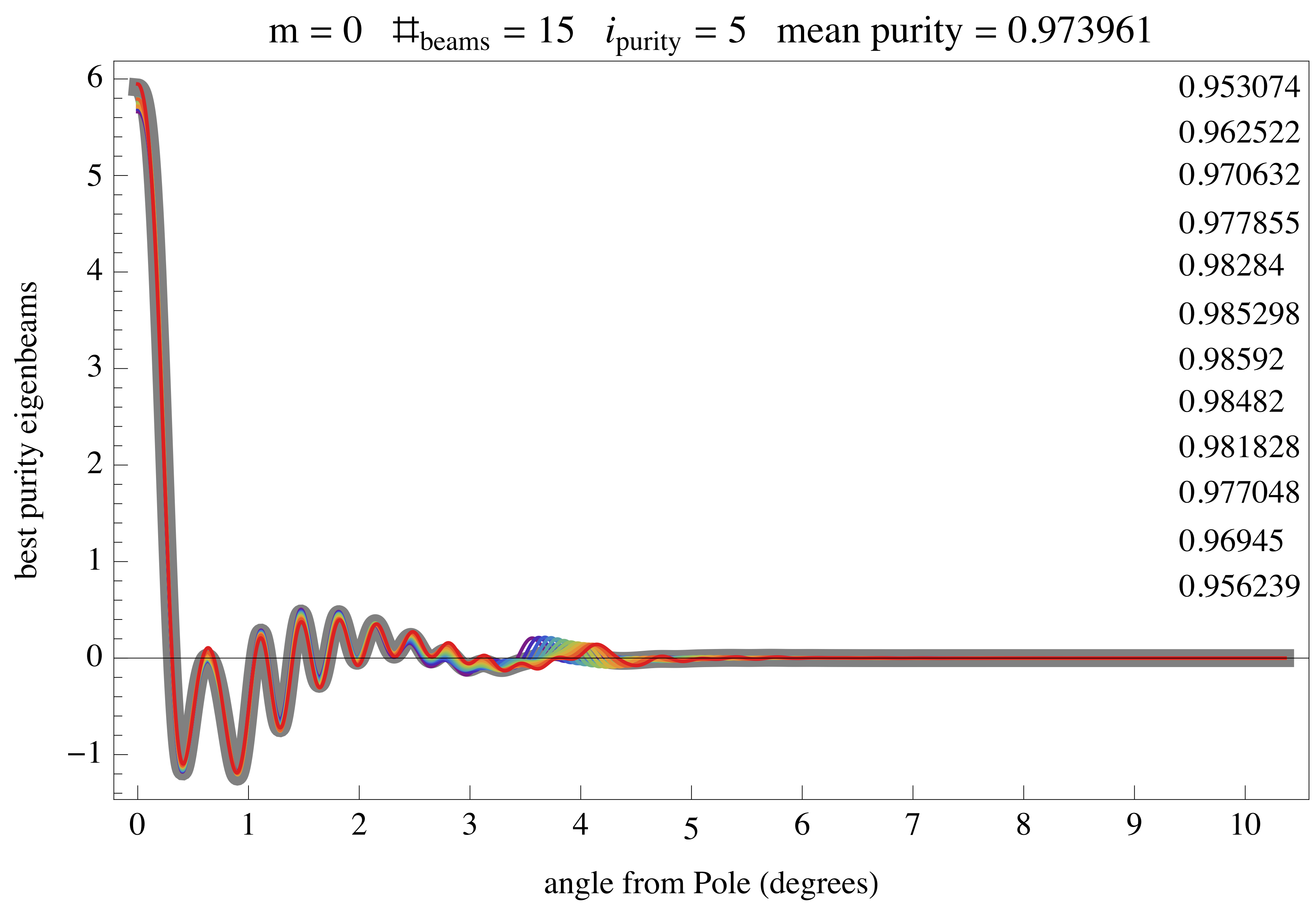
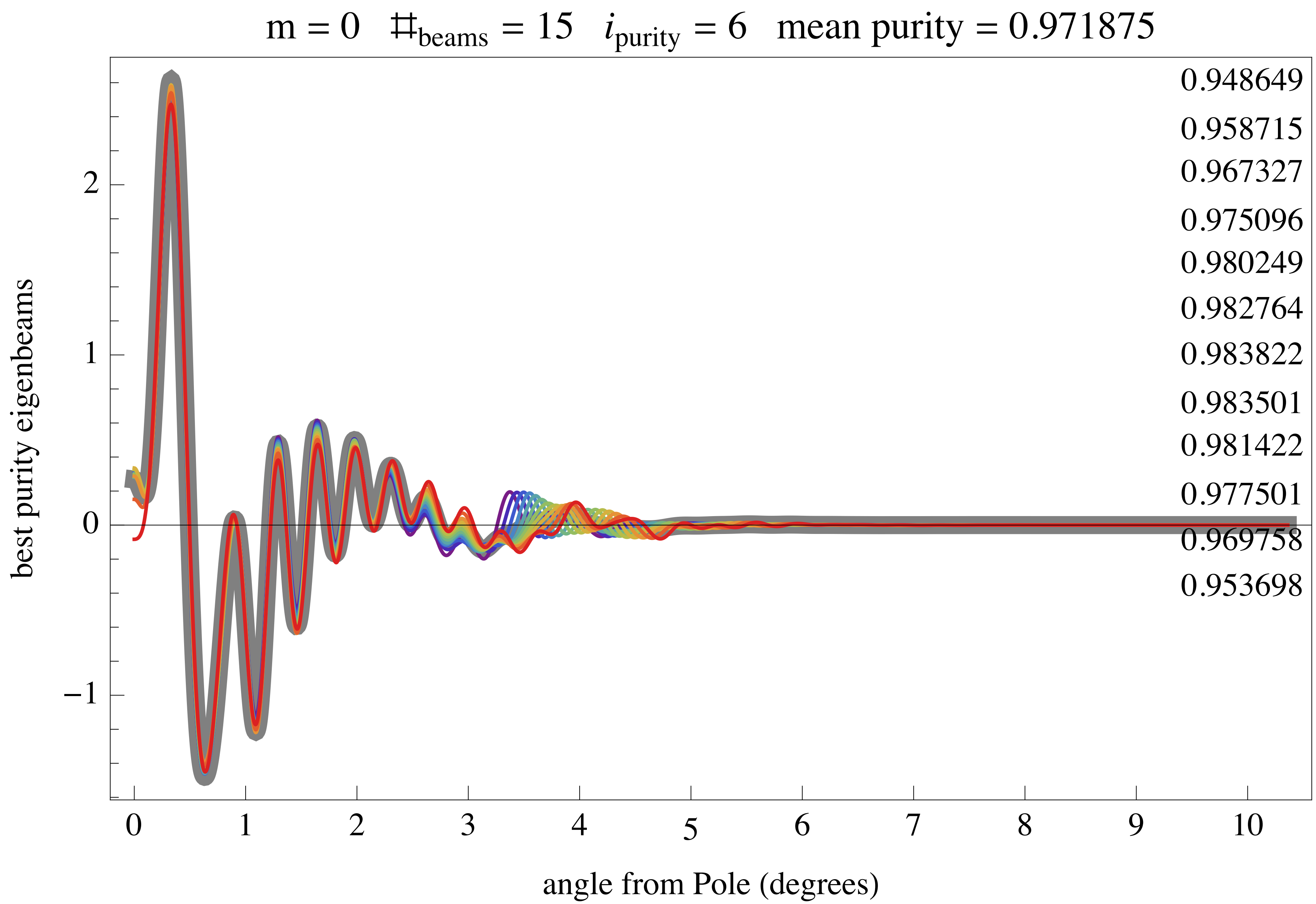
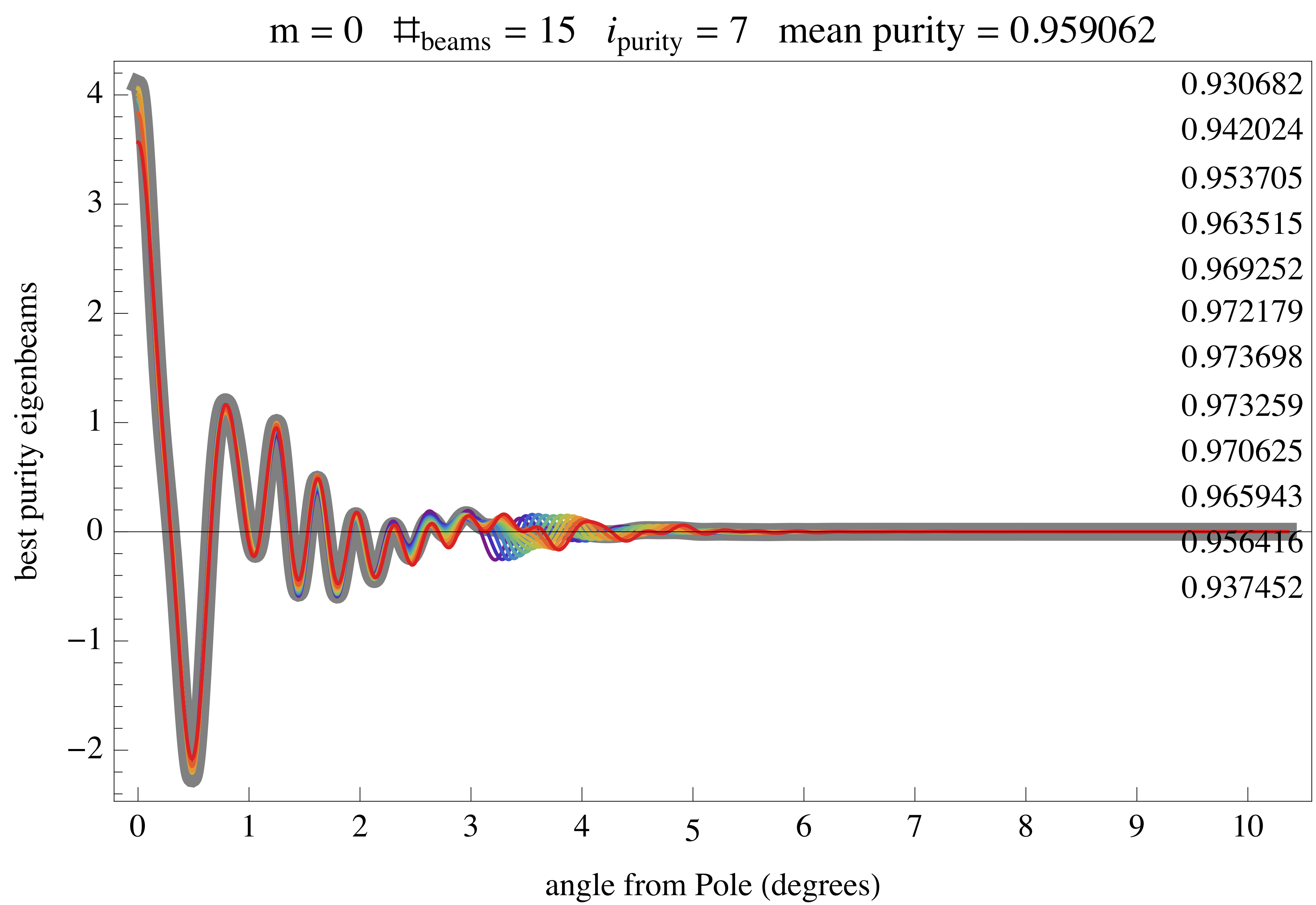
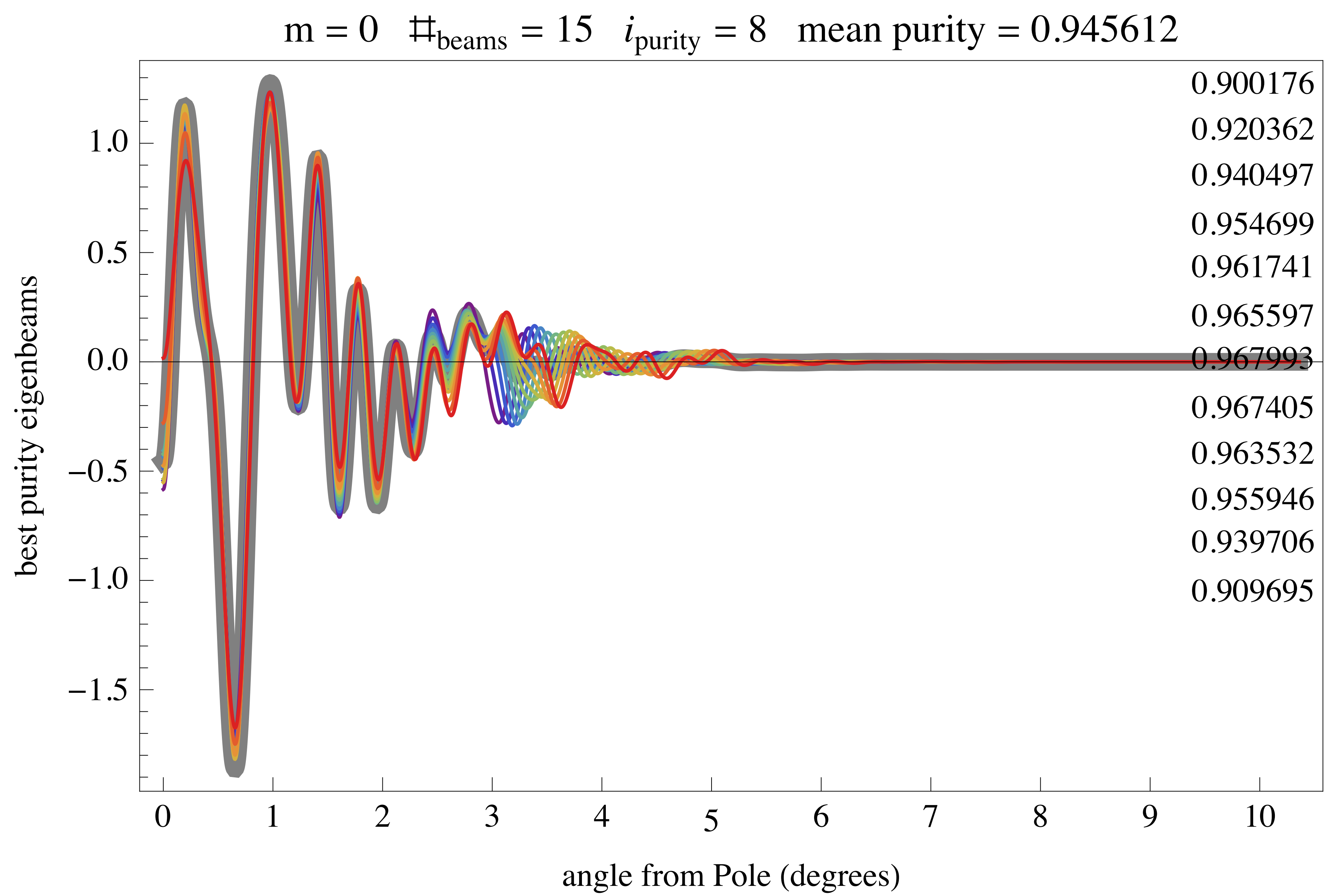
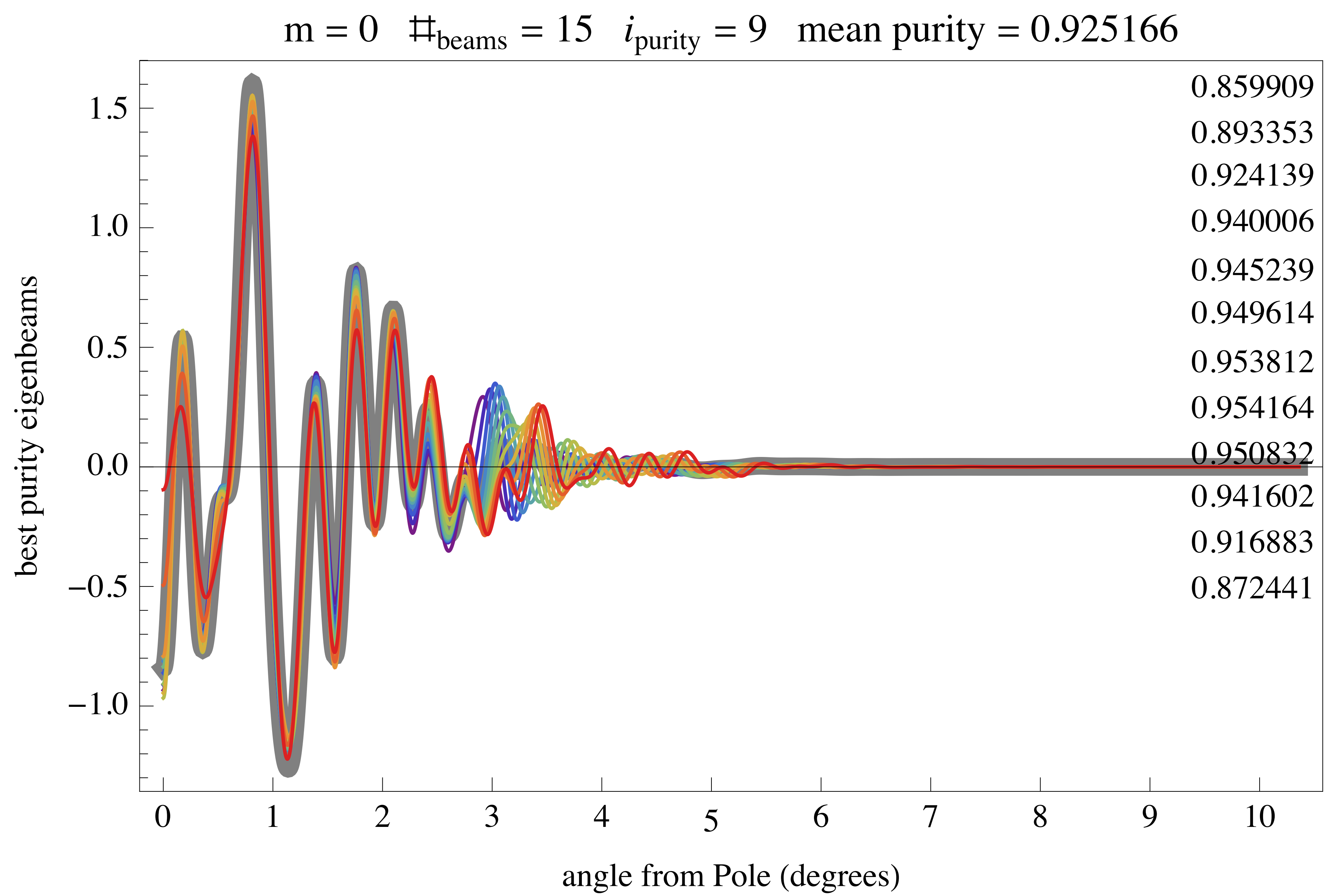
best purity eigenbeams
angle from Pole (degrees)
2
0
−2
−4
−6
0
1
2
3
4
5
6
7
8
9
10
0.97952
0.981025
0.980144
0.981555
0.983594
0.984601
0.985942
0.988471
0.991255
0.993787
0.99431
0.985713

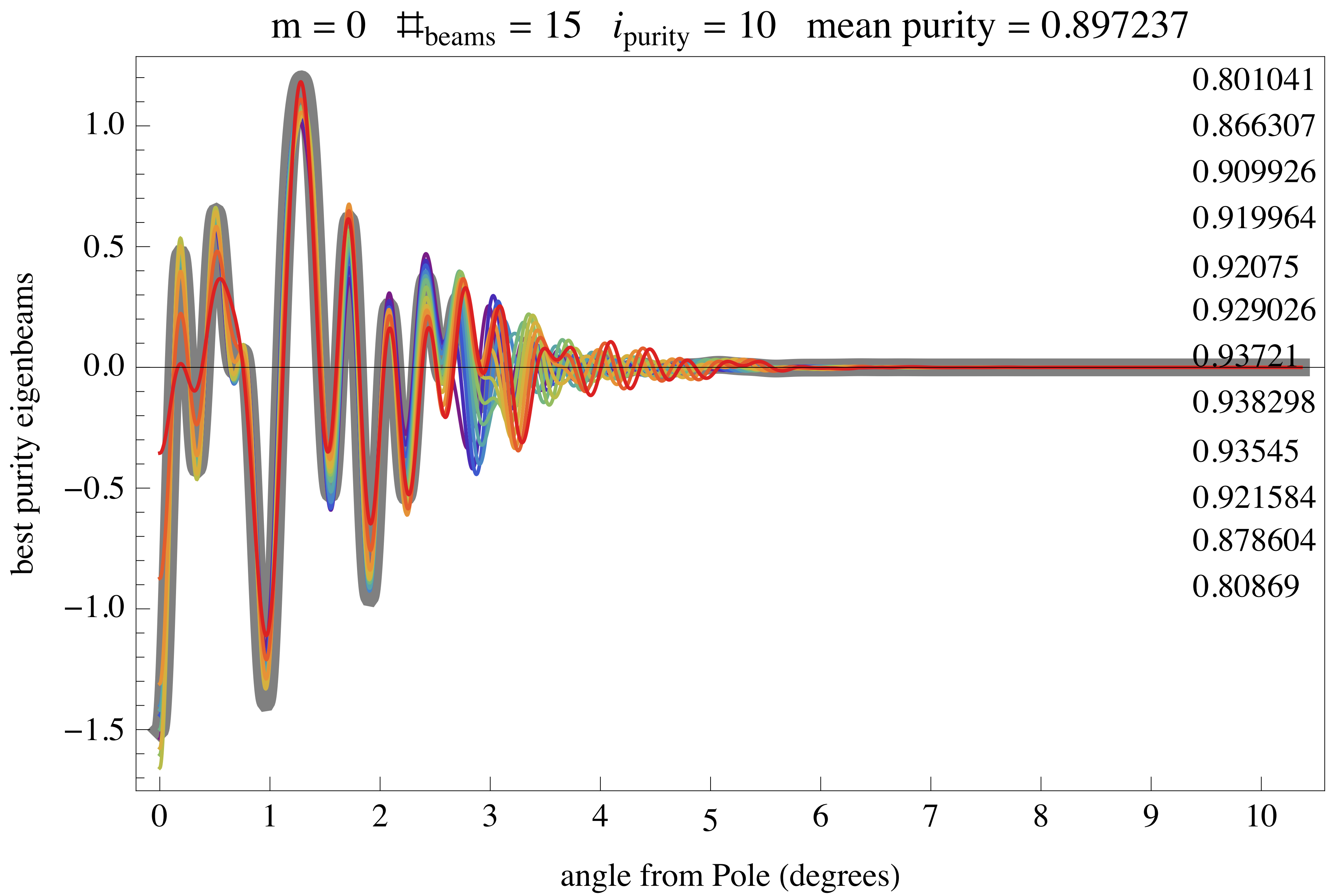

m = 0 ♯beams = 15 ipurity = 5 mean purity = 0.973961
0.953074
0.962522
0.970632
0.977855
0.98284
0.985298
0.98592
0.98482
0.981828
0.977048
0.96945
0.956239
best purity eigenbeams
angle from Pole (degrees)
6
5
4
3
2
1
0
−1
0
1
2
3
4
5
6
7
8
9
10


m = 0 ⌗beams = 15 ipurity = 6 mean purity = 0.971875
0.948649
0.958715
0.967327
0.975096
0.980249
0.982764
0.983822
0.983501
0.981422
0.977501
0.969758
0.953698
2
1
0
−1
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)


m = 0 ♯beams = 15 ipurity = 7 mean purity = 0.959062
best purity eigenbeams
angle from Pole (degrees)
0.930682
0.942024
0.953705
0.963515
0.969252
0.972179
0.973698
0.973259
0.970625
0.965943
0.956416
0.937452
4
3
2
1
0
−1
−2
0
1
2
3
4
5
6
7
8
9
10

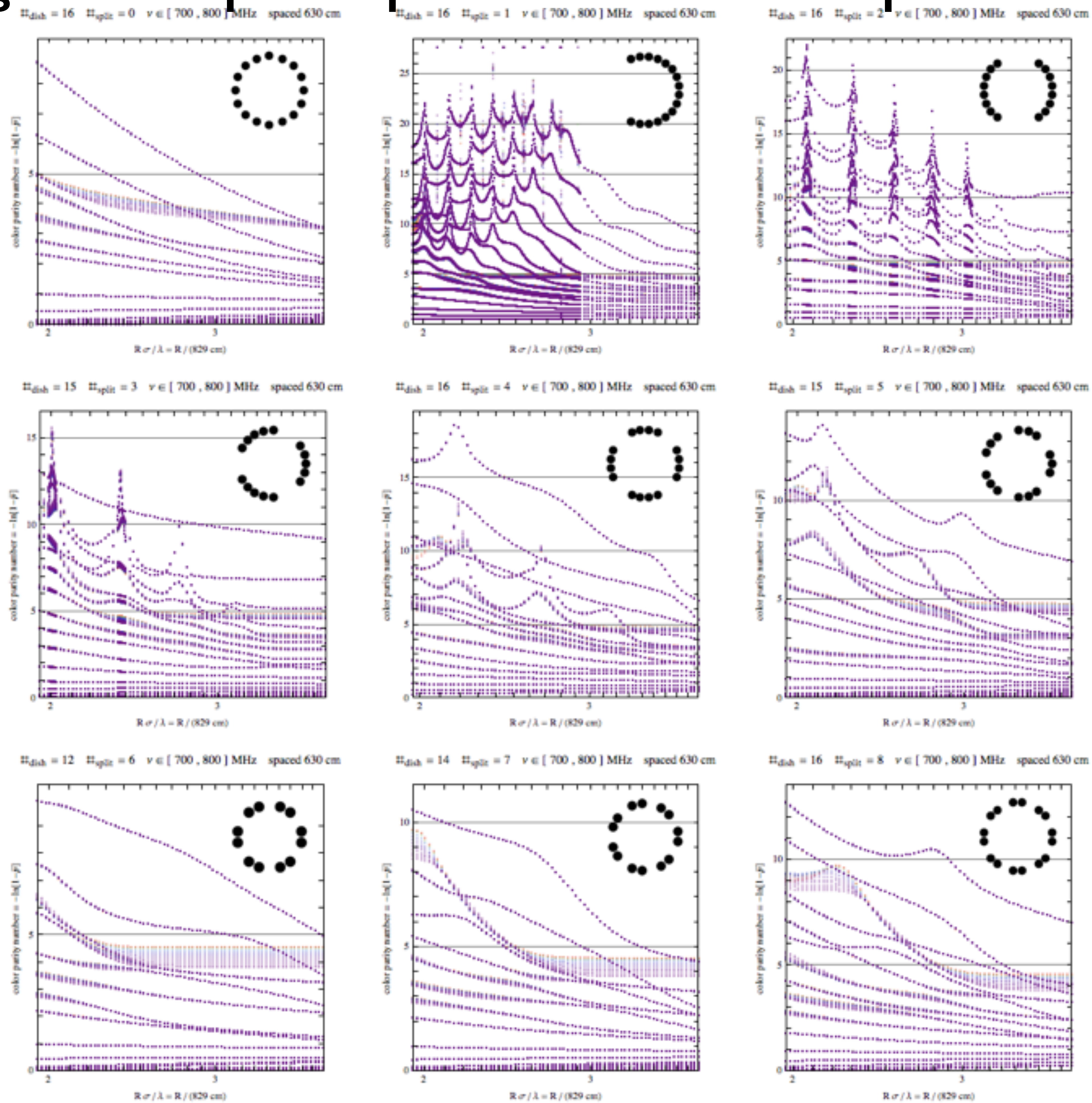

m = 0 ♯beams = 15 ipurity = 8 mean purity = 0.945612
0.900176
0.920362
0.940497
0.954699
0.961741
0.965597
0.967993
0.967405
0.963532
0.955946
0.939706
0.909695
best purity eigenbeams
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)


m = 0 ♯beams = 15 ipurity = 9 mean purity = 0.925166
0.859909
0.893353
0.924139
0.940006
0.945239
0.949614
0.953812
0.954164
0.950832
0.941602
0.916883
0.872441
best purity eigenbeams
angle from Pole (degrees)
1.5
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
6
7
8
9
10



m = 0 ♯beams = 15 ipurity = 10 mean purity = 0.897237
0.801041
0.866307
0.909926
0.919964
0.92075
0.929026
0.93721
0.938298
0.93545
0.921584
0.878604
0.80869
best purity eigenbeams
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)

configuration space: split circle into *n* compact subarrays



best performance: split into two compact subarrays

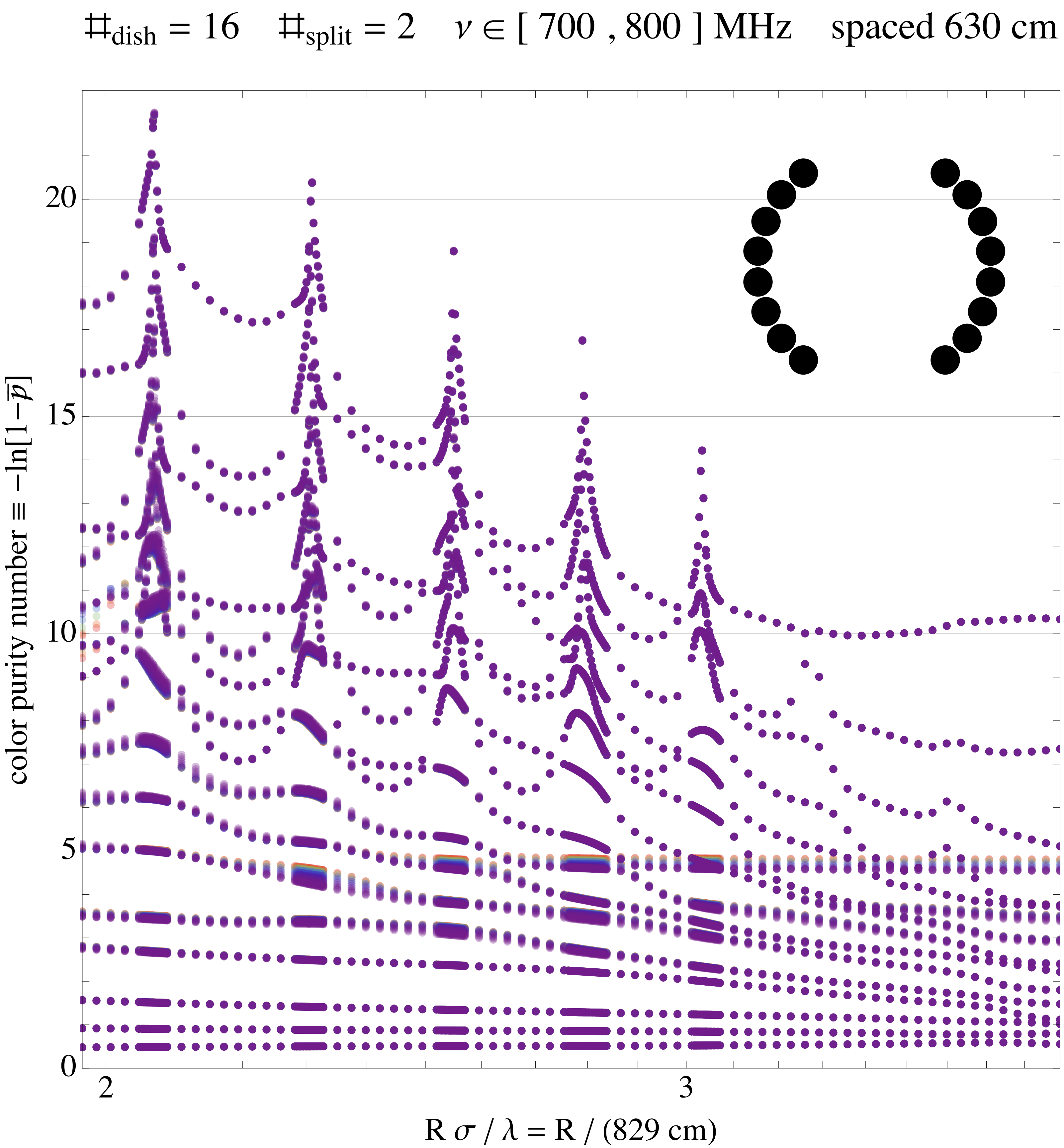
there exist purity "resonances" where astounding purity is attained.

resonances are "narrow" w/ few cm tolerance

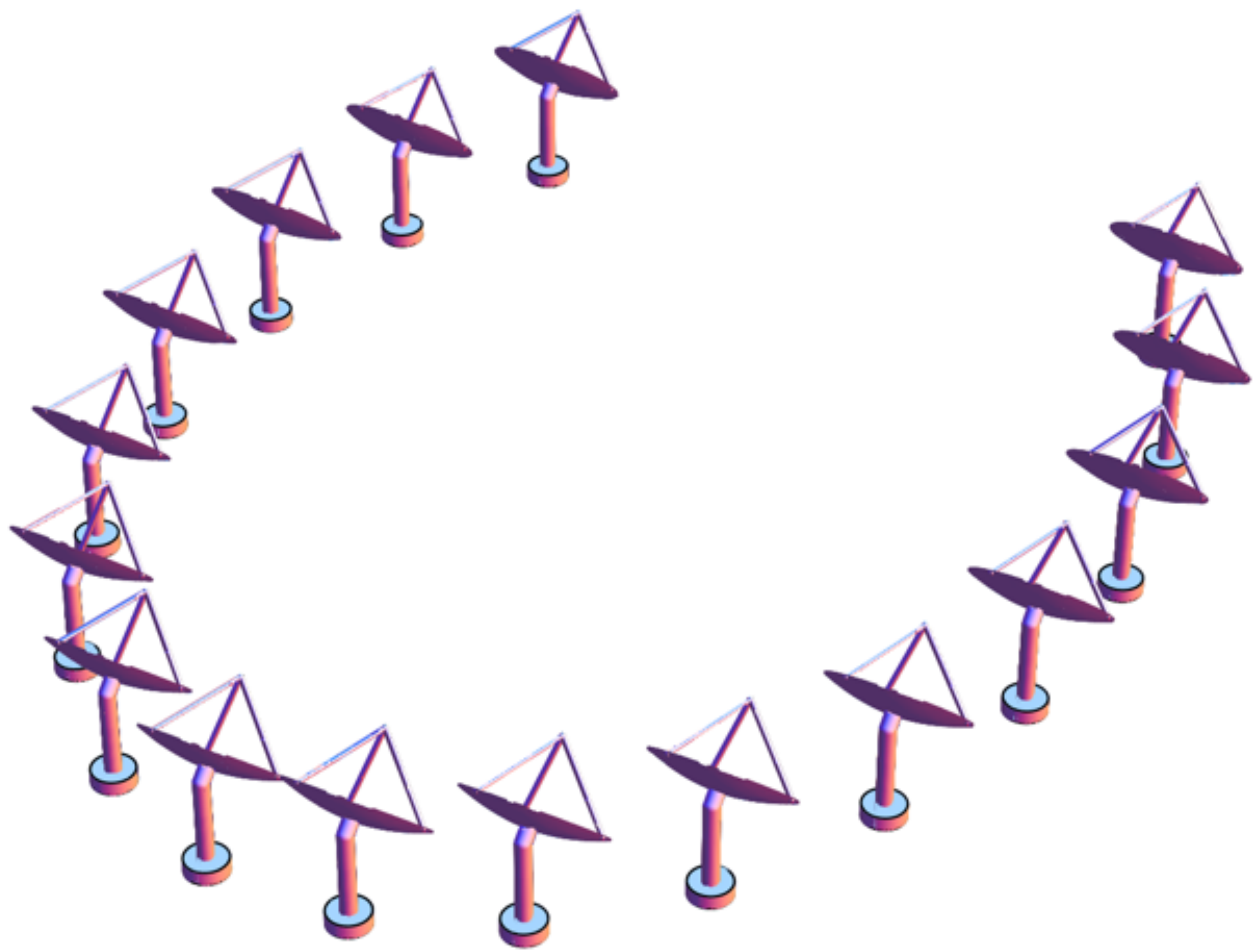
lowest purity attained near "singularities"

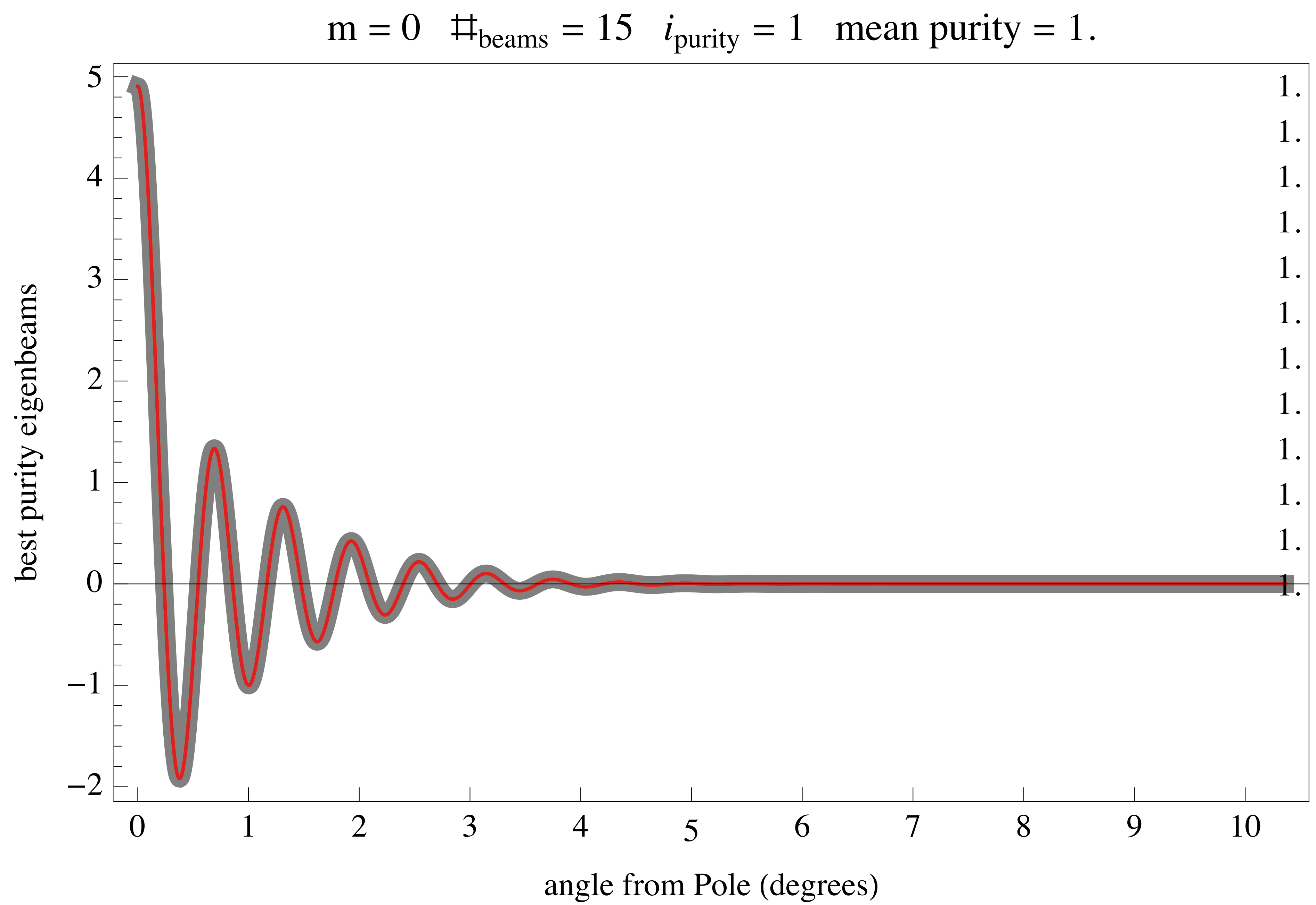
singularities are array configurations where two baselines become equal and the number of independent beams decreases.

resonances are not the most compact configuration



A Very Pure Polarscope





m = 0 ♯beams = 15 ipurity = 1 mean purity = 1.
best purity eigenbeams
angle from Pole (degrees)
5
4
3
2
1
0
−1
−2
0
1
2
3
4
5
6
7
8
9
10
1.
1.
1.
1.
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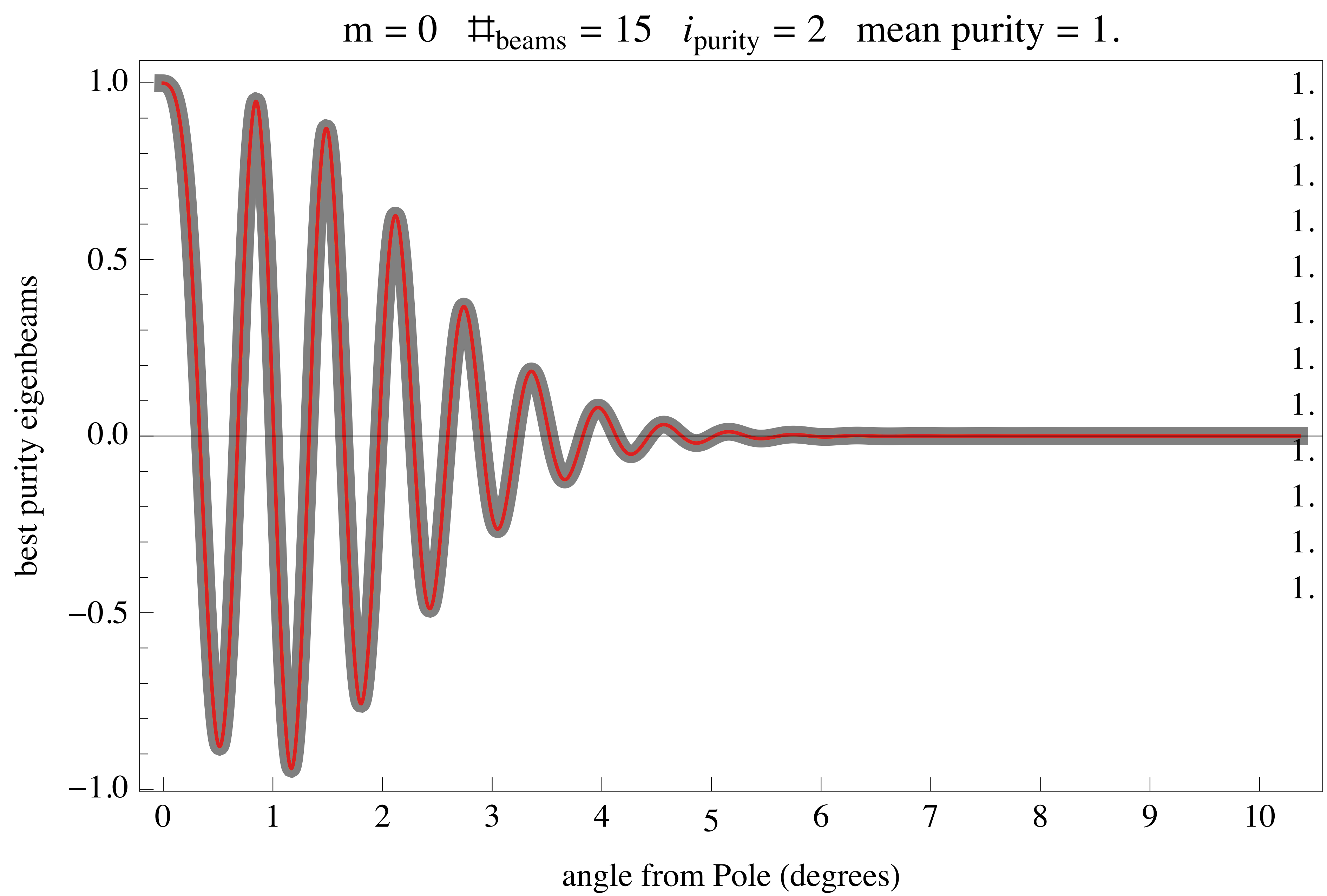
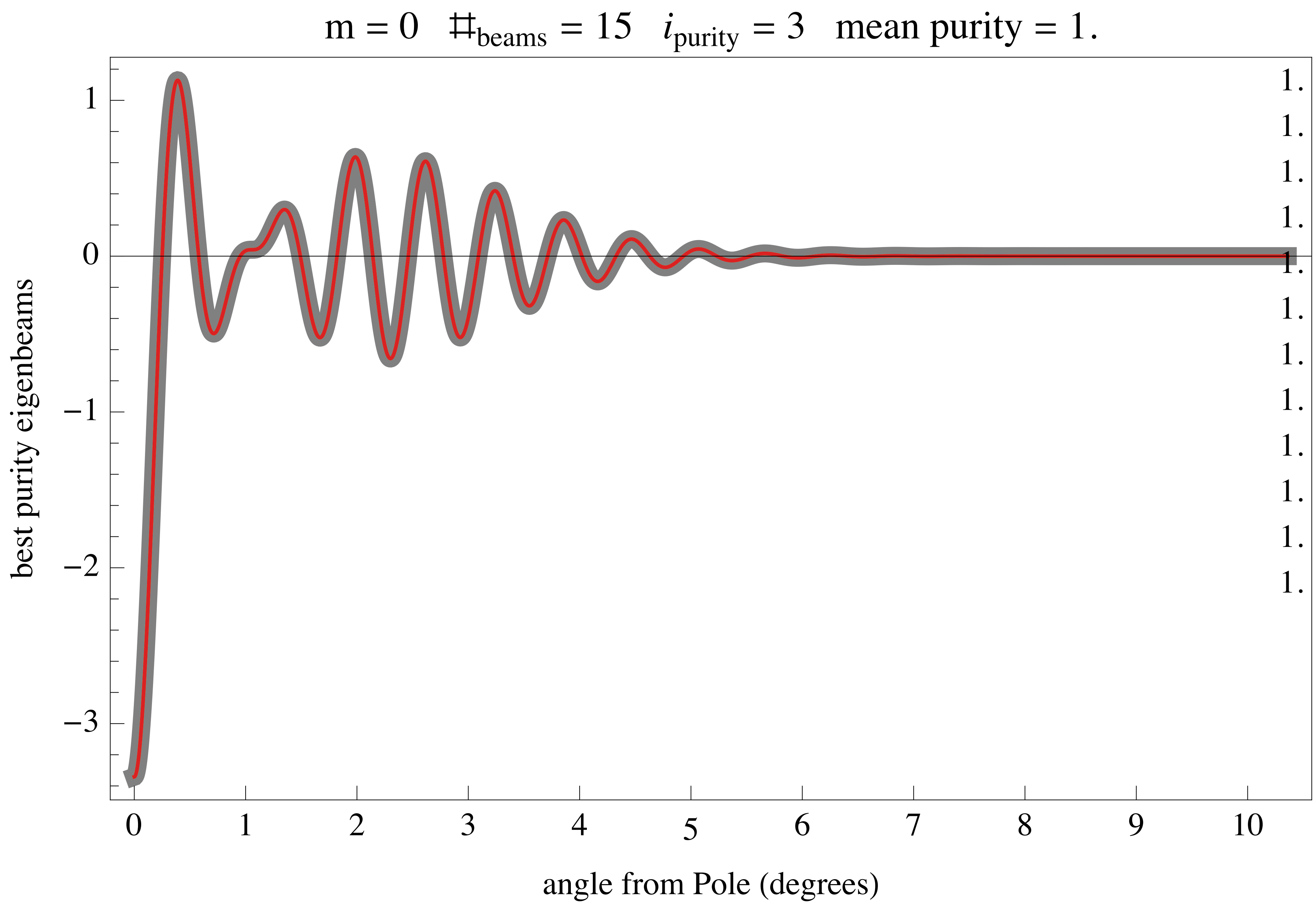
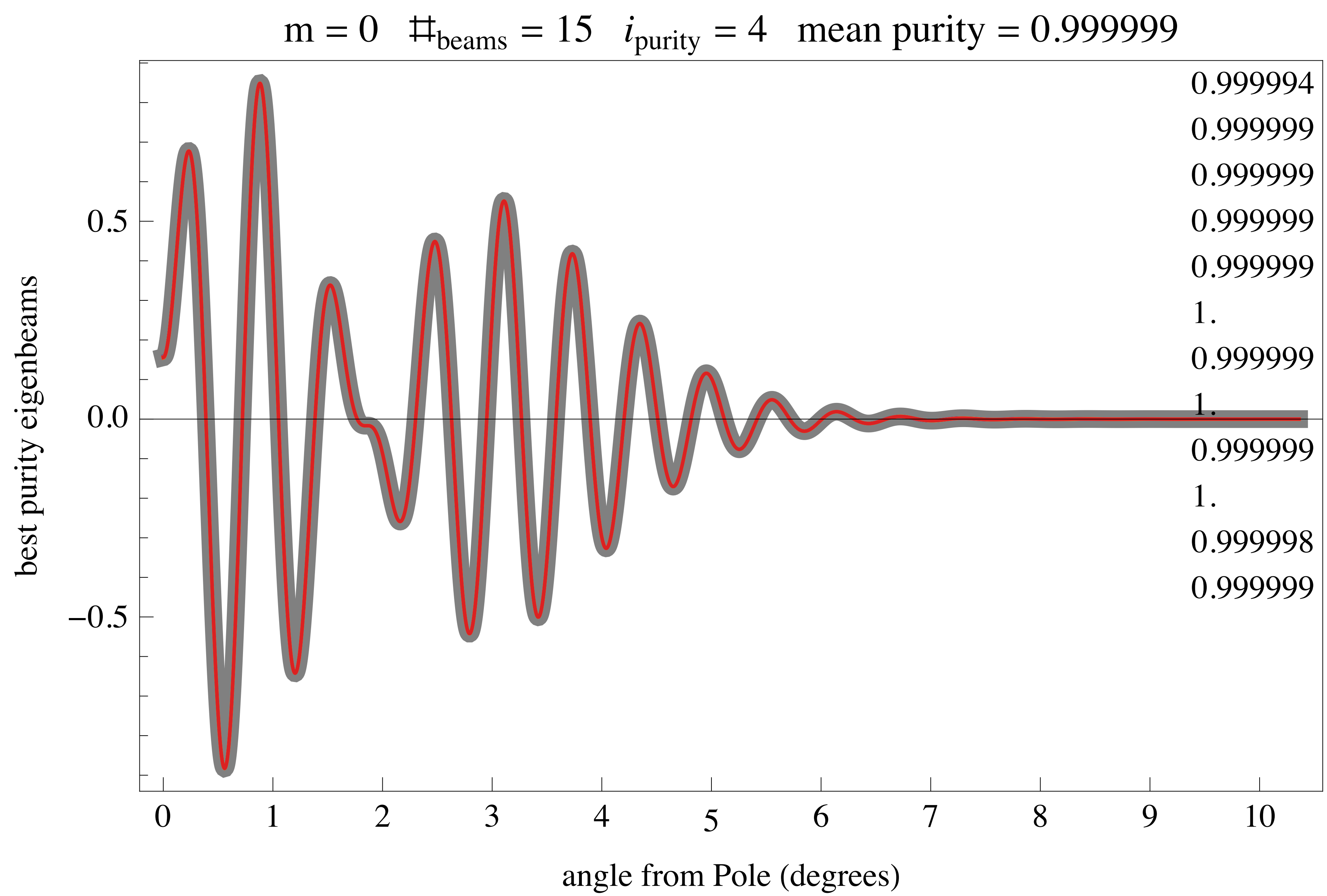
1.
1.
1.
1.
1.



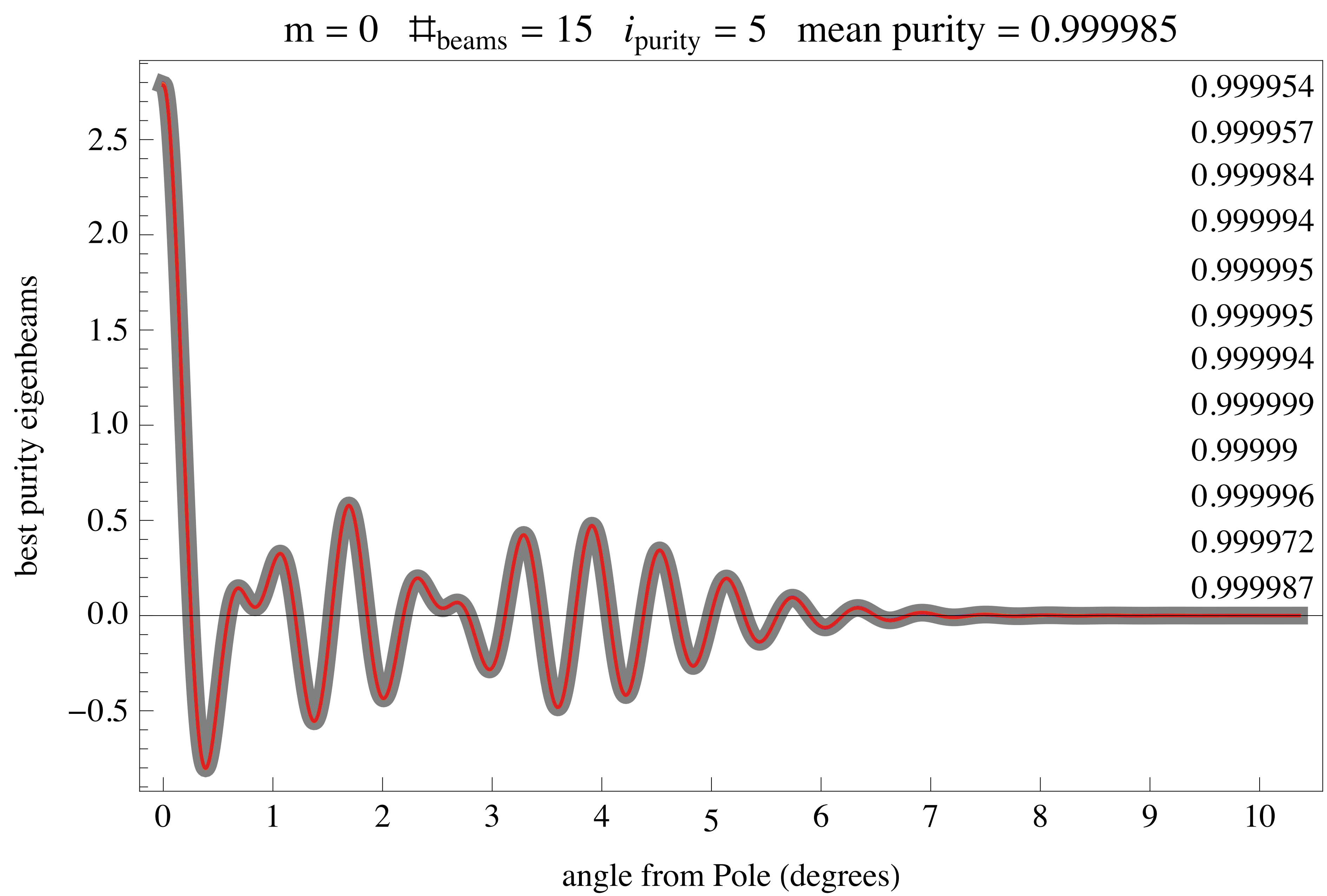
m = 0 ♯beams = 15 ipurity = 2 mean purity = 1.
1.0
0.5
0.0
−0.5
−1.0
0 1 2 3 4 5 6 7 8 9 10
best purity eigenbeams
angle from Pole (degrees)
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.



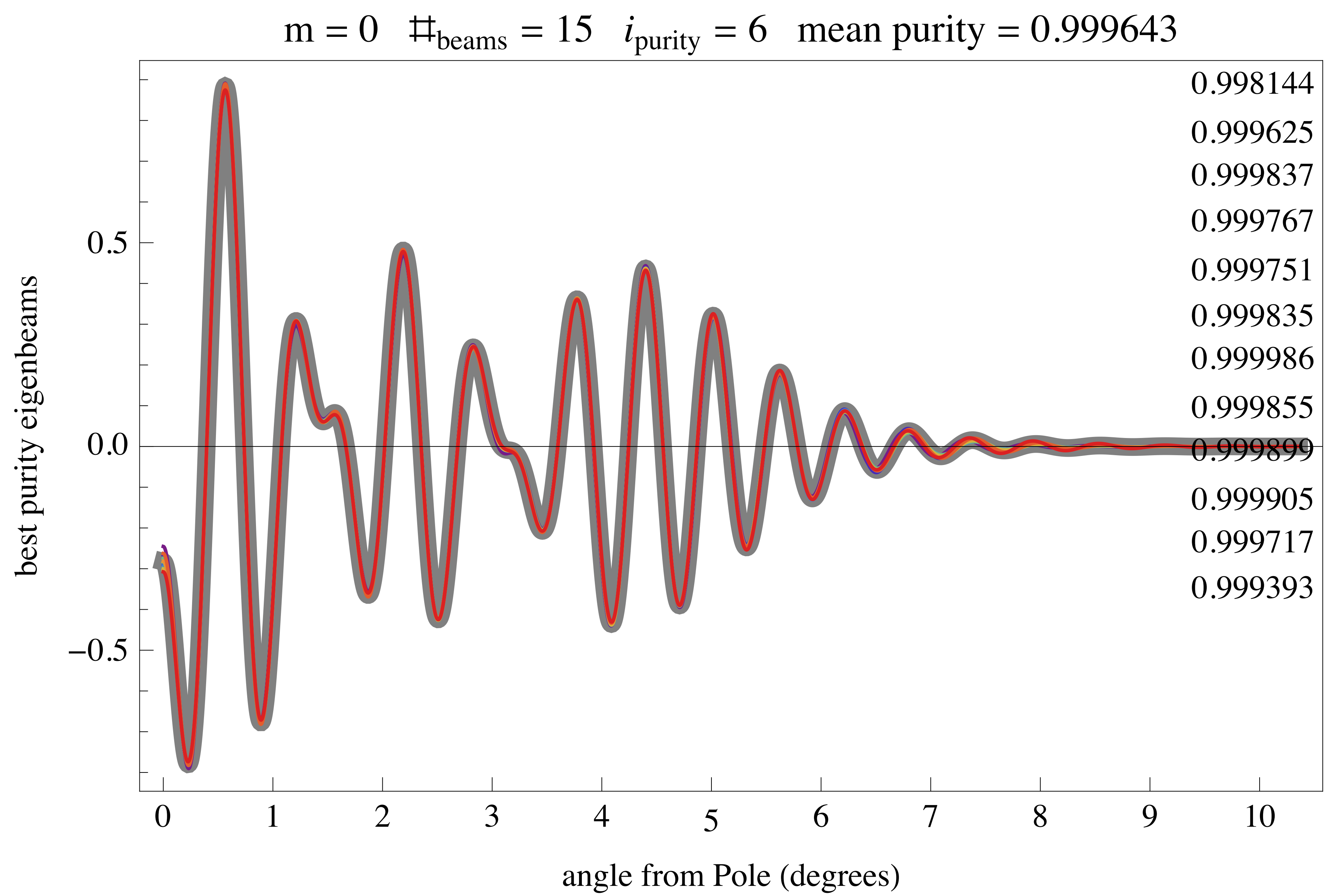
m = 0 ♯beams = 15 ipurity = 3 mean purity = 1.
1
0
−1
−2
−3
best purity eigenbeams
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.
1.
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)



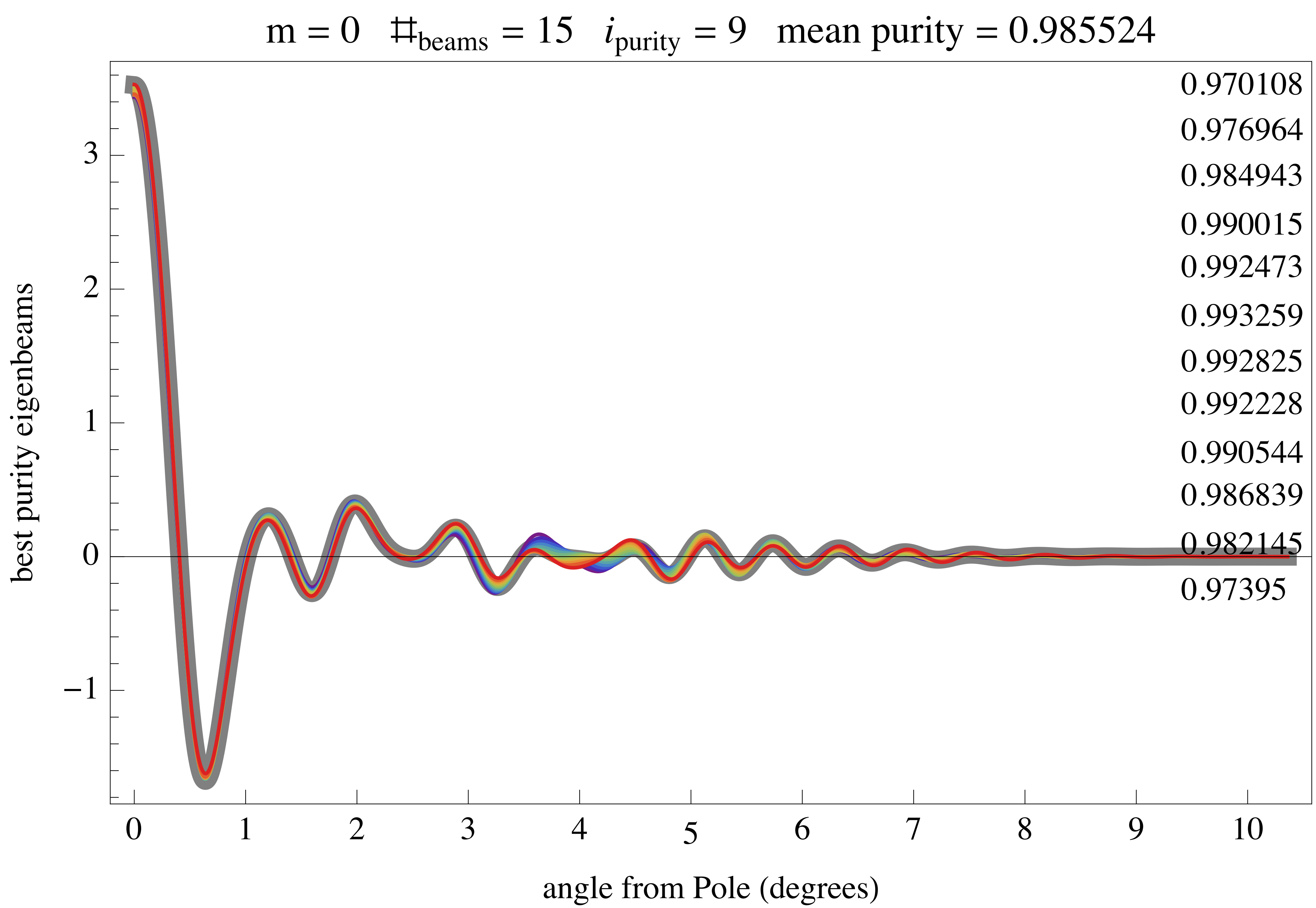
m = 0 ♯beams = 15 ipurity = 4 mean purity = 0.999999
0.999994
0.999999
0.999999
0.999999
0.999999
1.
0.999999
1.
0.999999
1.
0.999998
0.999999
0.5
0.0
−0.5
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)

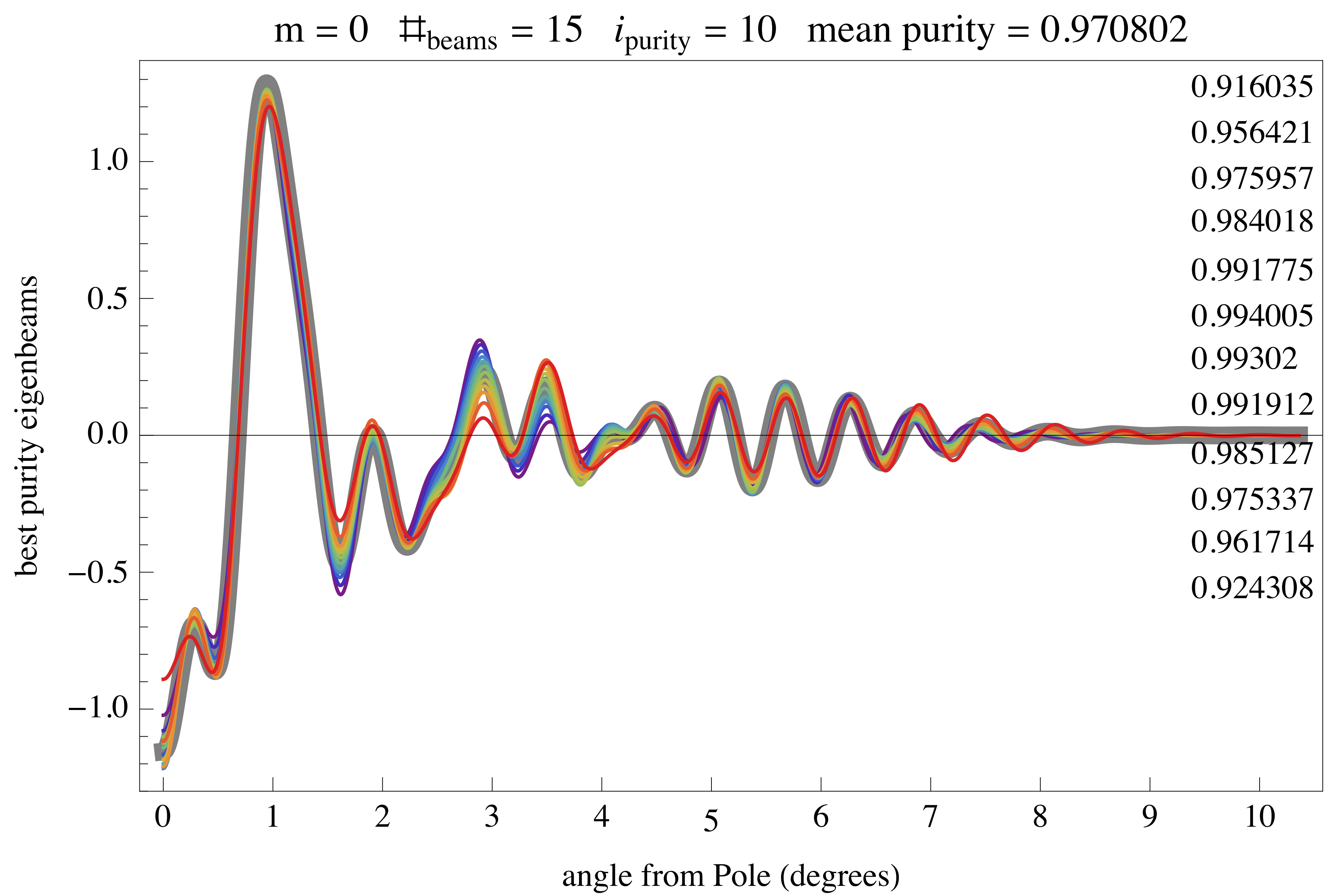


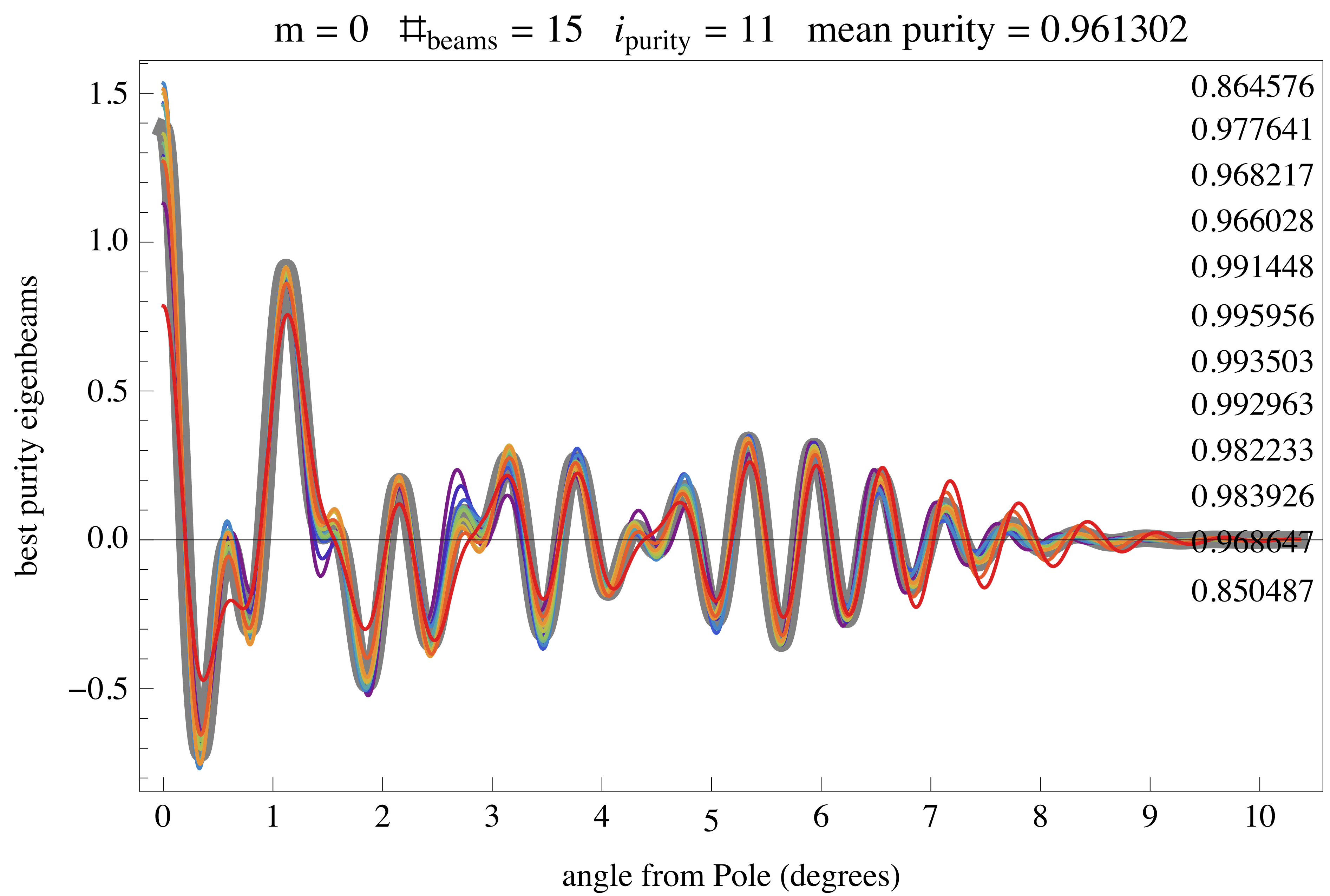
m = 0 ♯beams = 15 ipurity = 5 mean purity = 0.999985
0.999954
0.999957
0.999984
0.999994
0.999995
0.999995
0.999994
0.999999
0.99999
0.999996
0.999972
0.999987
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)


m = 0 ♯beams = 15 ipurity = 6 mean purity = 0.999643
0.998144
0.999625
0.999837
0.999767
0.999751
0.999835
0.999986
0.999855
0.999899
0.999905
0.999717
0.999393
best purity eigenbeams
0.5
0.0
−0.5
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)

Skip to 9th purity eigenmode




m = 0 ♯beams = 15 ipurity = 10 mean purity = 0.970802
0.916035
0.956421
0.975957
0.984018
0.991775
0.994005
0.99302
0.991912
0.985127
0.975337
0.961714
0.924308
1.0
0.5
0.0
−0.5
−1.0
best purity eigenbeams
0
1
2
3
4
5
6
7
8
9
10
angle from Pole (degrees)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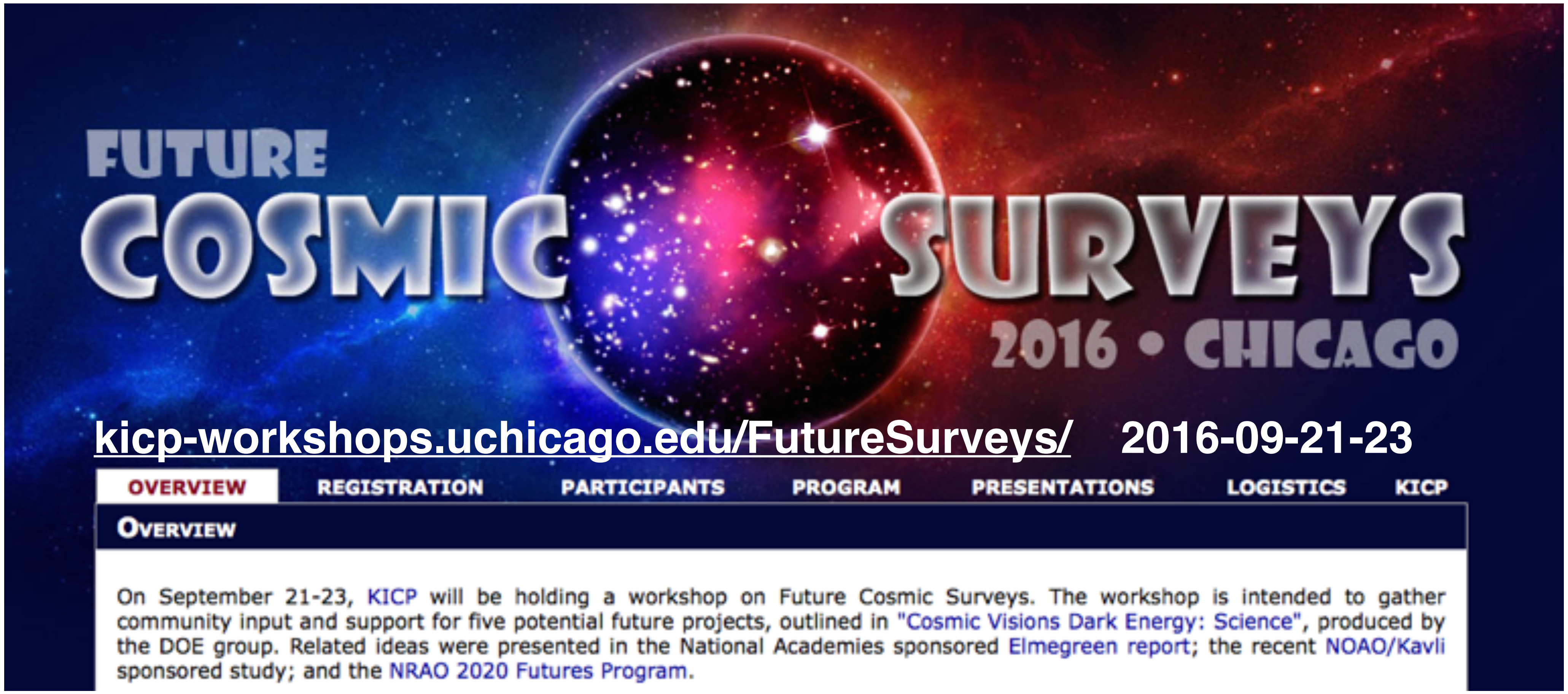
best purity eigenbeams
angle from Pole (degrees)
1.5
1.0
0.5
0.0
−0.5
0
1
2
3
4
5
6
7
8
9
10
0.864576
0.977641
0.968217
0.966028
0.991448
0.995956
0.993503
0.992963
0.982233
0.983926
0.968647
0.850487

Conclusions

- purity is one measure of the amount of mode mixing
- purity depends critically on details of array configuration
- similar configurations may have very different purity
- even a single high purity beam would allow one to *test* smooth spectrum hypothesis over entire bandwidth with high precision
- by pointing toward the NCP (e.g. a polarscope) one can integrate down rapidly to low noise levels [encounter and fix problems on shorter timescale]
- with dishes (and even cylinders) configuration space is large
- better simulations needed (realistic polarized beams)
- are we at the *science stage* or *proof of concept stage*?

Upcoming Events



The Tianlai Project – A Dark Energy Radio Observation Experiment

workshop & collaboration meeting 2016-09-26-29
location: Fermilab (near Chicago)
contact: stebbins@fnal.gov
website TBA